United States Patent

[11] 3,605,106

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[54] **SLOT FITTING OF CODED LINEAR ARRAY ANTENNA**
5 Claims, 4 Drawing Figs.

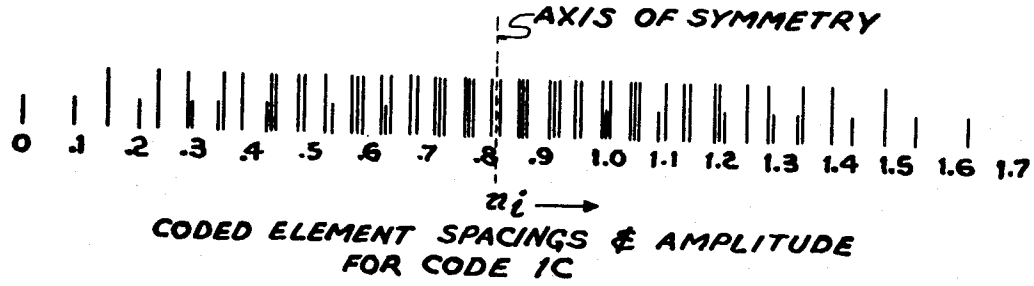
[52] **U.S. Cl.** **343/844**
[51] **Int. Cl.** **H01q 21/00**
[50] **Field of Search** 343/844, 853, 854

[56] **References Cited**
UNITED STATES PATENTS

3,182,330 5/1965 Blume 343/844

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ABSTRACT: There is provided a method for fitting the irregular radiator spacing as disclosed in U.S. Pat. No. 3,130,410 into a regular grid by approximation. This permits the construction of a space-coded array in a simpler and less costly manner.



CODED ELEMENT SPACINGS & AMPLITUDE FOR CODE 1C

PATENTED SEP 14 1971

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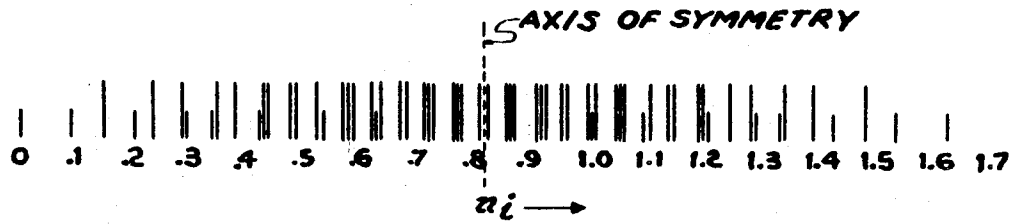
SHEET 1 OF 2

