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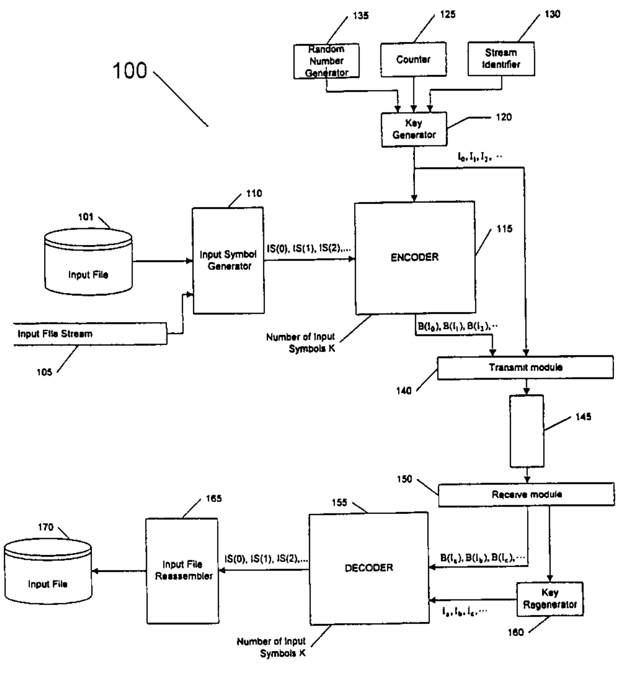


(57) Abstract: An encoder uses an input file of data and a key to produce a group of output symbols. A group of output symbols with key I is generated by determining a weight, W(I), for the group of output symbols to be generated, selecting W(I) of the input symbols associated with the group according to a function of I, and generating the output symbol values B(I) from a predetermined value function F(I) of the selected W(I) input symbols. An encoder can be called repeatedly to generate multiple groups of output symbols or multiple output symbols. The groups of output symbols are generally independent of each other, and an unbounded number (subject to the resolution of I) can be generated, if needed. A decoder receives some or all of the output symbols generated. The number of output symbols needed to decode an input file is equal to, or slightly greater than, the number of input symbols comprising the file, assuming that input symbols and output symbols represent the same number of bits of data.

INFORMATION ADDITIVE GROUP CODE GENERATOR AND DECODER FOR COMMUNICATION SYSTEMS

CROSS REFERENCE TO RELATED APPLICATIONS

The present application is a continuation-in-part of U.S. Patent Application 09/246,015 entitled "INFORMATION ADDITIVE CODE GENERATOR AND DECODER FOR COMMUNICATION SYSTEMS" and filed on February 5, 1999 (hereinafter "Luby I"). The present application also claims priority from co-pending U.S. Provisional Patent Application 60/101,473 entitled "ENCODING AND DECODING DATA IN COMMUNICATION SYSTEMS" and filed on September 23, 1998 hereinafter ("Luby Provisional"). The present application illustrates the use of group codes, such as Reed-Solomon codes, as the basis of output symbol representation in more detail than might be shown in Luby I.

BACKGROUND OF THE INVENTION

The present invention relates to encoding and decoding data in communications systems and more specifically to communication systems that encode and decode data to account for errors and gaps in communicated data, and to efficiently utilize communicated data emanating from more than one source.

Transmission of files between a sender and a recipient over a communications channel has been the subject of much literature. Preferably, a recipient desires to receive an exact copy of data transmitted over a channel by a sender with some level of certainty. Where the channel does not have perfect fidelity (which covers most all physically realizable systems), one concern is how to deal with data lost or garbled in transmission. Lost data (erasures) are often easier to deal with than garbled data (errors) because the recipient cannot always tell when garbled data is data received in error. Many error correcting codes have been developed to correct for erasures (so called "erasure codes") and/or for errors ("error-correcting codes", or "ECC's"). Typically, the particular code used is chosen based on some information about the infidelities of the channel through which the data is being transmitted and the nature of the data being transmitted. For example, where the channel is known to have long periods of infidelity, a burst error code might be best suited for that application. Where only short, infrequent errors are expected a simple parity code might be best.

File transmission between multiple senders and/or multiple receivers over a communications channel has also been the subject of much literature. Typically, file transmission from multiple senders requires coordination among the multiple senders to allow the senders to minimize duplication of efforts. In a typical multiple sender system sending one file to a receiver, if the senders do not coordinate which data they will transmit and when, but instead just send segments of the file, it is likely that a receiver will receive many useless duplicate segments. Similarly, where different receivers join a transmission from a sender at different points in time, a concern is how to ensure that all data the receivers receive from the sender is useful. For example, suppose the sender is continuously transmitting data about the same file. If the sender just sends segments of the original file and some segments are lost, it is likely that a receiver will receive many useless duplicate segments before receiving one copy of each segment in the file.

Another consideration in selecting a code is the protocol used for transmission. In the case of the global internetwork of networks known as the "Internet" (with a capital "I"), a packet protocol is used for data transport. That protocol is called the Internet Protocol or "IP" for short. When a file or other block of data is to be transmitted over an IP network, it is partitioned into equal size input symbols and input symbols are placed into consecutive packets. Being packet-based, a packet oriented coding scheme might be suitable. The "size" of an input symbol can be measured in bits, whether or not the input symbol is actually broken into a bit stream, where an input symbol has a size of M bits when the input symbol is selected from an alphabet of 2^M symbols.

The Transport Control Protocol ("TCP") is a point-to-point packet control scheme in common use that has an acknowledgment mechanism. TCP is good for one-to-one communications, where the sender and recipient both agree on when the transmission will take place and be received and both agree on which transmitters and receivers will be used. However, TCP is often not suitable for one-to-many or many-to-many communications or where the sender and the recipient independently determine when and where they will transmit or receive data.

Using TCP, a sender transmits ordered packets and the recipient acknowledges receipt of each packet. If a packet is lost, no acknowledgment will be sent to the sender and the sender will resend the packet. Packet loss has a number of causes. On the Internet, packet loss often occurs because sporadic congestion causes the buffering mechanism in a router to reach its capacity, forcing it to drop incoming packets. With protocols such as TCP/IP, the acknowledgment paradigm allows packets to be lost without total failure, since lost packets can just be retransmitted, either in response to a lack of acknowledgment or in response to an explicit request from the recipient. Either way, an acknowledgment protocol requires a back channel from the recipient to the sender.

Although acknowledgment-based protocols are generally suitable for many applications and are in fact widely used over the current Internet, they are inefficient, and sometimes completely infeasible, for certain applications. In particular, acknowledgment-based protocols perform poorly in networks with high latencies, high packet loss rates, uncoordinated recipient joins and leaves, and/or highly asymmetric bandwidth. High latency is where acknowledgments take a long time to travel from the recipient back to the sender. High latency may result in the overall time before a retransmission being prohibitively long. High packet loss rates also cause problems where several retransmissions of the same packet may fail to arrive, leading to a long delay to obtain the last one or last few unlucky packets.

"Uncoordinated recipient joins and leaves" refers to the situation where each recipient can join and leave an ongoing transmission session at their own discretion. This situation is typical on the Internet, next-generation services such as "video on demand" and other services to be offered by network providers in the future. In the typical system, if a recipient joins and leaves an ongoing transmission without coordination of the senders, the recipient will likely perceive a loss of large numbers of packets, with widely disparate loss patterns perceived by different recipients.

Asymmetric bandwidth refers to the situation is where a reverse data path from recipient to sender (the back channel) is less available or more costly than the forward path. Asymmetric bandwidth may make it prohibitively slow and/or expensive for the recipient to acknowledge packets frequently and infrequent acknowledgments may again introduce delays.

Furthermore, acknowledgment-based protocols do not scale well to broadcasting, where one sender is sending a file simultaneously to multiple users. For example, suppose a sender is broadcasting a file to multiple recipients over a satellite channel. Each recipient may experience a different pattern of packet loss. Protocols that rely on acknowledgment data (either positive or negative) for reliable delivery of the file require a back channel from each recipient to the sender, and this can be prohibitively expensive to provide. Furthermore, this requires a complex and powerful sender to be able to properly handle all of the acknowledgment data sent from the recipients. Another drawback is that if different recipients lose different sets of packets, rebroadcast of packets missed only by a few of the recipients causes reception of useless duplicate packets by other recipients. Another situation that is not handled well in an acknowledgment-based communication system is where recipients can begin a receiving session asynchronously, i.e., the recipient could begin receiving data in the middle of a transmission session.

