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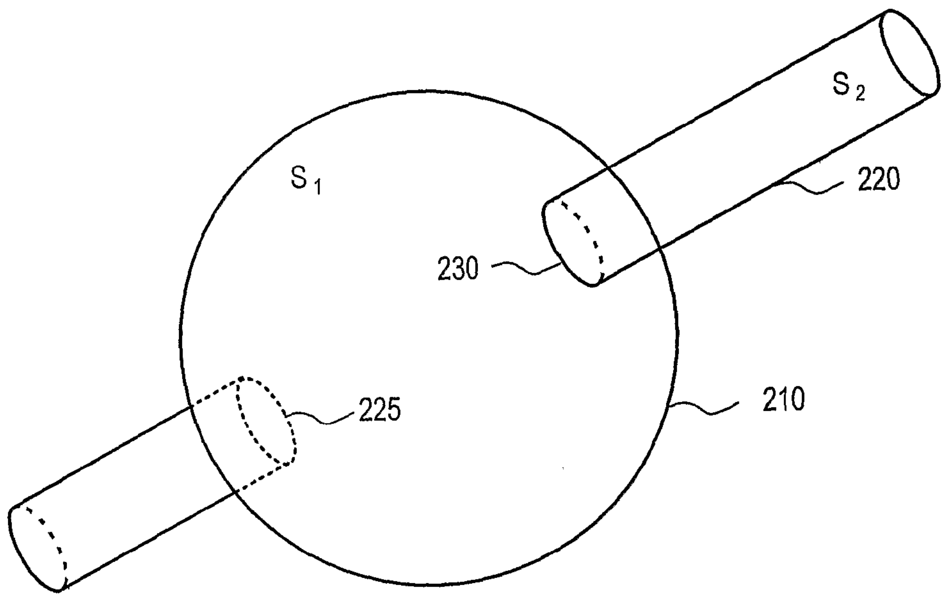
(54) **Title:** REPRESENTING IMPLICIT CURVES OF PROCEDURAL GEOMETRIC SURFACES



(57) **Abstract:** Compact and accurate piecewise parametric representations of implicit curves may be achieved by iteratively selecting ranges of parameterizing regions and testing each for satisfying an intervalized super convergence test. In one aspect, the implicit curves is represented as a compact form of one or more representations of such convergence regions. For memory and bandwidth constrained applications, starting points of convergence regions may not be stored but instead calculated at runtime prior to rendering a point on the implicit curve. Furthermore, not all endpoints relevant convergence regions of a selected implicit curve need be stored. Instead, based on at least one endpoint, the other endpoints can be derived via Newton iterations. To further reduce memory and bandwidth costs, coordinates can be stored in a quantized format and the points reflecting floating point accuracy can be derived at runtime again by Newton iteration.

WO 2007/024638 A1

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REPRESENTING IMPLICIT CURVES OF PROCEDURAL GEOMETRIC SURFACES

TECHNICAL FIELD

The technical field relates to modeling of graphical objects in computers graphics. More particularly, the field relates to modeling implicit curves of intersection in procedural geometric surfaces.

BACKGROUND

Procedural surface representations in computer graphics can be orders of magnitude smaller than polygonal or patch-based surface representations, which may be a desirable feature for memory constrained devices like game consoles and cell phones. Compactness is increasingly important because of some current trends in computer hardware. For instance, core processing speed has been increasing much faster than off chip memory bandwidth, so fetching data from memory is steadily becoming more expensive relative to computation. A compact representation can be faster to render than a larger one, even if much more computation is required to process the compact representation. Because of their compactness, procedural models would seem to be a better fit to this trend than polygon meshes or polynomial surfaces.

Constructive Solid Geometry (CSG) is a method of modeling procedural graphical surfaces, whereby more complex surfaces are modeled as a combination of simpler surfaces. For instance, in computer graphics, CSG methods provide a powerful way for defining surfaces of higher genus (e.g., a sphere with a hole) from surfaces of lower genus, such as a plain cylinder and plain sphere. CSG components, including primitives such as cylinders and spheres, can be described in terms of a procedure or a function that accepts some number of parameters. For instance, a spherical three-dimensional (3D) surface can be defined procedurally in terms of coordinates of its center and a value of its radius. More complex objects can then be modeled through CSG operations performed using such procedural objects. Virtually any manufactured object may be modeled using CSG operations in combination with surfaces of revolution and generalized extrusions, both of which are easily programmed procedurally. Also, the addition of CSG operations to procedural surfaces dramatically increases the scope of objects that can be modeled procedurally.

Defining parametric procedural surfaces of genus 0 (e.g., a sphere) and some surfaces of genus 1 (e.g., a torus) is usually straightforward since complexities, such as holes in the surface domain, are typically absent. However, for surfaces of higher genus, it is much more difficult since domains of some parametric surfaces may have holes whose boundaries are, in some sense, only implicitly defined (e.g., as an intersection of surfaces). These boundaries are difficult to program manually.

For instance, CSG operations may require computing and representing curves of intersection between the two or more surfaces being operated upon. These curves, in general, are defined only implicitly as the solution to an equation of the form $f(x_1,\ldots, x_n) = 0$. These equations are not easy to evaluate and typically require a sophisticated, slow, and computationally costly global zero finding solver. A more compact, exact, and resolution independent representation of such curves on the other hand may be efficiently evaluated at runtime on a graphics processing unit (GPU). Such representation may be a highly desirable feature for memory constrained devices, like game consoles or for bandwidth constrained applications. Also, such representations make it possible to represent high genus surfaces in an entirely procedural way at a significantly reduced memory cost while not significantly increasing the computational costs required for rendering such surfaces to be displayed.

Nevertheless it is desirable to have the opportunity to have a trade-off between reducing memory and bandwidth consumption and increasing computational costs. This will continue to hold true, at least so long as the trend of increasing computation capability of processors continues.

SUMMARY

Described herein are methods and systems for accurately generating compact parametric representations of implicit functions. In one aspect, the implicit functions relate to implicit curves of intersection generated by the way of constructive solid geometric operations involving a plurality of procedural surfaces of graphical objects.

In yet another aspect, the implicit curve is divided into parameterization regions comprising ranges of values of parameterization variables. In one aspect, an intervalized super convergence test comprising applying a Kantorovich condition may be applied to selected parameterization ranges in order to determine whether the dependent variables

associated therewith converge to a solution. Such ranges may be designated as convergence regions and used to generate a compact and accurate representation of the implicit curve.

In a further aspect, the compact representation of the implicit curve can be realized by storing sparse data indicating the convergence regions and sparse data for indicating starting points associated therewith. In one aspect, the sparse data related to starting points comprise data indicating a suitable method for deriving the starting points at runtime. In one aspect, runtime derivation of the starting points might involve retrieving starting point coordinates based on midpoints of the parameterization regions involved. In other aspects, the runtime derivation of the starting points might involve calculating the starting points based on approximations of the implicit curves. The curve approximations may be one of a straight-line approximation between known endpoints or a cubic approximation between the known endpoints of the curve.

In further aspects, the coordinate data of points on the curve, including endpoints of convergence regions or parameterization regions, can be stored in a compact manner. For instance, among a plurality of endpoints related to a plurality of regions describing an implicit curve, coordinates of just one endpoint may be sufficient. The coordinates of the other points can be iteratively calculated at runtime by applying Newton iterations starting with the known endpoint.

In one further aspect, stored coordinate information can be further pruned by quantizing the coordinate data by rounding the coordinates to the nearest integer value. Later, at runtime, the actual coordinates having floating point accuracy can be derived by the way of applying Newton iterations starting from the point defined by the quantized coordinates.

Additional features and advantages will become apparent from the following detailed description of illustrated embodiments, which proceeds with reference to accompanying drawings.

BRIEF DESCRIPTION OF THE FIGURES

FIG. 1 is a flow diagram describing exemplary overall methods for generating parametric representations of implicit functions.

FIG. 2 is a block diagram illustrating exemplary implicit curves of intersection generated by the intersection of two exemplary procedural objects.

FIG. 3 is a block diagram illustrating exemplary partitioning of a domain for determining parameterizing variables.

FIG. 4A is a flow diagram describing exemplary overall methods for generating parametric representations of implicit curves.

FIG. 4B is a flow diagram describing exemplary overall methods for processing parametric representations of implicit curves to render images comprising implicit curves on a computer display.

FIG. 4C is a block diagram describing an exemplary system for generating and processing parameterized representations of implicit curves to render images comprising implicit curves on a computer display.

FIG. 5A is a flow diagram describing exemplary overall methods for generating parametric representations of implicit curve functions comprising convergence regions.

FIG. 5B is a block diagram describing a system for generating parametric representations of implicit curve functions comprising convergence regions.

FIG. 6 is a flow diagram describing exemplary overall methods for determining convergence regions of selected parameterizing variables of an implicit function.

FIG. 7A is a block diagram for illustrating exemplary sub-divisions of exemplary parameterization regions for determining convergence regions therein.

FIG. 7B is a block diagram for illustrating exemplary sub-divisions of exemplary parameterization regions with different starting points identified for each new sub-division for determining the convergence regions.

FIG. 8 is a flow diagram illustrating an exemplary method for determining convergence regions using iteratively applied Newton steps to selected ranges.

FIG. 9 is a block diagram illustrating the use of Newton steps for determining the convergence regions.

FIG. 10 is a listing describing an algorithm for determining convergence regions based on applying a super convergence test to iteratively sub-divided portions of one or more parameterization regions.

FIG. 11 is a listing describing an exemplary algorithm comprising a super convergence test iteratively applied to determine convergence regions.

FIG. 12 is a flow diagram describing an exemplary method for determining sparse parameterized forms of an implicit curve.

FIG. 13 is a block diagram illustrating a plurality of options for calculating starting points associated with convergence regions.

FIG. 14 is a block diagram illustrating starting points associated with convergence regions calculated based on a straight line approximation of the implicit curve.

FIG. 15 is a block diagram illustrating starting points associated with convergence regions calculated based on a cubic approximation of the implicit curve.

FIG. 16 is a block diagram illustrating advantages and disadvantages of selecting one or more the different types of starting points associated with the convergence regions.

FIG. 17 is a flow diagram describing an overall method for determining an appropriate sparse form of representing starting point data.

FIG. 18 is a block diagram illustrating a method of determining a sparse representation of convergence regions by representing just one of the associated endpoints.

FIG. 19 is a block diagram illustrating a method of determining a sparse representation of coordinates of points related to an implicit curve by quantizing the coordinate values.

FIG. 20 is a diagram depicting a general-purpose computing device constituting an exemplary system for implementing the disclosed technology.

DETAILED DESCRIPTION

Exemplary overall methods for generating representations of implicit functions

FIG. 1 illustrates an exemplary method for generating representations of functions that are defined implicitly. According to this exemplary method, at 110, definitions of one or more implicit functions are received and at 120, one or more parametric representations corresponding to the one or more implicit functions are generated. Among other things, such parametric representations allow for expressing implicit functions in an efficient manner. Alternative implementations of implicit function representation generating methods can include fewer or more operations.

Exemplary implicit curves of intersection

FIG. 2 illustrates an exemplary implicit function representation. As shown in FIG. 2, an exemplary sphere surface S_1 at 210 intersects an exemplary cylinder surface S_2 at 220 to form curves of intersection at 225 and 230. The function defining the curves 225 and 230 can be defined implicitly as an intersection of the sphere surface S_1 210 and the cylinder surface S_2 at 220. For instance, if the sphere surface S_1 210 is a function f_1 (u_0, u_1) and the cylinder 220 is represented by some function f_2 (u_2, u_3), then the intersection of the two surfaces 225 and 230 are implicitly represented as some function $F = f_1 (u_0, v_0) - f_2 (u_1, v_1) = 0$. Thus, the primitives S_1 at 210 and S_2 at 220 are defined procedurally in terms of domain variables (u_0, v_0) and (u_1, v_1), respectively. As a result, the function $F(u_0, v_0, u_1, v_1)$ can also be defined procedurally. Once such curves are defined procedurally they can be manipulated much more easily by a Graphics Processing Unit (GPU) for accomplishing animation, for instance. Identifying the intersection and representing the intersection are typically computationally expensive, however, and may also consume large amounts of memory for storage. However, it is possible to generate parametric representations of such implicit curves that are exact, compact, resolution independent and easily evaluated at runtime in a procedural way.

Exemplary parametric representations

Given the procedural function $F(u_0, v_0, u_1, v_1)$ as described above where $F(\underline{x})$ is such that $(\underline{x})$ is a range of values of the 4-dimensional (4D) domain variables (u_0, v_0, u_1, v_1) and suppose there is a function $FP\ (\overline{x}_p) = \overline{x}_d$ such that $\overline{x}_p$ and $\overline{x}_d$ partition the domain $\underline{x}$ into parameterizing variables (also known as independent variables) and dependent variables, respectively. If the $F(\underline{x})$ is such that $F : R^m \longrightarrow R^n$, then *m-n* may denote the number of parameterizing variables in a transformation. Thus, the function $FP\ (\overline{x}_p)$ allows for the dependent variables $\overline{x}_d$ to be expressed in terms of the independent or parameterizing variables $\overline{x}_p$. The actual number of *m* variables compared to *n* variables in a function $F : R^m \longrightarrow R^n$ can vary. For instance, a *4* ⟶ *3* transformation is one typical transformation in CSG. The ability to parameterize an implicit function has the immediate advantage of reducing the memory needed to store a representation of such a function, since values of dependent variables can be derived from values of the independent or parameterizing variables.

Exemplary methods for determining parameterization

It may not be always possible to parameterize an implicit function. More particularly, not every variable of an implicit function can be used in an expression as a parameterizing variable to express other variables of the function. One way to prove the possibility of a parameterization by any of the domain variables of an implicit function is to apply a simple form of the implicit function theorem to determine which of the various dependent derivative matrices of such a function approach non-singularity. For instance, in the example above, wherein F is such that $F: R^4 \longrightarrow R^3$ with 4 domain variables and 3 range variables and $F(u_0, v_0, u_1, v_1) = F(f_x, f_y, f_z)$ will yield a derivative matrix as follows:

$$\begin{matrix} \frac{\partial f_x}{\partial u_0} & \frac{\partial f_x}{\partial v_0} & \frac{\partial f_x}{\partial u_1} & \frac{\partial f_x}{\partial v_1} \\ \frac{\partial f_y}{\partial u_0} & \frac{\partial f_y}{\partial v_0} & \frac{\partial f_y}{\partial u_1} & \frac{\partial f_y}{\partial v_1} \\ \frac{\partial f_z}{\partial u_0} & \frac{\partial f_z}{\partial v_0} & \frac{\partial f_z}{\partial u_1} & \frac{\partial f_z}{\partial v_1} \end{matrix}$$

Suppose a dependent derivative matrix with u_0 as the independent variable is as follows:

$$\begin{matrix} \frac{\partial f_x}{\partial v_0} & \frac{\partial f_x}{\partial u_1} & \frac{\partial f_x}{\partial v_1} \\ \frac{\partial f_y}{\partial v_0} & \frac{\partial f_y}{\partial u_1} & \frac{\partial f_y}{\partial v_1} \\ \frac{\partial f_z}{\partial v_0} & \frac{\partial f_z}{\partial u_1} & \frac{\partial f_z}{\partial v_1} \end{matrix}$$

According to a simple form of the implicit function theorem, if the above dependent derivative matrix is non-singular such that no one column of the matrix can be expressed as a weighted sum of the others, then the implicit function can be expressed in a parametric form with u_0 in this instance as the parameterizing variable. Dependent derivative matrices based on the other variables also may be non-singular, and thus, indicate the possibility of parameterization of the function based on these other variables such that these other variables also may serve as parameterizing variables. However, there may be advantages in selecting one variable as the parameterzing variable over others to parameterize different parts of a curve. As shown in FIG. 3, for instance, within a 2D domain, such as (u_0, v_0), parameterization regions, can be selected based on non-singularity property of the relevant dependent derivative matrices. For instance, in FIG. 3, at 310 (P_1), u_0 is chosen as the parameterizing variable, whereas at 320 (P_2) v_0 is chosen. This is at least partially dependent on the fact that within the region 310 (P_1) every value of the parameterizing or independent variable u_0 is guaranteed to have just one corresponding value of the dependent variable v_0. The same principle can confirm the choice of v_0 as the parameterizing variable within the P_2 region. The same holds true for regions P_3 and P_4 having parameterizing variable choices of u_0 and v_0, respectively. However, if the whole curve in FIG. 3 was chosen as a single parameterizing region, no one variable u_0 or v_0 is guaranteed to have each of their values on the curve correspond to a single value of the other variable that is dependent. As a result, the curve of FIG. 3 can be divided into exemplary parameterizing regions P_1 ,P_2, P_3, and P_4. The 2D parameterization regions shown in FIG. 3 are merely illustrative. The same principle holds true for domains with higher dimensionalities, as well.

