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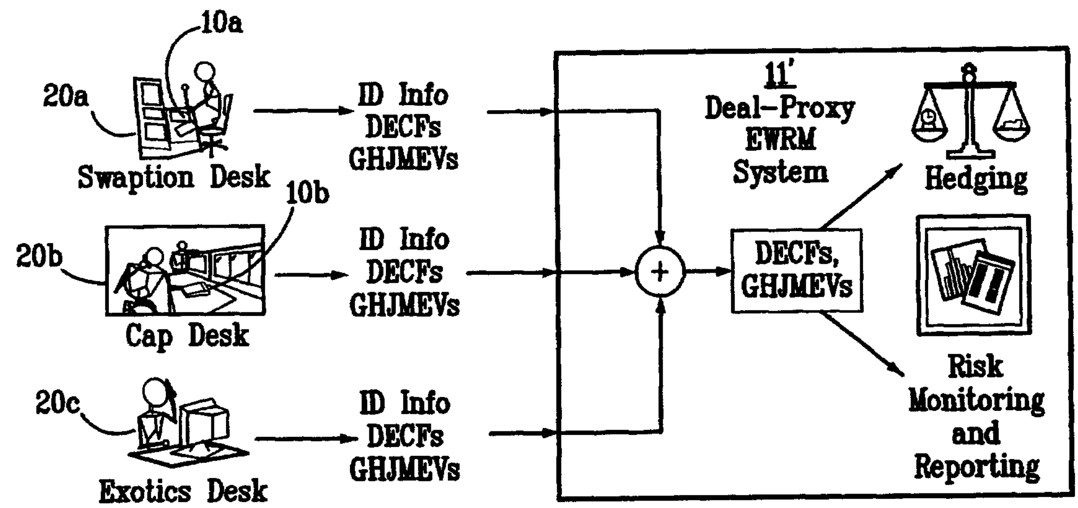
(54) Title: METHOD AND SYSTEM FOR PROVIDING UNIFIED VEGAS IN A RISK MANAGEMENT SYSTEM



(57) Abstract: A system employs a stochastic change-of-coordinates technique to represent parameter sensitivities for inconsistent mathematical models for financial derivatives in a commensurable way (20a, 20b, 20c). This includes the implementation of these calculations and the use of the derived representations in trading (front-office) systems as well as desk-level (10a, 10b) and EWRM systems implemented either on site or over a computer network. The financial instrument valuation engines are enhanced so that they can directly calculate commensurable exposure measures or the financial instrument valuation engine receives native exposures to calculate the commensurable exposure measure (11').

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METHOD AND SYSTEM FOR PROVIDING UNIFIED VEGAS IN A RISK MANAGEMENT SYSTEM

BACKGROUND OF THE INVENTION

Field of the Invention:

The present invention relates to computerized risk management systems. More particularly, the present invention relates to a method and system for providing consistent risk measures for financial derivatives when inconsistent mathematical models are used for their valuation.

Brief Description of Prior Developments:

Financial derivatives are contracts to exchange cash payments or assets depending on the level of one or more underlying financial variables, such as stock prices or interest rates. Financial derivatives may be used for both risk-mitigation purposes, termed hedging, as well as speculation. The ability to take highly leveraged positions using derivatives creates the potential for catastrophic losses. In addition, particularly with options, the nonlinearity of their payoffs with respect to the values of their underlying assets can make their potential behavior difficult to understand. The use of financial derivatives has therefore simulated the growth of quantitative risk-management techniques.

The value of many types of derivative contracts, in particular options, depends essentially on the degree to which the underlying financial variables fluctuate over time. These contracts are thus valued based on mathematical models of these fluctuations. These models typically contain one or more parameters, referred to as volatility parameters, that characterize the magnitude of these fluctuations. In addition, these models embed crucial assumptions about the dependence of absolute fluctuation levels on the underlying financial variables. For example, various models make different assumptions about how the absolute level of interest-rate fluctuations varies with the level of the underlying interest rate. One model might assume that the fluctuation level is independent of interest rates, while another model might assume that the fluctuation level is proportional to interest rates.

For hedging, risk management, and other reasons, traders and risk managers need to understand how the value of a portfolio of assets is effected by changes in fundamental market variables. For so-called primary assets, such as bonds and equities, the fundamental market variable can be simply taken to be the asset prices themselves. However, for financial-derivatives, the fundamental market variables are the underlying asset prices as well as their volatility parameters.

The most basic tools of risk management at the level of the individual trading desk are so-called "Greeks". These are simply the sensitivities, i.e., the partial (mathematical) derivatives, of the value of an individual instrument or a portfolio with respect to the parameters of the formula that is used to value them. For example, standard stock options may be valued using the celebrated Black-Scholes formula, which gives the value of a stock option in terms of the stock price, the volatility of the stock price, the interest rate, and the time to expiration of the option. The partial derivatives of the option value with respect to the stock price and its volatility parameter are termed "delta" and "vega", respectively. These and similar Greeks are widely used in the management of individual option positions. For example, delta gives an immediate approximation to the change in the value of an option with respect to a small change in the stock price. Greeks also tell one how to hedge, i.e., add a new position to neutralize the risk of an existing one. For example, to hedge exposure to a stock price, one needs to add a hedge position whose delta is the opposite of the delta of the portfolio.

Interest-rate options can be more complex than stock options because, at any given time, there is a term structure of interest rates. That is, interest rates depend on the length of time (the term) over which money is borrowed. In addition, there are numerous different ways in which interest rates are quoted, such are London Interbank Offered Rates (LIBOR) or swap rates. Many interest-rate options depend only on a single feature of the term structure of interest rates, such as a particular LIBOR or swap rate. Accordingly, it is common practice to apply the Black-Scholes formula to the particular feature of the term structure that is relevant to the interest-rate derivative being priced. For example, market practitioners value caps and swaptions using log-normal models of LIBOR and swap rates, respectively. Unfortunately, this practice makes it difficult to understand the risk exposure of the portfolio as a whole. This is because the Greeks for different instruments and models are sensitivities to different parameters and therefore cannot be sensibly simply added together.

Yet, since the parameters in the models all refer to the same underlying term structure of interest rates, the Greeks for the various models should be related. In the case of deltas, it is straightforward to obtain such relationships. For example, there is a well known formula that expresses swap rates in terms of LIBOR rates. One can then apply the chain rule from multivariate calculus to express the delta for a swaption as a linear function of deltas for caps. The coefficients in this relationship are given by the partial derivatives of the formula that expresses swap rates in terms of LIBOR rates. However, the situation for vega is more problematic. The problem is that the various Black-Scholes formulae can be mathematically inconsistent with each other. For example, the standard Black-Scholes formula for caps implies a model for the evolution of swap rates that is different from the one implied by the standard Black-Sholes formula for swaptions. Thus, unlike the case for delta, there isn't a formula relating the volatility parameters in the two models that can be differentiated to give the relation between the vegas. As a result, it has been difficult to reconcile the various vegas to get a unified picture of volatility risk at the overall portfolio level. The problem is compounded by exotic derivatives, which are typically valued using so-called short-rate models that are inconsistent with the various Black-Scholes formulae and thus have their own idiosyncratic vegas. The result is that practitioners are often faced with "Tower of Babel" vega reports, in which vegas for the various instrument classes and models are simply listed without any indication of the overall portfolio volatility position. Hence, there is a need to provide a system and method for describing the aggregate volatility exposure of a portfolio having a mixture of financial assets that may be valued using inconsistent valuation models.

SUMMARY OF THE INVENTION

The present invention provides commensurable "vegas" across inconsistent mathematical models that can be meaningfully added together to describe overall portfolio exposure to volatility risk.

Almost all mathematical models used in practice to value derivatives use a standard Brownian motion to model the essential randomness of financial variables over time. They differ in how the Brownian motion interacts with the financial variable to produce the observed degree of fluctuation in the financial variable. It turns out that a given model, say model A, can often be represented as a different model, termed a base model, by allowing the volatility parameter of the base model to depend on the sample value of the Brownian motion. We thus obtain a relationship between the "native" volatility parameter of model A and the volatility parameter of the base model that depends on the sample value of the driving Brownian motion. Thus, while there may not be a deterministic relationship between the volatility parameters for model A and the base model, we can often find a random or stochastic relationship. Mathematically differentiating this stochastic relationship between the volatility parameters in the two models allows one to convert the "native" vega for model A into an equivalent vega for the base model. Therefore, the present invention converts hitherto inconsistent vegas from the various models in use to vegas for the base model that can be added together to describe the overall exposure of a financial portfolio to volatility risk. It does so by applying what amounts to a stochastic change-of-coordinates that depends on the underlying source of randomness in the model.

According to another aspect of the invention, the resulting overall volatility exposure of the portfolio is graphically expressed in terms of a vega for a base model. The choice of a base model will depend on the asset class. For interest-rate instruments, the Heath, Jarrow, and Morton model (HJM), in which interest-rate volatility is parameterized as a function of maturity and time, is the preferred base model. For equities, the so-called Bachelier model, in which equity volatility is parameterized as a function of time, is preferred.

The base models have two significant features in common:

1. They are comprehensive in the sense that other models are easily expressed in terms of them. This comprehensiveness is achieved by having volatility parameters that can depend on time and, in the case of interest-rates, maturities. Of course, many of the models of interest will have a representation in the base model with stochastic volatility parameters.