Several complex schemes have been suggested to improve the performance of acknowledgment-based schemes, such as TCP/IP for multicast and broadcast. However none has been clearly adopted at this time, for various reasons. For one, acknowledgment-based protocols also do not scale well where one recipient is obtaining information from multiple senders, such as in a low earth orbit ("LEO") satellite broadcast network. In an LEO network, the LEO satellites pass overhead quickly because of their orbit, so the recipient is only in view of any particular satellite for a short time. To make up for this, the LEO network comprises many satellites and recipients are handed off between satellites as one satellite goes below the horizon and another rises. If an acknowledgment-based protocol were used to ensure reliability, a complex hand-off protocol would probably be required to coordinate acknowledgments returning to the appropriate satellite, as a recipient would often be receiving a packet from one satellite yet be acknowledging that packet to another satellite.

An alternative to an acknowledgment-based protocol that is sometimes used in practice is a carousel-based protocol. A carousel protocol partitions an input file into equal length input symbols, places each input symbol into a packet, and then continually cycles through and transmits all the packets. A major drawback with a carousel-based protocol is that if a recipient misses even one packet, then the recipient has to wait another entire cycle before having a chance at receiving the missed packet. Another way to view this is that a carousel-based protocol can cause a large amount of useless duplicate data reception. For example, if a recipient receives packets from the beginning of the carousel, stops reception for a while, and then starts receiving again at the beginning of the carousel, a large number of useless duplicate packets are received.

One solution that has been proposed to solve the above problems is to avoid the use of an acknowledgment-based protocol, and instead use erasure codes such as Reed-Solomon Codes to increase reliability. One feature of several erasure codes is that, when a file is segmented into input symbols that are sent in packets to the recipient, the recipient can decode the packets to reconstruct the entire file once sufficiently many packets are received, generally regardless of which packets arrive. This property removes the need for acknowledgments at the packet level, since the file can be recovered even if packets are lost. However, many erasure code solutions either fail to solve the problems of acknowledgment-based protocol or raise new problems.

One problem with many erasure codes is that they require excessive computing power or memory to operate. One coding scheme that has been recently developed for communications applications that is somewhat efficient in its use of computing power and memory is the Tornado coding scheme. Tornado codes are similar to Reed-Solomon codes in that an input file is represented by K input symbols and is used to determine N output symbols, where N is fixed before the encoding process begins. Encoding with Tornado codes is generally much faster than encoding with Reed-Solomon codes, as the average number of arithmetic operations required to create the N Tornado output symbols is proportional to N (on the order of tens of assembly code operations times N) and the total number of arithmetic operations required to decode the entire file is also proportional to N.

Tornado codes have speed advantages over Reed-Solomon codes, but with several disadvantages. First, the number of output symbols, N, must be determined in advance of the coding process. This leads to inefficiencies if the loss rate of packets is overestimated, and can lead to failure if the loss rate of packets is underestimated. This is because a Tornado decoder requires a certain number of output symbols (specifically, K + A output symbols, where A is small compared to K) to decode and restore the original file and if the number of lost output symbols is greater than N - (K + A), then the original file cannot be restored. This limitation is generally acceptable for many communications problems, so long as N is selected to be greater than K + A by at least the actual packet loss, but this requires an advance guess at the packet loss.

Another disadvantage of Tornado codes is that they require the encoder and decoder to agree in some manner on a graph structure. Tornado codes require a pre-processing stage at the decoder where this graph is constructed, a process that slows the decoding substantially. Furthermore, a graph is specific to a file size, so a new graph needs to be generated for each file size used. Furthermore, the graphs needed by the Tornado codes are complicated to construct, and require different custom settings of parameters for different sized files to obtain the best performance. These graphs are of significant size and require a significant amount of memory for their storage at both the sender and the recipient.

In addition, Tornado codes generate exactly the same output symbol values with respect to a fixed graph and input file. These output symbols comprise the K original input symbols and N-K redundant symbols. Furthermore, N can practically only be a small multiple of K, such as 1.5 or 2 times K. Thus, it is very likely that a recipient obtaining output symbols generated from the same input file using the same graph from more than one sender will receive a large number of useless duplicate output symbols. That is because the N output symbols are fixed ahead of time and are the same N output symbols that are transmitted from each transmitter each time the symbols are sent and are the same N symbols received by a receiver. For example, suppose N=1500, K=1000 and a receiver receives 900 symbols from one satellite before that satellite dips over the horizon. Unless the satellites are coordinated and in sync, the Tornado code symbols received by the receiver from the next satellite might not be additive because that next satellite is transmitting the same N symbols, which is likely to result in the receiver receiving copies of many of the already received 900 symbols before receiving 100 new symbols needed to recover the input file.

Therefore, what is needed is a simple erasure code that does not require excessive computing power or memory at a sender or recipient to implement, and that can be used to efficiently distribute a file in a system with one or more senders and/or one or more recipients without necessarily needing coordination between senders and recipients.

SUMMARY OF THE INVENTION

In one embodiment of a communications system according to the present invention, an encoder uses an input file of data and a key to produce a group of output symbols, wherein the input file is an ordered plurality of input symbols each selected from an input alphabet, the key is selected from a key alphabet, a group size is selected as a positive integer, and each output symbol is selected from an output alphabet. A group of output symbols with key I is generated by determining a group size, G(I), for the group of output symbols to be generated, wherein the group sizes, G, are positive integers that can vary between at least two values over the plurality of keys, determining a weight, W(I), for the group of output symbols to be generated, wherein the weights W are positive integers that vary between at least two values over the plurality of keys, selecting W(I) of the input symbols associated with the group of output symbols according to a function of I, and generating a group of G(I) output symbol values B(I) = B_1(I),...,B_G(I)(I) from a predetermined value function F(I) of the selected W(I) input symbols. The encoder may be called one or more times, each time with another key, and each such time it produces a group of output symbols. The groups of output symbols are generally independent of each other, and an unbounded number (subject to the resolution of I) can be generated, if needed.

In some embodiments, the number of output symbols in a group, G(I), varies from group to group depending on the key I. In other embodiments, G(I) is fixed at some positive integer value, b, for all groups of output symbols for all keys I.

In a decoder according to the present invention, output symbols received by a recipient are output symbols transmitted from a sender, which generated those output symbols in groups based on an encoding of an input file. Because output symbols can be lost in transit, the decoder operates properly even when it only receives an arbitrary portion of the transmitted output symbols. The number of output symbols needed to decode the input file is equal to, or slightly greater than, the number of input symbols comprising the file, assuming that input symbols and output symbols represent the same number of bits of data.

In one decoding process according to the present invention, the following steps are performed for each received group of output symbols: 1) identify the key I of the received group of b output symbols; 2) identify the group size G(I); 3) identify the received group of output symbol values B(I) = B_1(I),...,B_G(I)(I) for the group of output symbols; 3) determine the weight, W(I), of the group of output symbols, 4) determine positions for the W(I) associated input symbols associated with the group of output symbols, and 5) store B(I) = B_1(I) ,..., B_G(I)(I) in a table that holds the groups of output symbols and store the weight W(I) and the positions of the associated input symbols. The following process is then applied repeatedly until there are no more unused groups of output symbols with a key I such that the weight of the group is at most G(I): 1) for each stored group of output symbols with key I that has a weight of b' ≤ G(I) and is not denoted as a "used up" group of output symbols, calculate the positions J_1,...,J_b' of the unique remaining b' unrecovered input symbols associated with the group of output symbols based on its key I; 2) calculate the unknown values for input symbols J_1,...,J_b' from the group of output symbols and the known values of the other input symbols associated with the group; 3) identify the groups of output symbols in the group of output symbols table that have at least one of the input symbols J_1,...,J_b' as an associate; 4) decrement the weights of each of these identified groups of output symbols by one for each associate they have among J_1,...,J_b'; and 5) denote input symbols J_1,...,J_b' as recovered and the group of output symbols with key I as used up. This process is repeated until the ordered set of input symbols is recovered, i.e., until the entire input file is completely recovered.

One advantage of the present invention is that it does not require that the recipient begin receiving at any given point in the transmission and it does not require that the sender stop after a set number of groups of output symbols are generated, since the sender can send an effectively unbounded set of groups of output symbols for any given input file. Instead, a recipient can begin reception when it is ready, and from wherever it can, and can lose packets containing groups of output symbols in a random pattern or even an arbitrary pattern, and still have a good chance that the great majority of the data received is "information additive" data, i.e., data that helps in the recovery process rather than being duplicative of information already available. The fact that independently generated (often randomly unrelated) data streams are coded in an information additive way leads to many advantages in that it allows for multiple source generation and reception, erasure robustness and uncoordinated joins and leaves.