Exemplary overall methods for representing implicit curves

Thus, based on determining the possibility of parameterization of implicit curve functions and selecting parameterization regions and their corresponding parameterizing variables, as described above, a parameterized representation of an implicit curve can be calculated. FIG. 4A illustrates one such exemplary overall method for calculating parameterized representations of an implicit curve. As shown in FIG. 4A, at 410, a function defining an implicit curve is received, and at 420, a parameterized representation of the implicit curve is generated based on parameterizing the implicit curve function. Alternative implementations of this exemplary method may optionally omit the operations 410 and 420 or include additional operations.

Exemplary methods for processing parameterized implicit curve representations

FIG. 4B illustrates exemplary methods for processing parameterized implicit curve representations by a GPU, for instance. According to this method, at 425, the GPU receives a parameterized representation of an implicit curve, generated, for instance, as shown in FIG. 4A. At 430, the GPU processes the parameterized representations of the implicit curve to render an image comprising the implicit curves described by the parameterized representation thereof. Alternative implementations of this exemplary method may optionally omit the operations 425 and 430 or include additional operations.

Among other things, a compact representation of implicit curves reduces the memory bandwidth needed to transfer data related to the procedural CSG surfaces comprising the implicit curves based on which, the GPU renders images of the surface comprising the implicit curves. Thus, compact representations of implicit curves allow the GPU to process more image data at runtime to render a more rich set of images on a computer display.

An exemplary system for generating and processing parameterized implicit curve representations

FIG. 4C illustrates an exemplary system 400 for generating and processing a compact parameterized representation of an implicit curve. The system comprises an implicit curve representation generation processor 440, which receives data related to procedural surface representations of one or more graphical objects 435 and generates data 450 related to one or more parameterized representations of implicit curves formed based on the procedural representations of one or more graphical objects 435. For instance, the implicit curve representation generation processor 440 can use the methods of FIG. 4A to generate the data 450 related to one or more parameterized representations of implicit curves.

The image rendering processor 460 (e.g., a GPU) receives the data 450 related to one or more parameterized representations of implicit curves and processes the data 450 to generate image data 470 for displaying images comprising the implicit curves on the computer display 480. For instance, the image rendering processor 460 may be a GPU programmed for transforming the parameterized representations of the implicit curves through tessellation methods for generating polygon-based representations of implicit curves for displaying the implicit curves along with other image components on a computer screen 480.

Exemplary methods for generating parameterized representations of implicit curves comprising determining convergence regions

FIG. 5A illustrates a more detailed description of methods of calculating a parameterized representation of an implicit curve (e.g., 225 and 230 of FIG. 2). For instance, the implicit curve representation generation processor 440 in FIG. 4C can be programmed to implement the methods described below. According to this method, at 510, a function defining a first object (e.g., S_1 at 210) is received and at 520, a function defining a second object (e.g., S_2 at 220) is received. Then at 530, for an implicit curve function defined at least partially based on some operation on the functions of the first and the second objects (e.g., $f_1 (u_0, u_1) - f_2 (u_2, u_3) = 0$ defining a curve of intersection of procedural surfaces S_1 and S_2) parameterizing regions are first determined (e.g., as shown above with reference to FIG. 3).

The implicit function theorem relying on the non-singularity property of the appropriate dependent derivative matrices confirms that parameterization of dependent variables using the parameterizing variables is possible. Thus, solving for a range of values of the parameterizing variables over which parameterization of the dependent variables is possible based on the non-singularity of the relevant dependent derivative matrices yields parameterization regions. However, further calculations are needed to solve for dependent variable values that yield $f_1 (u_0, u_1) - f_2 (u_2, u_3) = 0$ for determining the implicit curve described above over a range of the parameterizing variable. Such a range of the parameterizing variable may be referred to as a convergence region.

For instance, if we have a parameterization region u_i such that there is some function $g(u_i) = u_d$ wherein u_d is a 3 vector of functions that define the dependent variables in terms of scalar parameterizing variable u_i. In that case, $g(u_i)$ would be a parametric form of some implicit curve function, such as $F_{csg}(u_0, v_0, u_1, v_1) = f_1(u_0, v_0) - f_2(u_1, v_1) = 0$. Suppose one can use the notation $u = [u_{ic}, u_d]$ to indicate a 4 vector point consisting of the scalar u_{ic}, then $F_{csg}(u_{ic}, u_{dc}) = 0$ can represent the implicit curve of intersection.

In that case, given the parameterizing region $\overline{u_{ic}}$, in order to solve for a point on the curve corresponding to the parameter value u_{ic}, such that $F_{csg}(u_{ic}, u_{dc}) = 0$, we need to find the unique 3 vector u_{dc} that satisfies $F_{csg}(u_{ic}, u_{dc}) = 0$. To do this, one could use Interval Newton to solve this equation, but it is orders of magnitude too slow for real time use. Conventional Newton iteration is faster and simpler, but in order to use it, two questions need answers: 1) what starting point, $[u_{ic}, u_s]$, should we use for a given u_{ic} and 2) for what range of values of the parameterzing variable $\overline{u_{ic}}$ can the Newton iteration be guaranteed to converge, starting from $[u_{ic} \in \overline{u_{ic}}, u_s]$ to the correct point on the curve?

Based on these answers, the parametric function $g(u_i)$ for one parametric region $\overline{u_i}$ can be constructed by partitioning $\overline{u_i}$ into intervals of guaranteed convergence $\overline{cr_k}$, each of which has an associated starting point u_s. To compute a point (u_{ic}, u_d) on the curve first, find such that $\overline{cr_k} | u_{ic} \in \overline{cr_k}$ so that scalar value u_{ic} is within the convergence region. Then, use the 4 vector $[u_{ic}, u_s]$ as the starting point for the Newton iteration as follows:

$$U_{dj+1} = U_{dj} - D^{-1}\left(f\left(u_{ic}, u_{dj}\right)\right) f\left(u_{ic}, u_{dj}\right)$$

On the average, three to four iterations may be enough. This Newton iteration is evaluated at runtime to compute points on the implicit curve based on definitions of convergence regions and associated starting points.

In one example, the entire parameterization region may also be the convergence region. Alternatively, several convergence regions and several starting points corresponding to these convergence regions may comprise a parameterization region. For an efficient and compact representation of an implicit curve, the lesser the data defining the convergence regions and their associated starting points, the better it is. As shown, at 540 in FIG. 5, one or more convergence regions are determined for at least one of the parameterization regions. Upon which, at 550, a parameterized representation of the implicit curve function is expressed in form of data representative of the parameterization regions, convergence regions therein, and associated starting points.

As noted above, an implicit curve representation generation processor 440 in FIG. 4C can be programmed to implement the methods described with reference to FIG. 5A for generating parameterized representation of implicit curves. For instance, the implicit curve representation generation processor 440 is programmed to execute the instructions of an exemplary implicit curve representation generator module 560 in FIG. 5B for processing procedural graphical objects from a modeling program 555. Such an implicit curve representation generator module 560 comprises a parameterization region generator 562 for generating parameterizing regions and a convergence region generator 565 for determining convergence regions from the parameterizing regions for representing implicit curves. The parameterized representations of implicit curves are stored as parameterized image data 570, which can be processed by an image rendering program 580 to display the images on a computer display.

Exemplary methods for determining convergence regions

As noted above with respect to FIG. 5A, once parameterization regions are determined, ranges of values of parameterizing variables within the parameterization region are determined, which yield values of the dependent variables that converge to a single solution (e.g., 540 and 550). FIG. 6 illustrates one overall method for determining such convergence regions. At 610, a definition of a parameterization region is received. Then at 620, starting with the entire parameterization region and with one starting point chosen (e.g., middle of the range of values of the parameterization region) from within the parameterization region, an intervalized super convergence test is conducted to determine whether the entire parameterization region converges.

An intervalized super convergence test comprises an interval extension to the simple single value form of the implicit function theorem. In general, an intervalized super convergence test is applied over an interval of values of a parametrizing variable, as opposed to a single value of the same.

Suppose, for instance, that the implicit function $F(u_0, v_0, u_1, v_1) = F(\underline{\bar{x}}) = 0$ defines an implicit curve of intersection and there is a function $FP\ (\bar{x}_p) = \bar{x}_d$ such that $\bar{x}_p$ and $\bar{x}_d$ partition the domain $\underline{\bar{x}}$ into parameterizing variables (also known as independent variables) and dependent variables, respectively. Then taking a starting point on the curve $\underline{\bar{x}}_o=(\underline{\bar{x}}_p, x_d)$ such that x_d is a starting point value of dependent variables and $\underline{\bar{x}}_p$ is a range of parameterizing variables if Kantorovich's theorem holds true for the starting point x_d, then the Newton iteration from the starting point will converge to a solution of the implicit function for the values of the parameterizing variable within the range $\underline{\bar{x}}_p$. For instance, Kantorovich's theorem in this example may be stated as follows:

> If $|f(\bar{\underline{x}}_o)|\ |\ D^{-1}f(\bar{\underline{x}}_o)\ |^2\ m < ½$; where m is provided by the Lipschitz given by $|\ Df(u_0) - Df(v_0)\ | \leq m * |u_0 - v_0$ for all u_0, and u_1 in U, then a Newton Iteration stated as $x_{i+1} = x_i + |Df(x_i)|\, f(x_i)$ super converges to a unique solution.

Also, the condition If $|f(\bar{\underline{x}}_o)|\ |\ D^{-1}f(\bar{\underline{x}}_o)\ |^2\ m < ½$ herein is referred to as Kantorovich condition. If Kantorovich condition equals to ½, that suggests a simple convergence. On the other hand, super convergence is reached if the same expression yields value < ½. Thus, as shown above, an intervalized super convergence test applies the Kantorovich condition to a range of values.

Nevertheless, a conventional application of Kantorovich's theorem to a range of values yields convergence regions that are very small. This may be so because, in general, Kantorovich's theorem is very pessimistic. Also, the Kantorovich condition is not a necessary condition, it is however a sufficient condition. Thus, if in a first iteration of the application of the super convergence test the Kantorovich condition does not hold true, then as shown at 630 in FIG. 6, the super convergence test, including the Kantorovich condition, can be applied iteratively based on new intervals of the parameterizing variables and new starting points or even the same starting point for that matter.

In one embodiment, new intervals are generated by sub-dividing the parameterizing variable intervals of previous iterations until some maximum number of iterations or until one sub-division of the intervals converges. Also, in one embodiment such sub-dividing may be by half.

Also, in one embodiment, new starting points on the curve associated with the new intervals can be selected by identifying a middle point of the new interval of the parameterizing variable and the corresponding values of the dependent variables. The sub-dividing and identification of new starting points is continued until one combination of a range of parameterizing variables and an associated starting point therein is determined that meets the super convergence test.

FIG. 7A illustrates one exemplary sub-division of the parameterization region that can lead to a convergence region. For instance, the entire parameterization region C_1, at 710, is tested for super convergence, and if the test is not satisfied, the region C_1 is sub-divided further to regions C_2 720 and C_3 730 and each such region is further tested again for super convergence. Furthermore, as shown in FIG. 7B, newer starting points at x_{s2} 750 and x_{s3} 740 are calculated to conduct further testing for super convergence. These processes can be repeated until determining one combination of intervals that meet the super convergence test or until some maximum number of iterations.

An accelerated super convergence test for determining convergence regions

FIG. 8 illustrates an exemplary method for determining convergence regions comprising a technique for accelerating the process of determining convergence regions with selected sub-divisions of the parameterization regions. At 810, super convergence test is applied to intervals within the parameterization regions. At 820, the super convergence test is re-applied to new intervals calculated, at least partially, based on intervalized Newton iterates of a previous iteration. The choice of Newton iterates further ensures that the range being evaluated is more likely to converge than a guess based on just simple sub-divisions of the parameterization regions and selecting middle points of sub-divisions as the starting points. Then, at 830, the process is continued for a maximum number of iterations or until super convergence test is met for some region being tested.

FIG. 9 illustrates the progressive Newton iterate steps, such as h_0 at 915, to determine new intervals with new starting points by starting at some point a_0 910 and progressing until identifying regions meeting the super convergence test. Newton iterate steps (e.g., h_0) are provided by Newton iteration expression stated as follows:

$$\overline{\underline{x_{i+1}}}=\overline{\underline{x_i}}-\overline{\underline{h_i}}$$
$$h_i=\left[D_{ui}\left(f\left(c,\overline{\underline{x_i}}\right)\right)\right]^{-1} f\left(c,\overline{\underline{x_i}}\right)$$

The regions that satisfy the convergence test are then used to represent a parameterized form of an implicit curve.

Exemplary algorithms for determining convergence regions

FIGs. 10 and 11 illustrate an exemplary algorithm for determining convergence regions by iteratively sub-dividing ranges of values of parameterization region and applying the super convergence test, as described with reference to FIGs. 6, 7, and 8. The implicit curve representation generation processor 440 of FIG. 4C of the system 400 for processing graphical data can be programmed to implement the methods described with reference to FIGs. 10 and 11 below. FIGs. 10 and 11 together illustrate two methods that are recursively called for a selected number of iterations or until a convergence region that satisfies the super convergence test is identified.

In one embodiment, as shown in FIG. 10, at 1010 the ConvergenceInRegion($\bar{\underline{x}}_o$) method is called for a selected range of values $\bar{\underline{x}}_o$ on the implicit curve for which convergence is being determined. For instance, the range of values $\bar{\underline{x}}_o$ may be expressed as $(\bar{\underline{x}}_p, x_d)$, which comprises a range of values of the parameterizing variables and a corresponding starting set of values for the dependent variables (e.g., those associated with a midpoint of $\bar{\underline{x}}_o$). Then at 1020, the recursive test for super convergence, which is referred to in this example as convergence($\bar{\underline{x}}_o$) method (1100 in FIG. 11) is called. If at 1020, the convergence($\bar{\underline{x}}_o$) (1100 at FIG. 11) method returns 'true', then the range of values $\bar{\underline{x}}_p$ identified as the range of parameterizing values associated with $\bar{\underline{x}}_o$ is determined to be the convergence region with x_d as the starting point. A typical initial value for $\bar{\underline{x}}_o = (\bar{\underline{x}}_p, x_d)$ may be one where $\bar{\underline{x}}_p$ is the entire parametrization region and x_d is associated with the midpoint of such a region. However, other ranges of values of $\bar{\underline{x}}_p$ and their corresponding starting values of dependent variables x_d may also be used. If at 1020, convergence($\bar{\underline{x}}_o$) (1100) method returns false, then ranges of parameterizing variables $\bar{\underline{x}}_p$ are iteratively sub-divided at 1030 and new starting points calculated for each region (e.g., $\bar{\underline{x}}_{pLow}$ and $\bar{\underline{x}}_{pHigh}$), at 1040, upon which, ConvergenceInRegion($\bar{\underline{x}}_o$) method is recursively called at 1050 for a selected number of times or until the convergence regions are determined.

