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INDUCTIVE ARTIFICIAL LINE

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4 Sheets-Sheet 2

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**FIG. 2**

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[Signature]

By Attorney
The object of my invention is to provide an improved method for duplex transmission of electrical signals over wave conductors, as for instance, over suboceanic cables.

The ordinary practice of duplex signalling over long submarine cables employs at the transmitting station a local sectional wave conductor, the so-called artificial line, which is placed on one side of a duplex bridge, the other side of the bridge containing the cable.

The electrical currents generated by the electrical pulses impressed upon the two sides of the bridge by a local generator are split into two parts; one part is transmitted over the side containing the cable and the other over the side containing the local artificial line. When the cable and its balancing artificial line have the same terminal resistance and reactance for every frequency which is of importance at the speed of signalling then they can be made to balance each other. It is obvious that when the local wave conductor is an ordinary artificial line consisting of resistances in series and capacities in parallel then at higher speeds of signalling such a balance cannot be obtained, because such an artificial line is non-inductive, whereas the cable has inductance the reaction of which becomes a determining factor at higher speeds. Inductance introduced into the artificial line will not improve the balance, unless this inductance and its internal resistance vary with the frequency according to the same law as the average inductance and resistance of the cable elements. The process of formulating the rules for designing such an inductive artificial line is disclosed in this specification and it consists of two distinct steps; the first step consists of preliminary experimental measurements, and the second is a mathematical theory which is based upon these measurements and furnishes definite mathematical formulae which guide the design.

I have discovered that the average effective resistance and inductance of a submarine cable vary as though each were comprised of two distinct parts, one constant, and one varying with the frequency, and that the variable part of the inductance decreases whereas the variable part of the resistance increases with the frequency of the impressed current. I have produced a novel network consisting of sections each of which has two inductances in series, one of which is shunted by a non-inductive resistance. The effective inductance and resistance of each section vary with the frequency of the impressed current in the same or substantially the same manner as the average effective inductance and resistance of a submarine cable vary. This improved network furnishes therefore a means for accurately balancing a submarine cable over a wide range of frequencies.

My invention comprises an inductive artificial line, preferably in the form of equal sections connected in series, the inductance and resistance of each section of which vary in accordance with the laws which I have discovered and above described.

In the drawings accompanying and forming part of this specification and in which like reference numerals designate corresponding parts throughout:

Fig. 1 is a diagrammatic representation by curves A, B and C of the effective terminal reactances and resistances of a submarine cable and of a balancing artificial line.

Fig. 2 is a diagrammatic representation by curves A and B of the effective inductances and resistance per nautical mile of the same cable.

Fig. 3 diagrammatically represents the constructional elements of a sectional artificial line made in accordance with my invention.

Fig. 4 diagrammatically represents a submarine cable balanced for duplex operation by means of my invention.
Fig. 5 is a plan view of an air-cored coil of toroidal symmetry.

Fig. 6 shows an end view of the same.

Fig. 7 is a sectional view of the same taken on the line 7–7 of Fig. 6.

Fig. 8 is a sectional view of the coil shown in Figs. 5, 6 and 7 after an insulating compound has been poured around it.

Fig. 9 is a plan view of a nickel cored coil of toroidal symmetry.

Fig. 10 is a sectional view of the same taken on the line 10–10 of Fig. 9.

Referring to the diagram of Fig. 1, curves A and B represent at various frequencies the effective terminal resistance $a_t$ and reactance $b_t$, respectively, of an actual submarine cable which has the following constants:

- Length: 1–974.23 nautical miles
- Resistance: $R = 2.45$ ohms
- Estimated inductance: $L = 7.4 \times 10^{-3}$ henrys per nautical mile at zero frequency.

Curve C represents the terminal resistance and reactance of its corresponding non-inductive artificial line of the ordinary type which is placed on one side of the duplex bridge, the cable being on the other side. The curves were plotted from experimental data obtained by Wheatstone bridge measurements on this cable and on its corresponding non-inductive artificial line.

