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(54) Title: OPTIMAL BEAM PATTERN SYNTHESIS VIA MATRIX WEIGHTING

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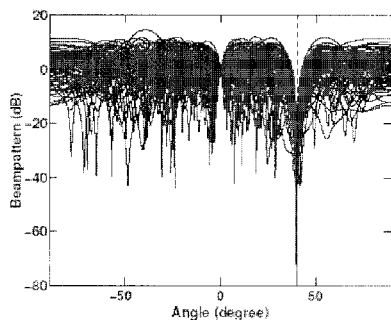
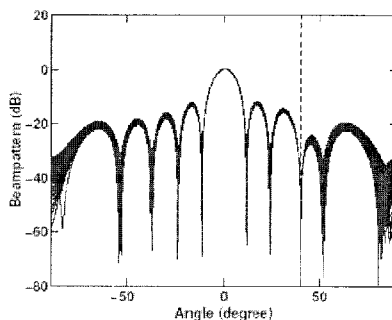
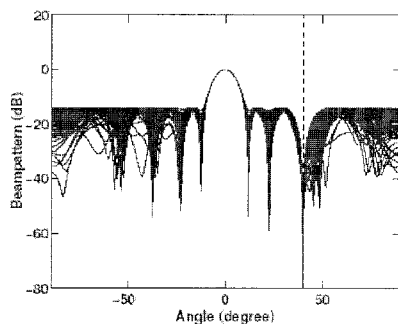


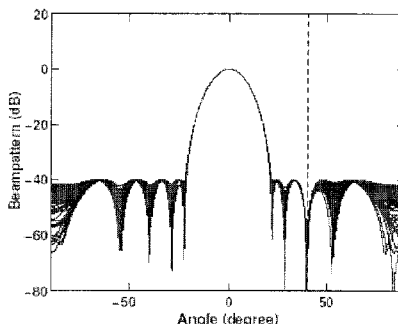
FIG. 1A



1B



1C



1D



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DESCRIPTIONOPTIMAL BEAM PATTERN SYNTHESIS VIA MATRIX WEIGHTING

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Cross-Reference to Related Application

10 The present application claims the benefit of U.S. Application Serial No. 60/833,948, filed July 28, 2006, which is hereby incorporated by reference herein in its entirety, including any figures, tables, or drawings.

Background of Invention

15 Antenna array plays an important role in a wide span of applications including, for example, radar, sonar, communications, aeroacoustic noise measurement, and biomedical imaging. One of the fundamental problems in array signal processing is array pattern synthesis. A conventional approach of using weight vectors at the array output for array
20 pattern synthesis, can be referred to as the Vector Weighting Approaches (VWA). In conventional formulation for this problem, a vector of complex-valued weights is designed, applied to the sensor outputs, and the weighted sensor outputs are coherently summed up to form a desired beampattern [1-7]. In the conventional VWA frame work, the solutions cannot be obtained efficiently and have no guarantee on their global optimality.

25 There is a long history of studying the array pattern synthesis problem using VWA (see, e.g., [1], and the references therein). Recently, numerical approaches based on convex optimization techniques [2], [5], [7] have received much attention. Compared to the early analytical approaches, the recent approaches can handle more complicated design specifications and are able to obtain the global optimal solutions efficiently, as long as the
30 design problems can be formulated as convex optimization problems. Applications of the convex optimization techniques to data-independent VWA beampattern synthesis were first introduced by Lebrete *et al.* in [2] and both narrowband and wideband Uniform Linear Arrays (ULA) were considered. The design of beampattern for ULA is equivalent to a power spectral density (PSD) constrained finite impulse response filter design problem. Once the optimal

PSD is designed, the optimal array pattern can be recovered using standard spectral factorization techniques [8]—[10]. However, these techniques are not suitable for non-uniform arrays. For non-uniform arrays, related designs with norm constraints on the weight vector were presented in [5]. However, the VWA based designs for non-uniform linear arrays are non-convex and hence are NP-hard. In [5], two iterative algorithms were used to obtain local solutions with no guarantee on their global optimality. The idea of using Semidefinite Relaxation (SDR) for array beampattern synthesis and for the related problem of two-dimensional filtering was presented in the recent work [7], [11]. However, their approaches are also based on weighting vectors and are obtained by modifying the relaxed solutions via some iterative methods, which provide no guarantee on their global optimality.

In the presence of strong interferences and jammers, data-adaptive methods are needed due to their adaptive nulling ability to suppress interferences and jammers. Adaptive arrays, such as the standard Capon beamformer [12], adjust the weight vector adaptively according to the incoming signals. Adaptive arrays can have much better resolution, lower sidelobe levels, and much better interference rejection capabilities than their data-independent counterparts [1]. The beampattern synthesis methods for adaptive arrays have been considered in [6], [13], [14] for linear arrays and 2-D non-uniform arrays. In [13] and [14], designs with the capability of peak sidelobe level control in adaptive arrays were presented using the analytical and convex optimization approaches, respectively. However, these designs do not control the main-beam shape. A method was presented in [6], where both the main-beam shape and the peak sidelobe level can be controlled to a certain extent with an iterative algorithm. However, it cannot control the peak sidelobe level and main-beam width precisely according to prescribed parameters.

Brief Summary

Embodiments of the subject invention relate to a method and apparatus for processing a plurality of signals from a sensor array requiring beam pattern synthesis. Embodiments of the invention relate to powerful and flexible beampattern synthesis techniques for antenna or ultrasound transducer arrays. Embodiments of the invention can be utilized with arrays in, for example, radar, sonar, communications, aeroacoustic noise measurement, and biomedical imaging. Examples of biomedical imaging array include ultrasound and microwave. Specific embodiments can incorporate a method that can be referred to as Matrix Weighting Approaches (MWA). In a specific embodiment, an optimal array pattern synthesis can be

achieved with an arbitrary array. Embodiments of the subject MWA can utilize weight matrices at the array output. The use of weight matrices can provide improved flexibility for optimal array pattern synthesis.

Embodiments of the subject method can be referred to as the data-adaptive MWA, which can also be referred to as the Adaptive WEighting of Signals via One Matrix Entity (AWESOME). Additional embodiments pertain to data-independent MWA designs, including, but not limited to, the beampattern matching, minimum sidelobe designs, and the constant beamwidth design for wideband array. In specific embodiments of these designs, globally optimal solutions can be determined efficiently for MWA. Convex optimization formulations can be used to determine globally optimal solutions. Certain embodiments of MWA can be considered as the Semidefinite Relaxation (SDR) of the VWA counterpart. Numerical examples presented show that embodiments of the data-adaptive AWESOME can allow for strict controls of main-beam shape and peak sidelobe level. The numerical examples also show that the capability of adaptive nulling of strong interferences and jammers can be maintained while controlling mainbeam shape and peak sidelobe level. Embodiments of the data-adaptive AWESOME have been shown to be robust against steering vector errors, small sample size problems, as well as interferences highly correlated or coherent with the signal-of-interest. Embodiments of data-adaptive AWESOME can have an improved performance in parameter estimations.

In addition, the numerical examples also show that embodiments of the data-independent MWA can achieve much better beampattern synthesis compared to the VWA counterpart. Embodiments of the invention can provide a convenient way to deal with the open problem of optimal shading design for arbitrary arrays under various practical constraints. Embodiments of the invention incorporating MWA can extend the working frequency band of a practical array, SADA, used for aeroacoustic wideband noise measurement, from the original 10 – 40 KHz to 8 – 65 KHz.

Brief Description of Drawings

Figures 1A-1D show a comparison of beam patterns from 100 Monte Carlo trials obtained via several adaptive beam forming methods.

Figures 2A-2B show a comparison of beam patterns formed by several data-adaptive methods.

Figures 3A-3B show a comparison of power estimates for the Signal of Interest (SOI) via several data-adaptive methods.

Figure 4 shows a comparison of power estimates for the Signal of Interest (SOI) versus the variance of the array steering vector perturbation when the interference power is 20 dB.

Figure 5 shows a comparison of SOI power estimates versus the correlation coefficient between the SOI and the 50 dB interference obtained via several data-adaptive methods using the theoretical $\mathbf{R}$.

Figures 6A-6B show a comparison of SOI power estimates versus the number of interferences obtained using the theoretical $\mathbf{R}$.

Figures 7A-7B show data-independent MWA and VWA beam patterns synthesized for 5-element Uniform Linear Arrays (ULA) via beampattern matching design, with each desired pulse width of 20° and under the uniform elemental gain constraint.

Figures 8A-8B show MWA and VWA beam patterns synthesized for 10-element ULA via minimum sidelobe level design with the 3dB main-beam width equal to 20° and under the uniform elemental gain constraint.

Figures 9A-9D show wideband constant beam width designs for ULA and MRA obtained via the MWA minimum side lobe level design under the total gain constraint.

Figures 10A-10H show beam patterns for Small Aperture Directional Array (SADA) at various frequencies.

Detailed Disclosure

Embodiments of the subject invention relate to a method and apparatus for processing a plurality of signals from a sensor array requiring beam pattern synthesis. Embodiments of the invention relate to powerful and flexible beampattern synthesis techniques for antenna or ultrasound transducer arrays. Embodiments of the invention can be utilized with arrays in, for example, radar, sonar, communications, aeroacoustic noise measurement, and biomedical imaging. Examples of biomedical imaging array include ultrasound and microwave. Specific embodiments can incorporate a method that can be referred to as Matrix Weighting Approaches (MWA). In a specific embodiment, an optimal array pattern synthesis can be achieved with an arbitrary array. Embodiments of the subject MWA can utilize weight matrices at the array output. The use of weight matrices can provide improved flexibility for optimal array pattern synthesis.

Embodiments of the subject method can be referred to as the data-adaptive MWA, which can also be referred to as the Adaptive WEighting of Signals via One Matrix Entity (AWESOME). Additional embodiments pertain to data-independent MWA designs, including, but not limited to, the beampattern matching, minimum sidelobe designs, and the constant beamwidth design for wideband array. In specific embodiments of these designs, globally optimal solutions can be determined efficiently for MWA. Convex optimization formulations can be used to determine globally optimal solutions. Certain embodiments of MWA can be considered as the Semidefinite Relaxation (SDR) of the VWA counterpart. Numerical examples presented show that embodiments of the data-adaptive AWESOME can allow for strict controls of main-beam shape and peak sidelobe level. The numerical examples also show that the capability of adaptive nulling of strong interferences and jammers can be maintained while controlling mainbeam shape and peak sidelobe level. Embodiments of the data-adaptive AWESOME have been shown to be robust against steering vector errors, small sample size problems, as well as interferences highly correlated or coherent with the signal-of-interest. Embodiments of data-dependent AWESOME can have an improved performance in parameter estimations.

In addition, the numerical examples also show that embodiments of the data-independent MWA can achieve much better beampattern synthesis compared to the VWA counterpart. Embodiments of the invention can provide a convenient way to deal with the open problem of optimal shading design for arbitrary arrays under various practical constraints. Embodiments of the invention incorporating MWA can extend the working frequency band of a practical array, SADA, used for aeroacoustic wideband noise measurement, from the original 10 – 40 KHz to 8 – 65 KHz.

Embodiments for implementing the optimal array pattern synthesis via Matrix Weighting Approaches (MWA) can focus on array power response designs since in many applications, such as for aeroacoustic noise measurement [15], [16], only the power response of the array is of interest. Additional embodiments involve the production of an output signal vector, which is useful for many beamspace processing applications (see, e.g., [17]).