FIG. 11 illustrates an exemplary super convergence test for determining convergence regions. In one embodiment, the super convergence test comprises applying the Kantorovich condition iteratively to selected ranges of values on an implicit curve, wherein the range may be changed iteratively by a Newton iterate step. Thus, as shown in the exemplary super convergence test convergence($\bar{\underline{x}}_o$), at 1110, while ($\bar{\underline{x}}_i$ are shrinking) is true, Newton iterate steps $\bar{\underline{h}}_i$ may be used to change the potential convergence region being tested and each new range is evaluated for compliance with the Kantorovich condition at 1120, and if true, the range of parameterizing variable values associated with the range $\bar{\underline{x}}_i$ changed by the Newton step (e.g., $\bar{\underline{h}}_i$) is identified as the convergence region. If the Kantorovich condition 1120 does not hold true, then the range of values of the parameterizing variables $\bar{\underline{x}}_p$ is sub-divided further at 1130 and the convergence() method 1100 is called on the sub-divisions recursively at 1140 for each of the sub-divisions with the same starting point x_d. This iteration can be continued up to a point where the convergence region is too small and returned back to the process of FIG. 10 to start with a new starting point.

The "while ($\bar{\underline{x}}_i$ are shrinking)" condition at 1110 allows for Newton steps to be taken until some point when the steps become so small that any rounding errors related to the floating point calculation would actually start to grow the $\bar{\underline{x}}_i$ range instead of shrinking it. The "while ($\bar{\underline{x}}_i$ are shrinking)", however, is an exemplary condition. Other conditions may be used to determine how many Newton steps are to be taken and in which direction.

In one alternative, the sub-divisions of ranges within parameterization regions which are called within methods the ConvergenceInRegion( ) 1000 and convergence( ) 1100 may be ½ divisions of such ranges determined during previous iterations. However, such sub-divisions need not be restricted to dividing the previous ranges by half. Other divisions (e.g., ¼) may be used.

Exemplary convergence data

Once convergence regions are determined, data descriptive of such regions are stored for later use. There are costs associated with storing and transferring such data descriptive of the convergence regions including such as data indicative of the range of values of the parmeterizing variable (e.g., C_2 and C_3 of FIG. 7B) and data indicative of the associated starting points (e.g., X_{s2} and X_{s3} in FIG. 7B). It is based on this data that the actual points on the related implicit curve will be derived at runtime. However, reducing the costs of transferring such data to the processor evaluating points on a curve (e.g., 460 in FIG. 4C) can be advantageous for memory bandwidth constrained applications (e.g., cellular phones).

In addition to memory, related costs, computational costs are also of concern. For instance, if the starting point guessed at the beginning of a convergence analysis, such as the midpoint of parameterization region being analyzed, happens to be far away from the point being evaluated, then it may take numerous Newton iteration steps to derive the point on the curve, even if convergence has been proven in an offline calculation.

FIG. 12 describes one method 1200 for determining convergence data that can be more accurate and sparse at the same time. At 1210, one or more convergence regions and their associated starting points are determined for a selected parameterization region. At

1220, descriptor data for regions of convergence and descriptor data of their associated starting points are stored. The starting point data need not be in terms of domain coordinates (e.g., u_{0s}, v_{0s}, u_{1s}, v_{1s}), which can be costly in terms of memory. Instead, it can be stored in a sparse form during static processing 1225, such that, later during runtime processing 1230 at step 1235, the actual starting point can be calculated, based at least in part on the sparse descriptor, for applying Newton iteration to determine the points on the associated curve.

FIG. 13 illustrates methods of selecting starting points that converge fast and can be represented sparsely in memory, for instance. For an exemplary implicit curve 1300, as shown with respect to FIGs. 7A-B one exemplary starting point 1315 relates to a midpoint 1315 of the parameterization region 1310. Typically, if convergence is proven for the entire parameterization region, then data describing the endpoints EP_1 at 1320 and EP_2 at 1325 are stored along with the coordinates for the midpoint based starting point X_p midpoint at 1315. First of all, the coordinate data consumes memory. Secondly, even if convergence is proven over the parameterization region 1300 from the starting point 1315, it may take an inordinate number of Newton iterations to derive most of the curve points, such as c_1 (e.g., 1305), that are far away from the starting point 1315.

However, approximations of the actual implicit curve 1300, such as a straight line approximation 1340 and a cubic approximation 1330, can yield starting points X_{s1} 1306 and X_{s2} 1307, for instance, which are much closer to the curve point (e.g., c_1 1305) to be determined. Thus, according to method 1200 of FIG. 12, one form of a sparse starting point descriptor data is data describing the selection of one or more approximations (e.g., 1330 and 1340) of the implicit curve, based on which, a starting point on the approximation curve can be calculated at runtime and used to fully derive the point (e.g., c_1 1305) on the curve 1300. Even though this requires additional computation during runtime, the savings in the reduced costs of memory required for storing convergence and transmitting data including starting point data may make this a tradeoff that is worthwhile for at least some graphics applications.

Exemplary implicit curve approximations

FIG. 14 illustrates a straight line approximation 1340 to the implicit curve 1300. The example shows that in a (u_0, v_0) domain, v_0 has appropriately been chosen as the parameterizing variable of an implicit function defining the curve 1300. The straight line approximation 1340 of such a curve is a straight line that is interpolated between the two known endpoints EP_1 at 1320 and EP_2 at 1325. A linear interpolation function can be used to calculate any point between the two known endpoints, EP_1 at 1320 and EP_2 at 1325. Many linear interpolations techniques exist, one such linear interpolation for interpolating between two known points say, exemplary (x_a,y_a) and (x_b,y_b), is given as follows:

$$f(x)=\frac{x-x_b}{x_a-x_b}y_a-\frac{x-x_a}{x_a-x_b}y_b$$

Thus, at runtime, based on known values of the parameterizing variable (e.g., 1341), the starting point X_{s2} at 1307 for converging to a solution of the curve point c_1 1305 can be calculated and applied in a Newton iteration analysis. At least because the starting point X_{s2} at 1307 is closer to the curve point of interest c_1 at 1305, the Newton iteration will converge faster. Also, since the starting point was calculated at runtime, any costs associated with storing and retrieving data related to starting points is avoided.

FIG. 15 illustrates a cubic approximation 1330 for the implicit curve 1300. The cubic approximation is a curved line that is also interpolated between two known points EP_1 at 1320 and EP_2 at 1325 based on the known tangents T_1 at 1521 and T_2 at 1526. Calculating the tangents (e.g., tangents T_1 at 1521 and T_2 at 1526) at runtime is not an additional runtime computational cost since it is part of any Newton iteration analysis (e.g., FIGs. 10 and 11). The curve approximation 1330 in one embodiment can be based on a Hermite curve spline that interpolates points between two know endpoints (e.g., EP_1 at 1320 and EP_2 at 1325) and their respective curves. The Hermite form consists of two control points and two control tangents (e.g., T_1 at 1521 and T_2 at 1526), one each for each polynomial. On each subinterval, given a starting point (e.g., EP_1 at 1320) and an ending point (e.g., EP_2 at 1525) with starting tangents (e.g., T_1 at 1521) and ending tangent (e.g., T_2 at 1326), the polynomial can be defined as follows:

$$p(t)=(2t^3-3t^2+1)p_0+(t^3-2t^2+t)m_0+(-2t^3+3t^2)p_1+(t^3-t^2)m_1$$

Thus, based on the cubic curve approximation and a known value 1342 of the parameterizing variable v_0, the starting point 1306 can be calculated at runtime. In this example, choosing to calculate a starting point based on a cubic approximation appears to be the better choice.

However, the choice of how best to calculate and, thereby, represent convergence regions and associated starting points is not always as clear as shown with respect to FIG. 13, for instance. In FIG. 13, the starting point X_{s1} 1306 generated by the way of the cubic approximation 1330 to the implicit curve 1300 appears to be the best overall choice for converging to most points along the curve 1300 including c_1 at 1305. This is so at least because the starting point X_{s1} at 1306 appears to be closest to the curve point c_1 at 1305. Thus, the cubic approximation can be chosen to calculate the starting points even though it appears that at least for some points of the curve 1300 the midpoint based starting point 1315 will be a better choice. Nevertheless, cubic approximation based starting point calculation is the better overall choice.

Among other factors, the shape of the implicit curve (e.g., 1300) affects the choice of how best to calculate and represent starting points associated with convergence regions. FIG. 16 illustrates this reasoning. For example, the curve 1600 can be divided into several parameterization regions. For instance, in this example, with u_0 as the parameterizing variable, it is possible to have a parameterizing region between the endpoints EP_1 at 1610 and EP_2 at 1620. The choice of an appropriate parametering region in this example is merely illustrative and does not consider all possible criteria for how best to divide an implicit curve into parameterizing regions as described with reference to FIG. 3, for instance. Nevertheless, suppose the curve point to be evaluated c_1 is at 1625, the starting point X_{mid} based on the parameterization region midpoint is at 1630, the starting point X_{s1}calculated from the straight line approximation 1635 is at 1632, and the starting point X_{s2} calculated from the cubic approximation 1640 is at 1642. In this example, calculating the starting point (e.g., X_{s2} at 1642) based on the cubic approximation 1640 appears to be a poor choice. However, the starting point X_{mid} at 1630 based on the midpoint of the parameterization region 1600 may be a better choice for at least some points on the curve and finally, calculating a starting point (e.g., X_{s1} at 1632) based on the straight line approximation 1635 appears to be the best choice in this example. This is so not only because it yields a starting point X_{s1} at 1632 closest to the curve point c_1 at 1625, but it also appears to yield the closer starting points for most other points on the curve 1600.

Such analysis related to determining how best to calculate starting points can be performed prior to runtime. Once this analysis is complete, some data indicating the choice of how best to calculate and represent convergence data, including starting points, can be stored so that at runtime, based on this descriptor data, a processor can calculate the starting point, or retrieve it in case of a midpoint (e.g., X_{mid} at 1630), to begin the process of applying Newton iteration to converge to a point on the implicit curve (e.g., 1600).

FIG. 17 illustrates one such method 1700 of static processing of convergence data. At 1710, for a selected parameterizing region an appropriate type of starting point descriptor data including a method of determining starting point data for runtime evaluation of the implicit curve using the Newton iteration is selected. Based on this selection, starting point data descriptors are stored that indicate the selected method of calculating starting point data. In one embodiment, the choices for starting point calculation methods include, retrieving a parameterization region midpoint based starting point, calculating a starting point based on a straight-line approximation of the implicit curve between two known curve points and calculating a starting point based on a cubic approximation of the implicit curve between two known curve points. Once a choice is identified, at 1720, data descriptive of the choice of how starting points descriptor data is stored in conjunction with the data (e.g., endpoints) of the associated convergence regions.

Data indicating a choice may be in form of a bitstream. For instance, for identifying among three different exemplary choices described above, two bits would suffice. In one embodiment, a midpoint based starting point choice is indicated by a value of "00", a straight line approximation based calculation is indicated by a value of "01", and a cubic curve approximation choice is indicated by a value of "10". Thus, convergence region data, including its endpoints, can be transferred to the processor at runtime along with a choice of how its associated starting point is to be determined. If it is "00", then the processor will be operable to retrieve a stored midpoint based starting point, which adds to the cost of memory bandwidth. However, if the choice is indicated as "01" or "10", for instance, then the actual starting point will be calculated at runtime based on the appropriate implicit curve approximation. Here, in this instant case (e.g., with choices of "01" and "10"), the memory bandwidth costs of transferring coordinate data of starting points can be avoided.

Exemplary sparse representations of implicit curve regions

Reducing costs associated with storing, transferring, and retrieving data related to regions of implicit curves, such as parameterization regions and convergence regions, for instance, is beneficial (e.g., for bandwidth constrained applications). As noted above, even with having starting points being calculated at runtime, instead of being stored and later retrieved, the cost of storing and retrieving curve points, such as endpoints (e.g., EP_1 at 1810 and EP_2 at 1820 in FIG. 18), remain. In one exemplary improvement, not every endpoint of the various parameterization regions need be stored and later retrieved for runtime use. For instance, a first selected starting point (e.g., EP_1 at 1810) may be stored and transferred to the appropriate processor. At runtime, the rest of the endpoints (e.g., EP_2 at 1820, EP_3 at 1830 and EP_4 at 1840) can be calculated by the way of applying Newton iterations with the known point (e.g., EP_1 at 1810) as the starting point. However, the entire range W_0 at 1850 may not converge with EP_1 at 1810 as the starting point. For instance, the range may need to be sub-divided further to first find a solution of an intermediate point, such as IP_1 at 1855 and it then may be used as a starting point to solve for EP_2 at 1820, for instance. This process may be continued to until all the required endpoints are calculated. This does add computational complexity at runtime. However, it also substantially reduces the amount of data that needs to be stored and later retrieved at runtime. In fact, in one embodiment, all that needs to be stored is a first endpoint (e.g., EP_1 at 1810) and a range W_0 at 1850 of the parameterizing variable v_0. Based on this information, the rest of the needed points can be derived.

For the rest of the parameterization regions (e.g., P_2, P_3, and P_4 in FIG. 18) even the information regarding which are the parameterizing variables associated with the respective parameterization regions need not be stored and retrieved at runtime. Instead, it can be deciphered from the parameterization region associated with the first endpoint (EP_1 at 1810) because, typically, the parameterizing variables alternate between the domain variables (u_0, v_0) for consecutive parameterization regions.

In one further improvement, costs associated with storing data related to regions of the implicit curve can be further reduced by quantizing the coordinate values of any point of relevance related to the implicit curves. FIG. 19 illustrates this further. For instance, if coordinate points of endpoints, such as EP_1 at 1810 and EP_2 at 1820, are to be stored. They need not be stored in a data structure floating points data type. Instead, they can be quantized to an integer data type. This might result in the stored point EP'_1 at 1910 not being exactly on the curve 1800, but offset a bit due to any rounding needed for quantization. The exact endpoint EP_1 at 1810 on the curve 1800 can be derived at runtime, as needed, by applying Newton iterations with the quantized endpoint EP'_1 at 1910 as the starting point. Again, this does add to the computation cost at runtime. However, rounded integer data types typically require less memory than associated floating point data types, and thus, the quantization as described above can reduce the costs associated with storing and retrieving coordinate data. This trade-off may be worthwhile for some memory constrained applications.

The memory saving features described, with reference to FIG. 18 and FIG. 19 above, can be used in combination with each other and in combination with any of the other methods described herein with respect to deriving and representing implicit curves. For instance, the memory costs for storing information related to parameterization region 1800 in FIG. 18 can be reduced by orders of magnitude, if the entire curve 1800 can be represented in terms of a single endpoint (e.g., EP_1 at 1810) by quantizing the coordinates therewith. The rest of the information can be derived at runtime based on this sparse information. This can present an enormous advantage to applications within environments where bandwidth and memory are not only expensive but, in fact, may be unavailable.

Exemplary computing environment

FIG. 20 and the following discussion are intended to provide a brief, general description of an exemplary computing environment in which the disclosed technology may be implemented. Although not required, the disclosed technology was described in the general context of computer-executable instructions, such as program modules, being executed by a personal computer (PC). Generally, program modules include routines, programs, objects, components, data structures, etc., that perform particular tasks or implement particular abstract data types. Moreover, the disclosed technology may be implemented with other computer system configurations, including hand-held devices, multiprocessor systems, microprocessor-based or programmable consumer electronics, network PCs, minicomputers, mainframe computers, and the like. The disclosed technology may also be practiced in distributed computing environments where tasks are performed by remote processing devices that are linked through a communications network. In a distributed computing environment, program modules may be located in both local and remote memory storage devices.

With reference to FIG. 20, an exemplary system for implementing the disclosed technology includes a general purpose computing device in the form of a conventional PC 2000, including a processing unit 2002, a system memory 2004, and a system bus 2006 that couples various system components including the system memory 2004 to the processing unit 2002. The system bus 2006 may be any of several types of bus structures including a memory bus or memory controller, a peripheral bus, and a local bus using any of a variety of bus architectures. The system memory 2004 includes read only memory (ROM) 2008 and random access memory (RAM) 2010. A basic input/output system (BIOS) 2012, containing the basic routines that help with the transfer of information between elements within the PC 2000, is stored in ROM 2008.

