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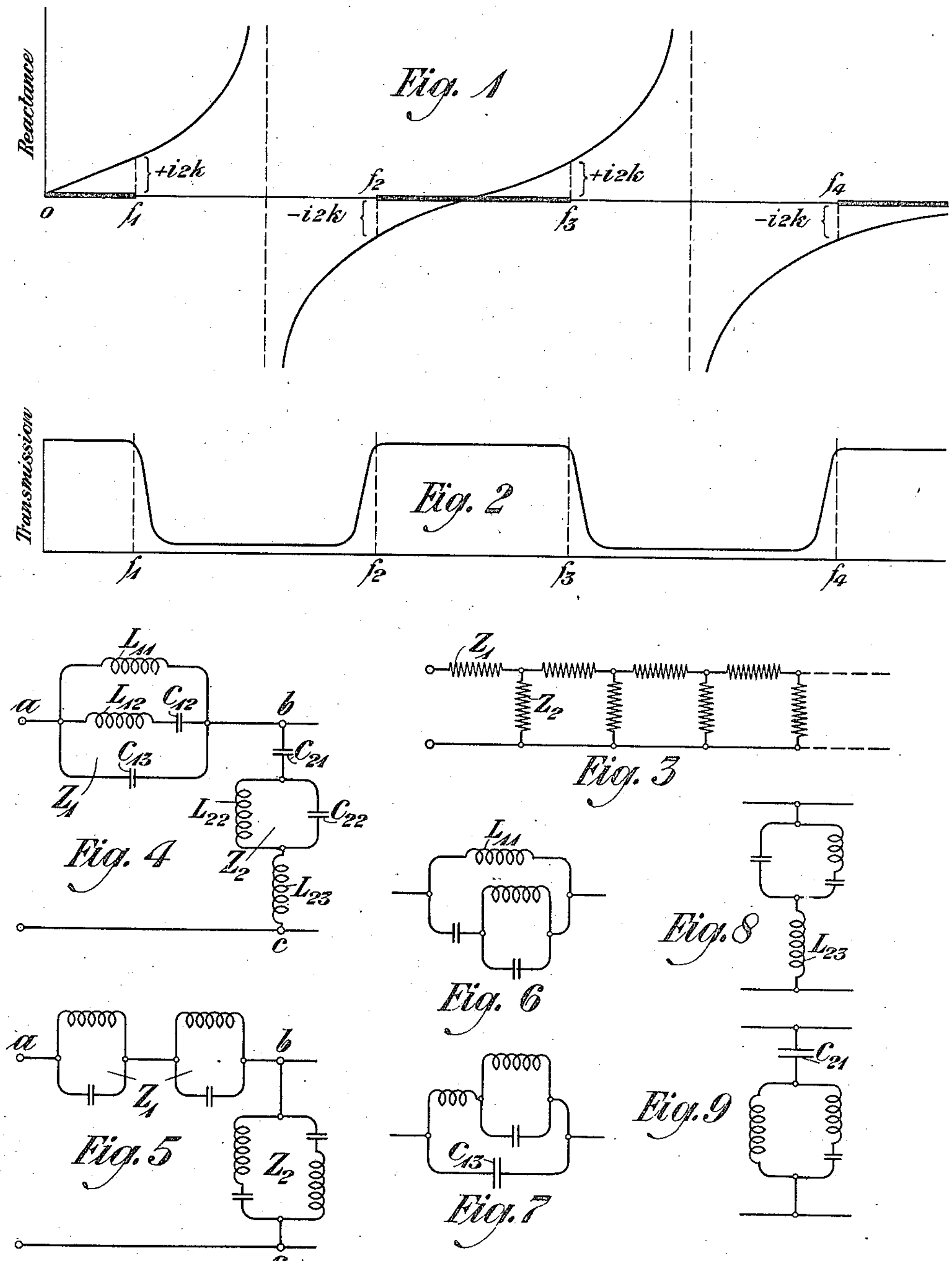
O. J. ZOBEL

MULTIPLE BAND WAVE FILTER

Filed April 30, 1920



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MULTIPLE-BAND WAVE FILTER.

Application filed April 30, 1920. Serial No. 377,963.

