Sept. 27, 1960 W. A. MENZEL 2,954,517

VOLTAGE DIVIDER

Filed Jan. 8, 1953 2 Sheets-Sheet 1

FIG. 1.

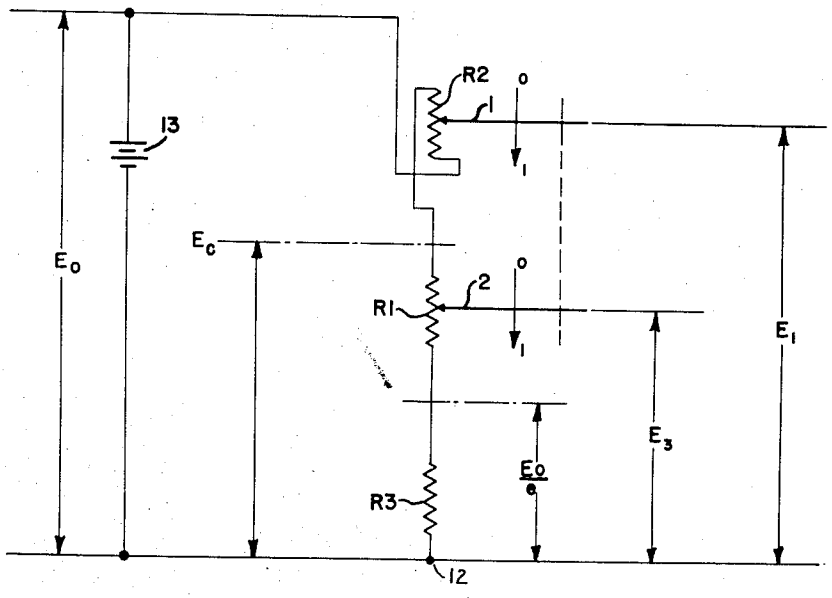
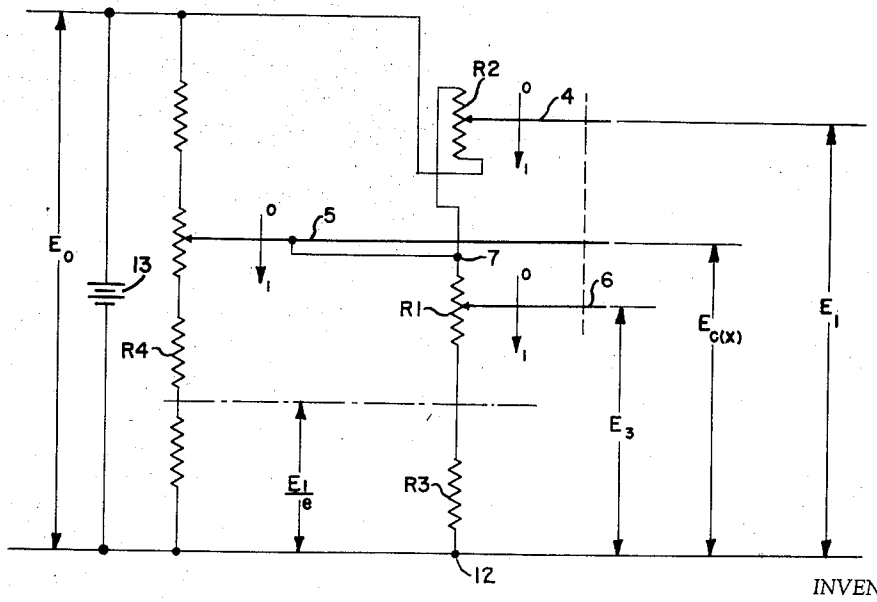


FIG. 2.



INVENTOR
WOLFGANG A. MENZEL

BY G. W. O'Brien
R. M. Hicks
ATTORNEYS

Sept. 27, 1960 W. A. MENZEL 2,954,517

VOLTAGE DIVIDER

Filed Jan. 8, 1953 2 Sheets-Sheet 2

FIG. 3.

