# United States Patent

**Langner**

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[54] **SPHERE-DOME CONSTRUCTION**


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[52] **U.S. Cl.** .................................. **52/80**, 52/DIG. 10

[51] **Int. Cl.** ............................................ **E04b 1/342**

[58] **Field of Search** ......52/80, 81, DIG. 10; 35/46 A, 35/47


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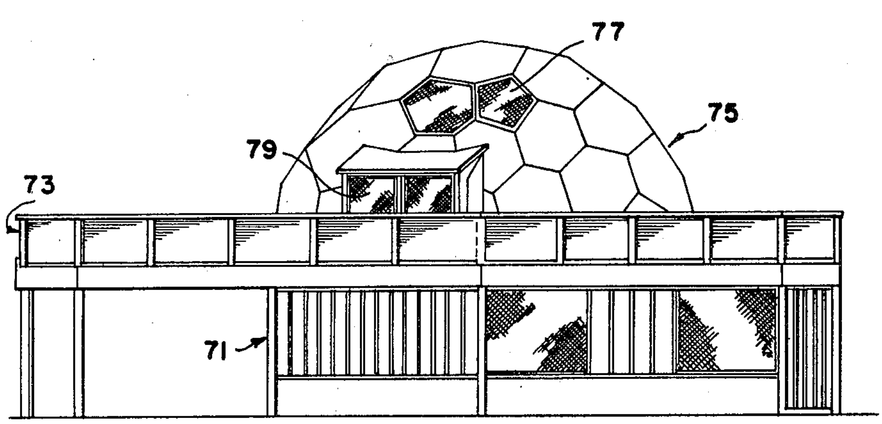
[57] **ABSTRACT**

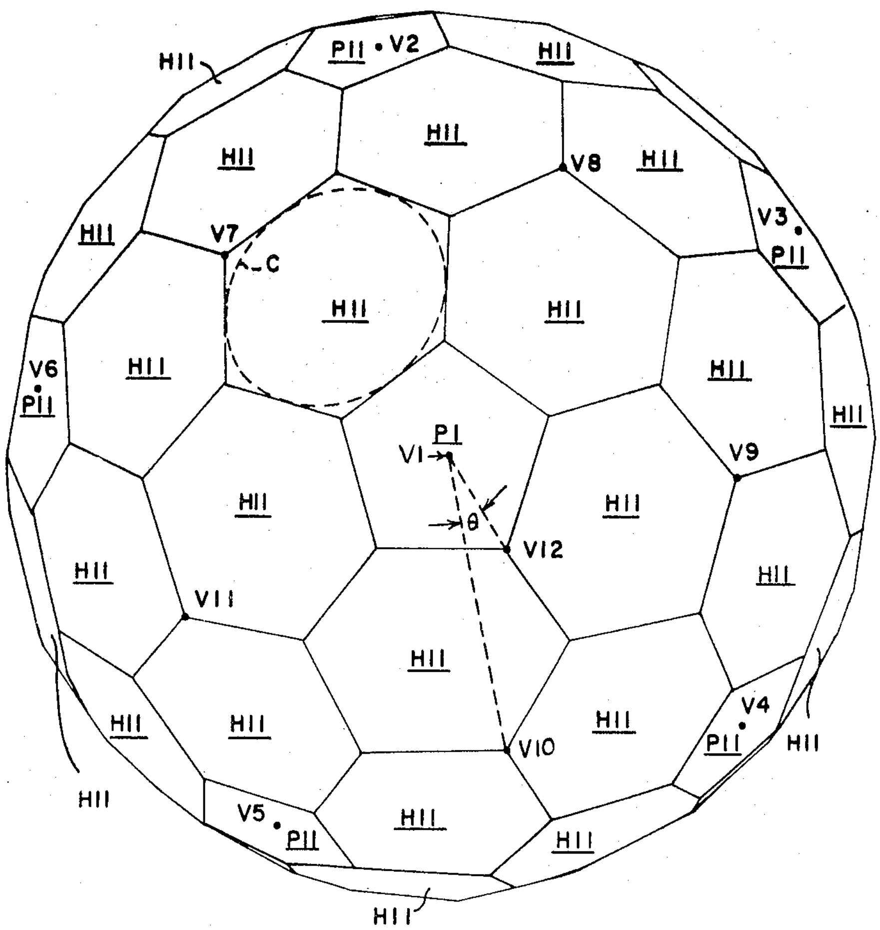
A self-supporting dome-like or sphere-like structure made of interconnected linear or planar elements, comparable to a geodesic dome and comprising, in the full sphere approximation, 12 regular pentagons and, in the simplest case, 60 irregular but identical hexagons. For higher orders of the structure, the number of pentagons remains 12 but the number of hexagons increases according to the formula

$$N_k = 3N_{k-1} + 20,$$

where $N_k$ is the number of hexagons in a sphere approximation of order $k$, and $N_{k-1}$ is the number of hexagons in a sphere approximation of order $(k-1)$.