The PC 2000 further includes a hard disk drive 2014 for reading from and writing to a hard disk (not shown), a magnetic disk drive 2016 for reading from or writing to a removable magnetic disk 2017, and an optical disk drive 2018 for reading from or writing to a removable optical disk 2019 (such as a CD-ROM or other optical media). The hard disk drive 2014, magnetic disk drive 2016, and optical disk drive 2018 are connected to the system bus 2006 by a hard disk drive interface 2020, a magnetic disk drive interface 2022, and an optical drive interface 2024, respectively. The drives and their associated computer-readable media provide nonvolatile storage of computer-readable instructions, data structures, program modules, and other data for the PC 2000. Other types of computer-readable media which can store data that is accessible by a PC, such as magnetic cassettes, flash memory cards, digital video disks, CDs, DVDs, RAMs, ROMs, and the like, may also be used in the exemplary operating environment.

A number of program modules may be stored on the hard disk 2014, magnetic disk 2017, optical disk 2019, ROM 2008, or RAM 2010, including an operating system 2030, one or more application programs 2032, other program modules 2034, and program data 2036. A user may enter commands and information into the PC 2000 through input devices such as a keyboard 2040 and pointing device 2042 (such as a mouse). Other input devices (not shown) may include a digital camera, microphone, joystick, game pad, satellite dish, scanner, or the like. These and other input devices are often connected to the processing unit 2002 through a serial port interface 2044 that is coupled to the system bus 2006, but may be connected by other interfaces such as a parallel port, game port, or universal serial bus (USB). A monitor 2046 or other type of display device is also connected to the system bus 2006 via an interface, such as a video adapter 2048. Other peripheral output devices, such as speakers and printers (not shown), may be included.

The PC 2000 may operate in a networked environment using logical connections to one or more remote computers, such as a remote computer 2050. The remote computer 2050 may be another PC, a server, a router, a network PC, or a peer device or other common network node, and typically includes many or all of the elements described above relative to the PC 2000, although only a memory storage device 2052 has been illustrated in FIG. 20. The logical connections depicted in FIG. 20 include a local area network (LAN) 2054 and a wide area network (WAN) 2056. Such networking environments are commonplace in offices, enterprise-wide computer networks, intranets, and the Internet.

When used in a LAN networking environment, the PC 2000 is connected to the LAN 2054 through a network interface 2058. When used in a WAN networking environment, the PC 2000 typically includes a modem 2060 or other means for establishing communications over the WAN 2056, such as the Internet. The modem 2060, which may be internal or external, is connected to the system bus 2006 via the serial port interface 2044. In a networked environment, program modules depicted relative to the personal computer 2000, or portions thereof, may be stored in the remote memory storage device. The network connections shown are exemplary, and other means of establishing a communications link between the computers may be used.

Having described and illustrated the principles of the invention with reference to the illustrated embodiments, it will be recognized that the illustrated embodiments can be modified in arrangement and detail without departing from such principles.

For instance, many of the examples used above describe algorithms for determining implicit curves of intersection of two or more procedural surfaces with known functions. However, the principles described herein may be applied to any implicitly defined functions. For instance, these same principles may be applied to motion in a machine with different functions defining different motions of different parts of the machine, which may need to be synchronized.

Furthermore, many of the examples illustrate functions wherein parametrization functions show a $R^4 \longrightarrow R^3$ domain to range transformation indicating that a single parameterizing variable parameterizes the expression of the dependent variables. However, other transformations are possible. For instance, two parameterizing variables may be chosen to parameterize the expression of the rest of the variables as dependent variables.

In addition to representing implicit curves, the principles described herein may be applied equally effectively to calculate compact piecewise parametric inverses of arbitrary functions and to compute exact efficient representations of arbitrary differentiable functions.

Furthermore, elements of the illustrated embodiment shown in software may be implemented in hardware and vice-versa. Also, the technologies from any example can be combined with the technologies described in any one or more of the other examples. In view of the many possible embodiments to which the principles of the invention may be applied, it should be recognized that the illustrated embodiments are examples of the invention and should not be taken as a limitation on the scope of the invention. For instance, various components of systems and tools described herein may be combined in function and use. We therefore claim as our invention all subject matter that comes within the scope and spirit of these claims.

Also, the alternatives specifically addressed in this sections are merely exemplary and do not constitute all possible alternatives to the embodiments described herein.

CLAIMS

I claim:

1. A computer implemented method for determining at least some coordinates of at least some points on an implicit curve resulting from an intersection of a plurality of graphical objects, the method comprising:

receiving a sparse parameterized representation of at least a portion of the implicit curve, the representation comprising: sparse convergence region descriptor data for describing one or more convergence regions of the implicit curve and sparse starting point descriptor data for deriving starting points associated with the one or more convergence regions;

at runtime, based at least on the sparse starting point descriptor data of the starting points associated with the one or more convergence regions, deriving the coordinates of at least some of the starting points associated with the one or more convergence regions; and

applying Newton iteration starting from the derived coordinates of the at least some of the starting points to determine at least some coordinates of one or more points on the implicit curve.

2. The computer implemented method of claim 1, wherein the sparse starting point descriptor data for determining starting points associated with the one or more convergence regions comprises data indicative of selection of a method for deriving the starting points at runtime.

3. The computer implemented method of claim 2, wherein the method for deriving the starting points at runtime comprises retrieving stored coordinates of one or more midpoints of one or more parameterization regions of the implicit curve.

4. The computer implemented method of claim 2, wherein the method for deriving the starting points at runtime comprises calculating coordinates of the starting points based on a straight line approximation of the implicit curve.

5. The computer implemented method of claim 2, wherein the method for deriving the starting points at runtime comprises calculating coordinates of the starting points based on a cubic approximation of the implicit curve.

6. The method of claim 5, wherein the cubic approximation of the implicit curve is a Hermite curve approximation.

7. The method of claim 1, wherein the sparse convergence region descriptor data for describing one or more convergence regions of the implicit curve comprises coordinates of just one known endpoint of one of the one or more convergence regions.

8. The method of claim 7, further comprising: calculating at runtime, endpoints of at least one of the one or more convergence regions by applying Newton iterations starting from the one known endpoint.

9. The method of claim 7, wherein the one know endpoint is represented in a quantized integer data type.

10. The method of claim 1, wherein the sparse convergence region descriptor data for describing the one or more convergence regions of the implicit curve comprises at least one endpoint of at least one of the one or more convergence regions represented in quantized integer data types and coordinates of the at least one endpoint is calculated at runtime by applying Newton iterations starting from the quantized representation of the at least one endpoint.

11. A computer implemented method for rendering an implicit curve of intersection related to a plurality of graphical objects by generating a parameterized representation of the implicit curve of intersection, the method comprising:

determining one or more parameterization regions of the implicit curve of intersection;

applying an intervalized super convergence test to determine one or more convergence regions of the one or more parameterization regions;

storing a parameterized representation of at least a portion of the implicit curve of intersection, the representation comprising: convergence region descriptor data for describing the one or more convergence regions and sparse starting point descriptor data for determining starting points associated with the one or more convergence regions;

at runtime, based at least on the sparse starting point descriptor data of the starting points associated with the one or more convergence regions, determining coordinates of the starting points associated with the one or more convergence regions; and

applying Newton iteration starting from the starting points to determine at least some coordinates of one or more points of the implicit curve of intersection for rendering at least some of the one or more points of the implicit curve of intersection on a display.

12. The method of claim 11, wherein the implicit curve is derived by a constructive solid geometry operation involving a plurality of geometric surfaces.

13. The method of claim 11, wherein applying the intervalized super convergence test comprises determining whether the selected ranges satisfy Kantorovich condition.

14. The method of claim 11, wherein the sparse starting point descriptor data for determining starting points associated with the one or more convergence regions comprises data indicative of a method for runtime determination of the starting points associated with the one or more convergence regions.

15. A computer system for processing parameterized representations of at least a portion of an implicit curve of intersection of a plurality of graphical objects to render the implicit curve of intersection on a display, the system comprising:

memory;

the memory communicative with a processor operable to:

receive the parameterized representation of at least a portion of the implicit curve from the memory, the representation comprising:

sparse convergence region descriptor data for describing one or more convergence regions of the implicit curve and data indicative of a selection of a method for a runtime calculation of starting points associated with the one or more convergence regions of the implicit curve; and

at runtime, to calculate coordinates of at least some of the starting points associated with the one or more convergence regions of the implicit curve by the method indicated in the parameterized representation.

16. The system of claim 15, wherein the processor is further operable to apply Newton iteration starting at the at least some of the runtime calculated starting points associated with the one or more convergence regions to derive at least some points on the implicit curve for rendering the at least some points on the implicit curve on the display.

17. The system of claim 15, wherein the method for the runtime calculation of the starting points associated with the one or more convergence regions of the implicit curve is selected from a group of starting point calculation methods consisting of retrieving stored coordinates of one or more midpoints of one or more parameterization regions of the implicit curve, calculating coordinates of the starting points based on a straight line approximation of the implicit curve and calculating coordinates of the starting points based on a Hermite cubic approximation of the implicit curve.

18. The system of claim 15, wherein the sparse convergence region descriptor data for describing the one or more convergence regions of the implicit curve comprises a quantized integer data type representation of at least one endpoint of at least one of the one or more convergence regions based on which coordinates of the at least one endpoint having floating point accuracy can be calculated at runtime by applying Newton iterations starting from the quantized representation of the at least one endpoint.

19. At least one computer-readable storage medium having stored thereon computer-executable instructions that describe a parameterized representation of at least a portion of an implicit curve of intersection of a plurality of graphical objects, the parameterized representation of the at least a portion of the implicit curve operable to be used by a processor for runtime processing related to rendering the implicit curve on a display, the parameterized representation comprising:

sparse convergence region descriptor data for describing one or more convergence regions of the implicit curve; and

sparse starting point descriptor data that indicates to the processor of a selection of a method for a runtime calculation to be performed by the processor of starting points associated with the one or more convergence regions of the implicit curve.

20. The at least one computer-readable storage medium of claim 19, wherein the method for the runtime calculation of the starting points associated with the one or more convergence regions of the implicit curve is selected from a group of runtime calculations of starting points consisting of retrieving stored coordinates of one or more midpoints of one or more parameterization regions of the implicit curve, calculating coordinates of the starting points based on a straight line approximation of the implicit curve and calculating coordinates of the starting points based on a Hermite cubic approximation of the implicit curve.

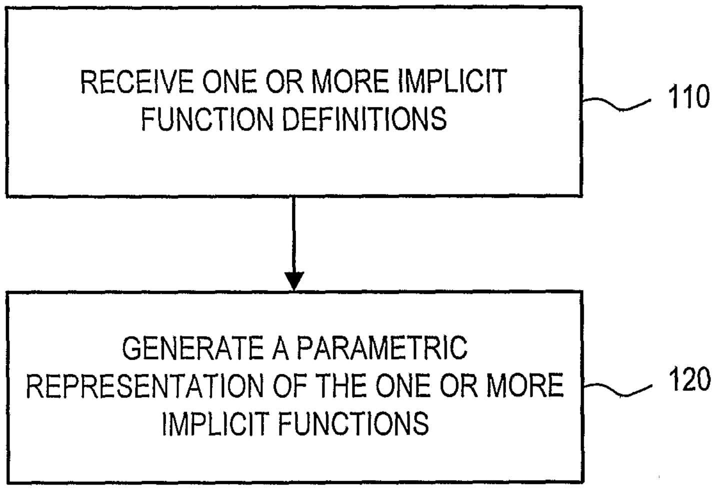

RECEIVE ONE OR MORE IMPLICIT FUNCTION DEFINITIONS
110
GENERATE A PARAMETRIC REPRESENTATION OF THE ONE OR MORE IMPLICIT FUNCTIONS
120

FIG. 1

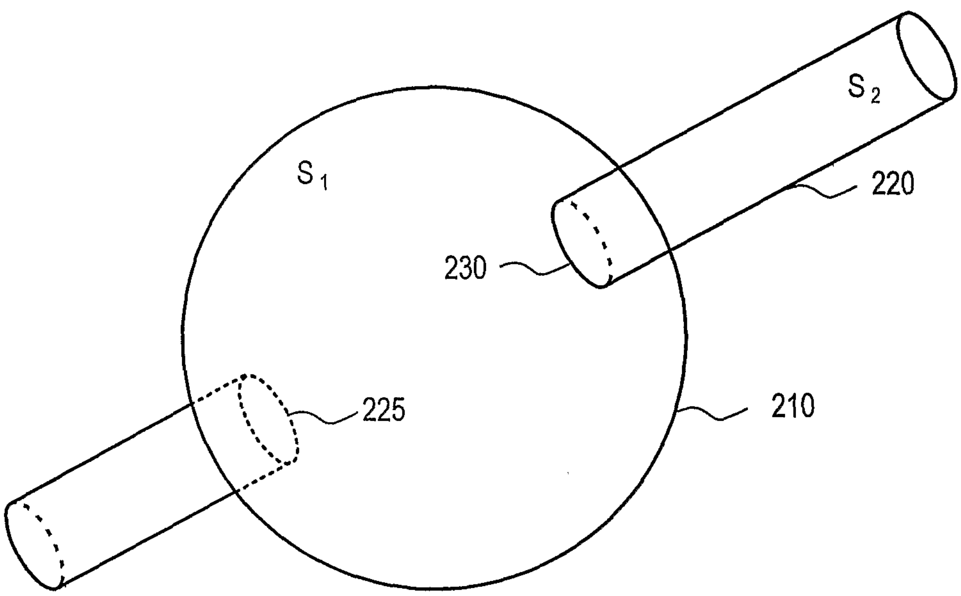
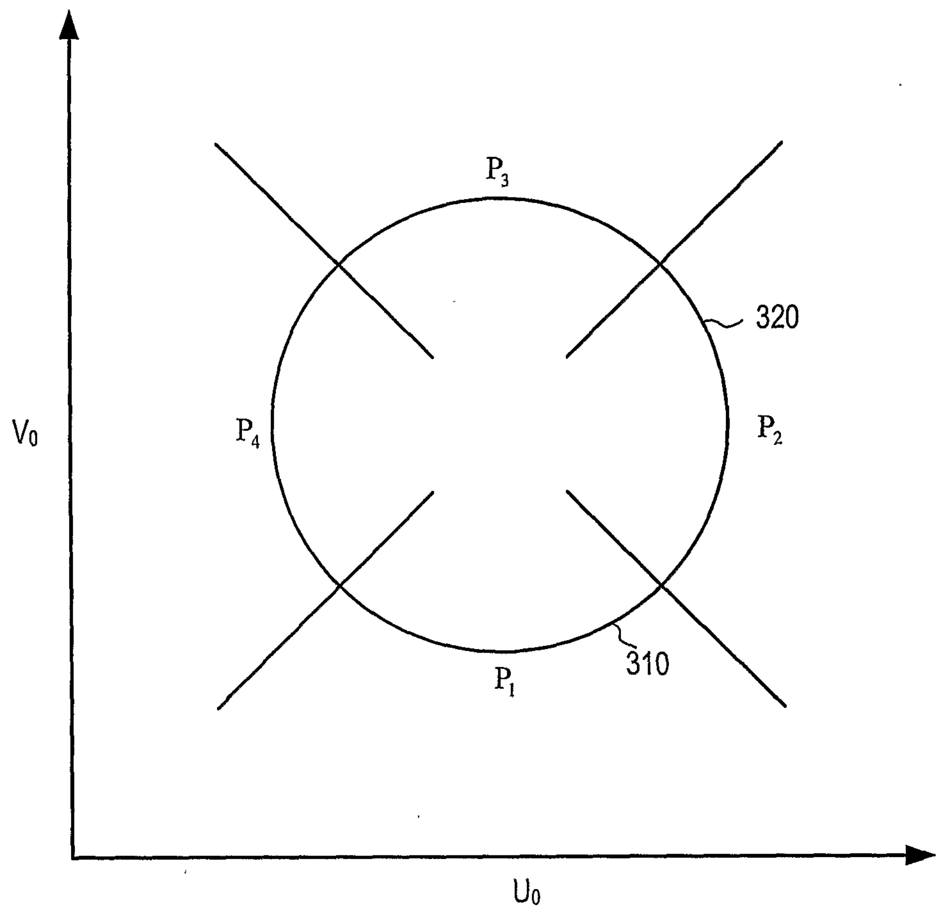
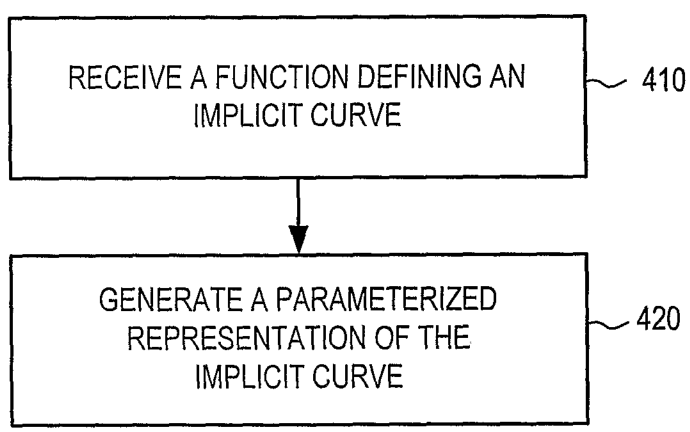