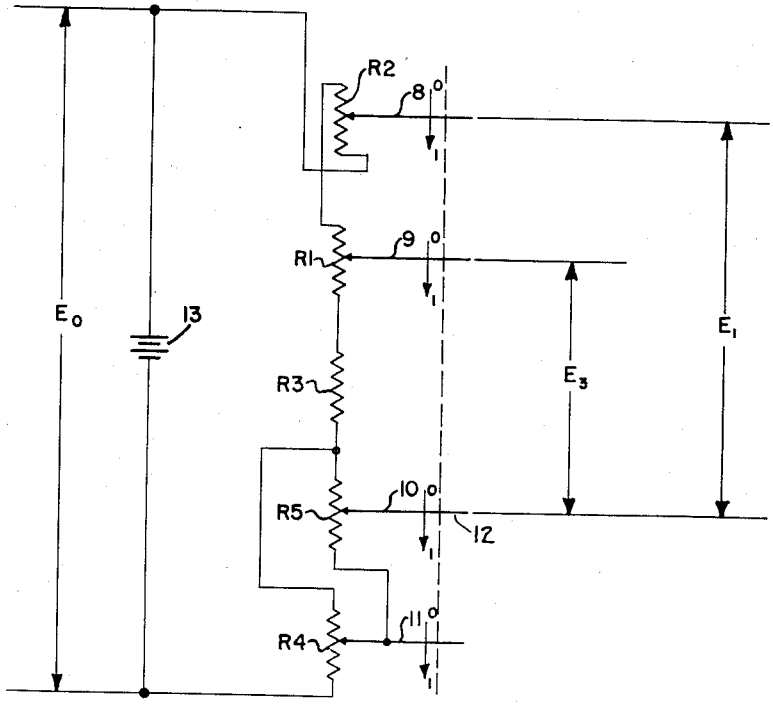
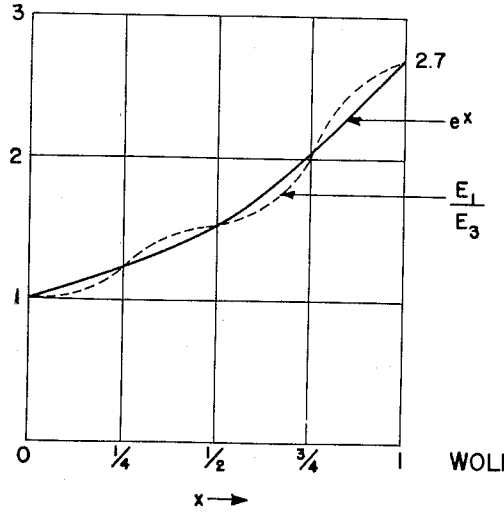


FIG. 4.



INVENTOR
WOLFGANG A. MENZEL

BY R. M. Hicks

ATTORNEYS

United States Patent Office

2,954,517

Patented Sept. 27, 1960

2,954,517

VOLTAGE DIVIDER

Wolfgang A. Menzel, Silver Spring, Md., assignor to the United States of America as represented by the Secretary of the Navy

Filed Jan. 8, 1953, Ser. No. 330,366

5 Claims. (Cl. 323—74)

(Granted under Title 35, U.S. Code (1952), sec. 266)

The invention described herein may be manufactured and used by or for the government of the United States of America for governmental purposes without the payment of any royalties thereon or therefor.

This invention relates to a voltage divider and more particularly to a potentiometer designed to produce two voltages having a ratio varying according to an exponential function.

For many purposes in electrical circuits the voltage change is desired to be exponential. For example, an analysis of the electric stabilized time fuze circuit shows that a linear time setting requires an exponential change in the ratio of two voltages. Heretofore, this has been accomplished by dividing a multiple turns potentiometer into equal segments and by inserting suitable parallel resistances converting the divider to the desired exponential ratio. This structure is delicate and expensive and does not provide sufficient accuracy.