**26 Claims, 7 Drawing Figures**






F/G. 1

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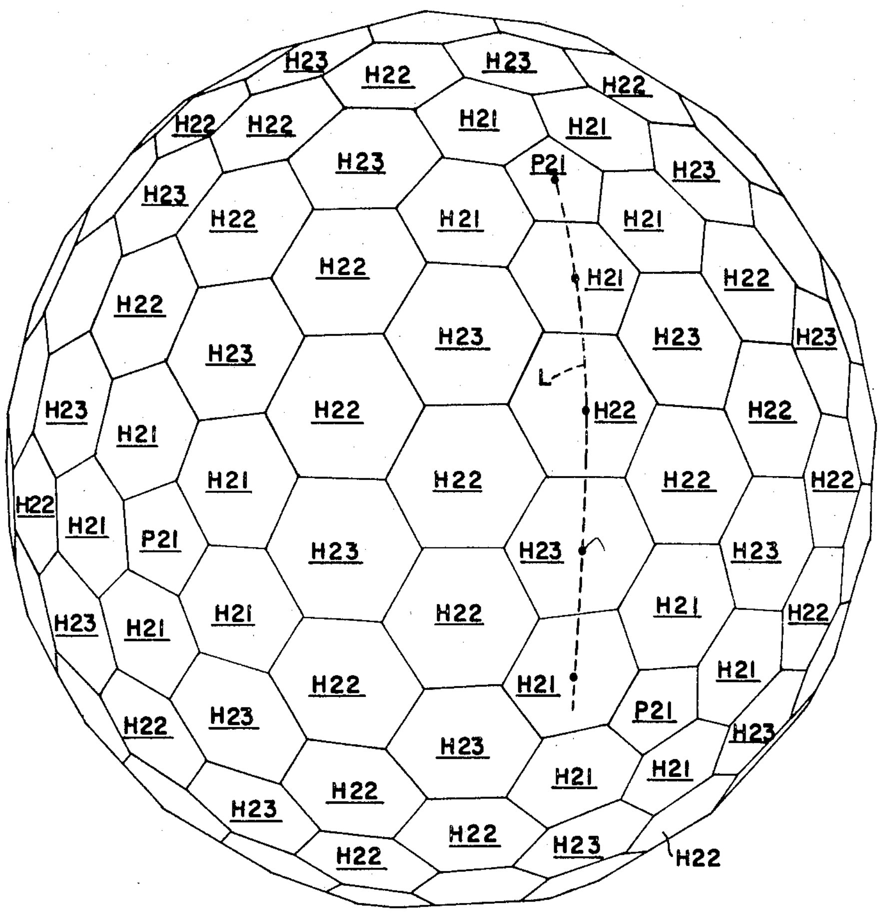


F I G. 2

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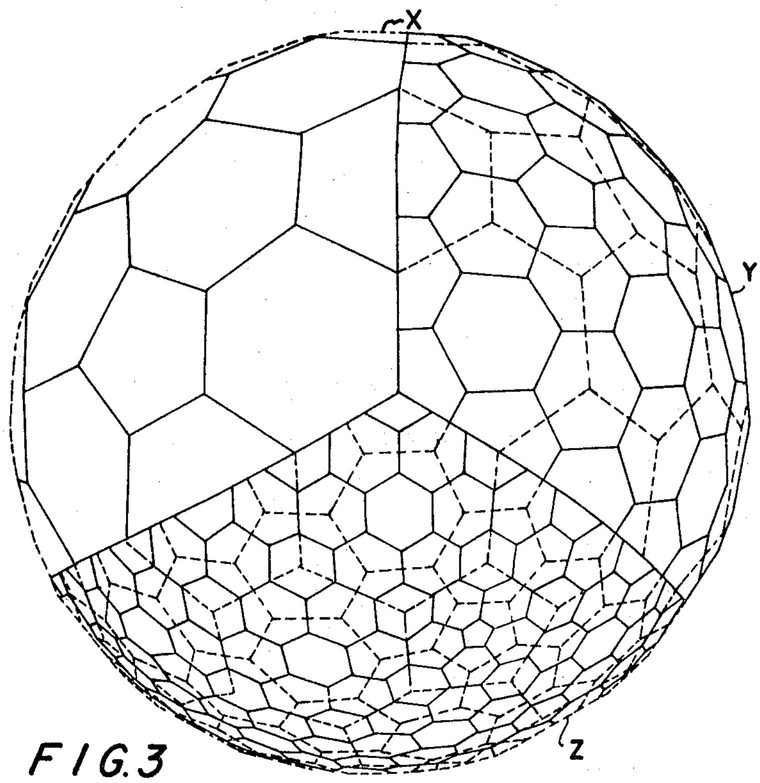


