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[21] Appl. No. **697,393**

[22] Filed **Jan. 12, 1968**

[45] Patented **Feb. 16, 1971**

[56]

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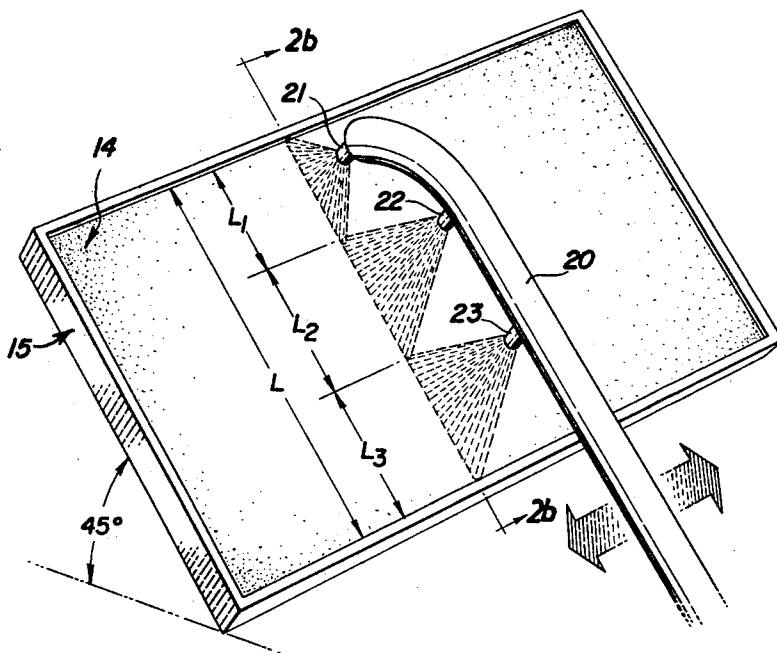
[54] **AIR FILTER WASH DEVICE**  
6 Claims, 13 Drawing Figs.

[52] U.S. Cl..... **239/561;**  
134/172; 239/566, 239/597

[51] Int. Cl..... **B05b 1/14**

[50] Field of Search..... 134/167,  
181, (Inquired); 239/(Inquired); 239/566, 561,  
597; 134/172, 198

**ABSTRACT:** Apparatus for cleaning filter elements comprising a plurality of nozzles for spraying water mounted asymmetrically with respect to the filter element to provide an asymmetric spray pattern whereby optimum utilization of the available water supply is obtained.



