United States Patent [19]

Brems

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[45] **Jan. 14, 1975**

[54] **CYCLICLY REPETITIVE MOTION GENERATING SYSTEM**

[76] Inventor: **John Henry Brems,** 32867 White Oaks Trl., Birmingham, Mich. 48010

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[21] Appl. No.: **412,469**

[52] **U.S. Cl.** **74/82,** 74/89.21, 74/25

[51] **Int. Cl.** **F16h 27/04**

[58] **Field of Search** 74/141.5, 242.1, 42, 112, 74/25, 89.2, 89.22, 96, 82

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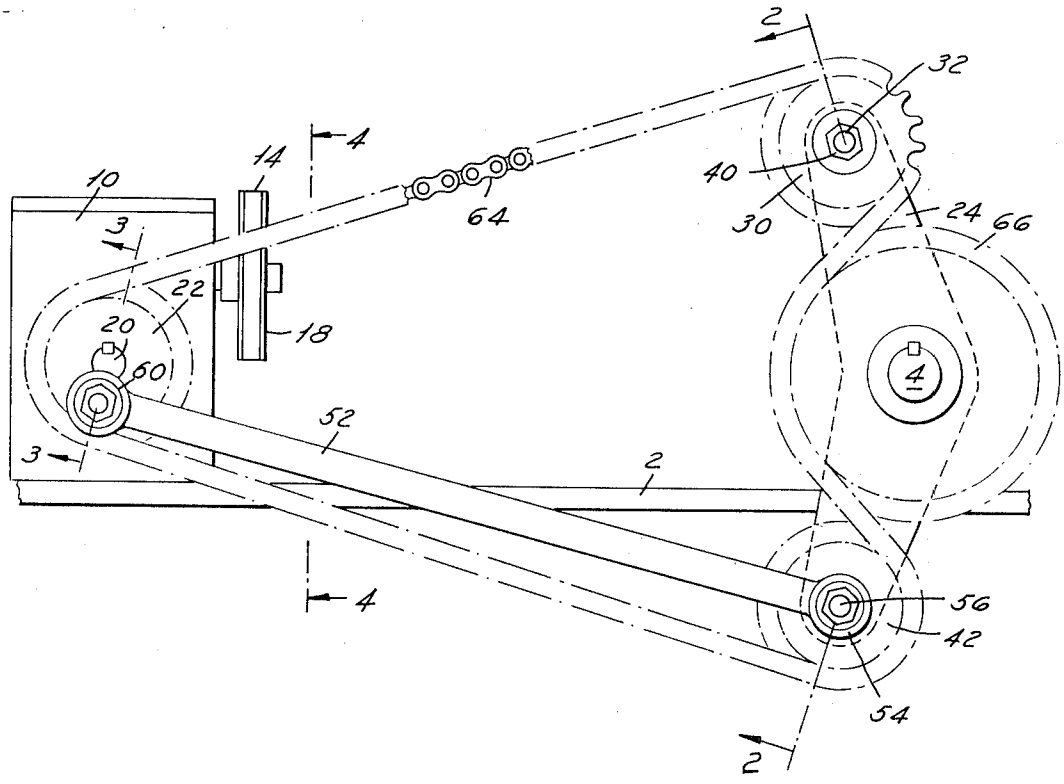
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Primary Examiner—Wesley S. Ratliff, Jr.
Attorney, Agent, or Firm—Barnes, Kisselle, Raisch & Choate

[57] **ABSTRACT**

A cyclicly repetitive motion generating system utilizing first and second drive members mounted respectively on first and second spaced axes and including an oscillating member pivotally mounted about the axis of one of said drive members carrying multiple idler means with a continuous loop of flexible drive means in tangential driving engagement with the first and second drive members and in engagement with said idler means, and a link connecting one of said drive members and said oscillating member wherein rotation of the drive members will cause an oscillation of the oscillating member about its axis. The result is a mechanism capable of generating repetitive index cycles consisting of a short dwell, a smooth acceleration to a maximum velocity and a smooth deceleration to a following dwell.