FIG.3

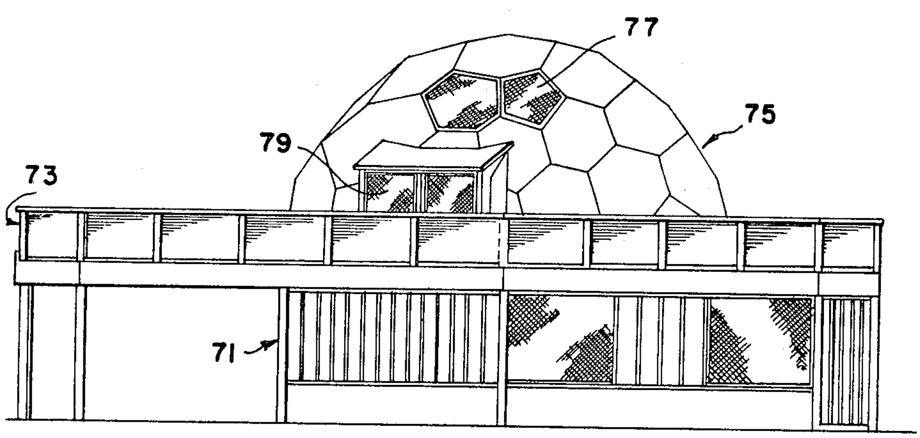


FIG. 7

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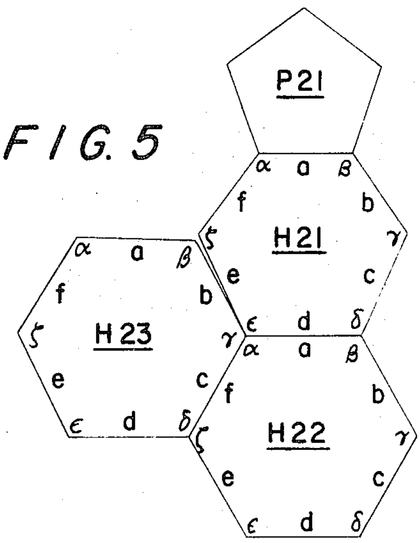