2. The base models with deterministic volatility parameters are so-called Gaussian models. This allows for a technical simplification that makes it much easier to do the change-of-coordinates calculations.

According to an aspect of the invention, functional Greeks, in particular vegas, for the various models used in practice are transformed into equivalent functional Greeks for the base model. (The adjective functional refers to the fact that the Greeks here are partial derivatives with respect to curve- and surface-valued, i.e., functional parameters.) The transformation operates by relating the volatility parameters in the various models by a stochastic change of coordinates. Greeks from inconsistent mathematical models can thus be expressed in a commensurable manner in terms of functional Greeks for the base model.

BRIEF DESCRIPTION OF THE FIGURES

Other features of the invention are further apparent from the following detailed description of presently preferred exemplary embodiments of the invention taken in conjunction with the accompanying drawings, of which:

Figure 1 is a block diagram representing a computer system in which aspects of the present invention may be incorporated;

Figure 2 is schematic diagram representing a computer network system wherein aspects of the invention may be incorporated;

Figure 3 is a diagram of a Full-Deal Enterprise-Wide Risk Management system wherein aspects of the present invention may be incorporated;

Figure 4 is a diagram of a Deal-proxy Enterprise-Wide Risk Management system wherein aspects of the present invention may be incorporated;

Figure 5 is an illustration of the Black-Scholes and Bachelier models for the evolution of stock prices wherein the resulting stock-price evolution is a function of the relevant volatility process as well as the driving Brownian motion;

Figure 6 is a graphical illustration of functional vegas for deterministic-volatility Black-Scholes and Bachelier models;

Figure 7 is a representation of the deterministic-volatility Black-Scholes model as a stochastic-volatility Bachelier model;

Figure 8 illustrates the effect of a deterministic perturbation in Bachelier volatility on a stochastic-volatility Bachelier model simply by applying the deterministic perturbation to the sample values of the stochastic Bachelier volatility process; and

Figure 9 graphically illustrates functional vegas with respect to Bachelier volatility for a 5-year European call option. Graph (a) is for an at-the-money option, while graph (b) is for an out-of-the-money option whose discounted strike is 10% above the current price wherein the dotted lines are the vega for the Black-Scholes model, while the solid lines are for the Bachelier model calibrated to agree with Black-Scholes for at-the-money options;

Figure 10 illustrates the Bachelier-equivalent vega for the deterministic-volatility Black-Sholes model applied to the deterministic Bachelier perturbation is equla to the native Black-Sholes vega applied to the stochastic Black-Sholes perturbation;

Figure 11 illustrates the interconversion of vegas with respect to LIBOR and swap-rate volatilities;

Figure 12 illustrates the EWRM systems of Figure 3 and 4 wherein the detail analytic conversion to a base model is further illustrated; and

Figure 13 provides examples of functional vega displays.

DETAILED DESCRIPTION OF THE INVENTION

OVERVIEW

While individual trading desks are the front line of risk management, most large institutions support a centralized enterprise-wide risk management (EWRM) function. While in part this is because of the need to exercise oversight and their fiduciary responsibilities, it is also necessary because risk is a portfolio property. On one hand, there is always the danger that risks can be magnified by different trading desks taking similar positions. On the other hand, the risk of the institution will generally be less than the sum of the risks of the trading desks due to diversification effects. It is therefore necessary for large institutions to gather together the information from their various risk-taking operations and put together a coherent picture of the risk profile of the institution as a whole.

Large financial institutions trade a bewildering variety of financial instruments. It is not unusual for a large institutions to use dozens of computerized trading systems each with its own mathematical model. As a result, the EWRM problem is notoriously difficult. As a first step, it is desirable in EWRM to describe the sensitivities of the institution as a whole to changes in foreign-exchange (FX) rates, interest rates, and stock prices, as well as their respective volatilities. While the individual trading desks produce such sensitivities, i.e., Greeks, it will generally not be sufficient to just add together the Greeks reported by the various trading desks. This is because, due to their use of different models, the reported Greeks will generally reflect sensitivities to different things. This has been termed the "apples and oranges" problem. The present invention solves the apples and oranges problem, particularly with respect to the Greek vega.

The present invention provides a financial risk-management system wherein uniform risk-factor sensitivities can be presented for a portfolio containing a variety of financial derivative assets. The system operates by converting risk exposures for the "native" valuation model for each financial asset into equivalent risk exposures for a base model. By converting the risk exposures for the various models into those for a common base model, commensurable risk sensitivities can be determined for each model. The converted sensitivities can then be combined together to present an overall risk evaluation for the portfolio.

EXEMPLARY OPERATING ENVIRONMENT

Figure 1 provides a block diagram of an exemplary environment in which the invention may be implemented. Moreover, the invention is described herein in the context of flow charts and computer-executable instructions that operate on a computer system such as the system of Figure 1. Generally, computer-executable instructions are contained in program modules such as programs, objects, data structures and the like that perform particular tasks. Those skilled in the art will appreciate that the invention may be practiced with other computer system configurations, including multi-processor systems, network PCs, minicomputers, mainframe computers and so on. The invention may also be practiced in distributed computing environments where tasks are performed by remote processing devices that are linked through a communications network.

Figure 1 includes a general-purpose computing device in the form of a computer system 20, including a processing unit 22, and a system memory 24. The system memory could include read-only memory (ROM) and/or random access memory (RAM) and contains the program code 10 and data 12 for carrying out the present invention. The system further comprises a storage device 16, such as a magnetic disk drive, optical disk drive, or the like. The storage device 16 and its associated computer-readable media provides a non-volatile storage of computer readable instructions, data structures, program modules and other data for the computer system 20.

A user may enter commands and information into the computer system 20 by way of input devices such as a keyboard 26 and pointing device 18. A display device 14 such as a monitor is connected to the computer system 20 to provide visual indications for user input and output. In addition to the display device 14, computer system 20 may also include other peripheral output devices (not shown), such as a printer.

It should be noted that the computer described above can be deployed as part of a computer network, and that the present invention pertains to any computer system having any number of memory or storage units, and any number of applications and processes occurring across any number of volumes. Thus, the present invention may apply to both server computers and client computers deployed in a network environment, having remote or local storage. Figure 2 illustrates an exemplary network environment, with a server in communication with client computers via a network, in which the present invention may be employed. As shown, a number of servers 11, 11', etc., are interconnected via a communications network 14 (which may be a LAN, WAN, intranet or the Internet) with a number of client computers 20a, 20b, 20c, etc. In a network environment in which the communications network 14 is the Internet, for example, the servers 11 can be Web servers with which the clients 20 communicate via any of a number of known protocols such as hypertext transfer protocol (HTTP).

Each client computer 20 and server computer 10 may be equipped with various application program modules 10, other program modules 37 and program data 38, and with connections or access to various types of storage elements or objects. Thus, each computer 10 or 20 may have financial information associated therewith, such as stock prices, interest rates, bond prices and so on. Each computer 20 may contain computer-executable instructions that model financial assets or risk associated with the assets. For example, one system may model financial derivatives based on a Black-Scholes model, whereas another may model interest rate derivatives based on a Heath-Jarrow-Morton model, and so on. These computers can then pass their respective data to the server computers 11 wherein an enterprise-wide risk management (EWRM) system may reside to determine overall risk across asset classes.

System for Unifying Vegas

Generally speaking, EWRM systems gather information from one or more source ("front-office") systems and use this information to construct a partial or complete risk analysis for a financial institution. There are at least two general architectures for such systems. For brevity, we term these the full-deal and deal-proxy architectures. In a full-deal system, such as SunGard's Panorama system, the source systems pass an essentially complete description of each deal to the EWRM. For example, for an interest-rate cap, this description would include the start and end dates for each period that the cap applies to, the level (termed the strike) of the cap, a precise description of how the payment amount is to be computed, and so forth. It is assumed that the EWRM system has the "intelligence" to value each deal using its native valuation model. In addition, the key task of the EWRM system is to construct a coherent description of the overall risk of the institution. In order to do so, it must construct commensurable descriptions of the risk characteristics of the various deals. As described more fully below, according to an aspect of the invention, Gaussian Heath-Jarrow-Morton equivalent vegas (GHJMEVs) are used as a uniform description of interest-rate-volatility exposures.

In a full-deal EWRM system incorporating the present invention, commensurable exposure measures, such as GHJMEVs, are computed internally, either directly or by converting native exposures. Such a deployment is shown in Figure 3. Here, computer systems 20a, 20b, and 20c gather information representative of a deal and pass that information to EWRM system 11 wherein analytical software extracts from the deal a vega of the GHJMEV form so that vegas for the deals in a portfolio can be added to each other by program 10d. For example, the data representative of the terms of a swaption deal are provided to the swaption analytics module 10a from swaption computer 20a. The swaptions analytics module 10a then computes the GHJMEV. Similarly, data representative of a cap deal are provided to the cap analytics module 10b, and so on.