To all whom it may concern:

Be it known that I, OTTO J. ZOBEL, residing at Maplewood, in the county of Essex and State of New Jersey, have invented certain Improvements in Multiple-Band Wave Filters, of which the following is a specification.

This invention relates to selective circuits and more particularly to selective circuits of the type known as wave filters.

In multiplex signaling, frequency selectivity has been heretofore obtained in cases where it has been desired to discriminate between different frequency bands by the employment of simple recurrent networks comprising arrangements of inductances and capacities. Of these networks, those which have been principally used are the so-called "low pass", "high pass" and "single band" filters described in the U. S. patents to George A. Campbell, Nos. 1,227,113 and 1,227,114 issued May 22, 1917. These filters are characterized by the fact that currents lying within certain ranges of frequencies are freely transmitted without substantial attenuation, while currents of other frequencies are very strongly attenuated.

In general when a filter of this character is connected to a line whose impedance is practically a constant resistance, large impedence irregularities are introduced between the wave filter and the line, in the range of frequencies to be transmitted, since the characteristic impedance of the filter at any termination varies with frequency. These impedance irregularities at the frequencies to be transmitted are objectionable, not only from the standpoint of maximum energy transferred from the line to the wave filter, but also from the standpoint of repeater balance.

One of the objects of this invention is to provide a wave filter of the so-called "constant *k*" type, that is a filter so characterized that the product of the impedance of the series element of any section into the impedance of the shunt element of that section will be proportional to the square of a constant and hence independent of frequency.

Another object of the invention is to produce a wave filter which will be in general capable of transmitting over any preassigned number of frequency ranges. The filter of the present invention is quite distinct from that described in the Campbell patent, in this respect, since the Campbell filters were designed to transmit over a single range of frequencies or at most over two ranges of frequencies while attenuating all other frequencies.

Other and further objects of the invention will be more fully understood from the following detailed description when read in connection with the accompanying drawing, Figures 1 and 2 of which illustrate curves characteristic of the filters of this invention, Fig. 3 of which is a simplified diagram of a recurrent network filter, while Figs. 4 to 9 inclusive are diagrams of elements which may be used in making up the filter of this invention.

The wave filters forming the subject matter of this invention have a singly periodic structure consisting of a plurality of sections, as indicated in Fig. 3, each section comprising a series impedance element z_1 and a shunt impedance z_2. These impedance elements are reactances, that is they are made up of inductances and capacities, in a manner more fully hereafter described. While the discussion of these wave filters is primarily on the basis that the elements are non-dissipative in character, the inevitable introduction of dissipation will not materially alter the designs obtained. In order to minimize the transmission losses in the wave filter, there should be provided as large time constants for the inductances and capacities as is practicable in each specific case.

The computation for a filter on the assumed basis that its reactances are non-dissipative is justified both by theoretical investigations and by practical tests. It is well known that the resistance of a coil or a condenser can be made very small compared to its inductive reactance or capacity reactance and therefore the performance of such a coil or condenser may be computed approximately with entire neglect of such slight resistance.

When the magnitude of the inductances and capacities have been obtained on this basis and the type of coils and condensers giving these magnitudes have been decided upon, the corresponding amounts of resistance necessarily introduced are then accurately taken account of in practice when computing the current losses thru the wave-filter. The effect of this dissipation is principally to cause small allowable current losses within the bands of free transmission.

It is well known among engineers that at the sending end a long transmission line behaves the same as if its length were infinite. The characteristic impedance of a smooth line is its impedance at the sending end, assuming that the line is so long that it is practically the same as if its length were infinite. There is an analogous situation for filters of the type having recurrent sections. Their behavior is most easily computed in the first instance on the assumption that they extend with an infinite number of sections from the sending end. In practice, a filter must have a finite number of sections and in that case it may be terminated at the drop end by a suitable network to simulate the discarded infinite extension. The matter of providing such a network is not treated in the present specification.

It is well known that a smooth transmission line having uniform series impedance distributions z_1 and uniform shunt impedance distributions z_2 per unit of length has the characteristic impedance

$$k=\sqrt{z_1 \cdot z_2} \quad (1)$$

Such a line also has a propagation constant:

$$\gamma=\sqrt{\frac{z_1}{z_2}} \quad (2)$$

A wave filter such as is indicated schematically in Fig. 3 of the drawing comprising a series of sections, each having a series impedance element z_1 and a shunt impedance element z_2 has a characteristic impedance at any termination which is a function of both the product and the ratio of z_1 and z_2 and a propagation constant which is a function of their ratio, hence it is convenient to express both the characteristic impedance and propagation constant of the wave filter in terms of k and γ, the parameters of the corresponding smooth line.

