MULTIPLE BAND WAVE FILTER

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INVENTOR.

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To all whom it may concern:

Be it known that I, Otto J. Zobel, residing at Maplewood, in the county of Essex and State of New Jersey, have invented certain improvements in Multiple-Band Wave Filters, of which the following is a specification.

This invention relates to selective circuits and more particularly to selective circuits of the type known as wave filters.

In multiplex signaling, frequency selectivity has been heretofore obtained in cases where it has been desired to discriminate between different frequency bands by the employment of simple recurrent networks comprising arrangements of inductances and capacities. Of these networks, those which have been principally used are the so-called "low pass", "high pass" and "single band" filters described in the U. S. patents to George A. Campbell, Nos. 1,297,113 and 1,297,114 issued May 22, 1917. These filters are characterized by the fact that currents lying within certain ranges of frequencies are freely transmitted without substantial attenuation, while currents of other frequencies are very strongly attenuated.

In general when a filter of this character is connected to a line whose impedance is practically a constant resistance, large impedance irregularities are introduced between the low band filter and the designed range of frequencies to be transmitted, since the characteristic impedance of the filter at any termination varies with frequency.

These impedance irregularities at the frequencies to be transmitted are objectionable, not only from the standpoint of maximum energy transferred from the line to the wave filter, but also from the standpoint of repeater balance.

One of the objects of this invention is to provide a wave filter of the so-called "constant k" type, that is a filter so characterized that the product of the impedance of the series element of any section into the impedance of the shunt element of that section will be proportional to the square of a constant and hence independent of frequency.

Another object of the invention is to produce a wave filter which will be in general capable of transmitting over any preassigned number of frequency ranges. The filter of the present invention is quite distinct from that described in the Campbell patent, in this respect, since the Campbell filters were designed to transmit over a single range of frequencies or at most over two ranges of frequencies while attenuating all other frequencies.

Other and further objects of the invention will be more fully understood from the following detailed description when read in connection with the accompanying drawing.

Figures 1 and 2 of which illustrate characteristic of the filters of this invention, Fig. 3 of which is a simplified diagram of a recurrent network filter, while Figs. 4 to 9 inclusive are diagrams of elements which may be used in making up the filter of this invention.

The wave filters forming the subject matter of this invention have a completely periodic structure consisting of a plurality of sections, as indicated in Fig. 3, each section comprising a series impedance element $z_s$ and a shunt impedance $z_s$. These impedance elements are reactances, that is they are made up of inductances and capacities, in a manner more fully hereafter described. While the discussion of these wave filters is primarily on the basis that the elements are non-dissipative in character, the inevitable introduction of dissipation will not materially alter the designs obtained. In order to minimize the transmission losses in the wave filter, there should be provided as large time constants for the inductances and capacities as is practicable in each specific case.

The computation for a filter on the assumed basis that its reactances are non-dissipative is justified both by theoretical investigations and by practical tests. It is well known that the resistance of a coil or a condenser can be made very small compared to its inductive reactance or capacity reactance and therefore the performance of such a coil or condenser may be computed approximately with entire neglect of such slight resistance.

When the magnitude of the inductances and capacities have been obtained on this basis and the type of coils and condensers giving these magnitudes have been decided upon, the corresponding amounts of resistance necessarily introduced are then accurately taken account of in practice when computing the current losses thru the wave.
filter. The effect of this dissipation is principally to cause small allowable current losses within the bands of free transmission.

It is well known among engineers that at the sending end a long transmission line behaves the same as if its length were infinite. The characteristic impedance of a smooth line is its impedance at the sending end, assuming that the line is so long that it is practically the same as if its length were infinite. There is an analogous situation for filters of the type having recurrent sections. Their behavior is most easily computed in the first instance on the assumption that they extend with an infinite number of sections from the sending end. In practice, a filter must have a finite number of sections and in that case it may be terminated at the drop end by a suitable network to simulate the discarded infinite extension. The matter of providing such a network is not treated in the present specification.

