

Jan. 16, 1962

D. C. KALBFELL
ELECTRONIC ANALOG MULTIPLIER

3,017,108

Filed June 2, 1958

5 Sheets-Sheet 1

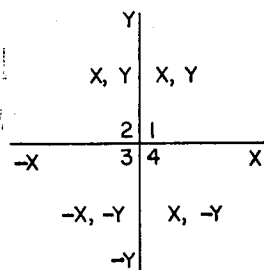


FIG. 1

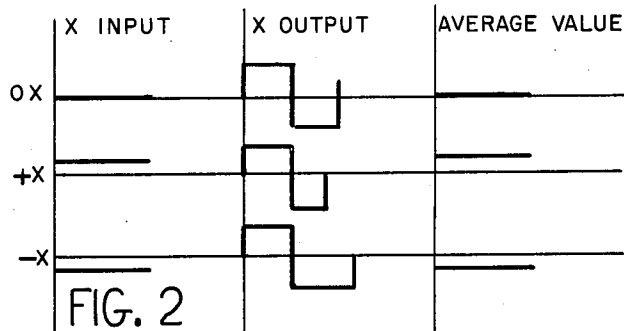


FIG. 2

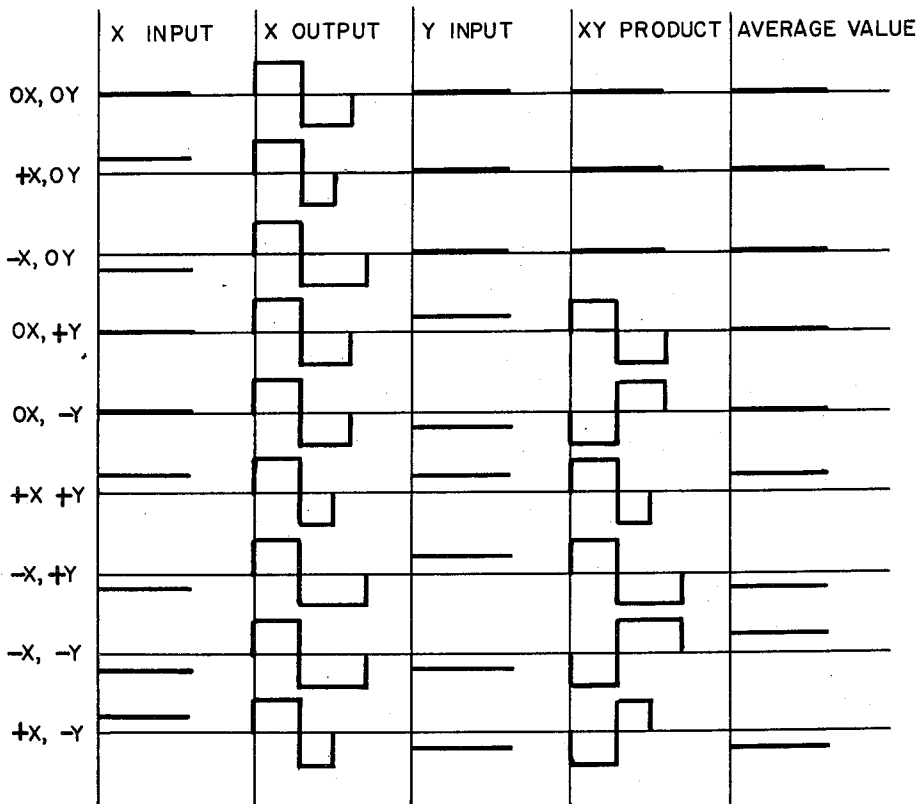


FIG. 3

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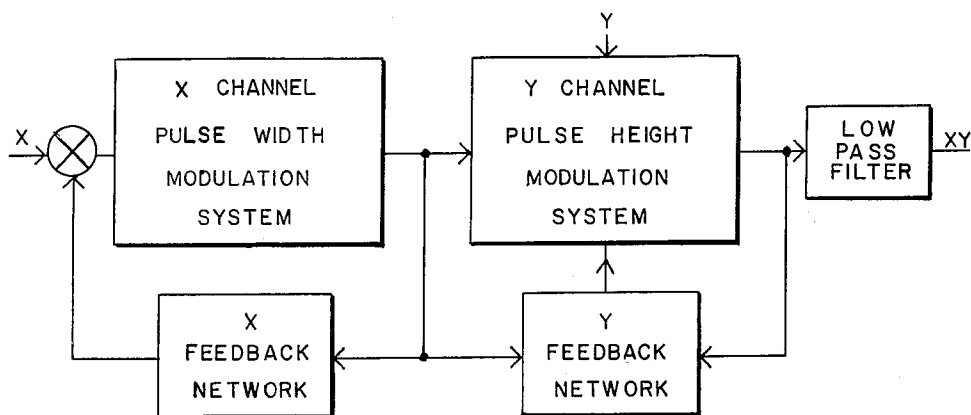


