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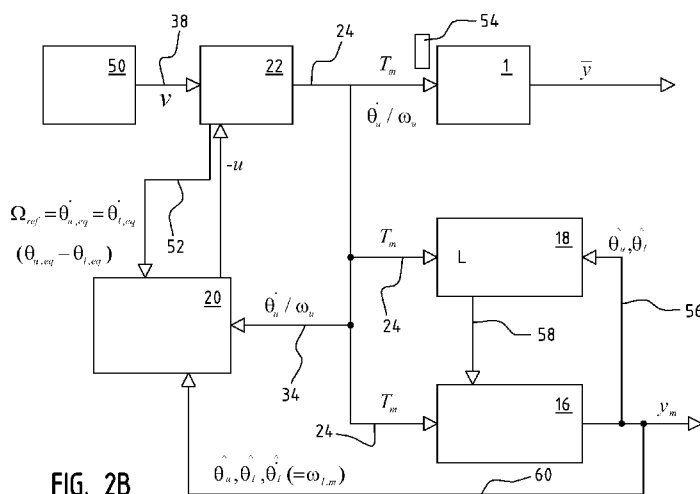


FIG. 2B

(57) Abstract: The invention provides a control system and method for limiting vibrations in a drilling system, including a drill string and a drive system for providing a drive torque for rotating the drill string at a reference frequency (Ω_{ref}). The control system comprises: - a sensor module for determining at least one uphole parameter of the drilling system; a model module provided with a model of the drilling system, the model module being adapted to provide modeled parameters of the drilling system using the drive torque as an input; - a model gain module for providing a model gain vector (L) to the model module in response to one or more of the modeled parameters and the drive torque (T_m), the model gain vector enabling the module to update the model thereby obtaining an updated model; and a control module for providing a torque correction factor (u) to the drive system depending on the modeled parameters, the uphole parameter, the reference frequency (Ω_{ref}), and the drive torque (T_m).

WO 2013/076184 A2

METHOD AND SYSTEM FOR CONTROLLING VIBRATIONS IN A
DRILLING SYSTEM

The present invention relates to a method and to a system for controlling vibrations in a drilling system.

Numerous vibrations can occur in an elongate body extending into a borehole formed in a subsurface formation, such as in a drill string operated to drill the borehole for the production of hydrocarbon fluid from the subsurface formation.

Drilling of an oil or gas wellbore is typically done by rotary drilling. Herein the wellbore may include vertical sections and/or sections deviating from vertical, e.g. horizontal sections. Rotary drilling generally employs a drill string including a drill bit at its downhole end. The drill string typically includes drill pipe sections which are mutually connected by threaded couplings. In operation, a drive system located at or near surface may provide torque to the drill string to rotate the drill string to extend the borehole. The drive system may include, for example, a top drive or a rotary table. The drill string transmits the rotational motion to the drill bit. Generally the drill string also provides weight on bit and may transmit drilling fluid to the drill bit.

As a drill string may be several kilometres long, e.g. exceeding 5 or 10 km, the drill string may have a very large length to diameter ratio. As a result, the drill string behaves as a rotational spring and can be twisted several turns during drilling. Different modes of vibration may occur during drilling, e.g. rotational,

lateral and/or longitudinal (axial) vibrations, possibly causing alternating slip-stick motions of the drill string or the drill bit relative to the borehole wall. Such vibrations are due to, for example, fluctuating bit-rock interactions and pressure pulses in the drilling fluid generated by the mud pumps.

In a model description, a drill string can often be regarded as a torsional pendulum wherein the top of the drill string rotates with a substantially constant angular velocity, whereas the drill bit performs a rotation with varying angular velocity. The varying angular velocity can have a constant part and a superimposed torsional vibration part. In extreme cases, the bit periodically comes to a complete standstill. Maintaining rotation of the drill string at surface builds up torque and eventually causes the drill bit to come loose and to suddenly rotate again, typically leading to a downhole angular velocity being much higher than the angular velocity at surface, typically more than twice the speed of the nominal speed of the motor at surface, e.g. a top drive or rotary table. The downhole angular velocity is dampened again whereafter the process is repeated, causing an oscillating behaviour of the lower part of the drill string. This phenomenon is known as stick-slip.

It is desirable to reduce or prevent these vibrations in order to reduce one or more of shock loads to the drilling equipment, excessive bit wear, premature tool failures and relatively poor drilling rate. High peak speeds occurring during the slip phase can lead to secondary effects like extreme axial and lateral accelerations and forces. Proper handling of downhole

vibrations can significantly increase reliability of the drilling equipment.

To suppress the stick-slip phenomenon, control methods and systems have been applied in the art to control the speed of the drive system such that the rotational speed variations of the drill bit are dampened or prevented.

One such method and system is disclosed in EP-B-443689, whereby the energy flow through the drive system of the drilling assembly is controlled to be between selected limits, the energy flow being definable as the product of an across-variable and a through-variable. The speed fluctuations are reduced by measuring at least one of the variables and adjusting the other variable in response to the measurement.

In EP-B-1114240 it is pointed out that the control system disclosed in EP-B-443689 can be represented by a combination of a rotational spring and a rotational damper associated with the drive system. To obtain optimal damping, the spring constant of the spring and the damping constant of the damper are to be tuned to optimal values, whereby the rotational stiffness of the drill string plays an important role in tuning to such optimal values. To aid this tuning, EP-B-1114240 discloses a method and system for determining the rotational stiffness of a drill string for drilling of a borehole in an earth formation.

WO 2010/063982 discloses a method and system for dampening stick-slip operations, wherein the rotational speed is controlled using a PI controller that is tuned such that the drilling mechanism absorbs torsional energy at or near the stick-slip frequency. The method can also comprise the step of estimating a bit speed, which is the

instantaneous rotational speed of a bottom-hole assembly. The bit speed is displayed at a driller's graphical interface and is regarded as a useful optional feature to help the driller visualize what is happening downhole.

5 A basic control theory for a non-smooth mechanical systems is described in A. Doris, Output-feedback design for non-smooth mechanical systems: Control synthesis and experiments, Ph.D. thesis, Eindhoven University of Technology, September 2007 (hereinafter referred to as
10 the Doris publication).

The Doris publication uses a dynamic rotor system, including an upper disc connected to the motor (the top drive) and a lower disc connected to the bit. Inputs to the model are the angular position (phase) and the speed
15 (first derivative of the phase) of both the upper disc and the lower disc. For the model to provide accurate results, the speed and phase of the lower disc will have to be measured using a downhole sensor.

Jens Rudat ET AL: "Development of an innovative
20 model-based stick/slip control system", SPE/IADC Drilling Conference and Exhibition, 139996, 1 March 2011, discloses a system using surface measured rotary speed Ω and hook load H as inputs to both the real drilling process and to a model thereof. The output of the model
25 y_m , comprising downhole rotary speed ω , weight-on-bit W and torque on bit T, is compared to the measured output vector y of the drilling process. The comparison is used to adapt parameters of the model to the real drilling system. The model itself however remains the same.

30 Disadvantage of the system is the requirement to measure downhole. As disclosed in Rudat et al., in the bottom hole assembly a dynamics measurement tool was positioned close to the bit enabling the measurement of

the downhole parameters downhole rotary speed ω , weight-on-bit W and torque on bit T . The model parameters related to the measured values have to be transmitted to surface, using limited bandwidth telemetry systems.

5 US-2009/229882 discloses a system for active vibration damping which relies on downhole sensors to measure motions of the drill string.

In practice, it is difficult to accurately measure the angular position and speed of the downhole disc, i.e. typically the drill bit. For instance, measurement of
10 angular position and rotational speed may for instance be measured using a two-dimensional gravity sensor. During a slip-phase, wherein the bit suddenly accelerates from a complete standstill to a rotational speed exceeding the
15 speed of the top drive, the phase accuracy is often lost. And not knowing the exact angular position will render further output of the model inaccurate. Herein, please note that data transmission rates between a downhole location and surface, using currently available
20 techniques, are very low, typically less than 1 Hz. These low transmission rates allow for instance only one sample per 15 seconds or less.

In addition, the exact initial angular position of the drill bit is not known, which implies there is always
25 an uncertainty or error in the measurement of the angular displacement of the bit. Since the drill string system is a non-linear system, for instance due to the friction, and exhibits multiple steady state solutions for the same excitation input, this error can drive the system in one
30 or the other solution. A steady state solution, for instance, is constant rotational velocity at the top drive and stick-slip behaviour at the bit. Another steady state solution, for instance, is constant rotational

velocity at the top drive and at the bit. Again, this is also due to the low data transmission rate as explained above.

5 The known methods and systems assume a specific frequency of stick-slip oscillations (vibrations), and tune the control system to that effect. Such control strategy is inadequate in case the stick-slip vibrations occur at a different frequency than the expected frequency, or when there are multiple vibration
10 frequencies which may change with operating conditions.

There is a need for an improved method of controlling vibrations in a drilling system, which overcomes the drawbacks of the prior art.