A further understanding of the nature and the advantages of the inventions disclosed herein may be realized by reference to the remaining portions of the specification and the attached drawings.

BRIEF DESCRIPTION OF THE DRAWINGS

Fig. 1 is a block diagram of a communications system according to one embodiment of the present invention.

Fig. 2 is a block diagram showing the encoder of Fig. 1 is greater detail.

Fig. 3 is an illustration of how a group of output symbols might be generated from a set of associated input symbols.

Fig. 4 is a block diagram of a basic decoder as might be used in the communications system shown in Fig. 1.

Fig. 5 is a block diagram of an alternative decoder.

Fig. 6 is a flowchart of a process that might be used by a decoder, such as the decoder shown in Fig. 5, to recover input symbols from a set of groups of output symbols.

Fig. 7 is a flowchart of process that might be used by a receive organizer, such as the receive organizer shown in Fig. 5, to organize received groups of output symbols.

Fig. 8(a) is a flowchart of process that might be used by a recovery processor, such as the recovery processor shown in Fig. 5, to process received groups of output symbols.

Figs. 8(b)-(c) form a flowchart of portions of a variation of the process of Fig. 8(a), with Fig. 8(b) showing steps performed in the recovery process including deferred processing and Fig. 8(c) showing the deferred processing.

Fig. 9 is a block diagram showing the associator of Fig. 2 in greater detail.

Fig. 10 is a flowchart of one process that might be used by an associator, such as the associator shown in Fig. 9, to quickly determine the association of input symbols with groups of output symbols.

Fig. 11 is a block diagram showing the weight selector of Fig. 2 in greater detail.

Fig. 12 is a flowchart of a process that might be used by a weight selector, such as the weight selector shown in Fig. 11, to determine a weight for a given group of output symbols.

Fig. 13 is a flowchart of a process for decoding for a decoder that does not need to be particularly efficient.

Fig. 14 is a block diagram of a more efficient decoder.

Fig. 15 is a flowchart of a process for decoding as might be implemented using the decoder of Fig. 14 for decoding more efficiently than the decoding described with reference to Figs. 12-13.

Fig. 16 is a diagram illustrating an example of a document and received groups of output symbols for the decoding process of Fig. 15.

Fig. 17 illustrates the contents of tables in a decoder during the decoding process shown in Fig. 15.

Fig. 18 is a diagram illustrating the contents of a weight sort table as might be used during the decoding process shown in Fig. 15.

Fig. 19 illustrates an execution list that might be formed during the decoding process shown in Fig. 15.

Fig. 20 illustrates the progress of the recovery process in the form of a plot of decodable set size versus number of input symbols recovered for an ideal distribution.

Fig. 21 illustrates the progress of the recovery process in the form of a plot of decodable set size versus number of input symbols recovered for a robust weight distribution.

Fig. 22 is an illustration of a point-to-point communication system between one sender (transmitter) and one receiver using an encoder and a decoder as illustrated in previous figures.

Fig. 23 is an illustration of a broadcast communication system between one sender and multiple receivers (only one of which is shown) using an encoder and a decoder as illustrated in previous figures.

Fig. 24 is an illustration of a communication system according to one embodiment of the present invention where one receiver receives groups of output symbols from multiple, usually independent, senders.

Fig. 25 is an illustration of a communication system according to one embodiment of the present invention where multiple, possibly independent, receivers receives groups of output symbols from multiple, usually independent, senders to receive an input file in less time than if only one receiver and/or only one sender is used.

Appendix A is a source code listing of a program for implementing a weight distribution.

DESCRIPTION OF THE PREFERRED EMBODIMENTS

In the examples described herein, a coding scheme denoted as "group chain reaction coding" is described, preceded by an explanation of the meaning and scope of various terms used in this description.

With group chain reaction coding, groups of output symbols are generated by the sender from the input file as needed. Each group of output symbols can be generated without regard to how other groups of output symbols are generated. At any point in time, the sender can stop generating groups of output symbols and there need not be any constraint as to when the sender stops or resumes generating groups of output symbols. Once generated, these groups of output symbols can then be placed into packets and transmitted to their destination, with each packet containing one or more groups of output symbols. In a specific example of group chain reaction coding, each group comprises one output symbol and this results in a system with similar performance and behavior as the chain reaction coding system described in Luby I.

As used herein, the term "file" refers to any data that is stored at one or more sources and is to be delivered as a unit to one or more destinations. Thus, a document, an image, and a file from a file server or computer storage device, are all examples of "files" that can be delivered. Files can be of known size (such as a one megabyte image stored on a hard disk) or can be of unknown size (such as a file taken from the output of a streaming source). Either way, the file is a sequence of input symbols, where each input symbol has a position in the file and a value.

Transmission is the process of transmitting data from one or more senders to one or more recipients through a channel in order to deliver a file. If one sender is connected to any number of recipients by a perfect channel, the received data can be an exact copy of the input file, as all the data will be received correctly. Here, we assume that the channel is not perfect, which is the case for most real-world channels, or we assume that the data emanates from more than one sender, which is the case for some systems, or we assume some portion of the transmitted data is purposely dropped en route, which is the case for some systems. Of the many channel imperfections, two imperfections of interest are data erasure and data incompleteness (which can be treated as a special case of data erasure). Data erasure occurs when the channel loses or drops data. Data incompleteness occurs when a recipient does not start receiving data until some of the data has already passed it by, the recipient stops receiving data before transmission ends, or the recipient intermittently stops and starts again receiving data.

As an example of data incompleteness, a moving satellite sender might be transmitting data representing an input file and start the transmission before a recipient is in range. Once the recipient is in range, data can be received until the satellite moves out of range, at which point the recipient can redirect its satellite dish (during which time it is not receiving data) to start receiving the data about the same input file being transmitted by another satellite that has moved into range. As should be apparent from reading this description, data incompleteness is a special case of data erasure, since the recipient can treat the data incompleteness (and the recipient has the same problems) as if the recipient was in range the entire time, but the channel lost all the data up to the point where the recipient started receiving data. Also, as is well known in the communication systems design, detectable errors can be the equivalent of erasures by simply dropping all data blocks or symbols that have detectable errors.

As another example, routers purposely drop packets when their buffers are full or nearly full (congested), and routers can also purposely drop packets to be fair to competing packets and/or to enforce rate limitations.

In some communication systems, a recipient receives data generated by multiple senders, or by one sender using multiple connections. For example, to speed up a download, a recipient might simultaneously connect to more than one sender to transmit data concerning the same file. As another example, in a multicast transmission, multiple multicast data streams might be transmitted to allow recipients to connect to one or more of these streams to match the aggregate transmission rate with the bandwidth of the channel connecting them to the sender. In all such cases, a concern is to ensure that all transmitted data is of independent use to a recipient, i.e., that the multiple source data is not redundant among the streams, even when the transmission rates are vastly different for the different streams, and when there are arbitrary patterns of loss.

In general, transmission is the act of moving data from a sender to a recipient over a channel connecting the sender and recipient. The channel could be a real-time channel, where the channel moves data from the sender to the recipient as the channel gets the data, or the channel might be a storage channel that stores some or all of the data in its transit from the sender to the recipient. An example of the latter is disk storage or other storage device. In that example, a program or device that generates data can be thought of as the sender, transmitting the data to a storage device. The recipient is the program or device that reads the data from the storage device. The mechanisms that the sender uses to get the data onto the storage device, the storage device itself and the mechanisms that the recipient uses to get the data from the storage device collectively form the channel. If there is a chance that those mechanisms or the storage device can lose data, then that would be treated as data erasure in the channel.

When the sender and recipient are separated by a data erasure channel, it is preferable not to transmit an exact copy of an input file, but instead to transmit data generated from the input file that assists with recovery of erasures. An encoder is a circuit, device, module or code segment that handles that task. One way of viewing the operation of the encoder is that the encoder generates groups of output symbols from input symbols, where a sequence of input symbol values represent the input file. Each input symbol would thus have a position, in the input file, and a value. A decoder is a circuit, device, module or code segment that reconstructs the input symbols from the groups of output symbols received by the recipient.