It is well known that a smooth transmission line having uniform series impedance distributions $z_1$ and uniform shunt impedance distributions $z_2$ per unit of length has the characteristic impedance

$$ k = \sqrt{z_1 z_2} \quad (1) $$

Such a line also has a propagation constant:

$$ \gamma = \sqrt{\frac{z_2}{z_1}} \quad (2) $$

A wave filter such as is indicated schematically in Fig. 3 of the drawing comprising a series of sections, each having a series impedance element $z_1$ and a shunt impedance element $z_2$, has a characteristic impedance at any termination which is a function of both the product and the ratio of $z_1$ and $z_2$ and a propagation constant which is a function of their ratio, hence it is convenient to express both the characteristic impedance and propagation constant of the wave filter in terms of $k$ and $\gamma$, the parameters of the corresponding smooth line.

In order that the filter may be of the "constant $k$" type, it is necessary that the product of the series impedance $z_1$ and the shunt impedance $z_2$, per section should be a constant independent of frequency, that is

$$ z_1 z_2 = k^2 = \text{constant} \quad (3) $$

As pointed out in the above mentioned patents to Campbell, free transmission takes place through such a network, for frequencies which give

$$ \gamma^2 = \frac{z_1}{z_2} \quad (4) $$

within the range whose limits are defined by

$$ \gamma^2 = 0 \quad \text{to} \quad \gamma^2 = -4. \quad (5) $$

By combining equation 5 with equation 3, it is apparent that the network transmits within a range of frequencies in which $\gamma^2$ lies between

$$ \gamma^2 = 0 \quad \text{and} \quad \gamma^2 = \pm 2k \quad (6) $$

There are two standpoints, transmission and attenuation, from which it is possible to synthetically construct any desired "constant $k$" wave filter. Let us first consider the construction of a filter from the transmission requirements.

From equation 6 it is apparent that a free transmission region occurs whenever the series element $z_1$ becomes resonant, that is $z_1 = 0$. Assuming then that the desired wave filter should have a transmission region, it is apparent that $z_1$ must have $n$ resonant points. These points may be obtained by constructing $z_1$ of $n$ simple reactance components $z_{11}, z_{12}, \ldots, z_{1n}$, all in parallel, wherein each component takes care of one desired resonant point of $z_1$. As a specific example, take the case of a filter which is desired to have three free transmission bands, one in the neighborhood of zero cycles, another in the neighborhood of infinity and still another at some intermediate frequency. For example, let one free transmission band extend from zero to frequency $f_1$, another free transmission band extend between frequencies $f_2$ and $f_3$, and a third free transmission band from frequency $f_4$ to infinity. The reactance curve for the element $z_1$ at the various frequencies will be as indicated in Fig. 1. In this figure the reactance is zero at zero frequency and increases in a positive direction until it becomes infinite at a point somewhere between frequency $f_1$ and $f_2$. When the reactance becomes infinite it suddenly changes sign and becomes negative and from this point the negative reactance decreases until it becomes zero somewhere between $f_2$ and $f_3$ for the second transmission band. The reactance now continues on the upper slope in a positive direction, again becoming infinite between frequencies $f_3$ and $f_4$, then changing sign from positive to negative and then decreasing until it again becomes zero at infinity.

The critical frequencies are determined from equation 6, above, and it will be seen that at the upper critical frequency for the first transmission band $z_1$, will be $\pm 2k$, as indicated in Fig. 1; at the lower critical frequency of the internal transmission band $z_1$ is $-2k$ at frequency $f_1$; at the higher critical frequency it becomes $+2k$ at frequency $f_2$; and at the lower critical frequency of the infinite frequency transmission region the value of $z_1$ becomes $-2k$ at frequency $f_4$.

The variation of transmission with frequency will now be as indicated in Fig. 2, which shows three bands of free transmission separated by two attenuation bands. A com-
Comparison of Fig. 2 with Fig. 1 shows that the reactance becomes zero at some point in each transmission band. Therefore, if we are to design the filter from the transmission requirements, it is apparent that the series impedance element \( z_1 \) may be constructed of three parallel elements, one of which includes a simple inductance \( L_{12} \). Such an element will have zero reactance at zero frequency.

Another parallel element may comprise a simple resonant circuit consisting of inductance \( L_{12} \) and capacity \( C_{12} \), since such an element may be made resonant at some desired frequency between the frequencies \( f_1 \) and \( f_2 \). Finally, the third element may consist of a simple capacity, \( C_{13} \), which will have zero reactance at an infinite frequency.