S 2
S 1
220
230
225
210

FIG. 2



P3
320
V0
P4
P2
310
P1
U0

FIG. 3



RECEIVE A FUNCTION DEFINING AN IMPLICIT CURVE
410
GENERATE A PARAMETERIZED REPRESENTATION OF THE IMPLICIT CURVE
420

FIG. 4A

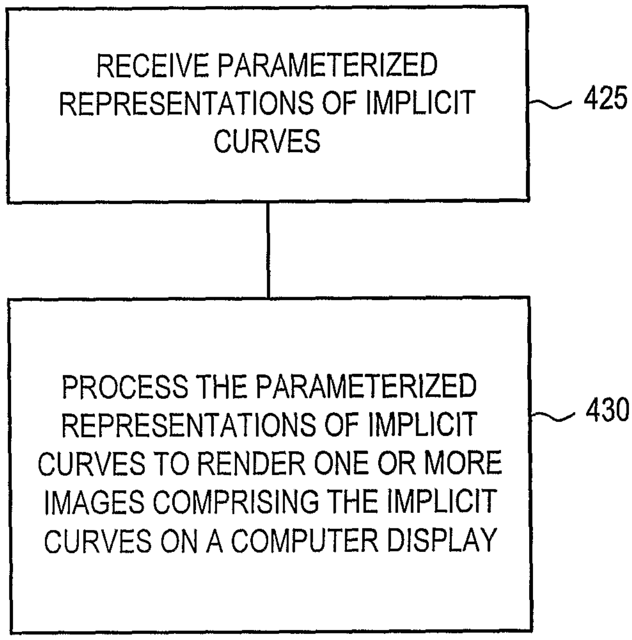

RECEIVE PARAMETERIZED REPRESENTATIONS OF IMPLICIT CURVES
425
PROCESS THE PARAMETERIZED REPRESENTATIONS OF IMPLICIT CURVES TO RENDER ONE OR MORE IMAGES COMPRISING THE IMPLICIT CURVES ON A COMPUTER DISPLAY
430

FIG. 4B

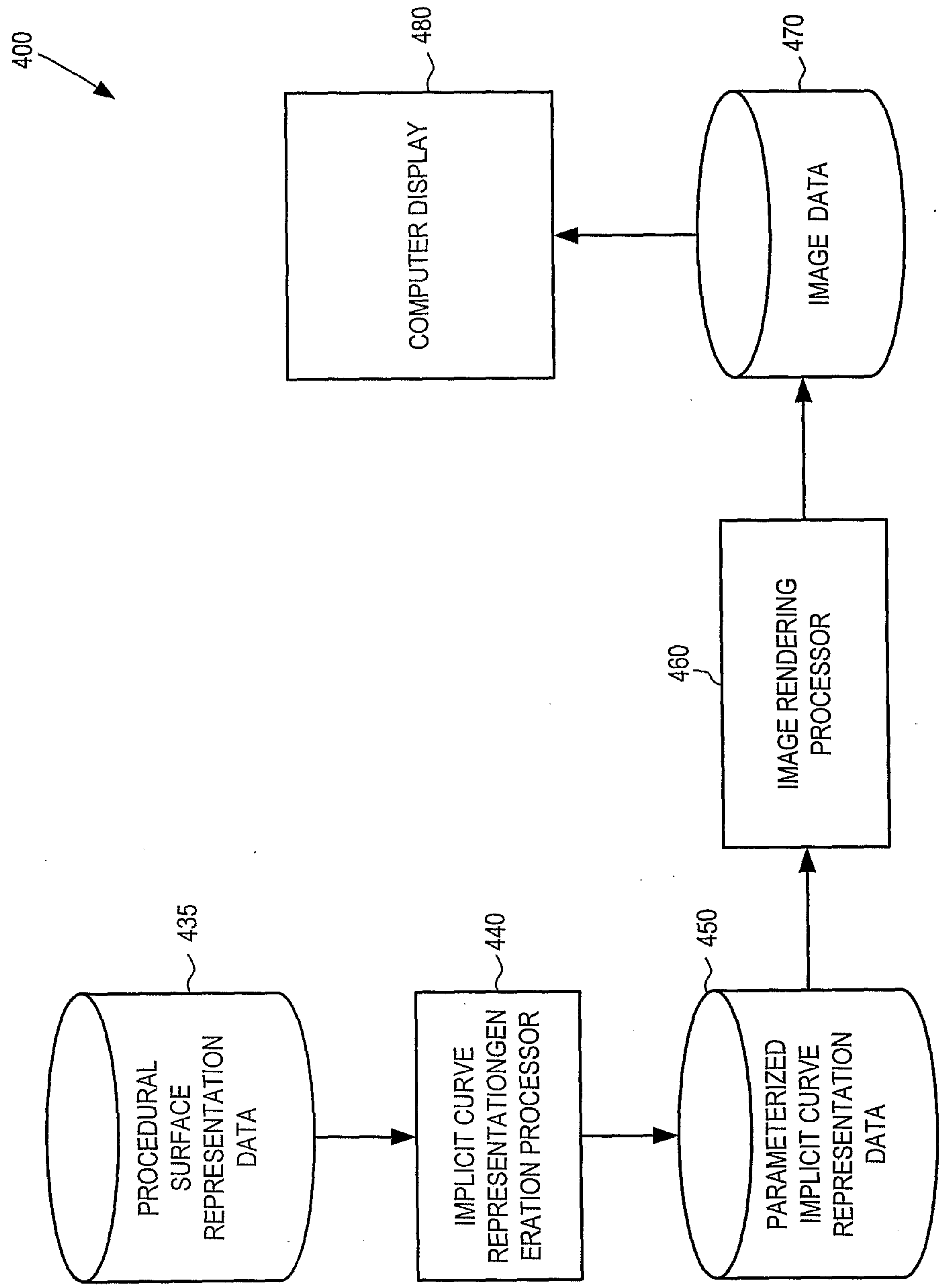

400
435
PROCEDURAL SURFACE REPRESENTATION DATA
440
IMPLICIT CURVE REPRESENTATIONGEN ERATION PROCESSOR
450
PARAMETERIZED IMPLICIT CURVE REPRESENTATION DATA
460
IMAGE RENDERING PROCESSOR
470
IMAGE DATA
480
COMPUTER DISPLAY

FIG. 4C

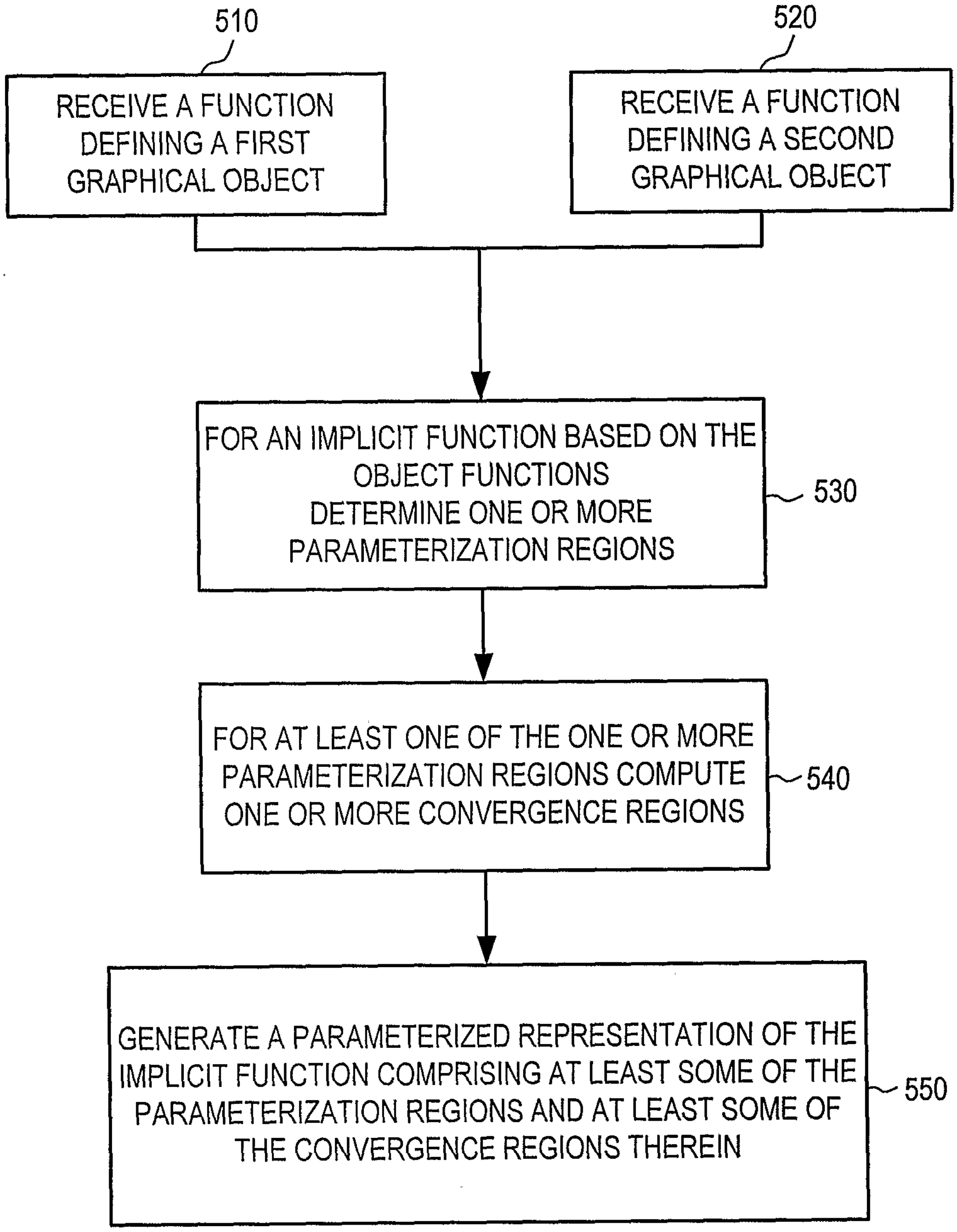

510
RECEIVE A FUNCTION DEFINING A FIRST GRAPHICAL OBJECT
520
RECEIVE A FUNCTION DEFINING A SECOND GRAPHICAL OBJECT
FOR AN IMPLICIT FUNCTION BASED ON THE OBJECT FUNCTIONS DETERMINE ONE OR MORE PARAMETERIZATION REGIONS
530
FOR AT LEAST ONE OF THE ONE OR MORE PARAMETERIZATION REGIONS COMPUTE ONE OR MORE CONVERGENCE REGIONS
540
GENERATE A PARAMETERIZED REPRESENTATION OF THE IMPLICIT FUNCTION COMPRISING AT LEAST SOME OF THE PARAMETERIZATION REGIONS AND AT LEAST SOME OF THE CONVERGENCE REGIONS THEREIN
550

FIG. 5A

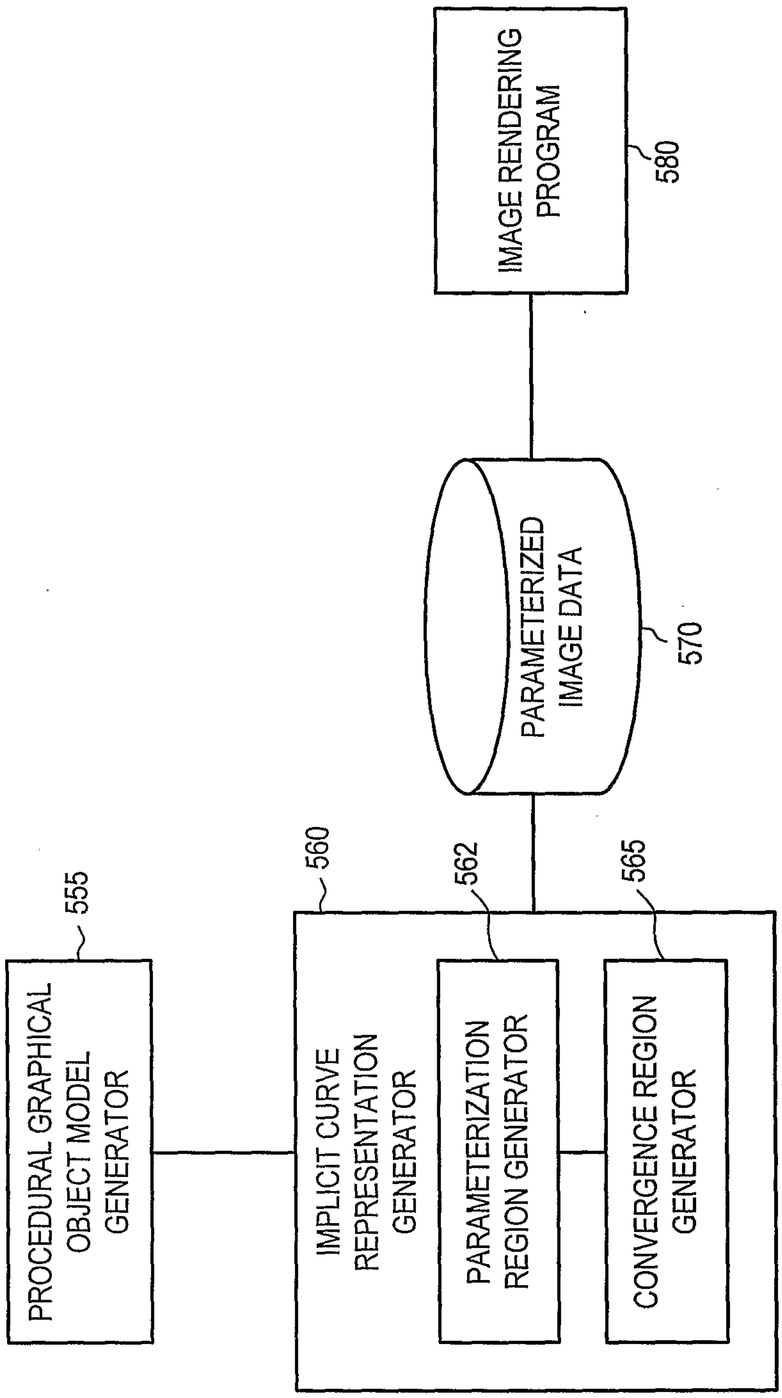


IMAGE RENDERING PROGRAM
580
PARAMETERIZED IMAGE DATA
570
555
PROCEDURAL GRAPHICAL OBJECT MODEL GENERATOR
560
IMPLICIT CURVE REPRESENTATION GENERATOR
562
PARAMETERIZATION REGION GENERATOR
565
CONVERGENCE REGION GENERATOR