The notations adopted hereafter are standard. $(\cdot)^T$ denotes the transpose of a matrix or vector. $(\cdot)^*$ denotes the conjugate transpose of a matrix or vector. $|\cdot|$ denotes the absolute value, and $\|\cdot\|$ denotes the Euclidean norm of a vector. $\mathbf{X}^{1/2}$ denotes the Hermitian square root of the matrix $\mathbf{X}$. $\text{tr}(\mathbf{X})$ denotes the trace, and X_{mm} denotes the m th diagonal element of the

matrix $\mathbf{X}$. $\mathbf{X} \geq 0$ means that $\mathbf{X}$ is a positive semi-definite matrix. $\text{vec}(\mathbf{X})$ denotes a vector obtained by stacking the columns of $\mathbf{X}$ on top of each other, and $\otimes$ denotes the Kronecker matrix product. $C^{m \times n}$ ($R^{m \times n}$) is the complex-valued (real-valued) matrix of dimension $m \times n$. $\mathbf{I}$ is the identity matrix with its dimension determined from the content.

Consider an M -element array with an arbitrary array geometry. Let $s(n)$ denote the unknown waveform of a narrowband signal-of-interest (SOI). The wideband signal case can be addressed by, for example, dividing waveform into narrowband frequency bins. Let θ_0 denote a generic source location parameter for the SOI, which may be the direction-of-arrival of SOI in the far-field of an array, or the 3-D coordinate of SOI in the near-field of an array.

The model for the received data vector is given by (see, e.g., [18]):

$$\mathbf{y}(n) = \mathbf{a}(\theta_0)s(n) + \mathbf{e}(n), \quad n = 1, \dots, N, \quad (1)$$

where $\mathbf{y}(n)$ is the n th received data vector or the n th snapshot, $n = 1, \dots, N$, with N denoting the snapshot number; $\mathbf{a}(\theta_0)$ is the array steering vector for the SOI, which is a known function of θ_0 , and $\mathbf{e}(n)$ is the residue term, which contains the noise as well as interferences and jammers. The array steering vector may have different expressions, depending on the array geometry and on whether the source is in the near- or far-field of the array. For the far-field linear array case, the steering vector $\mathbf{a}(\theta_0)$ is given by

$$\mathbf{a}(\theta_0) = \left[e^{-j\frac{2\pi f}{c}x_1 \cos(\theta_0)}, \dots, e^{-j\frac{2\pi f}{c}x_M \cos(\theta_0)} \right]^T, \quad (2)$$

where c is the speed of propagation, f is the carrier frequency, and x_m is the location of the m th array element.

In VWA, the signal received by the m th array element is weighted by a complex-valued scalar w_m^* ; the weighted signals are summed to yield the array output.

$$\mathbf{w} = [w_1, \dots, w_M]^T \in C^{M \times 1} \quad (3)$$

The corresponding power response of the array as a function of θ is the array beampattern (see, e.g., [2] and Chapter 6 in [18]):

$$P(\theta) = |\mathbf{w}^* \mathbf{a}(\theta)|^2 = \mathbf{a}(\theta)^* \mathbf{w} \mathbf{w}^* \mathbf{a}(\theta) \triangleq \mathbf{a}(\theta)^* \mathbf{T} \mathbf{a}(\theta), \quad (4)$$

$$\mathbf{T} \triangleq \mathbf{w} \mathbf{w}^* \in C^{M \times M}$$

where the matrix has rank one.

In VWA beampattern synthesis, the variable to be designed is the weight vector $\mathbf{w}$, or equivalently, the weight matrix $\mathbf{T}$ subject to the rank-one constraint. Due to the non-convexity of the rank-one constraint [19], the general VWA design problems cannot be solved in polynomial time and the solutions cannot be guaranteed to be globally optimal (in some cases with particular constraints, the VWA design problems can be formulated as convex optimization problems [14]).

In specific embodiment array pattern synthesis via MWA, we remove the rank-1 constraint on $\mathbf{T}$ and allow $\mathbf{T}$ to have rank higher than one. The resulting formulation is the Semidefinite Relaxation (SDR) [19], [20] of the corresponding VWA formulation for the same beampattern synthesis problem. SDR is often used to obtain approximate (in the sense that the solution has rank larger than one) solutions to rank constrained optimization problems [19], [20]. Although it seems that MWA has increased the system implementation complexity, our numerical examples show that the optimal solution of $\mathbf{T}$ is usually not full rank, and the number of its dominant eigenvalues K is often quite small, which means that specific embodiments utilizing MWA can require K times the hardware implementation cost of VWA. In specific embodiments, the matrix weighting can be done in parallel so as to require little, if any, extra processing time. Once $\mathbf{T}$ is determined, we let $\mathbf{T} = \mathbf{U}\Sigma\mathbf{U}^*$ and let $\mathbf{W} = \Sigma^{1/2}\mathbf{U}$, where the columns of $\mathbf{U}$ are the eigenvectors of $\mathbf{T}$ and the diagonal elements of the diagonal matrix Σ are the corresponding eigenvalues. Then

$$\mathbf{T} = \mathbf{W}\mathbf{W}^*, \quad (5)$$

where $\mathbf{W} = [\mathbf{w}_1, \dots, \mathbf{w}_K] \in \mathbb{C}^{M \times K}$, with $\mathbf{w}_k$ denoting the k th weight vector in Fig. 2. Then the power response of the array as a function of θ is the array beampattern of MWA:

$$P(\theta) = \mathbf{a}^*(\theta)\mathbf{T}\mathbf{a}(\theta) = \mathbf{a}^*(\theta)\mathbf{W}\mathbf{W}^*\mathbf{a}(\theta) = \sum_{k=1}^K \left| \mathbf{a}^*(\theta)\mathbf{w}_k \right|^2. \quad (6)$$

Therefore, embodiments of MWA can be viewed as a filter-bank approach. Specific embodiments of MWA can seek to find an optimal $\mathbf{T} \geq 0$ that gives a desired array beampattern.

The trace of $\mathbf{T}$ can be referred to as the total gain of the weight matrix $\mathbf{W}$ in MWA and can determine the power amplification at the array output. Note that:

$$\text{tr}(\mathbf{T}) = \text{tr}\left(\sum_{k=1}^K \mathbf{w}_k \mathbf{w}_k^*\right) = \sum_{k=1}^K \text{tr}(\mathbf{w}_k \mathbf{w}_k^*) = \sum_{k=1}^K \text{tr}(\mathbf{w}_k^* \mathbf{w}_k) = \sum_{k=1}^K \|\mathbf{w}_k\|^2 \quad (7)$$

where we have use the fact that $\text{tr}(\mathbf{AB}) = \text{tr}(\mathbf{BA})$. Hence the total gain constraint on
 5 the array weights is the trace constraint on $\mathbf{T}$. Embodiments of the invention can incorporate one or both of the following two types of gain constraints: i) the total gain constraint, which requires $\text{tr}(\mathbf{T}) = c$, where c is some given constant; ii) the elemental uniform gain constraint, i.e., $T_{mm} = c/M$, $m = 1, \dots, M$, which requires that each antenna element contributes equal gain to the array output. Both constraints are linear in $\mathbf{T}$ and hence are easy to incorporate
 10 into the optimal array designs.

In VWA, the design under the elemental uniform gain constraint becomes the phase-only problem [21], which is non-convex and much more difficult to solve. In an embodiment, the solution to MWA can be used as an initial solution to the phase-only problem.

The $\mathbf{W}$ determined from $\mathbf{T}$ via (5) is not unique since $\mathbf{T} = \mathbf{WPP}^*\mathbf{W}^*$ for any unitary
 15 matrix $\mathbf{P}$ with $\mathbf{P}^*\mathbf{P} = \mathbf{PP}^* = \mathbf{I}$. Hence we can pick $\mathbf{P}$ to be a diagonal matrix with each of its diagonal elements unit modulus but with the phase adjusted so that each element of $\mathbf{W}^*\mathbf{a}(\theta_0)$ is real-valued. For VWA, i.e., with $\mathbf{T}$ being rank-1, this choice of $\mathbf{P}$ together with $\mathbf{a}^*(\theta_0)\mathbf{T}\mathbf{a}(\theta_0) = 1$ yields the distortionless response constraint $\mathbf{w}^*\mathbf{a}(\theta_0) = 1$.

The output signal vector obtained via MWA can be used in many beamspace
 20 processing applications (see, e.g., [17]). Embodiments of the invention can utilize MWA to produce an output signal vector $\mathbf{W}^*\mathbf{y}(n)$, $n = 1, \dots, N$. These output signal vectors can be used in various beam space processing applications.

Adaptive arrays can be found in many applications including radar, sonar, aeroacoustics, communications, and medical imaging. One classical data-adaptive
 25 beamformer, the standard Capon beamformer [12], seeks to minimize the array output power, subject to the constraint that the SOI is passed through the beamformer without distortion. However, one inherent problem associated with the data-adaptive beamformer is the varying main-beam shape and the unmanageable peak sidelobe level, due to, for example, the inadequate estimation of the sample covariance matrix [13] or the changing interference
 30 environment. Yet in some applications, such as some applications in radar, sonar, and/or communications, the main-beam shape must be maintained and the peak sidelobe level must

be lower than a prescribed value [6], [13]. Another problem of Capon is that it is rather sensitive to the model errors, including the steering vector errors, the small sample size problems (which are shown in [22] to be equivalent to the steering vector mismatches), and the presence of interferences correlated (especially highly correlated or coherent) with SOI.

For more than three decades, many approaches have been proposed to make the adaptive beamformer robust [23]. However, the problems of maintaining the main-beam shape and controlling the peak sidelobe level of adaptive arrays still remain to be solved. Various embodiments of the invention utilizing the Adaptive WEighting of Signals via One Matrix Entity (AWESOME) algorithm can address one or more of these problems.

Consider the data model in (1). Similar to the formulation of the Capon beamformer, the array output power can be minimized under the constraint of unit power gain for the SOI. One or more additional constraints can be introduced to control the 3-dB main-beam width as well as the peak sidelobe level. A specific embodiment of AWESOME is formulated as follows:

$$\min_{\mathbf{T}} \quad \text{tr}(\hat{\mathbf{R}}\mathbf{T}) \quad (8)$$

$$\text{subject to} \quad \mathbf{a}^*(\theta_0)\mathbf{T}\mathbf{a}(\theta_0) = 1, \quad (9)$$

$$\mathbf{a}^*(\theta_i)\mathbf{T}\mathbf{a}(\theta_i) = 0.5, \quad i = 1, 2, \quad (10)$$

$$\mathbf{a}^*(\mu_l)\mathbf{T}\mathbf{a}(\mu_l) \leq \varsigma, \quad \mu_l \in \Psi_s, \quad (11)$$

$$\mathbf{a}^*(\mu_l)\mathbf{T}\mathbf{a}(\mu_l) \geq 0.5, \quad \mu_l \in (\theta_1, \theta_2), \quad (12)$$

$$\mathbf{T} \geq 0. \quad (13)$$

where θ_0 is the location parameter of the SOI, θ_1 and θ_2 are the prescribed 3-dB points, ς is the desired peak sidelobe level, Ψ_s denotes the sidelobe region, the interval (θ_1, θ_2) is the 3-dB main-beam region, and

$$\hat{\mathbf{R}} = \frac{1}{N} \sum_{n=1}^N \mathbf{y}(n)\mathbf{y}^*(n) \quad (14)$$

is the sample covariance matrix. Alternative embodiments can use another value in equation (9) rather than 1 and/or modify and use one or more of the constraints provided in equations (9), (10), (11), and (12). Additional constraints can be added to the AWESOME formulation

as well. The formulation in (8) - (13) is a Semi-Definite Program (SDP) [20] and can be solved efficiently via the SDP solvers (see, e.g, [24], [25]).

The 3-dB main-beam width cannot be arbitrarily narrow given a certain peak sidelobe level ζ and vice versa, due to the well-known tradeoff between the beamwidth and the peak sidelobe level. The problem in (8) may become infeasible if these parameters are not properly chosen.

No constraint on the trace or the diagonal elements of $\mathbf{T}$ is imposed in this formulation since we already have the explicit gain constraint in (9). Again, the gain constraint in (9) can be modified in other embodiments.

The constraints for the main-beam region (12) can be active when the desired main-beam width is very wide and, optionally inactive when the desired main-beam is not very wide. They are used to prevent main-beam splitting.