15 In accordance with the invention there is provided a method of controlling vibrations in a drilling system, the drilling system including an elongate body extending from surface into a borehole formed in an earth formation and a drive system for rotating the elongate body by providing a drive torque to the elongate body, the method
20 comprising:

- a) operating the drive system to provide a drive torque to the elongate body, and determining a system parameter that relates to an uphole parameter of the drilling system;
- 25 b) obtaining a model of the drilling system;
- c) applying the model to determine a modeled system parameter that corresponds to said system parameter;
- d) determining a difference between the system parameter and the modeled system parameter;
- 30 e) updating the model in dependence of said difference, thereby obtaining an updated model;
- f) determining from the updated model at least one modeled parameter of rotational motion, and adjusting the

drive torque in dependence of each modeled parameter of rotational motion so as to control vibrations of the elongate body.

The invention also relates to a control system for controlling vibrations in a drilling system, the drilling system including an elongate body extending from surface into a borehole formed in an earth formation and a drive system for rotating the elongate body by providing a drive torque to the elongate body, the control system comprising:

- an operating device for operating the drilling system to provide a drive torque to the elongate body, and for determining a system parameter that relates to an uphole parameter of the drilling system;

- a model of the drilling system; and
- computer means for applying the model to determine a modeled system parameter that corresponds to said system parameter, for determining a difference between the system parameter and the modeled system parameter, for updating the model in dependence of said difference thereby obtaining an updated model, for determining from the updated model at least one modeled parameter of rotational motion, and for adjusting the drive torque in dependence of each modeled parameter of rotational motion so as to control vibrations of the elongate body.

By using a model that predicts the responses of the drilling system, e.g. displacement, angular velocity, acceleration, frictional torque between drill string and rock formation, it is achieved that stick-slip vibrations of the drill string are eliminated for a range of angular velocities from angular velocities close to zero up to very high angular velocities. Furthermore, by updating the

model in dependence of a difference between the system parameter and the modeled system parameter, it is achieved that the parameters of the model converge rapidly to the parameters of the real drilling system so that the model accurately represents the state of the real drilling system. Also, with the method and control system of the invention, a constructive approach is used to adjust the drive torque delivered to the drill string so that there is no need for trial-and-error approach that can be time consuming. The model can include high system modes as many as required to simulate the drilling system accurately for the control purposes. It is robust in terms of model inaccuracies due to changes in the interaction between the rock formation and the drill bit or the drill string (frictional changes, damping changes, etc). The controller provides information to the drive system to adjust the drive torque in order to avoid undesirable drill string vibrations. The adjusted drive torque results in a winding/unwinding of the drill string able to eliminate the stick-slip vibrations of the bottom-hole assembly.

Suitable, said uphole parameter of the drilling system relates to an uphole torque in the drilling system. An example of a parameter related to uphole torque can be a torque parameter provided by a rotary drive coupled to an uphole end of the elongate body, for example as available in modern top drives. Alternatively or in addition a parameter related to uphole torque can be a torque parameter, such as torque, measured at an uphole position of the elongate body.

For example, said uphole parameter of the drilling system suitably relates to torque (T) in the elongate body at or near the earth's surface.

In one embodiment, said model of the drilling system includes a modelled torsional stiffness ($k_{\theta m}$) of the elongate body, and wherein said drilling parameter comprises a ratio of said torque (T) over said modelled torsional stiffness ($k_{\theta m}$).

Suitably, said modeled system parameter relates to a modeled difference between an uphole rotational position of the elongate body and a downhole rotational position of the elongate body.

In one embodiment, said uphole parameter of the drilling system is a first uphole parameter, and wherein step (c) comprises applying the model using an input parameter relating to a second uphole parameter of the drilling system.

Suitably, the drive system comprises a rotary drive coupled to an uphole end of the elongate body, and wherein said second uphole parameter is or comprises torque (T_m) provided by the rotary drive to said uphole end of the elongate body.

To accurately model the drilling system, the model suitably includes at least one modeled state parameter and wherein step (e) comprises adding to each modeled state parameter the product of said difference and a respective gain factor pertaining to the modeled state parameter.

In one embodiment, each modeled state parameter relates to a modeled parameter of rotational motion of the elongate body.

Suitably, said at least one modeled state parameter is selected from a modeled difference between an uphole angular velocity and a downhole angular velocity of the elongate body, a modeled uphole angular acceleration of

the elongate body, and a modeled downhole angular acceleration of the elongate body.

In one embodiment, step (b) comprises obtaining a state observer in which the model is included, the state observer further including a gain module for calculating each said gain factor.

Suitably, said at least one modeled parameter of rotational motion includes at least one of a modeled difference between an uphole rotational position and a downhole rotational position of the elongate body, a modeled uphole angular velocity of the elongate body, and a modeled downhole angular velocity of the elongate body.

The term "uphole" may refer to locations within, for example, 200 m from the earth surface or from a drilling rig used in the method of the invention. In case of an offshore operation, the earth surface is formed by the seabed. The term "downhole" may refer to locations within, for example, 200 m from the lower end of the elongate body. Suitably, the elongate body comprises a drill string having a drill bit at its downhole end.

The invention will be described hereinafter in more detail by way of example, with reference to the drawings, in which:

Figure 1 schematically shows a drilling system to be controlled by a preferred embodiment of the method and control system of the invention;

Figure 2A schematically shows an embodiment of the control system in modular form;

Figure 2B shows a schematic representation of an embodiment of the control system of the invention;

Figures 3a, 3b, 3c, 4a, 4b and 4c schematically show various results achieved using the method and control system of the invention.

In the description, like reference numerals relate to like components.

Fig. 1 shows a drilling system 1 including a drill string 2 extending from surface into a borehole (not shown) formed in an earth formation. The drill string 2 can be several thousand meters in length, and therefore behaves as a torsional spring. A drive system 4 is arranged at surface to rotate the drill string 2 in the borehole by providing a drive torque to the drill string 2. The drive system generally includes a motor arranged to drive a rotary table or a top drive (not shown). The drill string 2 typically includes a downhole end part 6. Said downhole end part may include a bottom hole assembly (BHA) 6 including a drill collar having an increased weight which provides the necessary weight on bit during drilling. Top drive may imply a drive system which rotates an upper end of the drill string. Upper end implies the end at surface, i.e. near the location where the drill string is suspended from a drilling rig.

Reference sign 7 represents torque resistance T_u of the upper part of the drill string, e.g. due to electrostatic forces in the motor, friction in the ball bearings, etc. Reference sign 8 represents torque resistance T_l of the lower part of the drill string due to interaction of the BHA with the rock formation and the drilling mud.

The following parameters are used in the discussion below:

T_m : drive torque provided by the drive system 4 to the drill string 2;

V : voltage input to a motor (not shown) of the drive system 4;

T : torque in the drill string 2 as determined at or near the earth surface;

u : an update value for controlling drive torque;

θ_u, θ_l : rotational position of the drill string 2 at
5 respective uphole and downhole locations;

$\dot{\theta}_u, \dot{\theta}_l$: rotational velocity of the drill string 2 at
respective uphole and downhole locations;

$\ddot{\theta}_u, \ddot{\theta}_l$: rotational acceleration of the drill string 2
at respective uphole and downhole locations;

10 $\bar{\theta}_u, \dot{\bar{\theta}}_u, \ddot{\bar{\theta}}_u$: model estimates of respective parameters
 $\theta_u, \dot{\theta}_u, \ddot{\theta}_u$;

$\hat{\theta}_u, \hat{\theta}_l, \dot{\hat{\theta}}_l, \ddot{\hat{\theta}}_l$: observer estimates of respective
parameters $\theta_u, \theta_l, \dot{\theta}_l, \ddot{\theta}_l$;

$\dot{\theta}_{u,eq}, \dot{\theta}_{l,eq} = \dot{\theta}_{eq}$: equilibrium values of $\dot{\theta}_u$ and $\dot{\theta}_l$;

15 $\theta_{u,eq}, \theta_{l,eq}$: equilibrium values of θ_u and θ_l .

Furthermore, arrow 9 (Fig. 1) refers to the parameters
 $\theta_u, \dot{\theta}_u, \ddot{\theta}_u$, arrow 10 refers to the parameter T_m , and arrow
12 refers to the parameters $\theta_l, \dot{\theta}_l, \ddot{\theta}_l$.

Generally, the subscript "u" ("upper") refers to an
20 uphole position, preferably at or near the surface of the
earth, and the subscript "l" refers to a downhole
position, preferably at or near the downhole end of the
elongate body. A bar above a symbol indicates a modelled
parameter. A dot above a symbol refers to a single time
25 derivative, i.e. a single dot indicates a velocity, and a
double dot indicates acceleration. A "hat" above a symbol
(such as $\hat{\theta}_l$) refers to a parameter of a state observer.
The subscript "eq" refers to an equilibrium value, that is
a value for a state in which the system is free of
30 vibrations. When the system is in equilibrium, the bit

will generally rotate at the same frequency as the drill string at the connection to the motor. Angular velocity is also referred to as rotational velocity.

Uphole parameters of the drilling system 1 are
5 determined at or near surface for use in the method of the invention. At or near surface implies that accurate measurements can be obtained using high-frequency sensors. High-frequency is for instance exceeding 1 kHz, i.e. more than 1000 samples per second. One such uphole
10 parameter relates to uphole torque T in the upper part of the drill string 2. In the practice of the invention, the torque T_m applied in a modern drive system or a parameter directly related to T_m is often available as a digital parameter. Generally T differs slightly from T_m due to,
15 for example, friction in the drive system itself and/or higher-frequency contributions that may not be transmitted between the drive system and the drill string. In case the drive system 4 includes a rotary table, such difference also can be due to transmission losses. In any event,
20 uphole torque T or a parameter directly related to T can be determined for example by measuring, e.g. by a torque sensor at a location at or near the earth surface. Further uphole parameters can be measured by suitable sensors.