Fig. 1

CODE 1C

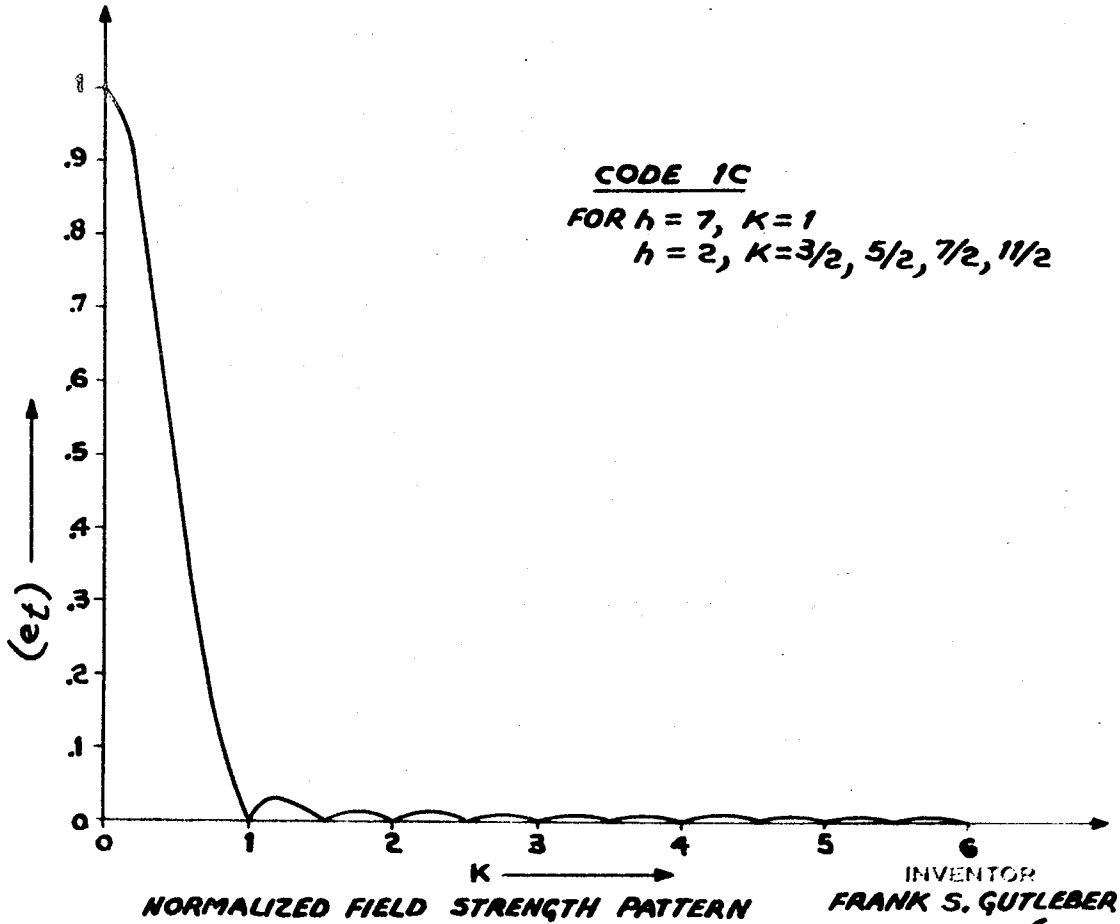
FOR $h=7$, $K=1$

$h=2$, $K=3/2, 5/2, 7/2, 11/2$



CODED ELEMENT SPACINGS & AMPLITUDE FOR CODE 1C

Fig. 2



CODE 1C

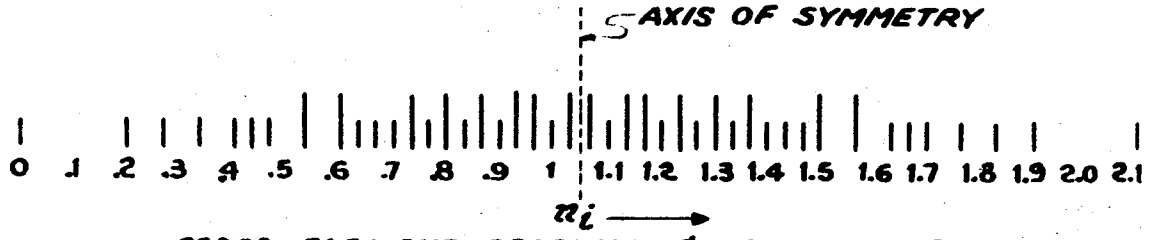
FOR $h=7$, $K=1$

$h=2$, $K=3/2, 5/2, 7/2, 11/2$

NORMALIZED FIELD STRENGTH PATTERN

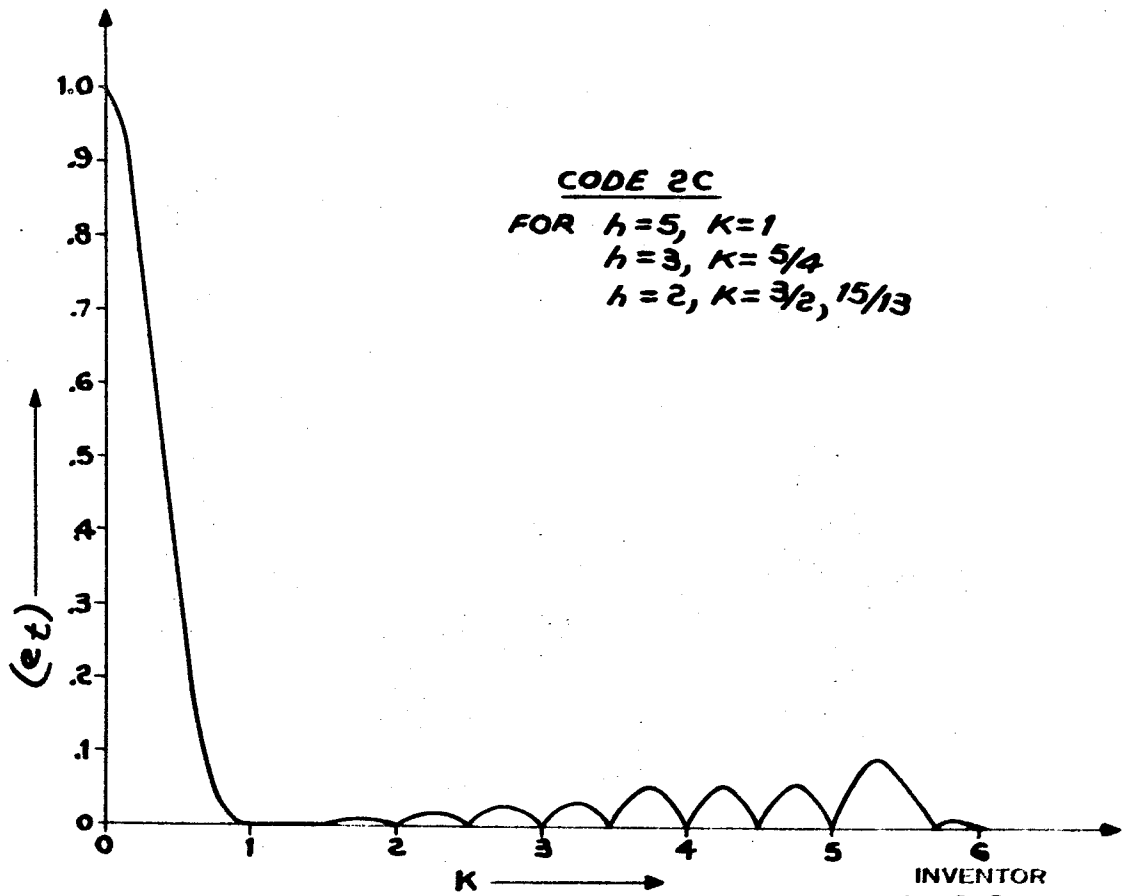
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Fig. 3



CODED ELEMENT SPACINGS & AMPLITUDES FOR CODE 2C

Fig. 4



NORMALIZED FIELD STRENGTH PATTERN

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SLOT FITTING OF CODED LINEAR ARRAY ANTENNA

CROSS-REFERENCE TO RELATED APPLICATIONS

This application is related to and incorporates by reference herein U.S. Pat. No. 3,130,410 issued to Frank S. Gutleber on Apr. 21, 1964, and entitled "Space Coded Linear Array Antenna."

BACKGROUND OF THE INVENTION

In general this invention is related to slot fitting as applied to a general analysis of designing a coded linear array antenna, and more particularly to a method of forcing antenna coded positions into existing increments for the general array design solution.

As a general rule, the forcing of zeros in the pattern of an antenna increases the reactive energy of the antenna, which results in increased complex mutual impedance among the elements This requires the design of a more complicated feed network to achieve the specified element currents. The application of a small amount of zero forcing may be helpful but as the number of zeros go up, there is forced an inefficient "-super gain" operation.

SUMMARY OF THE INVENTION

It is therefore an object of this invention to provide a method of fitting the irregular radiator spacing as defined by U.S. Pat. No. 3,130,410 into a regular grid by approximation.

It is also an object of the invention to provide a method which makes the construction of a coded array simpler and less expensive.

Accordingly, there is shown that instead of adding one additional element for each existing element in an array in order to force a zero in an antenna pattern, n elements may be added whose vector fields are the n–complex roots of unity.

BRIEF DESCRIPTION OF THE DRAWINGS

The following description will best be understood if reference is made to the drawings of U.S. Pat. No. 3,130,410 in conjunction with the following drawings in which:

FIG. 1 shows the code element positions for code 1C obtained by the repeated applications of equations (6);

FIG. 2 shows the resulting normalized field strength pattern for the code 1C of FIG 1;

FIG. 3 shows code element spacing and amplitudes for code 2C; and

FIG. 4 shows the resulting normalized plot of the antenna field strength pattern of FIG 3.