FIG. 4

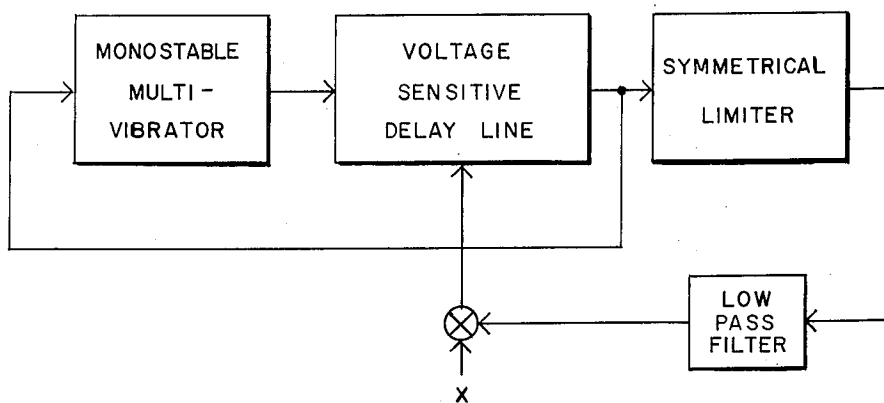


FIG. 6

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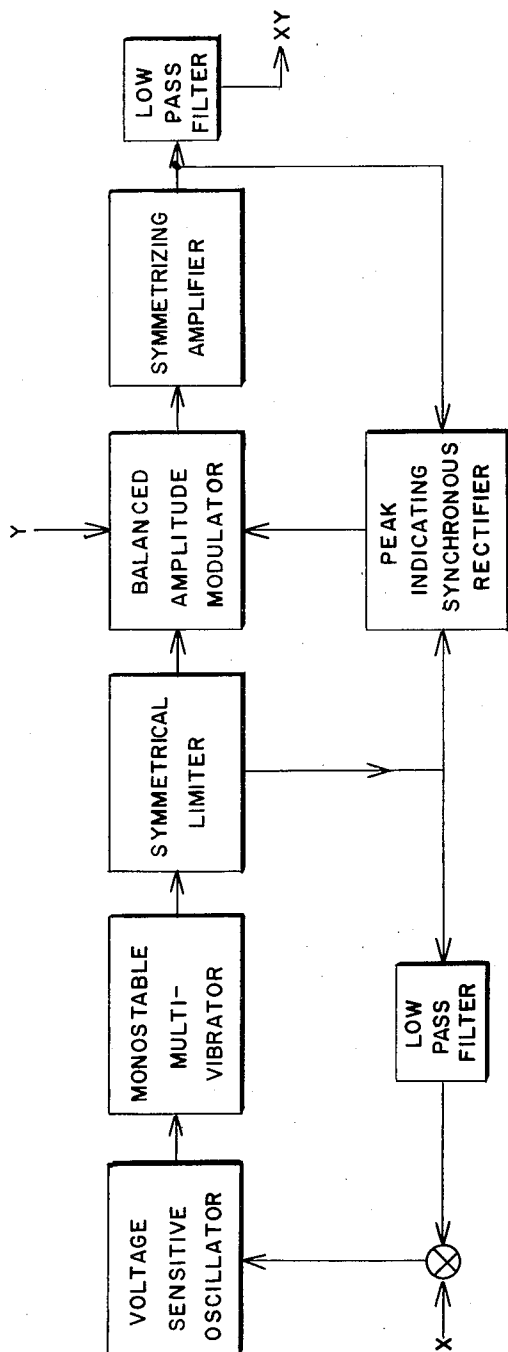


FIG. 5

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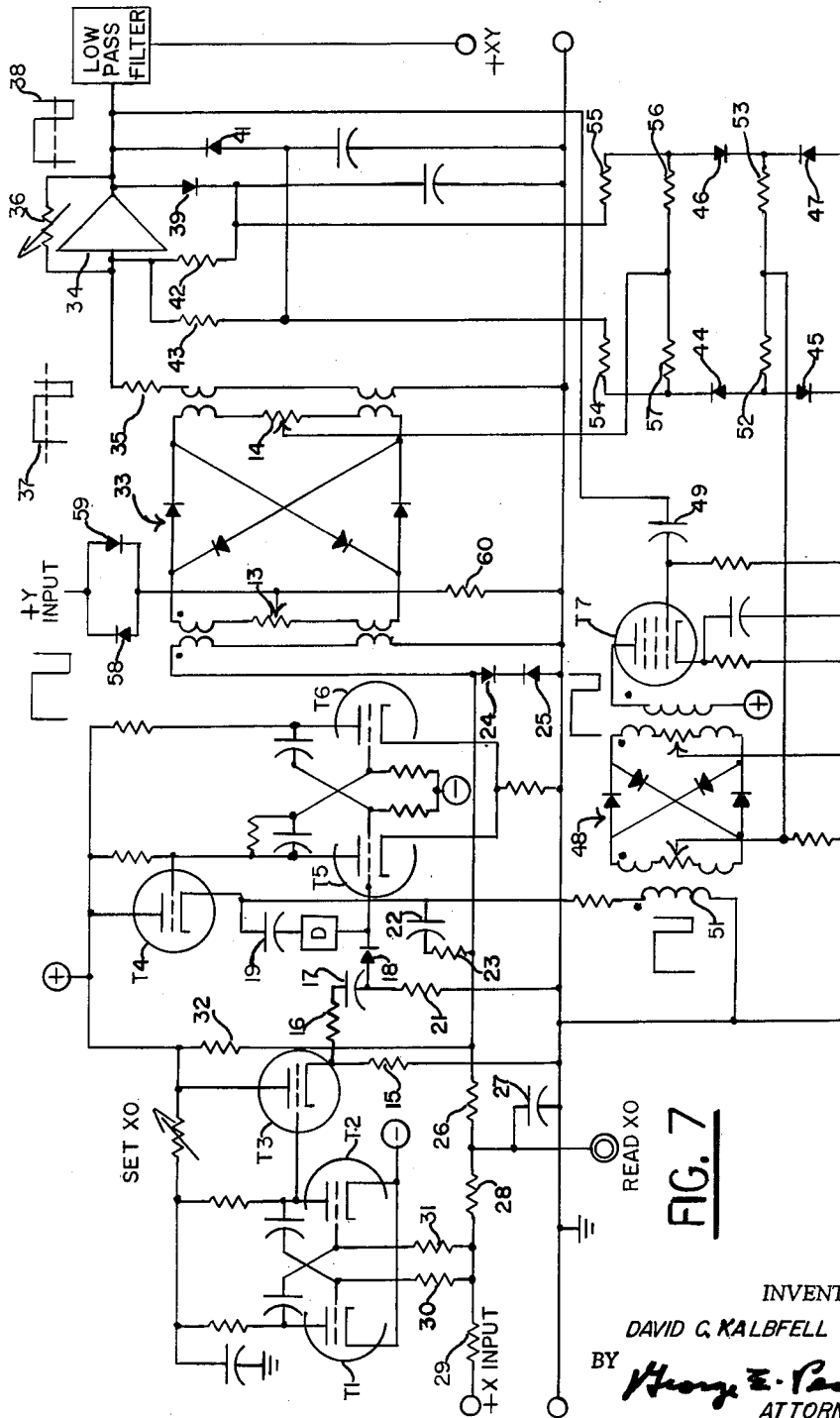
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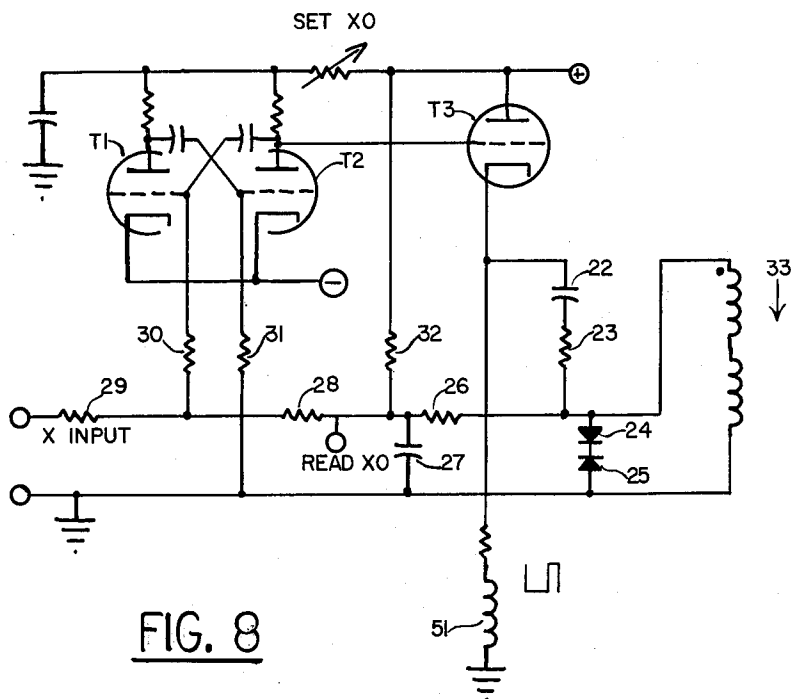