FIG. 5

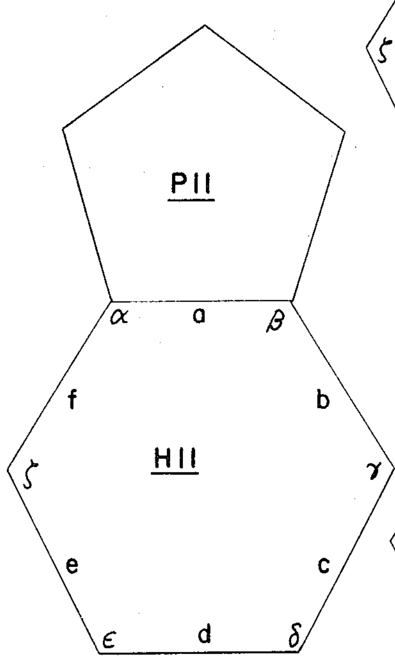


FIG. 4

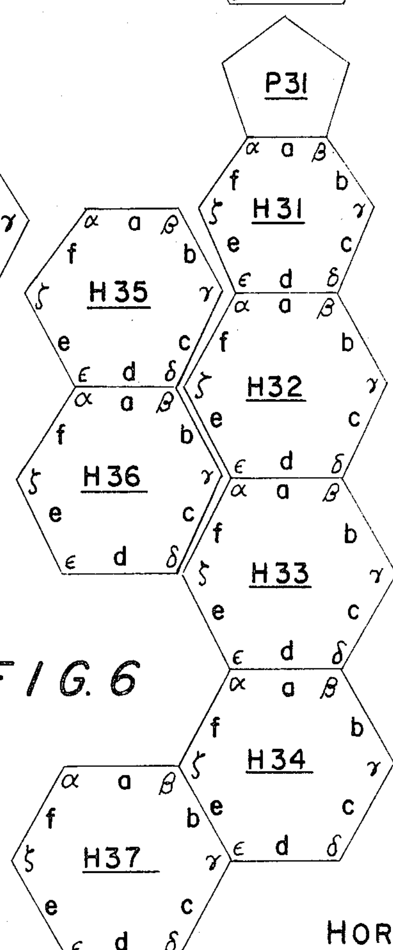


FIG. 6

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# SPHERE-DOME CONSTRUCTION

## BACKGROUND OF THE INVENTION

This invention relates to self-supporting building structures and particularly to structures approximating the shape of the sphere or a portion of a sphere (such as a dome), of the type composed of interconnected linear or planar elements.

A number of building structures are known in which a sphere or portion of a sphere is approximated by a plurality of interconnected linear ribs or interconnected plane panels. The structures have in common with structures of the present invention the fact that no internal supporting elements are used—the outer shell of interconnected panels or ribs is sufficient to support the structure.

The most famous of this class of building structure is the geodesic dome of Buckminster Fuller, the basic concept of which is described in his U.S. Pat. No. 2,682,235, issued June 29, 1954, and variants of which have been described in later patents and other literature. Among these is Fuller's U.S. Pat. No. 3,197,927 (Aug. 3, 1965) which teaches the use of hexagonal-pentagonal structural arrangements. Other literature also describes hexagonal-pentagonal sphere-approximating structures, for example U.S. Pat. No. 2,918,992 (J. Z. Gelsavage, Dec. 29, 1959). These known structures can all be classified as "geodesic" structures, i.e., if ribs are used, the ribs tend to align themselves along great circles of the sphere which is approximated by the structure, and if planar elements are used, the great circles are approximated by the side edges of the individual planar elements (or, by the lines joining the centers of the planar elements).

One of the disadvantages associated with such previously known structures is that the geodesic alignment of the structural elements creates fold lines about which the entire structure can collapse when subjected to sufficiently severe stresses. Regularity in the structural design is advantageous from the point of view of enabling the use of a small number of different modules, i.e., different sizes and shapes of component elements, for the composition of the structure, but this structural regularity can be disadvantageous, from the point of view of overall strength of the structure, if the regularity produces structural weaknesses such as fold lines along great circles of the sphere which the structure approximates.

Other proposals such as that contained in D. G. Emmerich's U.S. Pat. No. 3,341,989 (Sept. 19, 1967) have involved the use of quadrangles, hexagons, pentagons or more complex elements but have either failed to overcome the structural strength problem just mentioned or have failed to create a pleasing aesthetic effect. One of the contributing factors to aesthetic appeal is considered by the inventor to reside in the use of elements of approximately the same size. Construction of a building is usually facilitated as well when the component elements are of approximately the same size, and when the number of different sizes and shapes needed is small.

## SUMMARY OF THE INVENTION

It is an object of the present invention to combine in an aesthetically attractive sphere-approximating structure, or a portion thereof, a plurality of pentagons and



hexagons formed by interconnected planar or linear elements in a sphere-approximating configuration, wherein all of the component elements are of approximately the same size, wherein a small number of different sizes and shapes of elements are required, and wherein structural strength is achieved by the choice of the hexagonal or pentagonal shape of individual elements and also by the configuration of these elements in their sphere-approximating structural relationship.

In its broadest aspect, the invention provides the whole or a portion (a "portion" being, for example, a dome) of a self-supporting sphere-approximating structure comprising, in its completely enclosed sphere-approximating whole version, 12 equal regular pentagons having their centers at the vertexes of a regular icosahedron; and 60 (for a sphere approximation of order 1) or $N_k$ (for a sphere approximation of order $k$ ) hexagons, where

$$N_k = 3N_{k-1} + 20,$$

$k$ is an integer greater than 1, and, as indicated, $N_k$ is the number of hexagons in a whole sphere-approximating structure of order $k$. At least some of the hexagons depart slightly from regular hexagons to enable continuity of the structure.