The estimated SOI power $\hat{P}_s$ can be computed via

$$\hat{P}_s = \text{tr}(\hat{\mathbf{R}}\mathbf{T}). \quad (15)$$

If we include with this AWESOME formulation the constraints that $\text{rank}(\mathbf{T})=1$ and that the diagonal elements of $\mathbf{T}$ are equal, then (8) becomes the angle-only adaptive array problem discussed in [21]. The $\text{rank}(\mathbf{T})=1$ constraint makes the angle-only problem non-convex. By dropping the rank-one constraint, we can obtain the SDR solution to the angle-only problem, which can be used as the initial solution to the phase-only problem and refined via, e.g., a Newton-like search method [19].

Embodiments of the invention relate to the data-independent optimal designs based on MWA. Data-independent means we do not use the statistics of the received data.

A specific embodiment of data-independent MWA can utilize a beampattern match design, assume that we have a desired beampattern $P_d(\theta)$ defined over a region of interest Ω . Let $\{\mu_l\}_{l=1}^L$ be a fine grid of points covering Ω . An embodiment involves finding the matrix $\mathbf{T} \geq 0$ such that $P(\theta)$ matches or rather approximates (in a mean-squared error (MSE) sense) the desired beampattern $P_d(\beta)$, over the region of interest Ω under either the elemental uniform gain constraint or the total gain constraint. Mathematically, we can solve the following problem:

$$\min_{\alpha, \mathbf{T}} \frac{1}{L} \sum_{l=1}^L v_l \left[\alpha P_d(\mu_l) - \mathbf{a}^*(\mu_l) \mathbf{T} \mathbf{a}(\mu_l) \right]^2 \quad (16)$$

$$\text{subject to } T_{mm} = \frac{c}{M}, \quad m=1, \dots, M, \quad \text{or} \quad \text{tr}(\mathbf{T}) = c, \quad (17)$$

$$\mathbf{T} \geq 0, \quad (18)$$

where $v_l \geq 0$, $l=1, \dots, L$, is the weighting factor for the l th grid point, $\alpha > 0$ is a variable to control the magnitude of the desired beampattern $P_d(\theta)$. In (16), the scaling factor α is introduced for the consideration that typically $\phi(\theta)$ is given in a “normalized form” (e.g., satisfying $\sum_{l=1}^L P_d(\mu_l) = 1$), and an appropriately scaled version of $\phi(\theta)$ can be approximated, not $\phi(\theta)$ itself.

Techniques similar to those used in [26] can be used to show that the problem is a convex optimization problem. In certain embodiments, the problem is a Semi-definite Quadratic Programming (SQP) problem [27] since this received beampattern design problem is a dual problem to the MIMO transmit beampattern design problem considered in [26]. Let $\mathbf{r}$ denote the $M^2 \times 1$ real-valued vector made from T_{mm} ($m=1, \dots, M$) and the real and imaginary parts of T_{mp} , ($m, p=1, \dots, M; p > m$). Then, given the Hermitian symmetry of $\mathbf{T}$, we can write:

$$\text{vec}(\mathbf{T}) = \mathbf{J}\mathbf{r}, \quad (19)$$

for a suitable $M^2 \times M^2$ matrix $\mathbf{J}$ whose elements are easily derived constants ($0, \pm j, \pm 1$). Making use of (19) and of some simple properties of the vec operator, we have

$$\begin{aligned} \mathbf{a}^*(\mu_l)\mathbf{T}\mathbf{a}(\mu_l) &= \text{vec} \left[\mathbf{a}^*(\mu_l)\mathbf{T}\mathbf{a}(\mu_l) \right] \\ &= \left[\mathbf{a}^T(\mu_l) \otimes \mathbf{a}^*(\mu_l) \right] \mathbf{J}\mathbf{r} \\ &\triangleq -\mathbf{g}_l^T \mathbf{r}. \end{aligned} \quad (20)$$

Inserting (20) into (16) yields the following more compact form of the design cost function:

$$\frac{1}{L} \sum_{l=1}^L v_l \left[\alpha \phi(\mu_l) + \mathbf{g}_l^T \mathbf{r} \right]^2 \triangleq \boldsymbol{\rho}^T \boldsymbol{\Gamma} \boldsymbol{\rho}, \quad (21)$$

where the vector

$$\boldsymbol{\rho} = \begin{bmatrix} \alpha \\ \mathbf{r} \end{bmatrix}, \quad (22)$$

contains all the variables and

$$\Gamma = \frac{1}{L} \sum_{l=1}^L v_l \begin{bmatrix} \phi(\mu_l) \\ \mathbf{g}_l \end{bmatrix} \begin{bmatrix} \phi(\mu_l) & \mathbf{g}_l^T \end{bmatrix}. \quad (23)$$

The matrix Γ above could be rank deficient. For example, in the case of an M -sensor ULA with half-wavelength or smaller inter-element spacing, it can be verified that the rank of Γ is $2M$. The rank deficiency of Γ , however, does not pose any serious problem for the SQP solver we used (see, e.g., [25]).

By making use of (19) – (23), the beampattern matching design in (16) – (18) becomes the following SQP (see, e.g., [27]):

$$\begin{aligned} \min_{t, \rho} t \quad \text{subject to} \quad & \|\Gamma^{1/2} \rho\| \leq t \\ & T_{mm}(\rho) = \frac{c}{M}, \quad m = 1, \dots, M, \quad \text{or} \quad \text{tr}(\mathbf{T}) = c, \\ & \mathbf{T}(\rho) \geq 0, \end{aligned} \quad (24)$$

where we have indicated explicitly the (linear) dependence of $\mathbf{T}$ on ρ . For practical values of the array size M , the SQP above can be efficiently solved on a personal computer using public domain software (see, e.g., [24], [25]).

In some applications, we may wish that the synthesized beampattern at some given locations be exactly (or very close to) some values, or that one peak at a certain location to have exactly the same power (or of some ratio β) as a different peak at another location. The first requirement can be met by adding the following equality constraints to the original formulation (16):

$$\mathbf{a}^*(\tilde{\mu}_l) \mathbf{T} \mathbf{a}(\tilde{\mu}_l) = \zeta_l, \quad l = 1, \dots, \tilde{L}, \quad (25)$$

where $\{\zeta_l\}$ are the prescribed values and $\{\mu_l\}$ are the corresponding mesh grids. The second requirement can be added to (16) by using the following equality constraints:

$$\mathbf{a}^*(\tilde{\mu}_k) \mathbf{T} \mathbf{a}(\tilde{\mu}_k) = \beta \mathbf{a}^*(\tilde{\mu}_j) \mathbf{T} \mathbf{a}(\tilde{\mu}_j), \quad \text{for some } k \neq j, \quad (26)$$

where μ_k and μ_j are the locations at which we wish to have equal gain (or certain ratio). The extended problems with the above additional constraints remain SQP's and can be solved efficiently using the aforementioned SQP solvers.

Specific embodiments can incorporate minimum sidelobe level design. These embodiments can be useful when it is important to control the peak sidelobe level of the beam pattern (see, e.g., [6], [13]), while maintaining the shape of the main-lobe (e.g., direction, prescribed 3-dB beamwidth, etc.). For such applications, the following is an example of a minimum sidelobe level design that can be used. Assume that the main-beam is directed toward θ_0 , the prescribed 3-dB angles are θ_1 and θ_2 (the 3-dB beamwidth is $\theta_2 - \theta_1$, with $\theta_2 > \theta_0$ and $\theta_1 < \theta_0$), the 3-dB main-beam region is Ψ_m and the sidelobe region is Ψ_s . Such a minimum sidelobe level design problem can be formulated as follows:

$$\min_{\mathbf{T}} \quad -t \quad (27)$$

$$\text{subject to} \quad \mathbf{a}^*(\theta_0)\mathbf{T}\mathbf{a}(\theta_0) - \mathbf{a}^*(\mu_p)\mathbf{T}\mathbf{a}(\mu_p) > t, \quad \mu_p \in \Psi_s \quad (28)$$

$$0 < \mathbf{a}^*(\theta_0)\mathbf{T}\mathbf{a}(\theta_0) - \mathbf{a}^*(\mu_p)\mathbf{T}\mathbf{a}(\mu_p) \leq 0.5\mathbf{a}^*(\theta_0)\mathbf{T}\mathbf{a}(\theta_0), \quad \mu_p \in \Psi_m \quad (29)$$

$$0.5\mathbf{a}^*(\theta_0)\mathbf{T}\mathbf{a}(\theta_0) - \mathbf{a}^*(\theta_i)\mathbf{T}\mathbf{a}(\theta_i) = 0, \quad i = 1, 2, \quad (30)$$

$$T_{mm} = c/M, \quad m = 1, \dots, M, \quad \text{or} \quad \text{tr}(\mathbf{T}) = c, \quad (31)$$

$$\mathbf{T} \geq 0. \quad (32)$$

where the constraint (29) guarantees that the gain in the 3-dB main-beam region is at least half of the gain at θ_0 . The constraints (29) are used to prevent main-beam splitting and are active only in some cases. The formulation (27) – (32) is a SDP [20] and can be solved efficiently in polynomial time using public domain software (e.g., [25]). Again, specific embodiments may modify (28) and/or modify and use one or more of the constraints in (28), (29), (30), and (31). Additional constraints can be added to the formulation as well.

In specific embodiments, some of the constraints in the minimum sidelobe level design can be relaxed. We can relax somewhat the constraints in (30) defining the 3-dB mainbeam width; for instance, we can replace them by

$$(0.5 - \delta)\mathbf{a}^*(\theta_0)\mathbf{T}\mathbf{a}(\theta_0) \leq \mathbf{a}^*(\theta_i)\mathbf{T}\mathbf{a}(\theta_i) \leq (0.5 + \delta)\mathbf{a}^*(\theta_0)\mathbf{T}\mathbf{a}(\theta_0), \quad i = 1, 2,$$

for some small number $\delta > 0$. Such a relaxation can lead to designs with lower peak sidelobe levels, since in some designs, such as for 2-D arrays, the problem can become infeasible without proper relaxation. If needed, we can also relax the elemental uniform gain constraints somewhat by allowing the elemental gain to be within a certain range around c/M , while still maintaining the same total gain of c . Our numerical simulations show that such a relaxation can result in lower sidelobe levels and smoother beam patterns.

The VMA counterparts can be readily modified from the previously described beampattern matching or minimum sidelobe level beampattern designs by adding the constraint $\text{rank}(\mathbf{T})=1$ to (16) and (27), respectively. However, due to the non-convexity of the rank constraint, the problem becomes much harder to solve and no globally optimal solution is guaranteed.

In the numerical examples to follow, the Newton-like algorithm presented in [19] was used. This algorithm uses the solution to SDR as an initial solution, and then uses the tangent-and-lift procedure to iteratively find the solution satisfying the rank-one constraint. Although the convergence of the Newton-like algorithm is not guaranteed [19], we did not encounter any numerical problem in our simulations.

In specific embodiments, a constant beamwidth design for wideband arrays can be utilized. Beamformers with frequency-invariant main-beam widths are desirable in many wideband signal processing applications, such as aeroacoustics [15], [16], [28], radar, sonar, and communications [29]. A widely used wideband array processing approach, which can be utilized in embodiments of the invention, is to sample the spectrum of the wideband signal at each array element output to form narrowband frequency bins, and then process the samples in each frequency bin separately using narrowband array processing techniques [23]. Constant beamwidth beamformer can form consistent estimates of the source location and power across all frequency bins. However, the beamwidth of most beamformers, such as the Delay-And-Sum (DAS) and the Capon methods [18], decreases as the center frequency of the frequency bin increases, due to a larger effective array aperture at a higher frequency or a smaller wavelength. One way to mitigate this problem is to use shading, i.e., to apply frequency-dependent weights to the sensor outputs. Various shading schemes exist for regular arrays including ULA and Small Aperture Directional Array (SADA) [15]. Shading design for arbitrary arrays with various constraints still appears to be an open problem.