BRIEF DESCRIPTION OF PREFERRED EMBODIMENTS

Referring now to reference U.S. Pat. No. 3,130,410, it is shown that side-lobe control of array antennas can be accomplished through the expedient of amplitude and/or space coding the antenna elements, i.e. adding a second element of each existing element at a space position which ensures a 180° phase difference between each pair of elements for some specific value of space angle θ. A more general solution would enable adding n elements to each existing element to achieve a design zero at some desired value of space angle θ. It is apparent from the reference patent that the following conditions must exist in order to realize the unique properties contained in the original work (1) all original design zeros must be retained as new design zero sets are established; and (2) each design value corresponding to a specific value of the space angle θ should result in a known cyclic set of zero positions.

These conditions are satisfied for a class of vectors which comprise the h roots of unity. In fact, the cross reference involved adding vectors whose phase corresponded to the square root of unity (h=2) in the complex number domain. That analysis is in essence a case of the following more general analysis.

The referenced patent illustrated that the antenna field pattern can be represented by the following equation:

$$E_t = e^{j n_1 \psi} + e^{j n_2 \psi} + \ldots + e^{j n_m \psi} \tag{1}$$

where n_1 = the relative code positions,

$$\psi = \frac{2\pi}{\lambda} d \sin\theta$$

and since

$$K = \frac{d \sin\theta}{\lambda}$$

$$\psi = 2\pi K$$

Therefore,

$$E_t = e^{j n_1 2\pi K} + e^{j n_2 2\pi K} + \ldots + e^{j n_m 2\pi K} \tag{2}$$

now for each general term

$$e^{j n 2\pi K}$$

which exists, a set of vectors

$$\sum_{i=1}^{(h-1)} e^{j n'_i 2\pi K}$$

can be added such that the total summation of the added vectors with the original vector is equal to zero for a specific value of θ or K. This condition is realized when the arguments of the added vectors comprise the h^{th} roots of unity:

$$\sum_{i=1}^{(h-1)} e^{j n'_i 2\pi K} = \sum_{i=1}^{(h-1)} e^{j\left(n 2\pi K + \frac{2\pi}{h} i\right)} \tag{3}$$

therefore,

$$\sum_{i=1}^{(h-1)} n'_i 2\pi K = n 2\pi K + \sum_{i=1}^{(h-1)} \frac{2\pi i}{h} \tag{4}$$

and

$$\sum_{i=1}^{(h-1)} n'_i = n + \sum_{i=1}^{(h-1)} \frac{i}{hK} \tag{5}$$

Equation (5) identifies the required coded element positions which will result in a null or a zero at any specific value of θ or K desired. This equation is expanded into the following set for the purpose of calculating the exact required coded positions.

$$\begin{aligned} n'_1 &= n + \frac{1}{hK} \\ n'_2 &= n + \frac{2}{hK} \\ &\vdots \\ n'(h-1) &= n + \frac{(h-1)}{hK} \end{aligned} \tag{6}$$

Note that for h=2, the set reduces to the equation given by

$$n' = n + \frac{1}{2K}$$

this being the equation (12) derived in the referenced patent.

It will now be proved that zeros will occur for K = all integers times the design values of K (K=K_o) with the exclusion of $K_o h$ times all integers. The general characteristics of the h^{th} roots of unity are given by:

$$1,\ e^{j\frac{2\pi}{h}},\ \Sigma^{j\frac{2\pi}{h}(2)},\ \ldots,\ \Sigma^{j\frac{2\pi}{h}(h-1)}$$

And a group of vectors given by any of these roots satisfies the following conditions:

$$1+\gamma+\gamma^2+\text{----}+\gamma^{h1}=h \text{ for } \gamma=1 \tag{7}$$

$$1+\gamma+\gamma^2+\text{----}+\gamma^{h1}=0 \text{ for } \gamma\ 1 \tag{8}$$

where γ = any of the h^{th} roots of unity.