The corresponding elements of the shunt impedance \( z_2 \) may be readily determined from a consideration of the fact that whereas the series element must be resonant at a given frequency for free emission, the shunt element should be anti-resonant at the same frequency, in order that the least amount of current be diverted through the shunt path. Consequently, the shunt impedance element \( z_2 \) should contain a simple condenser \( C_{23} \), which will have an infinite reactance at zero frequency. An anti-resonant combination comprising inductance \( L_{23} \) and capacity \( C_{23} \) should be provided for a frequency between the critical frequencies \( f_1 \) and \( f_2 \) of the internal transmission band. Finally, a simple inductance \( L_{23} \) having an infinite reactance at infinite frequency should be provided to correspond with the capacity path \( C_{12} \) of the series impedance element.

Obviously, if additional internal transmission bands are to be provided, such bands may be accommodated by providing additional resonant circuits in parallel in the series element \( z_1 \) and corresponding anti-resonant circuits in series in the shunt impedance element \( z_2 \).

A filter producing the same results may be designed from a consideration of the attenuation requirements only, although in this case the physical form of the filter will be somewhat different. Referring to Figs. 1 and 2, it is apparent that the design of a filter from attenuation requirements involves providing a series impedance element which shall have an infinite reactance at some frequency in each of the bands of attenuation. In other words, at some point between frequency \( f_1 \) and \( f_2 \), the reactance should be infinite and also at some point between frequency \( f_3 \) and \( f_4 \), the reactance should be infinite. The obvious construction to satisfy this requirement would be to construct the series element \( z_1 \) of two anti-resonant elements in series, one anti-resonant element being resonant at a frequency between \( f_1 \) and \( f_2 \), and the other between \( f_3 \) and \( f_4 \). Such a design is illustrated in Fig. 3. Obviously from an attenuation standpoint the corresponding requirement of the shunt element is that its reactance be zero for the frequencies at which the series element is resonant.

The shunt element \( z_2 \) should therefore be constructed of two resonant circuits in parallel, one circuit being resonant at a frequency between \( f_1 \) and \( f_2 \), and the other being resonant at a frequency between \( f_3 \) and \( f_4 \).

While the filter of Fig. 5 is apparently somewhat different in construction from that of Fig. 4, it will function in a manner which is identical therewith, if dissipation be disregarded, though by appropriate design dissipation may be present and make the filters of Figs. 5 and 4 function identically. The equivalence of the two circuits will be more readily apparent when it is considered that in Fig. 4 the requirement was to provide one path between point \( a \) and \( b \), containing a pure inductance \( L_{12} \). Such a path is provided in Fig. 5 through the two inductance elements of the anti-resonant sets in series. In Fig. 4 it will be noted that there is also a path containing simple capacity. Such a path is provided in Fig. 5, through the two capacity elements of the anti-resonant sets in series. In Fig. 4 a path is also provided through a resonant circuit comprising inductance and capacity. Such a path is provided in Fig. 5 through the inductance of one anti-resonant set and the capacity of the other anti-resonant set. A comparison of the shunt elements of Figs. 4 and 5 will likewise show their equivalence. In Fig. 5, for instance, from \( b \) to \( c \), one path extends through a capacity \( C_{23} \) and inductance \( L_{23} \) and \( L_{24} \). Obviously in Fig. 5 there is a path including inductance and capacity between \( b \) and \( c \). In a similar manner in Fig. 4 there is a path including capacity \( C_{23} \), capacity \( C_{12} \) and inductance \( L_{23} \). An equivalent path including both inductance and capacity also occurs in Fig. 5.

