June 11, 1940. G. J. DASHEFSKY 2,203,674

MATHEMATICAL INSTRUMENT

Filed Dec. 28, 1938 5 Sheets-Sheet 1

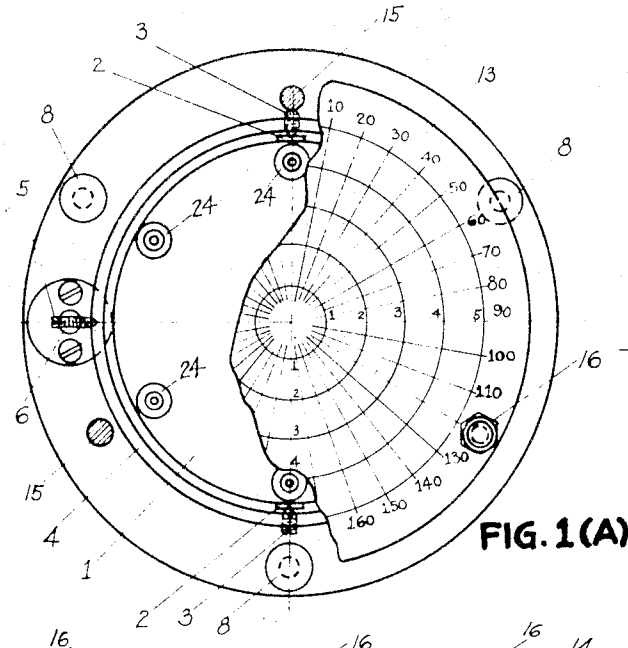


FIG. 1(A)

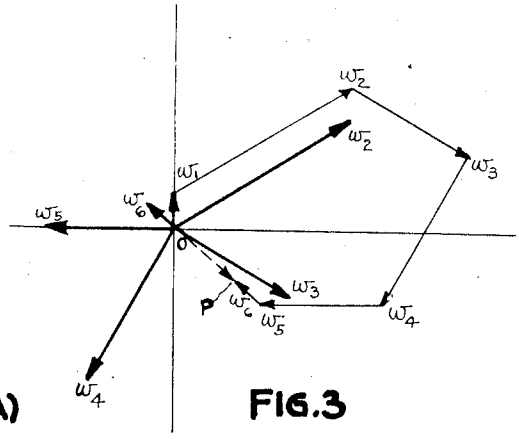


FIG. 3

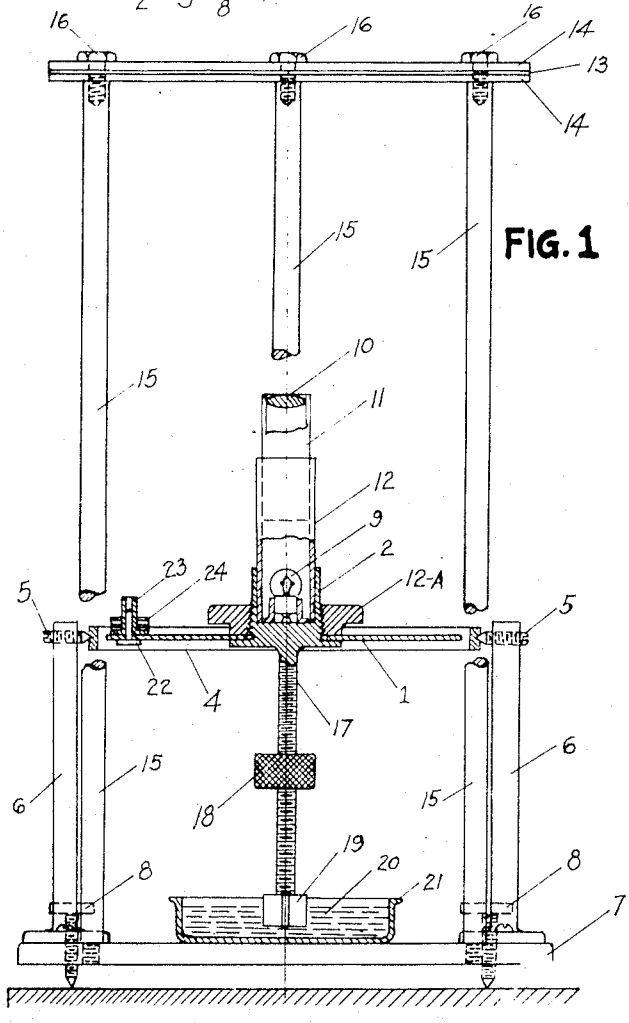


FIG. 1

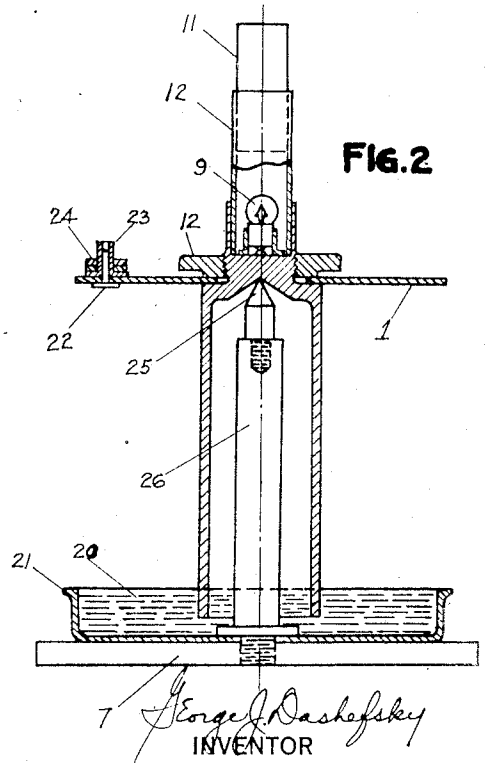


FIG. 2

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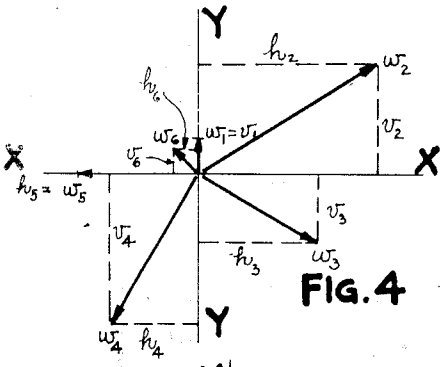