A set of vectors whose positions are established from equation (6) results in the following expression for the field strength pattern:

$$E=1+e^{j\frac{2\pi}{h}\frac{K}{(K_o)}}+e^{j\frac{2\pi}{h}(2)\frac{K}{(K_o)}}+\ldots+e^{j\frac{2\pi}{h}(h-1)\frac{K}{(K_o)}} \quad (9)$$

letting

$$\frac{K}{(K_o)}=(k+n)$$

results in

$$E=1+e^{jn\frac{2\pi}{h}}\left[e^{j\frac{2\pi}{h}k}+e^{j\frac{2\pi}{h}2k}+\ldots+e^{j\frac{2\pi}{h}(h-1)k}\right] \quad (10)$$

which from the properties of the roots of unity is equal to zero for:

$n=0, h, 2h,$ ---- etc.
when $k=1, 2, 3$----$(h-1)$

Therefore, zeros will occur for all K equal to

$K=K_ok, K_o(k+h), K_o(k+2h)$ ---- etc.,

this verifies that zeros occur at all integers times K_o with the exclusion of:

$K_oh, 2K_oh, 3K_oh,$ ------ etc.

Note that for $h=2$, the above result corresponds to zeros occurring at all odd integers times K_o which was proven in the referenced patent for the case of $h=2$.

It will now be shown that all original design zeros are retained as the design progresses. Consider the first design zero occurring at $K=a$, $h=h_1$. The antenna field strength would then be given by:

$$E=1+e^{j\frac{1}{h_1a}2\pi K}+\ldots+e^{j\frac{h_1-1}{h_1a}2\pi K} \quad (11)$$

If now a set of h vectors is added to each of the existing vectors to yield a zero at $K=b$, the field strength would be given by:

$$E=1\left[1+e^{j\frac{1}{h_2b}2\pi K}+\ldots+e^{j\frac{(h_2-1)}{h_2b}2\pi K}\right]+e^{j\frac{1}{h_1a}2\pi K}\left[1+e^{j\frac{1}{h_3b}2\pi K}+\ldots+e^{j\frac{(h_3-1)}{h_3b}2\pi K}\right]+\ldots+e^{j\frac{h_1-1}{h_1a}2\pi K}\left[1+e^{j\frac{1}{h_nb}2\pi K}+\ldots+e^{j\frac{(h_n-1)}{h_nb}2\pi K}\right] \quad (12)$$

And $E=0$ for $K=b, 2b, 3b$ · · · etc.,
with the exclusion of $bh_2, b2h_2, b3h_2$ · · · etc.;
provided that $h_2=h_3=\ldots=h_n$.
For this condition, Equation 12 can be expressed as:

$$E=\left[1+e^{j\frac{1}{h_1a}2\pi K}+\ldots+e^{j\frac{h_1-1}{h_1a}2\pi K}\right]\left[1+e^{j\frac{1}{h_2b}2\pi K}+\ldots+e^{j\frac{(h_2-1)}{h_2b}2\pi K}\right] \quad (13)$$

And more generally for "n" forced zeros:

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$$E=\left[1+e^{j\frac{1}{h_1a}2\pi K}+\ldots+e^{j\frac{h_1-1}{h_1a}2\pi K}\right]\left[1+e^{j\frac{1}{h_2b}2\pi K}+\ldots+e^{j\frac{(h_2-1)}{h_2b}2\pi K}\right]\left[1+e^{j\frac{1}{h_3c}2\pi K}+\ldots+e^{j\frac{(h_3-1)}{h_3c}2\pi K}\right]\ldots\left[1+e^{j\frac{1}{h_nn}2\pi K}+\ldots+e^{j\frac{(h_n-1)}{h_nn}2\pi K}\right] \quad (14)$$

Yielding zeros at all K equal to:
$K=a, 2a, 3a,$ --- etc.
$=b, 2b, 3b,$ ---- etc.
$=c, 2c, 3c,$ · · · etc.
·
·
·
$=n, 2n, 3n,$ · · · etc.

with the exclusion of the points given by
$K=h_1a, 2h_1a, 3h_1a,$ ---------etc.
$h_2b, 2h_2b, 3h_2b,$ ---------- etc.

$h_3c, 2h_3c, 3h_3c,$ · · · etc.
·
·
·
$h_nn, 2h_nn, 3h_nn,$ · · · etc.

The following design example (code 1C) is presented to verify the results of the analysis in addition to demonstrating what results are realized for complete equispacing of the zero points on the normalized plot of E vs. K.

The design K's are given by:
For $h=7$, $K=1$
$h=2$, $K=3/2, 5/2, 7/2$ and $11/2$.
This choice yields anticipated zeros at
$K=1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5, 5.5,$ and 6.