Still additional variations may be made in the design of either the series or the shunt impedance element. For instance, the series impedance element may be constructed of a simple inductance in parallel with a circuit including a simple capacity in series with an anti-resonant set, as indicated in Fig. 6. This arrangement provides three paths, one through a simple inductance, one through capacity only, a third through inductance and capacity in series. Still another modification may be provided, as indicated in Fig. 7, by arranging a simple capacity element in parallel with a circuit including an inductance serially related to an anti-resonant set. The lower path including the simple condenser in Fig. 7, corresponds to the path including the condenser \( C_{12} \) of Fig. 4, while the other two paths in Fig. 4 are replaced by an inductance in series with an anti-resonant circuit.
In a similar manner additional modifications of the shunt impedance element may be made, for instance, in Fig. 8 the inductance element \( L_{21} \) may be retained and the condenser \( C_{21} \) in series with an anti-resonant set may be replaced by a condenser in parallel with a resonant circuit. That such a substitution is warranted is at once apparent from a comparison of the lower half of Fig. 6 with the elements \( L_{21} \), \( C_{12} \) and \( C_{13} \) of Fig. 4 for which it was substituted. Likewise, as indicated in Fig. 9, the condenser \( C_{21} \) may be retained in the shunt impedance element \( z_2 \) and the combination \( L_{22}, C_{22} \) and \( L_{23} \) replaced by a simple inductance in parallel with the tuned circuit. The basis of this substitution will be at once seen from a comparison of the upper half of Fig. 7 with the elements \( L_{11}, L_{13} \) and \( C_{13} \) of Fig. 4.

As a matter of fact, any one of the series impedance elements illustrated in Figs. 4, 5, 6 and 7 may be used with any one of the shunt impedance elements of Figs. 4, 5, 8 and 9, consequently there are sixteen combinations possible where three transmission bands are present. If additional transmission bands are to be provided, with consequent additional resonant and anti-resonant circuits in the series and shunt impedance elements respectively, still further rearrangements of the elements may be made, so that a much larger number of combinations will be possible.

In order to illustrate the manner in which the various inductances and capacities making up the shunt and series elements of the filter may be determined, the equations will be given for determining the inductance and capacity elements of impedances \( z_1 \) and \( z_2 \) of Fig. 4, it being understood that the inductances and capacities may be worked out for the other cases described above, in a similar manner.

Referring to Fig. 4, the series element \( z_1 \) consists of a parallel arrangement of an inductive component \( L_{11} \), a simple resonant component of inductance \( L_{12} \) in series with the capacity \( C_{12} \) and a capacitative component \( C_{13} \). The corresponding shunt element \( z_2 \) is a series arrangement of a capacitative component \( C_{21} \), a simple anti-resonant component of inductance \( L_{22} \) in parallel with the capacity \( C_{22} \) and an inductive component \( L_{23} \). The constants of \( z_1 \) will first be determined and from these constants the values of the corresponding constants of \( z_2 \) may then be obtained.

In general, the value of \( k \) is put equal to the known resistance of the line in which the wave filter is to be placed. The critical frequencies separating the transmission and attenuation regions are also supposed to be known and will be designated as \( f_1, f_2, f_3 \) and \( f_4 \) (see Figs. 1 and 2). At these frequencies \( z_2 \) must have the values \( +2k, -2k, +2k \) and \( -2k \), respectively, as is apparent from consideration of equation 6.

The general expression for \( z_1 \) in terms of frequency may be readily determined from the frequencies of each of the three parallel circuits, thus the impedance \( z_{11} \) of the path including the inductance \( L_{11} \) may be expressed

\[
z_{11} = i L_{11} p \quad (7)
\]
where \( p = 2\pi f \).

Similarly the impedance \( z_{12} \) of the resonant path may be expressed

\[
z_{12} = i L_{12} p - \frac{i}{C_{12} p} \quad (8)
\]
and the impedance \( z_{13} \) of the path including the capacity \( C_{13} \) will be

\[
z_{13} = -\frac{i}{C_{13} p} \quad (9)
\]

From equations 7, 8 and 9 we may obtain as the impedance of the series element \( z_1 \)

\[
z_1 = \frac{z_{12} z_{13}}{z_{12} z_{13} + z_{11} z_{13} + z_{11} z_{12}} = \frac{i}{1 - \left(L_{11} C_{13} + L_{12} C_{13} + L_{12} C_{12}\right) p^2 + L_{11} L_{12} C_{12} C_{13} p^4} \quad (10)
\]

Equation 10 may be written:

\[
z_1 = \frac{i (a f - b) f}{1 - c f^2 + d f^4} k \quad (11)
\]
if we put for convenience,

\[
\begin{align*}
ak &= 2\pi L_{11} \\
bk &= 8\pi L_{11} L_{12} C_{12} \\
e &= 4\pi^2 (L_{11} C_{13} + L_{12} C_{12} + L_{12} C_{13}) \\
d &= 10\pi^2 L_{11} L_{12} C_{13} C_{13} \quad (12)
\end{align*}
\]