FIG. 4

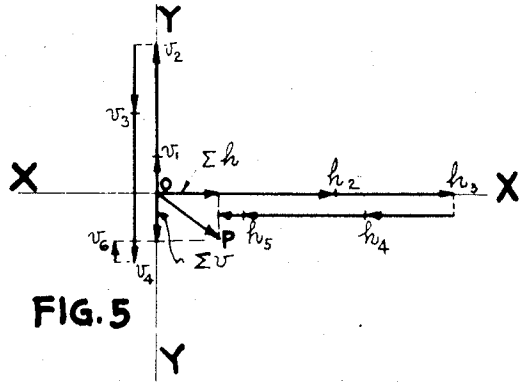


FIG. 5

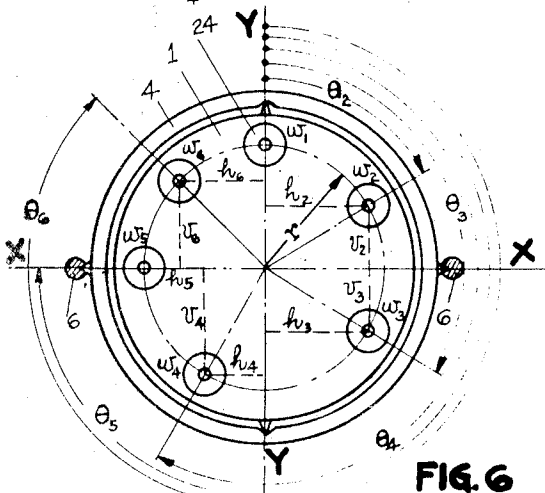


FIG. 6

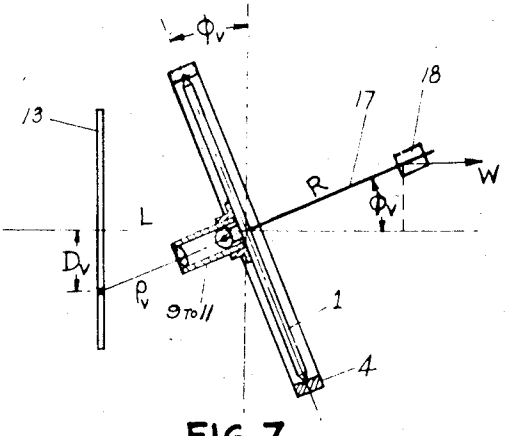


FIG. 7

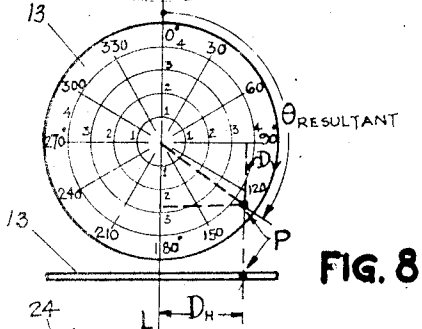
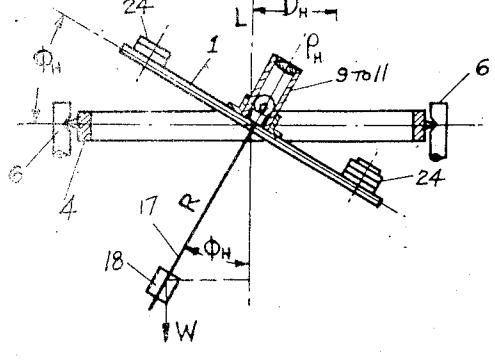


FIG. 8



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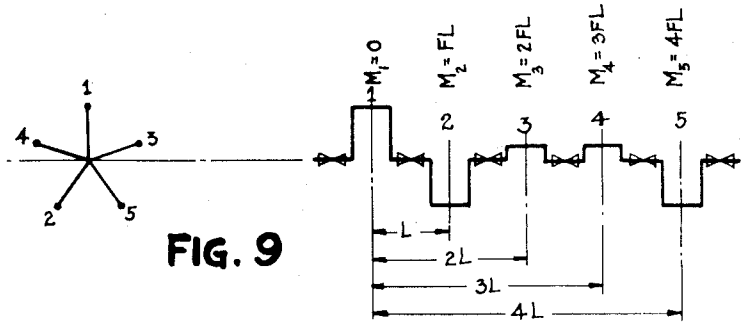


FIG. 9

FIG. 10

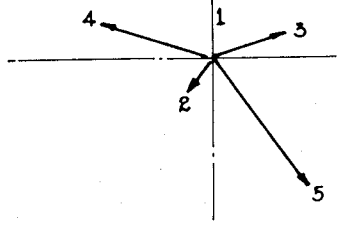


FIG. 11

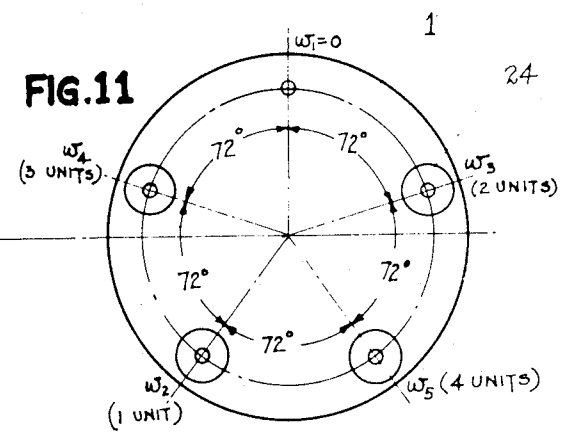


FIG. 12

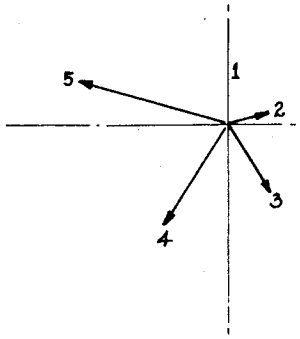
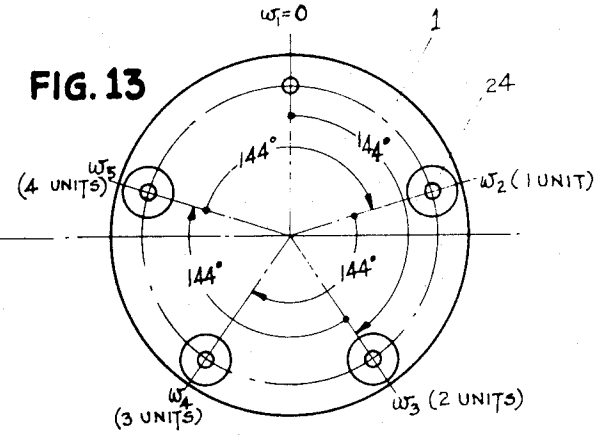