Uphole rotary velocity $\dot{\theta}_u$ or a related parameter may
25 also be measured by a sensor at or near surface. Such related parameter is for example a period of one rotation of the drill string 2 at an uphole position. The period of rotation is directly related to and representative of angular velocity.

30 Fig. 2A shows a block diagram of a control system for controlling vibrations in the drilling system 1. The control system comprises a state observer 14 for providing an estimate of the state of the drilling system

1. The state observer 14 may use measurements of the input and the output of the drilling system 1. The state observer 14 includes a mathematical model 16 of the drilling system and a gain module 18 for updating the model 16. The gain module may use input and output measurements of the drilling system 1. The model 16 may typically be implemented in a computer system running software, e.g. written in Matlab. It is known in the art how to build a model for a given drill string, and for the drill string in the borehole. The model 16 can be a simple two degree-of-freedom (DOF) model, e.g. similar to the one used in section 6.2.2. of the Doris publication. The model can also be a more complex multi degree-of-freedom model. It is also possible to derive a two degree-of-freedom model from a multi degree-of-freedom model using model reduction techniques known per se. The skilled person knows how to build a model that describes the dynamics of a specific drilling system accurately enough for the controller needs, by including sufficient eigenmodes of the drilling system. The control system further comprises a controller 20 arranged to control a motor 22 that drives the drill string 2.

The state observer 14 may receive an input signal 24 representing T_m . In practice, said motor torque T_m is available to the driller, as it may be derived from the current drawn by the top drive. Input signal 24 may also include T . The model 16 provides output signals 28, 30, 32 representing respective parameters $\hat{\theta}_u, \hat{\theta}_l, \dot{\hat{\theta}}_l$. $\hat{\theta}_u, \hat{\theta}_l$ are supplied to both the gain module 18 and the controller 20. $\dot{\hat{\theta}}_l$ is supplied to the controller 20. The controller 20 also receives an input signal 34 representing parameter $\dot{\theta}_u$ and an input signal 36 representing parameters

$\dot{\theta}_{eq}, \theta_{u,eq} - \theta_{l,eq}$. Furthermore, reference sign 38 represents voltage V supplied to motor 22, reference sign 40 represents drive torque T_m supplied by motor 22 to drill string 2, and reference sign 42 represents a gain vector L supplied by gain module 18 to model 16.

During normal use of the drilling system 1, voltage V is supplied to the motor 22 and as a result the motor produces torque T_m that drives the drill string 2 in rotation. T and $\dot{\theta}_u$ are measured at surface. T may be input to the observer. The observer output comprises $\hat{\theta}_u, \hat{\theta}_l$ and $\dot{\hat{\theta}}_l$. These parameters together with $\dot{\theta}_u$ and $\dot{\theta}_{eq}, \theta_{u,eq} - \theta_{l,eq}$ are input to the controller where they are multiplied by a controller gain. The controller output is $-u$ which is input back into the motor. The motor adapts T_m by $-u$ and supplies the adjusted torque to the drill string 2.

A more detailed description of the way in which the observer 14, the model 16 and the controller 20 are used, is presented below.

The equations of motions of the drilling system 1 are governed by two inertias J_u, J_l , a spring flexibility k_θ , two frictional torques T_{fu}, T_{fl} , and torque input from the motor T_m . J_u is rotational inertia of the top drive and part of the drill string, J_l is rotational inertia of the Bottom-Hole-Assembly (BHA) and the remaining part of the drill-pipe. k_θ is rotational stiffness of the drill string. T_{fu} represents torque resistance in torsional motion of the upper part of the drill string (e.g. electrostatic forces in the motor, friction in the ball bearings, etc.) and T_{fl} represents frictional interaction of the BHA with the formation and the drilling mud. The

differential equations that describe the torsional dynamics of this system are given in formulas (1)-(8):

$$J_u \cdot \ddot{\theta}_u + k_\theta \cdot (\theta_u - \theta_l) + T_{fu}(\dot{\theta}_u) - T_m = 0 \quad (1)$$

$$J_l \cdot \ddot{\theta}_l - k_\theta \cdot (\theta_u - \theta_l) + T_{fl}(\dot{\theta}_l) = 0 \quad (2)$$

$$T_{fu}(\dot{\theta}_u) \in \begin{cases} T_{cu}(\dot{\theta}_u) \cdot \text{sgn}(\dot{\theta}_u) & \text{for } \dot{\theta}_u \neq 0 \\ [-T_{su} + \Delta T_{su}, T_{su} + \Delta T_{su}] & \text{for } \dot{\theta}_u = 0 \end{cases} \quad (3)$$

$$T_{cu}(\dot{\theta}_u) = T_{su} + \Delta T_{su} \cdot \text{sgn}(\dot{\theta}_u) + b_u \cdot |\dot{\theta}_u| + \Delta b_u \cdot \dot{\theta}_u \quad (4)$$

$$T_{fl}(\dot{\theta}_l) \in \begin{cases} T_{cl}(\dot{\theta}_l) \cdot \text{sgn}(\dot{\theta}_l) & \text{for } \dot{\theta}_l \neq 0 \\ [-T_{sl}, T_{sl}] & \text{for } \dot{\theta}_l = 0 \end{cases} \quad (5)$$

$$T_{cl}(\dot{\theta}_l) = T_{sl} + (T_{sl} - T_{cl}) \cdot e^{-\frac{|\dot{\theta}_l|}{\omega_{st}}} \delta_{st} + b_l \cdot |\dot{\theta}_l| \quad (6)$$

5

wherein:

$T_{cu}, T_{su}, \Delta T_{su}, b_u, \Delta b_u, T_{cl}, T_{sl}, b_l$ are constant parameters governed by frictional torque in the upper part of the drill string, such as in the drive system, and in the Bottom-Hole-Assembly. Example values of these parameters are presented in Table 1 at the end of this section. The torque T in the drill string 2 at or near surface is:

10

$$T = k_\theta \cdot (\theta_u - \theta_l).$$

15

The torque T can be derived from the current in the motor 22, and in practice it is always available to the drill-string operator.

20

Equations (9), (10) below are a copy of equations (1), (2) however with some disturbances included in the parameters $k_\theta, J_u, J_l, T_{fu}$ and T_{fl} . These disturbances are used to simulate modelling inaccuracies when deriving a model for an oil-field drilling system. This set of equations (called: model of the drilling system) will be used to build the observer shown in the cascade of Figure 2A.

$$J_{um} \cdot \ddot{\bar{\theta}}_u + k_{\theta m} \cdot (\bar{\theta}_u - \bar{\theta}_l) + T_{fum}(\dot{\bar{\theta}}_u) - T_m = 0 \quad (9)$$

$$J_{lm} \cdot \ddot{\bar{\theta}}_l - k_{\theta m} \cdot (\bar{\theta}_u - \bar{\theta}_l) + T_{flm}(\dot{\bar{\theta}}_l) = 0 \quad (10)$$

where $k_{\theta m}$, J_{um} , J_{lm} , T_{fum} and T_{flm} are model values of respective parameters k_{θ} , J_u , J_l , T_{fu} , T_{fl} . T_{fum} and T_{flm} have the same structure as T_{fu} and T_{fl} respectively. The parameters of the frictional forces of the model that are different from those of the drilling system are T_{sum} (instead of T_{su}), b_{um} (instead of b_u), T_{clm} (instead of T_{cl}) and b_{lm} (instead of b_l).

In state-space form the dynamics of the drilling system can be written as:

$$\begin{aligned} \dot{x}_1 &= x_2 - x_3 \\ \dot{x}_2 &= \frac{1}{J_u} [-k_{\theta} x_1 - T_{fu}(x_2) + T_m] \\ \dot{x}_3 &= \frac{1}{J_l} [k_{\theta} x_1 - T_{fl}(x_3)] \end{aligned} \quad (11)$$

where $x_1 = \theta_u - \theta_l$, $x_2 = \dot{\theta}_u$ and $x_3 = \dot{\theta}_l$.

In state-space form the dynamics of the model of the drilling system can be written as:

$$\begin{aligned} \dot{\bar{x}}_1 &= \bar{x}_2 - \bar{x}_3 \\ \dot{\bar{x}}_2 &= \frac{1}{J_{um}} [-k_{\theta m} \bar{x}_1 - T_{fum}(\bar{x}_2) + T_m] \\ \dot{\bar{x}}_3 &= \frac{1}{J_{lm}} [k_{\theta m} \bar{x}_1 - T_{flm}(\bar{x}_3)] \end{aligned} \quad (12)$$

where $\bar{x}_1 = \bar{\theta}_u - \bar{\theta}_l$, $\bar{x}_2 = \dot{\bar{\theta}}_u$ and $\bar{x}_3 = \dot{\bar{\theta}}_l$.