The relations expressed by equations 12, also give

\[
L_{11} = \frac{a}{2\pi^2} k \\
L_{12} = \frac{ab}{2\pi^3} (abc - a^2d - b^2) k \\
C_{12} = \frac{1}{2\pi^2} \frac{1}{abc - a^2d - b^2} \quad (13)
\]

\[
C_{13} = \frac{d}{2\pi^2} \frac{1}{k} \quad (14)
\]
To obtain the constants \(a, b, c\) and \(d\) it is necessary to equate equation 11 to plus or minus \(2k\), as mentioned above, at the critical frequencies \(f\), \(f\), \(f\) and \(f\). This leads directly to a set of four linear equations in \(a, b, c\) and \(d\), viz:

\[
\begin{align*}
 f_a &= -f_a + b - 2f_c + c - 2f_d + d = -2 \\
 f_b &= a - f_a + b - 2f_c + c + 2f_d - d = -2 \\
 f_c &= a + f_a + b + 2f_c - c - 2f_d - d = 2 \\
 f_d &= a - f_a + b - 2f_c - c + 2f_d + d = 2
\end{align*}
\]  

(14)

The values of \(a, b, c\) and \(d\), which depend only upon the critical frequencies are obtained by solving this set of equations by the ordinary method of determinants.

\[z_{11}z_{22} (\frac{d}{C_{11}2\pi f}) \cdot \left(\frac{d}{C_{11}2\pi f}\right) = k^2 \]  

Equation 16 at once reduces to

\[
\frac{L_{11}}{C_{11}} = k^2
\]  

(19)

Similarly equation 18 reduces to

\[
\frac{L_{22}}{C_{22}} = k^2
\]  

(20)

Equation 17 may be simplified and written as follows:

\[
\frac{(L_{11}C_{22}4\pi^2 f^2 - 1)}{L_{22}C_{22}4\pi^2 f^2 - 1} = k^2
\]  

(21)

From the form of equation 21 it is apparent that if the equation is to be independent of the variable frequency, then \(L_{11}C_{22}\) must equal \(L_{22}C_{11}\) and \(L_{11}C_{12}\) must equal \(L_{22}C_{21}\). Through these relations we have at once

\[
\frac{L_{11}C_{22}}{C_{11}} = \frac{L_{22}C_{11}}{C_{22}} = k^2
\]  

(22)

and hence

\[
\frac{L_{11}}{C_{11}} = \frac{L_{22}}{C_{22}} = \frac{L_{11}}{C_{12}} = \frac{L_{22}}{C_{21}} = k^2
\]  

(23)

Equations 23 in conjunction with equations 13 give

\[
L_{22} = \frac{1}{2\pi^2 c^2 (a^2 c d - b^2)} c
\]

\[
L_{22} = \frac{d}{2\pi^2 a} c
\]

\[
C_{11} = \frac{a}{2\pi} \cdot \frac{1}{k}
\]

\[
C_{22} = \frac{1}{2\pi (a c d - a^2 b)} \cdot \frac{1}{k}
\]

It will thus be seen that the design of the elements making up a "constant \(k\)" filter is a fairly simple matter, although the algebra becomes more complicated as the number of transmission bands is increased. It will also be observed that in formulae 13 and 24 all the inductances are directly proportional to \(k\) and all the capacities are inversely proportioned to \(k\).

If it is desired to obtain the constants of some one of the other possible equivalent arrangements, such as those illustrated in Figs. 5 to 9 inclusive, the procedure is to obtain the impedance expression for \(z\) in terms of frequency, thereby obtaining an expression which is the same function of frequency as that expressed in equation 11. A set of relations similar to those set forth in equation 12 may then be obtained in terms of \(a, b, c\) and \(d\) and the constants will then be sufficient to determine the inductances and capacities of the new design in terms of \(a, b, c\) and \(d\). The constants of some other possible equivalent impedance design for \(z\) may be similarly determined, for since \(z, z_2\) equals \(k^2\)

\[
z = -i \cdot \frac{(1 - c f^2 + d f^2)}{a f^2 - b f^2} \cdot k
\]  

(25)

Equation 25 is the expression for the value of the shunt impedance \(z\) in terms of frequency and corresponds to equation 11 of the series impedance element. It will be obvious that the general principles herein disclosed may be embodied in many other organizations widely different from those illustrated, without departing
from the spirit of the invention as defined in the following claims.