20 Claims, 23 Drawing Figures

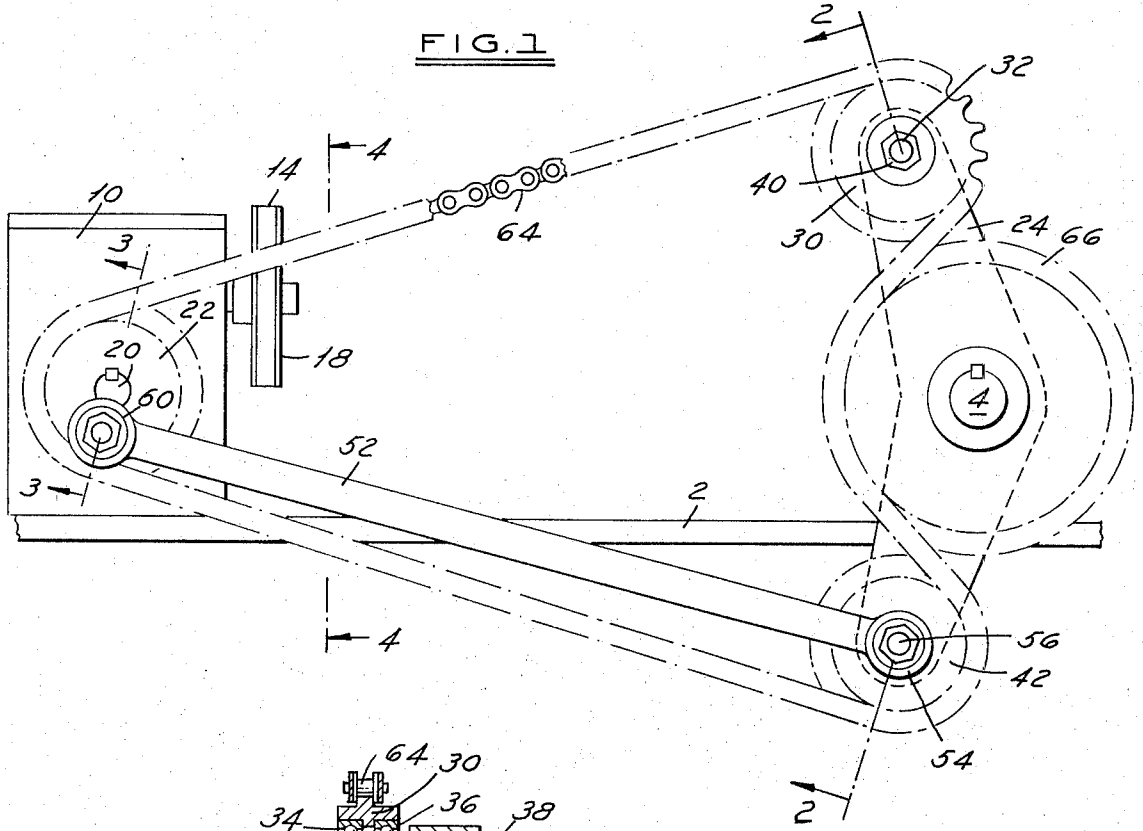


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SHEET 1 OF 4

FIG. 1



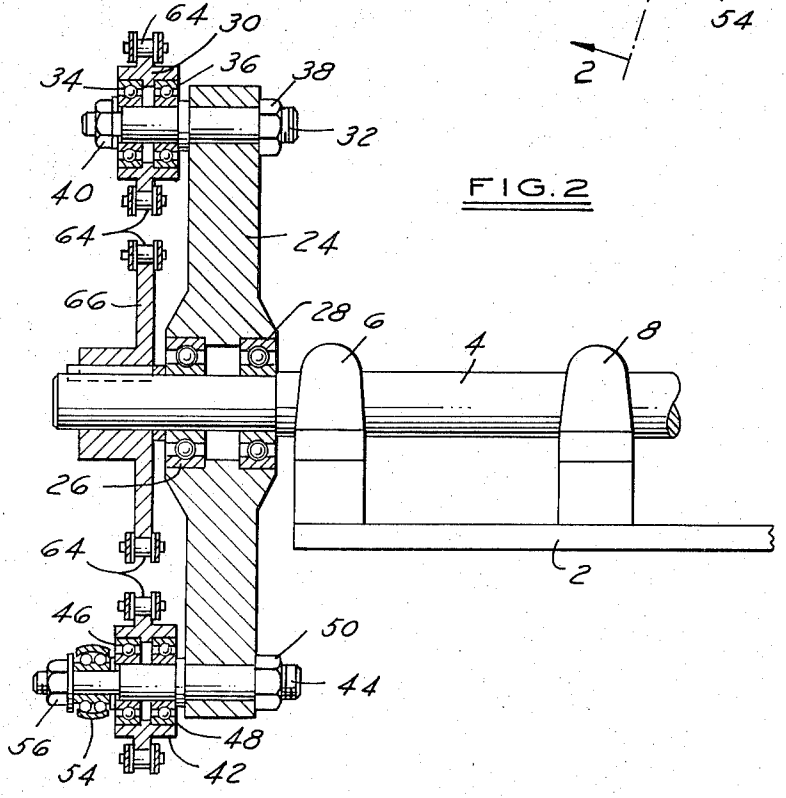


FIG. 2

FIG. 3

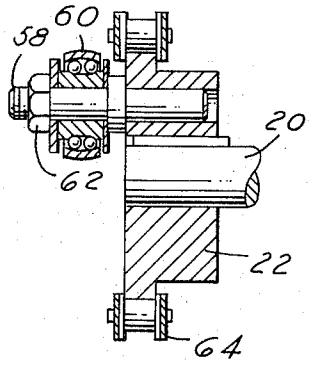


FIG. 4

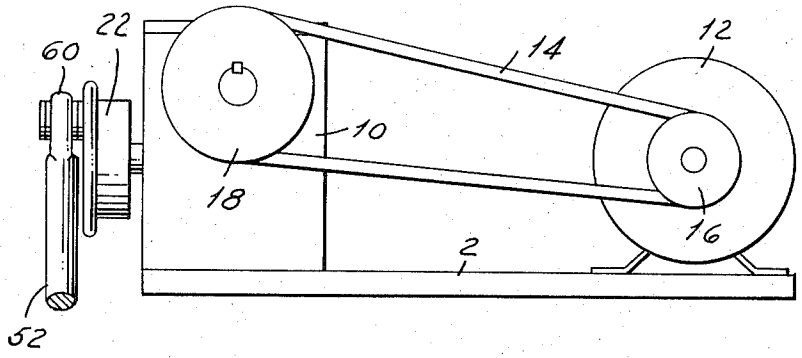


FIG. 5

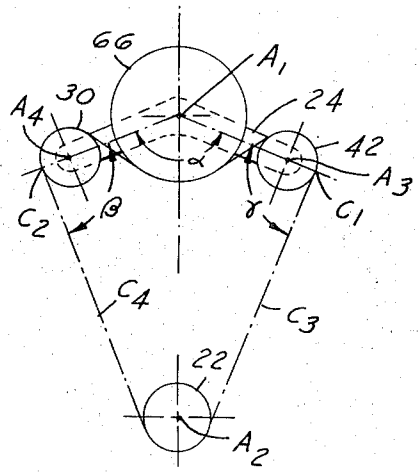


FIG. 6

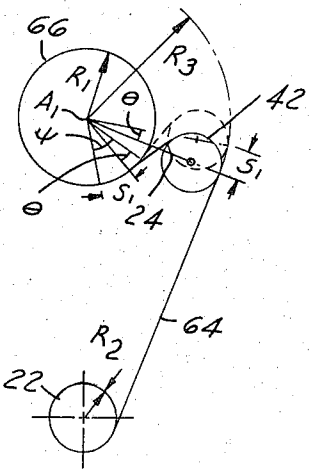


FIG. 7

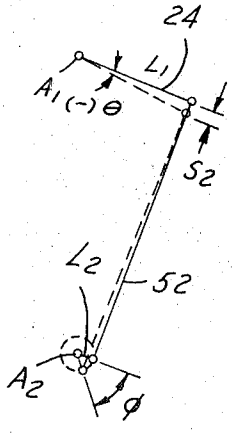


FIG. 8

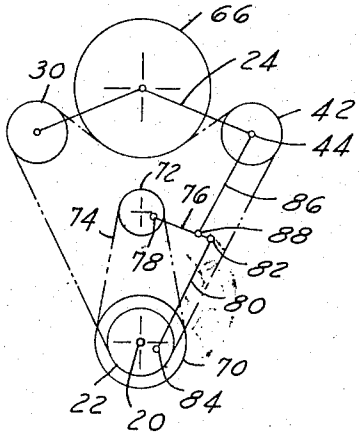


FIG. 12

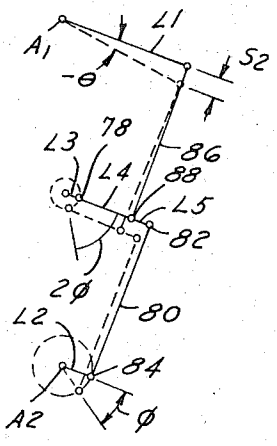
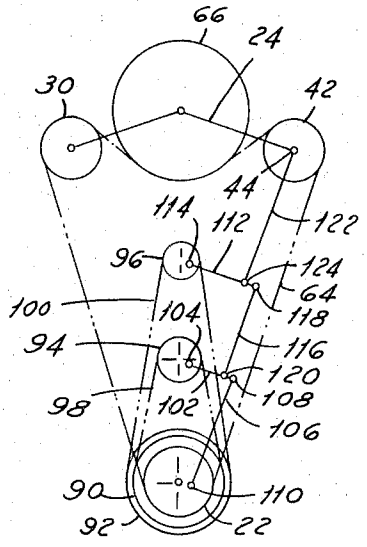


