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RESISTANCE NETWORK

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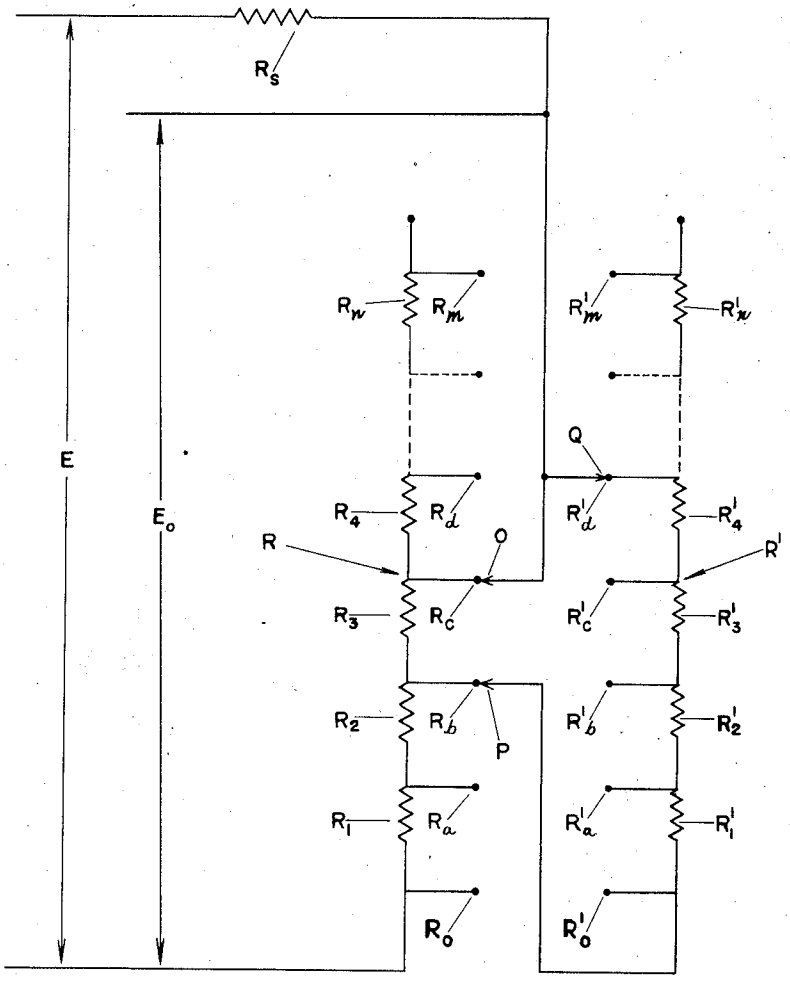

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RESISTANCE NETWORK

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The invention described herein may be manufactured and used by or for the Government for governmental purposes, without the payment to me of any royalty thereon.

My present invention has to do with resistance networks. More particularly, it relates to a device for adjusting electrical resistance whereby a voltage is varied in accordance with a predetermined non-linear mathematical curve.

It is the main object of my present invention to provide a device of the general character indicated wherein a variation of resistance and, therefore, a variation in voltage is made substantially smooth and continuous to closely approximate the exponential charging rate of a capacitance.

While not limited thereto, my present invention is admirably adapted to controlling the operation of a range-determining device.

In the accompanying specification I describe and in the annexed drawing I show an illustrative embodiment of the resistance network of the present invention.

In said drawing, the single figure is a schematic diagram of a resistance network assembled in accordance with the principles of the present invention.

Referring now more in detail to the present invention, with particular reference to the drawing illustrating a preferred embodiment thereof, the reference character R generally designates a group of series-connected resistors R_1, R_2, R_3, R_4, . . . R_n. Associated with said resistors are contacts R_o, R_a, R_b, R_c, R_d, . . . R_m, and selectively engageable with said contacts is an adjustable arm O.

The resistor R_1 is connected to one terminal of a source of voltage E and the arm O is connected, through a fixed high resistance R_s, to the other terminal of said source of voltage.

The value of each resistor R_1, R_2, R_3, R_4, . . . R_n is such that the voltage drop across the total resistance of the combination of resistors respectively corresponding to each of the contacts R_o, R_a, R_b, R_c, R_d, . . . R_m, is an exponential function of the number of resistors in each such combination.

Adjacent the group R of resistors, I provide a second group R′ of series-connected resistors R'_1, R'_2, R'_3, R'_4, . . . R'_n and associated contacts R'_o, R'_a, R'_b, R'_c, R'_d, . . . R'_m. The resistor R'_1 is connected to a second adjustable arm P selectively engageable with the contacts R_o, R_a, R_b, R_c, R_d, . . . R_m; and selectively engageable with any of the contacts R'_o, R'_a, R'_b, R'_c, R'_d, . . . R'_m is a third adjustable arm Q, the opposite end of which is connected with the adjustable arm O. The value of each resistor R'_1, R'_2, R'_3, R'_4, . . . R'_n is such that the equivalent resistance and, therefore, the voltage drop across the parallel combination of selected resistors R'_1, R'_2, R'_3, R'_4, . . . R'_n and any selected resistor of the group R is an approximately linear function of the number of resistors of the group R′ entering into the combination.