In view of the great difficulty in representing the variety of valuation models found in front-office systems in EWRM systems and the fact that many important calculations can be based entirely on linear approximations, an alternative system in which the present invention may be employed is an EWRM system architecture in which the front-office systems pass linear approximations rather than complete details of each deal to the EWRM system. We term such an architecture a deal-proxy system. This results in advantages on the software-engineering side, as the EWRM system no longer needs to understand all the internal models used by all the front-office systems in order to perform basic operations such as limit administration, hedging, and risk management. While the deal-proxy architecture is often used for managing interest-rate exposures, probably its biggest limitation at present has been the lack of a standard form for characterizing volatility exposures. An application of the vega methodology of the present invention is to provide a standard representation of volatility exposures in deal-proxy systems. Such an architecture is illustrated in Figure 4. Here, the computer systems 20a, 20b, etc. perform the analytics. The EWRM system 11' can then directly add the vegas across the financial instrument classes. The ability to capture volatility exposures will make the deal-proxy architecture much more attractive for many applications. Such an architecture is particularly attractive for real-time or near-real-time applications because of its simplicity and the fact that almost all analyses that are feasible to run in real time are based on linear approximations.

It should be recognized that one of the most important advantages of basing risk-management operations on first-order sensitivities such as GHJMEVs is that such abstractions can be defined in a financially precise and system-independent manner. Building EWRM systems is difficult because each of the various front-office systems is based on different financial abstractions. Each system encodes these financial abstractions in terms of character strings and numbers, but it is extremely difficult to reconcile the semantics of these character strings and numbers into a common coherent framework. GHJMEVs can serve as a financial lingua franca for expressing market exposures in a standard way. Having a semantically un-ambigious language for describing exposures greatly facilitates communication both across functional and geographic boundaries within an institution and to external parties.

In particular, having a standard for communicating exposures facilitates the "outsourcing" of risk-management calculations. In addition to calculation of exposures, risk-management calculations generally require extensive archives of historical market data and specialized computational engines. Given the ability to clearly communicate the exposures, it becomes feasible to have a third party maintain the historical data and specialized computational engines. For example, the exposures could be transmitted over the Internet to a specialized provider of risk-management services. Risk calculations would be performed by the risk-managment-service provider and results returned to the client back over the Internet.

The GHJMEV for an interest-rate instrument is a two-dimensional function of maturity and time. In practical calculations, it will be possible to divide the maturity-time rectangle into a finite number of subrectangles with the GHJMEV being constant on each subrectangle. Thus the GHJMEV can be conveniently represented in a relational database table with each subrectangle of the GHJMEV corresponding to a row in the table. For example, the columns of this database might be as follows:

Currency The 3-character SWIFT currency code of the currency for the interest rate.

DiscountCurve A string variable describing the particular interest-rate curve that is relevant to the instrument being valued, e.g., LIBOR or Treasury.

BeginningMaturityDate A date variable containing the beginning maturity date for the subrectangle in maturity and time.

EndingMaturityDate A date variable containing the ending maturity date for the subrectangle in maturity and time.

BeginningTimeDate A date variable containing the beginning time date for the subrectangle in maturity and time.

EndingTimeDate A date variable containing the ending time date for the subrectangle in maturity and time.

Sensitivity A real variable containing the partial derivative of the PV of the instrument with respect to the local Gaussian HJM volatility for the sub-rectangle.

Hence, a deal-proxy system could pass a data set of the above variables that represents the GHJMEV for the financial derivative. The EWRMS can then use that information to aggregate the sensitivities (i.e., vegas) across instrument classes.

Having provided an overview of the system of the present invention, the details of the implementation, i.e., the functions performed by software, are discussed below.

Vegas are used to indicate sensitivity of a particular financial derivative to changes in the model's volatility inputs. As described above, conventional financial models for financial derivative assets may be inconsistent with each other and thus have incommensurable vega values. Inconsistencies between the various interest-rate models arise because various features of the term structure are modeled in a piecemeal way. Hence, as a first step toward a unified approach to expressing vegas of interest-rate derivatives, more comprehensive models for the evolution of the term structure as a whole are needed.

The first step in the methodology of the present invention is to extend the notion of Greeks. Most of the option-valuation models used in finance are characterized by a small number of scalar parameters and the Greeks used in mathematical finance are partial derivatives with respect to these parameters. However, there are classes of more sophisticated models in which the volatility parameter is allowed to vary continuously over time, and, in the case of interest-rate models, with maturity as well. Thus these models, such as the HJM model, have curve- and surface-valued parameters in addition to scalar parameters.

Part of the analysis is to extend the notion of Greeks to the derivative of a valuation function with respect to a curve- or surface-valued parameter. The derivative of a function of a curve- or surface-valued argument is best understood in terms of its directional derivatives. A curve- or surface-valued argument is formalized by letting the argument take values in a infinite-dimensional vector space. Let X and Y be vector spaces and let v be a function from X to Y. The function v is said to have directional derivative $Dv|_x \cdot \Delta x$ in the direction Δx at the evaluation point x in X if

$$Dv|_x \cdot \Delta x = \lim_{s \to 0} \frac{v(x + s\Delta x) - v(s)}{s}. \tag{1}$$

In words, the directional derivative $Dv|_x \cdot \Delta x$ is the rate that v changes when the evaluation point x is perturbed in the direction Δx. If a function v has directional derivatives in all directions Δx, then the function from the perturbation Δx in X to $Dv|_x \cdot \Delta x$ in Y gives a linear approximation to the function v at x in the sense that

$$v(x + \Delta x) \approx v(x) + Dv|_x \cdot \Delta x.$$

In the particular case where v is a real-valued function of a curve-valued parameter σ, $Dv|_\sigma \cdot \Delta\sigma$ often has a representation of the form

$$Dv|_\sigma \cdot \Delta\sigma = \int \frac{dv}{d\sigma}(t)\Delta\sigma(t)dt, \tag{2}$$

where $dv/d\sigma$ is a real-valued curve termed the functional derivative of v with respect to σ. Intuitively, $dv/d\sigma(t)$ describes the sensitivity of v to a localized change in the curve σ at t. As $dv/d\sigma$ is simply a real-valued function, it provides a convenient graphical representation of $Dv|_\sigma$.

We will give some simple examples for stock-price models. In the well-known Black-Scholes model for stock prices, the stock price $\widetilde{S}(t)$ is given by the solution to the stochastic differential equation (SDE)

$$d\widetilde{S}(t) = \widetilde{S}(t)\sigma_{BS}(t)dW(t), \tag{3}$$

where W(t) denotes a standard Brownian motion and $\sigma_{BS}(t)$ is the Black-Scholes volatility as a function of time. In an alternative model, termed the Bachelier model, the stock price is given by the solution to the SDE

$$d\widetilde{S}(t) = \sigma_B(t)dW(t), \tag{4}$$

where $\sigma_B(t)$ is the Bachelier volatility as a function of time. We assume that the volatility functions are deterministic, i.e., that they have no functional dependence on the stock-price process. The stock-price processes for these models are illustrated in Figure 5.

Functional vegas for a stock option valued under the Bachelier and Black-Scholes models are shown in Figure 6. In each case, the value of functional vega at time t is the rate at which the option price changes with respect to a localized change in the volatility function at time t. In the example shown in Figure 6, the volatility parameter for the Bachelier model has been adjusted so that the prices of so-called at-the-money (ATM) options agree with those obtained from the Black-Scholes model. The fact that the vegas are so different is a manifestation of the apples-and-oranges problem, i.e., the incommensurability of vegas that are produced from different underlying financial models.

The key to the apples and oranges problem for deltas of caps and swaptions was the function that expressed swap rates in terms of LIBOR rates. This was a simple deterministic function between finite-dimensional vector spaces which can easily be differentiated to obtain the relation between the deltas. The key to the apples and oranges problem for vegas is finding analogous relationships between volatility processes in the various models. The inconsistency of the various models rules out the possibility of finding simple deterministic relationships between the parameters of the various models. The essence of our invention is that, nonetheless, there are relationships between the volatility parameters in the various models that can be differentiated to obtain relations between the vegas. In general, these relationships are given by maps between infinite-dimensional stochastic processes.

We give a simple example for stock-price models. The Black-Scholes and Bachelier models are inconsistent with each other when their volatility processes are taken to be deterministic. Indeed, it can be shown that stock prices are log-normally distributed under the Black-Scholes model, whereas they are normally distributed under the Bachelier model. However, the deterministic-volatility Black-Scholes model can be represented as a stochastic-volatility Bachelier model. This is most easily seen by comparing the SDEs given by equations 3 and 4. All we need to do is take σ_B equal to

$\tilde{S}_{BS}(\sigma_{BS})\sigma_{BS}$, where $\tilde{S}_{BS}(\sigma_{BS})$ denotes the solution to equation 3. This is illustrated in Figure 7.

Having represented the Black-Scholes model as a stochastic-volatility Bachelier model, we are now in a position to construct a Bachelier-equivalent vega for options valued under the Black-Scholes model. To see this, it should be understood that knowing the Bachelier vega for a given model amounts to knowing the effect of any given deterministic perturbation in Bachelier volatility on the option price. Even though the Bachelier volatility is now stochastic, the effect of a given deterministic perturbation in Bachelier volatility on the option price can still measured by applying it to each of the sample values of the stochastic Bachelier volatility process, as illustrated in Figure 8.