In order that the filter may be of the "constant k" type, it is necessary that the product of the series impedance z_1 and the shunt impedance z_2 per section should be a constant independent of frequency, that is

$$z_1 . z_2 = k^2 = \text{constant} \quad (3)$$

As pointed out in the above mentioned patents to Campbell, free transmission takes place through such a network, for frequencies which give

$$\gamma^2=\frac{z_1}{z_2} \quad (4)$$

within the range whose limits are defined by

$$\gamma^2=0 \text{ to } \gamma^2=-4. \quad (5)$$

By combining equation 5 with equation 3, it is apparent that the network transmits within a range of frequencies in which z_1 lies between

$$z_1=0 \text{ and } z_1=\pm i2k \quad (6)$$

There are two standpoints, transmission and attenuation, from which it is possible to synthetically construct any desired "constant k" wave filter. Let us first consider the construction of a filter from the transmission requirements.

From equation 6 it is apparent that a free transmission region occurs whenever the series element z_1 becomes resonant, that is $z_1=0$. Assuming then that the desired wave filter should have n transmission regions, it is apparent that z_1 must have n resonant points. These points may be obtained by constructing z_1 of n simple reactance components z_{11}, z_{12}—z_{1n}, all in parallel, wherein each component takes care of one desired resonant point of z_1. As a specific example, take the case of a filter which is desired to have three free transmission bands, one in the neighborhood of zero cycles, another in the neighborhood of infinity and still another at some intermediate frequency. For example, let one free transmission band extend from zero to frequency f_1, another free transmission band extend between frequencies f_2 and f_3 and a third free transmission band from frequency f_4 to infinity. The reactance curve for the element z_1 at the various frequencies will be as indicated in Fig. 1. In this figure the reactance is zero at zero frequency and increases in a positive direction until it becomes infinite at a point somewhere between frequency f_1 and f_2. When the reactance becomes infinite it suddenly changes sign and becomes negative and from this point the negative reactance decreases until it becomes zero somewhere between f_2 and f_3 for the second transmission band. The reactance now continues on the upper slope in a positive direction, again becoming infinite between frequencies f_3 and f_4, again changing sign from positive to negative and then decreasing until it again becomes zero at infinity.

The critical frequencies are determined from equation 6, above, and it will be seen that at the upper critical frequency for the first transmission band z_1 will be $+ i2k$, as indicated in Fig. 1; at the lower critical frequency of the internal transmission band z_1 is $- i2k$ at frequency f_2; at the higher critical frequency it becomes $+ i2k$ at frequency f_3; and at the lower critical frequency of the infinite frequency transmission region the value of z_1 becomes $- i2k$, at frequency f_4.

The variation of transmission with frequency will now be as indicated in Fig. 2, which shows three bands of free transmission separated by two attenuation bands. A comparison of Fig. 2 with Fig. 1 shows that the reactance becomes zero at some point in each transmission band. Therefore, if we are to design the filter from the transmission requirements, it is apparent that the series impedance element z_1 may be constructed of three parallel elements, one of which includes a simple inductance L_{11}. Such an element will have zero reactance at zero frequency. Another parallel element may comprise a simple resonant circuit consisting of inductance L_{12} and capacity C_{12}, since such an element may be made resonant at some desired frequency between the frequencies f_2 and f_3. Finally, the third element may consist of a simple capacity, C_{13}, which will have zero reactance at an infinite frequency.

The corresponding elements of the shunt impedance z_2 may be readily determined from a consideration of the fact that whereas the series element must be resonant at a given frequency for free transmission, the shunt element should be anti-resonant at the same frequency, in order that the least amount of current be diverted through the shunt path. Consequently, the shunt impedance element z_2 should contain a simple condenser C_{21}, which will have an infinite reactance at zero frequency. An anti-resonant combination comprising inductance L_{22} and capacity C_{22} should be provided for a frequency between the critical frequencies f_2 and f_3 of the internal transmission band. Finally, a simple inductance L_{23} having an infinite reactance at infinite frequency should be provided to correspond with the capacity path C_{13} of the series impedance element. Obviously, if additional internal transmission bands are to be provided, such bands may be accommodated by providing additional resonant circuits in parallel in the series element z_1 and corresponding anti-resonant circuits in series in the shunt impedance element z_2.