In state-space form the observer is represented by the following expression:

$$\begin{aligned}
\dot{\hat{x}}_1 &= \hat{x}_2 - \hat{x}_3 + l_1 \left(\frac{T}{k_{\theta m}} - \hat{x}_1 \right) \\
\dot{\hat{x}}_2 &= \frac{1}{J_{um}} [-k_{\theta m} \hat{x}_1 - T_{fum}(\hat{x}_2) + T_m + l_2 \left(\frac{T}{k_{\theta m}} - \hat{x}_1 \right)] \\
\dot{\hat{x}}_3 &= \frac{1}{J_{lm}} [k_{\theta m} \hat{x}_1 - T_{flm}(\hat{x}_3) + l_3 \left(\frac{T}{k_{\theta m}} - \hat{x}_1 \right)]
\end{aligned} \tag{13}$$

where l_1, l_2 and l_3 are observer gains and $\hat{x}_1 = \hat{\theta}_u - \hat{\theta}_l$,

$\hat{x}_2 = \hat{\theta}_u$ and $\hat{x}_3 = \hat{\theta}_l$. The gain vector $L = [l_1, l_2, l_3]$.

By applying the observer design techniques disclosed
5 in Chapters 5 and 6 of the Doris publication, the values
 l_1, l_2, l_3 can be computed. The goal of the observer is to
derive estimates of the drilling system states that are as
close as possible to the real drill-string system's
states. To derive the observer gains l_1, l_2, l_3 , a set of
10 linear matrix inequalities (LMIs) is to be solved, for
example using the software Matlab and in particular the
Matlab toolbox LMI-tool. The procedure to derive these
LMIs is described in the Doris publication.

In a further step, an adjustment to the drive torque
15 is applied for torque control so as to control
vibrations. The adjustment takes the following form in
this example:

$$u = -k_1 \cdot [\hat{\theta}_u - \hat{\theta}_l - (\theta_{u,eq} - \theta_{l,eq})] - k_2 \cdot [\dot{\hat{\theta}}_u - \dot{\theta}_{u,eq}] - k_3 \cdot [\hat{\theta}_l - \dot{\theta}_{l,eq}] \tag{14}$$

where the subscript eq refers herein to equilibrium values
20 of the model and the drill-string system. The adjustment u
to the drive torque is calculated using modelled downhole
parameters of rotational motion. $\dot{\theta}_{u,eq}$ and $\dot{\theta}_{l,eq}$ are equal to
each other, as these are the desired values of the
drilling system while drilling because no stick-slip
25 vibrations occur when they are equal. Hence the bit will

rotate at the same rotational speed as the drill string at the connection to the drive system.

In order to calculate $\theta_{u,eq}, \theta_{l,eq}$, the acceleration component in equations (9), (10) is nullified (i.e.

5 $\ddot{\theta}_{u,eq} = 0, \ddot{\theta}_{l,eq} = 0$), substitutions $\dot{\theta}_u = \dot{\theta}_{u,eq} = \dot{\theta}_{eq}, \dot{\theta}_l = \dot{\theta}_{l,eq} = \dot{\theta}_{eq}$ are performed, and equations (9), (10) are solved:

$$\begin{aligned} k_{\theta m} \cdot (\bar{\theta}_{u,eq} - \bar{\theta}_{l,eq}) + T_{fum}(\dot{\theta}_{eq}) - T_m &= 0 \\ -k_{\theta m} \cdot (\bar{\theta}_{u,eq} - \bar{\theta}_{l,eq}) + T_{flm}(\dot{\theta}_{eq}) &= 0 \end{aligned}$$

so that:

$$k_{\theta m} \cdot (\bar{\theta}_{u,eq} - \bar{\theta}_{l,eq}) + T_{fum}(\dot{\theta}_{eq}) - T_m = -k_{\theta m} \cdot (\bar{\theta}_{u,eq} - \bar{\theta}_{l,eq}) + T_{flm}(\dot{\theta}_{eq})$$

10 and:

$$(\bar{\theta}_{u,eq} - \bar{\theta}_{l,eq}) = \frac{T_{flm} - T_{fum}(\dot{\theta}_{eq}) + T_m}{2k_{\theta m}}$$

Herein, $(\bar{\theta}_{u,eq} - \bar{\theta}_{l,eq})$ is constant. Formulas (9) and (10) may be used to derive either $(\bar{\theta}_{u,eq} - \bar{\theta}_{l,eq})$, i.e. the output of the model 16 or $(\hat{\theta}_{u,eq} - \hat{\theta}_{l,eq})$, i.e. the output of the
 15 observer 14. Also, $(\theta_{u,eq} - \theta_{l,eq})$ may be derived. Regarding the latter, optionally a measured value for $\theta_{u,eq}$ may be included, for instance provided by sensor 54. In formula (14), k_1, k_2, k_3 are constants calculated according to the control theory of the Doris publication using the model
 20 (9), (10). Example values of these parameters are presented in Table 1.

Formula (14) provides a correction factor to the torque. The corrected torque T_c applied to the drill string, after the adjustment, is for instance:

25
$$T_c = T_m - u$$

This torque is then substituted in equation (1) replacing T_m .

Fig. 2B shows a schematic flow scheme, representing the above described control system of the invention in an alternative form.

5 A driller 50 operates a drilling rig (not shown) comprising the drive system 22. The driller 50 sets the voltage input V by signal 38 to the drive system 22. In response to the voltage signal 38, the drive system 22 will try to rotate the drill string 2 of the drilling system 1 at a reference rotation Ω_{ref} .

Thus, the reference rotation Ω_{ref} is set by the driller, by signal 38. When the drill string system would rotate in equilibrium, the equilibrium rotary speed at surface $\dot{\theta}_{u,eq}$ and downhole $\dot{\theta}_{l,eq}$ would be equal to the set reference rotation Ω_{ref} :

$$\dot{\theta}_{u,eq} = \dot{\theta}_{l,eq} = \Omega_{ref}$$

These values are therefore readily available, and may be provided for instance by the drive system 22, see signal 52.

To rotate the drill string, the drive system 22 provides a motor torque T_m to the drilling system 1. In response to the received motor torque T_m , the drill string and drill bit of the drilling system 1 will rotate. A resulting output vector y may include rotary position and rotary speed both at surface and downhole. However, in the system of the invention, downhole components of the output vector y of the drilling system may be disregarded. Only one or more uphole components, which can be accurately measured, are required. Downhole measurements are obviated.

It is for instance sufficient to measure the rotary speed $\omega_u = \dot{\theta}_u$ at the connection between the drive system and the drilling system, for instance using sensor 54. Sensor 54 may be a separate module, or may be included in the drive system 22. Said surface rotary speed $\omega_u = \dot{\theta}_u$ may be provided to the controller 20. See signal 34.

A measured torque value, for instance the motor torque T_m , may be provided to the model 16 and to the model gain module 18. See signals 24.

In response to receiving the motor torque T_m , the model provides a model output vector y_m . Said model output vector y_m may comprise angular position and rotary speed both at surface and downhole respectively.

The signal 52 may also comprise the value of $(\theta_u - \theta_l)$, which is available due to the relation thereof to the torque T in the drill string 2 at or near surface:

$$T = k_\theta \cdot (\theta_u - \theta_l) \text{ or}$$

$$(\theta_u - \theta_l) = \frac{T}{k_\theta}$$

The torque T can be derived from the current in the drive system 22. In practice the value T is available to the operator 50. Otherwise, T can be measured accurately at or near the connection between the drive system 22 and the drill string 2.

When the drilling system rotates in equilibrium or steady state, the above also provides:

$$(\theta_{u,eq} - \theta_{l,eq}) = \frac{T_{eq}}{k_\theta}$$

The value of $(\theta_{u,eq} - \theta_{l,eq})$ may thus be derived from the torque value when the drilling system operates at equilibrium.

The value $(\theta_u - \theta_l)$ may be provided to the control module 20 via signal 52. Alternatively, the control module 20 may calculate the value $(\theta_u - \theta_l)$ using the torque T as provided by the drive system.

5 The model module 16 may be provided with any suitable model of the drilling system 1. Using the input signal 24, which comprises the motor torque T_m , the model module provides the model output vector y_m .

The model output vector y_m comprises for instance the modelled rotary positions $\hat{\theta}_u, \hat{\theta}_l$ at surface and downhole respectively. Also, y_m may comprise modelled downhole rotary speed $\hat{\dot{\theta}}_l = \omega_{l,m}$. Herein, $\omega_{l,m}$ and $\hat{\dot{\theta}}_l$ are both representations of the modelled downhole rotary speed.

The modelled rotary positions $\hat{\theta}_u, \hat{\theta}_l$ at surface and downhole respectively may be provided to a model gain module 18. See signal 56. The model gain module 18 calculates the gain vector $L = [l_1, l_2, l_3]$, as described above relating to Formula (13). The model gain module provides the gain vector L to the model module 16. See 15 signal 58. The model module 16 uses the gain vector L to improve the parameters of the model, and consequently to improve the output vector y_m .

The values of $\hat{\theta}_u, \hat{\theta}_l, \omega_{l,m}$ as provided by the model module 16, which preferably have been adjusted and improved using 25 the input of the gain module 18, are provided to control module 20. See signal 60.