FIG. 13



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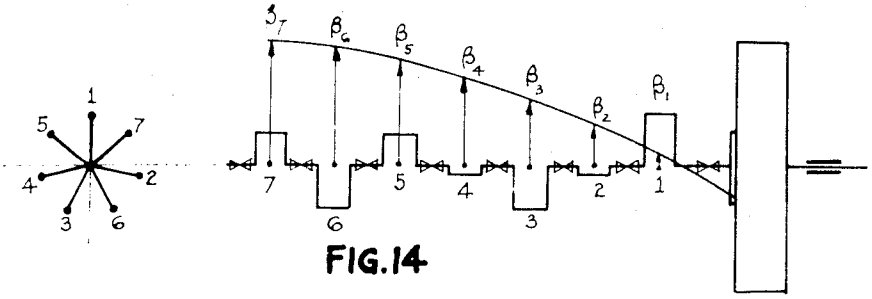


FIG. 14

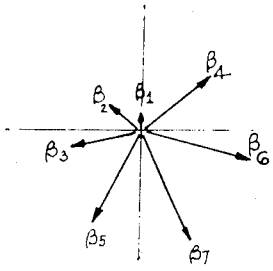


FIG. 15

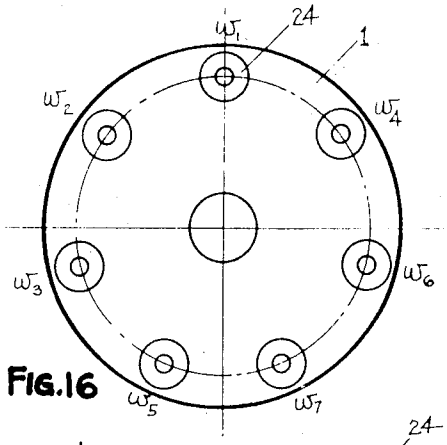


FIG. 16

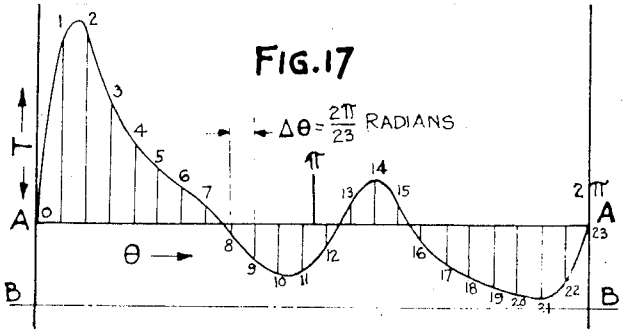


FIG. 17

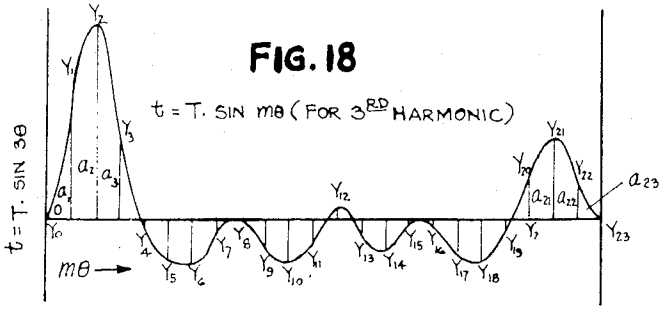


FIG. 18

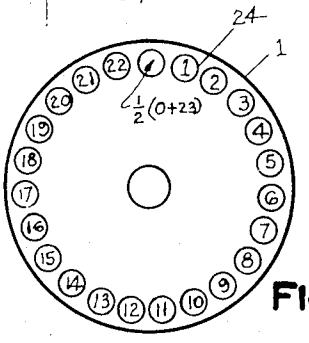


FIG. 19

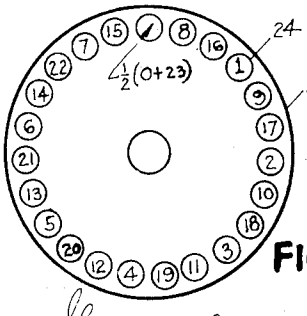


FIG. 20

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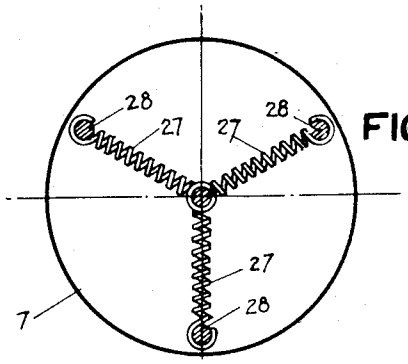


FIG. 21

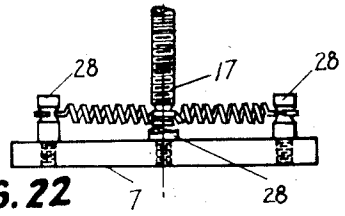


FIG. 22

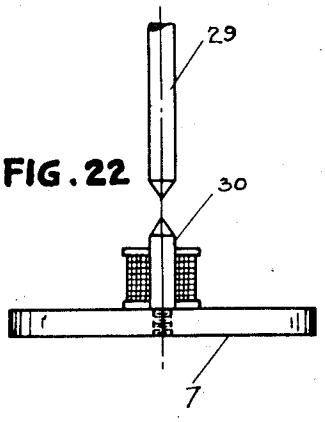


FIG. 22

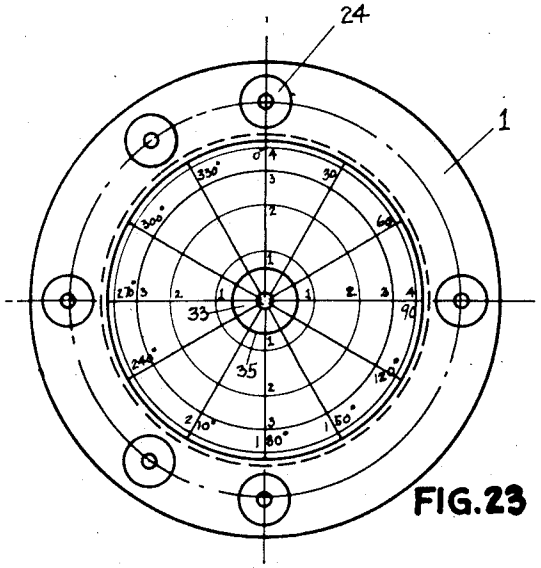


FIG. 23

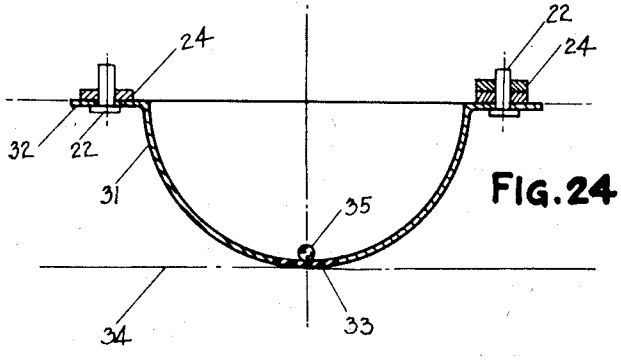


FIG. 24

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INVENTOR

Patented June 11, 1940 2,203,674

UNITED STATES PATENT OFFICE

2,203,674

MATHEMATICAL INSTRUMENT

George J. Dashefsky, Rockville Centre, N. Y.