As an example, in Figure 9 is a plot of Bachelier-equivalent functional vegas for stock options valued under the Black-Scholes model (shown in dashed lines). For comparison, Figure 9 also shows the functional vegas for the deterministic-volatility Bachelier model in which the volatility has been adjusted so that prices for at-the-money options agree with those from the Black-Scholes model (shown in solid lines). On the left are vegas for an at-the-money (ATM) option, while on the right are vegas for a so-called out-of- the-money (OTM) option. The differences in the curves, while small, reflect the fact that the two models have slightly different "views" on the effects of volatility perturbations. The vegas are more similar for the at-the-money option, as might be expected as the volatility parameter in the Bachelier model was adjusted to make its prices agree with Black-Scholes for at-the-money options.

The above sections describe how to compute a Bachelier-equivalent vega for an option valued under the Black-Scholes model by representing the Black-Scholes model as a stochastic-volatility Bachelier model. This next section describes how this Bachelier-equivalent vega is related to the native Black-Scholes vega by the derivative of the function that maps the Black-Scholes volatility to the corresponding stochastic Bachelier volatility.

Roughly speaking, a Black-Scholes model with volatility parameter σ_{BS} has a Bachelier model representation with volatility parameter

$$\sigma_B = \tilde{S}_{BS}(\sigma_{BS})\sigma_{BS}. \qquad (5)$$

Going the other way, it can be shown that if a Black-Scholes model has a Bachelier representation with volatility σ_B, then the Black-Scholes volatility is given by

$$\sigma_{BS} = \widetilde{S}_B^{-1}(\sigma_B)\sigma_B \qquad (6)$$

where $\widetilde{S}_B(\sigma_B)$ denotes the solution to equation 4. Applying the chain rule, using equation 5, says that the relation between the vegas with respect to Black-Scholes and Bachelier volatility is given by the formula

$$Dv\big|_{\sigma_B} \cdot \Delta\sigma_B = Dv\big|_{\sigma_{BS}} \circ D\sigma_{BS}\big|_{\sigma_B} \cdot \Delta\sigma_B$$

In particular, this equation says that the Bachelier-equivalent vega for the deterministic-volatility Black-Scholes model applied to the deterministic Bachelier perturbation $\Delta\sigma_B$ is equal to the native Black-Scholes vega applied to the stochastic Black-Scholes perturbation $D\sigma_{BS}\big|_{\sigma_B} \cdot \Delta\sigma_B$. This interpretation is illustrated in Figure 10, cf. Figure 8.

An important facet behind the invention is that the above reasoning can be given a rigorous mathematical justification. Equations (5) and (6) exhibit maps between infinite-dimensional stochastic processes and some care is needed in defining their derivatives. The standard techniques of calculus extend in a natural way from real functions of real variables to a class of infinite-dimensional spaces termed normed linear spaces. However, due to the somewhat "pathological" nature of Brownian motion, the maps given by equations (5) and (6) cannot be differentiated satisfactorily as maps between normed linear spaces. An important point is that the maps given by equations (5) and (6) can be differentiated satisfactorily as maps between more general spaces termed topological vector spaces. The mathematical derivation are further described in Kuruc Alvin "Commensurable ``Vegas" for Heterogeneous Volatility Models." Working paper presentation on April 5, 2000 at the Global Derivatives 00 Conference, which is hereby incorporated by reference.

With these technical tools in hand, we can now solve the apples-and-oranges problem for vega. One can use the chain rule to convert between the vegas with respect to LIBOR- and swap-rate volatilities. This is illustrated in Figure 11. This analysis allows us to reconcile vegas for two models when one model has a stochastic-volatility representation in another model. To achieve the goal of reconciling vegas across the various models used in practice, the various models used in practice need to be represented in terms of a single "base model". In the case of interest-rate models, it is well known that the commonly used interest-rate models can all be represented in a general class of models known as Heath-Jarrow-Morton (HJM) models. (In this regard, it is worth pointing out that the recent work on so-called market models details how the standard Black's models for caps and swaptions are represented by stochastic-volatility HJM models.) In general, the volatility in these HJM representations will be stochastic. The special case of HJM models in which the volatility is deterministic are termed Gaussian HJM models. We can compute vegas with respect to deterministic, i.e., Gaussian, HJM volatility perturbations even for models that have stochastic-volatility HJM representation using the approach discussed above. Since all of the commonly used interest-rate models can be represented as HJM models, we thereby obtain a uniform description of interest-rate-volatility exposures in terms of Gaussian-HJM-equivalent vegas (GHJMEVs). GHJMEVs may be used as a standard representation of interest-rate-volatility exposures. Such a use is shown in Figure 12. In the case of equity models, the corresponding base model is the Bachelier models. Again, the equity models commonly used in practice can be represented as stochastic-volatility Bachelier models.

Figure 12 illustrates further detail of the analytical functions of EWRM systems. In particular, each of the analytic portions converts the native vega for the native valuation model to an equivalent vega for the selected base model. Here the base model is the Gaussian HJM model.

We have shown that if one uses the Gaussian HJM model as a base model, then the vegas with respect to Gaussian-HJM volatility for the standard Black's models for caps and swaptions are given by simple closed-form approximations. The details of the GHJMEV calculations for caps and swaptions under the standard models are described in Kuruc, Alvin: "A Unified Approach to "Greeks" for Interest-Rate Derivatives." Working paper presented on November 30, 1999 at the Risk Management 99 Conference, which is hereby incorporated by reference. A key to this result is that the evolution of interest rates is linear in volatility in the Gaussian HJM model and therefore the derivative of the interest-rate evolution with respect to Gaussian HJM volatility is independent of the evaluation point. The implication of this is that the vega calculations can be done without explicitly involving the sample paths of the stochastic HJM-volatility process.

While an important application of the invention is in the area of interest-rate derivatives, there are also important applications to equity and foreign-exchange (FX) derivatives. Note that, from the mathematical point of view, equities and FX are essentially identical. In the case of equity models, computational advantages analogous to the ones described above for interest-rate models are obtained by using the Bachelier model as a base model. Details of the Bachelier-equivalent vega calculations for basic options under the standard Black-Scholes model are described in Kuruc Alvin "Commensurable ``Vegas" for Heterogeneous Volatility Models." Working paper presenton on April 5, 2000 at the Global Derivatives 00 Conference.To avoid duplication, in what follows we will mainly discuss interest-rate models and just mention equity models when particularly pertinent. However, it should be understood that the applications that we describe for interest-rate models generally apply to equity and FX models as well.

In either the full-deal or deal-proxy architecture, it is necessary to compute GHJMEVs for each instrument. GHJMEVs are two-dimensional, depending on both maturity and time. The GHJMEVs are therefore most naturally displayed in terms of a two-dimensional heat map, in which vega levels are associated with colors. Examples of such displays are shown in gray scale in Figure 11. For equities, the Bachelier-equivalent vega is one-dimensional, depending only on time. It can therefore be conveniently displayed in a standard two-dimensional graph.

The disclosed embodiments describe the use of the stochastic change-of-coordinates technique to represent parameter sensitivities for inconsistent mathematical models for financial derivatives in a commensurable way. This includes the implementation of these calculations and the use of the derived representations in trading (front-office) systems as well as desk-level and EWRM systems implemented either on site or over a computer network. The invention can be implemented in a variety of computer systems including, but not limited to, EWRM systems. The invention can be used to enhance financial instrument valuation engines so that they can directly calculate commensurable exposure measures or by having the financial instrument valuation engine pass native exposures to a separate computational engine that calculates the commensurable exposure measure. Additionally, the invention could be part of an Internet-based risk-management-service whereby a client transmits native exposures over the Internet and the risk-management-service provider calculates the commensurable exposure measures and provides the results to the client over the Internet.

Having described and illustrated the principles of the present invention with reference to an illustrated embodiment, it will be recognized that the illustrated embodiment can be modified in arrangement and detail without departing from such principles. It should be understood that the programs, processes, or methods described herein are not related or limited to any particular type of computer apparatus, unless indicated otherwise. Various types of general purpose or specialized computer apparatus may be used with or perform operations in accordance with the teachings described herein. Elements of the illustrated embodiment shown in software may be implemented in hardware and vice versa.

In view of the many possible embodiments to which the principles of the present invention may be applied, it should be recognized that the detailed embodiments are illustrative only and should not be taken as limiting the scope of my invention. Rather, the invention includes all such embodiments as may come within the scope and spirit of the following claims and equivalents thereto.

Claims:

What is claimed is:

1. A method for characterizing the volatility exposure of a portfolio of financial assets, comprising:

receiving a first volatility exposure characterization for a first financial asset valued under a first mathematical model;

receiving a second volatility exposure characterization for a second financial asset valued under a second mathematical model;

converting the first volatility exposure characterization to a volatility exposure characterization for a base financial model;

converting the second volatility exposure characterization to a volatility exposure characterization of the base financial model, whereby the first and second volatility exposure characterizations are commensurable; and

combining the converted first and second volatility exposure characterization into a volatility exposure characterization of the portfolio.

2. The method according to claim 1 wherein the first asset comprises an interest-rate derivative.

3. The method according to claim 2 wherein the first asset comprises one of a swaption and a cap.

4. The method according to claim 1 wherein the base model comprises a Gaussian Heath-Jarrow-Morton volatility model.

5. The method of claim 1 outputting the volatility exposure characterization of the portfolio.

6. The method as recited in claim 5 wherein the volatility exposure output comprises a two-dimensional heat map representation wherein the volatility exposure is indicated relative to a heat value on the graph.