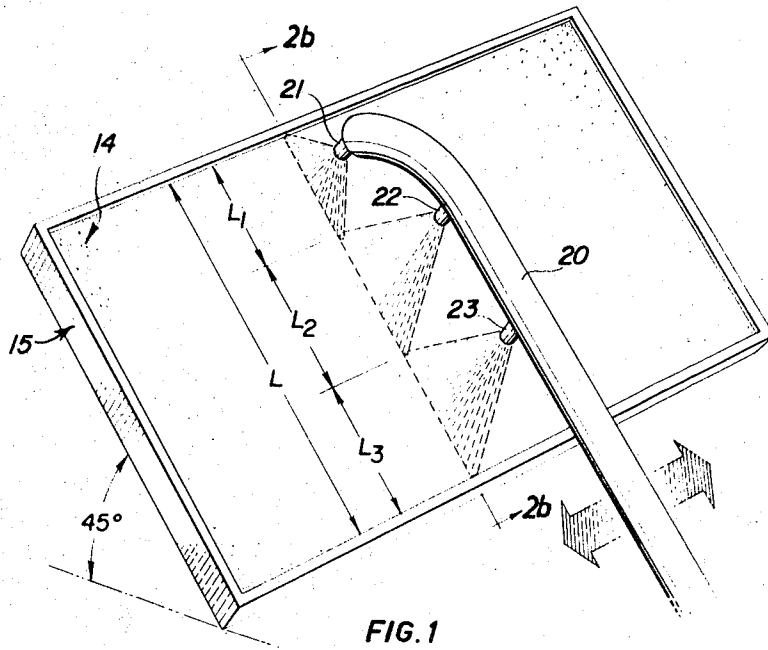


FIG. 1

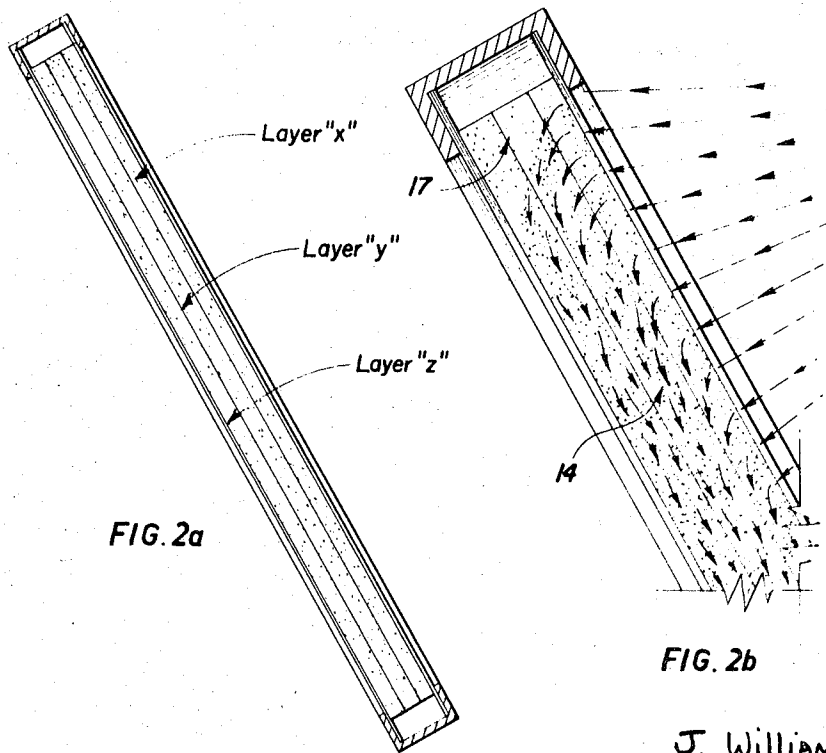


FIG. 2a

FIG. 2b

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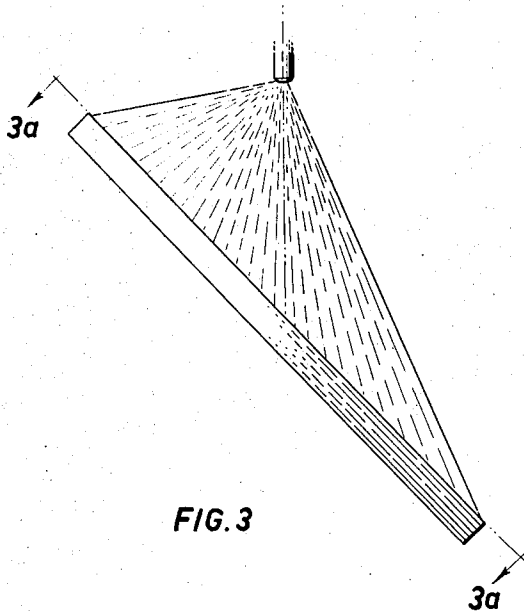