Application December 28, 1938, Serial No. 248,040

8 Claims. (Cl. 235—61)

(Granted under the act of March 3, 1883, as amended April 30, 1928; 370 O. G. 757)

This invention relates to an instrument for rapidly and accurately effecting the addition of vector quantities by mechanical means.

An object of the invention is to provide a device which reduces the addition of vectors to a mechanical process, requiring a minimum of skill and mental concentration.

A further object is to provide a simple instrument which is useful for the rapid evaluation of the resultants of a large number of vector diagrams such as are met in the study of properties of firing sequences of internal combustion engines, particularly as they affect their dynamic characteristics, such as unbalanced moments, torsional vibration, torque reaction, et cetera.

A further object is to provide an instrument, commonly known as a harmonic analyzer, for analyzing a periodic curve into its harmonic components, that is to obtain the mathematical coefficients of the Fourier Series frequently used in science and engineering.

A further object is to provide an instrument for summing up vector quantities met in the art of dynamic balancing of rotating machine members.

A further object is to provide an instrument useful in the solution of certain problems dealing with alternating current electric circuits.

This invention should be of value in quickly and accurately solving any of the problems met in various fields, which either inherently are, or may be, reduced to a process of vector addition. Other objects of the invention will be apparent from the following description:

Figures 1 and 1(A) indicate the general arrangement of the instrument in one of its preferred forms.

Figure 2 shows a second form, particularly as regards the means for supporting the disk 1.

Figure 3 shows a typical vector diagram and its resultant obtained by graphical closure of the vector polygon.

Figures 4 and 5 show a second method of summing up the same vectors shown in Figure 3, by resolving the original vectors into their co-ordinate components.

Figures 6, 7, and 8, showing the instrument in elemental form, illustrate the underlying mathematical principle.

Figure 9 is a diagrammatic representation of the crankshaft of a 5-cylinder, 2-cycle engine, used to illustrate an application of the invention to the problem of engine balance.

Figure 10 is the familiar vector diagram for primary moments for the crankshaft of Figure 9.

Figure 11 indicates the arrangement of weights on the disk of the instrument, corresponding to the vector diagram Figure 10.

Figure 12 is the vector diagram for secondary moments for the crankshaft of Figure 9.

Figure 13 is the arrangement of weights on the disk of the instrument, corresponding to the vector diagram, Figure 12.

Figure 14 represents a typical relative amplitude curve of torsional vibrations, for an installation comprising a 7-cylinder, 2-cycle reciprocating engine and one principal revolving mass. This representaive installation and curve are used to illustrate an application of the invention to the solution of problems in synchronous torsional vibration.

Figure 15 represents a vector diagram, whose resultant, $\Sigma\beta$ is a measure of the exciting torque causing torsional vibrations. As represented, $\Sigma\beta$ is for the 3rd harmonic of the turning effort.

Figure 16 shows the arrangement of the instrument disk and weights corresponding to the vector diagram of Figure 15.

Figure 17 represents a periodic curve to be analyzed into harmonic components, and used in illustrating application of the invention as a harmonic analyzer.

Figure 18 is a curve derived from Figure 17 in a manner later described, and is used in describing the mathematical principle of the instrument when used as a harmonic analyzer.

Figure 19 represents the arrangement of the instrument disk and weights to determine, as an example, the first harmonic coefficients.

Figure 20 represents the arrangement of the instrument disk and weights for obtaining the 3rd harmonic coefficients as a further example of the use of the invention as a harmonic analyzer.

Figs. 21 and 22 are plan and side elevational views, respectively, of an alternative means, similar to that of the pendular system shown in Fig. 1, for providing an equilibrating force to act on the disk.

Figures 23 and 24 indicate a basic form of the instrument to further illustrate the various mechanical arrangements which may be used to embody the purposes of the present invention.

One embodiment of the invention is shown in Figures 1 and 1(A). One of several interchangeable disks 1 is carried by member 2, which is mounted in gimbals in the manner of a nautical compass. Member 2 is free to rotate in a pair of pivot bearings 3 carried on the ring 4. The ring 4 in turn is also mounted in pivot bearings

5, carried by stanchions 6, permitting rotation about an axis at right angles to the axis of rotation of the disk 1. The pivots are arranged so that the mutually perpendicular axes of rotation pass through the center of the disk 1. The ring 4 is supported on a base 7, which may be levelled off by means of screws 8. Member 2 carries a light projection system comprising a lamp 9 with a filament of the concentrated type, a lense 10, carried by tube 11, which telescopes into tube 12, residing in member 2. The disks 1 are retained on 2 by means of nut 12A. Current is fed to the lamp by very flexible leads passing through holes in 2. An image of the lamp filament is focused on a translucent screen 13 graduated in polar co-ordinates. The graduated screen, sandwiched between two plates of glass, or plastic 14, is supported from the base by stanchions 15 and secured thereto by nuts 16, may readily be replaced with other screens graduated in any required manner to permit direct readings in units pertinent to the problem in hand. Deflections of the disk 1 are measured by the displacement of the light spot from the center of the screen 13 indicating both magnitude and direction.

A threaded rod 17, forming the lower part of 2, carries a movable weight 18, by means of which the instrument sensitivity and scale reading is controlled.

The disks 1 are provided with pegs 22 with appropriate angular spacing. Weight holders 23 carrying preweighed groups of washers 24 of 1 unit, 0.1 unit, and 0.01 unit may be shifted from peg to peg as requisite. The weights 24 representing the vector magnitudes are thus adjustable by a simple counting process rather than by weighing.

The oscillations of the disk in its gimbals are damped out by vanes 19, carried at the extremity of rod 17, and moving in a dish 21, containing a suitable damping fluid 20. The time required for the disk to come to rest is thus greatly reduced.

Referring again to Figure 1, it will be noted that the disk 1 is free to rotate about each of the mutually perpendicular axes. It may of course rotate simultaneously about both axes. Any tilting action of the disk due to weights 24 is balanced by the pendular weight 18, which assumes a deflected position about the two axes of rotation, in accordance with the magnitudes of weights 24 and their disposition on the disk 1. Actually the weight of the light projection system, 9, 10, 11, and 12, member 2, and threaded rod 17, must also be considered. However, there is a net weight, W, which represents the combined effect of all of these. See Figures 6, 7, and 8.

The deflection of the disk 1, in its gimbals, is equivalent to supporting it on a single effective pivot 25 carried on 26, as shown in Figure 2. Such an arrangement could be used in a second form of the instrument, and would possess all of the mathematical characteristics of the instrument shown in Figure 1.