FIG. 16



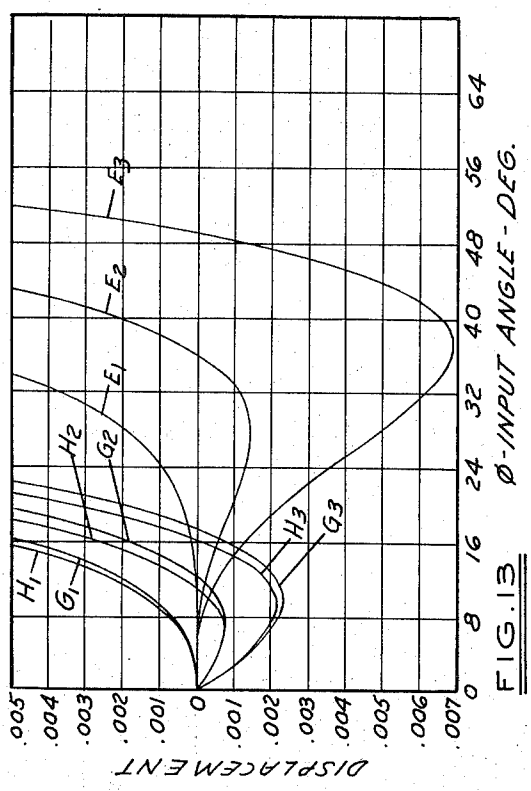


FIG. 13

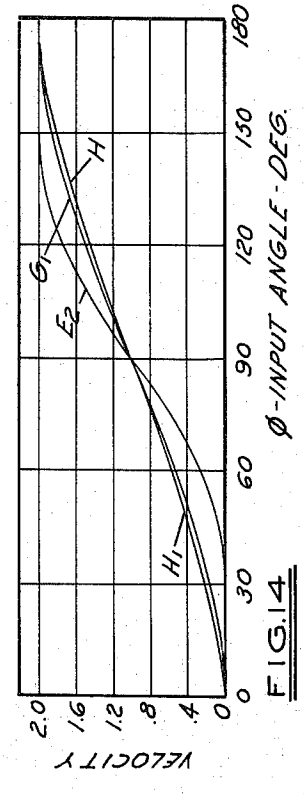


FIG. 14

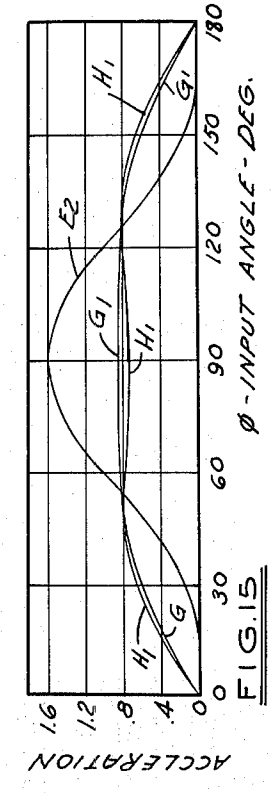


FIG. 15

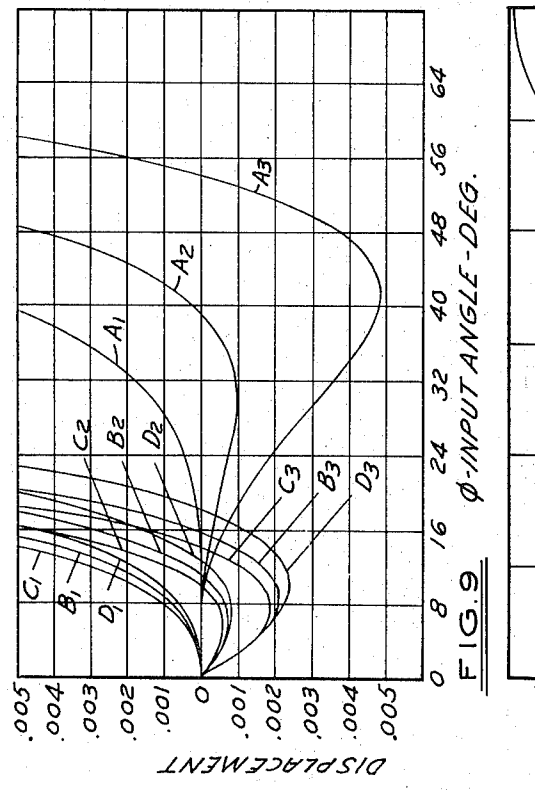


FIG. 9

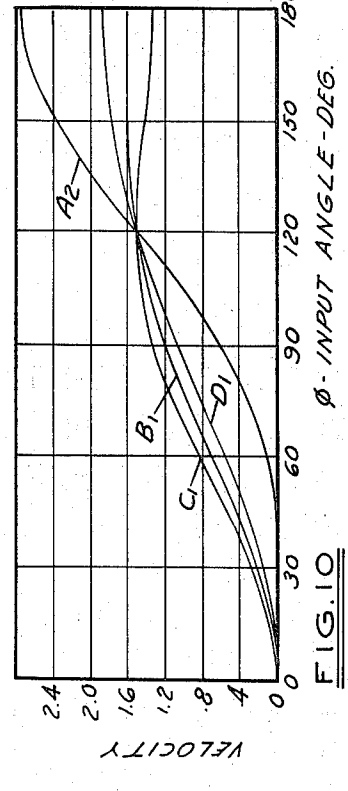


FIG. 10

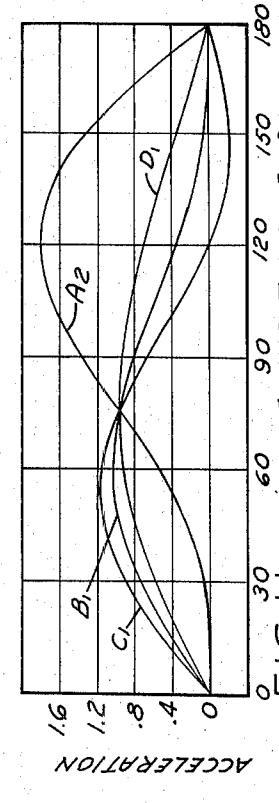


FIG. 11

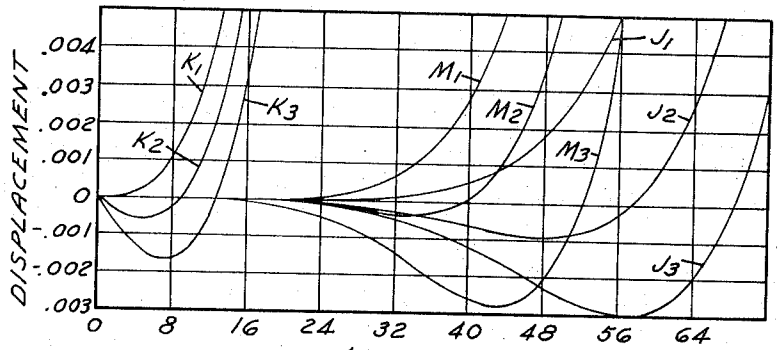


FIG. 17

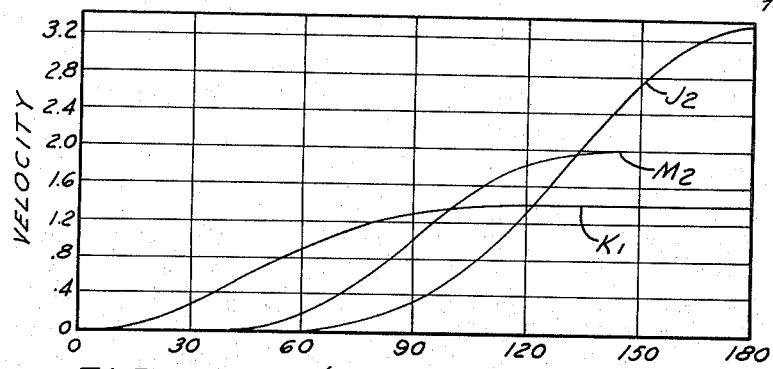


FIG. 18

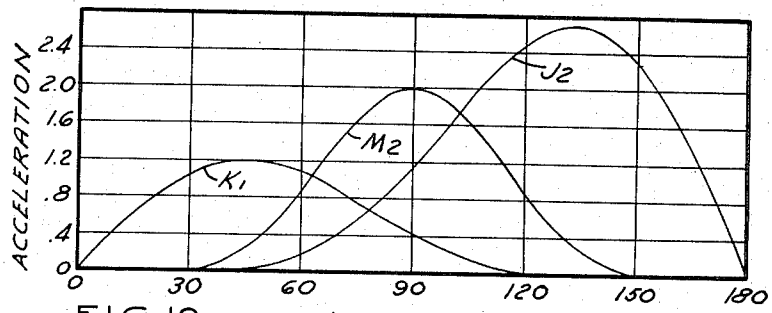


FIG. 19

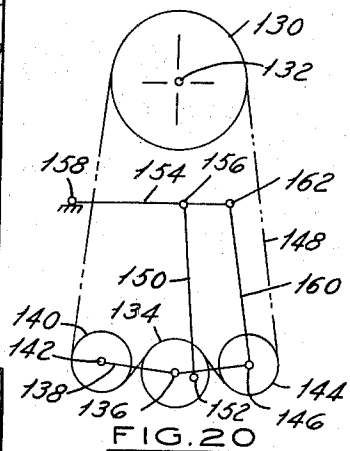


FIG. 20

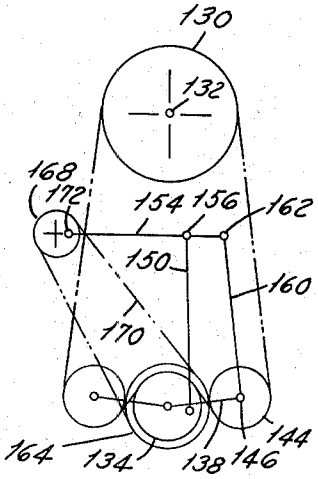


FIG. 21

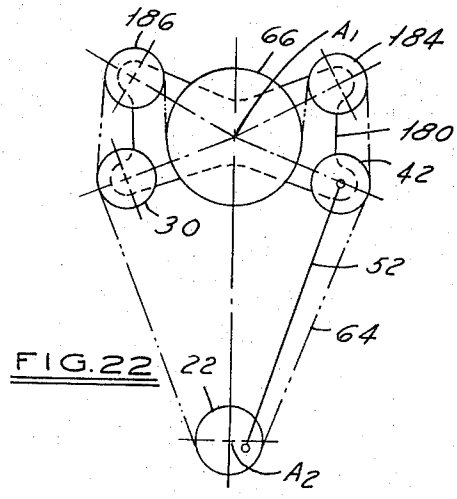


FIG. 22

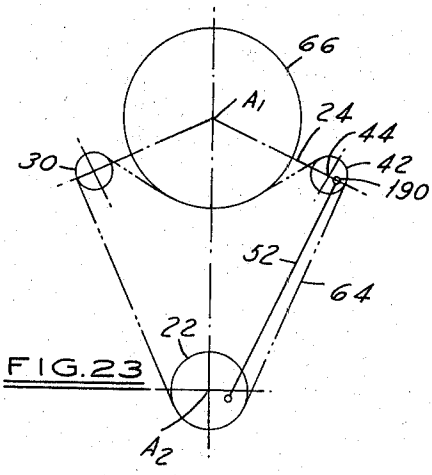


FIG. 23

CYCLICLY REPETITIVE MOTION GENERATING SYSTEM

This invention relates to a cyclicly repetitve motion generating system.

Various motion devices which utilize complicated cam grooves and followers can accomplish predetermined acceleration, deceleration and dwell and even reversing characteristics; but the cam configurations are very expensive to build and they admit of no flexibility or adjustment short of changing or substituting any particular cam section.

It is an object of the present invention to provide a motion generating system that can be constructed of relatively simple parts in the form of gears, levers, and chains which can generate repetitive cycles which can rather easily be altered in any portion without expensive machining.

Another object is to provide such a mechanism in which second and third harmonics may be readily added to increase the versatility of the mechanism.

It is a further object of this invention to provide a mechanism for superimposing a reversing motion on a substantially constant motion.

It is another object of this invention to provide a mechanism capable of generating repetitive index cycles consisting of a short dwell, smooth acceleration to a maximum velocity and smooth deceleration to a following short dwell.

A still further object of this invention is to provide a mechanism capable of generating a wide variety of repetitive motion patterns or profiles.

Other objects and features of this invention relating to the details of construction and operation will be apparent in the following description and claims in which the principles of operation are set forth together with the best modes presently contemplated for the practice of the invention.

Drawings accompany this disclosure and the various views thereof are briefly described as:

FIG. **1**, a side view of a mechanism for practicing the invention.

FIG. **2**, an end view of the mechanism illustrated in FIG. **1** taken on line **1—1**.

FIG. **3**, a view taken on line **3—3** of FIG. **1**.

FIG. **4**, a view of the power input taken on line **4—4** of FIG. **1**.

FIGS. **5** to **7**, diagrammatic views of the mechanism of FIG. **1** illustrating the motion.

FIG. **8**, a modified mechanism with the addition of a second harmonic.

FIGS. **9, 10, 11** illustrate in graph form, respectively, the relative displacement, velocity and acceleration which can be obtained by the previously illustrated mechanisms.

FIG. **12**, a schematic drawing of the mechanism illustrated in FIG. **8**.

FIGS. **13, 14** and **15** illustrate, respectively, in graph form the displacement, velocity, and acceleration utilizing a third harmonic.

FIG. **16**, a modified mechanism illustrating the addition of two distinct and higher harmonics.

FIGS. **17, 18, 19** illustrate, respectively, in graph form, the displacement, velocity and acceleration of a mechanism utilizing multiple harmonics.

FIG. **20** illustrates an embodiment in which the idler arm pivots about the input axis.

FIG. **21** illustrates a mechanism similar to that shown in FIG. **20** with a single higher harmonic added.

FIG. **22**, a mechanism for increasing the angle of chain wrap for high torque applications.

FIG. **23**, a modified arrangement for introducing a second or third harmonic into the output motion.

Referring to the drawings FIGS. **1, 2, 3** and **4**, a frame **2** supports an output shaft **4** through bearings **6** and **8**. It will be understood that this output shaft in turn drives any one of a variety of mechanisms, such as an indexing elevator, an indexing conveyor or other type of mechanism requiring an intermittent indexing rotary drive.

The frame **2** also supports a gear reducer **10** (FIG. **4**) driven by a suitable motor **12** through belt **14** and pulleys **16** and **18**. The output shaft **20** of the gear reducer **10** acts as the input shaft of this mechanism. It will be understood that the shaft **20** may be driven by any one of many other types of input systems, such as a direct drive from high torque prime movers such as air or hydraulic motors, or from some related and interconnecting mechanism. The shaft **20** has mounted on it an input sprocket **22**, which is the input member of the mechanism.

An idler arm **24** is mounted to the output shaft **4** through bearings **26** and **28**. An idler sprocket **30** is mounted to one end of the idler arm **24** through shaft **32**, bearings **34** and **36** and retainer nuts **38** and **40**. A second idler sprocket **42** is mounted to the other end of the idler arm **24** through shaft **44**, bearings **46** and **48** and retainer **50**.

An oscillating link **52** is connected at one end to the idler arm **24** through shaft **44**, bearing **54** and retainer **56**. At its other end the link **52** is driven by an eccentric shaft **58**, bearing **60** and retainer **62** (FIG. **3**) mounted on the input sprocket **22**. The shaft **58** revolves on an axis which is eccentric to the axis of the shaft **20**.

A flexible drive chain **64** interconnects the input sprocket **22** and an output sprocket **66** mounted on the output shaft **4**. The chain passes over the idler sprockets **30** and **42** in completing its driving loop.

If it is hypothetically assumed that the idler arm **24** is maintained in a stationary position (as by disconnecting the link **52** from the shaft **54**), it can be seen that the input sprocket **22** will drive the output sprocket **66** through a ratio determined by the numbers of teeth on the respective sprockets, and, further, that the speed ratio will be constant.

If it is assumed that the input sprocket **22** is stationary, while the idler arm **24** is moved through some small angle, the output sprocket **66** will rotate through some proportional angle.

Therefore, it can qualitatively be seen that if the idler arm **24** is connected to the eccentric shaft **58** on the input sprocket **22** through the link **52**, that the motion of the output sprocket **66** is the superposition or summation of motions caused by the rotation of the input sprocket and the oscillation of the idler arm. Indeed, if the proportions of the system are properly chosen, the output sprocket can be made to stop momentarily or to "dwell" once during each complete rotation of the input sprocket.

To construct a viable system based on this concept, it is necessary that the chain length remain substantially unaltered over the useful range of angular positions of the idler arm. How this is accomplished is represented schematically in FIG. **5**. The output sprocket **66** rotates about an axis A_1 and the input sprocket **22** rotates

about an axis A_2; the idler arm **24** also rotates about axis A_1; and the idler sprockets **30** and **42** rotate about axes A_3 and A_4 on the idler arm **24.** The line of centers from A_1 to A_3 is defined as C_1 and the line of centers of A_1 to A_4 is defined as C_2. If the angle α, between C_1 and C_2 is chosen such that, at the approximate center of the oscillation of idler arm **24,** the angle γ between C_1 and the chain centerline C_3 is 90°, and the angle β between C_2 and the chain centerline C_4 is also 90°, then the total developed chain length will remain substantially constant even through the idler arm **24** is rocked about an angle of ± 20° about axis A_1. The formula for the total developed chain length may be developed using straightforward trigonometric relationships but becomes extremely long and cumbersome. It has been evaluated with the aid of a computer and the total developed length of the chain found to vary less than 1 part in 10,000 for a total oscillation angle of ± 20°. This is negligibly small for all practical purposes. It should be noted that the pitch diameters of the idler sprockets **30** and **42** need not be identical, nor is it necessary that the distance from A_1 to A_3 be the same as the distance from A_1 to A_4.

The amplitude and character of oscillation of the idler arm **24** determines the dynamic characteristics of the output sprocket **66.** The quantitative analysis refers to FIGS. **6** and **7** which are the schematic kinematic drawings of the system. The variables are defined as follows:

θ = rotation of idler arm **24** about axis A_1
Ψ_1 = rotation of output sprocket **66** about axis A_1 caused by rotation of idler arm **24**
Ψ_2 = rotation of output sprocket **66** about axis A_1 caused by rotation of input sprocket **22**
Ψ = total rotation of output sprocket **66**
ϕ = rotation of input sprocket **22**
R_1 = pitch radius of output sprocket
R_2 = pitch radius of input sprocket
R_3 = pitch radius of the arc which is the envelope of the pitch circle of the idler sprocket during the oscillation of the idler arm **24**
L_1 = distance from axis A_1 to pivot connection of link **52** on arm **24**
L_2 = eccentric distance from axis A_2 to pivot connection of link **52** on input sprocket **22**

Referring to FIG. **6** which is a schematic drawing showing the rotation of the output sprocket **66** due to a rotation of the idler arm **24,** assuming the input sprocket **22** is stationary, it can be seen that the total rotation of the output sprocket **66** is caused by two effects. If the idler arm **24** rotated through an angle θ, the output sprocket rotates through that angle θ also, assuming there is no relative movement of the chain between idler sprocket **42** and output sprocket **66.** However, there is a movement of the chain shown as S_1 which in radian measure is

$$S_1 = R_3\,\theta$$

The chain length S_1 subtends an angle on output sprocket **66** equal to S_1/R_1

Therefore, the total angle of rotation of the output sprocket **66** is

$$\Psi_1 = \theta + (S_1/R_1)$$
$$\Psi_1 = \theta + (R_3/R_1)\,\theta \qquad (1)$$

where θ is the rotation of the idler arm **24.**

If we now consider the idler arm **24** stationary, it is easily seen that the rotation of the output sprocket **66** due to a rotation of the input sprocket **22** is

$$\Psi_2 = (R_2/R_1)\,\theta \qquad (2)$$

Therefore, the total rotation of the output sprocket **66** is

$$\Psi = \Psi_1 + \Psi_2$$
$$\Psi = [(R_3 + R_1)/R_1]\,\theta + (R_2/R_1)\,\theta \qquad (3)$$

FIG. **7** is a schematic drawing relating the rotation, θ, of the idler arm **24** due to a rotation θ of the input sprocket **22.** The chain and sprocket pitch diameters are omitted for clarity.