A filter producing the same results may be designed from a consideration of the attenuation requirements only, although in this case the physical form of the filter will be somewhat different. Referring to Figs. 1 and 2, it is apparent that the design of a filter from attenuation requirements involves providing a series impedance element which shall have an infinite reactance at some frequency in each of the bands of attenuation. In other words, at some point between frequency f_1 and f_2 the reactance should be infinite and also at some point between frequency f_3 and f_4 the reactance should be infinite. The obvious construction to satisfy this requirement would be to construct the series element z_1 of two anti-resonant elements in series, one anti-resonant element being resonant at a frequency between f_1 and f_2 and the other between f_3 and f_4. Such a design is illustrated in Fig. 5. Obviously from an attenuation standpoint the corresponding requirement of the shunt element is that its reactance be zero for the frequencies at which the series element is resonant. The shunt element z_2 should therefore be constructed of two resonant circuits in parallel, one circuit being resonant at a frequency between f_1 and f_2 and the other being resonant at a frequency between f_3 and f_4.

While the filter of Fig. 5 is apparently somewhat different in construction from that of Fig. 4, it will function in a manner which is identical therewith, if dissipation be disregarded, though by appropriate design dissipation may be present and make the filters of Figures 5 and 4 function identically. The equivalence of the two circuits will be more readily apparent when it is considered that in Fig. 4 the requirement was to provide one path between point *a* and *b*, containing a pure inductance L_{11}. Such a path is provided in Fig. 5 through the two inductance elements of the anti-resonant sets taken in series. In Fig. 4 it will be noted that there is also a path containing simple capacity. Such a path is provided in Fig. 5, through the two capacity elements of the anti-resonant sets in series. In Fig. 4 a path is also provided through a resonant circuit comprising inductance and capacity. Such a path is provided in Fig. 5 through the inductance of one anti-resonant set and the capacity of the other anti-resonant set. A comparison of the shunt elements of Figs. 4 and 5 will likewise show their equivalence. In Fig. 5, for instance, from *b* to *c*, one path extends through a capacity C_{21} and inductance L_{22} and L_{23}. Obviously in Fig. 5 there is a path including inductance and capacity between *b* and *c*. In a similar manner in Fig. 4 there is a path including capacity C_{21}, capacity C_{22} and inductance L_{23}. An equivalent path including both inductance and capacity also occurs in Fig. 5.

Still additional variations may be made in the design of either the series or the shunt impedance element. For instance, the series impedance element may be constructed of a simple inductance in parallel with a circuit including a simple capacity in series with an anti-resonant set, as indicated in Fig. 6. This arrangement provides three paths, one through a simple inductance, one through capacity only, a third through inductance and capacity in series. Still another modification may be provided, as indicated in Fig. 7, by arranging a simple capacity element in parallel with a circuit including an inductance serially related to an anti-resonant set. The lower path including the simple condenser in Fig. 7, corresponds to the path including the condenser C_{13} of Fig. 4, while the other two paths in Fig. 4 are replaced by an inductance in series with an anti-resonant circuit.

In a similar manner additional modifications of the shunt impedance element may be made, for instance, in Fig. 8 the inductance element L_{23} may be retained and the condenser C_{21} in series with an anti-resonant set may be replaced by a condenser in parallel with a resonant circuit. That such a substitution is warranted is at once apparent from a comparison of the lower half of Fig. 6 with the elements L_{12}, C_{12} and C_{13} of Fig. 4 for which it was substituted. Likewise, as indicated in Fig. 9, the condenser C_{21} may be retained in the shunt impedance element z_2 and the combination L_{22}, C_{22} and L_{23} replaced by a simple inductance in parallel with the tuned circuit. The basis of this substitution will be at once seen from a comparison of the upper half of Fig. 7 with the elements L_{11}, L_{12} and C_{12} of Fig. 4.

