April 14, 1925. 1,533,927

C. G. E. LARSSON

MULTIPLE SPAN BRIDGE

Filed Feb. 7, 1923

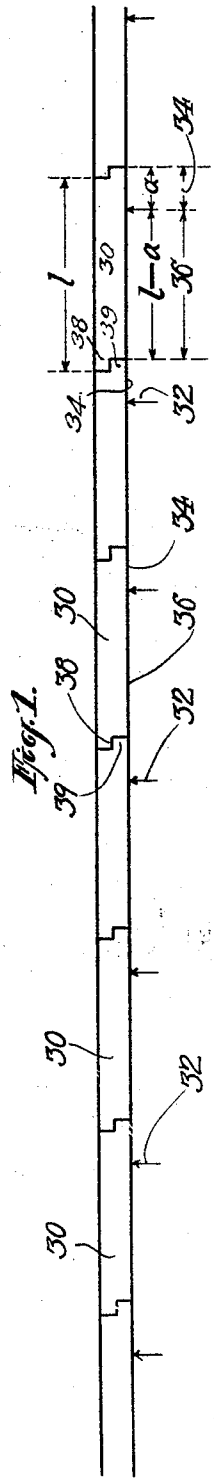


Fig. 1.

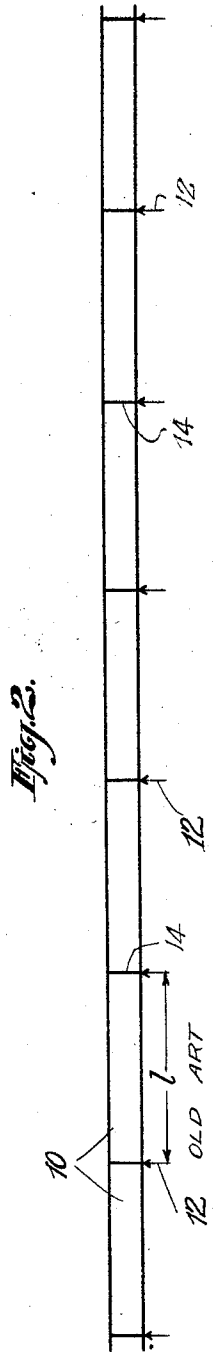


Fig. 2.

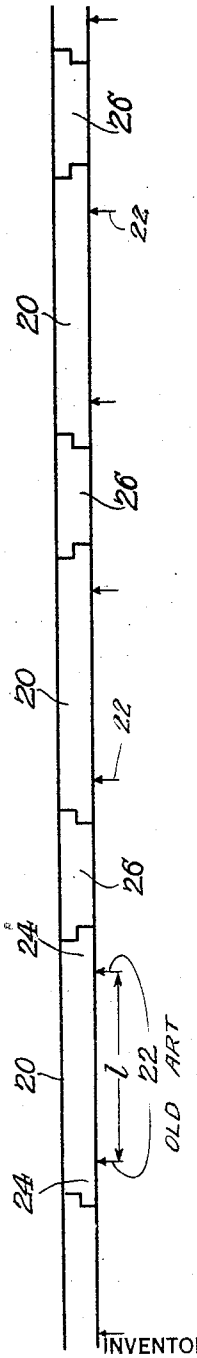


Fig. 3.

INVENTOR

C. G. Emil Larsson

BY

D. Anthony Usina

ATTORNEY

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CARL G. EMIL LARSSON, OF PLAINFIELD, NEW JERSEY.

MULTIPLE-SPAN BRIDGE.

Application filed February 7, 1923. Serial No. 617,547.

To all whom it may concern:

Be it known that I, CARL G. EMIL LARSSON, a subject of the King of Sweden, and resident of Plainfield, New Jersey, have invented certain new and useful Improvements in Multiple-Span Bridges, of which the following is a specification.

This invention relates to improvements in bridge construction and particularly to that type of bridge support—by a multiplicity of piers or towers or columns in the case of a viaduct or other elevated structure. The invention aims to provide means whereby a bridge for carrying a given load can be built with less material than bridges heretofore constructed and yet possess the same strength.

The invention is illustrated diagrammatically in the accompanying drawings in which—

Fig. 1 is a side elevation illustrating a bridge built in accordance with my invention;

Fig. 2 and Fig. 3 are diagrams showing known types of bridges and these figures are presented in order to make clear the calculations expressed hereinafter.

It has heretofore been the practice in building long bridges requiring a great number of piers to either arrange them as shown in Fig. 2 or Fig. 3. With the design shown in Fig. 2 the bridge comprises a plurality of sections 10 supported by piers 12, the sections abutting as at 14 over the centers of the piers.

Another known design as illustrated in Fig. 3 comprises a series of sections 20 supported near each end by piers 22 and having overhanging or cantilever portions 24. The sections 20 have interposed between them simple spans 26. With the above designs if the length of span between the piers 12 in the one instance or between the piers 22 in the other instance is represented by l and if the live load per unit of length is represented by w then the bending moment at the center is equal to $\frac{wl^2}{8}$.