It can be seen that if the input sprocket **22** is rotated through an angle ϕ about axis A_2, the link **52** is moved through a distance S_2 where

$$S_2 \cong L_2 \sin\phi$$

In radian measure

$$-\theta = S_2/L_1$$

Therefore,

$$\theta \cong -(L_2 \sin\phi/L_1) \qquad (4)$$

Substituting equation (4) in the general relationship established by equation (3), we obtain

$$\Psi \cong (R_2/R_1)\,\phi - (L_2/L_1)\,(R_3 + R_1/R_1)\sin\phi$$
$$\Psi \cong (R_2/R_1)\,\phi\,(L_2/L_1)\,(R_2/R_1)\,(R_3 + R_1/R_2)\sin\phi$$
$$\Psi \cong (R_2/R_1)\,[\phi - (L_2/L_1)\,(R_3 + R_1/R_2)\sin\phi]$$

If it is desired to obtain a cycloidal output motion characteristic in which the output sprocket **66** has a momentary dwell once for each revolution of the input sprocket **22,** the coefficient of the sin ϕ term must be made equal to 1, i.e.,

$$(L_2/L_1)\,(R_3 + R_1/R_2) = 1$$

OR

$$L_2 = (L_1\,R_2)/(R_3 + R_1) \qquad (5)$$

Then equation (4) becomes

$$\Psi = (R_2/R_1)\,(\phi - \sin\phi) \qquad (6)$$

which is the classical equation for cycloidal motion.

The relationship between the idler arm angle θ and the input angle ϕ need not be the simple harmonic established by the crank L_2 as represented by equation (4). It is possible to add a second harmonic to the output motion using the technique shown schematically in FIG. **8.**

Referring to FIG. **8,** the input sprocket **22,** output sprocket **66,** idler arm **24,** idler sprockets **30** and **42,** chain **64** and all their associated bearings, shafts and retainers remain the same as shown in FIGS. **1, 2, 3** and **4.** However, the technique for oscillating the idler arm

24 is modified through the addition of a second harmonic or Fourier component.

A secondary drive sprocket **70** is mounted parallel to and concentric with the input sprocket **22** on the gear reducer output shaft **20** and both sprockets rotate with the shaft. An intermediate sprocket **72** is mounted on a suitable shaft and bearings from the frame **2**; this intermediate sprocket **72** is driven by sprocket **70** through chain **74.** If there are twice as many teeth on sprocket **70** as on sprocket **72,** then sprocket **72** will rotate at twice the angular velocity of sprocket **70.** An idler link **76** is pivotally connected to the sprocket **72** through an eccentric pivot **78.** The other end of this link **76** is pivotally connected to a primary link **80** through a shaft **82**; and the other end of the link **80** is driven through an eccentric shaft **84** on the input sprocket **22.**

It can be seen, therefore, that the point **82** on the link **76** makes one vertical oscillation for each revolution of the sprocket **22,** and that the point **78** on the link **76** makes two vertical oscillations for each revolution of the sprocket **22.**

A drive link **86** is connected at one end to the idler arm **24** through the shaft **44** and at its other end to the link **76** through a pivot shaft **88.** It can be seen, therefore, that the idler arm **24** will make one complete oscillation for each rotation of the input sprocket **22,** to which is added a component of second harmonic whose magnitude is dependent upon the ratio of the distances along link **76** between points **78, 88** and **82,** and the relative eccentricities of shafts **78** and **84** on their respective sprockets.

The effect of the addition of this second harmonic component may best be illustrated through several examples. In the general case, through an extrapolation of formula 6, it may be hypothesized that the equation of motion for the output movement would become:

$$\Psi\ (R_1/R_2) = \phi + F_1 \sin \phi + F_2 \sin 2\ \phi$$

Note: If $F_1 = -1$ and $F_2 = 0$, we obtain equation (6).

For simplicity, $\Psi\ (R_1/R_2)$ is defined as relative output motion, U, and therefore:

$$U = \phi + F_1 \operatorname{Sin} \phi + F_2 \sin 2\ \phi \qquad (7)$$

By successive differentiation

$$dU/d\phi = 1 + F_1 \operatorname{Cos} \phi + 2\ F_2 \operatorname{Cos} 2\ \phi \ \text{(Velocity)} \qquad (8)$$

$$d^2U/d\phi^2 = -F_1 \operatorname{Sin} \phi - 4\ F_2 \sin 2\ \phi \ \text{(Acceleration)} \qquad (9)$$

$$d^3U/d\phi^3 = -F_1 \operatorname{Cos} \phi - 8\ F_2 \cos 2\ \phi \qquad (10)$$

$$d^4U/d\phi^4 = F \operatorname{Sin} \phi + 16\ F_2 \operatorname{Sin} 2\ \phi \qquad (11)$$

These relationships may now be utilized to establish the values of F_1 and F_2 that best meet the design requirements, and, subsequently, these values of F_1 and F_2 will be converted into usable geometric parameters.

EXAMPLE I

It is desired to design the mechanism to achieve a long dwell. Mathematical "dwell" consists of a single or, at best, a group of points; practical dwell, considering lost motions, bearing clearances, and the demands of the application may be a band whose width will vary from application to application. In a typical case, the dwell may be considered that portion of the cycle in which the output motion does not exceed one one-thousandth of the total output movement of an index cycle.

With this in mind, we examine the behavior of equation (7), (8), (9), (10), and (11). It will be noted that when $\phi = 0$, then

$$U = d^2U/d\phi^2 = d^4U/d\phi^4 = 0$$

for all values of F_1 and F_2.

To achieve the maximum dwell, it is clear that the velocity, $dU/d\phi$, should be set equal to 0; therefore, from equation (8)

$$1 + F_1 + 2\ F_2 = 0 \quad @\ \phi = 0 \qquad (12)$$

Noting that $d^3U/d\phi^3$ is the slope of the acceleration, we knowledgeably, but arbitrarily, may assign a value to equation (10) equal to $-M$ which indicates the slope of the acceleration is made slightly negative at $\phi = 0$. Therefore, the acceleration "starts out" from $\phi = 0$ and becomes slightly negative before starting its positive rise.

From equation (10), we obtain

$$-F_1 - 8\ F_2 = -M \qquad (13)$$

Solving equations (12) and (13) for the values of F_1 and F_2, the following are obtained:

$$F_1 = -1 - \tfrac{1}{3}\ (M + 1)$$

$$F_2 = (M + 1)/6$$

Then, for several values of M, the final values of F_1 and F_2 are as follows:

M	F_1	F_2	Curve in FIG. 9, 10, 11
0	−1.3333	.16667	A_1
.1	−1.3667	.18333	A_2
.2	−1.400	.20000	A_3

When these values of F_1 and F_2 are utilized in equation (7), and the values of U calculated for various values of ϕ, the curves A_1, A_2 and A_3 in FIG. **9** are obtained. It will be noted that the displacement axis is generally magnified and further that the output displacement is relative to a total cyclic displacement of 2π. Considering curve A_2, it will be noted that the output movement is negative for the first 31° of input movement, reaching a value of −0.001 at that point, and then not reaching +0.001 until 42° of input rotation.

Since these curves are symmetrical about the origin, it may be seen that the output "dwells" for 84° (2 × 42°) of input rotation if the dwell is defined as a band whose width is allowed to be $\pm 0.001 \times 1/2\pi \times$ output angle per index.

It will be further noted that if the definition of dwell width is widened and M made appropriately larger, the dwell is lengthened in terms of input angle. From curve A_3 where $M = 0.2$, the total negative movement is −0.0048 at 42° and crosses +0.0048 at 58°, and the total dwell becomes 116° of input movement if the dwell is a band of output movement equal to $\pm 0.0048 \times 1/2\pi \times$ output angle per index.

The corresponding velocity and acceleration for an entire half cycle are shown by curve A_2 in FIGS. **10** and **11**.

It will be understood that the velocity curves are symmetrical about the line where $\phi = 180°$ and that the acceleration curves are symmetrical about the point $U = 0, \phi = 180°$, i.e., the velocity curves remain positive between $\phi = 180°$ and 360°, while the acceleration becomes negative or becomes a deceleration.

The velocity and accceleration scales are relative to an index of 2π output units per unit of time.

It will be noted that only curve A_2 is presented in FIGS. **10** and **11** since curves A_1 and A_3 would be extremely close to it on the scale used.

EXAMPLE II

If it should be desired to make the output velocity as flat as possible near the center of the index stroke, while still retaining a dwell of some reasonable value, a different set of conditions must be placed on equations (7), (8), (9), (10) and (11), as follows:

In order to achieve a dwell at $\phi = 0$, $dU/d\phi = 0$ or to "widen" that dwell $dU/d\phi = -V$. Therefore,

$$1 + F_1 + F_2 = -V$$

Defining

$$V + 1 = K$$

$$F_1 + 2F_2 = -K \qquad (14)$$

In order to achieve a flat velocity at $\phi = 180°$, $d^2U/d\phi^2 = 0$, however $d^2U/d\phi^2 = 0$ at 180° for all values of F_1 and F_2. Therefore, the next condition may be employed, which is:

$$d^3U/d\phi\phi^3 = 0 \quad @ \quad \phi = 180°$$

Substituting in equation (10), the following is obtained.

$$F_1 - 8F_2 = 0 \qquad (15)$$

Solving equations (14) and (15):

$$F_1 = -0.8K$$

$$F_2 = -0.1K$$

Then for several values of K the final values of F_1 and F_2 are as follows:

K	F_1	F_2	Curve in FIGS. 9, 10, 11
1.00	−.8	−.1	B_1
1.01	−.808	−.101	B_2
1.02	−.816	−.102	B_3

When these values of F_1 and F_2 are utilized in equation (7), and the values of U calculated for various values of ϕ, the curves B_1, B_2, and B_3 in FIG. **9** are obtained. Here again, it is obvious that as the permissable dwell band width is increased, the total length of dwell in terms of input angles is increased.

Curve B_1 in FIG. **10** represents the relative velocity of the output for a standard output index of 2π, obtained by substituting the value of $F_1 = -0.8$ and $F_2 = -0.1$ in equation (8). The curves B_2 and B_3 are omitted in both FIGS. **9** and **10** because of their near coincidence with curve B_1. It should be noted that the velocity reaches a maximum of 1.6 and is nearly constant over a 70° span of input movement from $\phi = 145°$ to $\mathbf{100} = 215°$.

Curve B_1 in FIG. **11** represents the relative acceleration for these values of F_1 and F_2 when inserted in equation (10).