7. The method as recited in claim 6 wherein time and maturity of the volatility risk are represented along axes of the graph.

8. A risk management system for determining the volatility exposure of a portfolio of financial assets, comprising:

an electronic connection to a network for receiving a first volatility exposure characterization for a first financial asset valued under a first mathematical model and a second volatility exposure characterization for a second financial asset valued under a second mathematical model;

a module for converting the first volatility exposure characterization to a volatility exposure characterization for a base financial model;

a module for converting the second volatility exposure characterization to a volatility exposure characterization of the base financial model, whereby the first and second volatility exposure characterizations are commensurable; and

a module for combining the converted first and second volatility exposure characterization into a volatility exposure characterization of a portfolio.

9. The risk management system as recited in claim 8 wherein one of the first and second financial assets comprise an interest-rate derivative.

10. The risk management system as recited in claim 8 wherein one of the first and second financial assets comprise one of a swaption and a cap.

11. The risk management system according to claim 8 wherein the base model comprises a Gaussian Heath-Jarrow-Morton volatility model.

12. The risk management system as recited in claim 8 further comprising a module for outputting the volatility exposure characterization of the portfolio.

13. The risk management system as recited in claim 12 wherein the volatility exposure output comprises a three dimensional graphic representation wherein the volatility exposure is indicated by a shade on the graph.

14. The risk management system as recited in claim 13 wherein time and maturity of the volatility risk are represented along axes of the graph.

15. The risk management system of claim 8 wherein the volatility exposure is a vega with respect to a Heath-Jarrow-Morton volatility.

16. A method for use in a risk management system, comprising the acts of:

providing a sever computer coupled to a client system by way of a network;

receiving from said client system a first volatility exposure characterization for a first financial asset valued under a first mathematical model;

converting on the server the first volatility exposure characterization to a volatility exposure characterization for a base financial model;

receiving from said client system a second volatility exposure characterization for a second financial asset valued under a second mathematical model;

converting on the server the second volatility exposure characterization to a volatility exposure characterization of the base financial model, whereby the first and second volatility exposure characterizations are commensurable; and

combining the first and second volatility exposures characterizations into a portfolio volatility exposure characterization.

17. The method as recited in claim 16 wherein the portfolio volatility exposure characterization is expressed in terms of vega with respect to a Guassian Heath-Jarrow-Morton volatility parameter.

18. The method as recited in claim 17 wherein the vega is used to determine a volatility hedge for the portfolio.

19. The method as recited in claim 17 wherein the first and second mathematical models produce incommensurable vegas.

20. The method as recited in claim 17 wherein the base model is a Guassian Heath-Jarrow-Morton model.

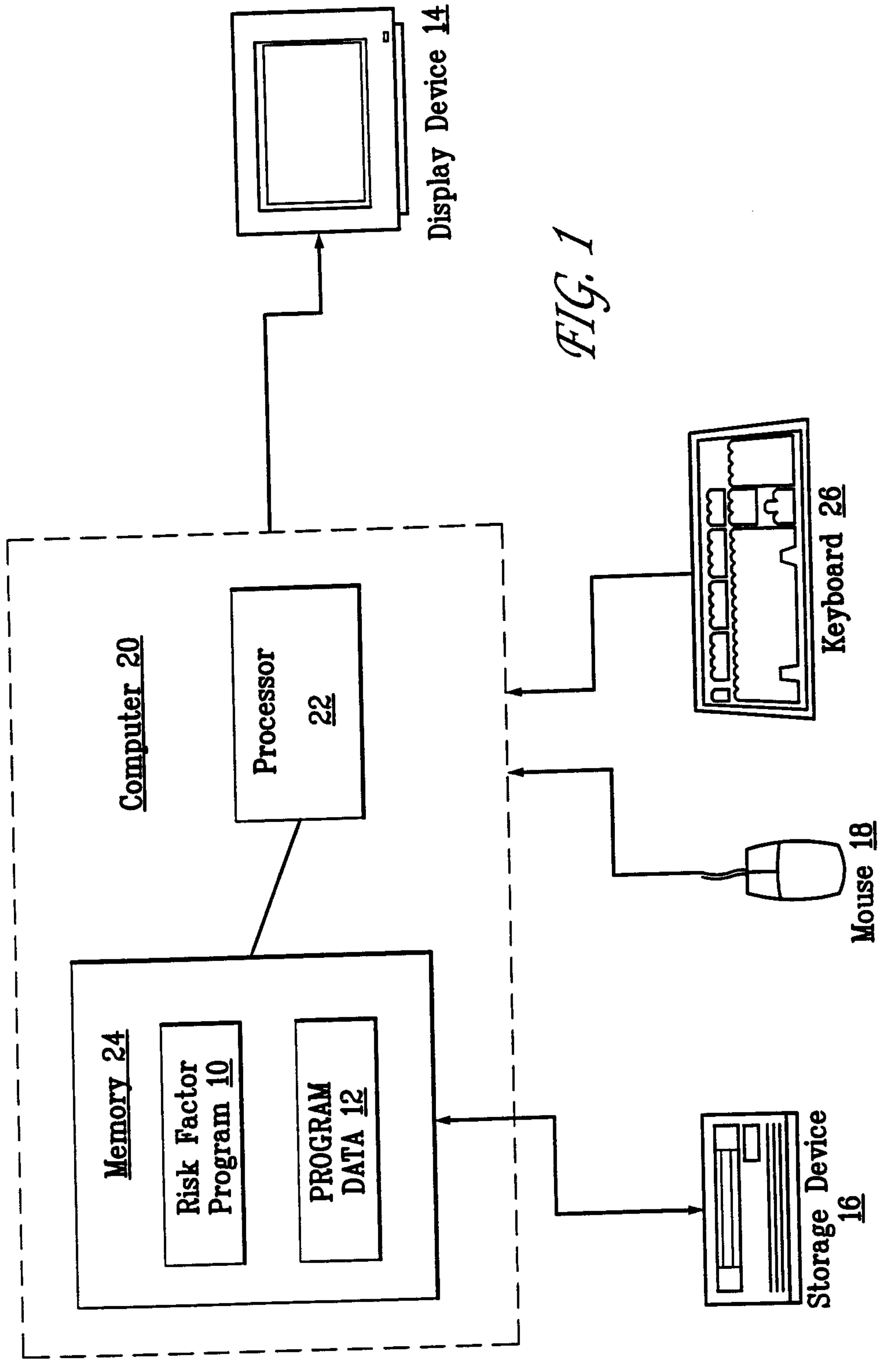

Display Device 14
Computer 20
Processor
22
Memory 24
Risk Factor
Program 10
PROGRAM
DATA 12
Keyboard 26
Mouse 18
Storage Device
16