The control module 20 uses the available inputs (included in signals 34, 52, and 60) in formula (14) to provide a torque correction factor u:

$$30 \quad u = -k_1 \cdot [\hat{\theta}_u - \hat{\theta}_l - (\theta_{u,eq} - \theta_{l,eq})] - k_2 \cdot [\hat{\dot{\theta}}_u - \dot{\theta}_{u,eq}] - k_3 \cdot [\hat{\dot{\theta}}_l - \dot{\theta}_{l,eq}]$$

Until the drilling system is in equilibrium, and given the available inputs, said formula can also be written as:

$$u = -k_1 \cdot [\hat{\theta}_u - \hat{\theta}_l - (\frac{T_m}{k_\theta})] - k_2 \cdot [\omega_u - \Omega_{ref}] - k_3 \cdot [\omega_{l,m} - \Omega_{ref}]$$

The torque correction factor u is provided to the drive system 22, which subtracts said factor u from the motor torque T_m , to arrive at a corrected torque value T_c :

$$T_c = T_m - u$$

The corrected torque T_c is then substituted in equation (1) replacing T_m .

Herein, please note that the reference frequency Ω_{ref} as set by the driller 50 is not affected by the above correction. Rather, the correction is applied to adjust the torque that the drive system applies to the drill string to arrive at said Ω_{ref} .

Reference is further made to Figs. 3a-c showing results for the drill string 2, the model 16 and the observer 14, whereby the controller 20 is de-activated in order to illustrate convergence of the drill string states as determined by the model 16 and the observer 14 to the real drill string states. Fig. 3a shows $\alpha = \theta_u - \theta_l$ as a function of time t . Fig. 3b shows $\omega_u = \dot{\theta}_u$ as a function of time t . Fig. 3c shows $\omega_l = \dot{\theta}_l$ as a function of time t . These figures indicate that the drilling system's state as determined by the observer 16 rapidly converges to the drilling system's real state.

Reference is further made to Figs. 4a-c showing results whereby the controller 20 is activated. Herein the control system operates in closed-loop with the drilling system so as to dampen stick-slip behaviour of the drill string 2. Fig. 4a shows $\alpha = \theta_u - \theta_l$ as a function of time t , Fig. 4b shows $\omega_u = \dot{\theta}_u$ as a function of time t , and Fig. 4c

shows $\omega_i = \dot{\theta}_i$ as a function of time t . These figures demonstrate that the control loop is able to rapidly eliminate the stick-slip behaviour of the drilling system. Rapidly herein implies for instance in less than a minute.

5 Example values of the various parameters discussed hereinbefore are presented in Table 1 below.

 The present invention is not limited by the above-described embodiments thereof, wherein many modifications are conceivable within the scope of the appended claims.
10 Features of respective embodiments may for instance be combined.

Table 1

	Parameter	Value	Unit
Drilling system	J_u	0.4765	$kg \cdot m^2$
	J_l	0.0414	$kg \cdot m^2$
	T_{su}	0.37975	$N \cdot m$
	ΔT_{su}	-0.00575	$N \cdot m$
	b_u	2.4245	$kg \cdot m^2 / rad \cdot s$
	Δb_u	-0.0084	$kg \cdot m^2 / rad \cdot s$
	k_θ	0.0775	$N \cdot m / rad$
	T_{sl}	0.2781	$N \cdot m$
	T_{cl}	0.0473	$N \cdot m$
	ω_{st}	1.4302	rad / sec
	δ_{st}	2.0575	[-]
	b_l	0.0105	$kg \cdot m^2 / rad \cdot s$
Model	$k_{\theta m}$	0.0787	$N \cdot m / rad$
	J_{um}	0.5003	$kg \cdot m^2$
	T_{sum}	0.3987	$N \cdot m$
	b_{um}	2.5457	$kg \cdot m^2 / rad \cdot s$
	J_{lm}	0.0455	$kg \cdot m^2$
	T_{clm}	0.052	$N \cdot m$
	b_{lm}	0.0116	$kg \cdot m^2 / rad \cdot s$
Controller	k_1	14.5	$N \cdot m / rad$
	k_2	1.5	$N \cdot m \cdot sec / rad$
	k_3	30	$N \cdot m \cdot sec / rad$
Observer	l_1	13.5751	-
	l_2	-4.458	-
	l_3	-152.204	-

C L A I M S

1. A control system for controlling vibrations in a drilling system, the drilling system (1) including an elongate body extending from surface into a borehole formed in an earth formation and a drive system (22) for providing a drive torque (T_m) to the elongate body for rotating said elongate body at a reference frequency (Ω_{ref}), the control system comprising:

- a sensor module (54) for determining at least one uphole parameter of the drilling system;

- a model module (16) provided with a model of the drilling system (1), the model module being adapted to provide modeled parameters of the drilling system using the drive torque (T_m) as an input;

- a model gain module (18) for providing a model gain vector (L) to the model module (16) in response to one or more of the modeled parameters and the drive torque (T_m), the model gain vector enabling the model module (16) to update the model thereby obtaining an updated model; and

- a control module (20) for providing a torque correction factor (u) to the drive system (22) depending on the modeled parameters, the uphole parameter, the reference frequency (Ω_{ref}), and the drive torque (T_m).

2. The system of claim 1, wherein the modeled parameters include:

- modeled uphole angular position ($\hat{\theta}_u$);

- modeled downhole angular position ($\hat{\theta}_l$); and

- modeled downhole rotary velocity ($\omega_{l,m}$).

3. The system of claim 1 or 2, wherein the uphole parameter of the drilling system as determined by the

sensor module (54) comprises the uphole rotary velocity (ω_u).

4. The system of any of the previous claims, wherein the control module (20) is adapted to determine a difference in uphole angular position and downhole angular position ($\theta_u - \theta_l$) using the drive torque (T_m).

5. The system of claim 4, wherein the difference in uphole angular position and downhole angular position ($\theta_u - \theta_l$) is determined by formula:

$$(\theta_u - \theta_l) = \frac{T_m}{k_\theta}$$

wherein k_θ is a constant.

6. The system of any of the previous claims, wherein the control module (20) is provided with the following formula to calculate the torque correction factor (u):

$$u = -k_1 \cdot [\hat{\theta}_u - \hat{\theta}_l - (\frac{T_m}{k_\theta})] - k_2 \cdot [\omega_u - \Omega_{ref}] - k_3 \cdot [\omega_{l,m} - \Omega_{ref}]$$

wherein k_1, k_2, k_3 and k_θ are constants.

7. A method of controlling vibrations in a drilling system, the drilling system (1) including an elongate body extending from surface into a borehole formed in an earth formation and a drive system (22) for providing a drive torque (T_m) to the elongate body for rotating said elongate body at a reference frequency (Ω_{ref}), the method comprising the steps of:

- providing the reference frequency (Ω_{ref}) to the drive system (22);
- the drive system (22) providing the drive torque (T_m) to the elongate body of the drilling system (1);
- determining at least one uphole parameter of the drilling system (1);

- providing the drive torque (T_m) to a model module (16) which is provided with a model of the drilling system (1), the model module providing modeled parameters of the drilling system;

5 - providing one or more of the modeled parameters and the drive torque (T_m) to a model gain module (18) for providing a model gain vector (L) to the model module (16) in response thereto;

10 - obtaining an updated model by the model module (16) using the model gain vector; and

- providing the modeled parameters, the uphole parameter, the reference frequency (Ω_{ref}), and the drive torque (T_m) to a control module (20);

15 - the control module (20) providing a torque correction factor (u) to the drive system (22) to correct the drive torque (T_m).

8. The method of claim 7, wherein the modeled parameters include:

- modeled uphole angular position ($\hat{\theta}_u$);

20 - modeled downhole angular position ($\hat{\theta}_l$); and

- modeled downhole rotary velocity ($\omega_{l,m}$).

9. The method of claim 7 or 8, wherein the uphole parameter of the drilling system comprises the uphole rotary velocity (ω_u).

25 10. The method of any of claims 7-9, wherein the control module (20) determines a difference in uphole angular position and downhole angular position ($\theta_u - \theta_l$) using the drive torque (T_m).

11. The method of claim 10, wherein the difference in uphole angular position and downhole angular position ($\theta_u - \theta_l$) is determined by formula:

$$(\theta_u - \theta_l) = \frac{T_m}{k_\theta}$$

5 wherein k_θ is a constant.

12. The method of any of claims 7-11, wherein the control module (20) calculates the torque correction factor (u) using formula:

$$u = -k_1 \cdot [\hat{\theta}_u - \hat{\theta}_l - (\frac{T_m}{k_\theta})] - k_2 \cdot [\omega_u - \Omega_{ref}] - k_3 \cdot [\omega_{l,m} - \Omega_{ref}]$$

10 wherein k_1, k_2, k_3 and k_θ are constants.

13. The method of any of claims 7-12, including the step of:

Replacing the drive torque (T_m) with a corrected drive torque (T_c), using formula:

15
$$T_c = T_m - u .$$

14. The method of any of claims 7-13, including the step of using only parameters which can be measured or modeled uphole.

15. A method of controlling vibrations in a drilling system, the drilling system including an elongate body extending from surface into a borehole formed in an earth formation and a drive system for rotating the elongate body by providing a drive torque to the elongate body, the method comprising:

20

25 a) operating the drive system to provide a drive torque to the elongate body, and determining a system parameter that relates to an uphole parameter of the drilling system;

b) obtaining a model of the drilling system;

c) applying the model to determine a modeled system parameter that corresponds to said system parameter;

d) determining a difference between the system parameter and the modeled system parameter;

5 e) updating the model in dependence of said difference, thereby obtaining an updated model;

f) determining from the updated model at least one modeled parameter of rotational motion, and adjusting the drive torque in dependence of each modeled parameter of rotational motion to control vibrations of the elongate body.