Group chain reaction coding is not limited to any particular type of input symbol, but the type of input symbol is often dictated by the application. Typically, the values for the input symbols are selected from an alphabet of 2^M symbols for some positive integer M. In such cases, an input symbol can be represented by a sequence of M bits of data from the input file. The value of M is often determined based on the uses of the application, on the channel and on the maximum size of a group. For example, for a packet-based Internet channel, a packet with a payload of size of 1024 bytes might be appropriate (a byte is eight bits). In this example, assuming each packet contains one group of output symbols and eight bytes of auxiliary information, and assuming all groups comprise four output symbols, an input symbol size of $M=(1024 - 8)/4$, or 254 bytes would be appropriate. As another example, some satellite systems use the MPEG packet standard, where the payload of each packet comprises 188 bytes. In that example, assuming each packet contains one group of output symbols and four bytes of auxiliary information, and assuming all groups comprise two output symbols, a symbol size of $M=(188 - 4)/2$, or 92 bytes would be appropriate. In a general-purpose communication system using group chain reaction coding, the application-specific parameters, such as the input symbol size (i.e., M, the number of bits encoded by an input symbol), might be variables set by the application.

Each group of output symbols has a value for each of the output symbols in the group. In one preferred embodiment, which we consider below, each group of output symbols has an identifier called its “key.” Preferably, the key of each group of output symbols can be easily determined by the recipient to allow the recipient to distinguish one group of output symbols from other groups of output symbols. Preferably, the key of a group of output symbols is distinct from the keys of all other groups of output symbols. Also preferably, as little data as possible is included in the transmission in order for a recipient to determine the key of a received group of output symbols.

In a simple form of keying, the sequence of keys for consecutive groups of output symbols generated by an encoder is a sequence of consecutive integers. In this case, each key is called a "sequence number". In the case where there is one group of output symbol values in each transmitted packet, the sequence number can be included in the packet. Since sequence numbers can typically fit into a small number of bytes, e.g., four bytes, including the sequence number along with the group of output symbol values in some systems is economical. For example, using UDP Internet packets of 1024 bytes each, allocating four bytes in each packet for the sequence number incurs only a small overhead of 0.4%.

In other systems, it is preferable to form a key from more than one piece of data. For example, consider a system that includes a recipient receiving more than one data stream generated from the same input file from one or more senders, where the transmitted data is a stream of packets, each containing one group of output symbols. If all such streams use the same set of sequence numbers as keys, then it is likely that the recipient will receive groups of output symbols with the same sequence numbers. Since groups of output symbols with the same key, or in this case with the same sequence number, contain identical information about the input file, this causes useless reception of duplicate data by the recipient. Thus, in such a situation it is preferred that the key comprise a unique stream identifier paired with a sequence number.

For example, for a stream of UDP Internet packets, the unique identifier of a data stream could include the IP address of the sender and the port number that the sender is using to transmit the packets. Since the IP address of the sender and the port number of the stream are parts of the header of each UDP packet, there is no additional space required in each packet to ensure that these parts of the key are available to a recipient. The sender need only insert a sequence number into each packet together with the corresponding group of output symbols, and the recipient can recreate the entire key of a received group of output symbols from the sequence number and from the packet header. As another example, for a stream of IP multicast packets the unique identifier of a data stream could include the IP multicast address. Since the IP multicast address is part of the header of each IP multicast packet, the remarks made above about UDP packets apply to this situation as well.

Keying by the position of the group of output symbols is preferred when it is possible. Position keying might work well for reading groups of output symbols from a storage device, such as a CD-ROM (Compact Disk Read-Only-Memory), where the key of a group of output symbols is its position on the CD-ROM (i.e., track, plus sector, plus location within the sector, etc.). Position keying might also work well for a circuit based transmission system, such as an ATM (Asynchronous Transfer Mode) system, where ordered cells of data are transmitted under tight timing constraints. With this form of keying, the recipient can recreate the key of a group of output symbols with no space required for explicitly transmitting the key. Position keying, of course, requires that such position information be available and reliable.

Keying by position might also be combined with other keying methods. For example, consider a packet transmission system where each packet contains more than one group of output symbols. In this case, the key of the group of output symbols might be constructed from a unique stream identifier, a sequence number, and the position of the group of output symbols within the packet. Since data erasures generally result in the loss of entire packets, the recipient generally receives a full packet. In this case, the recipient can recreate the key of a group of output symbols from the header of the packet (which contains a unique stream identifier), the sequence number in the packet, and the position of the group of output symbols within the packet.

Another form of keying that is preferred in some systems is random keying. In these systems, a random (or pseudo-random) number is generated, used as at least part of the key for each group of output symbols and explicitly transmitted with the group of output symbols. One property of random keying is that the fraction of keys that have the same value is likely to be small, even for keys generated by different senders at different physical locations (assuming the range of possible keys is large enough). This form of keying may have the advantage over other forms in some systems because of the simplicity of its implementation.

As explained above, group chain reaction coding is useful where there is an expectation of data erasure or where the recipient does not begin and end reception exactly when a transmission begins and ends. The latter condition is referred to herein as "data incompleteness". These conditions do not adversely affect the communication process when group chain reaction coding is used, because the group chain reaction coding data that is received is highly independent so that it is information additive. If most arbitrary collections of groups of output symbols are independent enough to be largely information additive, which is the case for the group chain reaction coding systems described herein, then any suitable number of packets can be used to recover an input file. If a hundred packets are lost due to a burst of noise causing data erasure, an extra hundred packets can be picked up after the burst to replace the loss of the erased packets. If thousands of packets are lost because a receiver did not tune into a transmitter when it began transmitting, the receiver could just pickup those thousands of packets from any other period of transmission, or even from another transmitter. With group chain reaction coding, a receiver is not constrained to pickup any particular set of packets, so it can receive some packets from one transmitter, switch to another transmitter, lose some packets, miss the beginning or end of a given transmission and still recover an input file. The ability to join and leave a transmission without receiver-transmitter coordination greatly simplifies the communication process.

A Basic Implementation

Transmitting a file using group chain reaction coding involves generating, forming or extracting input symbols from an input file, encoding those input symbols into one or more groups of output symbols, where each group of output symbols is generated based on its key independently of all other groups of output symbols, and transmitting the groups of output symbols to one or more recipients over a channel. Receiving (and reconstructing) a copy of the input file using group chain reaction coding involves receiving some set or subset of groups of output symbols from one of more data streams, and decoding the input symbols from the values and keys of the received groups of output symbols.

As will be explained, the decoder can recover an input symbol from the values of one or more groups of output symbols and possibly from information about the values of other input symbols that have already been recovered. Thus, the decoder can recover some input symbols from some groups of output symbols, which in turn allows the decoder to decode other input symbols from those decoded input symbols and previously received groups of output symbols, and so on, thus causing a "chain reaction" of recovery of input symbols of a file being reconstructed at the recipient.

Aspects of the invention will now be described with reference to the figures.

Fig. 1 is a block diagram of a communications system 100 that uses group chain reaction coding. In communications system 100, an input file 101, or an input stream 105, is provided to an input symbol generator 110. Input symbol generator 110 generates a sequence of one or more input symbols (IS(0), IS(1), IS(2), ...) from the input file or stream, with each input symbol having a value and a position (denoted in Fig. 1 as a parenthesized integer). As explained above, the possible values for input symbols, i.e., its alphabet, is typically an alphabet of 2^M symbols, so that each input symbol codes for M bits of the input file. The value of M is generally determined by the use of communication system 100, but a general purpose system might include a symbol size input for input symbol generator 110 so that M can be varied from use to use. The output of input symbol generator 110 is provided to an encoder 115.

Key generator 120 generates a key for each group of output symbols to be generated by the encoder 115. Each key is generated according to one of the methods described previously, or any comparable method that insures that a large fraction of the keys generated for the same input file are unique, whether they are generated by this or another key generator. For example, key generator 120 may use a combination of the output of a counter 125, a unique stream identifier 130, and/or the output of a random generator 135 to produce each key. The output of key generator 120 is provided to encoder 115.

From each key I provided by key generator 120, encoder 115 generates a group of output symbols, with a set of symbol values B(I), from the input symbols provided by the input symbol generator. The b values of each group of output symbols is generated based on its key and on some function of one or more of the input symbols, referred to herein as the group of output symbol's "associated input symbols" or just its "associates". The selection of the function (the "value function") and the associates is done according to a process described in more detail below. Typically, but not always, M is the same for input symbols and output symbols, i.e., an input symbol is the same length as an output symbol.

In some embodiments, the number K of input symbols is used by the encoder to select the associates. If K is not known in advance, such as where the input is a streaming file, K can be just an estimate. The value K might also be used by encoder 115 to allocate storage for input symbols.