FIG. 3

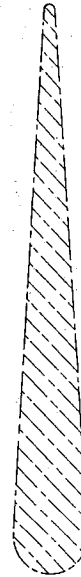


FIG. 3a

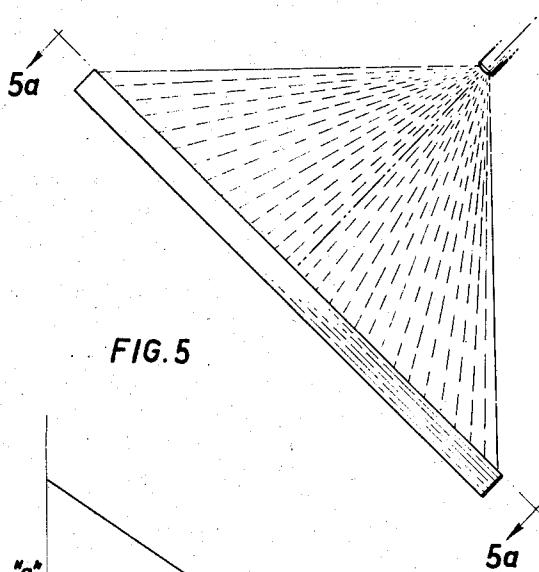


FIG. 5

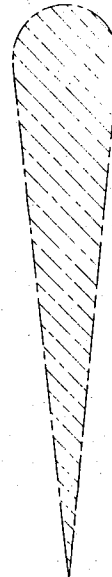


FIG. 5a

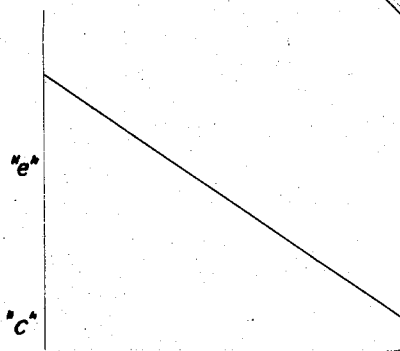


FIG. 4

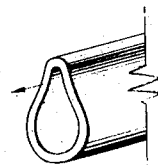


FIG. 10

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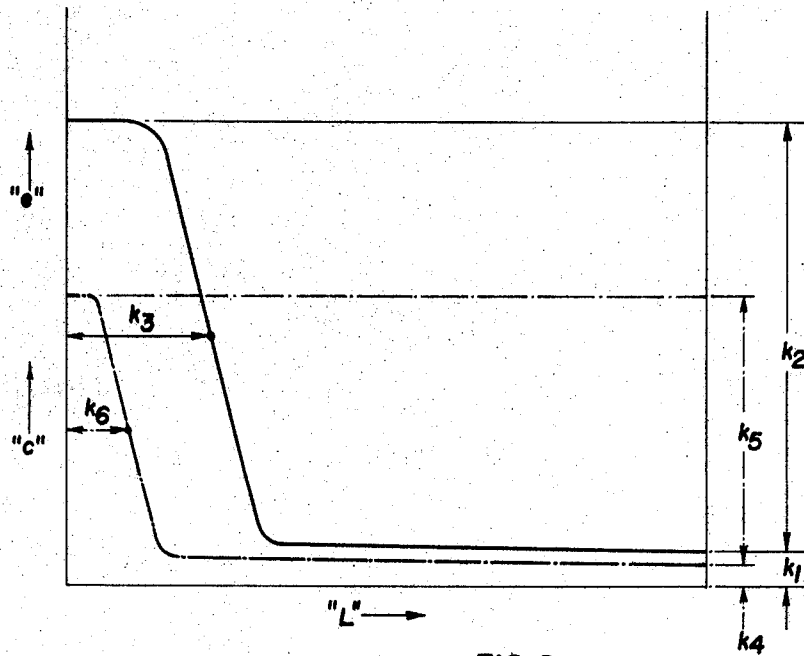


FIG. 6

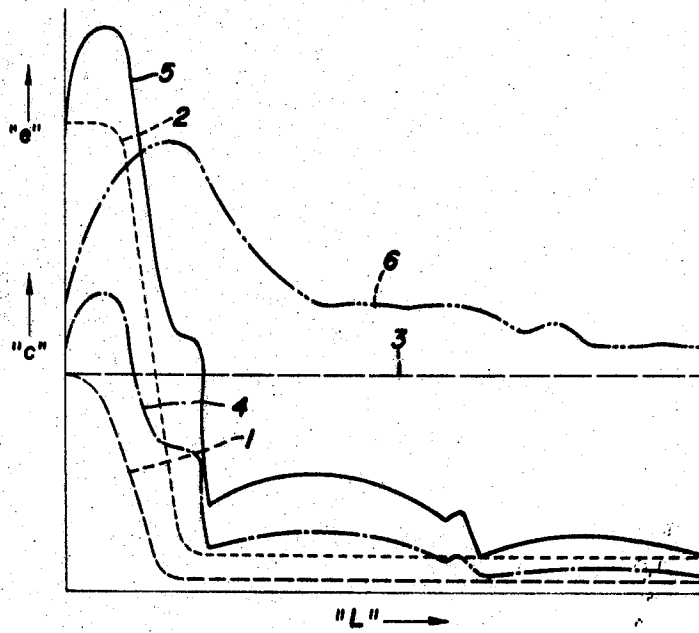


FIG. 7

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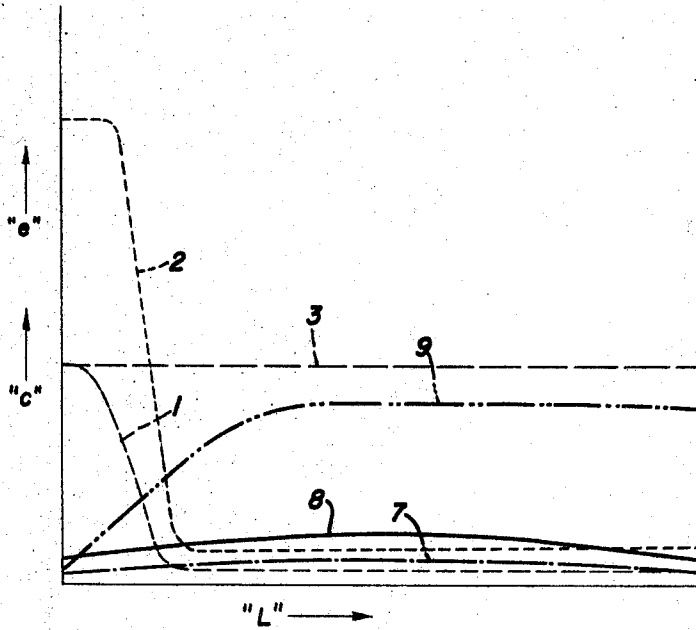