From the experimentally determined terminal resistance $a_t$ and reactance $b_t$ for any given frequency $f$ the effective resistance $R_t$ and inductance $L_t$, per nautical mile, for that frequency can be calculated as follows:

$$\left( a_t - b_t \right) = \sqrt{\frac{L_t}{C} + \frac{R_t}{pC}} \tag{1}$$

Where

- $p = 2\pi f$
- $C =$ capacity per nautical mile
- $L_t =$ effective average inductance per nautical mile
- $R_t =$ effective average resistance per nautical mile

From Equation (1) follows:

$$\left[ \frac{L_t}{C} = \frac{R_t}{pC} \right] \tag{2}$$

From curves A and B of Fig. 1, curves A and B of Fig. 2 were calculated by Equations (2). The latter curves, derived by calculation from experimental measurements give $L_t$ and $R_t$ and they represent graphically the law of variation of the average effective resistance and of the average effective inductance, respectively, per nautical mile of the cable. They furnish the data for the design of the inductive artificial line disclosed by this invention. If, therefore, a sectional artificial line can be constructed in which the effective resistance and the effective inductance per nautical mile are for a sufficiently broad frequency interval the same or very nearly the same as in the cable then the two structures will have the same or very nearly the same terminal impedance for that frequency interval, and can balance each other in the duplex bridge.

The sectional artificial line described here is a structure of this kind as the following mathematical analysis will show. It is made up of preferably equal sections, each section having two inductance coils in series, one of which is shunted by a non-inductive resistance. At each point of juncture of two consecutive sections a condenser of suitable capacity is inserted between this point and the ground. Fig. 3 gives a graphical representation of one of these sections. The two inductance coils 3 and 4 are connected in series, coil 4 being shunted by a non-inductive resistance 7. Equal condensers 5 and 6 are connected to points of juncture, 1 and 2, and to the ground through 8. Let each of these sections be an equivalent of 10 nautical miles of the cable; hence condensers 5 and 6 at the two terminals of the section will each have a capacity of $3.84 \times 10^4$ farads. The total capacity of the artificial line should, preferably, be equal to the total capacity of the cable. The length of one will then be an electrical equivalent to the length of the other.

Let $L_{10a}$ and $R_{10b}$ be the effective inductance and resistance, respectively, of the shunted coil for frequency $f=10s$; then it can be shown that:

$$L_{10a} = \frac{L(1-a)^2}{1 + s^2a^2} \tag{3}$$
$$R_{10b} = \frac{(a + s^2a^2)}{1 + s^2a^2} \tag{4}$$

Where

- $R =$ internal resistance
- $L =$ internal inductance
- $R =$ shunt resistance of the same coil.

$$a = \frac{R}{R + R} \tag{3a}$$
$$a_s = \frac{2\pi \times 10(1-a)}{L} = 2\pi \times 10L \tag{4a}$$

$s$ may be any number, so that when $s=5, 1, 1.5, 2, 2.5, 3$, etc., the corresponding values of $10s$ will be 5, 10, 15, 20, 25, 30, etc. and these will stand for the frequencies under consideration. Thus:

$$L_{20} = \frac{L(1-a)^2}{1 + 4a^2}; \quad L_{30} = \frac{L(1-a)^2}{1 + 9a^2} \tag{3b}$$
$$R_{20} = \frac{R(1 + 4a^2)}{1 + 4a^2}; \quad R_{30} = \frac{R(1 + 9a^2)}{1 + 9a^2} \tag{4b}$$

give the effective inductances and resistances, respectively for the frequencies 20, 30, etc. These values $L_{20}, L_{30}$, etc. and $R_{20}, R_{30}$, etc.,
can be made equal to the average effective inductances \( L'_{20} \), \( L'_{30} \), etc. and average effective resistances \( R'_{20} \), \( R'_{30} \), etc. of a 10 nautical miles length of the cable provided that another coil of a definite inductance \( L_0 \) and resistance \( R \) be connected in series with the shunted inductance as indicated in Fig. 3. With this addition, which is indispensable, Formulas (3b) and (4b) become now:

\[
L_{20} = L_0 + \frac{L(1-a)^2}{1 + 4a^2} \quad \text{etc.} \quad \text{and} \quad \text{(3c)}
\]

\[
R_{20} = R_0 + \frac{R(1+4a^2)}{1 + 4a^2} \quad \text{etc.} \quad \text{(4c)}
\]