In this specification and in the following claims, reference is made to the frequency range, and in accordance with the well-recognized usage of mathematics, it will readily be understood that zero frequency is to be looked upon as a frequency of the whole range at one end thereof, and infinite frequency is to be looked upon as another frequency at the other end of the range. By infinite frequency we mean a frequency so high that the phenomena are not materially different from what they would be if it were made considerably higher. In other words, an infinite frequency is a frequency so high that the results obtained by making it high approach to limiting values.

What is claimed is:

1. A wave filter comprising a plurality of periodic sections, each section including series impedance elements in parallel and shunt impedance elements in series so proportioned that the filter will have more than two bands of free transmission, the impedance of the filter being such that the product of the series impedance and the shunt impedance of any section will be constant.

2. A wave filter comprising a plurality of periodic sections, each section including series and shunt impedance elements, said series elements being arranged to provide a plurality of parallel paths, and all said elements being so proportioned that the filter will have more than two bands of free transmission.

3. A wave filter comprising a plurality of periodic sections, each section including series and shunt impedance elements constructed of inductances and capacities, the inductances and capacities of the series elements being arranged to provide a plurality of parallel paths of respectively vanishing reactance at different frequencies, and the inductances and capacities of the series and shunt elements being so proportioned and related that the filter as a whole will have more than two bands of free transmission, the impedance of the filter being such that the product of the series impedance and the shunt impedance of any section will be a constant.

4. A wave filter comprising a plurality of periodic sections, each section including series and shunt impedance elements constructed of inductances and capacities, the series elements being arranged to provide a plurality of parallel paths each of zero reactance at a respective frequency, the inductances and capacities of the series and shunt elements being so proportioned and related that the filter as a whole will have more than two bands of free transmission.

5. A wave filter having a plurality of free bands of transmission, said wave filter comprising a plurality of periodic sections, each section consisting of a series and a shunt impedance element, each series impedance element consisting of a plurality of paths, each path having zero reactance at a frequency lying within one of the transmission bands, and each shunt impedance element including a plurality of inductances and capacities so proportioned and related as to form a combination of infinite reactance at one frequency in each band of free transmission.

6. A wave filter having a plurality of free bands of transmission, said wave filter comprising a number of periodic sections, each section including series and shunt impedance elements, said shunt impedance elements being constructed of inductances and capacities so proportioned and related so as to form a combination of infinite reactance with apparatus having a substantially constant resistance for all frequencies of current, said wave filter being of the "constant R" type with k equal to the resistance of said apparatus.

7. The combination of a wave filter with apparatus having a substantially constant resistance for all frequencies of current, said wave filter being of the "constant R" type for which k is substantially equal to the said resistance of the line.

8. The combination of a wave filter and a long line, whose impedance is substantially constant resistance at all frequencies, and a wave filter of the "constant R" type for which k is substantially equal to the said resistance of the line.

9. The combination of a wave filter and a long line, whose impedance is substantially constant resistance at all frequencies, said filter having recurrent sections, each with series and shunt elements, the product of the impedances of such elements being constant and substantially equal to the square of said resistance of the line.

10. A wave filter comprising a plurality of periodic sections, each section including series and shunt impedance elements, the series elements arranged to provide a plurality of parallel paths, each of zero reactance at a certain frequency, and the shunt elements arranged in series, each being of infinite reactance at the same frequency as that for which a respective series element is resonant.

11. A wave filter comprising a series component impedance in parallel and shunt com-
ponent impedance in series, the product of the impedance of each series component by the impedance of the respective shunt component being constant for varying frequency.

13. A wave filter comprising a plurality of periodic sections, each section including a series impedance of zero reactance at two or more frequencies at least one of those frequencies being finite, and a shunt impedance of infinite reactance at these same frequencies and so proportioned that the product of the series and shunt impedances will be a constant, said wave-filter having two or more separate ranges of free transmission and two or more separate ranges of attenuation.

In testimony whereof, I, have signed my name to this specification this 28th day of April 1920.

OTTO J. ZOBEL.