FIG. 5B

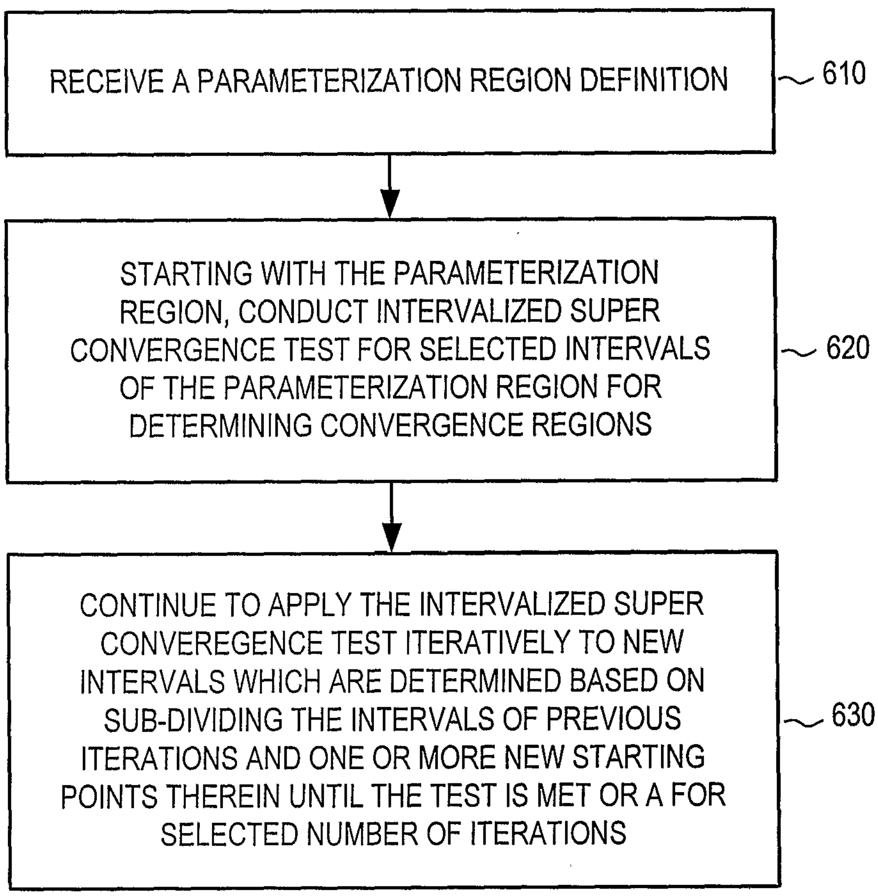
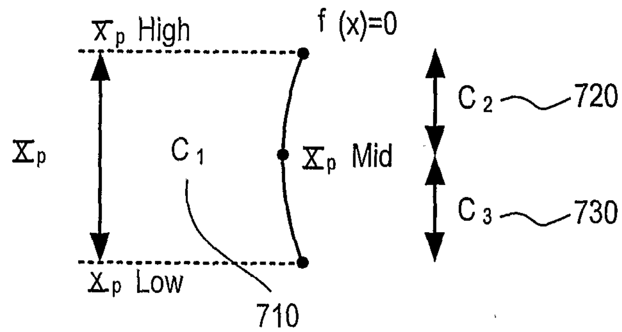
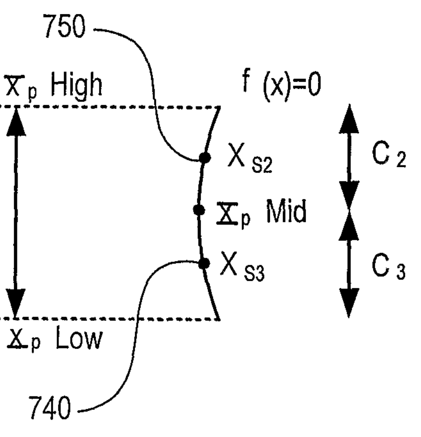

RECEIVE A PARAMETERIZATION REGION DEFINITION
610
STARTING WITH THE PARAMETERIZATION REGION, CONDUCT INTERVALIZED SUPER CONVERGENCE TEST FOR SELECTED INTERVALS OF THE PARAMETERIZATION REGION FOR DETERMINING CONVERGENCE REGIONS
620
CONTINUE TO APPLY THE INTERVALIZED SUPER CONVEREGENCE TEST ITERATIVELY TO NEW INTERVALS WHICH ARE DETERMINED BASED ON SUB-DIVIDING THE INTERVALS OF PREVIOUS ITERATIONS AND ONE OR MORE NEW STARTING POINTS THEREIN UNTIL THE TEST IS MET OR A FOR SELECTED NUMBER OF ITERATIONS
630

FIG. 6



Xp High
f (x)=0
C2
720
Xp
C1
Xp Mid
C3
730
Xp Low
710

FIG. 7A



750
Xp High
f (x)=0
XS2
C2
Xp
Xp Mid
XS3
C3
Xp Low
740

FIG. 7B

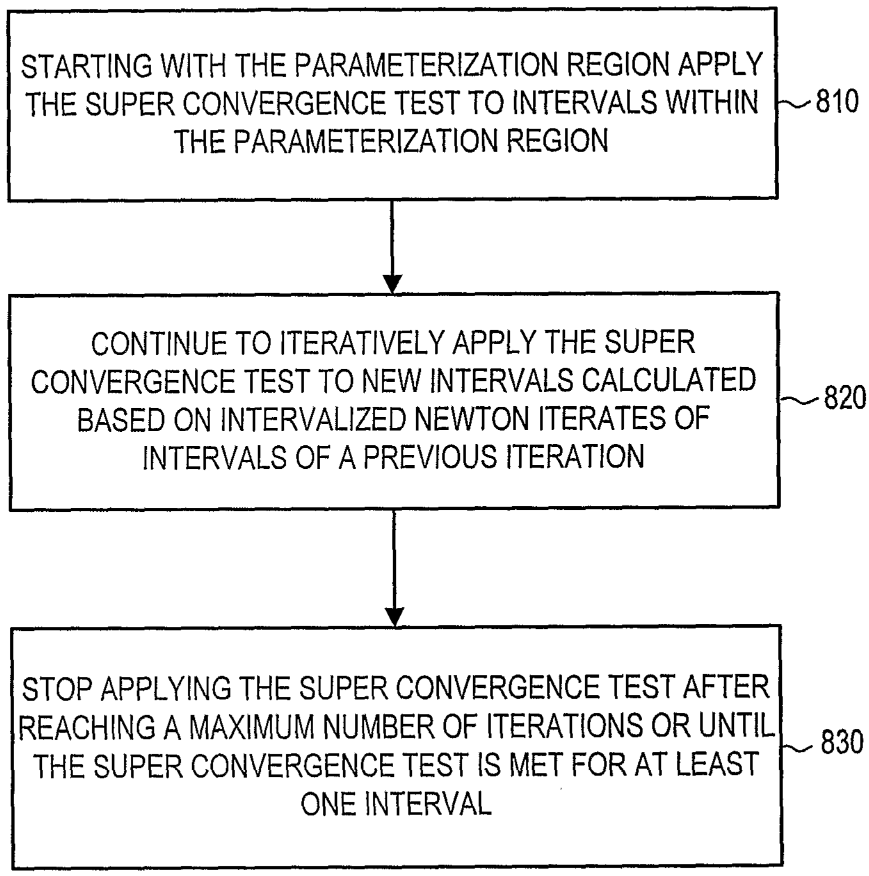

STARTING WITH THE PARAMETERIZATION REGION APPLY THE SUPER CONVERGENCE TEST TO INTERVALS WITHIN THE PARAMETERIZATION REGION
810
CONTINUE TO ITERATIVELY APPLY THE SUPER CONVERGENCE TEST TO NEW INTERVALS CALCULATED BASED ON INTERVALIZED NEWTON ITERATES OF INTERVALS OF A PREVIOUS ITERATION
820
STOP APPLYING THE SUPER CONVERGENCE TEST AFTER REACHING A MAXIMUM NUMBER OF ITERATIONS OR UNTIL THE SUPER CONVERGENCE TEST IS MET FOR AT LEAST ONE INTERVAL
830

FIG. 8

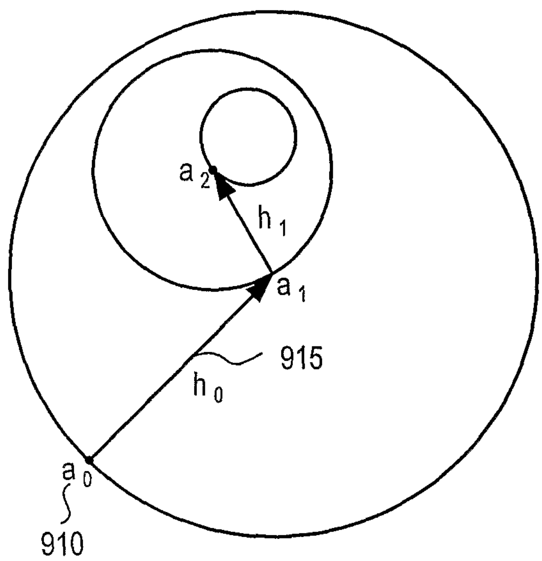

a_2
h_1
a_1
915
h_0
a_0
910

FIG. 9

1000

```
bool ConvergenceInRegion (x_o)                                            1010
  input: starting point (x_o)=(x_p,xd)
  if convergence (x_o) return true                                        1020
  else
    //subdivide x_p
    x_p low=(x_p,x_p.mid)     x_p high=(x_p.mid,x_p)                      1030
    //compute new starting points using Interval Newton
    start low=Interval Newton (x_p low.mid)
    start high=Interval Newton (x_p high.mid)                             1040
          x_o low = (x_p low, start low)
          x_o high = (x_p high, start high)                               1050
    return ConvergenceInRegion (x_o low) && ConvergenceInRegion (x_o high)
```

FIG. 10

1100

```
bool convergence (x_o = (x_p, x_d))
    x_i = x_o
    while(x_i are shrinking){                      1110
    h_i=D-1_D f(x_i)f(x_i)
    x_i+1 = x_i - h_i
    U = expand (x_i+1, h_i)
    M = Lipshitz Constant (U)
    if|f(x_i)11D-1f(x_i)|M<1/2                     1120
    return true;
  } //didn't prove convergence so have to subdivide
    //subdivide region but keep same starting point_d
    x low=([x_p,x_p.mid],x_d)
                                                   1130
    x high=([x_p.mid, x_p],x_d)
    return  convergence (x low) && convergence (x high)   1140
```

FIG. 11

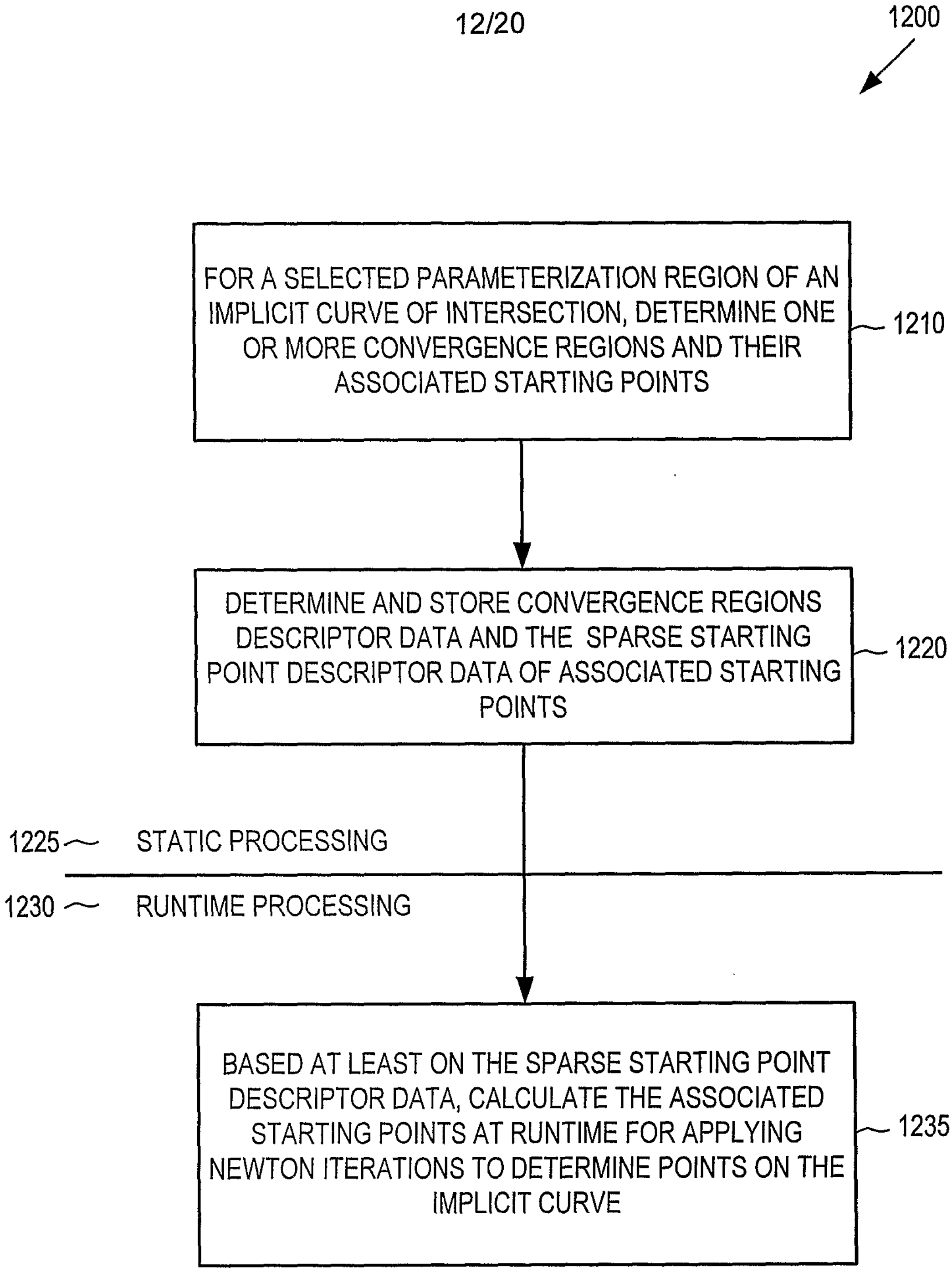
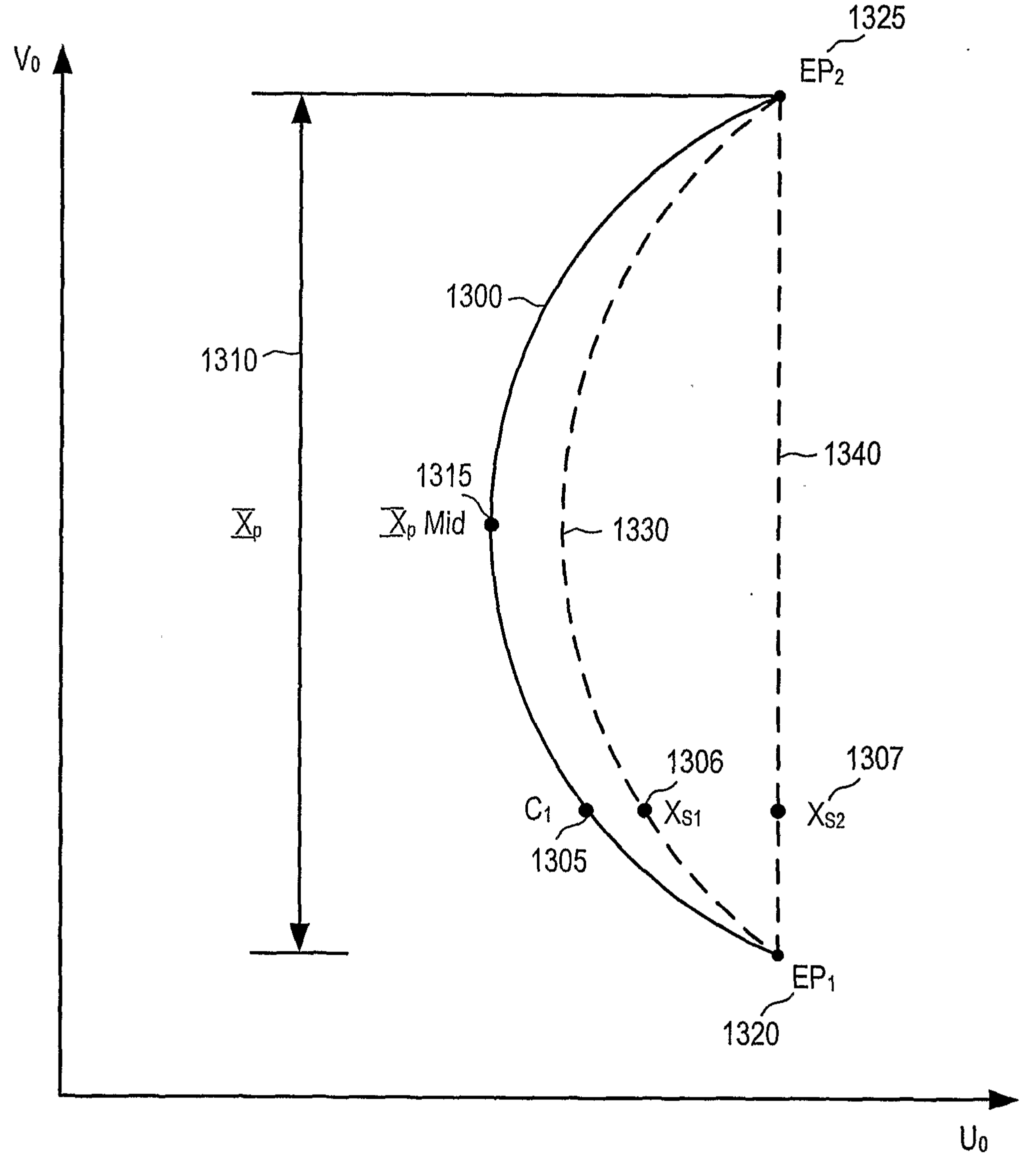