Or more generally:

\[
L_{20} = L_0 + \frac{L(1-a)^2}{1 + 4a^2} \quad \text{etc.} \quad \text{(3d)}
\]

\[
R_{20} = R_0 + \frac{R(1+4a^2)}{1 + 4a^2} \quad \text{etc.} \quad \text{(4d)}
\]

Let now:

\[
L'_{20} = 10L_{20}, \quad L'_{30} = 10L_{30}, \quad \text{etc.}
\]

\[
R'_{20} = 10R_{20}, \quad R'_{30} = 10R_{30}, \quad \text{etc.}
\]

Where \( L_{20}, L_{30} \), etc. and \( R_{20}, R_{30} \), etc. are the effective average inductances and resistances, respectively, of the cable, per nautical mile, as calculated from terminal impedance measurements by Formulas (4), and given in curves A and B of Fig. 2. If Formulas (3c) and (4c) represent inductances and resistances varying with the frequency in the same manner as the effective average inductances and resistances of a cable length of 10 nautical miles, then the following conditions must be satisfied:

\[
10L'_{20} = L_0 + \frac{L(1-a)^2}{1 + 4a^2} \quad \text{(5)}
\]

\[
10L'_{30} = L_0 + \frac{L(1-a)^2}{1 + 4a^2} \quad \text{(6)}
\]

\[
10R'_{20} = R_0 + \frac{R(1+4a^2)}{1 + 4a^2} \quad \text{(7)}
\]

\[
10R'_{30} = R_0 + \frac{R(1+4a^2)}{1 + 4a^2} \quad \text{(8)}
\]

Hence

\[
R'_{20} - R'_{30} = \frac{qL}{R_0(1-a)L} \quad \text{or} \quad R + R_1 = qL \quad \text{(9)}
\]

That is, if the curves represented by (3d) and (4d) are to coincide over a suitable frequency interval with curves A and B of Fig. 2 then they certainly must coincide at the points corresponding to frequencies 20 and 30. These frequencies have been selected, because they are the characteristic frequencies in the case of a cable which works at a speed of 600 letters per minute, as is the case in the cable which is considered here. The values calculated from the mathematical Formulas (5d) and (4d) for the effective inductance and resistance, respectively, at frequencies 20 and 30 will be the same as those derived from experiment for these frequencies, if Equation (9) is fulfilled; it will be fulfilled if for a suitably selected value of inductance \( L \) we select proper values for \( R \) and \( R_1 \); hence \( q \) is a constant, which has a definite value assigned to it by experimental measurement. The selection, however, must be such that not only is Equation (9) satisfied, but also Equations (5), (6), (7), and (8). This can be done as follows:

\[
a_0 = \frac{2\pi \times 10}{q} \text{ according to (4a) and (9).} \quad \text{(10)}
\]

Hence the constant \( a_0 \) just like \( q \) is fixed by the experimental measurements of the impedances at \( f = 20 \) and \( f = 30 \). It will be shown now that the value of \( L_0 \) is also fixed by these measurements. From (5) and (6) follows:

\[
10L'_{20} - L_0 = 1 + \frac{9a_0^2}{1 + 4a_0^2} \quad \text{(11)}
\]

\[
10L'_{30} - L_0 = 1 + \frac{9a_0^2}{1 + 4a_0^2} \quad \text{(12)}
\]

which gives \( L_0 \) in terms of the experimentally determined constants \( 10L'_{20}, 10L'_{30}, \) and \( a_0 \). That is to say, inductance \( L_0 \) is fixed by the experimental measurements; it cannot be selected arbitrarily.

Again from (5) it follows that

\[
(1-a)^2 = \frac{(10L'_{30} - L_0)(1 + 4a_0^2)}{L} \quad \text{(13)}
\]

so that having a coil of inductance \( L \), suitably selected, fixes the value of the constant \( a_0 \). But no matter how we select the value of \( L \) the product \( L(1-a)^2 \) must have a definite value. From (4d) follows:

\[
R + R_1 = \frac{2\pi \times 10L}{a_0} \quad \text{(14)}
\]

We also have

\[
(1-a) = \frac{R_1}{R + R_1} \quad \text{(15)}
\]

Therefore

\[
R_1 = \frac{2\pi \times 10L}{L(1-a)} \quad \text{(16)}
\]

With a given coil of inductance \( L \) the value of the shunt resistance \( R_1 \) is therefore fixed.