Embodiments of the invention incorporating MWA can be applied to achieve constant beamwidth beampattern designs for wideband arrays. For example, the minimum sidelobe level design of MWA can be used by specifying a common 3-dB main-beam width for all frequency bins, and then obtaining a weight matrix for each narrowband frequency bin using (27). The resulting beampattern for each frequency bin will have a constant main-beam width and the lowest possible sidelobe level. The beampattern matching design of MWA can also be modified for constant beamwidth design of wideband arrays. Embodiments of the invention can utilize AWESOME to achieve constant main-beam width across all frequency

bins while retaining its adaptive array capabilities of adaptively suppressing interferences and jammers. In the numerical examples in the next section, minimum sidelobe level designs via MWA are analyzed.

Numerical Examples

5 Several numerical examples are presented to demonstrate the performance of MWA compared with VMA and other approaches. Three types of arrays with different geometries are considered: ULA, Minimum Redundancy Array (MRA) [1], and SADA [15], [16]. The ULA comprises $M = 10$ antennas with half-wavelength spacing between adjacent antennas, and is used as the receiving array for far-field sources. The MRA includes $M = 5$ antennas,
10 and has the same physical aperture as that of the 10-element ULA. The inter-element spacings for the 5-element MRA are 1, 3, 3, 2 [1], in the unit of half-wavelength at the carrier frequency (for narrowband signals). SADA (see [15] for more details) is a directional array designed for aeroacoustic noise measurement, and incorporates 33 microphones arranged in four circles of eight microphones each and one microphone at the array center. The maximum
15 radius of the array is 3.89 inches. The weight vector for the DAS beamformer is $\mathbf{w} = \mathbf{a}^*(\theta_0)$ (see (2)) for ULA and MRA.

AWESOME

 The performance of the data-adaptive AWESOME is compared with those of the standard Capon beamformer [12] and the Robust Capon Beamformer (RCB) [30]. We collect
20 simulated data from the 10-element ULA is collected using the data model in (1). The noise is assumed to be spatially and temporally white circularly symmetric complex Gaussian random process with zero-mean and covariance matrix $\mathbf{Q} = \mathbf{I}$. One strong interference is present at 40° . For the RCB [30], the array steering vector is assumed to be within a spherical uncertainty set with uncertainty parameter $\varepsilon = 0.35M$. The SOI power estimates presented
25 below are obtained via averaging over 100 Monte-Carlo trials.

 First, the performance of AWESOME for maintaining the main-beam shape and controlling the peak sidelobe level is studied. The beampatterns in Figures 1A-1D are obtained with 100 Monte-Carlo trials when the snapshot number is $N = 50$, the SOI power is 10 dB, and the interference power is 60 dB. Figure 1A corresponds to the standard Capon
30 beamformer, where the peak sidelobe level is as high as about -1 dB and the pointing location of the main-beam varies from trial to trial. Figure 1B shows that RCB [30] has a peak

sidelobe level of about -10 dB, but the main-beam shape still varies slightly from one trial to another. Figure 1C corresponds to AWESOME, where the desired 3-dB points of AWESOME are set to be about the same as those of the standard Capon beamformer obtained in one of the trials (i.e., -5.5° and 5° and peak sidelobe level below $\zeta(\text{dB}) = -14$ dB), and the sidelobe region is $[-90^\circ, -15^\circ] \cup [15^\circ, 90^\circ]$. From Figure 1C, AWESOME can effectively control the peak sidelobe level to below -16 dB, and maintain a constant main-beam shape from trial to trail. If the peak sidelobe level must be very low, for example, lower than -40 dB, we must broaden the desired 3-dB main-beam width to be between -7.35° and 7.35° due to the well-known trade off between the sidelobe level and the main-beam width, and corresponding sidelobe region is also reduced to $[-90^\circ, -22^\circ] \cup [22^\circ, 90^\circ]$. As shown in Figure 1D, AWESOME can achieve this goal and maintain a constant main-beam shape from one trail to another.

Next, the robustness of AWESOME is examined in the presence of small sample size and steering angle error problems. The SOI power is 10 dB and the interference power is 60 dB. The horizontal dashed line corresponds to -3 dB. Figure 2A shows the beampatterns obtained from one Monte-Carlo trial in the absence of any steering angle error when the snapshot number is sufficiently large ($N = 100$). Figure 2B is obtained when the number of snapshots is small ($N = 10$) and the assumed SOI angle is 0° while the true angle is 2° . In the presence of these problems, the standard Capon beamformer fails to function properly and forms a null at 2° ; RCB's gain at 0° is above 0 dB; AWESOME maintains the main-beam shape and the peak sidelobe level.

Figure 3 shows the SOI power estimates when the SOI power is 20 dB and the interference power is 60 dB. The main-beam shape and the peak sidelobe level specifications of AWESOME are the same as those for Figure 2. Figs. 5(a) and 5(b) Figures 3A and 3B, respectively, show the SOI power estimates versus the snapshot number N with and without the 2° steering angle mismatch. The perturbing random variables are independent of each other. Figure 4 shows the SOI power estimates versus the variance of the perturbing random variables when $N = 100$. The steering vector varies from one Monte-Carlo trial to another for Figure 4. Note that AWESOME and RCB yield much more accurate SOI power estimates than the standard Capon beamformer. AWESOME performs similarly as RCB in most cases but outperforms RCB when the variance of the perturbing random variables is large. This advantage of AWESOME is a result of, at least in part, its strict main-beam shape control.

The next example shows that an embodiment of AWESOME can function properly even when the interference is highly correlated with the SOI, due to its strict main-beam shape and peak sidelobe level control. Figure 5 shows the SOI power estimates as a function of the correlation coefficient between the SOI and the interference obtained using the theoretical $\mathbf{R}$ (i.e., $N \rightarrow \infty$). The SOI power is 20 dB, and the interference power is 50 dB. The main-beam shape and the peak sidelobe level specifications of AWESOME are the same as those for Figure 5. Note that AWESOME significantly outperforms both RCB and the standard Capon beamformer in the presence of highly correlated or coherent interference since the latter two algorithms fail to function properly when the correlation coefficient becomes close to 1. When the highly correlated or coherent interference becomes even stronger, though, this embodiment of AWESOME will eventually fail since the peak sidelobe level of this embodiment of AWESOME is set to -40 dB and hence is not sufficient for the adequate suppression of the very strong interference.

Finally, the influence of the number of strong interferences that the adaptive array can handle is considered. The interference angles are uniformly distributed in the sidelobe region, at angles $-70^\circ, -60^\circ, \dots, -40^\circ$ as well as at their positive angle counterparts. Figures 6A and 6B show the SOI power estimates obtained using the theoretical $\mathbf{R}$ when the interference powers are 40 dB and 60 dB, respectively. Note that AWESOME outperforms RCB. AWESOME performs similarly as the standard Capon beamformer when the interference power is 40 dB, but the latter performs better when the interference power increases. Due to increased robustness of RCB and AWESOME, their abilities of suppressing many interferences decrease while the standard Capon beamformer performs at its best for this example because of the perfect conditions: using theoretical $\mathbf{R}$, no steering vector errors, and uncorrelated interferences.

An embodiment of the invention utilizes MWA: Beampattern Matching Design. Consider the beampattern matching design in (16) with the 5-element MRA under the elemental uniform gain constraint with $c=1$. The desired beampattern has three pulses centered at $\theta_1 = -40^\circ$, $\theta_2 = 0^\circ$, and $\theta_3 = 40^\circ$, each with a width of 20° . Fig. 8 is obtained with a mesh grid size of 0.1° , and the weight v_l in (16) is set to 100 when the corresponding μ_l is in the sidelobe region and is set to 1 when μ_l is in the pulse region. The mesh grid size has no significant impact on the synthesized beampattern. Setting v_l in the sidelobe region larger (such as 100 times larger) than that in the pulse region can bring down slightly the sidelobe

levels of the synthesized beampatterns. Note that from Fig. 8 that MWA can provide a much better beampattern matching than VWA, especially for this case of non-uniform MRA.

An embodiment of the invention utilizes MWA: Minimum Sidelobe Level Design. Consider the minimum sidelobe level design in (27) for the 10-element ULA. The main-beam is centered at $\theta_0 = 0^\circ$ with a 3-dB width equal to 20° ($\theta_1 = -10^\circ$, $\theta_2 = 10^\circ$). The sidelobe region is chosen to be $\Psi_s = [-90^\circ, -20^\circ] \cup [20^\circ, 90^\circ]$ to allow some roll-off regions for the beampattern. Fig. 9 is obtained with a mesh grid size of 0.1° . Note that VWA fails to produce a proper main-beam and that the peak sidelobe level of the VWA beampattern is more than 5 dB higher than that of MWA.

The worse performance of VWA, compared to that of MWA, in the above two examples is due to the fact that under the elemental uniform gain constraint, the number of degrees of freedom of VWA is equal to only $M-1$ (real-valued parameters), whereas that of MWA is $M^2 - M$. Under the total gain constraint, though, MWA and VWA perform similarly. The elemental gains, however, can vary significantly, which may be undesirable in some applications.

Embodiments of the invention can utilize MWA: Constant Beamwidth Design for Wideband Arrays.

Case I: Linear Array, far field sources

Consider first the wideband constant beamwidth design for both the ULA and MRA linear arrays. Assume that the frequency band of interest is $[f_l, f_u]$. To avoid the grating lobes, the inter-element spacing of ULA is chosen to be half-wavelength of the highest frequency component, i.e., $c/(2f_u)$. The inter-element spacings of MRA are still 1, 3, 3, 2, but in the unit of $c/(2f_u)$. For convenience, let $\tilde{f} = f/f_u$; then the corresponding normalized frequency band is $\tilde{f} \in [f_l/f_u, 1]$. Assume that $\tilde{f} \in [0.4, 1]$. The desired common 3-dB beamwidth for ULA and MRA is chosen to be 20° ($\theta_1 = -10^\circ$, $\theta_2 = 10^\circ$). The sidelobe and main-lobe regions are the same as for Figure 7. The beampatterns of MWA are obtained by using (27) for each frequency bin under the total gain constraint with a mesh grid size of 0.1° .

Figures 9A and 9B show the constant beamwidth beampatterns obtained via MWA, for ULA and MRA, respectively. The MWA beampatterns maintain a constant 3-dB

beamwidth across the frequency band. The sidelobe levels of MRA are higher than those of ULA because it is difficult to control the sidelobe levels of MRA. Figure 9C shows the 3-dB main-beam widths of MWA, which are compared with those of the DAS beamformer, as functions of the normalized frequency. The peak sidelobe level comparison is shown in Figure 9D. Note again that MWA can be used to achieve constant main-beam width across the frequency bins for both ULA and MRA. At lower frequencies, the peak sidelobe levels of MWA are slightly higher than those of DAS, but at higher frequencies, the peak sidelobe levels of MWA are lower (and much lower for ULA).

10 Case II: 2-D array, near-field sources

Consider now the wideband constant beamwidth design for SADA. The fact that SADA is a 2-D array and that it is used for near-field noise power measurements makes the problem more challenging. For a near-field source at location $\mathbf{r}_0$, the array steering vector for the k th narrowband frequency bin has the form:

$$15 \quad \mathbf{a}(\mathbf{r}_0, f_k) = \left[\frac{e^{-j \frac{2\pi f_k}{c} \|\mathbf{r}_1 - \mathbf{r}_0\|}}{\|\mathbf{r}_1 - \mathbf{r}_0\|}, \dots, \frac{e^{-j \frac{2\pi f_k}{c} \|\mathbf{r}_M - \mathbf{r}_0\|}}{\|\mathbf{r}_M - \mathbf{r}_0\|} \right]^T, \quad (33)$$

where $\mathbf{r}_m$ is the location of the m th sensor.