Generally, sphere-approximating structures of the first, second and third orders will be found to be satisfactory for most structural purposes. As can be seen from the above relationship, a sphere-approximating structure of the second order would have two hundred hexagons, and of the third order would have six hundred and twenty hexagons.

## SUMMARY OF THE DRAWINGS

FIG. 1 shows a sphere-approximating structure of the first order, in accordance with the present invention.

FIG. 2 shows a sphere-approximating structure of the second order, according to the invention.

FIG. 3 is a diagram illustrating the triangulation technique according to which sphere-approximating structures of higher order can be constructed from sphere-approximating structures of lower order according to the invention.

FIG. 4 illustrates the two component planar elements of a sphere-approximating structure of the first order, according to the invention.

FIG. 5 illustrates the component elements of a sphere-approximating structure of the second order according to the invention.

FIG. 6 illustrates the planar component elements of a sphere-approximating structure of the third order according to the invention.

FIG. 7 (on the same sheet as FIG. 3) illustrates in side elevation a structure including a portion of a sphere-approximating structure of the first order constructed in accordance with the present invention.

## DETAILED DESCRIPTION WITH REFERENCE TO THE DRAWINGS

FIG. 1 illustrates a sphere-approximating structure of the first order constructed in accordance with the invention. The complete sphere approximation includes 12 pentagons and 60 hexagons. The 12 pentagons, each of which shown in FIG. 1 is designated as P11, are regular pentagons placed on the sphere-approximating structure so that the centers of the pentagons are located at the vertexes of a regular icosahedron. The icosahedron and corresponding invert dodecahedron interrelationship with the first order structure as embodied in FIG. 1 can best be understood by observing that pentagon centers V1, V2, V3, V4, V5, V6 etc. define vertexes of a regular icosahedron, while points V7, V8, V9, V10, V11 are located at the vertexes of a regular dodecahedron which is the invert of such regular icosahedron.

In FIG. 1 each pentagon is surrounded by five contiguous irregular hexagons each designated as H11. The result is that, since there are five hexagons for each pentagon, there are 60 hexagons in the complete sphere-approximating structure, each of which is exactly the same size and shape, but in order that all elements be contiguous, the hexagons must part slightly from regular hexagons. However, they do not depart so markedly from regular hexagons that the overall aesthetic result of the structure of FIG. 1 is unpleasant.

In order that the relative size and shapes of the component elements of FIG. 1 be contiguous, so as to form a completely enclosed structure approximating a sphere, the interior angles and the lengths of the sides of hexagons H11 must be carefully selected. One approach that can be taken is to consider each of the pentagons P11 and hexagons H11 as circumscribing a circle (as for example the inscribed circle C in FIG. 1), and to consider that the circles, which will be tangent to one another in this hypothesis, are in turn tangent to the surface of a common sphere. On this assumption, it can be established that the circle inscribed in each of the pentagons P11 can be considered to be the circular face of a right regular cone whose apex angle is about 9°50′ The inscribed circle in each of the hexagons can be considered to be the circular planar face of a right regular cone whose apex angle is about 13°24½′ with the circular radius being about 1.37 times that of the inscribed circle in the pentagon. It can be shown that cones having these dimensions can be stacked so that their circular faces are in fact substantially tangent to one another and are substantially tangent to a common sphere which the entire structure approximates.

This is not to suggest that other arrangements of cones or contiguous circles could not be made to approximate a sphere nor should it be inferred that other arrangements of pentagons and hexagons could not be made to form a sphere-approximating structure. However, the specific arrangement herein described does have the advantage of requiring only two different sizes of planar components (the cone arrangement requires only two different sizes of cones or of contiguous circles) which has the advantage of simplicity for the purposes of mass production of components and also from the point of view of simplicity of erection of the structure by workmen.

FIG. 2 illustrates a sphere-approximating structure of the second order according to the invention. Again there are 12 pentagons arranged exactly in the same way as the pentagons of FIG. 1, namely with their centers located at the vertexes of a regular icosahedron. The pentagons are each designated P21 in FIG. 2. There are three different hexagonal modules, i.e. three different sizes of hexagons. The three different hexagonal modules are designated respectively as H21, H22, and H23 in FIG. 2.