EXAMPLE III

If it is desired that the maximum relative velocity, wherever it should be reached during the index cycle, be as low as possible, a more subtle technique is required. It will also still be desired to achieve the maximum practical dwell. Therefore, as before from equation (8):

$$(14) \quad F_1 + 2F_2 = -K$$

Then set

$F_1 = N F_2$ where N is an unknown parameter, whereupon:

$$F_1 = -NK/(N+2) \qquad (16)$$

$$F_2 = -K/(N+2) \qquad (17)$$

The point of maximum velocity, $dU/d\phi$ wherever it is reached in the cycle occurs at a point where $d^2U/d\phi^2 = 0$. Substituting equations (16) and and (17) into equation (N) and setting it equal to 0.

$$NK/(N+2)\ \text{Sin}\ \phi + 4K/(N+2)\ \text{Sin}\ 2\phi = 0 \qquad (18)$$

which may be solved to yield

$$\text{Cos}\ \phi = -N/8 \qquad (19)$$

This establishes a relationship between N and ϕ at any point where the velocity is at an intermediate peak. Equation (18) may then be substituted back into equation (8) to yield a value for maximum velocity at such a point in terms of N only, which is:

$dU/d\phi = 1 - [NK/(N+2)]\ (-N/8) - [2K/(N+2)]\ (2\ (N^2/64) - 1)$

which reduces to

$$dU/d\phi = 1 + K\ [(N^2 + 32)/16\ (N+2)] \qquad (20)$$

To find that specific point whwere $dU/d\phi$ is a minimum with respect to N, equation (20) is differentiated with respect to N and the result set equal to 0, which when solved, yields

$$N = 4$$

whereupon

$$F_1 = -0.66667K$$

$$F_2 = -0.16667K$$

Then for several values of K, the final values of F_1 and F_2 are as follows:

K	F_1	F_2	Curves in FIGS. 9, 10, 11
1.00	−.66667	−.166667	C_1
1.01	−.67333	−.16833	C_2
1.02	−.68	−.17	C_3

When these values are utilized in equation (7), and the values of U calculated for various values of ϕ, the curves C_1, C_2, and C_3 are obtained. It can be seen that for identical values of K, the dwell characteristics are slightly inferior to the B curves for flatest midstroke velocity.

Curve C_1 (curves C_2 and C_3 are again omitted because of near coincidence) in FIGS. **10** and **11** show the behavior of the relative velocity and acceleration throughout the cycle.

It will be noted that the peak relative velocity reaches a maximum value of only 1.5 at 120° then drops to an intermediate minimum of 1.33 at 180°, as compared to a constant 1.6 for curve B_1.

EXAMPLE IV

In this case, the objective is to find those values of F_1 and F_2 which achieve the lowest peak acceleration wherever it may occur during a cycle while still maintaining a maximum practical dwell. Therefore as before from equation (8)

$$(14) \quad F_1 + 2F_2 = -K$$

And again setting

$F_1 = N F_2$ (N is now a new unknown)

Then

$$(16) \quad F_1 = -NK/(N+2)$$

$$(17) \quad F_2 = -K/(N+2)$$

The point of maximum acceleration, $d^2U/d\phi^2$, wherever it occurs in the cycle must occur where $d^3U/d\phi^3 = 0$. Therefore, equations (16) and (17) are substituted into equation (10) which is then set equal to 0.

$$NK/(N+2)\ \text{Cos}\ \phi + 8K/N+2\ \text{Cos}\ 2\phi = 0 \quad (21)$$

which reduces to:

$$N = (8 - 16\ \text{Cos}^2\phi)/\text{Cos}\ \phi \quad (22)$$

This establishes a relationship between N and ϕ at any point where the acceleration is at an intermediate peak. Equations (22), (16) and (17) may then be substituted into equation (9) to yield a value for the maximum velocity at such a point in terms of ϕ only which reduces to:

$$d^2V/d\phi^2 = K\ [Sin^3\ \phi/(1 - 2\ \text{Cos}^2\ \phi + \text{Cos}\phi/4)] \quad (23)$$

This may now be differentiated with respect to ϕ and the result set equal to 0, which in turn reduces to:

$$-2\ \text{Cos}^3\ \phi + \tfrac{1}{2}\ \text{Cos}^2\ \phi - \text{Cos}\ \phi + \tfrac{1}{2} = 0$$

The only real root of this equation is:

$$\text{Cos}\ \phi = \tfrac{1}{4} \quad (24)$$

When this solution is substituted back into equation (22)

$$N = 28$$

And, therefore, from (16) and (17)

$$F_1 = -14/15\ K$$

$$F_2 = -K/30$$

Then for several values of K the final values of F_1 and F_2 are as follows:

K	F_1	F_2	Curves in FIGS. 9, 10, 11
1.00	−.93333	−.0333	D_1
1.01	−.94267	−.0336	D_2
1.02	−.952	−.034	D_3

As before, these values of F_1 and F_2 are utilized in equations (7), (8) and (9) to calculate the curves D_1, D_2, and D_3 in FIGS. **9**, **10** and **11**. As expected, the value of the peak relative acceleration is lower than that found in the other curves satisfying other conditions.

The foregoing examples are illustrative only to exemplify several pure mathematical techniques for evaluating the coefficients F_1 and F_2 to obtain the desired output conditions. Other conditions may be imposed on the output characteristics and other techniques may be employed with equal validity.

It will now be shown how these pure coefficients F_1 and F_2 may be converted into useful dimensions for the additive system shown in FIG. **8**. FIG. **12** represents the schematic drawing of the linkage and eccentricities which create the harmonic superposition and establish the relationship between the rotation, θ, of the idler arm **24** due to the rotation ϕ of the input sprocket **22** and the rotation 2ϕ of the sprocket **94**. The chain and sprocket pitch diameters are omitted for clarity.

It can be seen that if the input sprocket **22** is rotated through an angle ϕ about axis A_2 the link **80** is moved through a distance L_2 Sin ϕ. If it is temporarily assumed that pivot **78** is stationary, the resultant movement of link **86** is

$$[L_4/(L_4 + L_5)]\ L_2\ \text{Sin}\ \phi.$$

Similarly, it is temporarily assumed that the pivot **82** is momentarily stationary while the sprocket **72** rotates through an angle 2ϕ about axis A_3 the resultant movement of link **86** is

$$[L_5/(L_4 + L_5)]\ L_3\ \text{Sin}\ 2\phi.$$

By superposition, therefore, the total movement of link **86**, as represented by the distance S_2 is

$$S_2 \cong [L_4/(L_4 + L_5)]\ L_2\ \text{Sin}\ \phi + [L_5/(L_4 + L_5)]\ L_3\ \text{Sin}\ 2\phi$$

This is an approximation which is nearly perfect if the links **80** and **86** are substantially perpendicular to the link **76**.

As Before:

$$(4) \quad -\theta = S_2/L_1$$

Therefore

$$\theta = -[L_4/(L_4 + L_5)\ (L_2/L_1)]\ \text{Sin}\ \phi - [L_5/(L_4 + L_5)\ (L_3/L_1)]\ \text{Sin}\ 2\phi \quad (25)$$

Substituting equation (25) in the general relationship established by equation (3) and simplifying, the following is obtained

$$\psi = \frac{R_2}{R_1}\left[\phi - \frac{R_3 + R_1}{R_2}\frac{L_2}{L_1}\frac{L_4}{L_4 + L_5}\ \text{Sin}\ \phi - \frac{R_3 + R_1}{R_2}\frac{L_3}{L_1}\frac{L_5}{L_4 + L_5}\ \text{Sin}\ 2\phi\right]$$

Therefore

$$F_1 = -[(R_3 + R_1)/R_2]\ (L_2/L_1)\ [L_4/(L_4 + L_5)] \quad (26)$$

$$F_2 = -[(R_3 + R_1)/R_2]\ (L_3/L_1)\ [L_5/(L_4 + L_5)] \quad (27)$$

In a practical application, R_1, R_2, R_3, L_1, L_4, and L_5 will generally be chosen for mechanical convenience; the eccentricities L_2, and L_3 may then be calculated from these restatements of (26) and (27).

$$L_2 = -L_1\ [R_2/(R_3 + R_1)]\ [(L_4 + L_5)/L_4]\ F_1 \quad (28)$$

$$L_3 = -L_1\ [R_2/(R_3 + R_1)]\ [(L_4 + L_5)/L_5]\ F_2 \quad (29)$$

In all four mathematical examples, the value for F_1 was always minus, making L_2 positive, so that the direction of L_2 from axis A_2 is as shown in FIG. **12.** In the case of L_3, if it is positive, its direction from A_3 is as shown in FIG. **12**; if it is negative, its direction is 180° opposite from its indicated position in FIG. **12.**

While not shown in the examples, a phase shift may be added to the second harmonic component to meet those situations where non symmetry of output movement is desired.

Referring again to FIG. **8,** if the number of teeth on the sprocket **70** is made three times as great as the numer of teeth on the sprocket **72,** the sprocket **72** will rotate at three times the angular velocity of sprocket **70.** The motion of the idler arm **24** is now a sum of the fundamental and some component of third harmonic whose magnitude is dependent upon the ratio of the distance along link **76** between points **78, 88** and **82** and the relative eccentricities of shafts **78** and **84** on their respective sprockets, as was the case with the second harmonic addition.

The effect of the addition of this third harmonic component may again best be illustrated through several examples. The equation for the output motion of the system may now be hypothesized as follows:

$$U = \phi + F_1 \text{ Sin } \phi + F_3 \text{ Sin } 3\ \phi \quad \text{(Displacement)} \quad (30)$$

By successive differentiation:

$$dU/d\phi = 1 + F_1 \text{ Cos } \phi + 3\ F_3 \text{ Cos } 3\ \phi \text{ (Velocity)} \quad (31)$$

$$d^2U/d\phi^2 = -F_1 \text{ Sin } \phi - 9\ F_3 \text{ Sin } 3\ \phi \text{ (Acceleration)} \quad (32)$$

$$d^3U/d\phi^3 = -F_1 \text{ Cos } \phi - 27\ F_3 \text{ Cos } 3\ \phi \quad (33)$$

$$d^4U/d\phi^4 = F_1 \text{ Sin } \phi + 81\ F_3 \text{ Sin } 3\ \phi \quad (34)$$

These relationships may be utilized to establish values of F_1 and F_3 that best suit specific design requirements. Several illustrative example cases will be outlined.

EXAMPLE V

It is desired to design the mechanism to achieve a long dwell using the third harmonic (rather than the second as in Example I).

By a process completely identical with that used in Example I, it is found that

$$F_1 = -1 - \tfrac{1}{8}\ (M+1)$$

$$F_2 = M + 1/24$$

Then for several values of M, the final values of F_1 and F_3 are as follows:

M	F_1	F_2	Curves in FIGS. 13, 14, 15
0	−1.125	.04167	E_2
.2	−1.150	.05	E_2
.4	−1.175	.05833	E_3

When these values are utilized in equation (30), and the value of U calculated for various values of ϕ, the curves E_1, E_2, and E_3 in FIG. **13** are obtained. It can be seen that the dwell characteristics so achieved are slightly inferior to those achieved with the second harmonic addition as shown by curves A_1, A_2 and A_3 in FIG. **9.**

Only the curve E_2 is plotted in FIG. **14** from equation (31) representing the relative velocity using these values of F_1 and F_3, because of near identity of curves E_1 and E_3; similarly, curve E_2 is plotted alone in FIG. **15,** from equation (32) representing the relative acceleration using these values of F_1 and F_2. A significant reduction in the values of peak velocity and peak acceleration will be noted in comparison to the curves A_2 in FIGS. **10** and **11** which represent the velocity and acceleration for maximum dwell conditions using the second harmonic addition. Therefore, using the third harmonic instead of the second to increase the dwell is a compromise that can be chosen only in the light of a specific application's requirements.