FIG. 8

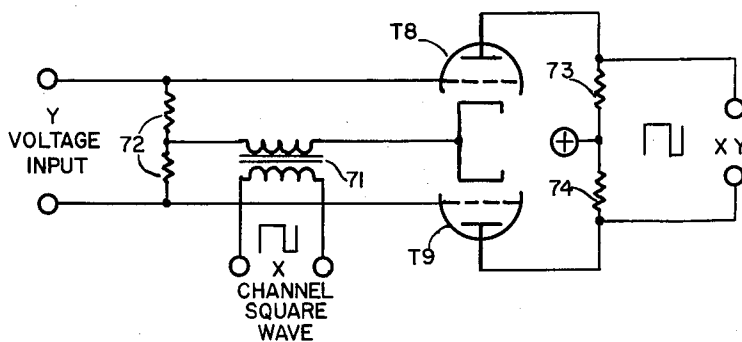


FIG. 9

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ELECTRONIC ANALOG MULTIPLIER

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17 Claims. (Cl. 235—194)

This invention relates generally to electronic computers and more particularly to an electronic analog multiplier in which the method of multiplying X and Y variable inputs utilizes the variable pulse area principle and employs phase sensitive circuitry to operate naturally in all four quadrants without offset or bias voltages, the output product being zero if either input is zero and of proper sign depending on whether the inputs are of like or unlike sign.

In accordance with the electronic multiplier method of the present invention, an X channel pulse width modulation system is utilized in which the sign of X retains its significance without requiring a D.C. biasing voltage in the input circuit. This system produces a pulse width modulated X wave or unsymmetrical square wave in which the ratio of the widths of the positive and negative portions depends upon the magnitude and sign of X. Otherwise expressed the generated square wave which is amplitude-symmetrical in its positive and negative portions but unsymmetrical in time with respect thereto, has a D.C. or average value that depends upon the magnitude of X and a polarity that depends upon the sign of X. This is accomplished by providing a constant time or fixed width for one portion or half cycle of the square wave and varying its total period in accordance with the X variable. Or the unsymmetrical square wave may be generated from a series of pulses in which the spacing therebetween varies in accordance with the X variable. In either case, the widths of the alternate half cycles of the square wave are made equal when X is zero, and then by symmetrically limiting the positive and negative excursions of the square wave, the average value thereof becomes directly proportional to X and reverses sign as X reverses sign. Thus, the phase of the X wave is preserved notwithstanding changes in the sign of X and phase inversion of this X wave under control of the Y variable may be utilized to give significance to the sign of Y in the XY product, as will hereinafter more fully appear.

The unsymmetrical square wave output of the X channel which is dependent on the magnitude and sign of the X variable, as aforementioned, is applied as one input to a Y channel modulator system and the Y variable input is applied as the other input thereto. This modulator system comprises a balanced modulator, or the like, which produces an output waveform having the same shape as the unsymmetrical square wave output of the X channel but having an amplitude proportional to Y and having a phase which either matches or opposes the phase of the X channel output depending on whether the sign of Y is positive or negative. The balanced modulator thus preserves phase as a measure of sign and makes it possible to utilize an X channel pulse width modulation system in which the sign of X retains its significance without a D.C. biasing voltage. Since the asymmetry of the X square wave depends upon the magnitude and sign of the X variable and the height of this square wave is merely modulated by the Y variable without changing the shape thereof, the sign of X is preserved in the square wave output of the balanced modulator

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which provides a measure of the XY product in that the average value of this square wave is proportional to the product of X and Y. When the sign of Y is positive, the phase of this square wave is unchanged and the sign of the product is positive or negative depending on the sign of X. When the sign of Y is negative, the phase of the output square wave is inverted such that the sign of the product is negative if X is positive and positive if X is negative. Thus, by the use of phase sensitive methods and circuitry, as aforescribed, the signs of X and Y are kept track of, so that bias voltages are unnecessary. The use of bias voltages to permit four-quadrant operation as in the case of prior art multipliers is undesirable since these bias voltages may drift and cause errors.

The average value of the amplitude modulated unsymmetrical XY square wave is extracted by passing the same through a low pass filter to thus provide a positive or negative D.C. voltage proportional to the product of X and Y.

It is desirable to improve the performance of the electronic analog multiplier aforescribed by incorporating negative feedback separately in the X and Y channels in order that the resulting product may have very high accuracy. Thus, each channel is independently linearized and compensated for drift. It is important that this be done in both channels since otherwise the larger of the two errors would predominate. The method of applying negative feedback to the X channel input comprises the steps of extracting the average value or D.C. component of the unsymmetrical X square wave after the same has been symmetrically limited in amplitude and then applying this D.C. component in opposition to the D.C. input signal so that only the net differential between the input and feedback D.C. signals is available for pulse width modulation purposes. In the application of negative feedback around the Y channel a peak indicating synchronous rectifier is employed which produces an output D.C. voltage whose sign depends upon the phase of the balanced amplitude modulator output with respect to its input and whose amplitude is proportional to the amplitude of the modulator output. This D.C. voltage is then applied in opposition to the Y input voltage so that only a small differential is available for application to the balanced amplitude modulator control terminals.