Encoder 115 provides groups of output symbols to a transmit module 140. Transmit module 140 is also provided the key of each such group of output symbols from the key generator 120. Transmit module 140 transmits the groups of output symbols, and depending on the keying method used, transmit module 140 might also transmit some data about the keys of the transmitted groups of output symbols, over a channel 145 to a receive module 150. Channel 145 is assumed to be an erasure channel, but that is not a requirement for proper operation of communication system 100. Modules 140, 145 and 150 can be any suitable hardware components, software components, physical media, or any combination thereof, so long as transmit module 140 is adapted to transmit groups of output symbols and any needed data about their keys to channel 145 and receive module 150 is adapted to receive groups of output symbols and potentially some data about their keys from channel 145. The value of K, if used to determine the associates, can be sent over channel 145, or it may be set ahead of time by agreement of encoder 115 and decoder 155.

As explained above, channel 145 can be a real-time channel, such as a path through the Internet or a broadcast link from a television transmitter to a television recipient or a telephone connection from one point to another, or channel 145 can be a storage channel, such as a CD-ROM, disk drive, Web site, or the like. Channel 145 might even be a combination of a real-time channel and a storage channel, such as a channel formed when one person transmits an input file from a personal computer to an Internet Service Provider (ISP) over a telephone line, the input file is stored on a Web server and is subsequently transmitted to a recipient over the Internet.

Because channel 145 is assumed to be an erasure channel, communications system 100 does not assume a one-to-one correspondence between the groups of output symbols that exit receive module 150 and the groups of output symbols that go into transmit module 140. In fact, where channel 145 comprises a packet network, communications system 100 might not even be able to assume that the relative order of any two or more packets is preserved in transit through channel 145. Therefore, the key of the groups of output symbols is determined using one or more of the keying schemes described above, and not necessarily determined by the order in which the groups of output symbols exit receive module 150.

Receive module 150 provides the groups of output symbols to a decoder 155, and any data receive module 150 receives about the keys of these groups of output symbols is provided to a key regenerator 160. Key regenerator 160 regenerates the keys for the received groups of output symbols and provides these keys to decoder 155. Decoder 155 uses the keys provided by key regenerator 160 together with the corresponding groups of output symbols, to recover the input symbols (again IS(0), IS(1), IS(2), ...). Decoder 155 provides the recovered input symbols to an input file reassembler 165, which generates a copy 170 of input file 101 or input stream 105.

A Basic Encoder

Fig. 2 is a block diagram of one embodiment of encoder 115 shown in Fig. 1. The block diagram of Fig. 2 is explained herein with references to Fig. 3, which is a diagram showing the logical equivalent of some of the processing performed by the encoder shown in Fig. 2.

Encoder 115 is provided with input symbols and a key for each group of output symbols it is to generate. As shown, the K input symbols are stored in an input symbol buffer 205. Key I (provided by the key generator 120 shown in Fig. 1) is an input to value function selector 210, group size selector 212, weight selector 215 and associator 220. The number of input symbols K is also provided to these four components, 210, 212, 215 and 220. A calculator 225 is coupled to receive outputs from value function selector 210, group size selector 212, weight selector 215, associator 220 and the input symbol buffer 205, and has an output for groups of output symbol values. It should be understood that other equivalent arrangements to the elements shown in Fig. 2 might be used, and that this is but one example of an encoder according to the present invention.

In operation, the K input symbols are received and stored in input symbol buffer 205. As explained above, each input symbol has a position (i.e., its original position in the input file) and a value. The input symbols need not be stored in input symbol buffer 205 in their respective order, so long as the position of stored input symbols can be determined.

If used, group size selector 212 determines a group size G(I) from key I and from the number of input symbols K. In one variation, the group size, G(I), is the same for all I (i.e., G(I)=b where b is a positive integer). In that variation, group size selector 212 is not needed and calculator 225 can be configured with the value of b. For example, the group size might be set to four for all I, i.e., G(I)=b=4 and each group of output symbols comprises four output symbols. In some embodiments, using small values of b, such as 2 or 3, or small varying values of G(I) makes for an efficient encoder. When b=1, the result is essentially the same as the encoder described in Luby I.

Using key I and the number of input symbols K, weight selector 215 determines the number W(I) of input symbols that are to be "associates" of the group of output symbols having key I. Using key I, weight W(I) and the number of input symbols K, associator 220 determines the list AL(I) of positions of input symbols associated with the group of output symbols. It should be understood that W(I) need not be separately or explicitly calculated if associator 220 can generate AL(I) without knowing W(I) ahead of time. Once AL(I) is generated, W(I) can be easily determined because it is the number of associates in AL(I).

Once I, W(I) and AL(I) are known, the G(I) values B(I) = B_1(I) ,..., B_G(I)(I) of the group of output symbols is calculated by calculator 225 based on a value function F(I) 210. One property of a suitable value function is that it would allow the unknown values for up to G(I) associates in AL(I) to be determined from the group of output symbol values B(I) and from the known values for the other associates in AL(I). One preferred value function used in this step is the Reed-Solomon value function, since it satisfies this property, is easily computed and easily inverted.

For the Reed-Solomon value function, the values B(I) are computed from the values of the associates in AL(I) according to a Reed-Solomon coding scheme. In that scheme the Reed-Solomon redundant symbols are B(I) and, together with the input symbols in AL(I), those symbols form a coded block coded using the Reed-Solomon coding scheme.

Other suitable value functions might be used instead, including using the XOR value function when the group size is equal to one, including methods based on polynomials over finite fields, including methods based on linear systems of equations, including methods based on Cauchy matrices over finite fields, or including other MDS codes (of which Reed-Solomon codes is one example).

A mix of different value functions that depends on the number of associates of the group of output symbols can also be used. For example, when the number of associates W(I) is equal to G(I), then the identity function could be used, i.e., for i = 1,…,G(I), the value of the i-th output symbol in the group is B_i(I) = IS(P_i), where P_1,…,P_G(I) are the positions of the G(I) associates. When the number of associates W(I) is equal to G(I)+1, then for i = 1,…,G(I), the value of the i-th output symbol in the group could be B_i(I) = IS(P_i) XOR IS(P_i+1) where P_1,…,P_G(I)+1 are the positions of the G(I) + 1 associates. When the number of associates W(I) is greater than G(I) + 1, then a Reed-Solomon value function could be used.

If used, value function selector 210 determines a value method or function F(I) from key I and from K, where F(I) is used to calculate a value for I. In one variation, value function selector 210 uses the same method or function F for all I. In that variation, value function selector 210 is not needed and calculator 225 can be configured with the value function F. For example, the value function might be a Reed-Solomon encoder for all I, i.e., the group of output symbols values comprises G(I) redundant symbols computed by a Reed-Solomon encoder based on the values of all of its associates.

For each key I, weight selector 215 determines a weight W(I) from I and K. In one variation, weight selector 215 selects W(I) by using the key I to first generate a random looking number and then uses this number to look up the value of W(I) in a distribution table that is stored within weight selector 215. A more detailed description of how such a distribution table might be formed and accessed is given below. Once weight selector 215 determines W(I), this value is provided to associator 220 and to calculator 225.

Associator 220 determines a list AL(I) of the positions of the W(I) input symbols associated with the current output symbol. The association is based on the value of I, on the value of W(I) and on K (if available). Once associator 220 determines AL(I), AL(I) is provided to calculator 225. Using list AL(I), weight W(I) and either the value function F(I) provided by value function selector 210 or a preselected value function F, calculator 225 accesses the W(I) input symbols referenced by AL(I) in input symbol buffer 205 to calculate the value, B(I) = B_1(I),…,B_b(I), for the current group of output symbols, where b is either the group size G(I) provided by the group size selector 212 or a preselected set value. An example of a procedure for calculating AL(I) is given below, but another suitable procedure might be used instead. Preferably, the procedure gives each input symbol a roughly even chance of being selected as an associate for a given group of output symbols and does the selection in a way that the decoder can replicate if the decoder does not already have AL(I) available to it.

Encoder 115 then outputs B(I) = B_1(I),...,B_b(I). In effect, encoder 115 performs the action illustrated in Fig. 3, namely, to generate a group of output symbol values B(I) = B_1(I),...,B_b(I) as some value function of selected input symbols. In the example shown, the value function is Reed-Solomon encoding with group size b set to 2, the weight W(I) of the group of output symbols is 3, and the associated input symbols (the associates) are at positions 0, 2, and 3 and have respective values IS(0), IS(2) and IS(3). Thus, the group of output symbols is calculated as:

$$B\_1(I) = RS\_1(IS(0),IS(2),IS(3))$$
$$B\_2(I) = RS\_2(IS(0),IS(2),IS(3))$$

for that value of I. Here, RS_i(*) is the symbol value produced by the Reed-Solomon encoder applied to the argument to produce the i-th redundant symbol.