The presently disclosed voltage divider provides a means for obtaining an exponential ratio between two voltages by combining a plurality of normally linear potentiometers. It consists of four potentiometers arranged on a single axis and having the contact arms connected in such a manner as to give two output voltages, the ratio of which follows the exponential function with an accuracy of one part in two million for range e^x, x varying from 0 to 1.

An object of the present invention is to provide a voltage divider which produces an exponential ratio between the resulting voltages.

A further object of this invention is to provide a voltage divider employing four linear potentiometers of such values that the resulting voltages vary according to an exponential ratio.

Other objects and many of the attendant advantages of this invention will be readily appreciated as the same becomes better understood by reference to the following detailed description when considered in connection with the accompanying drawings wherein:

Fig. 1 shows one form of a voltage divider constructed according to the present invention;

Fig. 2 illustrates another species of this invention;

Fig. 3 shows the preferred embodiment of a voltage divider; and

Fig. 4 is a graph showing the variation of

$$\frac{E_1}{E_3}$$

from e^x where x varies from 0 to 1 by employing the voltage divider of Fig. 3.

Referring now to the drawings and more specifically to Fig. 1 it can be seen that there is provided a voltage divider having ganged wiper contacts, or sliders **1** and **2**. Output voltages E_1 and E_3 are obtained across wiper contact **1** and the common output, or reference, point **12**, and across wiper contact **2** and reference point **12**, respectively. There are provided resistances R_1, R_2 and R_3 disposed in series as shown and across which is impressed a voltage E_0 which can be supplied by any suitable source, such as battery **13**. When $x=0$ the sliders are in the upper position and when $x=1$ the sliders are in the lower position. E_c is accepted as a fixed voltage as shown in Fig. 1. Accordingly when $x=1$, $E_1=E_0$ and

$$E_3=\frac{E_0}{e}$$

since

$$\frac{E_1}{E_3}=e^x$$

When $x=0$, $E_1=E_c$ and $E_3=E_c$.

In order to calculate the deviation of the two linear potentiometers shown from the theoretical values for

$$\frac{E_1}{E_3}=e^x$$

where x varies from 0 to 1, the following equation is assumed: $E_1=E_c+Ax$ and $E_3=E_c-Bx$ where A and B are coefficients chosen to meet the theoretical values of $x=0$ and $x=1$. The accuracy of the ratio

$$\frac{E_1}{E_3}=\frac{E_c+A_x}{E_c-B_x}$$

will be calculated for the case

$$E_1e^{-\frac{x}{2}}=E_3e^{+\frac{x}{2}}$$

in which E_1 and E_3 vary through one half the total range of e^x. By definition of the exponential function in

$$\frac{E_1}{E_c}=\ln\frac{E_c}{E_3}$$

From this equation E_c is found equal to

$$E_0\sqrt{\frac{1}{e}}$$

The values of A and B may be evaluated when $a=1$ as $A=E_0-E_c$ and

$$B=E_c-\frac{E_0}{e}$$

Substituting the derived value of E_c into the original equation the final equation for the voltage divider shown in Fig. 1 becomes:

$$\frac{E_1}{E_3}=\frac{1+x[\sqrt{e}-1]}{1-x\frac{[\sqrt{e}-1]}{\sqrt{e}}}$$

From this equation the ratio of

$$\frac{E_1}{E_3}$$

may be calculated for various values of x yielding the following results:

Table I

X	e^x [1]	$\frac{E_1}{E_3}$ [2]	Ratio $\frac{[1]}{[2]}$	$dx\cdot10^{+3}$
0.0	1.000000000	1.00000000	1.000000000	0.000
0.1	1.105170918	1.10848771	0.997007822	−2.992
0.2	1.221402758	1.22624197	0.996053623	−3.946
0.3	1.349858807	1.35450299	0.996571301	−3.429
0.4	1.491824698	1.49474265	0.998047856	−1.952
0.5	1.648721271	1.64872124	1.000000000	0.000
0.6	1.822118800	1.81856169	1.001956001	+1.956
0.7	2.013752707	2.00684808	1.003440532	+3.440
0.8	2.225540928	2.21675802	1.003962050	+3.962
0.9	2.459603111	2.45224342	1.003001207	+3.001
1.0	2.718281828	2.71828169	1.000000000	0.000