FIG. 1

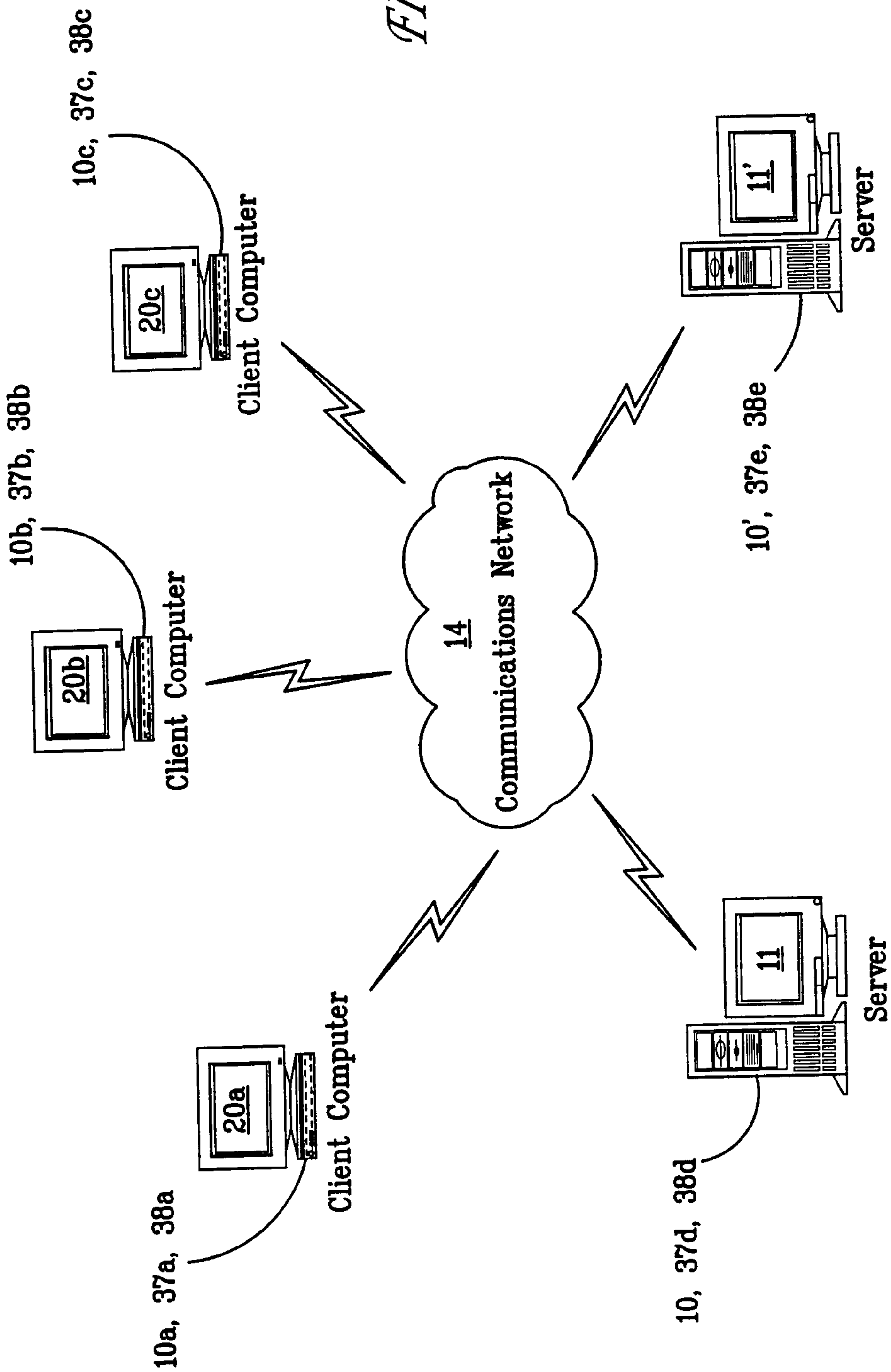

10c, 37c, 38c
20c
Client Computer
10b, 37b, 38b
20b
Client Computer
14
Communications Network
11'
Server
10', 37e, 38e
20a
Client Computer
10a, 37a, 38a
11
Server
10, 37d, 38d

FIG. 2

FIG. 3

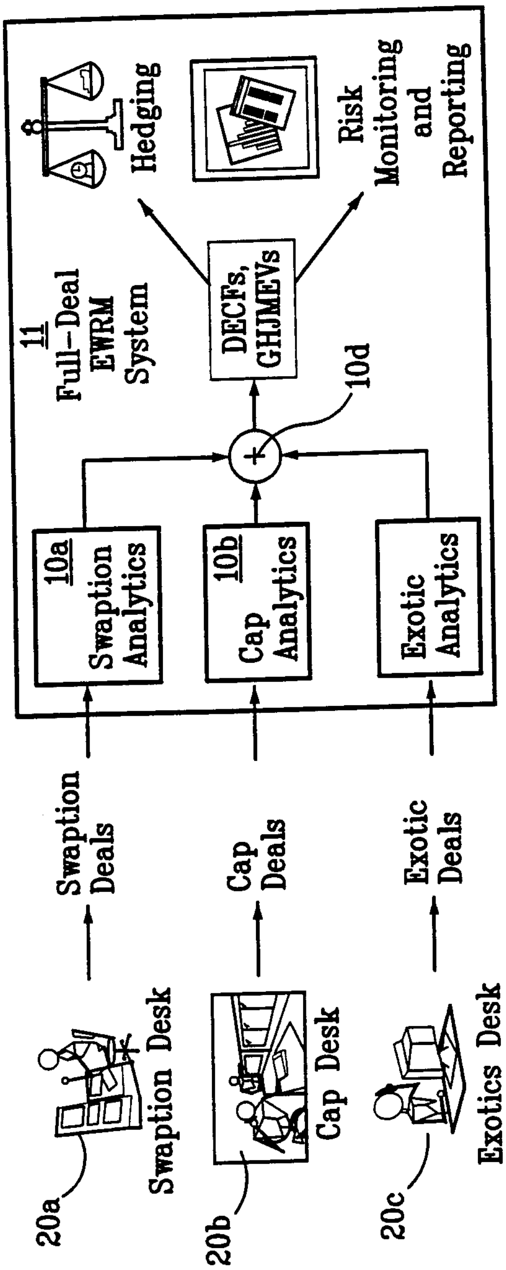
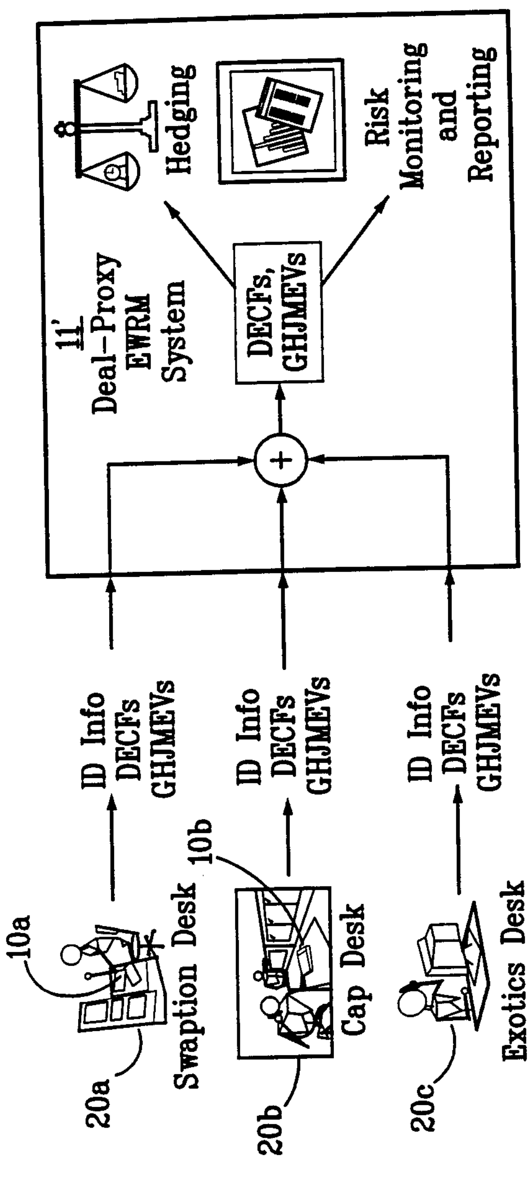

11
Full-Deal
EWRM
System
Hedging
DECFs,
GHJMEVs
Risk
Monitoring
and
Reporting
10d
10a
Swaption
Analytics
10b
Cap
Analytics
Exotic
Analytics
Swaption
Deals
Cap
Deals
Exotic
Deals
20a
Swaption Desk
20b
Cap Desk
20c
Exotics Desk

FIG. 4


11'
Deal-Proxy
EWRM
System
Hedging
DECFs,
GHJMEVs
Risk
Monitoring
and
Reporting
ID Info
DECFs
GHJMEVs
ID Info
DECFs
GHJMEVs
ID Info
DECFs
GHJMEVs
10a
10b
20a
Swaption Desk
20b
Cap Desk
20c
Exotics Desk

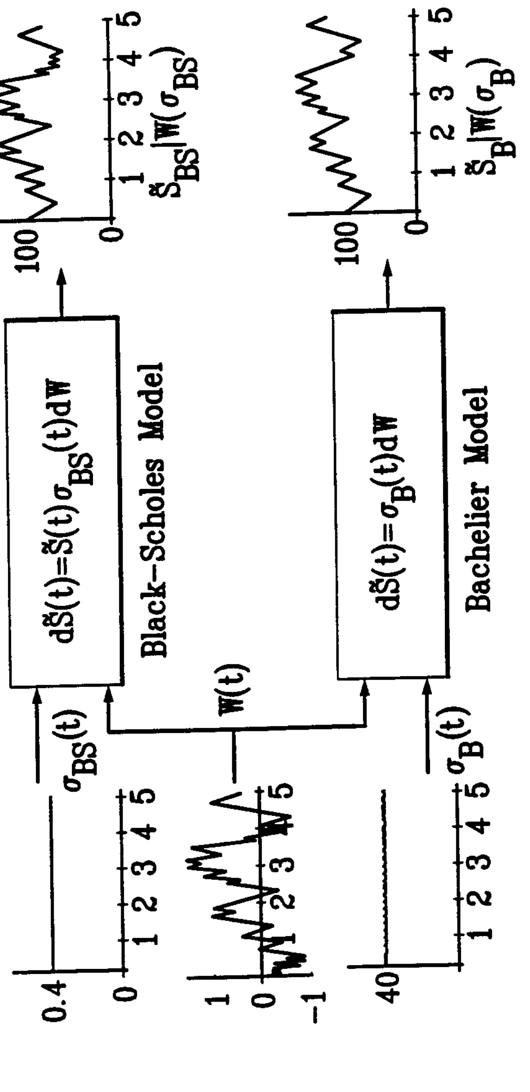

Black-Scholes Model
Bachelier Model
W(t)

FIG. 5

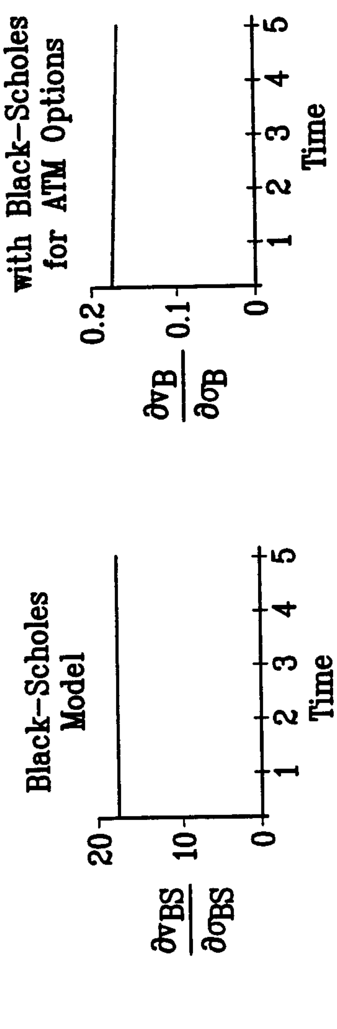

Black-Scholes Model
Bachelier Model Calibrated to Agree with Black-Scholes for ATM Options
Time
Time