16. The method of claim 15, wherein said uphole parameter of the drilling system relates to an uphole torque in the drilling system.

15 17. The method of claim 15 or 16, wherein said uphole parameter of the drilling system relates to torque (T) in the elongate body at or near the earth's surface.

18. The method of claim 16 or 17, wherein said model of the drilling system includes a modelled torsional stiffness ($k_{\theta m}$) of the elongate body, and wherein said drilling parameter comprises a ratio of said torque (T) over said modelled torsional stiffness ($k_{\theta m}$).

19. The method of claim 15, wherein said modeled system parameter relates to a modeled difference between an uphole rotational position of the elongate body and a downhole rotational position of the elongate body.

20. The method of claim 15, wherein said uphole parameter of the drilling system is a first uphole parameter, and wherein step (c) comprises applying the model using an input parameter relating to a second uphole parameter of the drilling system.

21. The method of claim 20, wherein the drive system comprises a rotary drive coupled to an uphole end of the

elongate body, and wherein said second uphole parameter is or comprises torque (T_m) provided by the rotary drive to said uphole end of the elongate body.

5 22. The method of claim 15, wherein the model includes at least one modeled state parameter and wherein step (e) comprises adding to each modeled state parameter the product of said difference and a respective gain factor pertaining to the modeled state parameter.

10 23. The method of claim 22, wherein each modeled state parameter relates to a modeled parameter of rotational motion of the elongate body.

15 24. The method of claim 22, wherein said at least one modeled state parameter is selected from a modeled difference between an uphole angular velocity and a downhole angular velocity of the elongate body, a modeled uphole angular acceleration of the elongate body, and a modeled downhole angular acceleration of the elongate body.

20 25. The method of claim 22, wherein step (b) comprises obtaining a state observer in which the model is included, the state observer further including a gain module for calculating each said gain factor.

25 26. The method of claim 15, wherein said at least one modeled parameter of rotational motion includes at least one of a modeled difference between an uphole rotational position and a downhole rotational position of the elongate body, a modeled uphole angular velocity of the elongate body, and a modeled downhole angular velocity of the elongate body.