As a matter of fact, any one of the series impedance elements illustrated in Figs. 4, 5, 6 and 7 may be used with any one of the shunt impedance elements of Figs. 4, 5, 8 and 9, consequently there are sixteen combinations possible where three transmission bands are present. If additional transmission bands are to be provided, with consequent additional resonant and anti-resonant circuits in the series and shunt impedance elements respectively, still further rearrangements of the elements may be made, so that a much larger number of combinations will be possible.

In order to illustrate the manner in which the various inductances and capacities making up the shunt and series elements of the filter may be determined, the equations will be given for determining the inductance and capacity elements of impedances z_1 and z_2 of Fig. 4, it being understood that the inductances and capacities may be worked out for the other cases described above, in a similar manner.

Referring to Fig. 4, the series element z_1 consists of a parallel arrangement of an inductive component L_{11}, a simple resonant component of inductance L_{12} in series with the capacity C_{12} and a capacitative component C_{13}. The corresponding shunt element z_2 is a series arrangement of a capacitative component C_{21}, a simple anti-resonant component of inductance L_{22} in parallel with the capacity C_{22} and an inductive component L_{23}. The constants of z_1 will first be determined and from these constants the values of the corresponding constants of z_2 may then be obtained.

In general, the value of k is put equal to the known resistance of the line in which the wave filter is to be placed. The critical frequencies separating the transmission and attenuation regions are also supposed to be known and will be designated as f_1, f_2, f_3 and f_4 (see Figs. 1 and 2). At these frequencies z_1 must have the values $+i2k$, $-i2k$, $+i2k$ and $-i2k$, respectively, as is apparent from consideration of equation 6.

The general expression for z_1 in terms of frequency may be readily determined from the impedances of each of the three parallel circuits, thus the impedance z_{11} of the path including the inductance L_{11} may be expressed

$$z_{11} = iL_{11}p \quad (7)$$

where p is $2\pi f$.

Similarly the impedance z_{12} of the resonant path may be expressed

$$z_{12} = iL_{12}p - \frac{i}{C_{12}p} \quad (8)$$

and the impedance z_{13} of the path including the capacity C_{13} will be

$$z_{13} = -\frac{i}{C_{13}p} \quad (9)$$

From equations 7, 8 and 9 we may obtain as the impedance of the series element z_1

$$z_1 = \frac{z_{11}z_{12}z_{13}}{z_{11}z_{12} + z_{11}z_{13} + z_{12}z_{13}} = i\frac{L_{11}p - L_{11}L_{12}C_{12}p^3}{1 - (L_{11}C_{12} + L_{11}C_{13} + L_{12}C_{12})p^2 + L_{11}L_{12}C_{12}C_{13}p^4} \quad (10)$$

Equation 10 may be written:

$$z_1 = i\frac{(a.f - b.f^3)}{1 - c.f^2 + d.f^4}.k \quad (11)$$

if we put for convenience,

$$\begin{aligned} ak &= 2\pi L_{11} \\ bk &= 8\pi^3 L_{11}L_{12}C_{12} \\ c &= 4\pi^2(L_{11}C_{12} + L_{11}C_{13} + L_{12}C_{12}) \\ d &= 16\pi^4 L_{11}L_{12}C_{12}C_{13} \end{aligned} \quad (12)$$

The relations expressed by equations 12, also give

$$\begin{aligned} L_{11} &= \frac{a}{2\pi}.k \\ L_{12} &= \frac{ab^2}{2\pi(abc - a^2d - b^2)}.k \\ C_{12} &= \frac{1}{2\pi a^2 b}(abc - a^2d - b^2).\frac{1}{k} \\ C_{13} &= \frac{d}{2\pi b}.\frac{1}{k} \end{aligned} \quad (13)$$

To obtain the constants a, b, c and d it is necessary to equate equation 11 to plus or minus $i2k$, as mentioned above, at the critical frequencies f_1, f_2, f_3 and f_4. This leads directly to a set of four linear equations in a, b, c and d, viz:

$$\begin{aligned} f_1.a-f_1^3.b+2f_1^2.c-2f_1^4.d&=+2 \\ f_2.a-f_2^3.b-2f_2^2.c+2f_2^4.d&=-2 \\ f_3.a-f_3^3.b+2f_3^2.c-2f_3^4.d&=+2 \\ f_4.a-f_4^3.b-2f_4^2.c+2f_4^4.d&=-2 \end{aligned} \quad (14)$$

The values of a, b, c and d, which depend only upon the critical frequencies are obtained by solving this set of equations by the ordinary method of determinants.