FIG. 8

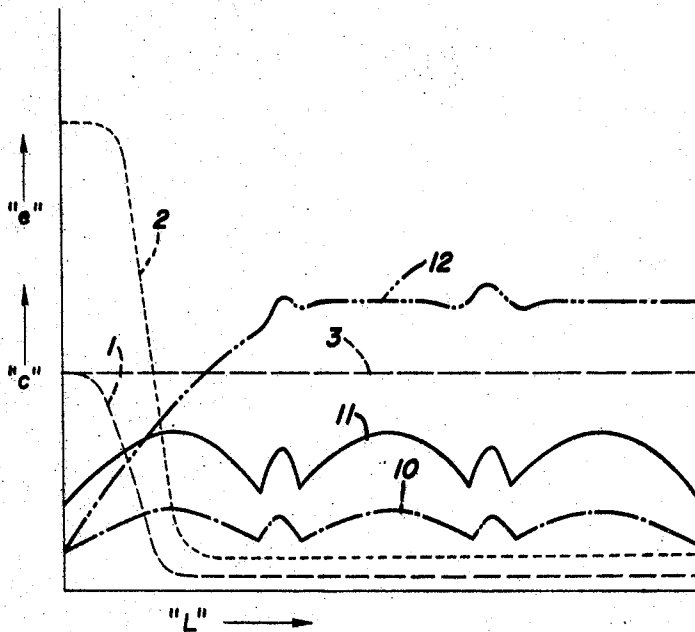


FIG. 9

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## AIR FILTER WASH DEVICE

This invention relates to the washing apparatus used to clean mechanical gas filter systems and relates more particularly to an improved washer spray header for cleaning the filter panels of such mechanical filter systems.

In a mechanical gaseous filter for removing particulate matter from a gas, the filter media or the collecting plates become loaded with dust. This leads to a reduction in the arrestive efficiency or an undesirable increase in the pressure drop across the filter or both.

Some mechanical gas filter systems require removal of the filter panels which may then be thrown away or may be washed and returned to the system. Other systems are arranged for manual washing in situ.

In the interest of reducing the manual labor required to maintain the efficiency of the filter bank, systems using disposable filter media have been developed which embody filter media in the form of a roll which can be advanced periodically to present fresh filter media to the airstream. These rolls can be of substantial length, allowing many changes of filter media in the air stream before the entire roll has to be removed and replaced manually. This method reduces the manual labor required but is costly because the filter media is not recoverable.

In other mechanical air filters employing permanent filter media, the filter panels have been arranged to pass down through the airstream into an oil sump where the collected dust sludge is removed and the filter recoiled before passing back up again into the airstream. In still other systems using permanent filter media, washing systems have been mounted on the filter bank to reduce the manual labor for washing. In these conventional washing systems, evenly disposed spray nozzles are used which are either fixed or mounted on a piece of pipe called a "standpipe" which may be disposed in a vertical plane reciprocally driven across the downstream side of the filter. These supposedly automatic in situ washing devices are not fully automatic as they require frequent manual attention to attend to the relatively ineffective degree of cleaning which is achieved and to attend to the relatively complex mechanical components.

10 years ago the most popular system were of the types embodying permanent media; however, due to the ineffectiveness and inefficiency of these types, the disposable roll type is now more often used. I propose a more effective and workable automatic cleaning system. This method is sufficiently efficient and reliable to allow the use of permanent filter media. Although the method used in this application is automatic, I do not propose to clean while air is passing through the filter, and it is necessary to either bypass the filter system, or arrange for cleaning of a portion of the filter bank while routing the dust-laden gas through another portion of the bank, or shut down the fans.

According to my present invention, filters are automatically cleaned in situ with a system of fixed nozzles attached to a moving standpipe for spraying water on the filters.

The nozzle arrangements that have been used up to the present time for washing filters in situ are imperfect because the available water supply is distributed in a regular pattern, which is generally insufficient on some parts of the filter and wasteful on others.

It is therefore an object of this invention to clean filters with a spray pattern of delivery which optimizes the water supply available by forming an asymmetrical spray pattern which matches the washing requirements of various parts of the filter and washes the various parts of the filter substantially evenly.

The proposed invention will function on normal water pressures, and, in fact, improves the efficiency of washing with normal pressures. It is readily seen that very high water pressures would have greater washing power, but in most plant installations, only normal water pressure is found and the best use must be made of it. Also, I have found that without adding detergent or hot water, effective cleaning can be attained on filters of synthetic material by the asymmetrical spray pattern.

The present invention would undoubtedly be made even more efficient with hot water or detergent.

Another limitation of conventional devices for cleaning filters in situ has been the relative complexity of the mechanism employed to move the standpipe header back and forth across the filters. The existing systems use chain, cable, or worm and screw, arranged to support the standpipe at both top and bottom in driving relationship with a track. I propose a system with standpipe or standpipes to be driven from the top only. This results in a substantial reduction in the amount of hardware and associated equipment required. In addition, the existing systems often have the drive mechanism in the airstream, which permits dust and cleaning water to enter the moving parts and clog them. It is accordingly a further aspect of my present invention to locate the drive mechanism for the standpipe out of the airstream to protect it from such clogging. One or more standpipes may be provided and may be driven reciprocally in a horizontal direction. More than one standpipe permits, among other things, a shorter distance of travel by each standpipe. It is a complementary object of this invention to use a mechanical system of such simplicity as to give it a high level of mechanical reliability so that maintenance and attention is minimized.