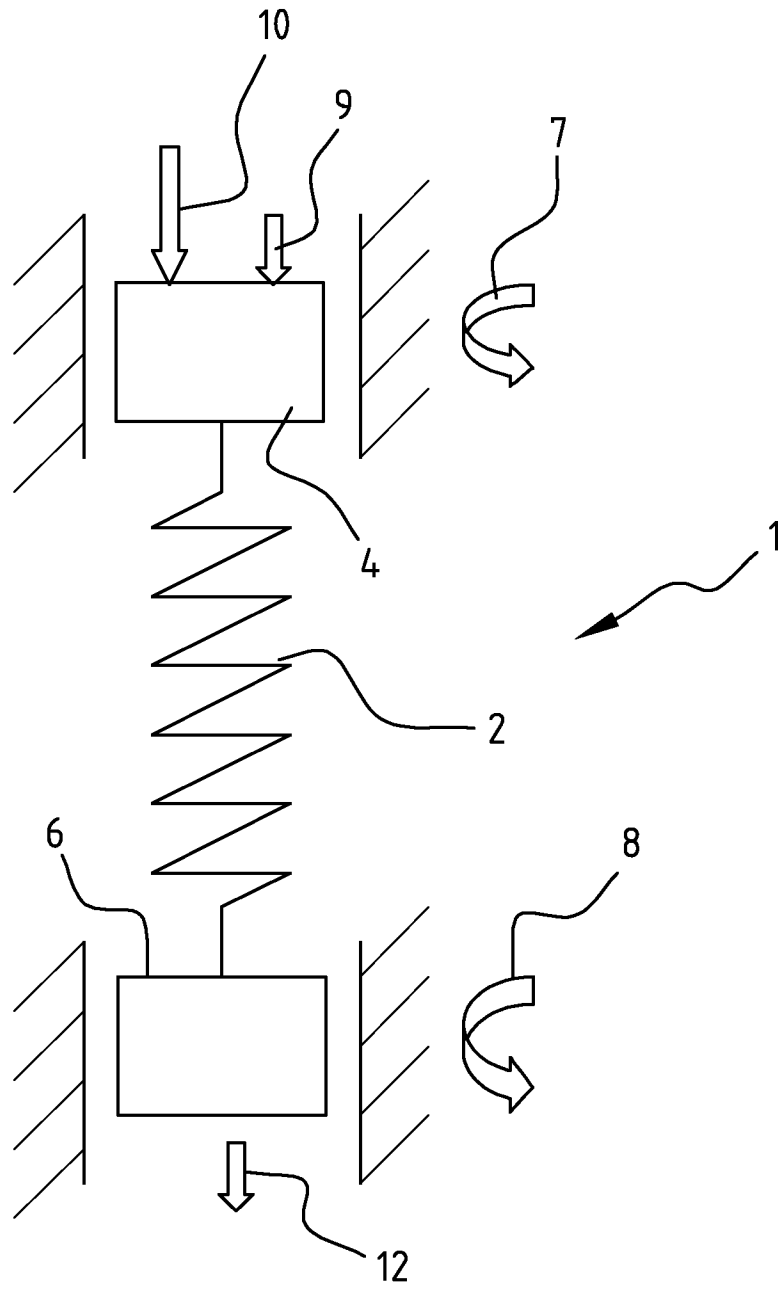


FIG. 1

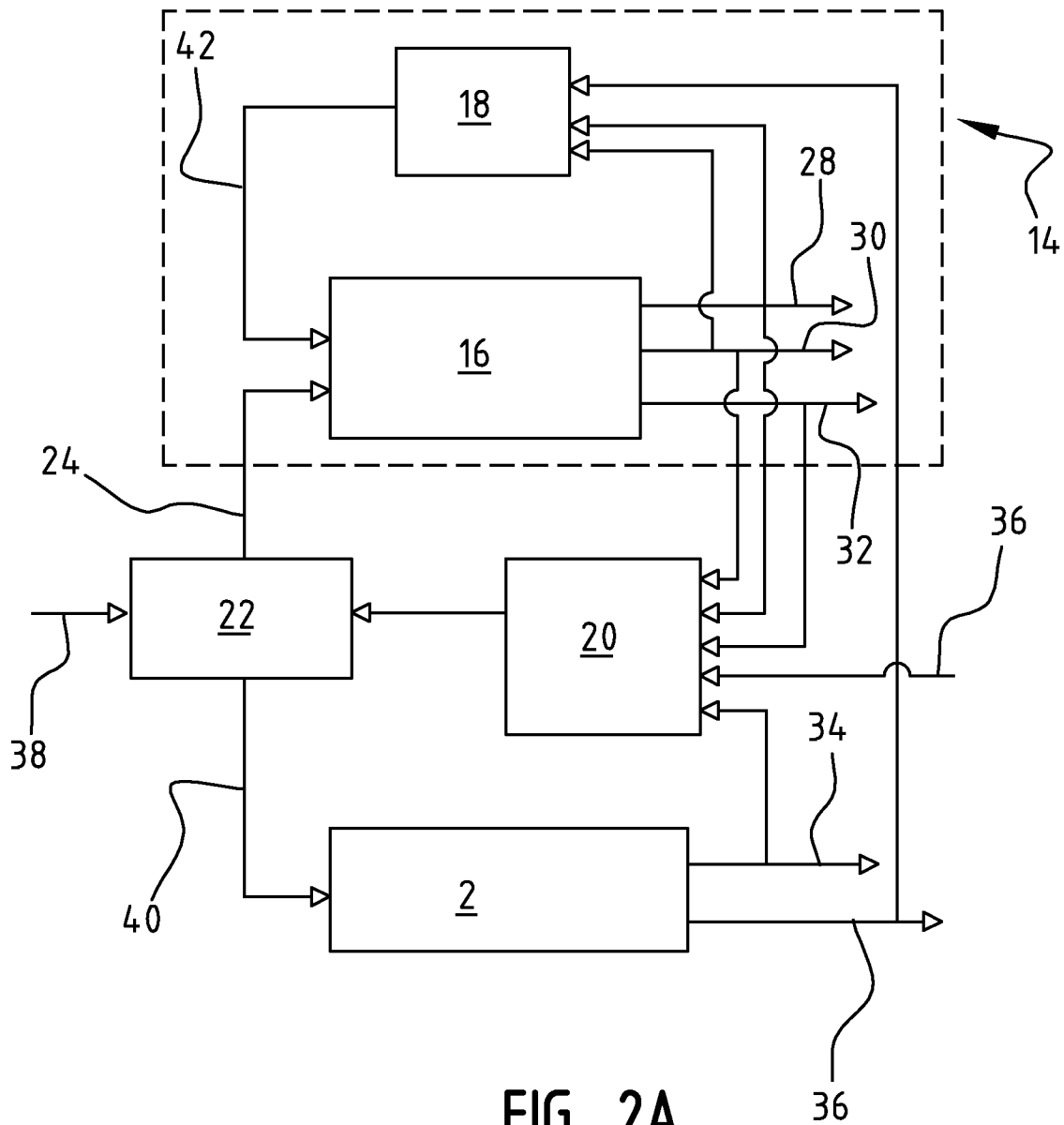


FIG. 2A

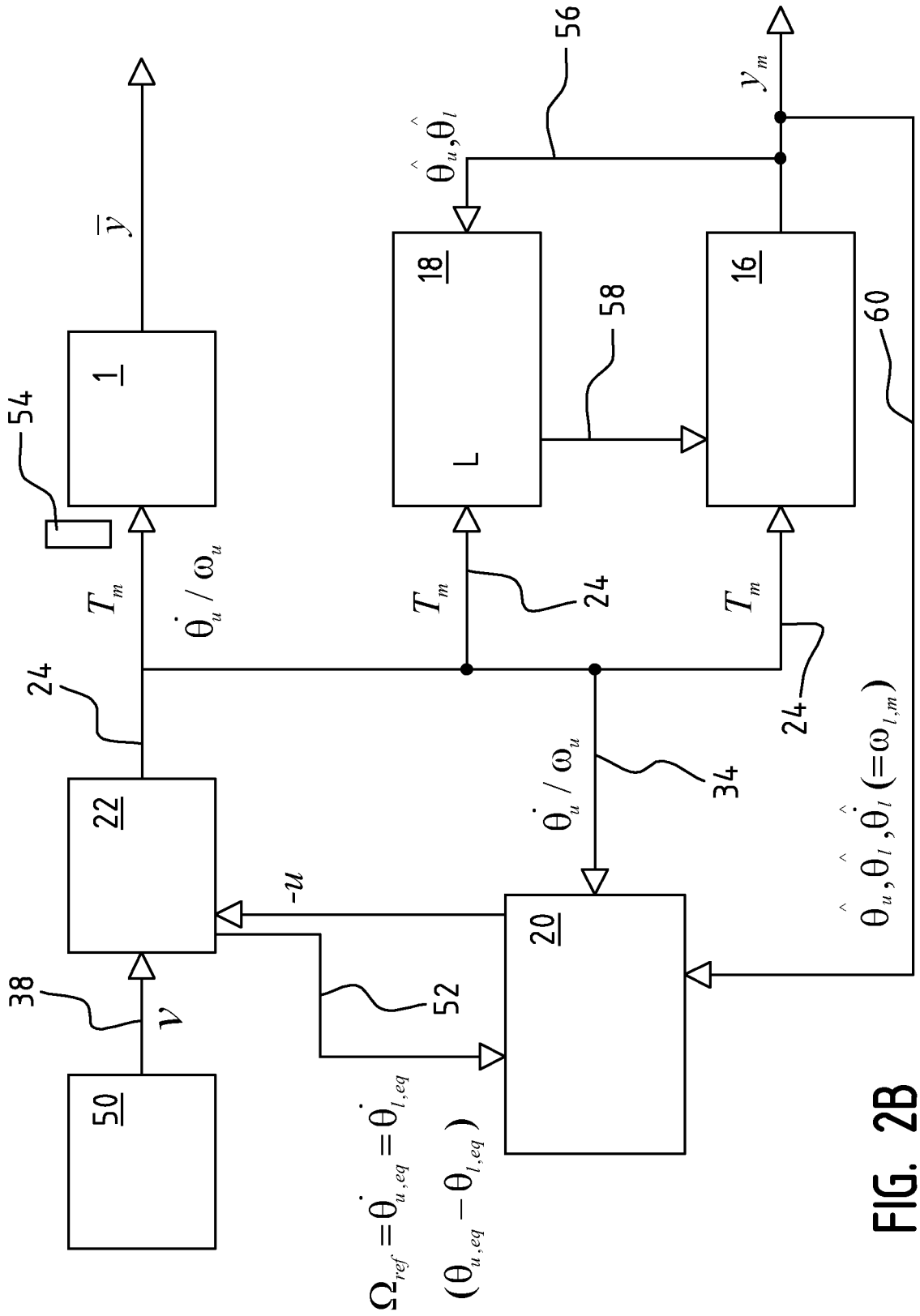


FIG. 2B

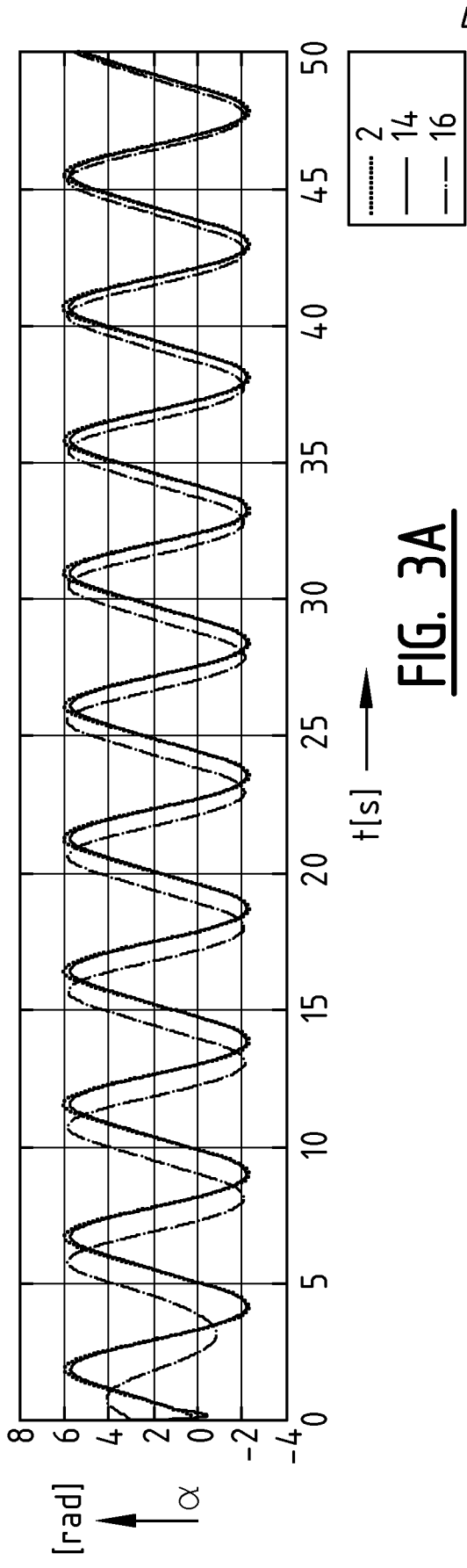


FIG. 3A

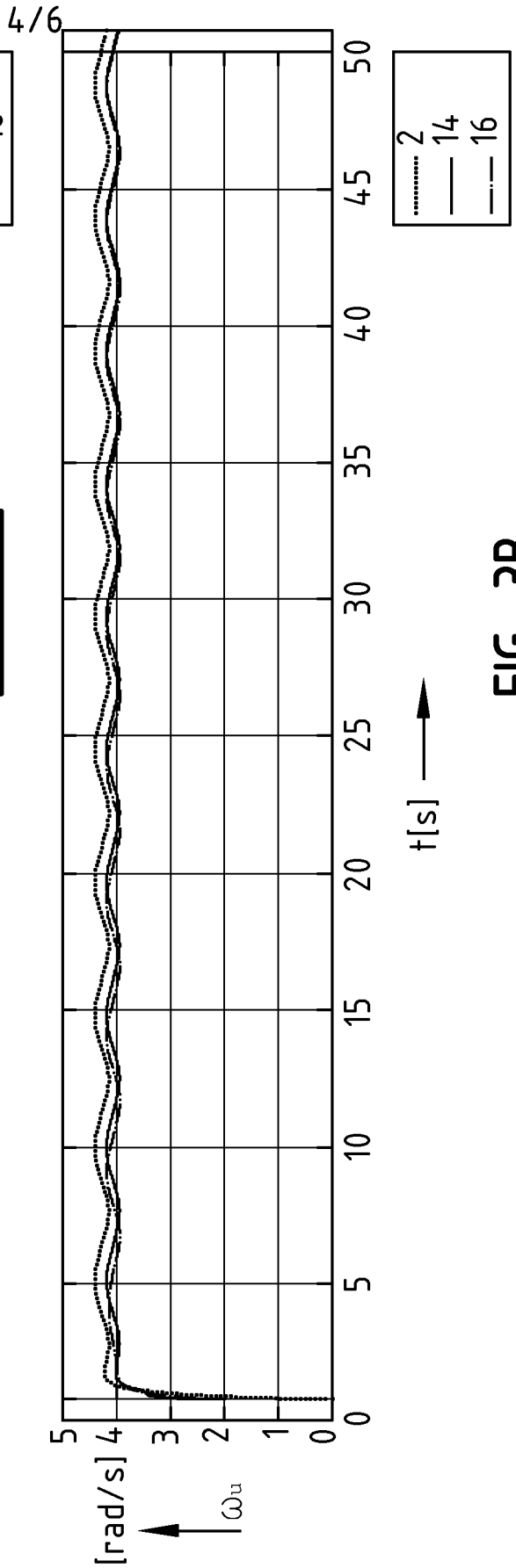