The beampatterns using the minimum sidelobe level design (27) under the total gain constraint are obtained. The beampatterns shown in Figure 10 are formed by scanning the locations on a plane parallel to the array and situated 4 feet above the array. The beam is steered to $\mathbf{r}_0 = [0, 0, 4]$ (feet), which is the center of the images shown in Figure 10. The ranges of the x-axis and y-axis of the images are from -2 to 2 feet. (For comparison purposes, these parameters are set to be the same as those in [15].) The mesh grid size on both the x- and y-axes is 0.5 inches. The radius of the desired 3-dB circle for MWA is selected to be 4 inches, which is roughly the same as that achieved by the shading scheme in [15] at 10 – 40 KHz. We use a real-valued $\mathbf{T}$ in this case. (The results obtained with complex-valued $\mathbf{T}$ are similar, but with slightly lower sidelobe levels.)

The beampatterns in Figs. Figures 10A-10D are obtained by using VMA shaded by the frequency-dependent weight vector designed in [15]. The frequency-dependent weight vector was designed to maintain a constant beamwidth within the frequency band of 10 – 40 KHz. The radius of the 3-dB circle at 8 KHz in Figure 10A is wider than those at 20 and 40

KHz in Figures 10B and 10C, respectively. The radius of the 3-dB circle at 65 KHz in Fig. Figure 10D is narrower than those at 20 and 40 KHz. The MWA beampatterns, on the other hand, have a constant 3-dB circle and low sidelobe levels throughout the 8 to 65 KHz frequency band, as shown in Figures 10E-10H. This demonstrates that MWA is capable of
5 extending the constant beamwidth working frequency band of SADA to 8 – 65 KHz. Interestingly, the optimal $\mathbf{T}$ determined by MWA has only one dominant eigenvalue, which means that better weighting vectors than those presented in [15] have been found.

All patents, patent applications, provisional applications, and publications referred to or cited herein are incorporated by reference in their entirety, including all figures and tables,
10 to the extent they are not inconsistent with the explicit teachings of this specification.

It should be understood that the examples and embodiments described herein are for illustrative purposes only and that various modifications or changes in light thereof will be suggested to persons skilled in the art and are to be included within the spirit and purview of this application.

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Claims

1. A method for processing a plurality of signals from a sensor array, comprising:

5 receiving a plurality of signals from a sensor array, wherein the plurality of signals is represented as $\mathbf{y}(n) = \mathbf{a}(\theta_0)s(n) + \mathbf{e}(n)$, $n = 1, \dots, N$, where $\mathbf{y}(n)$ is the n th received data vector or the n th snapshot, $n = 1, \dots, N$, with N denoting the snapshot number; $\mathbf{a}(\theta_0)$ is the array steering vector for the signal of interest (SOI), which is a known function of θ_0 , and $\mathbf{e}(n)$ is the residue term.

10 determining $\mathbf{T}$ subject to various constraints related to the power response of the sensor array as a function of θ , wherein the power response of the sensor array as a function of θ is the array beam pattern $P(\theta) \triangleq \mathbf{a}(\theta)^* \mathbf{T} \mathbf{a}(\theta)$, wherein $\mathbf{T}$ is allowed to have rank higher than one.

15 2. The method according to claim 1, further comprising:

providing the power of the signal of interest.

3. The method according to claim 1, further comprising:

calculating the power of the signal of interest as $\text{tr}(\hat{\mathbf{R}}\mathbf{T})$.

4. The method according to claim 3, wherein $\hat{\mathbf{R}}$ is based $\mathbf{y}(n)$.

5. The method according to claim 1, further comprising:

letting $\mathbf{T} = \mathbf{U}\Sigma\mathbf{U}^*$ and let $\mathbf{W} = \Sigma^{1/2}\mathbf{U}$, where the columns of $\mathbf{U}$ are the eigenvectors of

25 $\mathbf{T}$ and the diagonal elements of the diagonal matrix Σ are the corresponding eigenvalues, wherein

$$\mathbf{T} = \mathbf{W}\mathbf{W}^*,$$

where $\mathbf{W} = [\mathbf{w}_1, \dots, \mathbf{w}_K] \in C^{M \times K}$, with $\mathbf{w}_k$ denoting the k th weight vector, wherein the power response of the array as a function of θ is

$$P(\theta) = \mathbf{a}^*(\theta)\mathbf{T}\mathbf{a}(\theta) = \mathbf{a}^*(\theta)\mathbf{W}\mathbf{W}^*\mathbf{a}(\theta) = \sum_{k=1}^K \left| \mathbf{a}^*(\theta)\mathbf{w}_k \right|^2$$

$$\text{wherein } \text{tr}(\mathbf{T}) = \text{tr}\left(\sum_{k=1}^K \mathbf{w}_k \mathbf{w}_k^*\right) = \sum_{k=1}^K \text{tr}(\mathbf{w}_k \mathbf{w}_k^*) = \sum_{k=1}^K \text{tr}(\mathbf{w}_k^* \mathbf{w}_k) = \sum_{k=1}^K \|\mathbf{w}_k\|^2$$

selecting a unitary matrix $\mathbf{P}$ with $\mathbf{P}^*\mathbf{P} = \mathbf{P}\mathbf{P}^* = \mathbf{I}$, wherein $\mathbf{P}$ is a diagonal matrix with each of its diagonal elements unit modulus with the phase adjusted so that each element of $\mathbf{W}^*\mathbf{a}(\theta_0)$ is real-valued, where $\mathbf{T} = \mathbf{W}\mathbf{P}\mathbf{P}^*\mathbf{W}^*$.

5 6. The method according to claim 1, wherein determining $\mathbf{T}$ comprises $\min_{\mathbf{T}} \text{tr}(\hat{\mathbf{R}}\mathbf{T})$

subject to one or more constraints on $\mathbf{T}$, wherein $\mathbf{T} \geq 0$, wherein $\hat{\mathbf{R}}$ is sample covariance matrix.

7. The method according to claim 6, wherein the one or more constraints on $\mathbf{T}$ includes

10 $\mathbf{a}^*(\theta_0)\mathbf{T}\mathbf{a}(\theta_0) = 1$, where θ_0 is the location parameter of the SOI.

8. The method according to claim 6, wherein the one or more constraints on $\mathbf{T}$ includes

$\mathbf{a}^*(\theta_i)\mathbf{T}\mathbf{a}(\theta_i) = 0.5$, $i = 1, 2$, where θ_1 and θ_2 are the prescribed 3-dB points.

15 9. The method according to claim 6, wherein the one or more constraints on $\mathbf{T}$ includes

$\mathbf{a}^*(\mu_l)\mathbf{T}\mathbf{a}(\mu_l) \leq \varsigma$, where $\mu_l \in \Psi_s$, ς is the desired peak sidelobe level, and Ψ_s denotes the sidelobe region.

10. The method according to claim 6, wherein the one or more constraints on $\mathbf{T}$ includes

20 $\mathbf{a}^*(\mu_l)\mathbf{T}\mathbf{a}(\mu_l) \leq 0.5$, where $\mu_l \in (\theta_1, \theta_2)$, where the interval (θ_1, θ_2) is the 3-dB main-beam region.

11. The method according to claim 6, wherein $\hat{\mathbf{R}} = \frac{1}{N} \sum_{n=1}^N \mathbf{y}(n)\mathbf{y}^H(n)$.

12. The method according to claim 1,

25 wherein determining $\mathbf{T}$ comprises solving $\min_{\alpha, \mathbf{T}} \frac{1}{L} \sum_{l=1}^L v_l [\alpha P_d(\mu_l) - \mathbf{a}^*(\mu_l)\mathbf{T}\mathbf{a}(\mu_l)]^2$

subject to $\mathbf{T} \geq 0$, where $v_l \geq 0$, $l = 1, \dots, L$, is the weighting factor for the l th grid point, where $\alpha > 0$ is a variable to control the magnitude of the desired beampattern $P_d(\theta)$.

13. The method according to claim 12, wherein determining $\mathbf{T}$ is subject to the constraint

30 $T_{mm} = \frac{c}{M}$, $m = 1, \dots, M$, where c is a user parameter.

14. The method according to claim 12, wherein determining $\mathbf{T}$ is subject to the constraint $\text{tr}(\mathbf{T}) = c$, where c is a user parameter.

5 15. The method according to claim 12, wherein determining $\mathbf{T}$ comprises:

letting $\mathbf{r}$ denote the $M^2 \times 1$ real-valued vector made from T_{mm} ($m = 1, \dots, M$) and the real and imaginary parts of T_{mp} , ($m, p = 1, \dots, M; p > m$), wherein $\mathbf{T}$ is Hermitian symmetric and

$$\text{vec}(\mathbf{T}) = \mathbf{J}\mathbf{r},$$

10 for a suitable $M^2 \times M^2$ matrix $\mathbf{J}$ whose elements are derived constants ($0, \pm j, \pm 1$), wherein

$$\begin{aligned} \mathbf{a}^*(\mu_l)\mathbf{T}\mathbf{a}(\mu_l) &= \text{vec} \left[\mathbf{a}^*(\mu_l)\mathbf{T}\mathbf{a}(\mu_l) \right] \\ &= \left[\mathbf{a}^T(\mu_l) \otimes \mathbf{a}^*(\mu_l) \right] \mathbf{J}\mathbf{r} \\ &\triangleq -\mathbf{g}_l^T \mathbf{r} \end{aligned}$$

wherein the design cost function:

$$15 \quad \frac{1}{L} \sum_{l=1}^L v_l \left[\alpha \phi(\mu_l) + \mathbf{g}_l^T \mathbf{r} \right]^2 \triangleq \boldsymbol{\rho}^T \boldsymbol{\Gamma} \boldsymbol{\rho},$$

where the vector

$$\boldsymbol{\rho} = \begin{bmatrix} \alpha \\ \mathbf{r} \end{bmatrix},$$

contains all the variables and

$$20 \quad \boldsymbol{\Gamma} = \frac{1}{L} \sum_{l=1}^L v_l \begin{bmatrix} \phi(\mu_l) \\ \mathbf{g}_l \end{bmatrix} \begin{bmatrix} \phi(\mu_l) & \mathbf{g}_l^T \end{bmatrix}$$

wherein determining $\mathbf{T}$ comprising

$$\min_{t, \boldsymbol{\rho}} t \quad \text{subject to} \quad \|\boldsymbol{\Gamma}^{1/2} \boldsymbol{\rho}\| \leq t$$

$$\mathbf{T}(\boldsymbol{\rho}) \geq 0.$$

25

16. The method according to claim 15, wherein determining $\mathbf{T}$ is subject to

$$T_{mm}(\boldsymbol{\rho}) = \frac{c}{M}, \quad m = 1, \dots, M.$$

17. The method according to claim 15, wherein determining $\mathbf{T}$ is subject to $\text{tr}(\mathbf{T}) = c$.

18. The method according to claim 15, further comprising:

5 applying the constraint $\mathbf{a}^*(\tilde{\mu}_l)\mathbf{T}\mathbf{a}(\tilde{\mu}_l) = \zeta_l, \quad l = 1, \dots, \tilde{L},$
where $\{\zeta_l\}$ are the prescribed values and $\{\mu_l\}$ are the corresponding mesh grids.

19. The method according to claim 15, further comprising:

10 applying the constraint $\mathbf{a}^*(\tilde{\mu}_k)\mathbf{T}\mathbf{a}(\tilde{\mu}_k) = \beta\mathbf{a}^*(\tilde{\mu}_j)\mathbf{T}\mathbf{a}(\tilde{\mu}_j), \quad \text{for some } k \neq j,$
where μ_k and μ_j are the locations at which equal gain or certain ratio is desired.

20. The method according to claim 1, further comprising:

applying the constraint

15
$$\min_{\mathbf{T}} \quad -t$$

subject to $\mathbf{a}^*(\theta_0)\mathbf{T}\mathbf{a}(\theta_0) - \mathbf{a}^*(\mu_p)\mathbf{T}\mathbf{a}(\mu_p) > t, \quad \mu_p \in \Psi_s,$
 $\mathbf{T} \geq 0,$

wherein the main-beam is directed toward θ_0 , where the sidelobe region is Ψ_s .