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The output voltage of the network, designated in the drawing as E_o, is between the contact R_o and the meeting point of the adjustable arms O and Q.

I shall now describe a specific procedure for determining the values of the various resistors above referred to.

The potential across a resistance-capacitance combination builds up in accordance with the formula:

$$V=AE(1-e^{-BT})\equiv E\cdot F(T) \tag{1}$$

where A and B are constants depending upon any selected resistance and capacitance for the circuit under consideration and T is time measured from the instant of application of the potential E.

Now the group R of resistors is divided into n sections such that

$$V'=AE\left(1-e^{-B\left(\frac{k}{n}\right)}\right)\equiv E\cdot F\left(\frac{k}{n}\right) \tag{2}$$

for integral values of k up to and including n; and V′ is the output voltage across a selected number of resistors of the group R, excluding the effect of the parallel group R′, and k is said selected number of resistors.

From Ohm's law,

$$E=I(R_s+S_k) \quad \text{and} \quad V'=IS_k \tag{3}$$

where S_k is the value of the selected number of resistors of the group R. Therefore,

$$\frac{V'}{E}=\frac{S_k}{R_s+S_k} \tag{4}$$

and, inasmuch as from (2),

$$\frac{V'}{E}=F\left(\frac{k}{n}\right)$$

$$F\left(\frac{k}{n}\right)=\frac{S_k}{R_s+S_k} \tag{5}$$

Solving for S_k, we find

$$S_k=\frac{R_s\cdot F\left(\frac{k}{n}\right)}{1-F\left(\frac{k}{n}\right)} \tag{6}$$

The value of each resistor $R_1, R_2, R_3, R_4, \ldots R_n$ can be determined from the formula:

(7) $$R_k = S_k - S_{k-1}$$

Substituting therein for S_k and S_{k-1}, we have

(8) $$R_k = R_s \left(\frac{F\left(\frac{k}{n}\right)}{1-F\left(\frac{k}{n}\right)} - \frac{F\left(\frac{k-1}{n}\right)}{1-F\left(\frac{k-1}{n}\right)} \right)$$

and substituting still further, for

$$F\left(\frac{k}{n}\right) \text{ and } F\left(\frac{k-1}{n}\right)$$

and simplifying, we have

(9) $$R_k = AR_s \left(\frac{1-e^{-B\left(\frac{k}{n}\right)}}{1-A\left(1-e^{-B\left(\frac{k}{n}\right)}\right)} - \frac{1-e^{-B\left(\frac{k-1}{n}\right)}}{1-A\left(1-e^{-B\left(\frac{k-1}{n}\right)}\right)} \right)$$

where k takes on values from 0 to n.

Thus, having selected values for A, B, and R_s for the group R comprising n sections of resistors, we can immediately determine from Equation 9 the value of each of said resistors.

The procedure for calculating the values of the resistors $R'_1, R'_2, R'_3, R'_4, \ldots R'_n$, of the group R' is as follows:

The group R' consists of n sections, not necessarily the same number as included in the group R, having values such that the equivalent resistance across the parallel combination of R_k and S'_h is given as

(10) $$P_r = \frac{h}{n}\overline{R}$$

where S'_h is the total value of the first h resistors of the n resistors of the group R', h varies from 0 to n and $\overline{R}$ is the average value of the above-defined R_k. Therefore, by applying the law of parallel circuits, we have

(11) $$\frac{1}{S'_h} + \frac{1}{\overline{R}} = \frac{1}{\frac{h}{n}\overline{R}}$$

Solving for S'_h we obtain

(12) $$S'_h = \frac{\overline{R}}{n-h}$$

The value of each resistor $R'_1, R'_2, R'_3, R'_4, \ldots R'_n$ can be determined from the formula:

(13) $$R'_h = S'_h - S'_{h-1}$$

Substituting therein for S'_h and S'_{h-1}, and simplifying, we have

(14) $$R'_h = \frac{\overline{R}n}{(n-h)(n-h+1)}$$

Thus, knowing the average value $\overline{R}$ and the number of sections n of the group R', we can determine the value of each of the resistors R'_h.