FIG. 3B

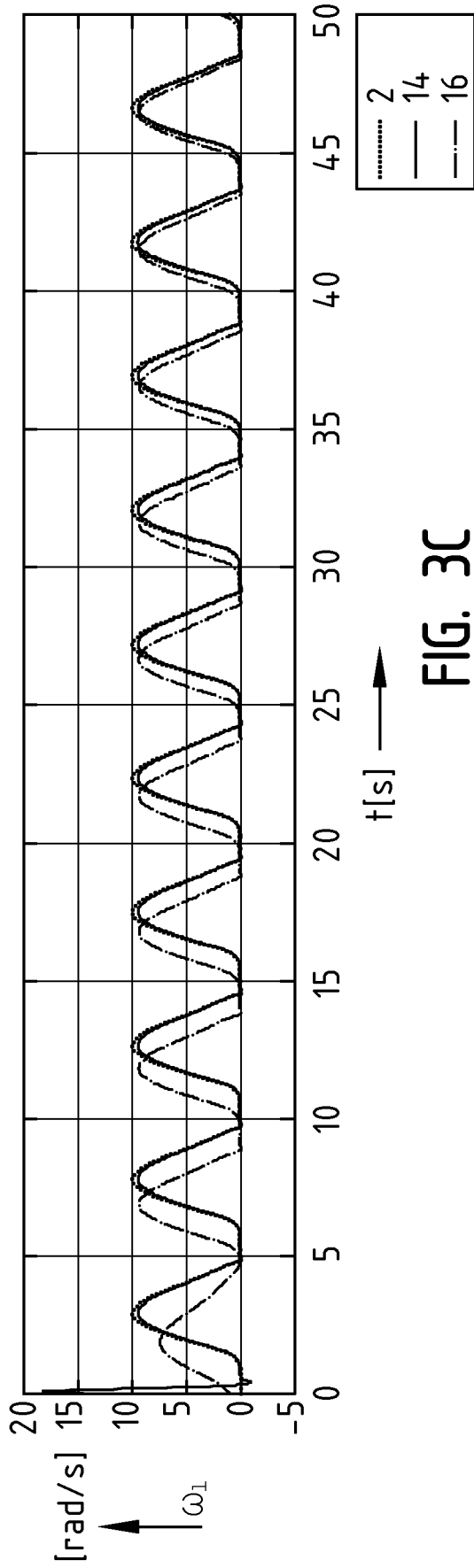


FIG. 3C

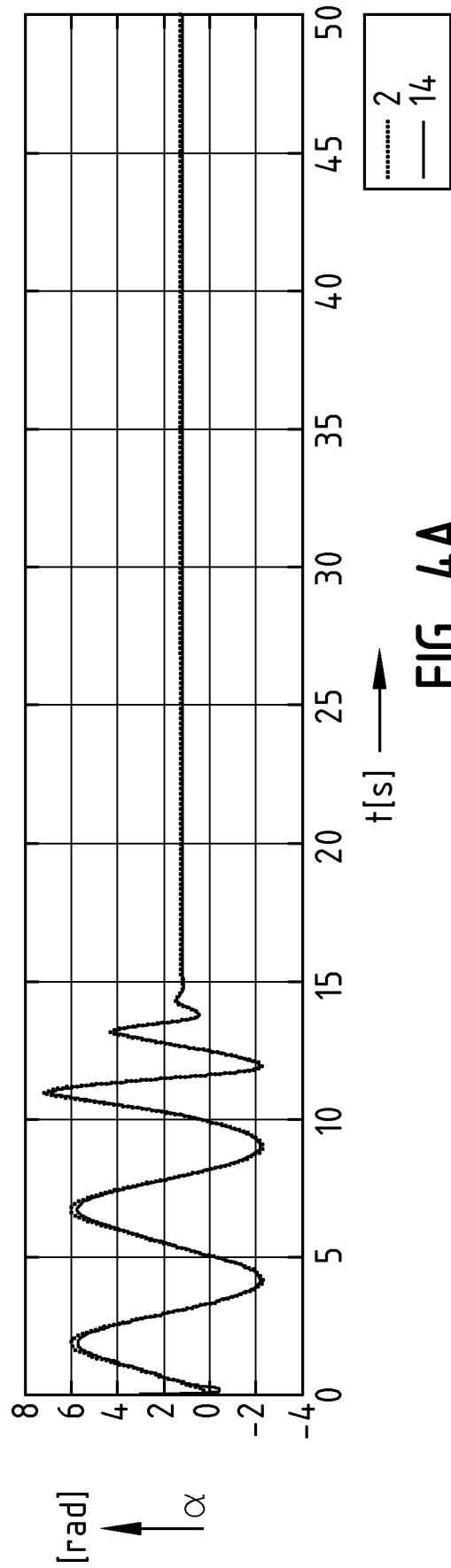


FIG. 4A

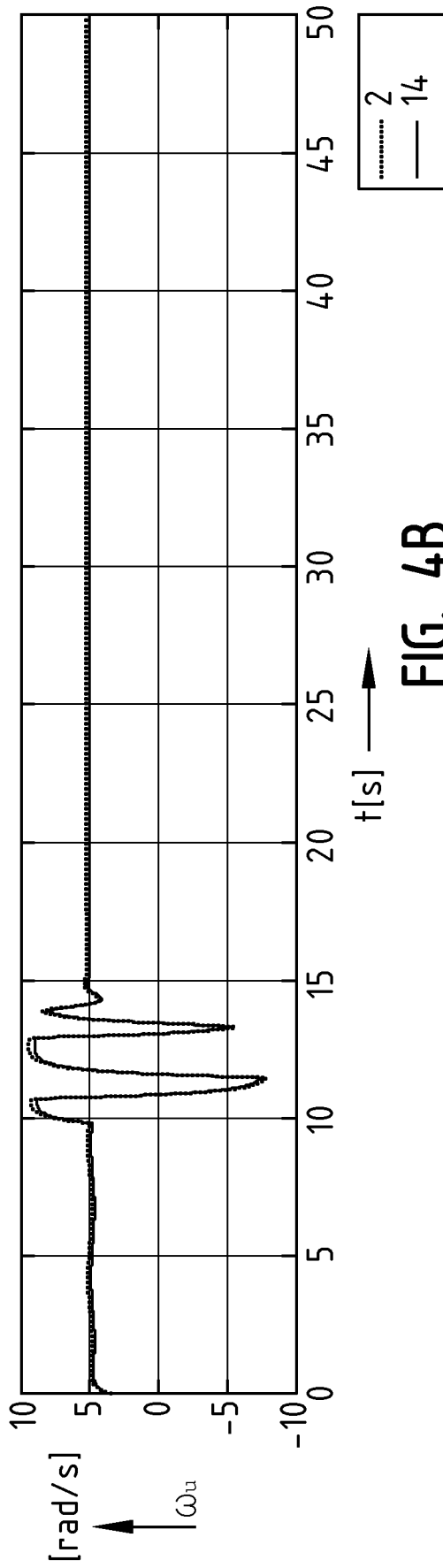


FIG. 4B

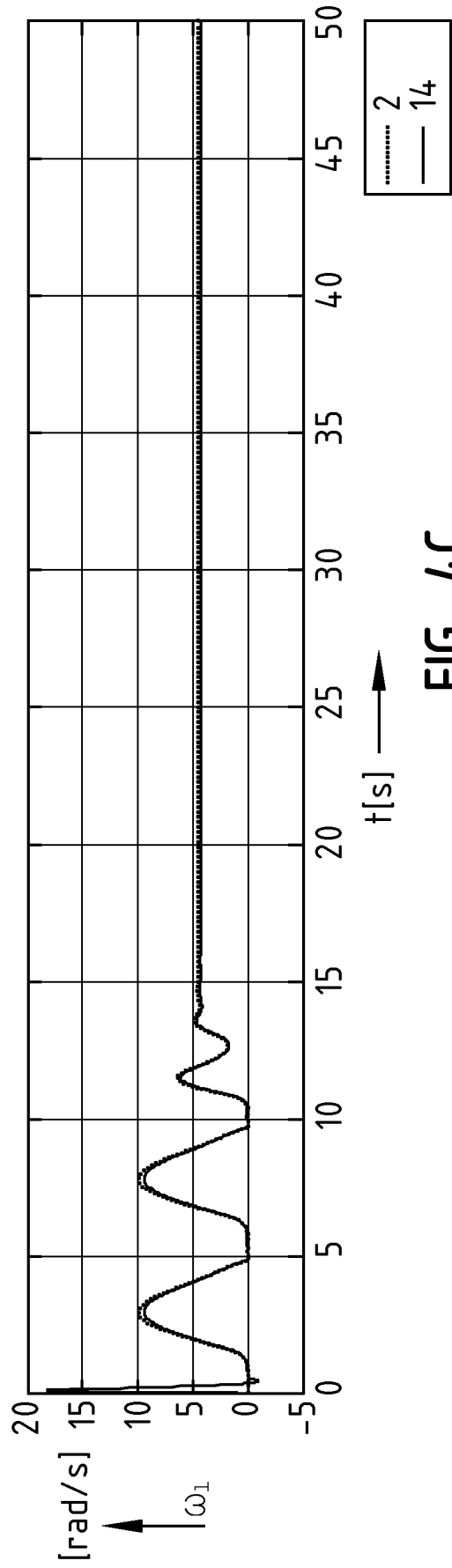


FIG. 4C