21. The method according to claim 20, further comprising:

20 applying the constraint $0 < \mathbf{a}^*(\theta_0)\mathbf{T}\mathbf{a}(\theta_0) - \mathbf{a}^*(\mu_p)\mathbf{T}\mathbf{a}(\mu_p) \leq 0.5\mathbf{a}^*(\theta_0)\mathbf{T}\mathbf{a}(\theta_0), \quad \mu_p \in \Psi_m,$
wherein the 3-dB main-beam region is Ψ_m .

22. The method according to claim 20, further comprising:

25 applying the constraint $0.5\mathbf{a}^*(\theta_0)\mathbf{T}\mathbf{a}(\theta_0) - \mathbf{a}^*(\theta_i)\mathbf{T}\mathbf{a}(\theta_i) = 0, \quad i = 1, 2,$ wherein the prescribed
3-dB angles are θ_1 and θ_2 (the 3-dB beamwidth is $\theta_2 - \theta_1$, with $\theta_2 > \theta_0$ and $\theta_1 < \theta_0$).

23. The method according to claim 20, further comprising:

applying the constraint $T_{mm} = \frac{c}{M}, \quad m = 1, \dots, M.$

24. The method according to claim 20, further comprising:
applying the constraint $\text{tr}(\mathbf{T}) = c$.

25. The method according to claim 5, further comprising:

5 outputting an output signal vector $\mathbf{W}^* \mathbf{y}(n)$ $n=1, \dots, N$.

26. The method according to claim 25, wherein the output signal vector is utilized in radar.

27. The method according to claim 25, wherein the output signal vector is utilized in sonar.

10

28. The method according to claim 25, wherein the output signal vector is utilized in
aeroacoustics.

29. The method according to claim 25, wherein the output signal vector is utilized in
15 communications.

30. The method according to claim 25, wherein the output signal vector is utilized in medical
imaging.

20 31. The method according to claim 5, further comprising:
outputting an estimate of the signal $\hat{s}(n) = \mathbf{w}^* \mathbf{y}(n)$, where $\mathbf{W}$ has rank-1 and is denoted as $\mathbf{w}$.

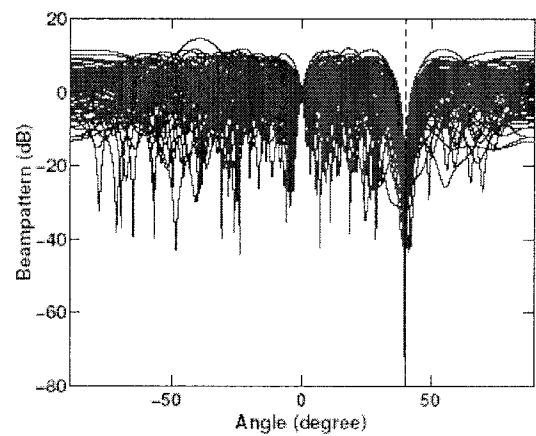


FIG. 1A

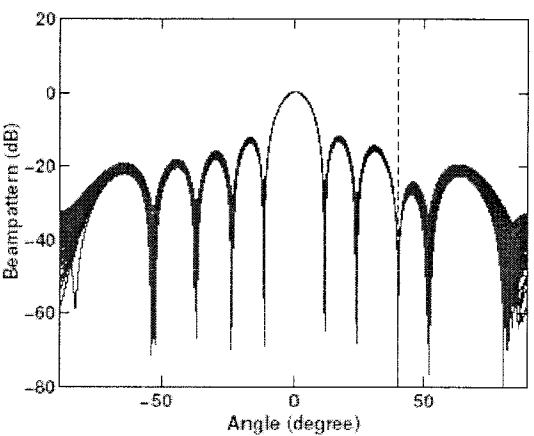


FIG. 1B

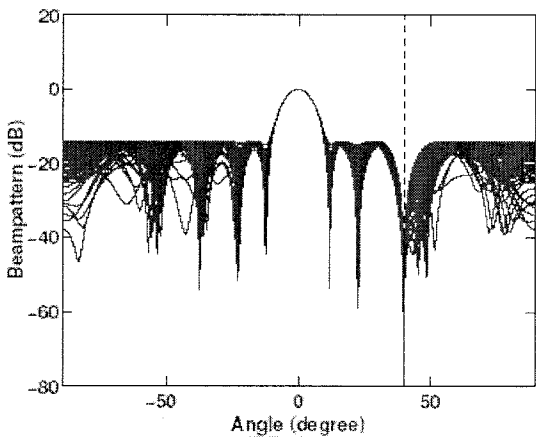


FIG. 1C

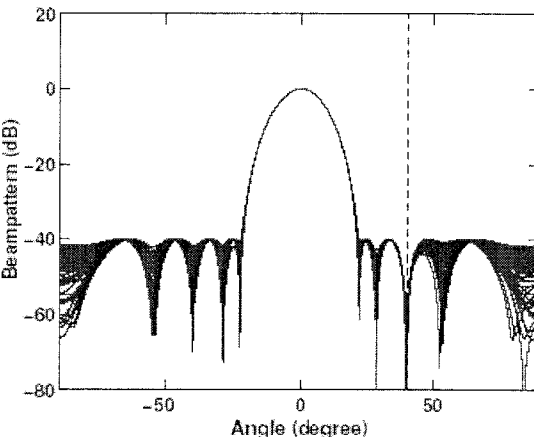


FIG. 1D

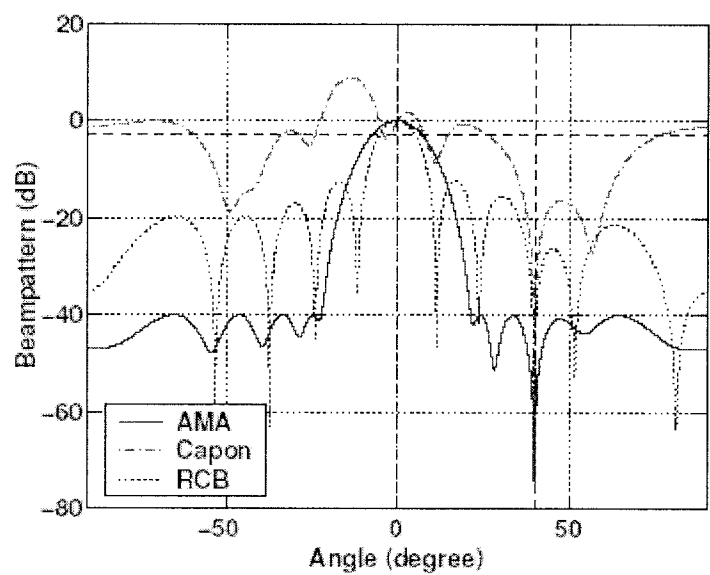


FIG. 2A

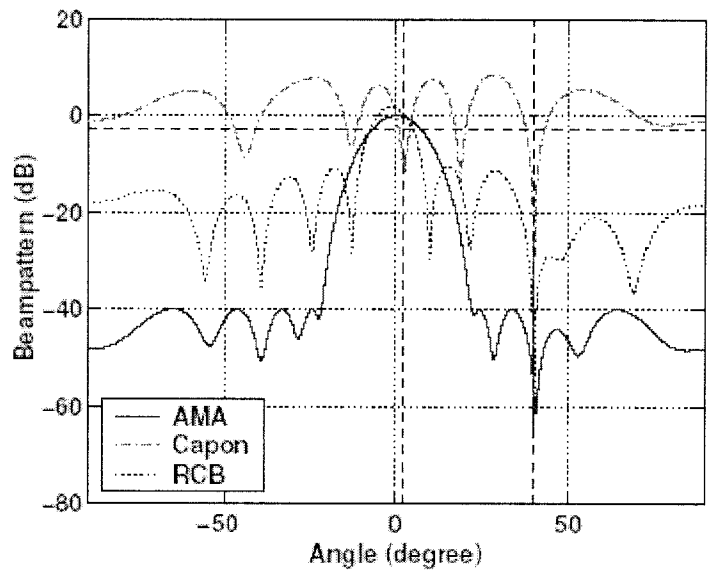


FIG. 2B

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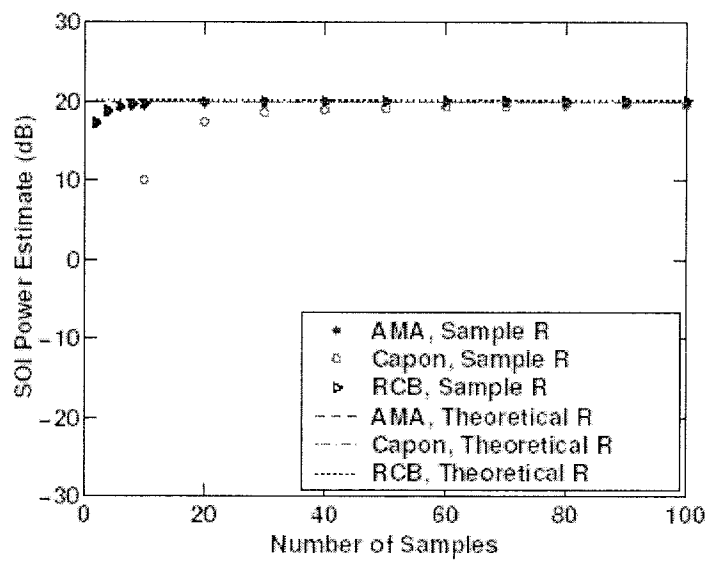


FIG. 3A

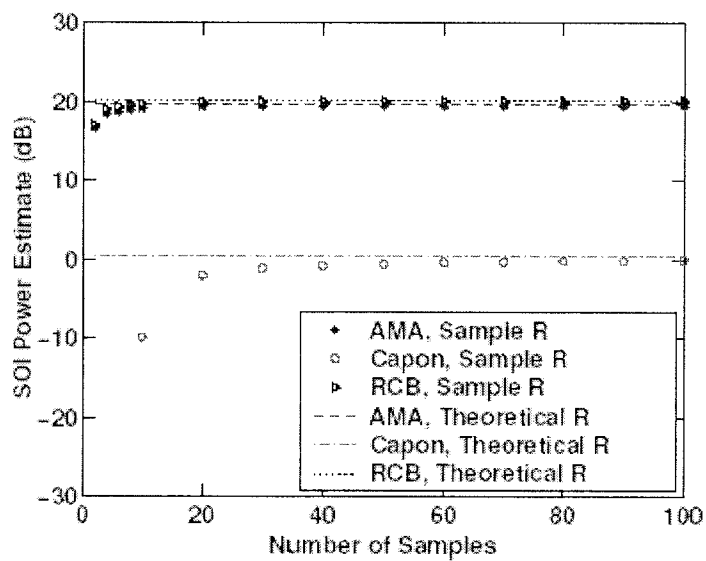


FIG. 3B

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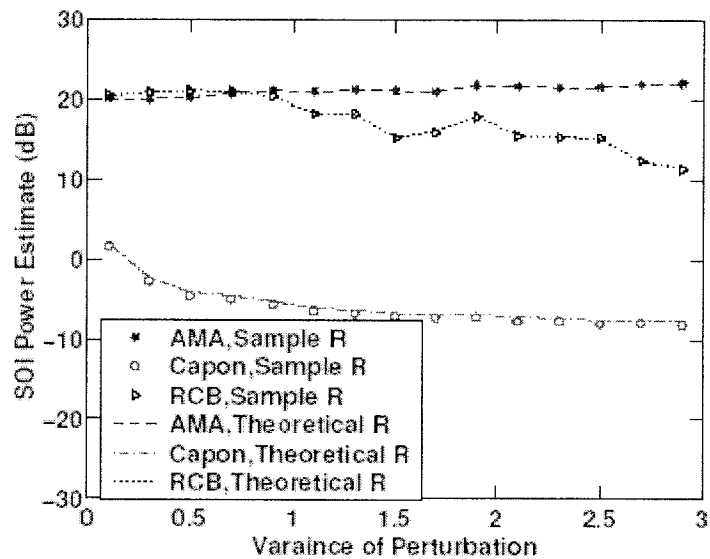