Having obtained these values the inductances and capacities in z_1 are then found from the group of equations 13.

To derive the constants of the shunt element z_2 from those of z_1, we may proceed as follows: In a "constant k" type of filter as here designed, there is a relation existing between every component in z_1, say z_{1n} and its corresponding component z_{2n} in z_2, such that

$$z_{1n}.z_{2n}=k_2=\text{constant} \quad (15)$$

Applying this relation to each of the three components of z_1, we have

$$z_{11}.z_{21}=(iL_{11}2\pi f)\cdot\left(\frac{-i}{C_{21}2\pi f}\right)=k^2 \quad (16)$$

$$z_{12}.z_{22}=\left(\frac{i(L_{12}C_{12}4\pi^2f^2-1)}{C_{12}2\pi f}\right)\cdot\left(\frac{-iL_{22}2\pi f}{L_{22}C_{22}4\pi^2f^2-1}\right)=k^2 \quad (17)$$

$$z_{13}.z_{23}=\left(\frac{-i}{C_{13}2\pi f}\right)\cdot(iL_{23}2\pi f)=k^2 \quad (18)$$

Equation 16 at once reduces to

$$\frac{L_{11}}{C_{21}}=k^2 \quad (19)$$

Similarly equation 18 reduces to

$$\frac{L_{23}}{C_{13}}=k^2 \quad (20)$$

Equation 17 may be simplified and written as follows:

$$\frac{(L_{12}C_{12}4\pi^2f^2-1)}{(L_{22}C_{22}4\pi^2f^2-1)}\left(\frac{L_{22}}{C^{12}}\right)=k^2 \quad (21)$$

From the form of equation 21 it is apparent that if the equation is to be independent of the variable frequency, then $L_{12}C_{12}$ must equal $L_{22}C_{22}$ and $\frac{L_{22}}{C_{12}}$ must equal k^2. Through these relations we have at once

$$\frac{L_{12}}{C_{22}}=\frac{L_{22}}{C_{12}}=k^2 \quad (22)$$

and hence

$$\frac{L_{11}}{C_{21}}=\frac{L_{12}}{C_{22}}=\frac{L_{22}}{C_{12}}=\frac{L_{23}}{C_{13}}=k^2 \quad (23)$$

Equations 23 in conjunction with equations 13 give

$$\begin{aligned} L_{22}&=\frac{1}{2\pi a^2b}(abc-a^2d-b^2).k \\ L_{23}&=\frac{d}{2\pi b}.k \\ C_{21}&=\frac{a}{2\pi}\cdot\frac{1}{k} \\ C_{22}&=\frac{ab^2}{2\pi(abc-a^2d-b^2)}\cdot\frac{1}{k} \end{aligned} \quad (24)$$

It will thus be seen that the design of the elements making up a "constant k" filter is a fairly simple matter, although the algebra becomes more complicated as the number of transmission bands is increased. It will also be observed that in formulae 13 and 24 all the inductances are directly proportional to k and all the capacities are inversely proportioned to k.

If it is desired to obtain the constants of some one of the other possible equivalent arrangements, such as those illustrated in Figs. 5 to 9 inclusive, the procedure is to obtain the impedance expression for z_1 in terms of frequency, thereby obtaining an expression which is the same function of frequency as that expressed in equation 11. A set of relations similar to those set forth in equation 12 may then be obtained in terms of a, b, c and d and the new constants will then be sufficient to determine the inductances and capacities of the new design in terms of a, b, c and d. The constants of some other possible equivalent impedance design for z_2 may be similarly determined, for since $z_1.z_2$ equals k^2

$$z_2=-i.\frac{(1-c.f^2+d.f^4)}{a.f.-b.f^3}.k \quad (25)$$

Equation 25 is the expression for the value of the shunt impedance z_2 in terms of frequency and corresponds to equation 11 of the series impedance element.