Refer to Figure 3 representing the addition of six vectors. The resultant vector is obtained by the usual graphical construction of the vector polygon in which line OP represents the vector sum or resultant.

A second method of adding these vectors is shown in Figures 4 and 5. Each vector is resolved into its co-ordinate components as shown in Figure 4, the respective components being added in the manner indicated in Figure 5. The two resultant components Σh and Σv may then be combined to give the resultant vector.

The following example involving the addition of vectors shown in Figure 3 will demonstrate the mathematical principle of the instrument. Refer to Figures 6, 7, and 8, showing the instrument in elemental form. Each vector is represented by a weight placed on the disk at a common radius, r. These weights have the same relative values as the original vector magnitudes, while the direction of the vectors is cared for by disposing the weights on the disk in the same angular relationship as the original vectors. The weights so disposed result in a net moment about axes X—X and Y—Y, producing rotation about these mutually perpendicular axes, displacing the net pendular weight W from its previously plumb position. The net moments of weights w_1 to w_6 inclusive, about the two axes, are balanced by the moment of the displaced weight W. For this condition of equilibrium,

About Y—Y:

$$WR \sin \phi_H = (w_1 r \sin \theta_1 + w_2 r \sin \theta_2 + w_3 r \sin \theta_3 + w_4 r \sin \theta_4 + w_5 r \sin \theta_5 + w_6 r \sin \theta_6) \cos \phi_H \quad \text{(Equa. 1)}$$

In general for any number of weights and orientation,

$$WR \sin \phi_H = r (\cos \phi_H) \Sigma (w \sin \theta) \quad \text{(Equa. 2)}$$

About X—X:

$$WR \sin \phi_V = (w_1 r \cos \theta_1 + w_2 r \cos \theta_2 + w_3 r \cos \theta_3 + w_4 r \cos \theta_4 + w_5 r \cos \theta_5 + w_6 r \cos \theta_6) \cos \phi_V \quad \text{(Equa. 3)}$$

In general for any number of weights and orientation,

$$WR \sin \phi_V = r (\cos \phi_V) \Sigma (w \cos \theta) \quad \text{(Equa. 4)}$$

The optical system 9 to 11, attached to the disk 1 projects a spot of light P, on to the graduated screen 13. The co-ordinate displacements of this point of light from the center of the screen as indicated in Figures 6, 7, and 8, are:

$$D_H = \rho_H \sin \phi_H \quad \text{(Equa. 5)}$$

$$D_V = \rho_V \sin \phi_V \quad \text{(Equa. 6)}$$

Now,

$$\sin \phi_H = \frac{D_H}{\rho_H}, \cos \phi_H = \frac{L}{\rho_H} \quad \text{(Equa. 7)}$$

$$\sin \phi_V = \frac{D_V}{\rho_V}, \cos \phi_V = \frac{L}{\rho_V} \quad \text{(Equa. 8)}$$

Therefore substituting Equations 7 and 8 into Equations 2 and 4, it follows that:

$$D_H = \frac{Lr}{WR} \Sigma (W \sin \Theta) \quad \text{(Equa. 9)}$$

$$D_V = \frac{Lr}{WR} \Sigma (W \cos \Theta) \quad \text{(Equa. 10)}$$

It is apparent from Equations 9 and 10, that D_H and D_V are measures of the co-ordinate components of the vector sum of the six original vectors shown in Figure 3, and represented by equivalent weights w_1 to w_6 inclusive, as shown in Figure 6. Comparing Figure 8 with Figures 4 and 5, it may be noted that the lines D_H and D_V in Figure 8 correspond to Σh and Σv in Figure 5, while OP in Figure 8 corresponds to the resultant vector OP in Figure 5. The direction of the resultant is also indicated directly on the screen 13.

Examining Equations 9 and 10, it may be noted that the scale or sensitivity of the instrument may be controlled by varying WR. Moving the pendular weight 18 toward the disk, decreases R and magnifies the readings. This offers a convenient means for calibrating the scale in terms of

any one of the vectors. One of the weights, say w_1 is placed on the disk, (see Figures 6, 7, and 8). This weight has a moment about X—X, but not about Y—Y. The vertical position of the pendular weight is now adjusted until the spot of light reaches a point on the screen, which yields convenient readings in terms of the vector represented by w_1. As an example, if W_1 represents a vector of 105. units, adjust the vertical position of the pendular weight until a scale reading 1.05 obtains. Subsequent readings of vector sums are therefore readily obtained by multiplying scale readings by 100.

The actual size of the weights w_1 and w_2 representing the vectors is not important, it is only necessary that they be in the same proportion as the original vectors.

The condition of engine balance is intimately associated with the choice of firing order. It is beyond the scope of this application to discuss this problem except as it is necessary to demonstrate application of the instrument to such problems.

Figure 9 shows the crankshaft of a 5-cylinder, 2-cycle engine taken as an illustrative example. Due to the reciprocating weight and crankshaft angularity there exists at each crank inertia forces acting in the line of piston stroke. There is no horizontal component.

The inertia force at each crank is expressed by:

$$F_I=\frac{4\pi^2N^2r_c}{3600g}\left(\cos\Theta+\frac{r_c}{l}\cos 2\Theta+\ \ldots\right) \qquad \text{(Equa. 11)}$$

This indicates that the inertia force is equivalent in effect, to a series of harmonic forces recurring once per revolution, twice per revolution, etc. The first two harmonics are the most important, but the 4th, 6th, etc. harmonics, though progressively smaller, also exist. In very high speed engines the 4th harmonic may be of importance. Limiting discussion to the first two harmonics, the presence of two separate forces may be considered.

$$F_I=\frac{4\pi^2N^2r_c}{3600g}\cos\Theta \qquad \text{(Equa. 12)}$$

$$F_{II}=\frac{4\pi^2N^2r_c}{3600g}\cdot\frac{r_c}{l}\cdot\cos 2\Theta \qquad \text{(Equa. 13)}$$

Where:

F_I=Primary force—lbs.
F_{II}=Secondary force—lbs.
N=Engine R. P. M.
l=Length connecting rod—inches
r_c=Crank radios—inches
g=Gravitational constant—inches/sec./sec.
θ=Angular displacement of crank from T. D. C. of cylinder No. 1 taken as reference position.

Take crank number 1 as a reference point for expressing moments. The same force exists at each cylinder. The moment of any cylinder is proportional to its distance from the reference point. The phase relation for successively firing cylinders is 72° for the primary forces and moments and $2\times72=144°$ for secondary forces and moments. Assuming equal cylinder spacing, the moments of cranks 2, 3, 4, and 5, are in the proportion of 1:2:3:4. The moment of crank number 1 is zero. Vector diagrams for primary and secondary moments are shown in Figures 10 and 12.