Since filters are typically installed in isolated location, any maintenance or attention is inconvenient and costly. My invention also lends itself to even greater automation. By the use of timers and due to the cleaning efficiency and simplicity of the drive mechanism, it may not require attention for possibly a year or more, whereas conventional systems often require attention weekly.

It is yet another object of my present invention to provide a suitable asymmetrical nozzle arrangement.

The foregoing and other objects and advantages of my present invention will in part be stated in and in part become apparent from the following detailed description, when read in conjunction with the accompanying drawings, in which:

FIG. 1 is a perspective view of a filter disposed at an angle of  $45^\circ$  to a vertical with a spray impinging thereon;

FIG. 2a is a cross-sectional view of a typical filter showing filter layers therein;

FIG. 2b is an enlarged sectional view of a typical filter showing the waterflow pattern typically obtained with conventional spray-type headers;

FIG. 3 is a schematic view of a slot-type nozzle mounted at an angle to the surface of the filter to produce an asymmetrical washing pattern on a typical filter;

FIG. 3a is a schematic plan view of the water pattern produced by the nozzle in FIG. 3, along the line 3a-3a of FIG. 3;

FIG. 4 is a graph showing the relationship of the energy of impact  $e$  and the concentration  $c$  plotted against  $L$ , the overall length of the filter measured along the face of the filter from the top, for the nozzle shown in FIG. 3;

FIG. 5 is a schematic view depicting a nozzle with an irregularly shaped orifice vertically disposed to the face of the filter;

FIG. 5a is a schematic plan view of the water pattern produced by the nozzle in FIG. 5 along the line 5a-5a of FIG. 5;

FIG. 6 is a graph showing the function  $e(L)$  represented by a solid line and the function  $c(L)$  represented by a phantom line, both plotted against  $L$ ;

FIG. 7 is a composite graph showing for the three nozzle patterns shown in FIG. 7 the energy of impact and concentration functions, against the distance of  $L$  from the top of the filter for an asymmetrical arrangement of nozzles;

FIG. 8 is a composite graph showing a typical nozzle pattern described with reference to its respective concentration energy and distance functions using a single symmetrical nozzle;

FIG. 9 is a composite graph showing typical nozzle patterns described with reference to their respective concentration, energy and distance functions using a symmetrical arrangement of three nozzles; and

FIG. 10 is a cutaway perspective view of an irregularly shaped nozzle.

Referring to the drawings, the device illustrated in FIG. 1 comprises an angularly inclined header 20, having nozzles 21, 22 and 23 located in spaced relationship thereon. Nozzle 21 is mounted closer to filter 15 than nozzles 22 and 23 and delivers water at greater impact on filter 15 than nozzles 22 and 23 by reason of its location on said header. The filter 15, collects dust from an airstream which impinges on its lower side *c* so that the dust particles in theory are retained by filter 15 and the cleaned air passes through on the upstream side adjacent to the header. The media 14 for filter 15 may be made from any of a number of known commercial materials, and does not comprise a feature of this invention.

Wash water emitted by the nozzles 21, 22 and 23 impinges on filter media 14 in a predetermined pattern so that the latter receives complete water coverage. Nozzle 21 covers a smaller area than nozzles 22 and 23 and provides a flushing effect along the upper portion of the filter, and dust particles and wash water are then carried downwards along the filter and through the lower side. Nozzle 21 is so designed that the water emitted thoroughly washes the filter.

FIG. 1 also shows the relationship of  $L_1$ ,  $L_2$ , and  $L_3$  to  $L$  as discussed in the mathematical description herein.

Referring to FIG. 2a, layer *x* (on the header or downstream side) of inclined filter 15 consists of a layer of filtering medium. Layer *y* also comprises a filtering medium and layer *z* on the upstream side consists of a form of screen.

FIG. 2b shows an unwetted area 17 occurring at the upper end of the filter on the opposite side from the spray. This unwetted area 17 remains unwashed by conventional spray-type headers.

Since filter banks tend to be quite large, often 100 or more square feet, there is a practical limitation on the quantity of water that can be delivered to the installation per second. There are also practical limitations to the amount of mainline pressure that can be made available and maintained during the washing cycle. Therefore, the design of an effective washing spray pattern can be viewed in terms of maximizing the usefulness of the pressure and volume available to the installation as well as in terms of the cost of volume and pressure required. To a large extent it has been the ineffective use of the volume and pressure provided or available which has resulted in less than satisfactory performance of the many filter washing systems described in the prior art.

The effectiveness of the washing action depends on the volume of water supplied per second, the velocity of the water and the total amount of water used.

A volume of water per second per unit area of the filter or low concentration *c* will tend to allow the water to trickle through the paths of least resistance, leaving unwetted pockets, whereas a high concentration *c* will tend to flood through even hydrophobic areas and float the dirt away.