It will be noted that the side edges of the hexagonal and pentagonal elements of FIGS. 1 and 2 are not aligned along great circles or approximate great circles. This is also true of the line joining centers of adjacent elements—the line may tend to follow a great circle for a few elements (see line L, for example, in FIG. 2) but a departure from the great circle eventually occurs.

In order to arrive at the FIG. 2 arrangement, the process of triangulation may be applied to the configuration of FIG. 1. This is illustrated in FIG. 3. FIG. 3 is a diagram which divides the sphere-approximating surface shown into three sectors designated X, Y, and Z respectively. In the sector X, the configuration is that of FIG. 1, namely a sphere-approximating structure of the first order. In sector Y, the first order configuration is shown in broken lines. From this structure, by conventional triangulation, one can derive a breakdown of the structural units into smaller structural units, again comprising only pentagons and hexagons. These are shown in solid lines in sector Y of FIG. 3. Finally, in sector Z of FIG. 3, the configuration of order 2 is shown in broken lines and, by conventional triangulation, the solid line configuration shown in sector Z of FIG. 3 represents a sphere-approximating structure of the third order, according to the invention.

Adjustments may have to be made in the sizes and shapes of the hexagons proceeding by triangulation from a configuration of one order to the configuration of the next highest order, in order to bring about proper contiguity of adjacent elements in a three-dimensional array. However, the adjustments required going merely from one order to the next highest order are relatively minor.

It can be shown that the number of hexagons for sphere-approximating structures of any given order according to the invention is twenty more than three times the number of hexagons in the next lowest order of sphere-approximating structure according to the invention, thus:

$$N_k = 3N_{k-1} + 20,$$

where $N_k$ is the number of hexagons for a sphere-approximating structure according to the invention of order $k$, and $k$ is an integer greater than 1. As explained above, the number of hexagons in the first order structure is 60. The above relationship implies that the second order sphere approximating structure will have 200 hexagons and the third order structure will have 620 hexagons. Regardless of the order, the number of pentagons is always 12.

FIGS. 4, 5, and 6 show examples of the planar modules than can be used in sphere-approximating structures according to the invention of the first, second, and third order respectively. The pentagonal modules P11 (first order—FIG. 4) P21 (second order—FIG. 5) and P31 (third order—FIG. 6) are all regular pentagons. For each of the hexagonal modules, sides $a$, $b$, $c$, $d$, $e$, and $f$ and interior angles $\alpha$, $\beta$, $\gamma$, $\delta$, $\epsilon$, and $\zeta$, can have the following relationships to one another, in accordance with exemplary embodiments of the present invention:

Relative
Side

| Lengths | a | b | c | d | e | f |
|---|---|---|---|---|---|---|
| First Order | | | | | | |
| H11 | 248 | 263 | 278 | 278 | 278 | 263 |
| Second Order | | | | | | |
| H21 | 127 | 134 | 150 | 159 | 150 | 134 |
| H22 | 159 | 159 | 159 | 159 | 159 | 159 |
| H23 | 150 | 150 | 159 | 159 | 159 | 159 |
| Third Order | | | | | | |
| H31 | 64.8 | 67.9 | 77.8 | 84.5 | 77.8 | 67.9 |
| H32 | 84.5 | 84.5 | 87.3 | 90.1 | 87.3 | 84.5 |
| H33 | 84.5 | 87.3 | 90.7 | 91.2 | 92.1 | 88.2 |
| H34 | 91.2 | 90.3 | 91.2 | 92.1 | 92.1 | 92.1 |
| H35 | 90.1 | 90.7 | 90.3 | 90.3 | 90.3 | 90.7 |
| H36 | 77.8 | 77.8 | 84.5 | 84.5 | 84.5 | 84.5 |
| H37 | 92.1 | 92.1 | 92.1 | 92.1 | 92.1 | 92.1 |
| Angles | α | β | γ | δ | ε | ζ |
| First Order | | | | | | |
| H11 | 123°44′ | 123°44′ | 118°08′ | 118°08′ | 118°08′ | 118°08′ |
| Second Order | | | | | | |
| H21 | 125°23′ | 125°23′ | 120°19′ | 114°18′ | 114°18′ | 120°19′ |
| H22 | 120° | 120° | 120° | 120° | 120° | 120° |
| H23 | 123°51′ | 123°44′ | 123°51 | 118°08′ | 118°08′ | 118°08′ |
| Third Order | | | | | | |
| H31 | 125°51′ | 125°51′ | 121°28′ | 112°41′ | 112°41′ | 121°28′ |
| H32 | 121°05′ | 121°05′ | 121°05′ | 117°50′ | 117°50′ | 121°05′ |
| H33 | 121°05′ | 121°05′ | 123°29′ | 118°59′ | 119°51′ | 119°51′ |
| H34 | 119°51′ | 120°38′ | 120°38′ | 119°51′ | 119°51′ | 119°51′ |
| H35 | 118°05′ | 118°05′ | 123°57′ | 118°08′ | 118°08′ | 123°57′ |
| H36 | 125°53′ | 116°40′ | 125°53′ | 117°18′ | 117°18′ | 117°39′ |
| H37 | 120° | 120° | 120° | 120° | 120° | 120° |