From

\[
(1-a) = \frac{R_1}{R + R_1} \quad \text{(17)}
\]

follows that

\[
R = \frac{aR_1}{(1-a)} \quad \text{(18)}
\]

so that the internal resistance \( R \) of the shunted coil is also fixed. The only remaining constant to be determined is \( R_0 \).
From (7) follows
\[ R_v = 10R_v' - \frac{(a + 4a^2)R_v}{1 + 4a^2} \quad \cdots (15) \]
This shows that the two impedance measurements at frequencies 20 and 30 and the selection of the value of \( L \) determine in a perfectly definite manner all of the constants in the fundamental Equations (3d) and (4d).

Nothing is arbitrary except the selection of the value of \( L \). In other words, if a sectional artificial line with shunted inductance in its sections is to imitate a submarine cable which is to be balanced by it then the theory and the experimental measurements described here furnish a complete and unique information for the design of these sections. The following numerical calculations will illustrate this:

**Numerical calculations of the constants**

Curves A and B Fig. (2) give:
\[ 10L_{20} = 42.6 \times 10^{-3} \quad 10R_{20} = 27.3 \]
\[ 10L_{30} = 32.5 \times 10^{-3} \quad 10R_{30} = 28.4 \]
Hence
\[ R_{20}' = - R_{30}' = 1.1 \times 10^3 \quad L_{20} - L_{30} = 10.1 \]

From Equation (10)
\[ a_v = \frac{2\pi \times 10}{109} = 0.576 \]
Therefore
\[ a_v^2 = (0.576)^2 = 0.332 \]

From Equation (11)
\[ L_v = 18.3 \times 10^{-3} \]
If we make \( L_v = 0.15 \), then from Equation (12) and (13),
\[ (3a_v) = 6.135 \]

Therefore
\[ a_v = 0.3865 \]
\[ L_v(1 - a_v^2) = 56.62 \text{ millihenry's} \]
\[ \frac{R}{R_v} = 0.63 \]

From Equation (12)
\[ R + R_v = \frac{2\pi 10 \times 1.15}{a_v} \quad 0.576 = 16.5 \]

Hence \( R_v = 10.2 \) and \( R = 6.3 \)

There remains only one constant to determine, that is \( R_v \). Equation (15) gives
\[ R_v = 10R_v' - \frac{(a + 4a^2)R_v}{(1 + 4a^2)} = 19.84 \]
It will be shown now how the effective inductances and resistances calculated from Formulas (3d) and (4d) compare with those obtained from cable measurements and recorded in curves A and B, of Fig. 2.
\[ L_v(1 - a_v^2) = 56.62 \times 10^{-3} \]
\[ L_v = 74.92 \]
\[ L_v = \frac{56.62}{1 + 0.083 + 18.3} = 70.60 \quad (74.00) \]
\[ L_v = \frac{56.62}{1 + 0.332 + 18.3} = 60.73 \quad (62.40) \]
\[ L_v = \frac{56.62}{1 + 2.25 \times 332 + 18.3} = 50.65 \quad (50.90) \]
\[ L_v = \frac{56.62}{1 + 4 \times 332} + 18.3 = 42.60 \quad (42.60) \]
\[ L_v = \frac{56.62}{1 + 6.25 \times 332 + 18.3} = 36.50 \quad (36.50) \]
\[ L_v = \frac{56.62}{1 + 9 \times 332 + 18.3} = 32.45 \quad (32.45) \]
\[ L_v = \frac{62}{1 + 16 \times 332 + 18.3} = 27.30 \quad (28.30) \]
\[ L_v = \frac{56.62}{1 + 25 \times 332 + 18.3} = 24.40 \quad (26.00) \]
All these inductances are in millihenrys.
The numerals in brackets are those from curves A and B, of Fig. 2. The results calculated from the formulae of the mathematical theory disclosed here are, therefore, in splendid agreement with the observed results for a band of frequencies between \( f = 5 \), and \( f = 50 \). This is the band which includes all the important frequencies in ordinary cable signalling at the speed of 600 letters per minute. The curves plotted from these calculated data indicated by dots (.) in Fig. 2 practically coincide with curves A and B of Fig. 2. Their deviation is visible on account of the large scale employed in the diagram of this figure.