The code element positions were obtained with the repeated application of equations (6) and are shown in FIG. 1. The above choice of design K's and h's yielded 64 code positions with a digitized coded amplitude level of 2. For some applications this might not represent a practical solution, however, the intent was to demonstrate the validity of the analysis in addition to the versatility contained in the general design of procedure. The resulting normalized field strength pattern for this code is shown in FIG. 2. Note that the side lobe levels are very nearly equal in the region of interest with the exception of the first side lobe. This has a value of 0.0298 or is down from the main lobe by approximately 31 db.

The remaining lobes are down by more than 40 db. Another zero formed in the vicinity of unity would reduce the adjacent lobe by a very large additional factor. This is apparent from the results of the reference patent, and shows that the general theory does perform as anticipated.

A modification of the cross-referenced analysis pertaining to space-coding a linear array antenna yielded a solution which forced the code positions to occupy some integral value of a specified minimum separation. Now in a similar manner, this analysis can be extended to ensure that resulting code positions fall into preselected code spacings. The analysis and constraints for realizing a preselecting code-spacing design is presented below. In addition, a specific design example is included to verify the theory and illustrate the increased flexibility contained in the solution. The general equation which established the relative code spacings was given by:

$$n'_i=n+\frac{i}{hk} \quad (15)$$

where $i=1, 2, \ldots, (h-l)$
Hence $n'd=nd+\frac{id}{hK}$
Or the difference between two code positions "ΔS" is:
$\Delta S=n'd-nd=\frac{id}{hK}$
Since "ΔS" is a minimum for $i=1$, $\Delta S=\frac{d}{hK}$
Also

$$d=\frac{\lambda}{\sin\theta_o}$$

Therefore

$$S=\frac{\lambda}{hK\sin\theta_o} \quad (16)$$

Now to accomplish slot fitting requires that ΔS be restricted to some specified fraction of a wavelength times an integer.
i.e., $S=(\lambda/X)\times N$ (17)
where λ = wavelength
λ/X = desired minimum spacing expressed as a fraction of a wavelength
N = any integer
Combining equations (16) and (17) results in:

$$K=\frac{X}{hN\sin\theta_0} \tag{18}$$

This is the desired constraint equation for the general array coding technique. Let us write this more compactly by substituting

$$D=\frac{X}{\sin\theta_0}$$

Then

$$K_{avail.}=\frac{D}{hN} \tag{19}$$

Equation (19) defines all the available design K's which can be utilized for a given value of minimum spacing λ/X and a given value of space angle θ. In this solution, different available design K's may exist as different quantities of vectors are added to existing vectors to force the desired zero points in the antenna pattern.

A typical constrained design example will now be considered to clarify the significance of the constraint equation in addition to illustrating that better antenna patterns may be realized by this solution. In order to be able to compare the results obtained here with that obtained before, (code 2B), a value of $D=30$ will be considered. This corresponds to a minimum spacing equal to $\lambda/5$ when $\sin\theta_0=1/6$.

now, or $h=2$,

$$K_{avail.}=\frac{30}{2N}=\frac{15}{N}$$

$$=15,\ \frac{15}{2},\ 5,\ \frac{15}{4},\ 3\frac{5}{2},\ \frac{15}{7},\ \frac{15}{8},\ \frac{5}{3},\ \frac{3}{2},\ \frac{15}{11},\ \frac{5}{4},\ \frac{15}{13},\ \frac{15}{14},\ 1.$$

For $h=3$

$$K_{avail.}=\frac{30}{3N}=\frac{10}{N}$$

$$=10,\ 5,\ \frac{10}{3},\ \frac{5}{2},\ 2,\ \frac{5}{3},\ \frac{10}{7},\ \frac{5}{4},\ \frac{10}{9},\ 1.$$

For $h=5$,

$$K_{avail.}=\frac{30}{5N}=\frac{6}{N}$$

$$=6,\ 3,\ 2,\ \frac{3}{2},\ \frac{6}{5},\ 1.$$

Utilizing the following design values for K and h results in equispacing the zeros on the normalized plot of field strength vs. K with the exception of not obtaining a zero in the vicinity of 5.5 This code is designated as code 2C.

Code 2C

For $h=5$ $K=1$

For $h=3$ $K=5/4$

For $h=2$ $K=3/2$ and $15/13$.