Positive and negative excursions of the variable amplitude output wave of the balanced modulator are made symmetrical by taking feedback from positive and negative peak indicating rectifiers to the input of an operational amplifier. Either of these positive and negative voltages, may be selected depending on the sign of the Y input, for negative feedback to the balanced amplitude modulator. To accomplish this selection, the phase of the output of the operational amplifier is compared with the phase of the asymmetrical X square wave in a phase sensitive modulator which produces a positive or negative voltage output depending on the relative phases of its inputs. The detector output voltage is applied to two pairs of clamping diodes which are connected respectively to the peak indicating rectifiers through buffer resistors, one of these pairs of diodes being driven to cut-off and the other being driven to conduction by the detector output voltage. The conducting diodes short the rectifier connected thereto to ground. The other

rectifier, however, faces only the blocked diodes and hence is able to apply voltage to the feedback input to the balanced Y modulator.

In addition to the aforescribed feedback arrangement for linearizing the Y channel, non-linear resistors such as diodes may be employed in series with the Y input to correct for the non-linearity of the balanced modulator itself at very low levels.

An object of the present invention is to provide a new and improved electronic analog multiplier circuit and method which operates on the variable pulse area principle.

Another object is to provide an electronic analog multiplier method and system which employs phase sensitive circuitry to operate naturally in all four quadrants without bias or offset voltages, thereby avoiding the errors introduced by drift in the prior art systems.

Another object is to multiply X and Y variable inputs by width and height modulation of a square wave in a manner which retains the sign significance of the inputs without requiring bias or offset voltages.

Another object in an electronic analog computer is to vary the time asymmetry of a square wave in accordance with the magnitude and sign of one variable input and to vary the amplitude and phase of the square wave in accordance with the magnitude and sign of a second variable input to be multiplied by the first input.

Another object in an electronic analog multiplier is to produce an asymmetrical square wave having an average value corresponding in magnitude and sign to that of one of two variable inputs to be multiplied by each other and to produce a second asymmetrical square wave having the same shape as the first and having an amplitude and phase corresponding to the magnitude and sign of the other variable whereby the second square wave has an average value proportional to the product of the variables and a sign which is proper according to the signs of the variables.

Another object in an electronic analog multiplier method and system is to provide X and Y channel pulse width and height modulation and to provide negative feedback separately in both channels in order to provide high accuracy in the XY product.

Still another object in an electronic analog multiplier method and system is to vary the time asymmetry of a square wave in accordance with the magnitude and sign of one variable input, to produce an identically shaped square wave and vary the amplitude and phase thereof in accordance with the magnitude and sign of another variable input, and to apply a negative feedback to the second input in accordance with the relative phase of the two square waves.

An additional object is to restore the direct current component of a square wave which is unsymmetrical in time following amplitude modulation thereof in a transformer coupled balanced modulator.

Still other objects, features, and advantages of this invention will be readily appreciated as the same becomes better understood by reference to the following detailed description when considered in connection with the accompanying drawings wherein:

FIG. 1 is a plot illustrating how the signs of X and Y coordinates differ in the four quadrants;

FIG. 2 is a series of graphs illustrating how the asymmetry of an X square wave varies with the magnitude and sign of the X variable input;

FIG. 3 is a series of graphs illustrating how the asymmetry and phase of the X and Y square waves change for X and Y inputs of different magnitude and sign to produce XY products having the proper magnitude and sign;

FIGS. 4 and 5 are block diagrams illustrating X and Y channel systems suitable for practicing the electronic analog multiplier method of the present invention;

FIG. 6 is a block diagram of an alternative form of an X channel system;

FIG. 7 is a schematic circuit diagram of the block diagram system of FIG. 5;

FIG. 8 is a schematic circuit diagram of an alternative form of an X channel system suitable for use with the Y channel circuit of FIG. 7; and

FIG. 9 is a schematic circuit diagram illustrating an alternative Y channel system suitable for use with the X channel system of the present invention.

Referring now to the drawings wherein like characters of reference designate like parts throughout the several views and more particularly to FIG. 1, there is shown thereon a plot of quadrants 1 to 4 from which it may be seen that in each of quadrants 1 and 3 the X, Y coordinates have the same sign, these being both positive in the first and both negative in the third quadrant. Hence the sign of the XY product for all values of X and Y in each of quadrants 1 and 3 will be positive. In each of quadrants 2 and 4, the X, Y coordinates are of unlike polarity, X being negative and Y positive in the second while X is positive and Y negative in the fourth. Hence the sign of the product for all values of X and Y in each of quadrants 2 and 4 will be negative. It is in this sense of proper sign of the product in relation to the signs of X and Y that the electronic analog multiplier of the present invention is said to operate naturally in all four quadrants.

According to the method of the present invention, the time asymmetry of an amplitude symmetrical square wave is varied with changes in the magnitude and sign of the X variable input as illustrated in FIG. 2 wherein it may be seen that the time of the positive portion or positive one half cycle of the square wave is constant for each of the three values of X disclosed, namely, X is zero (OX), X is positive (+X), and X is negative (-X). When the X input is zero, the X output wave is symmetrical and, consequently, the average value of the square wave is zero. When the X input is positive, the negative portion of the X output wave is narrowed in proportion to the magnitude of the X input and the average value of the square wave, accordingly, is positive and directly proportional to the input. In like manner, when the X input is negative, the negative portion of the X output wave is wider than the positive portion thereof in proportion to the magnitude of the X input and the average value of the square wave is negative and directly proportional to the input.

According to the second step in the multiplier method of the present invention, a second amplitude symmetrical square wave is produced having the same shape as the X output wave, except when the Y variable input is zero, and having an amplitude proportional to the magnitude of the Y variable input, this wave being produced with or without phase inversion selectively in accordance with the polarity of the Y input. The manner in which the magnitude and sign of the XY product varies for various magnitude and sign combinations of X and Y is illustrated in FIG. 3, although not to scale, wherein it may be seen that when the Y input is zero for any of the three X conditions illustrated in FIG. 2, there is no square wave output indicative of an XY product and, accordingly, there also is no average value, the product being zero as it should be.