for

$$\frac{E_1}{E_3}=\frac{1+x[\sqrt{e}-1]}{1-x\frac{[\sqrt{e}-1]}{\sqrt{e}}}$$

It can be seen from this table that the ratio

$$\frac{E_1}{E_3}$$

meets the theoretical values for $x=0$, $x=1$ and $x=.5$. The maximum error is $dx=.004$. With $x=t/\tau$, this means a maximum timing error of 120 milliseconds for $\tau=30$ seconds. In other words a time fuze for 30 seconds maximum time has no error due to the voltage divider at settings of 0, 15, 30 seconds, but the deviation is +120 milliseconds at 7.5 seconds and −120 milliseconds at 22.5 seconds. While this accuracy is sufficient for many purposes, it is not adequate for fuze application.

In Fig. 1, E_c is accepted as a constant. However, it can be shown that E_c varies between

$$E_0\left[\frac{e-1}{e}\right]$$

when $x=0$ and

$$E_0\left[\frac{1}{e-1}\right]$$

when $x=1$. The deviation of E_c from a linear function between these values is slight and therefore a linear potentiometer may be provided to vary E_c as E_1 and E_3 are varied. Such a voltage divider is shown in Fig. 2. There are provided sliders **4**, **5** and **6** which are ganged as shown. Slider **5** serves the function of impressing at point **7** a varying voltage corresponding to a linear variation of E_c. The output voltages E_1 and E_3 are obtained across wiper arm **4** and reference point **12**, and across wiper arm **6** and reference point **12**, respectively. This voltage divider may be expressed by the following equation:

$$\frac{E_1}{E_3}=\frac{\left[\frac{x}{e-1}+\frac{[e-1]}{e}[1-x]\right][1-x]+x}{\left[\frac{x}{e-1}+\frac{e-1}{e}[1-x]\right][1-x]+\frac{x}{e}}$$

$$=\frac{1+x\left[\frac{e^2}{[e-1]^2}-2\right]+x^2\left[1-\frac{e}{[e-1]^2}\right]}{1-x\left[2-\frac{2e-1}{[e-1]^2}\right]+x^2\left[1-\frac{e}{[e-1]^2}\right]}$$

or by the general equation

$$\frac{1+Ax+Cx^2}{1-Bx+Cx^2}$$

This equation yields the following results:

Table II

X	e^x [1]	$\frac{E_1}{E_3}$ [2]	Ratio $\frac{[1]}{[2]}$	$dx\cdot10^{+4}$
0.00	1.000000000	1.000000000	1.000000000	0.0000
0.1	1.105170918	1.105146026	1.000022523	+0.2252
0.2	1.221402758	1.221311619	1.000074623	+0.7462
0.3	1.349858808	1.349677057	1.000134662	+1.3466
0.4	1.491824698	1.491550042	1.000184141	+1.8414
0.5	1.648721271	1.648376609	1.000209091	+2.091
0.6	1.822118800	1.821751561	1.000201585	+2.01585
0.7	2.013752707	2.013427700	1.000161419	+1.614
0.8	2.225540928	2.225322877	1.000097986	+0.98
0.9	2.459603111	2,459523379	1.000032417	+0.324
1.0	2.718281828	2.71828182	1.000000000	+0.0000

for

$$\frac{E_1}{E_3}=\frac{\left[\frac{x}{e-1}+\frac{e-1}{e}[1-x]\right][1-x]+x}{\left[\frac{x}{e-1}+\frac{e-1}{e}[1-x]\right][1-x]+\frac{x}{e}}$$