According to Fig. 1 sections 30 are supported on the piers 32 and the spacing of the piers and the length of the sections is substantially equal and they are so arranged that each section comprises a cantilever portion 34 (as shown at the right) and a simple span portion 36 which is supported at 38 on the end 39 of the cantilever portion 34 of the next adjacent section.

It will thus be seen that the bridge comprises a multiplicity of sections each having a cantilever at one end which supports the end of an adjacent section which near its opposite end is supported on a pier. In other words, each of the sections comprises a simple span portion and a cantilever portion.

If we represent the length of the entire section by l as shown in Fig. 1 and the length of the cantilever by a then the length of the balance of the span is equal to l minus a $(l-a)$. If we represent the uniform live load per unit of length by w, the maximum bending moment on the simple span (that is, the moment at the center thereof) is expressed by the formula

$$\frac{w(l-a)^2}{8}.$$

Comparing this formula with the one above given in connection with the example shown in Figs. 2 and 3, it is clear that the maximum bending moment in my improved bridge structure is smaller than the maximum bending moments produced with the structures built according to Figs. 2 and 3.

It follows that a bridge can be built according to my improved design as shown in Fig. 1 with less material than is necessary to build such a bridge in accordance with former designs, such for example as shown in Figs. 2 and 3, using the same loading and the same unit stresses. In addition to requiring less material, a bridge built according to my invention can be built more quickly and cheaply because of the fact that the several sections 30 are duplicates (excepting end sections). It can also be more cheaply erected.

According to the above formula, the maximum bending moment under live load is least when the length a of the cantilever extension is equal to l multiplied by $(3-\sqrt{8})$; which is equal to $(0.172)l$. Thus by making the cantilever extension about one-sixth of the total span l (or one-fifth of the simple span) we have the best application of the principle of this invention.

The derivation of the expression l multiplied by $(3\sqrt{8})$ will be understood from the following:—

w=Uniform live load per unit of length.

Let l=the total length of a section.
Let $l-a$=the length of the simple span.
Let a=the length of the cantilever.
Then the maximum bending moment (at center) of the simple span=

$$\frac{w(l-a)^2}{8}$$

The load of the simple span produces a moment about the cantilever support expressed as,

$$\frac{w(l-a)a}{2}$$

The moment of the cantilever about its support=

$$\frac{wa^2}{2}$$

The sum of moments taken about cantilever support=

$$\frac{w(l-a)a}{2}+\frac{wa^2}{2}$$

Now to obtain the least maximum bending moment, the respective lengths of the simple span $(l-a)$ and the cantilever, (a) are so proportioned that their respective maximum bending moments are equal, or expressed algebraically,

$$\frac{w(l-a)^2}{8}=\frac{w(l-a)a}{2}+\frac{wa^2}{2}$$

Factoring,

$$\frac{(l-a)^2}{4}=(l-a)a+a^2$$

Simplifying,

$$\frac{l^2+a^2-2al}{4}=al-a^2+a^2$$

Solving,

$$l^2+a^2-2al=4al$$

Or,

$$a^2-6al=-l^2$$

Or,

$$a=3l\pm\sqrt{9l^2-l^2}$$

Or,

$$a=3l\pm l\sqrt{8}$$

Hence,

$$a=l(3\pm\sqrt{8})$$

Or,

$$a=.172l$$

The drawing diagrammatically illustrates plate girder bridge construction but it is to be understood that the same principles may be embodied in a bridge in which the several sections are of truss formation. Or the invention may be embodied in the girder and column arrangement of steel framed buildings or to the girder-supported beams of such buildings. In such cases the columns could be arranged like the piers 32 and the girders could have overhanging cantilever portions such as 34. In the case of floor beams, or the like they could bear on girders and have overhanging cantilever portions 34 as will be understood. The term bridge in the claims is intended to include such girders or beams and the term pier is intended to include columns, towers or other supports. Various expedients may be adopted to carry out the invention described without departing from the same as defined in the appended claims.

What I claim is:

1. A bridge comprising a plurality of sections and supports so spaced that each section comprises a simple span portion and a cantilever portion whose respective lengths are so proportioned that their least maximum bending moments are substantially or approximately equal.

2. A bridge comprising a plurality of sections supported on piers, the length of the sections and the spacing of the piers being such that a cantilever portion of one section supports a simple span portion of an adjacent section which rests on a pier, the length of the cantilever portions each being substantially one sixth of the length of the section.

3. A bridge comprising a plurality of sections all of which are supported intermediate their ends by a single pier and each section having an end supported by a cantilever approximately or substantially one-sixth the length of the section.

4. A bridge comprising a multiplicity of sections of equal length, all of which sections are supported intermediate their ends by single piers spaced at equal intervals, and each of said sections having one end supported by a cantilever extension of an adjacent section, so that each section comprises a simple span portion and a cantilever portion, said cantilever portion of each section being substantially one-fifth of the length of the simple span portion.

In witness whereof, I have hereunto signed my name.

C. G. EMIL LARSSON.