The generated groups of output symbols are then transmitted and received as described above. Herein, it is assumed that some of the groups of output symbols might have been lost or gotten out of order, or were generated by one or more encoders. It is assumed, however, that the groups of output symbols that are received have been received with an indication of their key I and some assurance their values B(I) = B_1(I),...,B_b(I) are accurate. As shown in Fig. 1, those received groups of output symbols, together with their corresponding keys reconstructed from their indication by key regenerator 160 and the value of K, are the input to decoder 155.

The number of bits, M, encoded in an input symbol (i.e., its size) is dependent on the application. The size of an output symbol and the number b of output symbols in a group can also depend on the application, but might also be dependent on the channel. For example, if the typical input file is a multiple megabyte file, the input file might be broken into thousands, tens of thousands, or hundreds of thousands of input symbols with each input symbol comprising a few, tens, hundreds or thousands of bytes.

In some cases, the coding process might be simplified if the output symbol values and the input symbol values were the same size (i.e., representable by the same number of bits or selected from the same alphabet). If that is the case, then the input symbol value size is limited when the output symbol value size is limited, such as when it is desired to put groups of output symbols in packets and each group of output symbols must fit into a packet of limited size and the size b of each group is set to a fixed value. If some data about the key were to be transmitted in order to recover the key at the receiver, the group of output symbols would preferably be small enough to accommodate the value and the data about the key in one packet.

As described above, although the positions of the input symbols are typically consecutive, in many implementations, the keys are far from consecutive. For example, if an input file were divided up into 60,000 input symbols, the positions for the input symbols would range from 0 to 59,999, while in one of the implementations mentioned previously, each key might be independently chosen as a random 32-bit number and the groups of output symbols might be generated continuously and transmitted until the sender is stopped. As shown herein, group chain reaction coding allows the 60,000 symbol input file to be reconstructed from any sufficiently large collection (60,000 + some increment A) of output symbols regardless of where in the output sequence those groups of output symbols were taken.

A Basic Decoder

Fig. 4 shows one embodiment of decoder 155 in detail, with many parts in common with encoder 115 shown in Fig. 2. Decoder 155 comprises a value function selector 210, a group selector 212, a weight selector 215, an associator 220, a buffer 405 that stores a group of output symbols, a reducer 415, a reconstructor 420 and a reconstruction buffer 425. As with the encoder, the group selector 212 and the space in buffer 405 allocated for storing the group size is optional and might not be used if the group size is the same for all groups of output symbols. Similarly, the value function selector 210 and the space in buffer 405 allocated for storing the description of the value function is optional and might not be used if the value function was the same for all groups of output symbols. Several entries of reconstruction buffer 425 are shown, with some input symbols reconstructed and with others as yet unknown. For example, in Fig. 4, the input symbols at positions 0, 2, 5 and 6 have been recovered and the input symbols at positions 1, 3 and 4 have yet to be recovered.

In operation, for each received group of output symbols with key I and values B(I) = B_1(I),...,B_b(I), decoder 155 does the following. Key I is provided to value function selector 210, group selector 212, weight selector 215 and associator 220. Using K and key I, weight selector 215 determines weight W(I). Using K, key I and W(I), associator 220 produces the list AL(I) of W(I) positions of input symbols associated with the group of output symbols. Optionally, using K and I, group selector 212 selects a group size G(I). Optionally, using K and I, value function selector 210 selects value function F(I). Then, I, B(I), W(I) and AL(I), optionally G(I) and optionally F(I), are stored in a row of buffer 405. Group selector 212, value function selector 210, weight selector 215 and associator 220 perform the same operation for decoder 155 as described for encoder 115. In particular, the group size G(I), the value function F(I), the weight W(I) and the list AL(I) produced by group selector 212, by value function selector 210, by weight selector 215 and by associator 220 in Fig. 5 are the same for the same key I as for the corresponding parts shown in Fig. 4. If K varies from input file to input file, it can be communicated from the encoder to the decoder in any conventional manner, such as including it in a message header.

Reconstructor 420 scans buffer 405 looking for groups of output symbols stored there that have weight at most their group size. Those symbols are referred to herein as members of a "decodable set." For value functions with the properties described above, groups of output symbols of weight at most their group size are in the decodable set because the unknown values of up to their group size input symbols can be determined from that group of output symbols and the known values of the other input symbols that are associates. Of course, if a value function were used that would allow input symbols to be decoded by a group of output symbols under a condition other than having a weight of at most their group size, that condition would be used to determine whether a group of output symbols is in the decodable set. For clarity, the examples described here assume that the decodable set is those groups of output symbols with weight at most their group size, and extensions of these examples to other value function decodable conditions should be apparent from this description.

When reconstructor 420 finds a group of output symbols with key I that is in the decodable set, the group of output symbol values are B(I) = B_1(I),...,B_G(I)(I) and optionally the value function F(I) is used to reconstruct the W(I) values of unknown input symbols listed in AL(I) and the reconstructed input symbols are placed into reconstruction buffer 425 at the appropriate position for those input symbols. If W(I) is strictly less than the group size G(I), then the reconstructor 420 could reconstruct input symbol values that have already been reconstructed in an independent way. In this case, the reconstructor 420 could drop the independently reconstructed input symbols, overwrite the existing reconstructed input symbols, or compare the values of the input symbols independently obtained and issue an error if they differ. Reconstructor 420 thus reconstructs input symbols, but only from groups of output symbols in the decodable set. Once a group of output symbols from the decodable set is used to reconstruct an input symbol it can be deleted to save space in buffer 405. Deleting the "used up" group of output symbols also ensures that reconstructor 420 does not continually revisit that group of output symbols.

Initially, reconstructor 420 waits until at least one group of output symbols is received that is a member of the decodable set. Once that one group of output symbols is used to reconstruct unknown input symbol values, the decodable set would be empty again, except for the fact that some other group of output symbols might be a function of some of these just reconstructed input symbols and up to their group size other not yet reconstructed input symbols. Thus, reconstructing input symbols from a member of the decodable set might cause other groups of output symbols to be added to the decodable set. The process of reduction of groups of output symbols to add them to the decodable set is performed by reducer 415.

Reducer 415 scans buffer 405 and reconstruction buffer 425 to find groups of output symbols that have lists AL(I) that list positions of input symbols that have been recovered. When reducer 415 finds such a "reducible" group of output symbols with key I, it recomputes W(I) to be equal to the number of input symbol values that have not yet been reconstructed among the input symbols in AL(I).

The action of reducer 415 reduces the weights of groups of output symbols in buffer 405. When a group of output symbols weight is reduced to at most its group size (or other decodable condition occurs for other value functions), then that group of output symbols becomes a member of the decodable set, which can then be acted upon by reconstructor 420. In practice, once a sufficient number of groups of output symbols are received, reducer 415 and reconstructor 420 create a chain reaction decoding, with reconstructor 420 decoding the decodable set to recover more input symbols, reducer 415 using those freshly recovered input symbols to reduce more groups of output symbols so they are added to the decodable set, and so on, until all input symbols from the input file are recovered.

The decoder shown in Fig. 4 reconstructs input symbols in a straightforward manner, without much consideration to memory storage, computation cycles or transmission time. Where the decoder memory, decoding time or transmission time (which constrains the number of groups of output symbols that are received) are limited, the decoder can be optimized to better use those limited resources.

A More Efficient Decoder

Fig. 5 shows a preferred embodiment of a more efficient implementation of a decoder 500 in detail. Here, the value function is assumed to be the Reed-Solomon value function. Similar implementations that can potentially be more efficient apply with respect to value functions other than the Reed-Solomon value function. Referring to Fig. 5, decoder 500 comprises group of output symbols data structure 505 (hereafter referred to as GOSDS 505), input symbol data structure 510 (hereafter referred to as ISDS 510), decodable set stack 515 (hereafter referred to as DSS 515), receive organizer 520, and recovery processor 525.

GOSDS 505 is a table that stores information about groups of output symbols, where row R of GOSDS 505 stores information about the R-th group of output symbols that is received. A variable R keeps track of the number of groups of output symbols that have been received, and it is initialized to zero. GOSDS 505 stores the fields KEY, VALUE, and WEIGHT for each row, with the fields shown organized in columns. The KEY field stores the key of the group of output symbols. The VALUE field stores the group of output symbol values. The WEIGHT field stores the initial weight of the group of output symbols. The WEIGHT of a group of output symbols is reduced over time until it becomes at most the group size and then can be used to recover input symbols.