The voltage divider of Fig. 1 yields the general equation

$$\frac{E_1}{E_3}=\frac{1+ax}{1-bx}$$

to meet the curve e^x at three points. If $x=0$, .5 and 1 are the meeting points then a and b can be calculated and the resulting equation gives coincidence at these points. In order to improve the accuracy of the voltage divider it would be necessary to increase the number of points of coincidence thereby increasing the order of the rational function by which

$$\frac{E_1}{E_3}$$

is expressed. This gives the equation

$$\frac{E_1}{E_3}=\frac{1+a_1x+b_1x^2}{1-a_2x+b_2x^2}$$

In this case the coefficients a_1, b_1, a_2, b_2 must be determined to give coincidence points with the curve e^x. Assuming these points to be at $x=0$, .25, .5, .75 and 1 then:

$a_1=.54115855$ $\quad b_1=.10755897$
$a_2=.45871032$ $\quad b_2=.06523960$

Comparing

$$\frac{E_1}{E_3}=\frac{1+a_1x+b_1x^2}{1-a_2x+b_2x^2}$$

with e^x, with the above calculated coefficients gave values shown in the following table:

Table III

X	e^x [1]	$\frac{E_1}{E_3}$ [2]	Ratio $\frac{[1]}{[2]}$	$dx\cdot10^{+6}$
0.0	1.000000000	1.000000000	1.000000000	0.0000
0.1	1.105170918	1.105165522	1.000004882	+4.882
0.2	1.221402758	1.221400527	1.000001826	+1.826
0.3	1.349858808	1.349860571	0.999998693	−1.307
0.4	1.491824698	1.491827289	0.999998263	−1.737
0.5	1.648721271	1.648721268	1.000000001	+0.001
0.6	1.822118800	1.822115631	1.000001739	+1.739
0.7	2.013752707	2.013750077	1.000001306	+1.306
0.8	2.225540928	2.225545003	0.999998168	−1.832
0.9	2.459603111	2.459615131	0.999995113	−4.887
1.0	2.718281828	2.718281828	1.000000000	0.000

for

$$\frac{E_1}{E_3}=\frac{1+a_1x+b_1x^2}{1-a_2x+b_2x^2}$$

$a_1=0.54115855$ $\quad b_2=0.1075589$
$a_1=0.45871032$ $\quad b_2=0.06523960$

It is apparent that the voltage divider of Fig. 1 must be modified to include a potentiometer arrangement varying proportional to mx^2. The voltage divider of Fig. 3 provides such an arrangement. There are provided sliders **8**, **9**, **10** and **11** which are ganged as shown. In this embodiment, the output voltages E_1 and E_3 are obtained across wiper arm **8** and common output point **12**, and wiper arm **9** and common output point **12**, respectively. Sliders **10** and **11** with resistors R_5 and R_4 respectively are added to yield the square function hereinbefore referred to. This voltage divider gives the following ratio for

$$\frac{E_1}{E_2}$$

$$\frac{E_1}{E_3}=\frac{R_1+R_3+xR_2+x^2\frac{R_4R_5}{xR_4+R_5}}{R_1+R_3-xR_1+x^2\frac{R_4R_5}{xR_4+R_5}}$$

or

$$\frac{1+x\frac{R_2R_5+R_4[R_1+R_3]}{[R_1+R_3]R_5}+x^2\frac{R_4[R_2+R_5]}{[R_1+R_3]R_5}}{1-x\frac{R_1R_5-R_4[R_1+R_3]}{[R_1+R_3]R_5}+x^2\frac{R_4[R_5-R_1]}{[R_1+R_3]R_5}}$$

The coefficients of this equation must equal the coefficients a_1, b_1, a_2, b_2 previously calculated. However, all resistance values can be expressed in terms of the single resistor R_1 as follows: With

$$\frac{R_2}{R_1}=r_2,\ \frac{R_3}{R_1}=r_3,\ \frac{R_4}{R_1}=r_4,\ \frac{R_5}{R_1}=r_5$$

there is:

r_2=.99560588	r_4=.17253453
r_3=.99586759	r_5=4.07642806

Since r_2 and r_3 are close to 1, for simplification purposes R_1 may be assumed equal to R_2 and R_3. A voltage divider with R_1=R_2=R_3 gives good accuracy with an error of not more than .0043 percent.