Since a regular hexagon has interior angles of 120°, it can be seen from the above that the hexagons used in the sphere approximation according to the invention depart only slightly from regularity. The near approximation of regular hexagons is believed by the inventor to contribute to the aesthetic appeal of the structures.

When the component planar elements have been assembled to form a sphere or portion of a sphere, it has been found, referring to FIG. 1, that the angle between the line joining the center (e.g., V1) of a pentagon to one of the vertexes (e.g., V10 in FIG. 1) of the corresponding invert dodecahedron and the line joining the center V1 to the pentagon vertex (V12) nearest V10, designated $\theta$ in FIG. 1, is about 13°6′ for the examples herein discussed. This appears to be a constant angle at least for the first, second, and third orders.

In a building structure, the pentagons and hexagons can be formed by planar sheets of the required shape, or the structure may be composed of ribs following the side edges of the pentagons and hexagons illustrated in the drawings. (Obviously, curved rather than planar sheets could be used so as to simulate a sphere more exactly). It is useful to consider the geometric properties of the structure according to the invention on the assumption that planar sheets are used but it will be understood that structures can be built according to the invention without using planar elements.

FIG. 7 illustrates an exemplary use of the principles of the present invention in a composite building structure. It can be seen that the design postulates a lower rectangular structure **71**, an elongated terrace **73**, and a dome **75** constructed in accordance with the principles of the present invention. The dome takes the form of a segment of a sphere-approximating structure according to the invention. The percentage of an entire sphere-approximation chosen by a designer for any given structural use is largely a matter of desired volume or area and aesthetics. It is contemplated in accordance with the invention that there may be openings in or additions to the basic structure—for example, FIG. 7 illustrates two different types of design modifications. The first type of modification involves a breaking of the structure along the side edges of component portions of the structure to form a window **77**. The designer may additionally choose to modify the structure by departing completely from the side edges of the structure and adding geometry of quite a different kind—for example, the rectangular porch **79**.



The structure according to the invention may be used as a tank or for other bulk storage, or for housing, arenas or other commercial buildings. The intended uses and the materials suitable for construction are analogous to those appropriate to geodesic domes and spheres.

What I claim as my invention is:

**1.** A self-supporting sphere-approximating structure comprising, interconnected structural elements forming twelve equal regular pentagons having their centers located at the vertexes of a regular icosahedron, and, for a sphere approximation of order **1**, 60 hexagons; and for a sphere approximation of order $k$, $N_k$ hexagons at least some of which depart from regular hexagons to enable contiguity of adjacent ones of said structural elements, where $k$ is an integer greater than 1, where $N_k$ equals $3N_{k-1} + 20$, and where $N_{k-1}$ is the number of hexagons for a sphere approximation of order one less than $k$.

**2.** A self-supporting sphere-approximating structure, comprising interconnected structural elements forming twelve equal regular pentagons and $n$ hexagons at least some of which hexagons are irregular to enable contiguity of adjacent modules, the centers of the pentagons defining vertexes of a regular icosahedron, where $n$ is an integer selected from the following group: 60, 200, and 620.

**3.** A structure as defined in claim **2** wherein the structural elements are planar pentagonal and hexagonal modules.

**4.** A structure as defined in claim **2** wherein the structural elements are ribs which lie along the edges of the hexagons and pentagons.

**5.** A structure as defined in claim **2**, wherein $n = 60$ and all of the hexagons are of the same size and the same shape.