EXAMPLE VI

In this case, it is desired that acceleration be as flat as possible at an input angle of 90° and that the practical dwell again be as large as possible. It is obvious from the form of equation (32) that the relative acceleration must be symmetrical about the line $\phi = 90°$; it is further obvious that the slope of the acceleration, as expressed by equation (33), is always 0 at $\phi = 90°$ regardless of the values of F_1 and F_3. The curvature of the acceleration is proportional to $d^4U/d\phi^4$; therefore, by setting $d^4U/d\phi^4 = 0$ at $\phi = 90°$ the optimum flatness of the acceleration is achieved; therefore, from equation (32)

$$F_1 - 81\ F_3 = 0$$

By again setting the slope slightly negative at $\phi = 0$, from equation (31)

$$F_1 + 3\ F_3 = -K$$

Therefore

$$F_1 = -27/28\ K$$

$$F_3 = -1/84\ K$$

Then for several values of K, the final values of F_1 and F_3 are as follows:

K	F_1	F_3	Curves in FIGS. 13, 14, 15
1.00	−.96429	−.011905	G_1
1.01	−.97393	−.012024	G_2
1.02	−.98357	−.012143	G_3

When these values are utilized in equations (30), (31) and (32) and the output displacement, velocity, and acceleration calculated the curves G_1, G_2 and G_3 in FIGS. **13, 14** and **15** are obtained. It will be noted that in FIG. **15,** the acceleration curve G_1 is substantially

flat from $\phi = 60°$ to $\phi = 120°$ at a value of 0.85, meeting the set-up boundary condition.

EXAMPLE VII

In this case, it is desired that the peak acceleration be as low as possible wherever it occurs in a cycle and that the practical dwell again be as large as possible. This situation occurs when an intermediate peak is reached at some input angle less than 90°, and at some symmetrical angle beyond 90° (due to the aforementioned acceleration symmetry about 90°). It is, therefore, hypothesized that:

$$F_1 = N F_2 \text{ (where } N \text{ is again a new unknown)} \quad (35)$$

It is also known that wherever the acceleration is a maximum that $d^3U/d\phi^3 = 0$. Therefore, by substituting (35) into equation (33), setting equation (33) equal to 0 and clearing, the following is obtained:

$$\text{Sin}^2\,\theta = 27{=}N/108 \quad (36)$$

This now establishes a relationship between N and θ at any point of maximum acceleration. As before, to achieve a slight negative velocity at $\phi = 0$

$$F_1 + 3\,F_3 = -K$$

By combining with equation (35), the following are obtained:

$$F_1 = -NK/N{+}3 \quad (37)$$

$$F_3 = -K/N{+}3 \quad (38)$$

Equations (36), (37) and (38) are substituted into equation (32), to establish a relationship between N and K and $d^2U/d\phi^2$ at all points where $d^2U/d\phi^2$ is a maximum. In other words, this establishes $(d^2U/d\phi^2)_{max} = f(N,K)$. After clearing, this is found to be

$$(d^2U/d\phi^2)_{max} = \tfrac{2}{3}\,[K/(108)^{1/2}]\,[(N{+}27)^{3/2}/N{+}3] \quad (39)$$

In order to find at what specific value of N, $(dU/d\phi^2)_{max}$ has is least value, equation (39) is differentiated with respect to N, and the result set equal to 0. After clearing

$$N = 45$$

When this is substituted back into equations (37) and (38)

$$F_1 = -\,45/48\,K$$

$$F_3 = -\,1/48\,K$$

Then for several values of K, the final values of F_1 and F_3 are as follows:

K	F_1	F_2	Curves in FIGS. 13, 14, 15
1.00	−.9375	−.020833	H_1
1.01	−.9469	−.02104	H_2
1.02	−.95625	−.02125	H_3

When these values are utilized in equations (30), (31) and (32), and the output displacement, velocity, and acceleration calculated, the curves H_1, H_2 and H_3 are obtained. In FIG. **15** it will be noted that the acceleration curve H_1 reaches a peak of 0.81 at $\phi = 50°$ and $\phi = 130°$, which is some 5 percent less than the more flattened top of G_1.

Examples V, VI and VII are again intended as illustrative only. Whether the value of the factors F_1 and F_3 are obtained by these methods or by other techniques to meet other requirements, they may again be converted to usable gometric parameters for the linkage using the methods outlined above in connection with the first and second harmonic system.

While the mechanism illustrated in FIG. **8** is capable of superimposing either a second, third or any other single higher harmonic to the output, additional adder links may be added so that multiple higher harmonics may be incorporated into the output characteristics. The schematic mechanism for adding **2** distinct and separate higher harmonic is shown in FIG. **16.**

Referring to FIG. **16,** the input sprocket **22,** output sprocket **66,** idler arm **24,** idler sprockets **30** and **42,** chain **64** and all associated bearings, shafts and retainers again remain the same as shown in FIGS. **1, 2, 3** and **4,** except that the center distance from the output sprocket **66** to the input sprocket **22** may be increased slightly to provide space for the addition of multiple additional sprockets, and as will be shown, the technique for oscillating the idler arm **24** is again modified.

A secondary drive sprocket **90** and a tertiary drive sprocket **92** are mounted parallel to and concentric with the input sprocket **22** on and to rotate with the gear reducer output shaft **20.** A first intermediate sprocket **94** and a second intermediate sprocket **96** are each mounted to the frame **2** through suitable shafts and bearings. The sprocket **94** is driven by sprocket **90** through chain **98,** and sprocket **96** is driven by sprocket **92** through chain **100.** As shown in FIG. **16,** sprocket **96** rotates at three times the angular velocity of the input sprocket **22,** and sprocket **94** rotates at twice the angular velocity of the input sprocket **22.**

A first adder link **102** is pivotally connected to the sprocket **94** through an eccentric pivot **104.** The other end of the link **102** is pivotally connected to a primary link **106** through a shaft **108;** the other end of the link **106** is driven through an eccentric shaft **110** on the input sprocket **22.**

A second adder link **112** is pivotally connected to the sprocket **96** through an eccentric pivot **114.** The other end of the link **112** is pivotally connected to a secondary link **116** through a shaft **118;** the other end of the link **116** is driven through a pivot connection **120** on the link **102.**

A drive link **122** is connected at one end to the idler arm **24** through a shaft **44** and at its other end to the link **112** through a pivot **124.**

It can be seen, therefore, that for each revolution of the input sprocket **22,** the idler arm **24** will make one complete oscillation, which is the summation of a fundamental component dependent on the eccentricity of the pivot **110** on the input sprocket **22,** a second harmonic component dependent on the eccentricity of the pivot **104** on the sprocket **94** and the ratio of the distances between points **104, 120** and **108** on the link **102,** and a third harmonic component dependent on the eccentricity of the pivot **114** on the sprocket **96** and the ratio of the distances between points **114, 124** and **118** on the link **112.**

The effect of the addition of second and third harmonic may again best be illustrated through several examples. The equation of motion of the output movement may now be hypothesized as follows:

$$U = \phi + F_1 \text{ Sin } \phi + F_2 \text{ Sin } 2\phi + F_3 \text{ Sin } 3\phi \quad \text{(Displacement)} \tag{40}$$

By successive differentiation:

$$dU/d\phi = 1 + F_1 \text{ Cos } \phi + 2F_2 \text{ Cos } 2\phi + 3 \text{ Cos } 3\phi \quad \text{(Velocity)} \tag{41}$$

$$d^2U/d\phi^2 = -F_1 \text{ Sin } \phi - 4F_2 \text{ Sin } 2\phi - 9 \text{ Sin } 3\phi \quad \text{(Acceleration)} \tag{42}$$

$$d^3U/d\phi^3 = -F_1 \text{ Cos } \phi - 8F_2 \text{ Cos } 2\phi - 27 \text{ Cos } 3\phi \tag{43}$$

$$d^4U/d\phi^4 = F_1 \text{ Sin } \phi + 16F_2 \text{ Sin } 2\phi + 81 \text{ Sin } 3\phi \tag{44}$$

$$d^5U/d\phi^5 = F_1 \text{ Cos } \phi + 32F_2 \text{ Cos } 2\phi + 243 \text{ Cos } 3\phi \tag{45}$$

These relationships may be utilized to meet an extremely wide variety of requirements; two examples will be shown.

EXAMPLE VIII

It is again desired to design the mechanism to achieve a long dwell using both the second and third harmonics (rather than either the second or third harmonic as in the earlier cases).

It will be noted that when $\phi = 0$, equations (40), (42) and (44), representing U, $d^2U/d\phi^2$, and $d^4U/d\phi^4$ are equal to 0 for all values of F_1, F_2 and F_3. It will be assumed that the maximum dwell can be achieved by requiring that the velocity, $dU/d\phi = 0$ at $\phi = 0$. Therefore, from equation (41),

$$1 + F_1 + 2F_2 + 3F_3 = 0 \tag{46}$$

It will be further assumed that the maximum dwell will be achieved by requiring that the slope of the acceleration, $d^3U/d\phi^3 = 0$ at $\phi = 0$. Therefore, from equation (43)

$$-F_1 - 8F_2 - 27F_3 = 0 \tag{47}$$

Since three unknowns, F_1, F_2 and F_3 are to be derived, a third independent relationship between these unknowns must be established. To do this, equation (45) is utilized with the stipulation that $d^5U/d\phi^5 = -H$, where $-H$ in the physical sense represents the curvature of the slope of the acceleration, and H becomes the controlling parameter of the dwell characteristics. Therefore, from equation (45),

$$H + F_1 + 32 F_2 + 243 F_3 = 0 \tag{48}$$

When equations (46), (47) and (48) are solved, the following values are found

$$F_1 = -(H+36)/24$$
$$F_2 = (H+9)/30$$
$$F_3 = -(H+4)/120$$

Then, for several values of H, the final values of F_1, F_2 and F_3 are as follows:

H	F_1	F_2	F_3	Curves in FIGS. 17, 18, 19
0	−1.5	.3	−.0333	J_1
1	−1.5417	.3333	−.0417	J_2
1.5	−1.5625	.35	−.0458	J_3

When these values are utilized in equation (40), and the values of U calculated for various values of ϕ, the curves J_1, J_2 and J_3 in FIG. **17** are obtained. It will be noted that the dwell can be made appreciably greater than was possible using only a single harmonic addition.

Curve J_2 in FIGS. **18** and **19** represents the relative velocity and relative acceleration characteristics under these conditions. Curves J_1 and J_3 are omitted in these two figures because of their near identity with curve J_3.