If, however, from these calculated values of the effective inductances and resistances of the artificial line per section its terminal resistances and resistances are calculated and the curves plotted then their deviations from curves A and B in Fig. 1, are scarcely visible. The points of these curves are given by the dots (.) in this figure. Differences can be detected by very accurate impedance measurements, and these differences are found to be extremely small; they are not detectable at all in the vicinity of frequency 20, which is the characteristic frequency at the speed of 600 letters per minute. Calculation shows that even at frequencies 5 and 50 these differences are very small. Thus if we denote by \( a_{10}, a_{40} \), and \( b_{10}, b_{40}, b_{50} \), the terminal resistances and reactances, respectively, at frequencies 10, 40, and 50 we shall have:

\[
\begin{align*}
a_{10} &= 247 (248) \quad a_{40} = 137.4 (138) \quad a_{50} = 125.3 (125) \\
b_{10} &= 212 (212) \quad b_{40} = 108.8 (108.5) \quad b_{50} = 96.7 (96)
\end{align*}
\]

The numerals in brackets denote the values obtained by Wheatstone bridge measurements, and recorded in curves A and B in Fig. 1. Even at frequency \( f = 5 \) the agreement between the terminal resistance and reactance of the inductive artificial line described here is to within less than 1% with those obtained on the cable by Wheatstone bridge measurements.

The remarkable agreement between the mathematical theory disclosed here and the experimental measurements reveals the following physical fact: The effective average resistance and inductance of cable elements of any submarine cable consist of two parts. One part is independent of the frequency and it is taken care of by the unshunted coil in each section of the artificial line; the other part varies with the frequency according to the same curves of variation as the effective resistance and inductance of a suitably shunted inductance coil, and this is taken care of by the shunted coil in each section. The curves which exhibit this physical fact must be determined by Wheatstone bridge measurements as explained here. They are the physical basis of this invention. Previous attempts to construct an inductive artificial line which imitates its corresponding cable did not as far as applicant is aware recognize the existence of this physical fact and hence they failed.

The cable under consideration here, when in commercial operation, has a length of 974.33 n. m. Since its constants are:

\[
C = 0.384 \times 10^{-8} \text{farad} \\
R = 2.45 \text{ ohms}
\]

it is obvious that the signal components corresponding to frequencies smaller than 6 p. p. s. and reflected at the receiving terminal of the cable will be appreciable when they return to the transmitting end. This will disturb the balance if suitable provisions are not made. Hence, having constructed the inductive artificial line in the manner described here, and having made it equivalent to 974.33 n. m. of the cable, it is necessary to provide additional means, in order to make the low frequency part of the signal reflected at the receiving end of the cable equal to the corresponding reflected part at the furthest terminal of the inductive artificial line. This is accomplished by the well known method which has been in practice for many years. The method consists in attaching to the terminal of the artificial line a network of conductors which has the same impedance for the low frequencies as the network which is employed at the receiving end of the cable. This is indicated in the diagram of Fig. 4. Referring to this diagram it represents two cable stations employing the same cable for electrical exchange of messages by the duplex method of operation, and balanced with my improved artificial line.

The numerals 1, 2, 3, etc. refer to the apparatus at one end of the cable, and 1', 2', 3', etc. refer to similar apparatus at the other end. The apparatus at one end, only, will be described; the same description applies to the other end.

I denotes the sending generator; 2, 3 denote two condensers in the two sides of the Wheatstone bridge, having in one of its arms the cable 7 and in the other the inductive artificial line described above and represented here by coils 11, 12, 13, 14, etc., and condensers 23, 24, 25, etc. In each section of the inductive artificial line there are two inductance coils, 11, 12, etc., one of which is shunted by a non-inductive resistance, 19, etc. The artificial line and the sending generator are connected as usual through so-called sea-earths at 8 and 9 to ground. An impedance 5 is connected in series with the cable 10 and serves to balance the impedance which the sea-earth 9 places in series with the inductive artificial line. Numeral 4 denotes the signal receiving apparatus. At the end of the inductive artificial line is a variable capacity 29 and a variable resistance 30 by the adjustments of which their impedance is
made approximately equal to the impedance of the network which connects the cable to ground at the receiving station. The receiving apparatus in cross-arm 4 may be any suitable curbing and amplifying system of conductors with local sources of energy supply.