Consider a disk 1, shown in Figure 11, having 5 pegs, disposed 72 degrees apart, and weights in the proportion of 0, 1, 2, 3, and 4 selected to represent respectively, the moments of the cranks 1, 2, 3, 4, and 5. Each of these weight groups is marked with the cylinder number. To evaluate the vector diagrams the following procedure is followed. Temporarily place one of the weights, preferably w_2 which represents one weight unit, on the peg at $\theta=0$ that is, the top peg shown in Figure 11. The pendular weight 18 Figure 1, is now adjusted until a convenient scale reading is obtained in terms of the primary moment:

$$\frac{4\pi^2N^2r_c}{3600g}\times L \qquad \text{(Equa. 14)}$$

Having thus calibrated the instrument, place the weights on the pegs in proper phase relation with respect to firing order. There is no weight for w_1 since crank No. 1 has zero moment. For the primary moment the vectors are 72 degrees (one interval on the disk) apart and the weights are disposed as shown in Figure 11. The vector sum and the direction of the resultant are indicated directly on the screen in the manner shown in Figure 8.

To obtain the resultant secondary moment, that is, to evaluate the vector diagram, Figure 12, readjust the scale, in the manner described for primary moments. Since the phase relation is 2×72 degrees for secondary moments, dispose the weights, in firing sequence, on every other peg as shown in Figure 13.

In similar manner, the 4th harmonic may be obtained by putting these same weights on every fourth peg. Higher harmonics are similarly summed up by distributing the weights counting off each time a number of peg spaces equal to the harmonic under consideration.

The moment of any crank is determined only by its distance from crank No. 1, and is independent of firing order. The resultant however is influenced by firing order. Once having calibrated the instrument and fixed the weights representing cylinder moments, resultant unbalanced moments for a large number of firing orders may be quickly investigated.

For the 5-cylinder, 2-cycle engine the number of firing orders possible is,

$$\text{factorial}\ \frac{(n-1)}{2}=\frac{4\times3\times2\times1}{2}=12$$

By systematic shifting of the weights, a reading for each of the possible firing orders is quickly obtained. A similar procedure is now carried out for the secondary moments, remembering that in reading off the firing order, successively firing cylinders are displaced by two peg spaces, or 144 degrees. If required, similar procedures may be carried out for the higher harmonics.

Thus far, moments due only to the inertia of reciprocating weights have been considered. These inertia forces act vertically only, having no horizontal component. Centrifugal forces due to unbalanced weights also produce moments of importance, having components in both vertical and horizontal planes of the engine. They are of primary frequency only. Higher harmonics do not exist. The response of the engine and structure in the two planes is usually different, and it is desirable and customary to determine the horizontal and vertical components, rather than the resultant unbalance. The vector diagram of moments, and the procedure in the use of the instrument to evaluate these moments is identical with that previously described. The horizontal and vertical components may be read off directly from the screen. A listing of the instru-

ment readings permits choice of a suitable firing order.

Disks may be arranged to suit any particular configuration of cranks and cylinders, such as V engines, radial, opposed piston engines, etc.

The instrument is useful in solving a certain phase of the torsional vibration problem. Reviewing the problem briefly:

The input of vibration energy (inch pounds) in a multi-cylinder engine is:

$$K_I=\frac{\pi^2}{180}M_V\alpha(\Sigma\beta) \qquad \text{(Equa. 15)}$$

Where,

α=Amplitude of vibration at the free end of the crankshaft-degrees.

M_V=Harmonic torque of cylinder for particular order of vibration under consideration—pound inches.

$\Sigma\beta$=Vector sum of the amplitudes of vibration at the respective cylinders, their relative phase being that of the successively firing cylinders.

The phase relation for a given order of vibration is:

$$\gamma=m\theta \qquad \text{(Equa. 16)}$$

Where,

γ=Phase angle—degrees.

m=Order of vibration (number of vibrations per revolution of engine).

θ=Crank displacement between successively firing cylinders.

In a critical speed a balance exists between the input energy, K_I, and the damping energy, K_D. Assume the following law of damping, where α is the amplitude of vibration, in radians,

$$K_D=K_d\alpha^\lambda$$

$$K_D=K_I$$

$$\frac{\pi^2}{180}M_V\alpha(\Sigma\beta)=K_d\alpha^\lambda$$

$$\alpha=\lambda-1\sqrt{\frac{\pi^2}{180}\times\frac{M_V(\Sigma\beta)}{K_d}} \qquad \text{(Equa. 17)}$$

It is apparent that the amplitude of vibration depends on $\Sigma\beta$. Examining Equation 16 as applied to the 7-cylinder 4-cycle engine of Figure 14,

$$\theta=\frac{720}{7}\text{degrees}$$

When m=3.5, 7, or 10.5, γ=360°, 720°, and 1080°. For these orders of vibration, known as majors, the β's add up arithmetically, since their phase relation is 360° or a multiple thereof. All other orders, known as minors, have a phase relation, γ, as listed in Table I below. Groups marked with the same letter are equivalent as regards the magnitude of the resultant vector, but of different resultant phase. The $\Sigma\beta$ for major orders is independent of firing order, whereas $\Sigma\beta$ for minor orders is very much affected.

Table I

$\gamma=m\theta$

Orders of Vibration (m)	½, 6½ 7½, 13½	1, 6 8, 13	1½, 5½ 8½, 12½	2, 5 9, 12	2½, 4½ 9½, 11½	3, 4 10, 11
Phase angle in terms of number of peg intervals (each interval=720° m)	1 (A)	2 (B)	3 (C)	4 (C)	5 (B)	6 (A)

In Figure 15 is shown a typical vector diagram for the third order vibration associated with the relative amplitude curve of Figure 14, and the firing order 1—2—3—5—7—6—4. To evaluate this vector diagram with the instrument, in the now familiar manner, weights in proportion to the β values are placed on the seven peg disk, in proper phase relation. The screen is calibrated by placing the weight representing β on the disk and adjusting the pendular weight 18 to a position giving a suitable scale.

For each firing order, the whole range of orders may be investigated by simply shifting the weights to their appropriate angular positions. Starting with the one order, each weight, in sequence of firing, is placed on the pegs in succession, progressing in a clockwise direction. For the second order, the weights are placed on every second peg; for the third order, every third peg, and so on. Usual interest would center about the magnitude of the resultant, the phase being unimportant, so that in the 7-cylinder engine there are only 3 groups of minor orders to be evaluated.

In any particular problem, the β values are fixed, and a change in firing order does not call for readjustment of the weights representing β values. Thus to investigate the whole range of possible firing orders calls merely for a proper shifting of these weights. The values of $\Sigma\beta$ for a whole set of firing orders may thus be quickly and accurately determined. In the 7-cylinder engine there are 360 possible firing orders. Since there are 3 groups of minors, the complete investigation involves 1080 shifts of weights. With the aid of the present invention this appears to be an entirely feasible task requiring a minimum of skill and care.