1200
FOR A SELECTED PARAMETERIZATION REGION OF AN IMPLICIT CURVE OF INTERSECTION, DETERMINE ONE OR MORE CONVERGENCE REGIONS AND THEIR ASSOCIATED STARTING POINTS
1210
DETERMINE AND STORE CONVERGENCE REGIONS DESCRIPTOR DATA AND THE SPARSE STARTING POINT DESCRIPTOR DATA OF ASSOCIATED STARTING POINTS
1220
1225
STATIC PROCESSING
1230
RUNTIME PROCESSING
BASED AT LEAST ON THE SPARSE STARTING POINT DESCRIPTOR DATA, CALCULATE THE ASSOCIATED STARTING POINTS AT RUNTIME FOR APPLYING NEWTON ITERATIONS TO DETERMINE POINTS ON THE IMPLICIT CURVE
1235

FIG. 12



V_0
1325
EP_2
1300
1310
1340
1315
$\overline{X}_p$
$\overline{X}_p$ Mid
1330
1306
1307
C_1
X_{S1}
X_{S2}
1305
EP_1
1320
U_0

FIG. 13

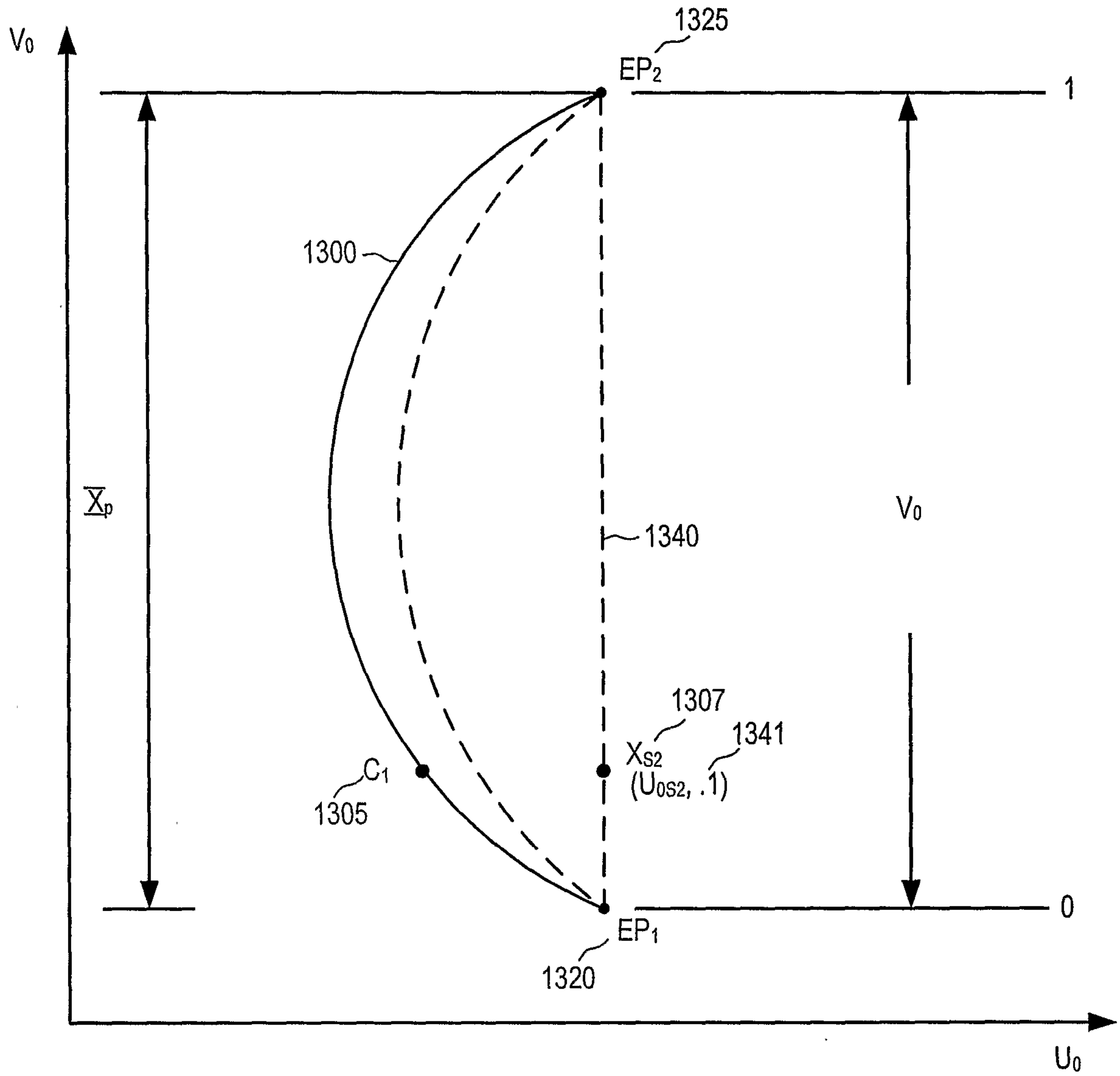

V0
1325
EP2
1
1300
X̄p
1340
V0
1307
XS2
1341
(U0S2, .1)
C1
1305
0
EP1
1320
U0

FIG. 14

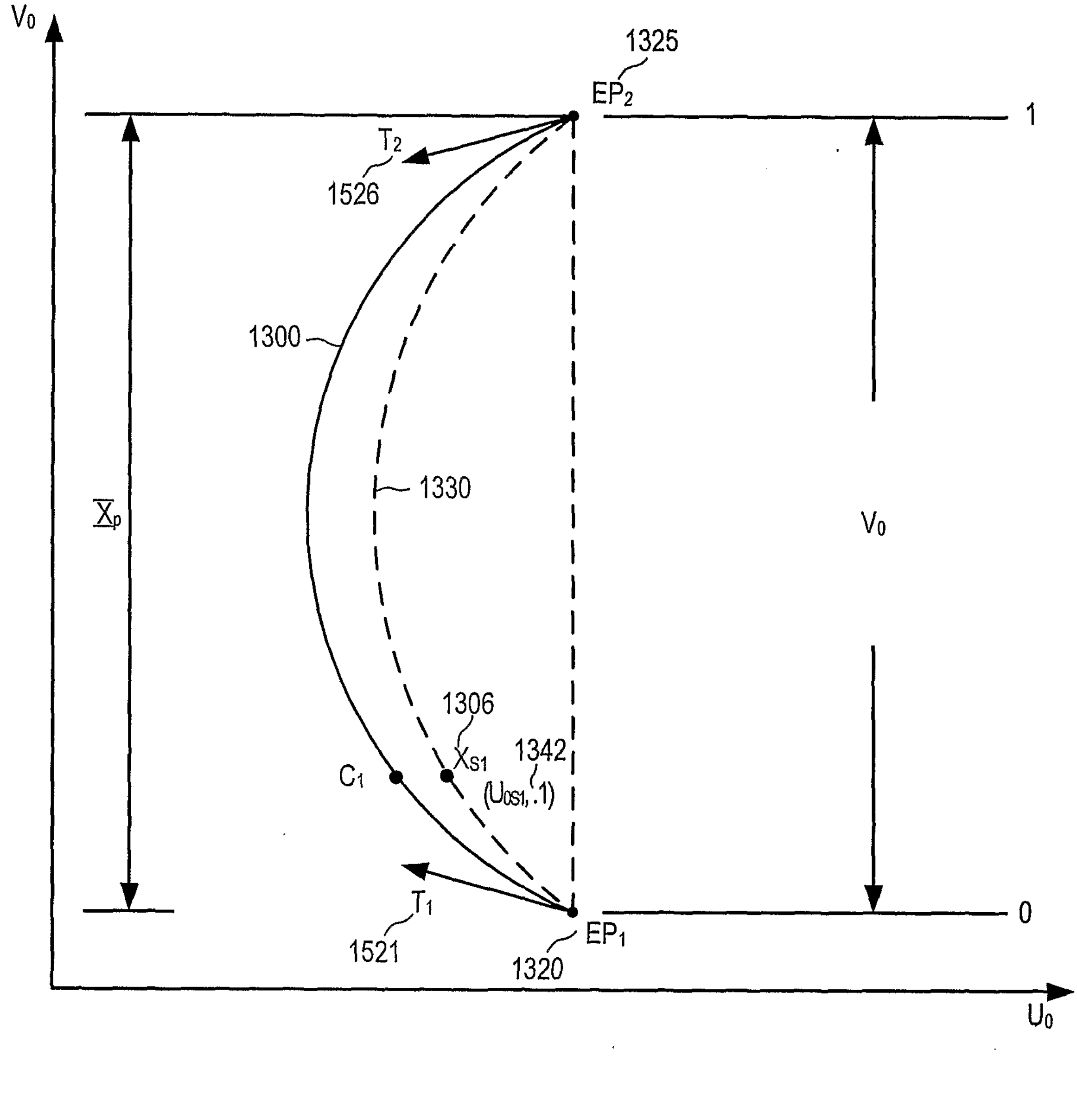

V0
1325
EP2
1
T2
1526
1300
1330
X̄p
V0
1306
XS1
1342
C1
(U0S1,.1)
T1
1521
EP1
1320
0
U0

FIG. 15

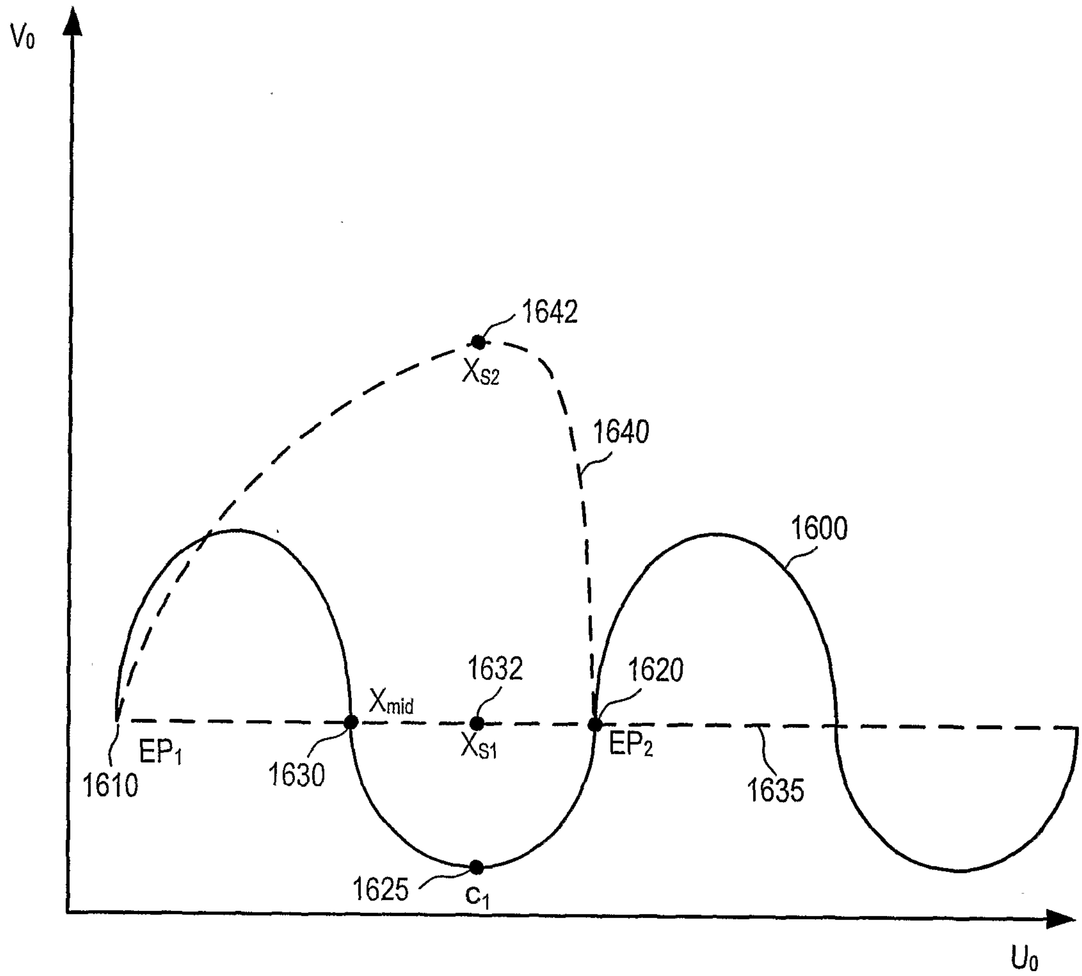

V0
1642
XS2
1640
1600
1632
1620
Xmid
EP1
1630
XS1
EP2
1635
1610
1625
c1
U0

FIG. 16

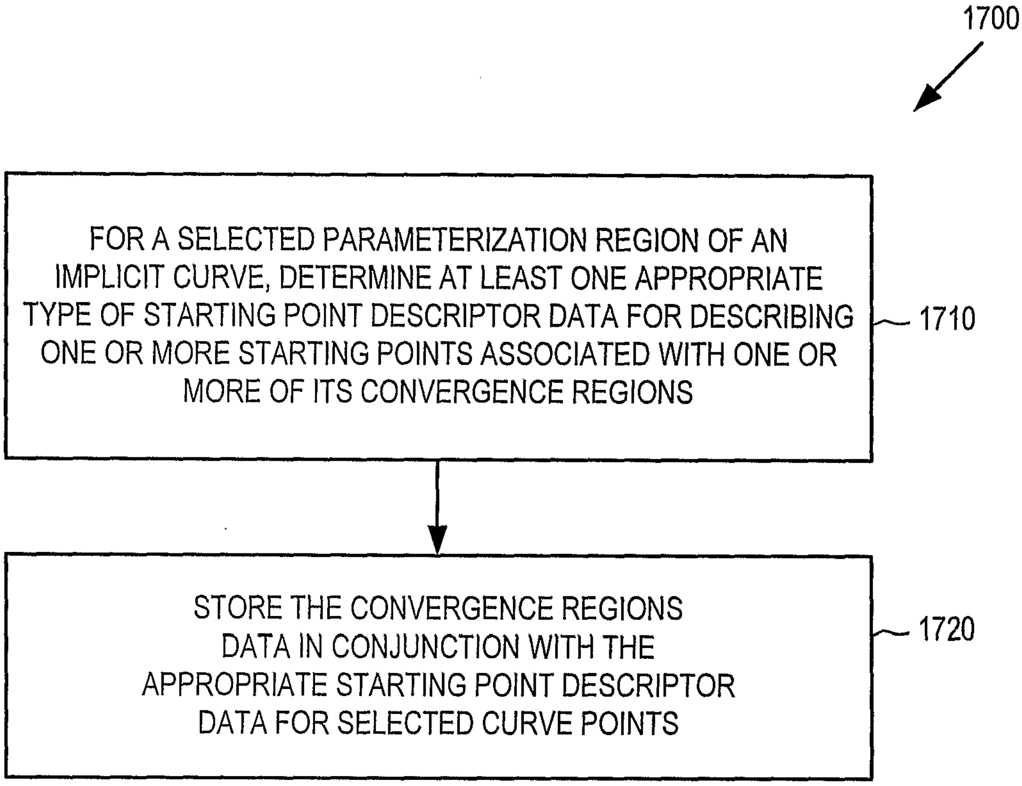
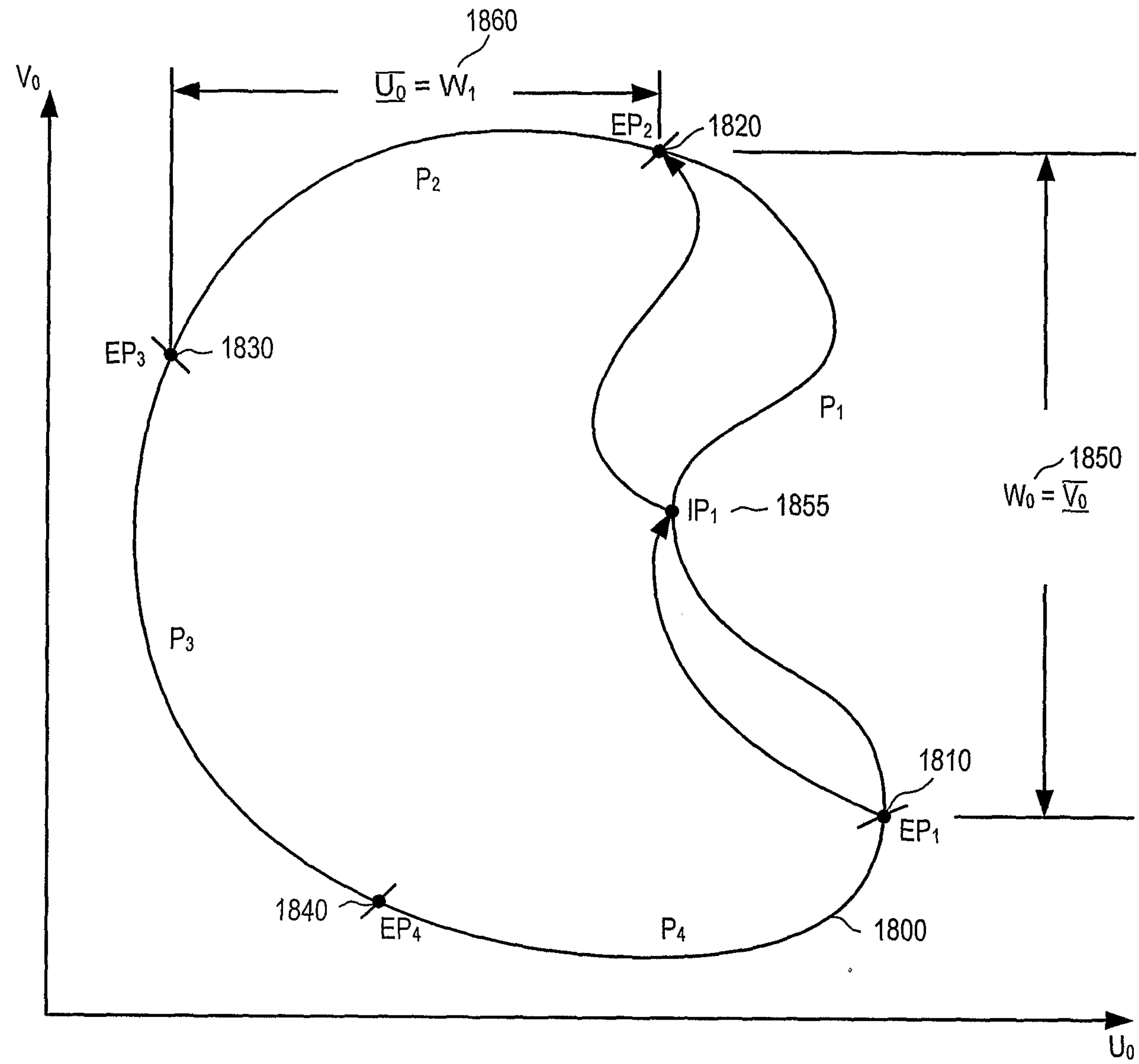

1700
FOR A SELECTED PARAMETERIZATION REGION OF AN IMPLICIT CURVE, DETERMINE AT LEAST ONE APPROPRIATE TYPE OF STARTING POINT DESCRIPTOR DATA FOR DESCRIBING ONE OR MORE STARTING POINTS ASSOCIATED WITH ONE OR MORE OF ITS CONVERGENCE REGIONS
1710
STORE THE CONVERGENCE REGIONS DATA IN CONJUNCTION WITH THE APPROPRIATE STARTING POINT DESCRIPTOR DATA FOR SELECTED CURVE POINTS
1720

FIG. 17



1860
U0 = W1
V0
EP2
1820
P2
EP3
1830
P1
1850
W0 = V0
IP1
1855
P3
1810
EP1
1840
EP4
P4
1800
U0

FIG. 18

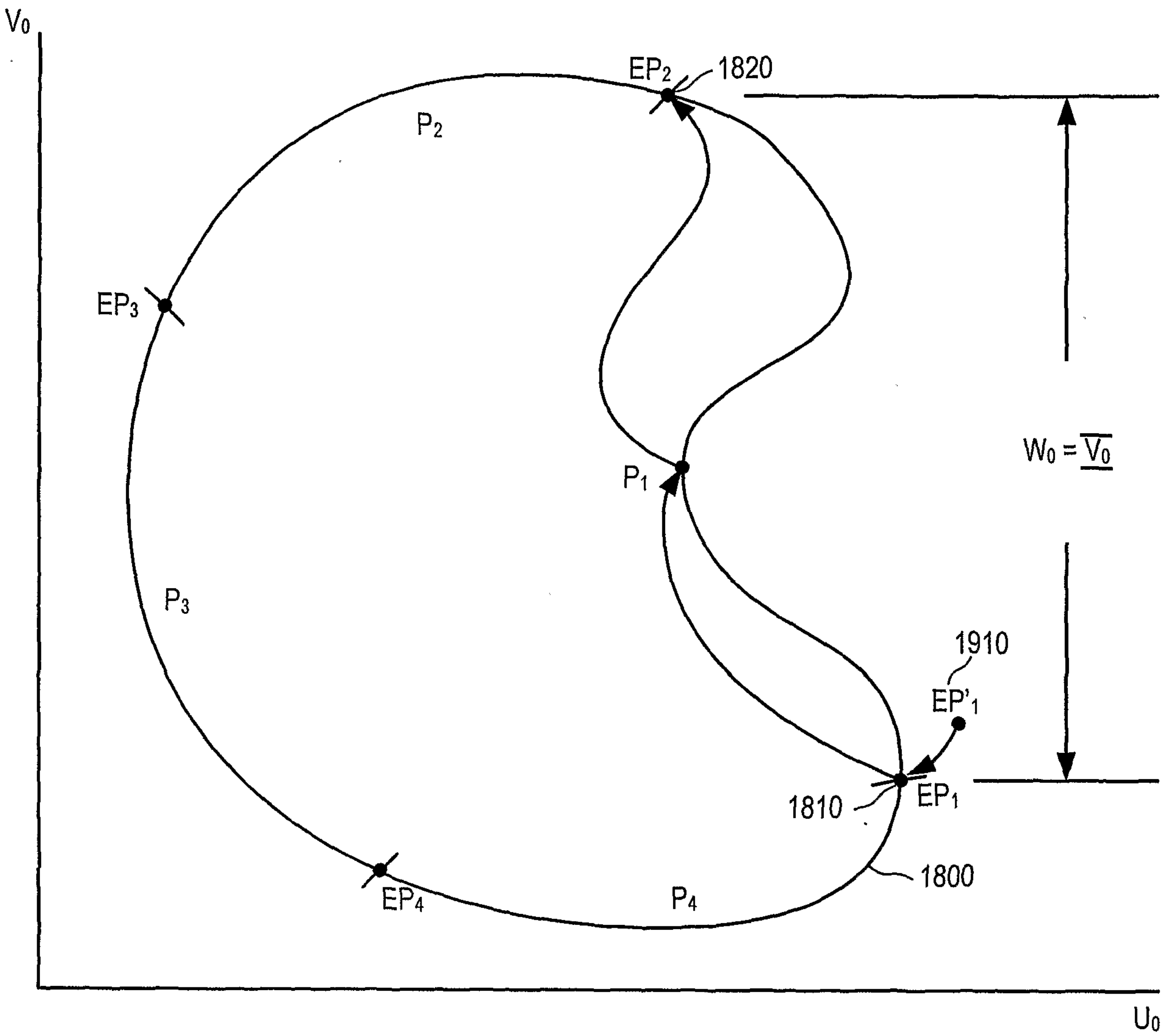


FIG. 19

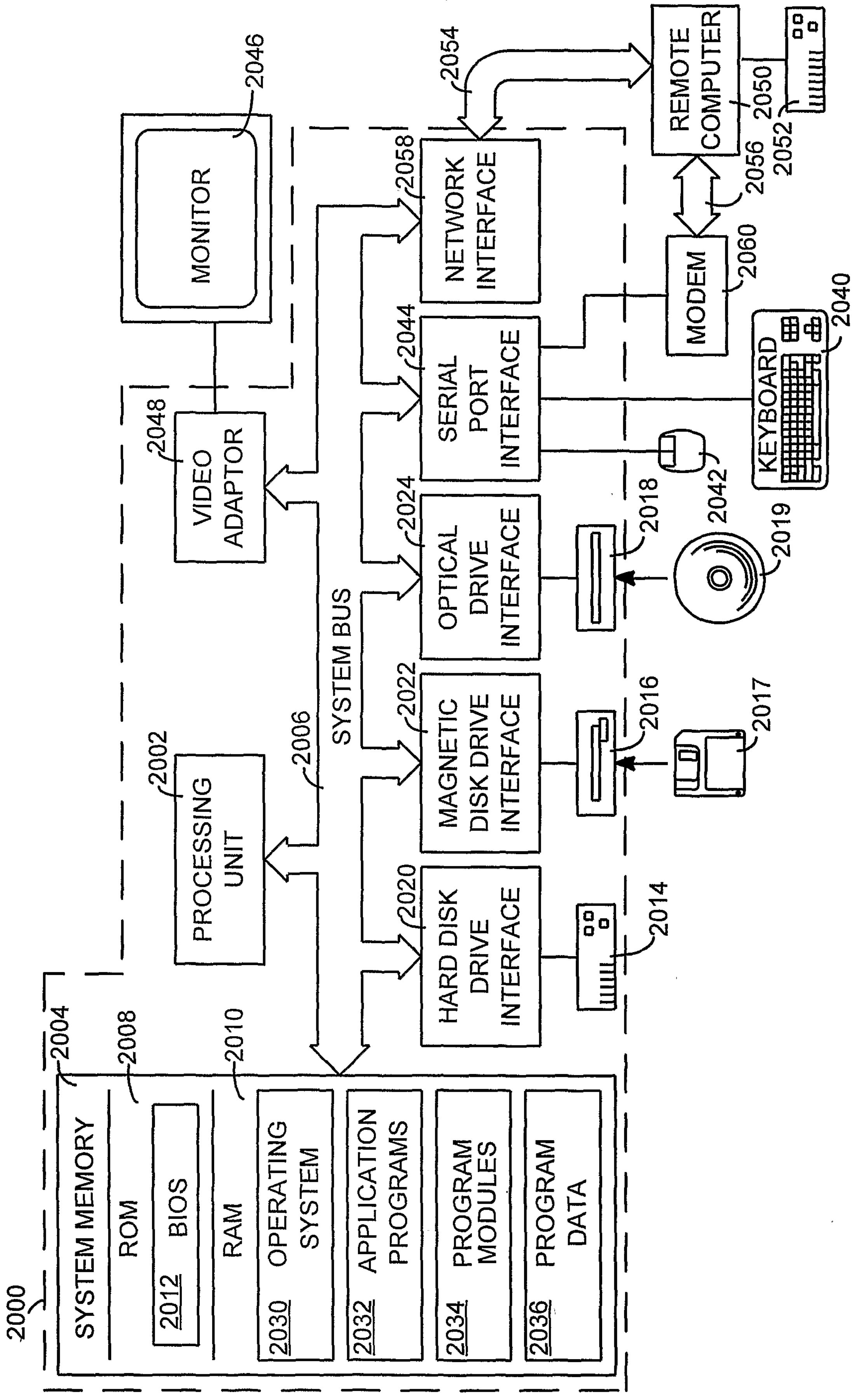

2000
SYSTEM MEMORY
2004
ROM
2008
2012 BIOS
RAM
2010
2030 OPERATING SYSTEM
2032 APPLICATION PROGRAMS
2034 PROGRAM MODULES
2036 PROGRAM DATA
PROCESSING UNIT
2002
2006
SYSTEM BUS
VIDEO ADAPTOR
2048
MONITOR
2046
HARD DISK DRIVE INTERFACE
2020
2014
MAGNETIC DISK DRIVE INTERFACE
2022
2016
2017
OPTICAL DRIVE INTERFACE
2024
2018
2019
SERIAL PORT INTERFACE
2044
2042
KEYBOARD
2040
MODEM
2060
NETWORK INTERFACE
2058
2054
2056
REMOTE COMPUTER
2050
2052

FIG. 20

INTERNATIONAL SEARCH REPORT