The very object of this invention is to increase the speed of transmission and by improved balancing reduce the degree of unbalance to such an extent that the reduced strength of the signal due to increased speed will still be considerably greater than the strength of the unbalance. Under these conditions powerful vacuum tube amplifiers can do excellent service and this invention makes it possible to employ this service with much effect. It is, therefore, understood that vacuum amplifiers form a part of the signaling system described here.

The inductive artificial line described here has been assumed to consist of equal sections, each section being equivalent to a cable length of 10 n. m. This assumption simplifies the explanation of the structure; it also simplifies its manufacture. But the well-known mathematical theory of sectional wave conductors makes it obvious that the sections may be much longer at signalling speeds of 600 letters per minute. It is also obvious that the artificial line may be divided into groups of sections in which each section of one group represents a cable length of say 10 n. m.; each section of the next group represents a cable length of 15 n. m.; and each section of the next group represents a cable length of 20 n. m. or even 30 n. m. It is obvious, however, that each group must have its inductance coils adjusted in accordance with the mathematical theory given above.

Mathematical theory demands also that the capacity at the transmitting end of the artificial line must be half as large as the other capacities; that is, it must be $1.92 \times 10^6$ farads in the case described above.

It is essential that the inductance coils used in the balancing network have a negligible external field. Otherwise they will have mutual inductance with external circuits and will therefore be sensitive to external disturbances. This is specified in claim 7 by the words "closed magnetic circuit"; the word "closed" meaning that the lines of force of the magnetic circuit have no inter-linkage with external electrical circuits. A toroidal coil is the simplest illustration of a closed magnetic field of this kind, but there are other forms of coils with paramagnetic cores which have a negligibly small external field and therefore no mutual inductance with external circuits. Another essential requirement is that the effective inductance and its accompanying effective resistance must not vary appreciably with the intensity of the signaling current. An inductance varying with the strength of the signaling current makes the artificial line unfit for balancing a submarine cable, because the effective average inductance and resistance of ordinary cables do not vary with the strength of the signaling current. Such inductance coils will be described now.

Fig. 5 represents ten coils 31, 31, etc. wound upon ten wooden blocks 32, 32, etc. Figs. 6 and 7 show these blocks with holes indicated by numerals 3. These holes permit the blocks to be slipped over a metal rod and clamped so as to form a rigid wooden rectangular pattern. The ten blocks forming this pattern are permanently coupled by a flexible coupling consisting of a strip of linen indicated by the black line 34 in Fig. 7 and by the dotted lines 34 in Fig. 5. Coils 31, 31, etc., are then wound by a machine and thus it is possible to make the individual coils equal to each other in every respect.

When all the ten coils have been wound the clamping rod is removed from the rectangular pattern and the ten coils, connected in series and coupled to each other by the linen ribbon, are then distributed symmetrically around a circular wooden cylinder forming a single symmetrical coil consisting of a plurality of individual coils as represented in Figs. 8, 9, and 7. Such a coil may be made to retain its form permanently by pouring around it an insulating compound made fluid by heat which hardens upon cooling. Fig. 8 shows a cross-sectional view of the coil after the insulating compound has been applied. An air-cored coil of this construction has no appreciable external magnetic field except at points in the immediate vicinity of the surface of the winding. The insertion into this part of the magnetic field of a thin sheet of magnetic material like silicon steel offers a means of a small adjustment of the inductance of the air-cored coil. The air-cored coil acts like an ideal toroidal coil. Its inductance and resistance do not vary with the strength of the signaling current. The inductance coil shown in detail in Figs. 5, 6, 7 and 8 is the inductance coil 3, Fig. 3; and 11, 13, 15, 17, etc., Fig. 4. It is an unshunted inductance coil, and has magnetically neutral core material as indicated above.