FIG. 6

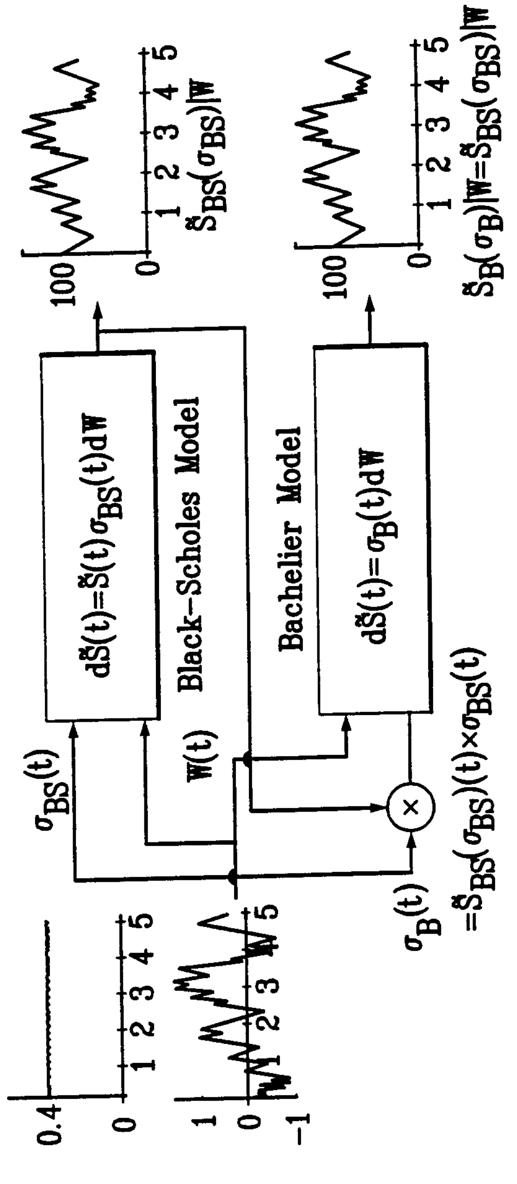

$\sigma_{BS}(t)$
$d\tilde{S}(t)=\tilde{S}(t)\sigma_{BS}(t)dW$
Black-Scholes Model
W(t)
Bachelier Model
$d\tilde{S}(t)=\sigma_B(t)dW$
$\sigma_B(t)$
$=\tilde{S}_{BS}(\sigma_{BS})(t)\times\sigma_{BS}(t)$
$\tilde{S}_{BS}(\sigma_{BS})|W$
$\tilde{S}_B(\sigma_B)|W=\tilde{S}_{BS}(\sigma_{BS})|W$
0.4
0
1
-1
100
1 2 3 4 5

FIG. 7

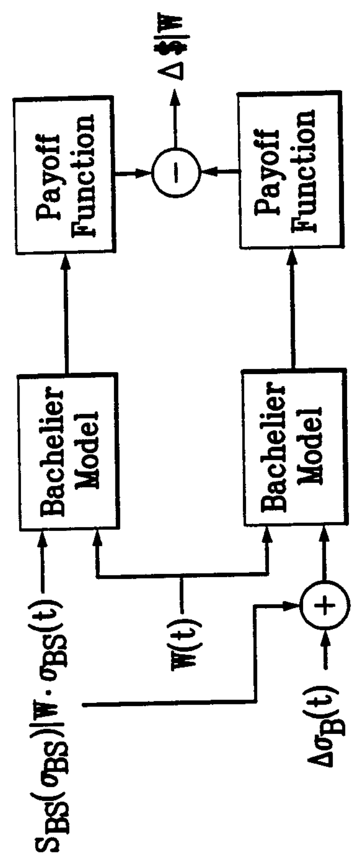

$S_{BS}(\sigma_{BS})|W\cdot\sigma_{BS}(t)$
W(t)
$\Delta\sigma_B(t)$
Bachelier Model
Bachelier Model
Payoff Function
Payoff Function
$\Delta \$|W$

FIG. 8

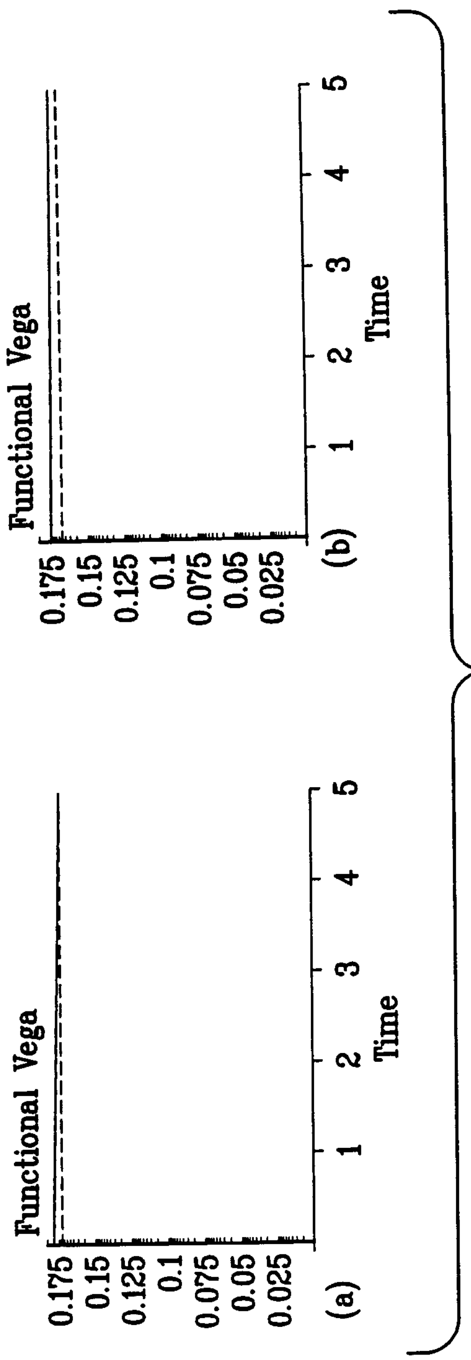

Functional Vega
0.175
0.15
0.125
0.1
0.075
0.05
0.025
(a)
1
2
3
4
5
Time
Functional Vega
0.175
0.15
0.125
0.1
0.075
0.05
0.025
(b)
1
2
3
4
5
Time

FIG. 9

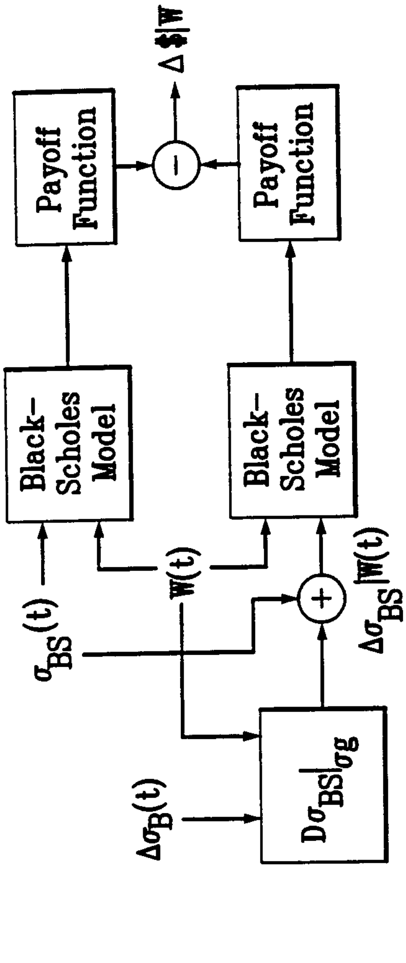

Black-Scholes Model
Payoff Function
Black-Scholes Model
Payoff Function
$\Delta \$|W$
$\sigma_{BS}(t)$
$W(t)$
$\Delta\sigma_B(t)$
$D\sigma_{BS}|_{\sigma g}$
$\Delta\sigma_{BS}|W(t)$