It will be obvious that the general principles herein disclosed may be embodied in many other organizations widely different from those illustrated, without departing from the spirit of the invention as defined in the following claims.

In this specification and in the following claims, reference is made to the frequency range, and in accordance with the well-recognized usage of mathematics, it will readily be understood that zero frequency is to be looked upon as a frequency of the whole range at one end thereof, and infinite frequency is to be looked upon as another frequency at the other end of the range. By infinite frequency we mean a frequency so high that the phenomena are not materially different from what they would be if it were made considerably higher. In other words, an infinite frequency is a frequency so high that the results obtained by making it high approach to limiting values.

What is claimed is:

1. A wave filter comprising a plurality of periodic sections, each section including series impedance elements in parallel and shunt impedance elements in series so proportioned that the filter will have more than two bands of free transmission, the impedance of the filter being such that the product of the series impedance and the shunt impedance of any section will be constant.

2. A wave filter comprising a plurality of periodic sections, each section including series and shunt impedance elements, said series elements being arranged to provide a plurality of parallel paths, and all said elements being so proportioned that the filter will have more than two bands of free transmission.

3. A wave filter comprising a plurality of periodic sections, each section including series and shunt impedance elements constructed of inductances and capacities, the inductances and capacities of the series elements being arranged to provide a plurality of parallel paths of respectively vanishing reactance at different frequencies, and the inductances and capacities of the series and shunt elements being so proportioned and related that the filter as a whole will have more than two bands of free transmission, the impedance of the filter being such that the product of the series impedance and the shunt impedence of any section will be a constant.

4. A wave filter comprising a plurality of periodic sections, each section including series and shunt impedance elements constructed of inductances and capacities, the series elements being arranged to provide a plurality of parallel paths each of zero reactance at a respective frequency, the inductances and capacities of the series and shunt elements being so proportioned and related that the filter as a whole will have more than two bands of free transmission.

5. A wave filter having a plurality of free bands of transmission, said wave filter comprising a plurality of periodic sections, each section consisting of a series and a shunt impedance element, each series impedance element consisting of a plurality of paths, each path having zero reactance at a frequency lying within one of the transmission bands, and each shunt impedance element including a plurality of inductances and capacities so proportioned and related as to form a combination of infinite reactance at one frequency in each band of free transmission.

6. A wave filter having a plurality of free bands of transmission, said wave filter comprising a number of periodic sections, each section including series and shunt impedance elements, said series impedance elements being constructed of capacities and inductances so related that there will be a path of zero reactance at a frequency lying within each of the bands of free transmission.

7. A wave filter having a plurality of free bands of transmission, said wave filter comprising a number of periodic sections, each section including series and shunt impedance elements, said shunt impedance elements being constructed of inductances and capacities so proportioned and related that a combination of infinite reactance will be formed for a frequency lying within each band of free transmission.

8. The combination of a wave filter with apparatus having a substantially constant resistance for all frequencies of current, said wave filter being of the "constant k" type with k equal to the resistance of said apparatus.

9. The combination of a long line of substantially constant resistance at all frequencies, and a wave filter of the "constant k" type for which k is substantially equal to the said resistance of the line.

10. The combination of a wave filter and a long line, whose impedance is substantially a constant resistance at all frequencies, said filter having recurrent sections, each with series and shunt elements, the product of the impedances of such elements being constant and substantially equal to the square of said resistance of the line.

11. A wave filter comprising a plurality of periodic sections, each section including series and shunt impedance elements, the series elements arranged to provide a plurality of parallel paths, each of zero reactance at a certain frequency, and the shunt elements arranged in series, each being of infinite reactance at the same frequency as that for which a respective series element is resonant.

12. A wave filter comprising a series component impedance in parallel and shunt com-

ponent impedance in series, the product of the impedance of each series component by the impedance of the respective shunt component being constant for varying frequency.

13. A wave filter comprising a plurality of periodic sections, each section including a series impedance of zero reactance at two or more frequencies at least one of those frequencies being finite, and a shunt impedance of infinite reactance at these same frequencies and so proportioned that the product of the series and shunt impedances will be a constant, said wave-filter having two or more separate ranges of free transmission and two or more separate ranges of attenuation.

In testimony whereof, I, have signed my name to this specification this 28th day of April 1920.

OTTO J. ZOBEL.