EXAMPLE IX

In this example, it is desired that the velocity be as uniform as possible over the center portion of a cycle and that the dwell be as long as possible consistent with this first requirement. To meet the latter condition, the dwell effect parameter K is again introduced into equation (41) at $\phi = 0$ whereupon:

$$K + F_1 + F_2 + 3 F_3 = 0 \tag{49}$$

In order to achieve the most uniform possible velocity over the center portion of the cycle, equations (42), (43), (44) and (45) must be examined at $\phi = 180°$; at $\phi = 180°$, it will be seen that equations (42) and (44), representing $d^2U/d\phi^2$ and $d^4U/d\phi^4 = 0$ for all values of F_1, F_2 and F_3.

Therefore, the second relationship established to find F_1, F_2 and F_3 is to set $d^3U/d\phi^3 = 0$ at $\phi = 180°$ in equation (43), whereupon

$$F_1 - 8 F_2 = 27 F_3 = 0 \tag{50}$$

The final relationship is established through equation (45). Here it is hypothesized that the curvature of the slope of the dwell, $d^5U/d\phi^5$, as represented by factor $-C$ should become the controlling parameter; therefore, at $\phi = 180°$ from equation (45):

$$C - F_1 + 32 F_2 - 243 F_3 = 0 \tag{51}$$

Solving equations (49), (50) and (51), the following values are found:

$$F_1 = -(180 K + 13 C)/264$$
$$F_2 = (C - 9 K)/66$$
$$F_3 = (5C - 12 K)/792$$

Through numerical representative calculations for several values of C, an excellent solution of near per-

fect constant velocity over the center of the cycle is found for $C = -1$. Therefore, the factors become:

$$F_1 = (-180K + 13)/264$$

$$F_2 = (-1 + 9K)/66$$

$$F_3 = (-5 + 12K)/792$$

Then, for several values of K, the final values of F_1, F_2 and F_3 are as follows:

K	F_1	F_2	F_3	Curves in FIGS. 17, 18, 19
1.00	−.63258	−.15152	−.02146	K_1
1.01	−.63939	−.15288	−.02162	K_2
1.02	−.64621	−.15424	−.02177	K_3

When these values are utilized in equation (40), the curves K_1, K_2 and K_3 in FIG. 17 are obtained, indicating the variation in dwell characteristics for the various values of K.

Only the curves K_1 are shown in FIGS. 18 and 19 because of the near identity of curves K_2 and K_3 to K_1, representing the relative velocity and acceleration under these conditions. It will be noted that the relative velocity is substantially constant at a value of 1.4 from $\phi = 115°$ to $\phi = 245°$, thereby meeting the original design requirements.

The sprocket ratios shown in FIG. 16 are two and three times the fundamental. Any other usable combination could also be used such as 2 and 4 or 3 and 5. Furthermore, one or more additional sprockets and adder links may be added if specific requirements demand. Using this technique it becomes possible to construct an almost limitless variety of kinematic output characteristics using Fourier analysis as a mathematical basis to determine the Fourier coefficients required to approximate the desired kinematic profile.

A single example will be presented to characterize a usage of third and fifth harmonic addition. In this case the equation of motion would become

$$U = \phi + F_1 \operatorname{Sin} \phi + F_3 \operatorname{Sin} 3\phi + F_5 \operatorname{Sin} 5\phi \quad (52)$$

By successive differentiation

$$dU/d\phi = 1 + F_1 \operatorname{Cos} \phi + 3F_3 \operatorname{Cos} 3\phi + 5F_5 \operatorname{Cos} 5\phi \quad (53)$$

$$d^2U/d\phi^2 = -F_1 \operatorname{Sin} \phi - 9F_3 \operatorname{Sin} 3\phi - 25F_5 \operatorname{Sin} 5\phi \quad (54)$$

$$d^3U/d\phi^3 = -F_1 \operatorname{Cos} \phi - 27F_3 \operatorname{Cos} 3\phi - 125F_5 \operatorname{Cos} 5\phi \quad (55)$$

$$d^4U/d\phi^4 = F_1 \operatorname{Sin} \phi + 81F_3 \operatorname{Sin} 3\phi + 625F_5 \operatorname{Sin} 5\phi \quad (56)$$

$$d^5U/d\phi^5 = F_1 \operatorname{Cos} \phi + 243F_3 \operatorname{Cos} 3\phi + 3125F_5 \operatorname{Cos} 5\phi \quad (57)$$

These relationships may again be utilized to satisfy many requirements in addition to their usage in the following example.

EXAMPLE X

The design requirements are the same as those for Example VIII, i.e., achieve as long a dwell as possible, but in this instance using the third and fifth harmonic in place of the second and third.

By a process completely analogous to that of Example VIII, the following values are found for F_1, F_3 and F_5.

$$F_1 = -(H+225)/192$$

$$F_3 = (H+25)/384$$

$$F_5 = -(H+9)/1920$$

Then, for several values of H, the final values for F_1, F_3 and F_5 are as follows:

H	F_1	F_3	F_5	Curves in FIGS. 17, 18, 19
0	−1.1719	.0651	−.0047	M_1
3	−1.1875	.0729	−.0063	M_2
6	−1.2031	.0807	−.0078	M_3

When these values are utilized in equations (52), (53), and (54), curves M_1, M_2, and M_3 in FIG. 17 are obtained indicating the dwell characteristics, curve M_2 in FIG. 18 representing the relative velocity characteristics and curve M_2 in FIG. 19 representing the relative acceleration characteristics. It is especially noteworthy that when using the third and fifth harmonic, those factors which give the best dwell also provide a very flat velocity characteristic through the center portion of the stroke.

In all the foregoing mechanisms, the idler arm was arranged to pivot about the axis of the output shaft. Under some conditions, it becomes more convenient to have the idler arm pivot about the input axis as is shown in FIG. **20.**

Referring to FIG. **20,** an output sprocket **130** rotates about an axis **132** on a suitable shaft mounted in suitable bearings in the frame **2.** An input sprocket **134** is driven by external means about an axis **136** on a suitable shaft. An idler arm **138** is independently mounted to pivot on axis **136.** At one end the idler arm **138** has connected to it, through a suitable shaft and bearing, the idler sprocket **140** rotating on an axis **142.** At its other end the idler arm **138** has connected to it through a suitable shaft and bearing another idler sprocket **144** rotating on an axis **146.** A drive chain **148** is in driving engagement with sprocket **134** and sprocket **130** and in engagement with the idler sprockets **140** and **144.** It can be seen that in the absence of motion of the idler arm **138,** the angular velocity ratio between the input sprocket **134** and the output sprocket **130** is the ratio of their pitch diameters.

A link **150** is connected to the input sprocket **134** through a suitable eccentric shaft and bearings on an eccentric axis **152;** the other end of the link **150** is connected to a beam link **154** through a pivot joint **156** at some suitable point along its length. One end of the beam link **154** is connected to the frame **2** through a suitable shaft and bearings on an axis **158;** the other end of the beam link **154** is pivot connected to a drive link **160** with a pivot joint **162.** The other end of the drive link **160** is connected to the idler arm **138** at the pivot joint **146.**

It can be seen that as the input sprocket **134** rotates about its axis **136,** the beam link **154** is caused to oscillate about pivot **158** as driven by link **150;** this in turn causes the idler arm **138** to oscillate about axis **136** as driven by link **160** from beam link **154.**

The magnitude of oscillation of the idler arm **138** is determined by the eccentricity of pivot **152** from axis **136,** and the ratio of distances between points **158, 156** and **162** on beam link **154.** By an analysis comparable to that employed in the earlier description, it is possible to show that the motion of the outut sprocket may be made to be substantially cycloidal through the proper selection of parameters. Therefore, the choice of positioning the idler arm to oscillate on the output axis or on the input axis is dependent only on the design factors of a particular application.

As was the case with the designs with the idler arms oscillating about the output axis, it is equally possible to employ superposition to construct Fourier additions when the idler arm is pivoted about the input axis.

A typical mechanism to add a single higher harmonic is shown in FIG. **21.** The basic mechanism is the same as in FIG. **20** with the following modifications and additions. A secondary drive sprocket **164** is mounted parallel to and concentric with the input sprocket **134** on the input shaft. An intermediate sprocket **168** is mounted on a suitable shaft and bearings from the frame **2;** this intermediate sprocket **168** is driven by sprocket **164** through chain **170.** As shown in FIG. **21,** the sprocket **168** rotates at twice the angular velocity of sprocket **164** but any other multiple ratio may be employed.

The beam link **154** is connected to the sprocket **168** through a pivot joint **172** which is eccentric to the axis of rotation of the sprocket **168.** It can be seen, therefore, that a second harmonic is added to the motion of link **160** and idler arm **138** whose magnitude is determined by the eccentricity of pivot **172** on the sprocket **168** and the ratio of the lengths between points **172, 156** and **162** on the beam link **154.**

A second intermediate sprocket and associated linkage may be added just as in the mechanisms in which the idler arm pivoted on the output sprocket.

In all the cases where the intermediate sprockets are used, it is possible to apply the input power from some external power source to that intermediate sprocket, rather than to the input sprocket directly. For example, in FIG. **21,** the power may be applied to sprocket **168,** in which case sprocket **164** is driven through chain **170** and in turn drives sprocket **134** which is the input sprocket of the variable loop.

The system is also available for situations in which the intermediate sprockets are larger than their associated primary sprocket, in which case subharmonics are generated. Therefore, a given cycle will not repeat until the slowest turning of the intermediate sprockets has made one complete revolution.

The drive link which causes the oscillation of the idler link was, in all cases, shown as being attached to the idler arm on an axis which was coincident with the axis of one of the idler sprockets. This was done for mechanical convenience and not kinematic necessity. The link may be attached to the idler arm at any point where an effective torque on the idler arm can be generated about the axis of rotation of the idler arm.

For light duty applications, any or all chain loops in all mechanisms may be replaced by cogged belts and suitably matching pulleys.

For high torque applications, it may become desirable to increase the angle of chain wrap about the output sprocket. A system for accomplishing this is shown schematically in FIG. **22.** The input sprocket **22** again rotates on the axis A_2 and the output sprocket **66** rotates on the axis A_1 as in FIGS. **1** and **5.** The idler arm **180** is enlarged to support two additional idler sprockets **184** and **186,** through suitable shafts and bearings, in addition to the original idler sprockets **30** and **42.** The idler arm **180** is free to pivot on axis A_1 and is oscillated by the link **52** from input sprocket **22** as before. The chain **64** passes from input sprocket **22** to the idler sprocket **30,** then to the added idler sprocket **186,** then to the output sprocket **66,** then to the other added idler sprocket **184,** then to the idler sprocket **42,** and finally back to the input sprocket **22.** It can be seen that the addition of the additional idler sprockets **184** and **186** significantly increases the angle of chain wrap about the output sprocket **66,** and that this addition will have no effect on the performance of the system if sprockets **30** and **42** are positioned as earlier described.