When X is zero and Y is positive, a square wave output is produced indicative of XY and having the same shape as the X output wave. Since this wave is symmetrical its average value is zero, being indicative of an XY product which is zero, as it should be, since X is zero. When X is zero and Y is negative, the symmetry of the X wave is reproduced in the XY wave, but the phase of the XY wave is inverted since Y is negative. The average value of the XY wave is zero, however, due to the symmetry of the wave, as when X is zero and Y is positive.

When X and Y are both positive, the XY wave is

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asymmetrical and of the same shape as, and in phase with, the X wave. Hence, the average value of this wave is positive, as it should be for multiplication of X and Y coordinates in the first quadrant.

When X is negative and Y is positive, the negative asymmetry of the X wave is repeated in the XY wave without phase inversion. Hence, the XY product is negative as it should be for operation in the second quadrant.

When both X and Y are negative, the negative asymmetry of the X wave results in positive asymmetry in the wave due to the phase inversion which is produced when Y is negative. Hence, the average value is positive as it should be for operation in the third quadrant. When X is positive and Y is negative, the positive asymmetry of the X wave is inverted to give negative asymmetry in the XY wave and the average value is negative as it should be for operation in the fourth quadrant.

According to the third principal step of the electronic multiplier method of the present invention, the XY wave is filtered to extract the average value thereof, thereby to provide an analog voltage output proportional to the product of X and Y and having the proper sign depending on whether X and Y have like or unlike signs.

FIG. 4 discloses circuitry in block diagram form suitable for practicing the aforescribed steps of the electronic multiplier method, the X output wave being generated in the X Channel Pulse Width Modulation System in response to the X input, the XY output wave being generated in the Y Channel Pulse Height Modulation System with or without phase inversion in response to the Y input and in accordance with the sign thereof, and the average value of the XY wave being extracted by the Low Pass Filter to provide a direct current voltage of the proper sign and directly proportional to the product of X and Y.

The X and Y channels each have a separate Feedback Network for independently linearizing the same and compensating for drift. The appropriate sign of the feedback voltage for the Y channel is obtained, however, by a method of comparing the phase of the XY product wave with that of the X output wave as will hereinafter more fully appear. The feedback network for the X channel may simply comprise a low pass filter for extracting the average value of the X output wave, as will be described hereinafter in connection with other circuitry disclosed elsewhere herein.

The X Channel Pulse Width Modulation System may take the form illustrated in FIG. 5 wherein a Monostable Multi-Vibrator generates a series of constant width pulses under control of a Voltage Sensitive Oscillator, the result of this arrangement being that the spacing between the pulses varies with the frequency of the oscillator. Otherwise expressed, as the total period of the oscillator determines the total period of a pulse and the space between this pulse and the succeeding one. The pulse being of fixed width, however, the variation proportional to frequency must occur in the spacing between pulses. Since the pulses and the spaces therebetween constitute a square wave, any asymmetry resulting from differences in the widths of alternate half cycles provides a measure of frequency deviation from a predetermined frequency having a total period equal to twice the time of the fixed pulses. Thus, when the frequency increases, the half cycle of fixed width is wider than its alternate half cycle. When the frequency decreases, the fixed width half cycle is narrower than its alternate half cycle.

The frequency of the Voltage Sensitive Oscillator is controlled by the X variable input voltage such that the predetermined or quiescent frequency is set when X equals zero so that the frequency increases as X increases positively and decreases as X increases negatively. Thus, the asymmetry in the X output wave is directly proportional to the magnitude of X and the decrease or increase in the total period of the X wave relative to that

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which obtains when the X wave is symmetrical depends upon whether X is positive or negative.

The positive and negative excursions of the X wave are symmetrically limited by the Symmetrical Limiter such that the average value or D.C. component of the resulting amplitude symmetrical, time asymmetrical square wave has a magnitude and polarity corresponding to that of the X input voltage. This D.C. component when extracted by passing of the X wave through the Low Pass Filter serves as negative feedback for the X channel.

The channel may comprise circuitry as disclosed in FIG. 6 wherein a Voltage Sensitive Delay Line responsive to the X input voltage interposes variable delays in the pulses generated by the Monostable Multi-Vibrator which is triggered by the pulses appearing at the output of the delay line. These pulses also pass through the Symmetrical Limiter and Low Pass Filter in the same manner as in FIG. 5. The Voltage Sensitive Delay Line may be of any type suitable for the purpose such, for example, as a lumped constant L-C delay line employing saturable reactors in which a D.C. saturating current responsive to the X input voltage controls the inductance of the inductors and hence the time delay effected thereby in the line.

Referring again to FIG. 5, the Y Channel Pulse Height Modulation System is disclosed as comprising a Balanced Amplitude Modulator and Symmetrizing Amplifier. The Balanced Modulator is made responsive to the Y variable input and also to a negative feedback voltage supplied under control of the Peak Indicating Synchronous Rectifier, the operation of which is best understood by reference to FIG. 7 wherein circuitry exemplifying the various blocks of FIG. 5 is disclosed.

Referring now to FIG. 7, the X voltage controls the frequency of a voltage sensitive-oscillator such, for example, as the conventional multivibrator comprising tubes T1 and T2 and the circuit elements associated therewith. This oscillator is set to operate at a suitable frequency such, for example, as 100 kc. when X equals zero. The output of the oscillator is buffered through a cathode follower circuit comprising tube T3 and cathode load resistor 15 and applied by way of resistor 16, condenser 17, and diode 18 to the grid of tube T5. Tubes T5 and T6 and the circuit elements associated therewith comprise a conventional mono-stable multivibrator, the duration of the metastable state of which is determined by the inert delay line D which may be of any conventional type suitable for the purpose such, for example, as an L-C lumped circuit line. The delay line should interpose a delay of 5 microseconds, this being half the total period at 100 kc.