The energy of impact *e* on any given area which is a function of the volume of water per second falling on that area and the velocity squared, will determine how far into the medium the water will penetrate unassisted by gravity and will also affect the scrubbing and flooding action.

The total volume *q* that is required across each surface will be a function of *c* and *e* and all the characteristics of the surfaces to be cleaned.

If the filter was mounted horizontally, the analysis to find the required volume would be quite simple. Values could be assigned to *c*, *e* and *q* making it possible through experimentation to develop a series of values for the three variables and to determine the optimum combination with the lowest total volume of water through the filter to give effective washing. However, since the filters to be washed are generally mounted at an angle other than the horizontal and since the filter has a finite depth, the values of *c* and *e* are different in various parts of the filter and analysis becomes more complex. For example, referring to FIG. 2b it can be noted that:

1. The energy of impact *e* diminishes rapidly as the water penetrates the filter.
2. Concentration *c* in the lower parts of the filter 15 are reinforced by water running down from above.

3. While gravity will carry water through the filter in lower portions, impact energy alone must carry it through to the topmost rear part of the filter, (17).

4. The impact absorbing and permeable characteristics of the various layers of material in the filter will effect *c* and *e* in different ways.

It follows that the minimum values of *c*, and *e* required for effective washing will be higher in the top part of the filter than those required in the lower parts. This suggests an experimental procedure for determining these values in which the behavior of the water is examined in segments down the filter from the top.

#### EXPERIMENTAL PROCEDURE

To determine the most effective arrangement of nozzle or nozzles in a header delivering a pattern of water along a vertical plane to the filter face:

1. Fix the pressure  $p_1$  at the nozzles, this pressure being the same for all the nozzles.
2. For different  $s_1$  (describing the size and shape of the top nozzle) find in each case the longest possible  $L$  (the vertical dimension of the area of wash) leading to satisfactory cleaning of the top part of the filter. These results can be expressed by the formula  $L_1 = f_1(s_1)$ .
3. For fixed  $s_1$  using the corresponding  $L_1 = f_1(s_1)$  and a different  $s_2$ , find the longest possible  $L_2$  leading to satisfactory cleaning of the second portion of filter. This leads to the formula  $L_2 = f_2(s_1, s_2)$ . Several such formulas, depending on different  $s_1$  can be expressed by the formula  $L_2 = f_2(s_1, s_2, L)$ .
4. Similarly for fixed  $s_1$  and  $s_2$  with corresponding  $L_1 = f_1(s_1)$  and  $L_2 = f_2(s_1, s_2)$  obtain formula  $L_3 = f_3(s_1, s_2, s_3)$  leading to  $L_3 = f_3(s_1, s_2, s_3)$ . It is a condition of the system that  $L_1 + L_2 + L_3 = L$  (see FIG. 1). The number of nozzles *n* required will depend on the sizes  $s_1, s_2, s_3$ .

#### TECHNIQUE

The total cost or efficiency of washing the filter will be the sum of the cost of water/1,000 cubic feet *Q* plus the cost of maintaining the mainline supply pressure *P*; the latter will be the pressure required at the nozzles, *p*, plus the pressure drop from the mainline to the nozzles for the volume/second required. Having developed a cost formula in terms of *s* and *p* which will have the form

$$f_{\text{cost } p_1}(s_1 s_2 s_3)$$

it is possible to optimize the cost of various values of  $n_1, n_2, n_3$ —where *n* equals the number of nozzles necessary to cover *L*.

The experimental procedure can be repeated for a different pressure  $p_2$  and the costs optimized with

$$f_{\text{cost } p_2}(s_1 s_2 s_3)$$

The optimum *s* and *p* can be finally refined by

$$f_{\text{cost}}(n, s) \text{ where}$$

where *n* represents the initial cost of *n* nozzles and *s* represents the operating maintenance cost as a function of nozzle size; the smaller the nozzle, the higher the risk of plugging, and hence the higher the maintenance cost.

The above procedure establishes the optimum asymmetrical pattern in a vertical plane through the filter under static conditions, i.e. no horizontal movement of the nozzle assembly. It is now necessary to calculate the maximum rate of horizontal travel *h* possible.

The value of *h* flows from an examination of each vertical segment of the filter under the conditions of the optimum pattern for the value of *q* and the width of the spray pattern at that segment. The maximum rate of horizontal travel will be the rate which will deliver the minimum *q* required per unit

area. It is obvious that if the rate of horizontal travel is too great, the filter media will not be sufficiently flushed throughout its thickness— $q$  per unit area will be too low. If the rate of travel is too slow,  $q$  will be greater than necessary, and water and time to clean the filter will be wasted.