**6.** A structure as defined in claim **2**, wherein $n = 200$ and 80 hexagons are regular and of equal size, while 60 hexagons are of a first predetermined size and shape, and 60 hexagons are of a second predetermined size and shape.

**7.** A structure as defined in claim **5** wherein a circle may be inscribed in each hexagon and in each pentagon, and the structural elements are aligned such that circles inscribed in the hexagons and pentagons would be tangent to a common sphere.

**8.** A structure as defined in claim **6** wherein the structural elements are aligned such that closed curved figures, selected from the class of figures comprising circles and approximate circles, inscribed in the hexagons and pentagons, would be approximately tangent to a common sphere.

**9.** A structure as defined in claim **6** wherein the arrangement of the structural elements is obtained substantially by triangulation of that structure obtained where the following conditions are satisfied: $n$ is selected to be 60 and all of the hexagons are of the same size and same shape.

**10.** A structure as defined in claim **2** wherein the angle between the line joining the center of any of said pentagons to a vertex of a dodecahedron corresponding in invert relationship to said icosahedron and the line joining the center of the last mentioned pentagon to the vertex of the pentagon nearest said dodecahedron vertex is substantially 13°6′.

**11.** A structure as defined in claim **2**, wherein $n$=620 and the arrangement of the structural elements is obtained by triangulation from that structure wherein $n$=60.

**12.** A structure as defined in claim **2**, wherein all of the interior angles of all of the irregular hexagons lie in the range 114° to 126°.

**13.** A segment of a self-supporting sphere-approximating structure comprising, in its complete sphere-approximating version, interconnected structural elements forming twelve equal regular pentagons having their centers located at the vertexes of a regular icosahedron, and, for a sphere approximation of order **1**, 60 hexagons; and for a sphere approximation of order $k$, $N_k$ hexagons at least some of which depart from regular hexagons to enable contiguity of adjacent ones of said structural elements, where $k$ is an integer greater than 1, where $N_k$ equals $3N_{k-1} + 20$, and where $N_{k-1}$ is the number of hexagons for a sphere approximation of order one less than $k$.

**14.** The structure defined in claim **13** wherein the segment approximates a dome shape.

**15.** A segment of a self-supporting sphere-approximating structure comprising in its whole version, interconnected structural elements forming twelve equal regular pentagons and $n$ hexagons at least some of which hexagons are irregular to enable contiguity of adjacent modules, the centers of the pentagons defining vertexes of a regular icosahedron, where $n$ is an integer selected from the following group: 60, 200, and 620.

**16.** A structure as defined in claim **15**, wherein $n$=620 and the arrangement of the structural elements is obtained by triangulation from that structure wherein $n$=60.

**17.** A structure as defined in claim **15**, wherein all of the interior angles of all of the irregular hexagons lie in the range 114° to 126°.

**18.** The structure defined in claim **17** wherein the segment approximates a dome shape.

**19.** A structure as defined in claim **18** wherein the structural elements are planar pentagonal and hexagonal modules.

**20.** A structure as defined in claim **18**, wherein the structural elements are ribs which lie along the edges of the hexagons and pentagons.

**21.** A structure as defined in claim **18**, wherein $n$=60 and all of the hexagons are of the same size and the same shape.

**22.** A structure as defined in claim **21** wherein a circle may be inscribed in each hexagon and in each pentagon, and the structural elements are aligned such that circles inscribed in the hexagons and pentagons would be tangent to a common sphere.

**23.** A structure as defined in claim **18**, wherein $n$=200 and 80 hexagons are regular and of equal size, while 60 hexagons are of a first predetermined size and shape, and 60 hexagons are of a second predetermined size and shape.

**24.** A structure as defined in claim **23** wherein the structural elements are aligned such that closed curved figures, selected from the class of figures comprising circles and approximate circles, inscribed in the hexagons and pentagons, would be approximately tangent to a common sphere.

**25.** A structure as defined in claim **23** wherein the arrangement of the structural elements is obtained substantially by triangulation of that structure obtained where the following conditions are satisfied: $n$ is selected to be 60 and all of the hexagons are of the same size and same shape.

**26.** A structure as defined in claim **18** wherein the angle between the line joining the center of any of said pentagons to a vertex of the invert dodecahedron corresponding to the said icosahedron and the line joining the center of the last mentioned pentagon to the vertex of the last mentioned pentagon nearest said invert dodecahedron vertex is substantially 13°6′.

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