This invention with the foregoing calculations demonstrates that a combination of five linear resistance units, four of which are potentiometers assembled on one axis can produce output voltages having a ratio which follows the exponential function with an accuracy of one part in two million for the range $e^x[x:0 \ldots 1]$.

Obviously many modifications and variations of the present invention are possible in the light of the above teachings. It is therefore to be understood that within the scope of the appended claims the invention may be practiced otherwise than as specifically described.

What is claimed as new and desired to be secured by Letters Patent is:

1. An electrical circuit comprising a potential energy source, a plurality of series connected linear resistance elements connected across said source, a common output terminal defined in the electrical circuit, and ganged wiper contacts individually coupled to two of said resistance elements, the instantaneous potentials appearing across each of said ganged contacts and said common output terminal as said contacts are moved across their respective resistance elements exhibiting an exponential relationship therebetween.

2. An electrical circuit comprising a potential energy source, a plurality of series connected linear resistance elements connected across said source, a common output terminal defined in the electrical circuit, ganged wiper contacts individually coupled to two adjoining resistance elements of said plurality of resistance elements, the effective resistance between said adjoining resistance elements being selectively varied by said contacts being electrically isolated from each other, the movement of said ganged wiper contact in a manner to provide instantaneous output potentials between each of said ganged contacts and said common terminal exhibiting an exponential relationship therebetween.

3. An electrical circuit comprising a potential energy source, a bank having a plurality of serially connected linear resistance elements connected across said source, a common output terminal defined by one juncture between said source and said bank, a first movable output terminal electrically coupled to one of said elements for developing a variable output potential between said first and said common output terminals correlative to the movement of said first terminal on said one element, a second movable output terminal electrically coupled to

the resistance element adjoining said one element for developing a variable output potential between said second and said common output terminals correlative to the movement of said second terminal on said adjoining element, said movable output terminals being ganged for movement in unison on their respective resistance elements in a manner to selectively vary the effective resistance between the adjoining elements to effect an exponential relationship between said output potentials.

4. An electrical circuit comprising a potential energy source, a first bank of a plurality of serially connected linear resistance elements connected across said source, a second bank of a plurality of serially connected linear resistance elements connected across said source, a common output terminal defined by one juncture between said first bank and said source, a first movable output terminal electrically coupled to one of the elements in said first bank for developing a variable output potential between said first and said common terminals correlative to the movement of said first terminal on said one element, a second movable output terminal electrically coupled to the element adjoining said one element in said first bank for developing a variable output potential between said second and said common terminals correlative to the movement of said second terminal on said adjoining element, a movable contact electrically coupled to one of the elements in said second bank and to the juncture between said one and adjoining elements on said first bank, said movable contact and terminals being ganged for movement in unison on their respective elements to effect an exponential relationship between said output potentials.

5. An electrical circuit comprising a potential energy source, a bank of interconnected linear resistance elements coupled across said source, said bank including a plurality of serially connected elements and one element connected in parallel with one of said serially connected elements, a movable contact electrically coupled to the one serially connected element and to said parallel connected element, a first movable output terminal electrically coupled to one of said plurality of serially connected elements for developing a variable output potential between said first terminal and a common terminal correlative to the movement of said first terminal on the one element, a second movable output terminal electrically coupled to the element adjoining said one of said serially connected elements for developing a variable output potential between said second terminal and a common terminal correlative to the movement of said second terminal on said adjoining element, a movable common terminal electrically coupled to said parallel connected element, said movable contact and terminals being ganged for movement in unison on their respective elements to effect an exponential relationship between said output terminals.

References Cited in the file of this patent

UNITED STATES PATENTS

1,858,364	Koenig	May 17, 1932
2,417,442	Parkinson	Mar. 18, 1947
2,448,885	Hooven	Sept. 7, 1948
2,453,462	Sellers	Nov. 9, 1948
2,573,280	Schmidt	Oct. 30, 1951

FOREIGN PATENTS

698,560	Germany	Nov. 13, 1940