International application No.
PCT/US2006/032227

A. CLASSIFICATION OF SUBJECT MATTER

G06F 17/00(2006.01)i, G06F 9/44(2006.01)i

According to International Patent Classification (IPC) or to both national classification and IPC

B. FIELDS SEARCHED

Minimum documentation searched (classification system followed by classification symbols)
IPC8 G06F, G06Q

Documentation searched other than minimum documentation to the extent that such documents are included in the fields searched
Korean Patents and applications for inventions since 1975
Korean Utility models and applications for Utility models since 1975
Japanese Utility models and applications for Utility models since 1975

Electronic data base consulted during the intertnational search (name of data base and, where practicable, search terms used)
PAJ, FPD, USPAT, eKIPASS(KIPO internal)

C. DOCUMENTS CONSIDERED TO BE RELEVANT

Category*	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
A	US 6,300,958 B1 (T-SURF CORP.) 09 October 2001 See abstract; claim 1	1-20
A	WO 2004-44689 A2 (METRIC INFORMATICS INC.) 27 May 2004 See abstract; claim 1	1-20
A	US 6,806,875 B2 (RENESAS TECHNOLOGY CORP.) 19 October 2004 See abstract; claim 1	1-20
A	KR 2005-44964 A (LG ELECTRONICS INC.) 16 May 2005 See abstract; claims 1, 2, 4	1-20

☐ Further documents are listed in the continuation of Box C.   ☒ See patent family annex.

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"A" document defining the general state of the art which is not considered to be of particular relevance
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"&" document member of the same patent family

Date of the actual completion of the international search	Date of mailing of the international search report
28 DECEMBER 2006 (28.12.2006)	**28 DECEMBER 2006 (28.12.2006)**

Name and mailing address of the ISA/KR	Authorized officer
Korean Intellectual Property Office 920 Dunsan-dong, Seo-gu, Daejeon 302-701, Republic of Korea Facsimile No. 82-42-472-7140	PARK, Sung Woo Telephone No. 82-42-481-5790

INTERNATIONAL SEARCH REPORT

Information on patent family members

International application No.

PCT/US2006/032227

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		RU2004131630A	10.04.2006
		US2005099415AA	12.05.2005