ISDS 510 is a table that stores information about input symbols, where row P stores information about the input symbol at position P. For each row ISDS 510 includes storage for a REC_VAL field, which eventually becomes the value of the recovered input symbol, a REC_IND field, which is initialized to all values "no" and indicates whether or not input symbols have been recovered, and an RL field. When an input symbol is recovered, the REC_IND of the input symbol is changed to "yes". The RL column is initialized to all "empty list" values. As groups of output symbols are received that have an input symbol as an associate, the row number in GOSDS 505 of the group of output symbols is added to the RL list for the input symbol.

DSS 515 is a stack that stores information about the decodable set. A variable S keeps track of the size of the decodable set, and it is initialized to zero. In DSS

515, column OUT_ROW stores row numbers in GOSDS 505 of groups of output symbols.

In one embodiment, decoder 500 operates as follows and as shown in the flowchart of Fig. 6 with the corresponding steps of Fig. 6 indicated parenthetically in the description of the process. First, ISDS 510 is initialized as described above, and both R and S are initialized to zero (605). When a new group of output symbols is received (610), i.e., the key I and the group of output symbol values B(I) = B_1(I),...,B_G(I)(I), KEY(R) is set to I and VALUE(R) is set to B(I) = B_1(I),...,B_G(I)(I) in GOSDS 505 (615). Receive organizer 520 is then called to process the received group of output symbols with key I stored in row R of GOSDS 505 (620). This includes adding information to GOSDS 505 and DSS 515 appropriately using information stored in ISDS 510, as shown in the flowchart of Fig. 7. Then, R is incremented by one (625) to cause the next group of output symbols to be stored in the next row of GOSDS 505. Recovery processor 525 is then called to process groups of output symbols in the decodable set and to add new groups of output symbols to the decodable set (630). This includes adding to and deleting from the decodable set stored in DSS 515, using and modifying portions of ISDS 510 and GOSDS 505 appropriately, as shown in the flowchart of Fig. 8(a) and/or 8(b). Decoder 500 keeps track of the number of input symbols that have been recovered, and when this number reaches K, i.e., all input symbols have been recovered, decoder 500 terminates successfully, otherwise it returns to step 610 to receive the next group of output symbols, as shown in 635 and 640.

A flowchart that describes the operation of receive organizer 520 is shown in Fig. 7, which refers to Figs. 9-12. When a group of output symbols with values B(I) = B_1(I),...,B_G(I)(I) and key I arrives, receive organizer 520 performs the following operations, referring to Fig. 7. Weight W(I) is computed from I and K (705) and list AL(I) of positions of associates is computed from I, W(I), and K (710). Figs. 11-12 show the details of one computation of W(I) and Figs. 9-10 show the details of one computation of AL(I).

Referring again to Fig. 7, for each position P on list AL(I), if input symbol P is not recovered, i.e., if REC_IND(P) = "no" in ISDS 510, then R is added to list RL(P) in ISDS 510, otherwise if input symbol P is recovered, i.e., if REC_IND(P) = "yes" in ISDS 510, then W(I) is decremented by one (720). Then, WEIGHT(R) is set to W(I) in GOSDS 505 (725). WEIGHT(R) is then compared to G(I) (730). If WEIGHT(R) is at most G(I), then the group of output symbols is added to the decodable set, i.e., OUT_ROW(S) is set to R in DSS 515 and the value of S is incremented by one (735). Finally, receive organizer 520 returns (740).

A flowchart that describes one operation of recovery processor 525 is shown in Fig. 8(a), which refers to Figs. 9-12. In that operation, recovery processor 525 first checks to see if the decodable set is empty, i.e., if S = 0, and if so it immediately returns (805, 810). If the decodable set is not empty, then S is decremented by one (815) and the row number R' of the group of output symbols is loaded from DSS 515 (820). Then, the original key of the output symbol, KEY(R') from GOSDS 505, is loaded into I (835) in order to compute the original weight W(I) and the original list of associates AL(I) of the group (840, 845). If all the input symbols in AL(I) have already been recovered, i.e., if REC_IND(P) = "yes" for all P in AL(I) (847), then recovery processor 525 stops processing this element of the decodable set and continues on to process the next. Otherwise, the group of output symbols stored at row number R' in GOSDS 505 is used to recover all the unknown values of input symbols that are in AL(I) in ISDS 510. This is performed by utilizing the Reed-Solomon decoder to recover the up to G(I) unknown values of input symbols in AL(I) using the group of output symbol values and the known values of input symbols in AL(I) (850). Then, for all positions P for which the input symbol value is recovered, REC_VAL(P) is set to the recovered input symbol value in ISDS 510 (852) and REC_IND(P) is set to "yes" to indicate the input symbol value has been recovered in ISDS 510 (852).

A variation of the process shown in Fig. 8(a) is shown in Fig. 8(b). There, instead of performing steps 850, 852, 855 and 860 for each group of output symbols as it is processed, the value of R' can be stored in an execution schedule for later processing, as shown in steps 851, 853, 856 and 861 of Fig. 8(b). An example of deferred execution processing is shown in Fig. 8(c). In this variation, the flowchart shown in Fig. 6 is modified by initializing E to zero in step 605. The deferred processing of the execution schedule can occur after the decoder determines that the received symbols are enough to decode the entire file, e.g. at step 640 after it is known that all input symbols are recoverable. In some cases, especially where the input symbols are large, the execution of the schedule could be deferred until the input file, or portions thereof, are needed at the receiver.

In either variation, i.e. in either Fig. 8(a), or respectively Fig. 8(b), at step 855, or respectively 856, the groups of output symbols that still have the just recovered, or respectively just could have been recovered, input symbols as associates are modified to reflect that these input symbols have been recovered, or respectively could have been recovered. For each position P of a just recovered, or respectively just could have been recovered, input symbol value, the row numbers of these groups of output symbols in GOSDS 505 are stored in RL(P). For each row number R" in RL(P), WEIGHT(R") is decremented by one to reflect the recovery of the input symbol in position P as an associate of the group of output symbols in row R" of GOSDS 505. If this modification causes the group of output symbols in row R" of GOSDS 505 to become weight equal to the group size, i.e., WEIGHT(R") = G(KEY(R")), then this group of output symbols is added to the decodable set by setting OUT_ROW(S) to R" and incrementing S by one (855, respectively 856). Finally, the space used for storing row numbers of groups of output symbols on list RL(P) is returned to free space (860, respectively 861), and processing continues at step 805.

An Associator Implementation

The mapping from a group of output symbols key to associated input symbols (i.e., the determination of the weight W(I) and the list AL(I) of positions of associates for a key I) can take a variety of forms. W(I) should be chosen to be the same value by both the encoder and decoder for the same key I (in the sender and the recipient, respectively). Similarly, AL(I) should be chosen to contain the same list of positions by both encoder and decoder for the same key I. The associator is the object that calculates or generates AL(I) from I and usually W(I) and K.

In one embodiment, W(I) and AL(I) are determined in a manner designed to mimic a random process. To satisfy the requirement that the encoder and decoder produce the same result with respect to the same key, a pseudorandom sequence could be generated by both encoder and decoder seeded with the key. Instead of a pseudorandom sequence, a truly random sequence might be used for the generation of W(I) and/or AL(I), but for that to be useful, the random sequence used for generating W(I) and AL(I) would need to be communicated to the recipient.

In the decoder shown in Fig. 4, the buffer 405 requires storage for each group of output symbols list of positions of associates, i.e., storage in the column labeled

AL(I). The more efficient decoder shown in Fig. 5 does not require this storage, because a list of associates is recalculated as it is needed, e.g., as shown in Figs. 9-10. There is an advantage in recalculating associate lists each time in order to save storage only if these calculations can be done quickly as needed.

A preferred embodiment of associator 220 is shown in Fig. 9 and operates according to the process shown in Fig. 10. This associator can be used in the encoder as well as in the decoder. Although memory storage for AL(I) at the encoder is not much of a concern, because the encoder does not normally need to store more than one AL(I) at a time, the same process should be used at both the encoder and decoder to ensure that the values for AL(I) are the same in both places.

The input to associator 220 is a key I, the number of input symbols K, and a weight W(I). The output is a list AL(I) of the W(I) positions of associates of the group of output symbols with key I. As shown in Fig. 9, the associator comprises a table ASSOC_RBITS 905 of random bits and a calculator ASSOC_CALC 910. Before a specific AL(I) is generated, the size of the input file is adjusted so that the number of input symbols is prime. Thus, if the input file begins with K input symbols, the smallest prime number, P, greater than or equal to K is identified. If P is greater than K, P-K blank (e.g., set to zero) input symbols are added to the input file and K is reset to P. For this modified input file, lists AL(I) of positions of associates are computed as shown in Fig. 9 and 10.