Applying equation (15) to these choices of design K's and h's results in the antenna element coded amplitudes and spaces shown in FIG. 3. The number of equal amplitude design positions corresponds to the product of the values of h raised to a power corresponding to the number of design K's utilized at the particular h value. The quantity of positions for this example is given by $5\times3\times2^2=60$. Due to element coincidence between 16 of these positions, 44 final code positions resulted with two digitized levels contained in the amplitudes.

The normalized field strength pattern for this code is shown in FIG 4. The large lobe between $K=5$ and $K=6$ was anticipated since a design zero did not exist near a value of $K=5.5$. This condition would, however, be desired in many practical designs since the overall physical antenna pattern consists of the product of this plot (the antenna space factor) with the individual antenna element pattern. In addition, this pattern yields a smaller lobe between $K=3.5$ and $K=4$, this is because a zero at $K=4$ was achieved by utilizing a value of $h=5$ for $K=1$.

In conclusion, the results for Code 2C demonstrate that the added versatility in this solution can be applied to slot fitting and results in improved antenna patterns in a simpler and less costly manner.

I claim:

1. A method of defining all available design K parameters for securing a predetermined directive radiation pattern for a linear array antenna having individual antenna units, in which relative spacing between said individual antenna elements is according to the equation

$$n'_i=n+\frac{i}{hK},\ \text{where } i=1,\ 2,\ \ldots\ (h-1)$$

the design K's being utilized for a given value of minimum spacing λ/X, where λ/X is desired minimum spacing expressed as a fraction of a wavelength, and a given value of space angle θ_o, the design K's existing according to the equation

$$K=\frac{X}{hN\sin\theta_0}$$

where N is any integer, h is the roots of unity, X is an index number assuming the values 1, 2, 3 etc., and θ_o is a given space angle associated with the null appearing between the main and first side lobes.

2. A method of establishing available design K parameters for linear array antenna coding, in which positioning of individual antenna elements from each other is according to a predetermined code given by the equation

$$n'_i=n+\frac{i}{hK},\ \text{where } i=1,\ 2,\ \ldots\ (h-1)$$

and in which the relative code spacing S is restricted to a specified fraction of a wavelength times an integer according to equation

$$K=\frac{\lambda}{hS\sin\theta_0}$$

where λ = wavelength, θ_o is a given value of space angle associated with the null appearing between the main and first side lobes, and h is the h^{th} roots of unity.

3. A method of space coding a linear array antenna in which positioning of the individual antenna elements is according to the equation

$$n'_i=n+\frac{i}{hK},\ \text{where } i=1,\ 2,\ \ldots\ (h-1)$$

wherein the code positions are forced to occupy an integral value of a specified minimum spacing λ/X expressed as a fraction of wavelength λ, and the slot fitting requires that the spacing S be restricted to this specified fraction times an integer N, the method according to the equation

$$S=\frac{\lambda}{hK\sin\theta_0}=\frac{\lambda}{X}\ (N)$$

where K represents available design parameters, θ_o is a given value of space angle, h is the h^{th} roots of unity.

4. A method of defining all available design K parameters which can be used to secure predetermined directive radiation pattern for a linear array antenna by forcing the code positions to occupy some integral value of a specified minimum separation, in which the individual antenna elements are positioned according to the equation

$$n'_i=n+\frac{i}{hk},\ \text{where } i=1,\ 2,\ \ldots\ (h-1)$$

the method according to the equation

$$K=\frac{\lambda}{h\left(\frac{\lambda}{X}\right)N\sin\theta_0}$$

where λ = wavelength, λ/X desired minimum spacing expressed as a fraction of wavelength, N = any integer, θ_o = a given value of space angle, and h is the h^{th} roots of unity.

5. A method of securing an exceedingly sharp central lobe and arbitrarily small side lobes for a linear array antenna by forcing the code positions to occupy some integral value of a specified minimum separation, in which positioning of individual antenna element is according to a predetermined code given by the equation

$$n'_i = n + \frac{i}{hK}, \text{ where } i = 1, 2, \ldots (h-1)$$

the design K parameters are established according to the equation

$$K = \frac{\lambda}{h\left(\frac{\lambda}{X}\right) N \operatorname{Sin} \theta_0}$$

where λ = wavelength, λ/X = desired minimum spacing expressed as a fraction of wavelength, N = any integer, θ_o = a given value of space angle, and h = the h^{th} roots of unity.