Tube T5 is normally cut off, since its grid leak, as well as that of tube T6, is returned to a negative bias voltage designated $\ominus$. Tube T6 is normally conducting, however, by reason of the plate of T5 being at this time at substantially the potential of the positive supply designated $\oplus$. A positive pulse from T3 causes T5 to fire, which cuts off T6. When T5 fires, its plate voltage drops and a negative pulse is fed through tube T4 and condenser 19 to delay line D. When the negative pulse comes out of the delay line, it cuts off T5, thus resetting the mono-stable multivibrator to its initial state. The energy in the negative pulse is partly drained to ground through diode 18 and resistor 21 as it comes out of the delay line, and this is minimized by picking resistor 21 to be the optimum terminating resistor for the delay line.

A positive increase in X above zero increases the frequency above 100 kc. and decreases the OFF time of T5 relative to its ON time, these being equal when X is zero. A negative change in X below zero decreases the frequency below 100 kc. and increases the OFF time of T5 relative to its fixed ON time. The asymmetry in the resulting square wave appearing at the cathode of T4 is thus proportional to X and follows its sign. In the

waveforms indicated in FIG. 7, both X and Y are assumed to be positive.

The time asymmetrical waveform voltage at the cathode of T4 is fed through series connected blocking condenser 22 and resistor 23 to a symmetrical clamp comprising back-to-back connected diodes 24 and 25, these being known in the art as a Zener Diode Clamp, which symmetrically limits the positive and negative excursions of the time asymmetrical waveform by symmetrically clamping the same with respect to ground. This clamped voltage is fed through a low pass filter comprising resistor 26 and condenser 27 which may cut off in the vicinity of 1 kc. and extracts the D.C. component due to the asymmetry in the waveform. This D.C. component is directly proportional to the original X modulating voltage except for such nonlinearities as might have been introduced by the X channel system up to this point. This D.C. component is fed back as negative feedback to the X input by way of resistors 28 and 29. The cathodes of T1 and T2 are connected to negative potential $\ominus$ which would normally make their grids negative and cause current to flow through their grid leaks 30 and 31 from an input point at ground potential. Resistor 32, however, is inserted to provide current from positive supply $\oplus$ to just equalize this normal grid current so that the signal input point is normally at ground potential. If X is positive, it causes current to flow into these grid leaks. The feedback signal, however, is then negative and causes a negative current to flow into the grid leaks, largely canceling the X input current.

This X channel system may be adjusted for drift by setting the X input point at ground and adjusting the variable resistor designated Set XO to make the potential at the terminal designated Read XO come to ground potential. This system is then adjustable and tends to be self-compensating.

For ordinary computer applications, X would have a full-scale value such, for example, as 100 volts, although 10 volts would probably be sufficient to produce the desired degree of modulation of the oscillator. If the feedback voltage is 90 volts when the input voltage is 100 volts, this will act like 20 db of feedback and will compensate for drifts and nonlinearities. For a superior system, the X voltage may be fed to the voltage-sensitive oscillator through a preamplifier. If the feedback voltage is then mixed with X at the input of the preamplifier, a larger feedback factor can be employed and the frequency of the oscillator could be deviated through a wider range to thus improve the accuracy of the multiplier.

The X channel functions as a unit, feeding the modulator 33 with a waveform which is symmetrically clamped in amplitude but is unsymmetrical in time. The Y input voltage is applied through balancing potentiometer 13, while the Y feedback voltage is applied to potentiometer 14 as will be described later. Modulator 33 has an output proportional to the magnitude of Y which is in phase with the X channel wave if Y is positive, and vice-versa.

Since the output of the modulator appears across a transformer, it contains no D.C. component. In order to make the positive and negative half cycles deviate equally from ground, a symmetrizing amplifier generally designated 34 is employed. This is a high gain operational type of amplifier having a signal input resistor 35 and a variable feedback resistor 36 to set the full scale output.

The input wave 37 may be thought of as made up of a symmetrical component having equal positive and negative excursions, and an unbalance component. The output wave 38 will have nearly equal positive and negative excursions, whose unbalance can be made as small as desired by selecting suitable resistors in the feedback network. The positive and negative excursions of the output wave are rectified by diodes 39 and 41 to give D.C. voltages whose difference is the unbalance component of the output wave. These are fed back to the

summing junction of the operational amplifier through resistors 42 and 43, which are assumed to be equal. The net gain of the amplifier for the unbalance component is then the ratio of resistors 42 to 35. The net gain for the symmetrical component is the ratio of resistors 36 to 35. Hence the symmetrizing amplifier suppresses the unbalance component with respect to the symmetrical component by the ratio of resistors 36 to 42. By making resistors 42 and 43 small compared to resistor 35, the unbalance voltage component will be fed back strongly, and the system will come to equilibrium with an unbalance component which may be made as small as desired by decreasing resistors 42 and 43.

The symmetrized signal now contains a D.C. component equal to the product of X and Y which may be extracted by passing the signal through the Low Pass Filter which eliminates the carrier frequency components and gives a pure D.C. output.

The accuracy of this system may be greatly increased by applying negative feedback around the Y channel to reduce non-linearity introduced in modulator 33. For this purpose, the voltages appearing at diodes 39 and 41 each have the proper magnitude proportional to Y, but the proper one must be selected depending on whether Y is positive or negative. This selection is accomplished by the diode gating network 44, 45, 46, and 47 in cooperation with the balanced modulator generally designated 48 which is driven on one side thereof by amplifier tube T7 which, in turn, is driven through condenser 49 from the output of amplifier 34.

The other side of modulator 48 is driven by the X channel signal appearing at the cathode of tube T4, primary winding 51 providing the D.C. return path therefor. Thus, when the phases of all voltages are as indicated for X and Y, both being positive, and the polarities of the transformer windings are as indicated by the dots, the two sides of modulator 48 will be in phase, and a positive voltage will be fed to the junction of resistors 52 and 53. This positive voltage will cause current to flow through resistor 52 and diode 45 to ground, so that the junction of diodes 44 and 45 is held near ground and the upper side of diode 44 cannot go negative with respect to ground although rectifier 41 can produce only negative voltages. Resistor 54 insures that this clamping will not overload rectifier 41. Diodes 46 and 47 are back biased by the positive voltage from modulator 48 with the result that voltage from rectifier 39 causes feedback current to flow into potentiometer 14 by way of resistors 55 and 56. Resistor 57 serves to prevent the feedback current through resistor 56 from being shorted to ground through clamping diode 44.

When Y is negative, diodes 46 and 47 perform the clamping function, and the rest of the circuit operates analogously to the positive case.