It can be deduced, and experimentation verifies, that a plot of the minimum values for  $c$  and  $e$  required to give effective cleaning can be described by the formulas:

$$(1) \quad e(L) = k_1 + \frac{k_2}{1 + \left(\frac{L}{k_3}\right)^\alpha}$$

$$(2) \quad c(L) = k_4 + \frac{k_5}{1 + \left(\frac{L}{k_6}\right)^\beta}$$

where  $L$  is the distance measured down the face from the top of the filter (see FIGS. 1 and 6).  $k_1$  will depend on the minimum energy of impact  $e$  required to effectively wash the first layer  $x$  (see FIG. 2b) and to impart sufficient turbulent energy to the water in the center of the filter  $y$ . For best results  $k_2$  will be more than twice  $k_1$  and all values of  $k$  must be positive.  $k_1 + k_2$  will be the energy  $e(0)$  required to force water through the filter unassisted by gravity at the top where  $L = 0$  as in (1) above.

$\alpha$  and  $\beta$  depend on the shape of the spray pattern and depend on the nature of the filter, and have arbitrarily assigned values. Generally  $\alpha$  and  $\beta$  are each greater than 6 for thick filters and less than 6 for thinner filters, but they are not critical values.

$k_3$  will be a function of gravity, the angle of the filter to the horizontal and the energy absorbing characteristics of the various layers of materials in the filter.

$k_4$  will depend on the minimum concentration  $c$  of water to flood all parts of the first layer  $x$  and to supplement the water running downwards inside the filter and escaping through layer  $z$ .

$k_5$  will be the amount of water required in the top portions to prime the rundown phenomenon in the filter.

$k_6$  will be a function of gravity, the angle of the filter to the horizontal and the permeability characteristics of the various layers of material in the filter.

It will be noted in selecting various nozzles for evaluation that for any given pressure and quantity of water the greatest energy of impact  $e$  and concentration  $c$  for any given zone extending from the upper to the lower end of the filter will be achieved with nozzles producing a flat, slot-like spray pattern. Such a slot-like pattern can be given 10 to 20 times the values that the same volume and pressure would be given with a round or square pattern.

With these facts in mind, and in particular knowing the general form of the optimum curves for  $e$  and  $c$ , as described above and shown in FIG. 6, it is possible to approximate the values of  $k_1$ ,  $k_2$ ,  $k_3$ ,  $k_4$ ,  $k_5$ , and  $k_6$  for a particular filter mounted at a certain angle to the horizontal which will greatly reduce the number of experiments in the experimental procedure required to obtain the pertinent data for the efficiency optimizing calculations.

FIG. 1 shows an arrangement embodying three nozzles designed to give an asymmetrical delivery. In the event the nozzle number 21 delivers 3 g.p.m. at 40 p.s.i. from a distance of 2 inches, nozzle number 22 delivers 2 g.p.m. at 40 p.s.i. from a distance of 3.5 inches and nozzle number 23 delivers 1 g.p.m. at 40 p.s.i. from a distance of 3.5 inches. The benefits that accrue from this asymmetrical delivery pattern can be seen in FIG. 7 where curve number 1 represents the minimum values of  $c$  and curve number 2 represents the maximum values of  $e$  required to give effective cleaning for a particular filter panel. Curve 3 is the minimum value of  $c$  with the rundown water added. Curve number 6 is the resultant  $c$  with run-

down water added for the asymmetrical nozzles. The resultant  $c$  and  $e$  from the asymmetrical heads of FIG. 1 are plotted as curves 4 and 5 respectively.

A single nozzle mounted vertically over the center of the filter delivering the same volume, i.e. 6 g.p.m. at 40 p.s.i. would give the results shown by curves 7, 8 and 9 in FIG. 8, which is not quite adequate in the bottom 70 percent of the filter and totally ineffective in the top 30 percent. Theoretically, a single nozzle would have to deliver 60 to 120 g.p.m., 10 to 20 times that of the asymmetrical arrangement to effectively clean the same filter.

The results obtained from three nozzles of 2 g.p.m. each at 40 p.s.i. mounted without asymmetry are shown by curves 10, 11 and 12 in FIG. 9. While this is better than the results from a single nozzle, it will be noted that it falls 75 percent short of providing effective cleaning in the top 20 percent of the filter and cleaning water is wasted in the bottom 80 percent. Three nozzles without asymmetry would have to deliver a total of 20 to 30 g.p.m. to effectively clean all the filter.

FIGS. 3, 4 and 5 illustrate other methods of obtaining an asymmetrical delivery of water to the filter face.

I claim:

1. In filter cleaning mechanisms for a rectangular gas filtering medium having a flat surface in a plane inclined with respect to the horizontal, the combination including header assembly means to be reciprocally moved in a horizontal direction parallel with the surface of the filtering medium, and means for connecting said header assembly means with a source of cleaning fluid during said reciprocal movement, said header assembly means also including nozzle means for directing a band of said cleaning fluid to the upper surface of the filtering medium extending across the entire surface between the upper and lower margins thereof and generally transverse to the direction of said reciprocal movement of the header assembly means, said band being narrow with respect to the height of said filtering medium, said nozzle means also including means for directing a greater concentration of fluid in said narrow band to areas of the filtering medium adjacent the upper margin of the filtering medium than to areas of the filtering medium lying therebelow.