FIG. 4

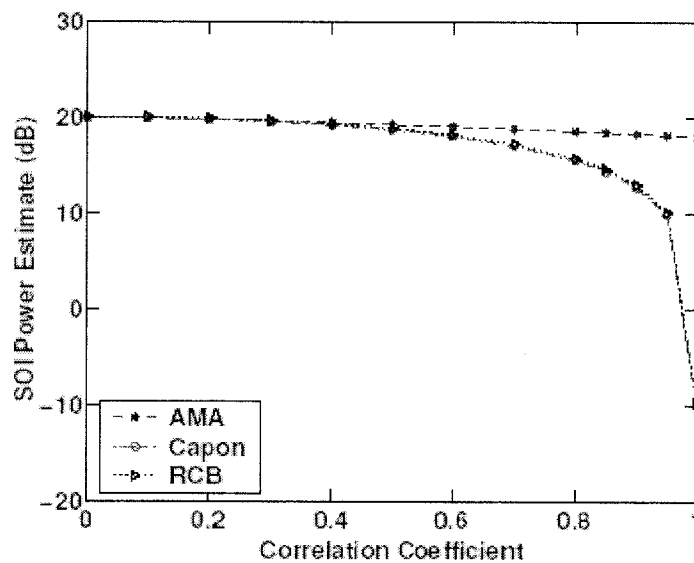


FIG. 5

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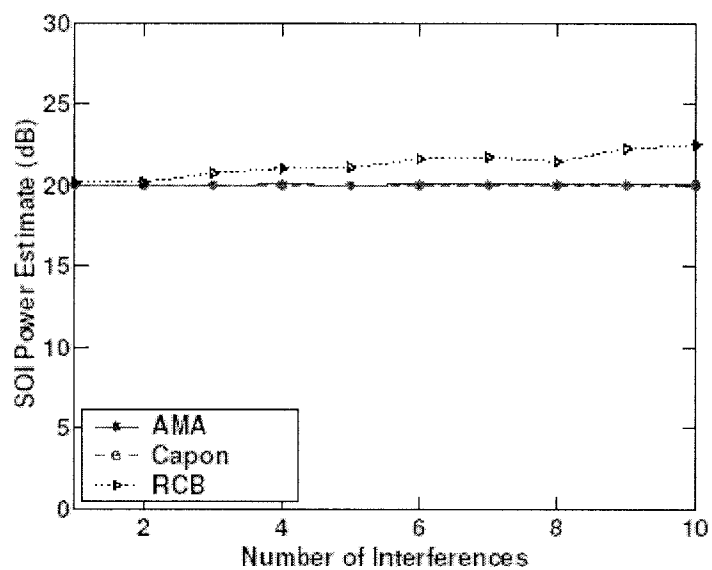


FIG. 6A

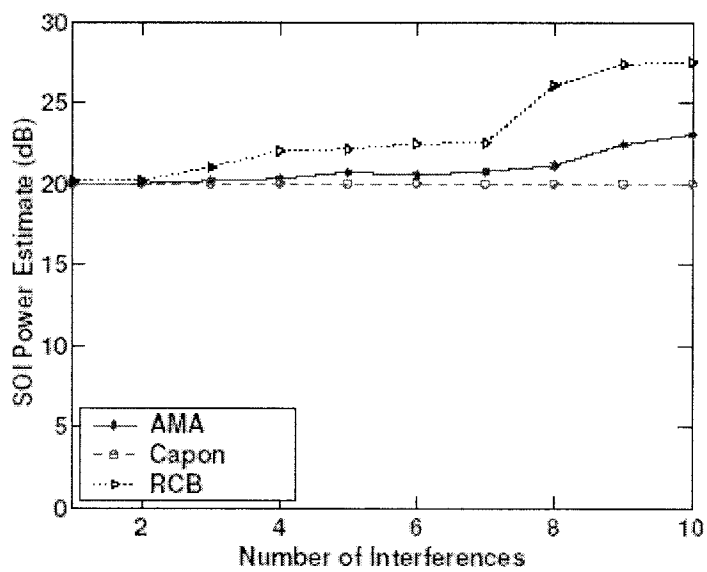
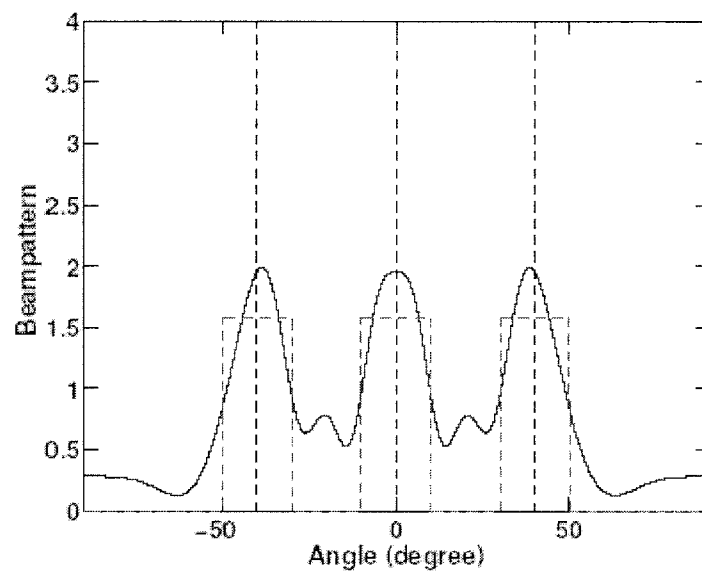
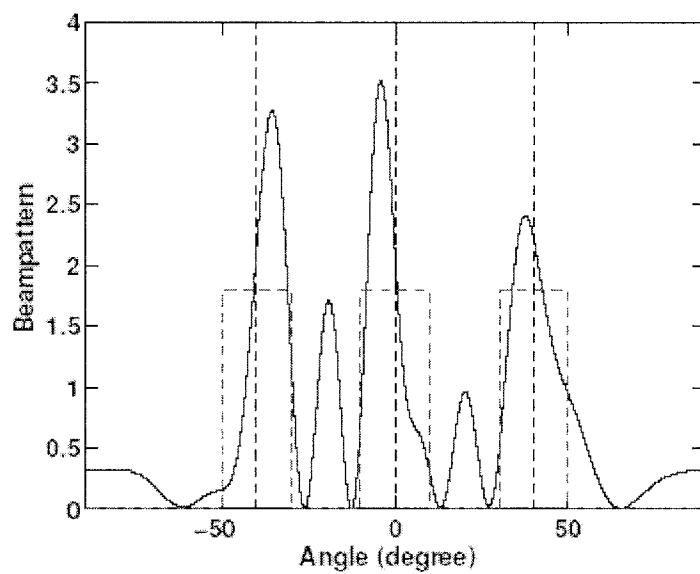


FIG. 6B

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**FIG. 7A****FIG. 7B**

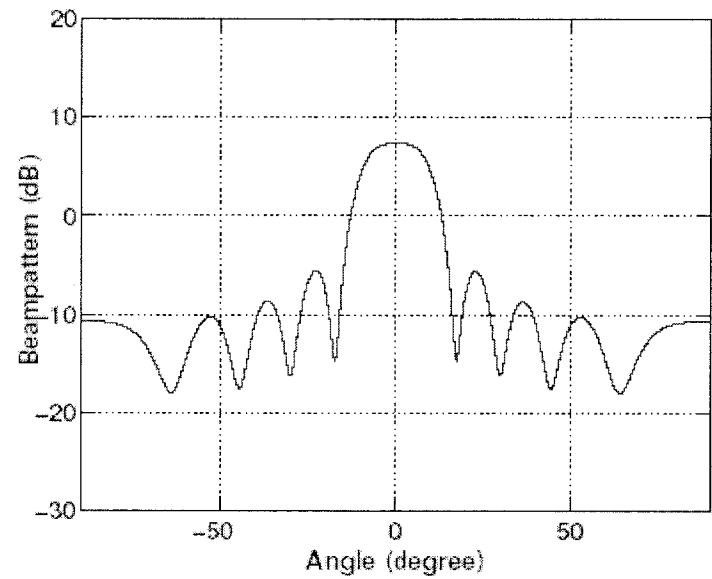


FIG. 8A

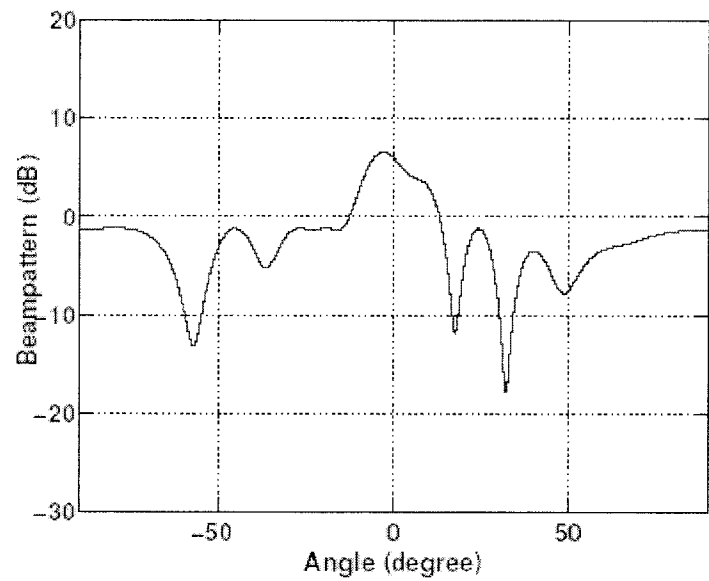


FIG. 8B

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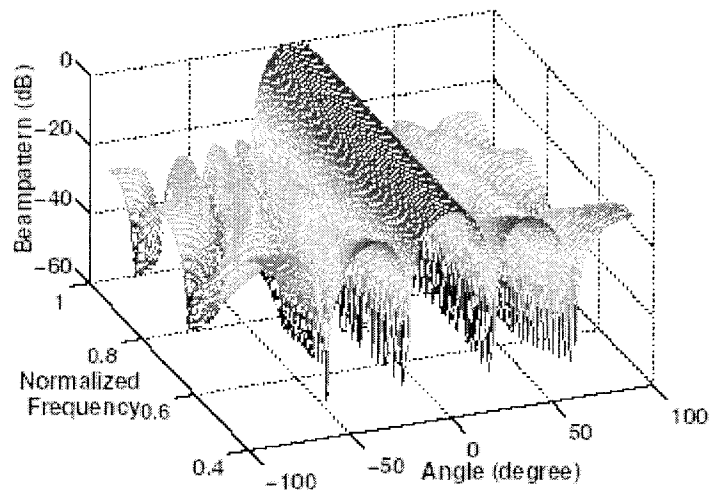


FIG. 9A

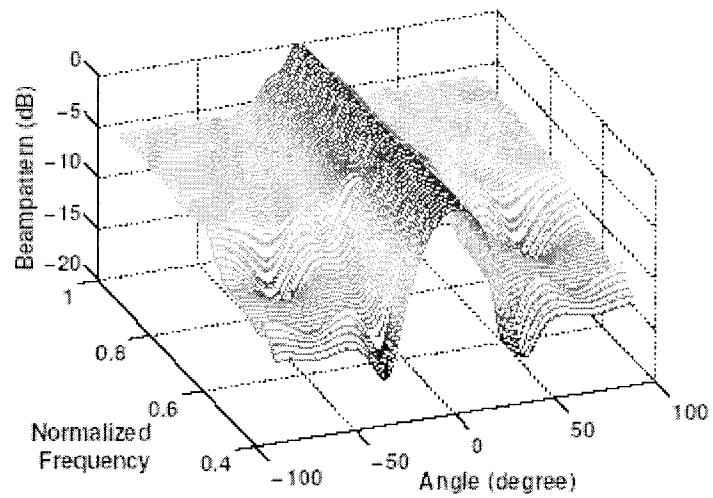


FIG. 9B

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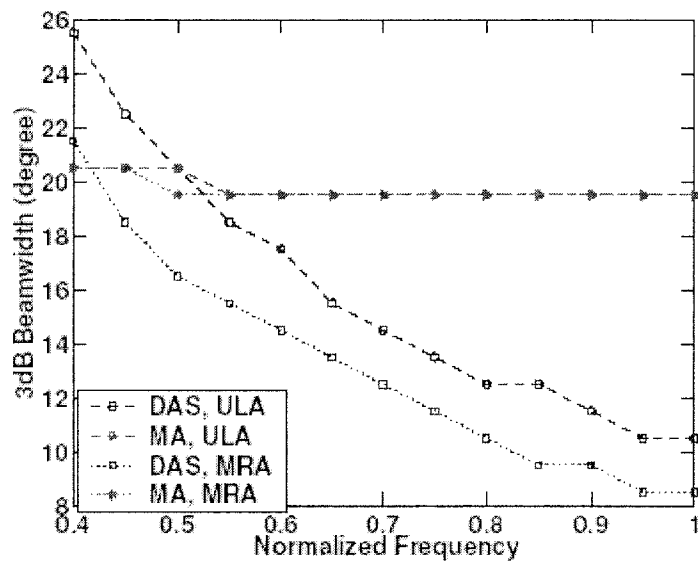