A similar coil but wound upon a paramagnetic core is shown in Figs. 9 and 10. Fig. 9 is a top view and Fig. 10 represents the cross-section of the coil. The core 35 consists of circular rings of cold rolled sheet nickel having a permeability of about 11. The plates are about 14 mils thick. The external field of such a coil is vanishingly small. Neither this coil nor the air-cored coil have any mutual inductance with external circuits and, hence, no external electrical disturbances can affect the circuit of such coils. The nickel cored inductance coil is employed in every section of the inductive artificial line as a
shunted inductance, the air-cored coil being the unshunted inductance. The nickel cored inductance coil, shown in detail in Figs. 9 and 10 has a core of paramagnetic material whose permeability varies less with changing magnetizing forces than does that of iron, and is the shunted inductance coil 4, Fig. 3; and 12, 14, 16, 18, etc., Fig. 4.

The nickel cored coil being shunted by resistance 7 (Fig. 3) receives a part, only, of the signaling current which passes through the air-cored coil, whereby the magnetizing force of this current upon the nickel core is diminished. It can be shown that if A be the amplitude of the signaling current passing through the winding of the air-cored coil and B the amplitude of the current which passes through the shunted winding of the nickel cored coil then

\[ B = A \left( \frac{L_1 - L_2}{L} \right) \]

where

\[ L'' \text{ = effective inductance for frequency } 10^{10}, \text{ of the cable length which is equivalent to one section of the inductive artificial line.} \]

\[ L_0 = \text{inductance of the air-cored coil.} \]

\[ L = \text{inductance of the nickel cored coil before being shunted.} \]

The intensity of the magnetizing force of B can be easily calculated by the mathematical theory disclosed here, when the amplitude A of the signaling current is given. Ordinarily A = 10^{-9} in absolute units.

Let the dimensions and the permeability of the nickel core be as follows:

Outside diameter = 9 x 2.54 cm.

Inside = 6 x 2.54 cm.

Height = 4 x 2.54 cm.

Permeability \( \mu = 11 \)

When A = 10^{-9}, then a simple calculation by means of the mathematical theory given here will give \( H = 1 \), where H is the mean intensity of the magnetizing force.

The permeability of cold rolled nickel sheet varies very little in the interval between H = 0 and H = 1, very much less than that of cold rolled sheet steel or even that of ordinary iron dust cores; hence the desirability of employing nickel cores for the inductance coils, the inductance of which shall not vary with the intensity of the magnetizing force, employed in cable telegraphy.

Having thus described the physical and the mathematical foundation of the invention and the best means of carrying the same into effect, I claim:

1. In a submarine cable duplex telegraph system a balancing artificial line comprising a plurality of sections, each section containing a plurality of independent inductance coils connected in series, one at least of which is shunted by a non-inductive resistance element.

2. In a submarine cable duplex telegraph system a balancing artificial line comprising a plurality of sections, each section containing a plurality of simple independent inductance coils connected in series, one at least of which is shunted by a non-inductive resistance, and shunt capacity elements.

3. In a submarine cable duplex telegraph system a balancing artificial line comprising a plurality of sections, each section comprising a series arm and a shunt arm and containing in its series arm two simple and independent inductance coils connected in series, one of which is shunted by a non-inductive resistance element, and containing in its shunt arm a capacity element.

4. In a submarine cable duplex telegraph system a balancing artificial line comprising a plurality of sections, each section comprising a series arm and a shunt arm and containing in its series arm two simple and independent inductance coils connected in series, one of which is shunted by a non-inductive resistance element, and containing in its shunt arm a capacity element.

5. Means as set forth in claim 2 further characterized in this, that the unshunted inductance coils have magnetically neutral core material, and the shunted coils have cores of a paramagnetic material the variation in permeability with varying magnetizing forces of which is less than that of iron.

6. Means as set forth in claim 2 further characterized in this, that the unshunted inductance coils have cores of neutral magnetic material and the shunted inductance coils have nickel cores.

7. Means as set forth in claim 2 further characterized in this, that each inductance coil has a closed magnetic circuit whereby its external magnetic field is minimized.

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