Balancing potentiometers are provided in the center portions of the transformers in both balanced modulators. These compensate the differences in the resistances of the individual diodes, as is standard practice with such modulators.

Although negative feedback is very useful in reducing non-linearity in the Y channel, it may be ineffective at very small values of Y. A diode network such as shown by 58, 59 and resistor 60 helps to linearize at small signals by compensating the natural non-linearities of the modulator, and its feedback network.

An alternative circuit arrangement for the X channel system is disclosed in FIG. 8 wherein the multivibrator comprising tubes T1 and T2 as disclosed in FIG. 7 has the input thereof modified so that an asymmetrical X square wave dependent on X is produced in the output thereof and appears at the cathode of T3 in the same manner as occurs at the cathode of T4 in FIG. 7. This is accomplished by returning grid leak 31 of T2 to ground, with the result that when X is grounded, the multivibrator operates at 100 kc. as in the case of the circuit of FIG. 7. When X increases positively, the por-

tion of the square wave produced by T1 decreases with the accompanying increase in frequency. Similarly, when X increases negatively, this portion of the square wave increases with the decrease in frequency, the portion of the square wave produced by T2 remaining constant in each case. Except as aforescribed, the operation is the same as in FIG. 7.

An alternative circuit arrangement for the Y Channel Pulse Height Modulation System is disclosed in FIG. 9, wherein the asymmetrical X wave is applied to both input grids of a pair of tubes T8 and T9 through a transformer 71, and the Y input is applied across a center tapped resistor 72 in series with the X wave input. The results of this arrangement is that the grids swing in the same direction in response to the X wave which thus cancels out across output resistors 73 and 74 when the Y input across center tapped resistor 72 is zero.

Whereas variable D.C. voltage inputs to the electronic multiplier have been disclosed herein, it will be understood by those skilled in the art that any physical variables may readily be multiplied in accordance with the principles of the present invention merely by converting the same to voltage inputs.

Obviously many modifications and variations of the present invention are possible in the light of the above teachings. It is therefore to be understood that within the scope of the appended claims the invention may be practiced otherwise than as specifically described.

What is claimed and desired to be secured by Letters Patent of the United States is:

1. In an electronic analog multiplier, the combination of a pulse width modulation system responsive to an X variable input of either sign for generating a square wave and introducing time asymmetries therein proportional to the magnitude of X and indicative of the sign of X; and a pulse height modulation system comprising a balanced modulator responsive to said square wave and to a Y variable input of either sign for amplitude modulating said square wave in proportion to the magnitude of Y and with or without phase inversion selectively depending on the sign of Y.

2. In an electronic analog multiplier, the combination of a pulse width modulation system responsive to an X variable input of either sign for generating a square wave and introducing time asymmetries therein proportional to the magnitude of X and indicative of the sign of X, a pulse height modulation system comprising a balanced modulator responsive to said square wave and to a Y variable input of either sign for amplitude modulating said square wave in proportion to the magnitude of Y and with or without phase inversion selectively depending on the sign of Y, and means for symmetrizing said amplitude modulated square wave so that the average value thereof may be taken as a measure of the algebraic product of X and Y.

3. An electronic analog multiplier comprising, in combination, a pulse width modulation system responsive to an X variable input of either sign for generating a square wave and introducing asymmetries therein proportional to the magnitude of X and indicative of the sign of X, a pulse height modulation system comprising a balanced modulator responsive to said square wave and to a Y variable input of either sign for amplitude modulating said square wave in proportion to the magnitude of Y and with or without phase inversion selectively depending on the sign of Y, means for symmetrizing said amplitude modulated square wave, and means for extracting the D.-C. component of the symmetrized square wave to provide a directly proportional measure of the algebraic product of X and Y.

4. In an electronic analog multiplier, the combination of means for generating a series of pulses of constant width, means responsive to an X variable input of either sign for increasing or decreasing the spacing between said pulses relative to the width thereof in proportion to the magnitude of X and selectively in accordance with the

sign of X, and balanced modulator means responsive to said pulses and to a Y variable input of either sign for amplitude modulating said pulses in proportion to the magnitude of Y and with or without phase inversion selectively depending on the sign of Y.

5. In an analog multiplier, the combination of means for generating a square wave, means for holding the time of one half cycle of said square wave constant, means responsive to an X variable input of either sign for increasing or decreasing the total period of said square wave in proportion to the magnitude of X and selectively according to whether the sign of X is negative or positive, and balanced modulator means responsive to said square wave and to a Y variable input of either sign for amplitude modulating said square wave in proportion to the magnitude of Y and selectively with or without phase inversion depending on whether the sign of Y is negative or positive.

6. In an electronic multiplier, the combination of means for generating a square wave, means responsive to an X variable input of either sign for decreasing the width of one half cycle of said square wave relative to the alternate half cycle thereof in proportion to the magnitude of X when X is positive and for increasing the width of said one half cycle relative to said alternate half cycle in proportion to the magnitude of X when X is negative, and balanced modulator means responsive to said square wave and to a Y variable input of either sign for amplitude modulating said square wave in proportion to the magnitude of Y without phase inversion when the sign of Y is positive and with phase inversion when the sign of Y is negative whereby the average value of said amplitude modulated square wave is directly proportional to the product of X and Y and has the proper sign for any values of X and Y in all four quadrants.

7. In an electronic multiplier, the combination of means for generating a square wave, means responsive to an X variable input of either sign for varying the relative widths of alternate half cycles of said square wave in proportion to changes in the magnitude of X, means for holding the width of one of said half cycles constant at the equal width of the other corresponding to X equals zero such that the average value of said square wave is zero when X is zero and changes sign when X changes sign, and balanced modulator means responsive to said square wave and to a Y variable input for amplitude modulating said square wave in proportion to the magnitude of Y and with or without phase inversion selectively depending on the sign of Y.

8. In an electronic analog multiplier, the combination of an X channel pulse width modulation system responsive to an X variable input of either sign for generating a square wave and introducing time asymmetries therein proportional to the magnitude of X and indicative of the sign thereof, an X channel feedback network responsive to said square wave for deriving a negative feedback signal from said asymmetries introduced therein, a Y channel pulse height modulation system comprising a balanced modulator responsive to said square wave and to a Y variable input of either sign for amplitude modulating said square wave in proportion to the magnitude of Y with or without phase inversion selectively depending on the sign of Y, and a Y channel feedback network responsive to the input and output square waves of said Y channel modulation system for deriving a negative feedback signal proportional to Y and having the same sign thereof.