FIG. 9C

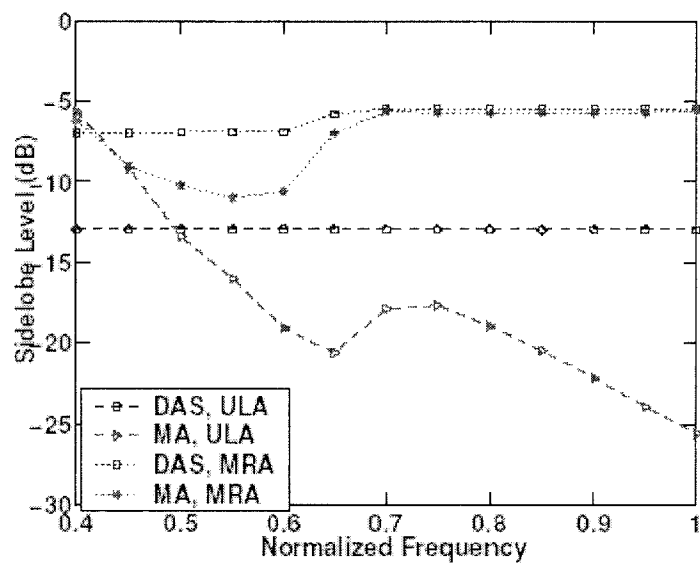


FIG. 9D

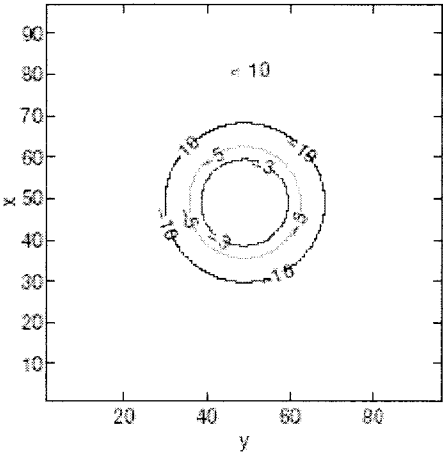


FIG. 10A

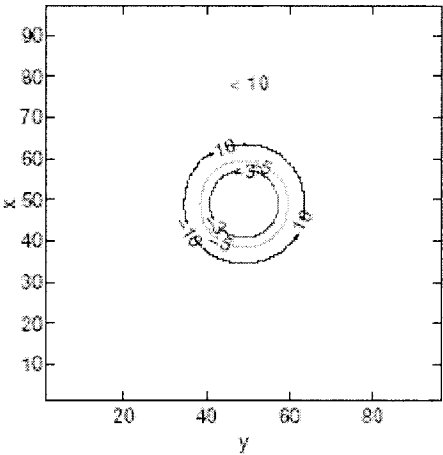


FIG. 10B

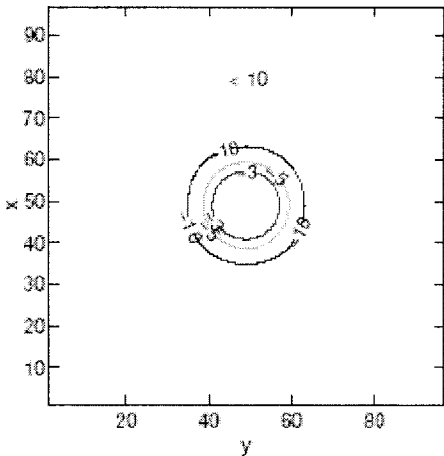


FIG. 10C

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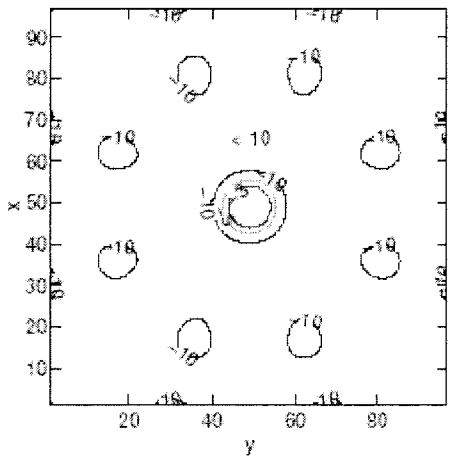


FIG. 10D

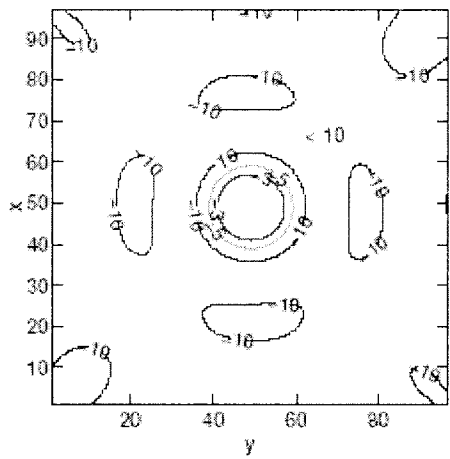


FIG. 10E

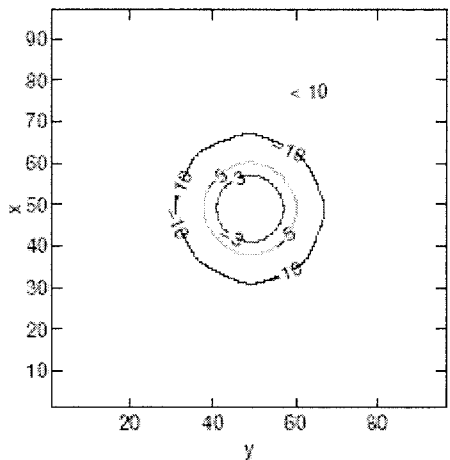


FIG. 10F

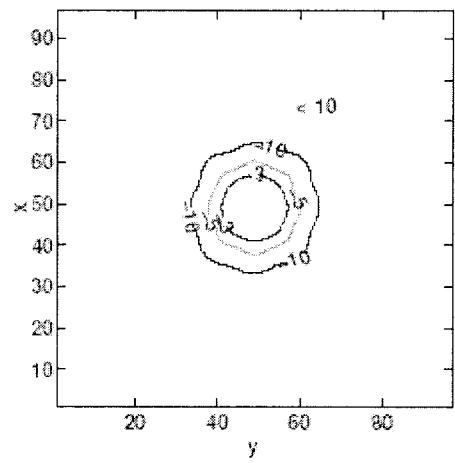


FIG. 10G

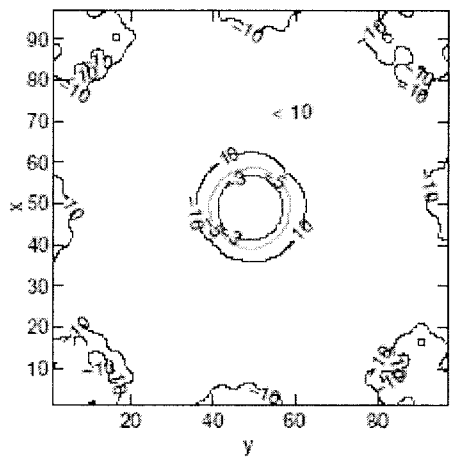


FIG. 10H

DECLARATION OF NON-ESTABLISHMENT OF INTERNATIONAL SEARCH REPORT

(PCT Article 17(2)(a), Rules 13ter.1(c) and Rule 39)

Applicant's or agent's file reference UF-521XC1 PCT	IMPORTANT DECLARATION	Date of mailing(day/month/year) 18/08/2008
International application No. PCT/US2007/074739	International filing date(day/month/year) 30/07/2007	(Earliest) Priority date(day/month/year) 28/07/2006
International Patent Classification (IPC) or both national classification and IPC H01Q3/30		
Applicant UNIVERSITY OF FLORIDA RESEARCH FOUNDATION INC		

This International Searching Authority hereby declares, according to Article 17(2)(a), that **no international search report will be established** on the international application for the reasons indicated below

1. ☒ The subject matter of the international application relates to:

- a. ☐ scientific theories
- b. ☐ mathematical theories
- c. ☐ plant varieties
- d. ☐ animal varieties
- e. ☐ essentially biological processes for the production of plants and animals, other than microbiological processes and the products of such processes
- f. ☐ schemes, rules or methods of doing business
- g. ☒ schemes, rules or methods of performing purely mental acts
- h. ☐ schemes, rules or methods of playing games
- i. ☐ methods for treatment of the human body by surgery or therapy
- j. ☐ methods for treatment of the animal body by surgery or therapy
- k. ☐ diagnostic methods practised on the human or animal body
- l. ☐ mere presentations of information
- m. ☐ computer programs for which this International Searching Authority is not equipped to search prior art

2. ☐ The failure of the following parts of the international application to comply with prescribed requirements prevents a meaningful search from being carried out:

- ☐ the description ☐ the claims ☐ the drawings

3. ☐ A meaningful search could not be carried out without the sequence listing; the applicant did not, within the prescribed time limit:

- ☐ furnish a sequence listing on paper complying with the standard provided for in Annex C of the Administrative Instructions, and such listing was not available to the International Searching Authority in a form and manner acceptable to it.
- ☐ furnish a sequence listing in electronic form complying with the standard provided for in Annex C of the Administrative Instructions, and such listing was not available to the International Searching Authority in a form and manner acceptable to it.
- ☐ pay the required late furnishing fee for the furnishing of a sequence listing in response to an invitation under Rule 13ter.1(a) or (b).

4. ☐ A meaningful search could not be carried out without the tables related to the sequence listings; the applicant did not, within the prescribed time limit, furnish such tables in electronic form complying with the technical requirements provided for in Annex C-bis of the Administrative Instructions, and such tables were not available to the International Searching Authority in a form and manner acceptable to it.

5. Further comments:

Name and mailing address of the International Searching Authority



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NL-2280 HV Rijswijk
Tel. (+31-70) 340-2040, Tx. 31 651 epo nl,
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Authorized officer

Javor Ribarov

FURTHER INFORMATION CONTINUED FROM PCT/ISA/ 203

A meaningful search is not possible on the basis of all claims because all claims are directed to - Scheme, rules and method for performing mental acts - Rule 39.1(iii) PCT

The applicant's attention is drawn to the fact that claims relating to inventions in respect of which no international search report has been established need not be the subject of an international preliminary examination (Rule 66.1(e) PCT). The applicant is advised that the EPO policy when acting as an International Preliminary Examining Authority is normally not to carry out a preliminary examination on matter which has not been searched. This is the case irrespective of whether or not the claims are amended following receipt of the search report or during any Chapter II procedure. If the application proceeds into the regional phase before the EPO, the applicant is reminded that a search may be carried out during examination before the EPO (see EPO Guideline C-VI, 8.2), should the problems which led to the Article 17(2)PCT declaration be overcome.