FIG. 10

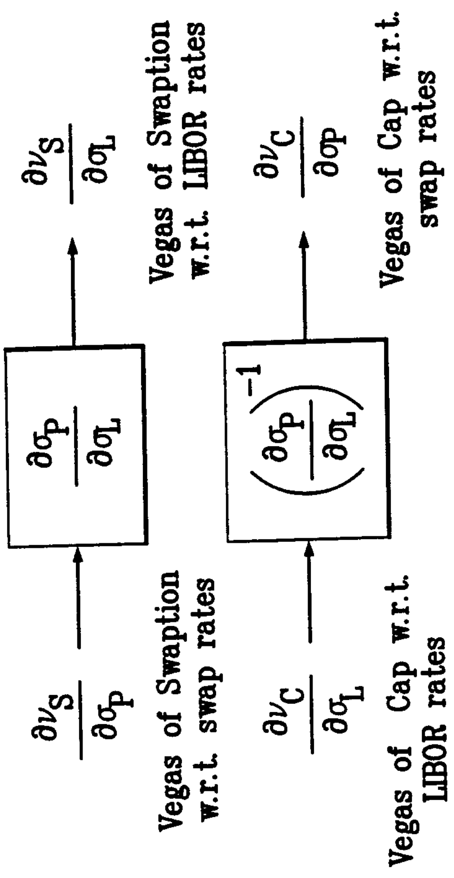

$\frac{\partial \nu_S}{\partial \sigma_P}$
Vegas of Swaption w.r.t. swap rates
$\frac{\partial \sigma_P}{\partial \sigma_L}$
$\frac{\partial \nu_S}{\partial \sigma_L}$
Vegas of Swaption w.r.t. LIBOR rates
$\frac{\partial \nu_C}{\partial \sigma_L}$
Vegas of Cap w.r.t. LIBOR rates
$\left(\frac{\partial \sigma_P}{\partial \sigma_L}\right)^{-1}$
$\frac{\partial \nu_C}{\partial \sigma_P}$
Vegas of Cap w.r.t. swap rates

FIG. 11

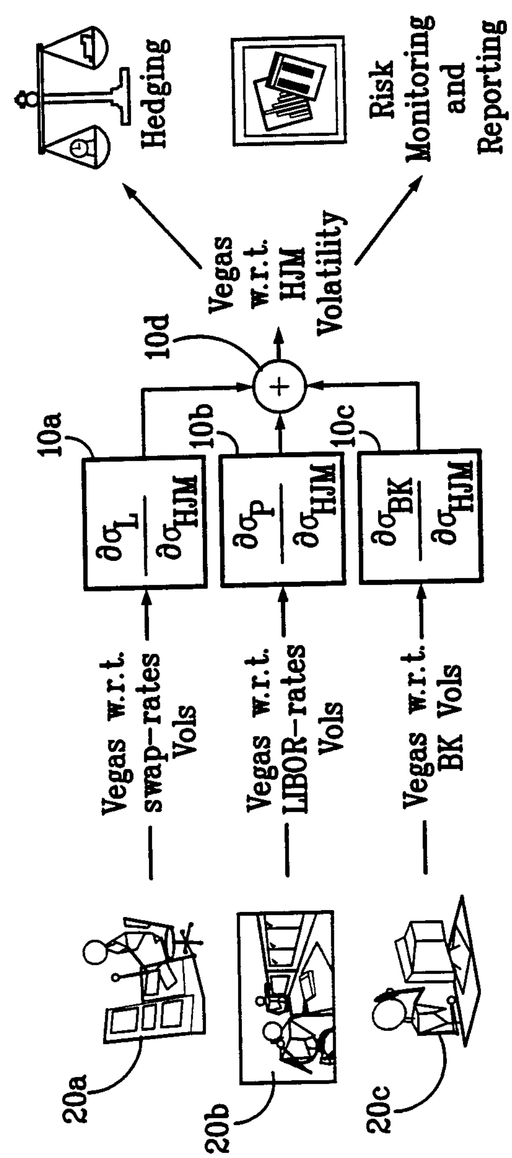
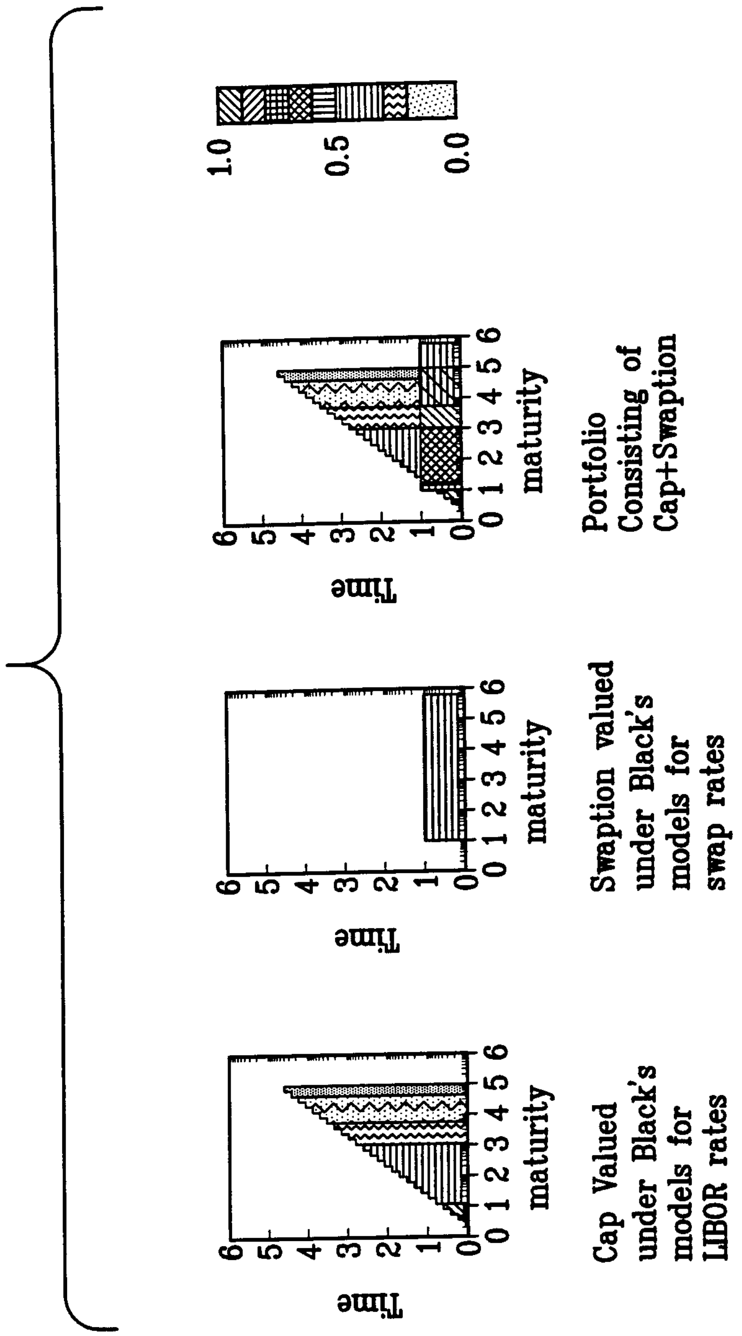

20a
20b
20c
Vegas w.r.t. swap-rates Vols
Vegas w.r.t. LIBOR-rates Vols
Vegas w.r.t. BK Vols
10a
10b
10c
10d
$\frac{\partial \sigma_L}{\partial \sigma_{HJM}}$
$\frac{\partial \sigma_P}{\partial \sigma_{HJM}}$
$\frac{\partial \sigma_{BK}}{\partial \sigma_{HJM}}$
+
Vegas w.r.t. HJM Volatility
Hedging
Risk Monitoring and Reporting

FIG. 12

FIG. 13


Time
maturity
0 1 2 3 4 5 6
Cap Valued under Black's models for LIBOR rates
Swaption valued under Black's models for swap rates
Portfolio Consisting of Cap+Swaption
1.0
0.5
0.0

INTERNATIONAL SEARCH REPORT

International application No.
PCT/US00/25292

A. CLASSIFICATION OF SUBJECT MATTER

IPC(7) :G06F 157/00
US CL :705/36, 35, 37
According to International Patent Classification (IPC) or to both national classification and IPC

B. FIELDS SEARCHED

Minimum documentation searched (classification system followed by classification symbols)

U.S. : 705/36, 35, 37

Documentation searched other than minimum documentation to the extent that such documents are included in the fields searched

Electronic data base consulted during the international search (name of data base and, where practicable, search terms used)

C. DOCUMENTS CONSIDERED TO BE RELEVANT

Category*	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
Y	US 5,799,287 A (DEMBO) 25 August 1998, col. 6, line 65-col. 11, line 44, col. 11, line 53-col. 16, line 67, col. 17, line 60-col. 18, line 20.	1-20
Y	US 5,774,880 A (GINSBERG) 30 June 1998, col. 4, line 30-col. 6, line 36, col. 6, line 50-col. 8, line 65, col. 9, line 4-col. 10, line 52.	1-20
A	US 5,930,762 A (MASCH) 27 JULY 1999, the whole document	1-20

☐ Further documents are listed in the continuation of Box C. ☐ See patent family annex.

* Special categories of cited documents:

"A" document defining the general state of the art which is not considered to be of particular relevance

"E" earlier document published on or after the international filing date

"L" document which may throw doubts on priority claim(s) or which is cited to establish the publication date of another citation or other special reason (as specified)

"O" document referring to an oral disclosure, use, exhibition or other means

"P" document published prior to the international filing date but later than the priority date claimed

"T" later document published after the international filing date or priority date and not in conflict with the application but cited to understand the principle or theory underlying the invention

"X" document of particular relevance; the claimed invention cannot be considered novel or cannot be considered to involve an inventive step when the document is taken alone

"Y" document of particular relevance; the claimed invention cannot be considered to involve an inventive step when the document is combined with one or more other such documents, such combination being obvious to a person skilled in the art

"&" document member of the same patent family

Date of the actual completion of the international search

13 DECEMBER 2000

Date of mailing of the international search report

16 JAN 2001

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