9. In an electronic analog multiplier, the combination of an X channel pulse width modulation system responsive to an X variable input of either sign for generating a square wave and introducing time asymmetries therein proportional to the magnitude of X and indicative of the sign thereof, an X channel feedback network responsive to said square wave for deriving a negative feedback signal from said asymmetries introduced therein, a Y channel pulse height modulation system comprising a

balanced modulator having said square wave as one input thereto and a Y variable input of either sign as the other input thereto, said balanced modulator having a square wave output of the same shape as said input square wave except when Y is zero, said output wave having an amplitude proportional to the amplitude of Y and having a phase which either matches that of said input wave or selectively is 180 degrees out of phase therewith depending on whether Y is positive or negative, symmetrizing means responsive to said output wave for equalizing positive and negative excursions therein, and means responsive to said input wave and to the symmetrized output wave for deriving a Y channel negative feedback signal from either of said positive or negative excursions selectively according to whether Y is positive or negative.

10. The combination in a multiplier as in claim 9 wherein said symmetrizing means comprises an operational amplifier having an input circuit for receiving said output square wave and a plurality of feedback circuits for suppressing the unbalance component of said output wave with respect to the symmetrical component thereof whereby said positive and negative excursions are made substantially equal and differ only by a magnitude of error signal as may be required for feedback.

11. The combination in a multiplier as in claim 9 wherein said symmetrizing means comprises a pair of peak indicating rectifiers having D.C. voltages appearing thereon corresponding to said positive and negative excursions respectively, and said Y channel negative feedback means comprises a diode clamping network for grounding one of said rectifiers and applying the D.C. voltage on the other as said feedback signal selectively in accordance with the sign of Y.

12. The combination in a multiplier as in claim 11 wherein said Y channel feedback means comprises a balanced modulator having said X channel square wave as one input thereto and the symmetrized square wave as the other input thereto whereby a D.C. voltage output is produced thereby having a sign matching that of said Y input, said diode network comprising a first pair of back-to-back connected diodes interconnected between ground and one of said rectifiers and a second pair of back-to-back connected diodes interconnected between ground and the other of said rectifiers, a first pair of resistors connected in series between said rectifiers and having the junction thereof connected to the Y input circuit, a second pair of resistors connected in series between the junctions of said pairs of diodes and having said D.C. output voltage applied to the junction thereof whereby one of said pairs of diodes is grounded and the other is back biased thereby selectively depending on whether said D.C. output voltage is positive or negative.

13. The combination in a multiplier as in claim 9 wherein said balanced modulator has an input circuit for said Y input including a pair of mutually shunted crystal diodes connected in series therein with mutually reversed polarities, said diodes compensating for nonlinearities of the balanced modulator for small values of Y.

14. An electronic analog multiplier comprising, in combination, a balanced modulator having a transformer input for receiving a square wave having a time asymmetry proportional to the magnitude of an X variable and indicative of the sign thereof, said transformer having a secondary circuit including a potentiometer of receiving a Y variable input voltage of either sign, said modulator having an output transformer including a primary circuit having a potentiometer for receiving a Y feedback voltage, a symmetrizing amplifier coupled to said output transformer and driven by the amplitude modulated square wave appearing therein, a second balanced ring modulator having one transformer input for receiving said asymmetrical X wave and a second transformer input coupled to said symmetrizing amplifier and driven by the output signal thereof, said amplifier having a pair of rectifiers for developing D.C. voltages proportional to the positive and negative excursions of said output signal, two pairs of clamping

diodes for respectively clamping said rectifiers to ground, a pair of resistors connected in series between the junctions of said clamping diodes, said second modulator having an output circuit including said pair of resistors, a second pair of resistors connected in series between said rectifiers and connected at the junction thereof to said Y feedback potentiometer, and means connected to the output of said symmetrizing amplifier for extracting the D.C. component from said output signal.

15. In an electronic analog multiplier, the combination of a voltage sensitive multivibrator responsive to an X variable input of either sign, said multivibrator being set so that the frequency thereof increases and decreases from a predetermined frequency as X increases and decreases from zero respectively, a monostable multivibrator arranged to be fired by the output of said voltage sensitive multivibrator and having an inert delay line for setting the metastable state thereof, said delay line interposing a delay corresponding to half the total period at said predetermined frequency whereby the square wave output of the monostable multivibrator has a D.C. component which is zero when X is zero and changes sign when X changes sign, a diode clamp for symmetrically limiting the positive and negative excursions of said square wave, means for extracting said D.C. component, and means for applying said D.C. component as negative feedback to the input of said voltage sensitive multivibrator.

16. The combination in a multiplier as in claim 9 wherein said output wave has an amplitude symmetrical component and an unbalance component and said symmetrizing means comprises, in combination, an operational amplifier having an input and an output, a first resistor connected to said input for applying said output wave thereto, a second resistor interconnecting the input and output of said amplifier for applying negative feedback to the input, said feedback being in the ratio of said first and second resistors, a pair of peak indicating rectifiers and a pair of condensers respectively connected in series therewith across said output, said rectifiers being connected to said output with reversed polarities, a third resistor and a fourth resistor respectively interconnecting said input and the junctions of said series connected rectifiers and condensers for applying as negative feedback to the input that portion of the output signal corresponding to said unbalance component, said third and fourth resistors being equal and the feedback applied thereby being in the ratio of said first and third resistors, said unbalance component being suppressed relative to said symmetrical component in the ratio of said third resistor to said second resistor, said third resistor being small in relation to said second resistor to suppress said unbalance component and thereby equalize said positive and negative excursions.

17. The combination in a multiplier as in claim 9 wherein said output wave has an amplitude symmetrical component and an unbalance component and said symmetrizing means comprises, in combination, an amplifier having an output and an input circuit for receiving said output wave, and a plurality of feedback circuits for said amplifier for separately applying to said input circuit portions of the output signal corresponding respectively to said symmetrical and unbalance components, said feedback circuits individual to said unbalance component having rectifier means therein, said feedback circuits respectively individual to said unbalance and symmetrical components having impedance means in the ratio of desired suppression of said unbalance component relative to said symmetrical component sufficient to equalize said positive and negative excursions.

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