From Ohm's law,

(15) $$E_0 = I(S_{k-1} + P_r) \quad \text{and} \quad E = I(R_s + S_{k-1} + P_r)$$

Eliminating I, we obtain as a final expression of the voltage output of the entire network,

(16) $$E_0 = E\frac{(S_{k-1} + P_r)}{(R_s + S_{k-1} + P_r)}$$

I shall now describe the mode of operation of the resistance network of the present invention. I will assume, as shown in the drawing, that the arms O and P respectively engage the contacts R_c and R_b, the arm Q engages the contact R'_d

and that it is desired to decrease the output voltage E_0. First, the arm Q is moved through R'_c toward R'_0. The result is a progressive decrease in said output voltage. Assuming that the arm Q has reached the contact R'_0 and it is desired to further reduce the output, the arms O and P are moved toward the contact R_0 in such a manner that the arm O is engaged with the contact R_b before the arm P is disengaged therefrom. This movement is continued until the arm P is disengaged. At this time the arm Q is moved so as to engage the contact R'_m and the arm P is moved into engagement with the contact R_a. During all of these movements the arm O remains engaged with the contact R_b and no change occurs in the output voltage E_0. Now the arm Q can be moved toward R'_0 as previously described in order to decrease the output voltage E_0. The entire procedure outlined can be continued until the desired reduction in the output has been attained.

To increase the output voltage E_0 all of the foregoing movements are reversed.

This completes the description of the present invention and it will be noted that I have provided a simple resistance network whereby a voltage drop can be varied substantially smoothly and continuously so as to closely approximate a non-linear mathematical curve, such as the exponential curve of a charging capacitance.

Other objects and advantages of the present invention will readily occur to those skilled in the art to which the same relates.

I claim:

1. In a resistance network, a first group of resistors each having a value such that the total resistance of selected combinations thereof is a given non-linear function of the number of resistors comprising each such combination, and a second group of resistors each having a value such that the equivalent resistance of a parallel combination of selected resistors of said second group and any selected resistor of said first group is an approximately linear function of the number of resistors of said second group entering into said parallel combination.

2. In a resistance network, a first group of resistors each having a value such that the total resistance of selected combinations thereof is an exponential function of the number of resistors comprising each such combination, and a second group of resistors each having a value such that the equivalent resistance of a parallel combination of selected resistors of said second group and any selected resistor of said first group is an approximately linear function of the number of resistors of said second group entering into said parallel combination.

3. In a resistance network, a first group of resistors each having a value such that the total resistance of selected combinations thereof is a given non-linear function of the number of resistors comprising each such combination, and a second group of resistors connected in series each of whose value is given by the expression

$$\frac{\overline{R}n}{(n-h)(n-h+1)}$$

where $\overline{R}$ is the average value of the resistors of said first group, n is the number of resistors in said second group, and h is the number of the resistor of said second group whose value is being determined.

4. In a resistance network, a first group of resistors each having a value such that the total

resistance of selected combinations thereof is an exponential function of the number of resistors comprising each such combination, and a second group of resistors connected in series each of whose value is given by the expression

$$\frac{\overline{R}n}{(n-h)(n-h+1)}$$

where $\overline{R}$ is the average value of the resistors of said first group, n is the number of resistors in said second group, and h is the number of the resistor of said second group whose value is being determined.

5. In a resistance network, a first group of series-connected resistors each of whose value is given by the expression

$$AR_s\left(\frac{1-e^{-B\left(\frac{k}{n}\right)}}{1-A\left(1-e^{-B\left(\frac{k}{n}\right)}\right)}-\frac{1-e^{-B\left(\frac{k-1}{n}\right)}}{1-A\left(1-e^{-B\left(\frac{k-1}{n}\right)}\right)}\right)$$

where A, B, and R_s are constants of the network, n is the number of resistors in said first group, and k is the number of the resistor whose value is being determined, and a second group of resistors connected in series each of whose value is given by the expression

$$\frac{\overline{R}n}{(n-h)(n-h+1)}$$

where $\overline{R}$ is the average value of the resistors of said first group, n is the number of resistors in said second group, and h is the number of the resistor of said second group whose value is being determined.

6. In a resistance network, a first group of series-connected resistors each of whose value is given by the expression

$$AR_s\left(\frac{1-e^{-B\left(\frac{k}{n}\right)}}{1-A\left(1-e^{-B\left(\frac{k}{n}\right)}\right)}-\frac{1-e^{-B\left(\frac{k-1}{n}\right)}}{1-A\left(1-e^{-B\left(\frac{k-1}{n}\right)}\right)}\right)$$

where A, B, and R_s are constants of the network, n is the number of resistors in said first group, and k is the number of the resistor whose value is being determined, a second group of resistors connected in series each of whose value is given by the expression

$$\frac{\overline{R}n}{(n-h)(n-h+1)}$$

where $\overline{R}$ is the average value of the resistors of said first group, n is the number of resistors in said second group and h is the number of the resistor of said second group whose value is being determined, and adjustable means engageable with said first and second groups of resistors for selectively connecting in parallel any resistor of said first group with selected resistors of said second group.

JAMES R. MOORE.

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