It is further evident that a marked increase in wrap may be accomplished through the addition of only one additional idler sprocket, either **184** or **186.**

This increase in chain wrap around the output sprocket, through the addition of one or two idler sprocket, is also applicable to all the embodiments in which one or more higher harmonics is added. Furthermore, it may be seen that in those embodiments in which the idler arm **138** pivots about the axis A_2 of the input sprocket **134,** the angle of chain wrap about that input sprocket **134** may be similarly increased through the addition of one or more idler sprockets to the idler arm **138.**

Another arrangement for introducing a second or third harmonic into the output motion is shown in FIG. **23** (see also FIGS. **1** and **5**). As shown, the idler sprocket **42** has one-half the pitch diameter of the input sprocket **22;** therefore, the idler sprocket **42** will make two complete revolutions during the interval that the input sprocket **22** makes one revolution. It will be noted that the link **52** is no longer attached to the arm **24** through the shaft **44,** but is attached to an eccentric pivot shaft **190** mounted on sprocket **42.** This then superimposes a modified second harmonic component on the motion of the arm **24** and, consequently, to the output motion of the output sprocket **66.** The modification of the second harmonic arises out of the fact that the sprocket **42** does not rotate at a constant angular velocity about shaft **44** but undergoes cyclical velocity changes as arm **24** oscillates about axis A_1.

A modified third harmonic may be introduced into the output motion by designing the system such that the sprocket **42** is one-third the pitch diameter of the sprocket **22.**

I claim:

1. A multiple step reversible intermittent motion mechanism comprising:

a. a frame,

b. a first drive member rotatably mounted on said frame to rotate on a first axis,

c. a second drive member rotatably mounted on said frame to rotate about a second axis spaced from said first axis,

d. an oscillating member pivotally mounted about the axis of one of said drive members,

e. multiple idler means mounted on said oscillating member,

f. a continuous loop of flexible drive means in tangential driving engagement with said first and second drive members and in engagement with said idler

means having a path such that one of said drive members is external to the topological loop of said flexible drive means and said idler means and said other drive member is internal to said topological loop,

g. a drive link,

h. means pivotally mounting said drive link at one point to said first drive member at an axis spaced from said first axis, and

i. means connecting said link at another point to said oscillating member, said other point being spaced from said first point on said link and spaced from the mounting axis of said oscillating member, wherein rotation of said first drive member will cause an oscillation of said oscillating member about its mounting axis.

2. A mechanism as defined in claim **1** in which said idler means are positioned on said oscillating member such that the line of said flexible drive means from either side of that said drive member within the said topological loop of said flexible drive means to the tangency point with the first engaged of said multiple idler means, and the line from said tangency point to the axis of pivot of said oscillating member, form an angle whose mean value during a given oscillation cycle of said oscillating member lies within the range of 70° to 110°.

3. A multiple step reversible intermittent motion mechanism comprising:

a. a frame,

b. an output member rotatably mounted on said frame to rotate about a first axis,

c. an input member rotatably mounted on said frame to rotate about a second axis spaced from said first axis,

d. an oscillating member pivotally mounted on said first axis,

e. multiple idler means mounted on said oscillating member,

f. a continuous loop of flexible drive means in tangential driving engagement with said input and output members and in engagement with said idler means such that said input member and said idler means lie within the topological loop of said flexible drive means and said output member lies external to the topological loop of said flexible drive means,

g. an eccentric drive shaft mounted on said input member, and

h. means connecting said eccentric drive shaft with said oscillating member at a point spaced from the mounting axis of said oscillating member, wherein rotation of said input member will cause an oscillation of said oscillating member about its mounting axis.

4. A mechanism as defined in claim **3** in which said idler means are positioned on said oscillating member such that the line of said flexible drive means from either side of said input member to the tangency point of contact with the first of said idler means, and the line from said tangency point to the axis of pivot of said oscillating member, form an angle whose mean value during a given oscillation cycle of said oscillating member lies within the range of 70° to 110°.

5. A mechanism as defined in claim **3** in which said idler means comprise two idler members, with one of said idler members rotatably mounted on one side of said oscillating member with respect to said first axis and the other of said idler members rotatably mounted on the other side of said oscillating member with respect to said first axis.

6. A mechanism as defined in claim **3** in which said idler means comprise multiple idler members, with one or more of said idler members rotatably mounted on one side of said oscillating member with respect to said first axis and one or more of said idler members rotatably mounted on the other side of said oscillating member with respect to said first axis.

7. A mechanism as defined in claim **3** in which said idler means comprise multiple idler members, with multiple said idler members rotatably mounted on one side of said oscillating member with respect to said first axis and multiple said idler members rotatably mounted on the other side of said oscillating member with respect to said first axis.

8. A mechanism as defined in claim **3** in which said means connecting said eccentric drive shaft with said oscillating member comprises a link member rotatably connected at one end to said eccentric shaft and rotatably connected at its other end to said oscillating member at a point spaced from said first axis.

9. A mechanism as defined in claim **3** in which said means connecting said eccentric drive shaft with said oscillating member comprises:

a. an auxiliary drive member rotatably mounted on said frame on an axis spaced from said first axis and said second axis,

b. means driving said auxiliary drive member from said input member,

c. a second eccentric shaft mounted on said auxiliary drive member,

d. a beam link rotatably mounted on said second eccentric shaft,

e. a first link rotatably connected at one end to said first eccentric drive shaft and pivotally connected at its other end to said beam link at a point spaced from said second eccentric shaft, and

f. a second link pivotally connected at one end to said beam link at a point spaced from said second eccentric shaft and said first link connection and pivotally connected at its other end to said oscillating member at a point spaced from said first axis.

10. A mechanism as defined in claim **3** in which said means connecting said eccentric drive shaft with said oscillating member comprises:

a. multiple auxiliary drive members, each independently and rotatably mounted on said frame on axes spaced from each other and spaced from said first and second axes,

b. multiple means driving said auxiliary drive members at different angular velocities from said input member,

c. multiple auxiliary eccentric shafts each mounted on one of said auxiliary drive members,

d. multiple beam links, each rotatably mounted on one of said auxiliary eccentric shafts,

e. a first link rotatably mounted at one end to said first eccentric drive shaft and pivotally connected at its other end to the first of said beam links,

f. connector links, being one less in number than the number of said beam links, each pivotally connected at one end to one of said beam links and pivotally connected at its other end to another of said beam links forming a series of connections

from the first of said beam links to the last of said beam links, and

g. a last link pivotally connected at one end to the last of said beam links and pivotally connected at its other end to said oscillating member at a point spaced from said first axis.

11. A mechanism as defined in claim **3** in which said means connecting said eccentric drive shaft with said oscillating member comprises:

a. a second eccentric shaft mounted on one of said idler means, and

b. a link member rotatably connected at one end to said first eccentric drive shaft and rotatably connected at its other end to said second eccentric shaft.

12. A multiple step reversible intermittent motion mechanism comprising:

a. a frame,

b. an output member rotatably mounted on said frame to rotate about a first axis,

c. an input member rotatably mounted on said frame to rotate about a second axis spaced from said first axis,

d. an oscillating member pivotally mounted on said second axis,

e. multiple idler means mounted on said oscillating member,

f. a continuous loop of flexible drive means in tangential driving engagement with said input and and output members and in engagement with said idler means such that said output member and said idler means lie within the topological loop of said flexible drive means and said input member lies external to the topological loop of said flexible drive means,

g. an eccentric drive shaft mounted on said input member, and

h. means connecting said eccentric drive shaft with said oscillating member at a point spaced from the mounting axis of said oscillating member, wherein rotation of said input member will cause an oscillation of said oscillating member about its mounting axis.

13. A mechanism as defined in claim **12** in which said idler means are positioned on said oscillating member such that the line of said flexible drive means from either side of said output member to the tangency point of contact with the first of said idler means and the line from said tangency point to the axis of pivot of said oscillating member form an angle whose mean value during a given oscillation cycle of said oscillating member lies within a range of 70° to 110°.

14. A mechanism as defined in claim **12** in which said idler means comprise two idler members, with one of said idler members rotatably mounted on one side of said oscillating member with respect to said second axis and the other of said idler members rotatably mounted on the other side of said oscillating member with respect to said second axis.

15. A mechanism as defined in claim **12** in which said idler means comprise multiple idler members, with one or more of said idler members rotatably mounted on one side of said oscillating member with respect to said second axis and one or more of said idler members rotatably mounted on the other side of said oscillating member with respect to said second axis.

16. A mechanism as defined in claim **12** in which said means connecting said eccentric shaft with said oscillating member comprises:

a. a beam link pivotally supported from said frame,

b. a first link rotatably connected at one end to said eccentric drive shaft and pivotally connected at its other end to said beam link at a point spaced from said frame, and

c. a second link pivotally connected at one point to said beam link at a point spaced from said frame and pivotally connected at another point to said oscillating member, and other point being spaced from said second axis.

17. A mechanism as defined in claim **12** in which said eccentric drive shaft with said oscillating member comprises:

a. an auxiliary drive member rotatably mounted on said frame on an axis spaced from said second axis,

b. means driving said auxiliary drive member from said input member,

c. a second eccentric shaft mounted on said auxiliary drive member,

d. a beam link rotatably mounted on said second eccentric shaft,

e. a first link rotatably connected at one point to said first eccentric shaft and pivotally connected at another point to said beam link at a point spaced from said second eccentric shaft, and

f. a second link pivotally connected at one point to said beam link at a point spaced from said second eccentric shaft and pivotally connected at another point spaced from said first point to said oscillating member, said other point being spaced from said second axis.

18. A multiple step reversible intermittent motion mechanism comprising:

a. a frame,

b. a first drive member rotatably mounted on said frame to rotate on a first axis,

c. a second drive member rotatably mounted on said frame to rotate about a second axis spaced from said first axis,

d. an oscillating member pivotally mounted about the axis of one of said drive members,

e. multiple idler means mounted on said oscillating member,

f. a continuous loop of flexible drive means in tangential driving engagement with said first and second drive members and in engagement with said idler means having a path such that one of said drive members is external to the topological loop of said flexible drive means and said idler means and said other drive member is internal to said topological loop,

g. an eccentric drive shaft mounted on said first drive member, and

h. means connecting said eccentric drive shaft with said oscillating member at a point spaced from the mounting axis of said oscillating member, wherein rotation of said first drive member will cause an oscillation of said oscillating member about its mounting axis.

19. A mechanism as defined in claim **18** in which said means connecting said eccentric drive shaft with said oscillating member comprises:

a. multiple auxiliary drive members, each independently and rotatably mounted on said frame on

axes spaced from each other and spaced from said second axis,

b. multiple means driving said auxiliary drive members at different angular velocities from one of said drive members,

c. multiple auxiliary eccentric shafts each mounted respectively on one of said auxiliary drive members,

d. multiple beam links each rotatably mounted on one of said auxiliary eccentric shafts,

e. a first link rotatably mounted at one end to said first eccentric drive shaft and pivotally connected at its other end to the first of said beam links,

f. connector links, being one less in number than the number of said beam links, each pivotally connected at one end to one of said beam links and pivotally connected at its other end to another of said beam links forming a series of connections from the first of said beam links to the last of said beam links, and

g. a last link pivotally connected at one end to the last of said beam links and pivotally connected at its other end to said oscillating member at a point spaced from said second axis.

20. A mechanism as defined in claim **18** in which said means connecting said eccentric drive shaft with said oscillating member comprise:

a. a second eccentric shaft mounted on one of said idler means,

b. a beam link pivotally supported from said frame,

c. a first link rotatably connected at one end to said eccentric drive shaft and pivotally connected at its other end to said beam link at a point spaced from said frame, and

d. a second link pivotally connected at one end to said beam link at a point spaced from said frame and rotatably connected at its other end to said second eccentric shaft.

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