The instrument may be used as a harmonic analyzer. Any periodic function such as the curve illustrated in Figure 17 may be represented by Fourier's trigonometric series, so that,

$$T=f(\theta)=A_0+A_1\sin\theta+A_2\sin 2\theta+A_3\sin 3\theta+\ldots A_n\sin N\theta+B_1\cos\theta+B_2\cos 2\theta+B_3\cos 3\theta+\ldots B_n\cos N\theta \qquad \text{(Equa. 18)}$$

The coefficient, A_0, represents the mean value of the function while the coefficients A_1, A_2, etc. and B_1, B_2, etc. represent respectively the sine and cosine harmonic components of the curve.

These are important coefficients in many engineering problems and it frequently becomes necessary to evaluate them. Schematic tabulations and mechanical devices are available for performing harmonic analyses. The present device is suited to a speedy determination of one or many of these coefficients by a simple mechanical procedure requiring a minimum of care and concentration.

Representing the periodic function as occurring over 2π radians it can be shown that,

$$A_0=\frac{1}{2\pi}\int_0^{2\pi}T.d\theta \qquad \text{(Equa. 19)}$$

$$A_m=\frac{1}{\pi}\int_0^{2\pi}T.\sin m\theta.d\theta \qquad \text{(Equa. 20)}$$

$$B_m=\frac{1}{\pi}\int_0^{2\pi}T.\cos m\theta.d\theta \qquad \text{(Equa. 21)}$$

A_m, B_m, represent the harmonic coefficients for the mth harmonic of the Fourier series of Equation 18.

The coefficient A_0, is obtained from evaluation of Equation 19, where the integral represents the net area under the original curve. The integral

involved in the expression for A_m is the area under the curve, $t=T.\sin m\theta$. This latter function is determined by multiplying each ordinate of the original function by the appropriate sin $(m\theta)$. A graph of $t=T.\sin m\theta$ is shown on Figure 18. The area under this curve multiplied by

$$\frac{1}{\pi}$$

is the value of A_m. The values of B_m may be obtained in a similar manner, excepting of course, that the auxiliary curve $t=T.\cos m\theta$ is obtained by multiplying ordinates of the original curve by the appropriate cosines of $m\theta$.

As a general matter, the process for determining the areas representing the intervals of Equations 19, 20, and 21, is not important. However, in adapting the present instrument as a harmonic analyzer, the following method of obtaining areas is desirable.

The abscissa of the original function is divided so as to give r equally spaced ordinates at $(r-1)$ equal intervals of $\Delta\theta$. The ordinates at each point are multiplied by the appropriate sine or cosine value giving a new series of ordinates on the curves, $t=T.\sin m\theta$, and $t=T.\cos m\theta$. Refer to Figure 18. The first elementary area a_1 assumed as a trapezoid may be closely approximated by,

$$\left(\frac{Y_0}{2}+\frac{Y_1}{2}\right)\Delta\theta$$

Adding up all of the elementary areas, a_1, a_2, a_3, etc. gives the net area of the curve, representing the value of the integrals of Equations 20 and 21. The total area under the curve is therefore:

$$\int_0^{2\pi} T.\sin m\theta.d\theta=\left(\frac{Y_0}{2}+\frac{Y_1}{2}\right).\Delta\theta+\left(\frac{Y_1}{2}+\frac{Y_2}{2}\right).\Delta\theta\ \ldots\ +\left(\frac{Y_{r-1}}{2}+\frac{Y_r}{2}\right).\Delta\theta$$

$$A_m=\frac{1}{\pi}\int_0^{2\pi} T\sin m\theta.d\theta=\frac{\Delta\theta}{\pi}\left(\frac{Y_0}{2}+Y_1+Y_2+\ldots\frac{Y_r}{2}\right)$$

$$A_m=\frac{\Delta\theta}{\pi}\left[\frac{T_0}{2}\sin 0^\circ+T_1\sin(m.\Delta\theta)+T_2\sin(m.2.\Delta\theta)+\ \ldots\ +T_{r-1}\sin\{m(r-1).\Delta\theta\}+\frac{T_r}{2}\sin 2\pi\right] \qquad \text{(Equa. 22)}$$

Similarly:

$$B_m=\frac{\Delta\theta}{\pi}\left[\frac{T_0}{2}\cos 0^\circ+T_1\cos m.\Delta\theta+T_2\cos(m.2.\Delta\theta)+\ \ldots\ +T_{r-1}\{m(r-1).\Delta\theta\}+\frac{T_r}{2}\sin 2\pi\right] \qquad \text{(Equa. 23)}$$

It will be noted that Equations 22 and 23 for A_m and B_m are of the form which can be evaluated by the instrument.

By dividing the curve into a prime number of intervals, the possibility of several weights piling up for some of the harmonics is avoided.

The modus operandi for loading the disk for making harmonic analyses is illustrated in Figures 17, 18, 19, and 20. It is desired to analyze the curve shown in Figure 17. Draw an auxiliary line, B—B, below the lowest point on the curve. For the purposes of illustration, the curve is to be analyzed into its harmonic components by using 24 ordinates. A disk with 23 pegs such as shown in Figures 18 and 19 is therefore to be used. Weights corresponding to the 24 equally spaced ordinates on the curve are prepared, the first and last ordinates being represented by weights corresponding to half of these ordinates, ordinates being measured from line B—B, Figure 17.

Weights should be suitably marked for ordinate identification. The instrument is adjusted in the manner previously described, to give a suitable scale. A_0 is merely the algebraic sum of the ordinates measured from A—A, Figure 17, multiplied by

$$\frac{1}{2\pi}$$

and the use of the harmonic analyzer will not be required. To obtain A_1 and B_1 as well as their resultant and its phase, place the weights in order on the pegs: w_1 on peg 1, w_2 on peg 2, etc. Having distributed the weights, the disk will deflect, moving the spot of light on the screen. Horizontal and vertical coefficients as well as the resultant coefficient and its phase are indicated directly, except for multiplication by

$$\frac{\Delta\theta}{\pi}$$

which may however be incorporated into the scale reading, if so desired.

The second harmonic is obtained by redistributing the weights, placing w_1 on the first peg, w_2 on the third peg (2 peg angles apart), w_3 on the 5th peg, etc., until all weights have been distributed.

Thus any harmonic, say the mth one, may be obtained by distributing the weights in succession, counting off m peg intervals each time. Figures 19 and 20 show, as examples, the distribution of weights for the 1st and 3rd harmonics of the curve of Figure 17, for a 24-ordinate analysis.