2. The invention defined in claim 1, wherein said nozzle means includes an outlet orifice having an irregular cross section, whereby a greater amount of fluid is directed toward the end of said narrow band adjacent the upper margin of the filtering medium.

3. The invention defined in claim 1, wherein said nozzle means includes at least two outlet orifices disposed in alignment transverse with respect to the direction of said reciprocal movement, the orifice disposed adjacent to the upper margin of the filtering medium being disposed closer to the surface thereof than another orifice.

4. The invention defined in claim 1, wherein said nozzle means includes a generally vertical header assembly provided with more than one nozzle for communicating with cleaning fluid supply.

5. In a system for cleaning filter elements having a header assembly, support structure for said header assembly and drive means for producing reciprocal movement of said header assembly across the surface of a filter element as defined in claim 4, the improvement comprising a series of more than one nozzle mounted on said vertical header assembly and communicating with a water supply, said series having at least one nozzle disposed in a spaced relationship with the filter so that the energy of impact and the concentration of water at the surface of the filter are described according to the following relationship:

$$(1) \quad e(L) = k_1 + \frac{k_2}{1 + \left(\frac{L}{k_3}\right)^\alpha}$$

$$(2) \quad c(L) = k_4 + \frac{k_5}{1 + \left(\frac{L}{k_6}\right)^\beta}$$

where

$e(L)$  is energy of impact;  
 $k_1 + k_2$  is energy  $e(0)$  and are as follows: the energy required to force water through the filter unassisted by gravity at the top where  $L=0$ ;

$L$  is the distance measured down the face from the top of the filter;

$k_3$  is a function of gravity the angle of the filter to the horizon and the permeability characteristics of the various layers of material in the filter;

$\alpha$  is a constant;

$c(L)$  is the concentration at a given point;

$k_4 + k_5$  is the concentration  $c(0)$ ;

$k_4$  will depend on the minimum concentration  $c$  of water to flood all parts of the first layer  $x$  and to supplement the water running down inside the filter for which is lost at the back (layer  $z$ );

$k_5$  will be the amount of water required in the top portions to prime the rundown phenomenon in the filter;

$k_6$  is a function of gravity, the angle of the filter to the horizon and the permeability characteristics of the various layers of material in the filter; and

$\beta$  is a constant.

6. In a system for cleaning filter elements having a header assembly, support structure for said header assembly and means for producing reciprocal movement across the surface of a filter element, drive said means adaptable to advance and to retard said header assembly in said support structure as defined in claim 4, the improvement comprising more than

one nozzle whose apertures produce a spray pattern whose general characteristics of energy of impact and concentration are described in accordance with the formulas set out below:

$$e(L) = k_1 + \frac{k_2}{1 + \left(\frac{L}{k_3}\right)} \alpha$$

$$c(L) = k_4 + \frac{k_5}{1 + \left(\frac{L}{k_6}\right)} \beta$$

$e(L)$  is energy of impact;

$k_1 + k_2$  is energy  $e(0)$ ;

$L$  is the distance measured down the face from the top of the filter;

$\alpha$  is a constant;

$c(L)$  is the concentration at a given point;

$k_4$  will depend on the minimum concentration  $c$  of water to flood all parts of the first layer  $x$  and to supplement the water running down inside the filter for which is lost at the back (layer  $z$ );

$k_5$  will be the amount of water required in the top portions to prime the rundown phenomenon in the filter;

$k_6$  is a function of gravity, the angle of the filter to the horizon and the permeability characteristics of the various layers of material in the filter; and

$\beta$  is a constant.

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UNITED STATES PATENT OFFICE  
CERTIFICATE OF CORRECTION

Patent No. 3,563,474 Dated February 16, 1971

Inventor(s) Joseph William Robinson

It is certified that error appears in the above-identified patent and that said Letters Patent are hereby corrected as shown below:

column 3, line 48, insert --low-- before "volume";  
column 3, line 49, cancel "low";  
column 5, formula (1), that portion of the formula

reading  $+ \frac{k_2}{1+} \left( \frac{L}{k_3} \right) \alpha$  should read  $+ \frac{k_2}{1+ \left( \frac{L}{k_3} \right)} \alpha$

column 5, formula (2), that portion of the formula

reading  $+ \frac{k_5}{1+} \left( \frac{L}{k_6} \right) \beta$  should read  $+ \frac{k_5}{1+ \left( \frac{L}{k_6} \right)} \beta$

Signed and sealed this 1st day of May 1973.

(SEAL)  
Attest:

EDWARD M. FLETCHER, JR.  
Attesting Officer

ROBERT GOTTSCHALK  
Commissioner of Patents