Greater accuracy will, of course, attend the use of larger number of ordinates, but the use of 24 ordinates should give results of sufficient accuracy for most purposes and types of curve. However the number used should be dictated by the particular problem in hand, and its requirements for accuracy. The highest harmonic required must also be considered in selecting the number of ordinates used.

The uses of the instrument are not confined to the specific problems discussed in this specification. These were selected solely for the purpose of illustrating the principle and use of the invention. The instrument is adaptable to many fields of engineering.

Several groups of weights, each group on a concentric circle, may be used to evaluate families of vectors.

The instrument may also be used to resolve a vector into components in any desired directions. The vector to be resolved, represented by a weight, is placed on one of the pegs. On the other pegs representing the direction of the component vectors, (but shifted through 180° on the disk) are placed weights until the disk returns to its undeflected position. The weights required to balance the disk represent the magnitudes of the component vectors and the direction diametrically opposite these weights are the directions of the component vectors.

Likewise the construction of the instrument should not be limited to the forms disclosed in this specification. The light projection system need not be used, a simple pointer extending from the disk could be used to indicate deflections. Also the light system could be so arranged that the projector is a fixed element, while the disk carries a mirror at its center, which reflects the beam of light upon a screen in the same manner

as used in certain types of galvanometer. Again, instead of a pendular mass attached to the disk, a movable weight could be arranged, adjustable both as to magnitude and angular position. This weight could be varied until the unbalanced moment of the disk were counterbalanced and the disk brought back to its initial position in a manner analogous to that employed in the ordinary beam scale. Or the unbalanced force could actually be measured by direct-reading weighing scales.

An optional system of equilibrating forces is shown in Figures 21 and 22. Three springs 27, of the same load-deflection characteristic connect the rod 17 and lugs 28 secured to the base 7. All springs are under the same initial tension. The manner of loading the disk is not limited to the system of weights at constant radius used in the forms disclosed. Fixed weights may be used and the radius varied. It is only necessary that the moment of loads on the disk, representing a system of vectors, be such that the moments of these loads about the center of the disk are in the same proportion as the vectors. Therefore hydraulic, magnetic, or any desired form of loading may be used.

As a further example of the diverse ways in which the purposes of the present invention may be accomplished, reference is made to Figure 24. This embodiment comprises a hemisphere 31, terminating in a plane flange 32, which serves as the disk 1 of Figure 1. As previously described, the disk is loaded by weights 24, carried on appropriately located pegs 22. The hemisphere 31 and flange 32 are made of relatively light material and provided with a heavy inset 33. The concave spherical surface is graduated into meridians and lines of latitude as indicated. The sphere is free to roll on a smooth lever surface 34. Under action of the weights 24, the disk will roll to a position of equilibrium between the displaced center of gravity of the sphere and flange and the weights. This displacement is a measure of the unbalanced moment on the disk, and is consequently, as previously discussed, a measure of the resultant vector sum of the vectors represented by the weights 24. The displacement of the sphere is indicated by the position of the ball 35, which remains always at the lowest point of the concave surface.

The description of the invention has been restricted to the embodiments shown in the specification and to illustrative uses described therein. The scope of the invention is not limited to these embodiments and uses, but only to the scope of the following claims.

The invention described herein may be manufactured and/or used by and for the Government of the United States of America for governmental purposes without the payment of any royalties thereon or thereafter.

Having thus set forth and disclosed the nature of this invention, what is claimed is:

1. An instrument for mechanically obtaining the resultant of a plurality of quantities having the properties of magnitude and directional relationship with respect to a common reference point; comprising a disk pivoted in effect at its polar center, means for loading said disk in proportion to the magnitudes of said quantities and in the same angular relationship as obtains there-among, and means for measuring the unbalanced moment on said disk as an index of said resultant quantity.

2. An instrument for mechanically obtaining the resultant of a plurality of quantities having the properties of magnitude and directional relationship with respect to a common reference point; comprising a disk pivoted in effect at its polar center, means for loading said disk in proportion to the magnitude of said quantities and in the same angular relationship as obtains there-among, means for providing an equilibrating force on said disk, and means for indicating the deflection of said disk, said deflection being a measure of the magnitude and direction of said resultant quantity.

3. An instrument for mechanically obtaining the resultant of a plurality of quantities having the properties of magnitude and directional relationship with respect to a common reference point; comprising a disk pivoted in effect at its polar center; means for providing a counterbalancing force on said disk, weights representing the magnitudes of said quantities, means for retaining the weights on said disk in the same angular relationship as that of said quantities, and means for indicating the deflection of said disk, the deflection being a measure of the magnitude and direction of said resultant quantity.

4. An instrument for mechanically obtaining the resultant of a plurality of quantities having the properties of magnitude and directional relationship with respect to a common reference point; comprising a disk pivoted in effect at its polar center, a pendular weight attached to said disk, weights in the same proportion as the magnitudes of said quantities, means for retaining said weights in the same angular relationship as exists among said quantities and means for indicating the deflection of said disk as a measure of the magnitude and direction of said resultant quantity.

5. An instrument for mechanically obtaining the resultant of a plurality of quantities having the properties of magnitude and directional relationship with respect to a common reference point; comprising a disk pivoted in effect at its polar center, a pendular weight attached to said disk, weights representing the magnitudes of said quantities, means for retaining said weights in the same angular relationship as that existing among said quantities, a screen, and means for projecting a spot of light on said screen for indicating deflections of said disk, said deflections representing the magnitude and direction of said resultant quantity.

6. An instrument for mechanically obtaining the resultant of a plurality of quantities having the properties of magnitude and directional relationship with respect to a common reference point; comprising a disk mounted in gimbals, a pendular weight attached to said disk, weights representing the magnitudes of said quantities, means for retaining said weights on said disk in the same angular relationship as exists among said quantities, and means for indicating the deflection of said disk as a measure of the magnitude and direction of said resultant quantity.

7. An instrument for mechanically obtaining the resultant of a plurality of quantities having the properties of magnitude and directional relationship with respect to a common reference point; comprising a disk mounted in gimbals, a pendular weight attached to said disk, weights representing the magnitudes of said quantities, means for retaining said weights on said disk in the same angular relationship as that which exists among said quantities, a screen and means movable with said disk for projecting a spot of

light on said screen as a measure of the magnitude and direction of said resultant quantity.

8. An instrument for mechanically obtaining the resultant of a plurality of quantities having the properties of magnitude and directional relationship with respect to a common reference point; comprising a disk mounted in gimbals, a pendular weight attached to said disk, weights representing the magnitudes of said quantities, means for retaining said weights on said disk in the same angular relationship as that which exists among said quantities, a screen, means movable with said disk for projecting a spot of light on said screen as a measure of the magnitude and direction of said resultant quantity, and means for adjusting the distance of said pendular weight from said disk, thereby varying the sensitivity and scale of said instrument.

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