In this embodiment, ASSOC_CALC 910 operates as described below and as shown in the flowchart of Fig. 10. The first step is to use the key I, the number of input symbols K and the table of random bits ASSOC_RBITS 905 to generate two integer values X and Y that have the property that X is at least one and at most K-1, and Y is at least zero and at most K-1 (1005). Preferably, X and Y are independently and uniformly distributed within their respective ranges. Next, an array V[] with W(I) entries is initialized for storage of AL(I) as its members are calculated (1010). Since V[] is just temporary storage for one list, it would occupy much less memory than the AL(I) column of buffer 405 (see Fig. 4). V[0] (this is the first element of the list AL(I)) is set to Y (1015). Then, for all values of J starting at 1 and ending at W(I)-1, the value of V[J] is set to (V[J-1] + X) mod K, as shown in steps 1020-1050. Since K is prime and W(I) is at most K, all of the V[] values will be unique. As shown, the "mod K" operation can be a simple compare and subtract operation, i.e., steps 1035 and 1040. Thus, the process of producing the list of positions of associates of a given group of output symbols is very efficient.

One advantage of the above approach to calculating AL(I) is that it produces enough variety in the distribution on the positions of the associates to ensure that the decoding algorithm works with high probability with minimal reception overhead (i.e., the input file is recovered after receiving only slightly more than K/b groups of output symbols, assuming input symbols and output symbols are the same length) when coupled with a good procedure for selecting W(I).

If AL(I) is calculated as shown in Fig. 10 and the table of random bits ASSOC_RBITS 905 is kept secret between the sender and the receiver, then communications system 100 could be used as a secure communications system because, without knowing the sequence of AL(I), decoding the received stream is difficult, if not impossible. This feature would also apply to a communications system that uses other methods of calculating or generating AL(I), so long as the method of calculating or generating AL(I), or preferably the seed used in a method of calculating generator AL(I), is kept secret.

A Weight Selector Implementation

The performance and efficiency of the encoder/decoder is dependent on the distribution of weights and some distributions are better than others. The operational aspects of weight selection are discussed below, followed by a description of some important weight distributions. The block diagram of Fig. 11 and the flowchart of Fig. 12 are used to illustrate these concepts.

As shown in Fig. 11, the weight selector comprises two processes WT_INIT 1105 and WT_CALC 1110, and two tables WT_RBITS 1115 and WT_DISTRIB 1120. Process WT_INIT 1105 is invoked only once when the first key is passed in to initialize table WT_DISTRIB 1120. The design of WT_DISTRIB 1120 is an important aspect of the system, and is considered later in much more detail. Process WT_CALC 1110 is invoked on each call to produce a weight W(I) based on a key I. As shown in the flowchart of Fig. 12, WT_CALC 1110 uses the key and random bits stored in table WT_RBITS to generate a random number R (1205). Then the value of R is used to select a row number L in table WT_DISTRIB 1120.

As shown in Fig. 11, the entries in the RANGE column of WT_DISTRIB 1120 are an increasing sequence of positive integers ending in the value MAX_VAL. The set of possible values for R are the integers between zero and MAX_VAL - 1. A desirable property is that R is equally likely to be any value in the range of possible values. The value of L is determined by searching the RANGE column until an L is found that satisfies $\text{RANGE}(L-1) \leq R < \text{RANGE}(L)$ (1210). Once an L is found, the value of W(I) is set to WT(L), and this is the returned weight (1215, 1220). In Fig. 11 for the example table shown, if R is equal to 38,500, then L is found to be 4, and thus W(I) is set to WT(4) = 8.

Other variations of implementing a weight selector and associator include generating I pseudorandomly and generating W(I) and AL(I) directly from I. As should be apparent, W(I) can be determined by examining AL(I), since W(I) is the equal to the number of associates in AL(I). It should be apparent from this description that many other methods of generating W(I) values are equivalent to the just-described system to generate a set of W(I) values with the distribution defined by WT_DISTRIB.

If W(I) is calculated as shown in Fig. 12 and the table of random bits WT_RBITS 1115 is kept secret between the sender and the receiver, then communications system 100 could be used as a secure communications system because, without knowing the sequence of W(I), decoding the received stream is difficult, if not impossible. This feature would also apply to a communications system that uses other methods of calculating or generating W(I), so long as the method of calculating or generating W(I), or preferably the seed used in a method of calculating generator W(I), is kept secret.

<u>An Alternative Decoder</u>

Upon reading this disclosure, it should be clear to those of skill in the art that a receiver can work more efficiently than the implementations described above. For example, the receiver might be more efficient if it buffered packets and only applied the recovery rule once a group of packets arrived. This modification reduces computational time spent in doing subsequently unnecessary operations and reduces overhead due to context switching. In particular, since the decoder cannot hope to recover the original file of K input symbols before at least K output symbols (assume same size input and output symbols) arrive in packets, it can be beneficial to wait until at least K output symbols arrive before beginning the decoding process.

Fig. 13 shows a different method of decoding, which includes the concepts expressed above and which is a modification of the process used by the decoder of Fig. 6. The primary difference between the two methods is that the method of Fig. 13 receives groups of output symbols in batches, as shown in 1315. The size of the first batch is set so that the total number of output symbols that arrive is K + A, where A is a small fraction of the number of input symbols K (1310). After the first batch of groups of output symbols is received, the groups of output symbols are processed as before, using receive organizer 520 (1340) to process groups of output symbols intermingled with using recovery processor 525 (1350) to process the decodable set and recover input symbols from groups of output symbols of reduced weight at most their group size. If recovery of all K input symbols is not achieved using the first batch of groups of output symbols, then additional batches of output symbols are received, each batch containing an additional G output symbols, and processed until all input symbols are recovered.

It is advantageous to minimize the storage required for the decoder's auxiliary data structures as much as possible. As already described, storage for the associates list for each group of output symbols is not needed, since associator 220 can be used to quickly calculate those lists as needed. Another storage need is for storing, for each as yet unrecovered input symbol, the row number in GOSDS 505 of the groups of output symbols that have the input symbol as an associate, i.e., the space for the lists shown in the RL column in table ISDS 510 of Fig. 5. As already described in step 855 of Fig. 8, one use of this storage is to be able to quickly identify which groups of output symbols are reducible when a given input symbol is reconstructed. Unless it is done efficiently, the storage required for these lists would be proportional to the total number of associates of all groups of output symbols used to recover all the input symbols.

A Presorting Decoder

A more preferred embodiment of the decoder is now described, referring to Fig. 14 and Fig. 15. Fig. 14 shows the parts that comprise the decoder, which is the same as those shown in Fig. 5 except for the addition of a table WEIGHT SORT 1405 and the EXECUTION LIST 1420 used to store the execution schedule formed as described in Fig. 8(b). Table WEIGHT SORT is used to store batches of row numbers in

GOSDS 505 of groups of output symbols as they are received sorted in order of increasing weight. The WT_VAL column is used to store weights, and the ROW_LIST column is used to store row numbers of groups of output symbols in GOSDS 505. In general, the row numbers of all groups of output symbols with weight WT_VAL(L) are stored in ROW_LIST(L). This table is used to process the batch of groups of output symbols in order of increasing weight, as shown in step 1520 of Fig. 15. Low weight groups of output symbols are less computationally intensive to use to recover an input symbol, and it is likely, as the larger weight groups of output symbols come along, that most of their associates will already be recovered, and thus it saves substantially on the link storage space (the decoder can recover space used by recovered input links, and groups of output symbols that are being processed will have few associates not yet recovered).

Processing groups of output symbols in batches of appropriate sizes in order of increasing weight lowers the memory requirements as well as the processing requirements.

As shown in Fig. 15, slightly more than K output symbols in groups of output symbols (denoted by K + A output symbols in the figure) are allowed to arrive before any processing begins (1515). Here, we assume the same size input and output symbols and K input symbols in the input file. Initially, the decoder simply waits for receipt of the K + A output symbols in groups, since the decoder should not expect to be able to recover the input file from less than K + A output symbol output symbols anyway, and cannot possibly recover an arbitrary input file from less than K output symbols. In practice, $5\cdot\sqrt{K}$ was found to be a good value for A.

The row numbers in GOSDS 505 of received groups of output symbols are stored in table WEIGHT SORT 1405 of Fig. 14 in order of increasing weight, as shown in step 1515 of Fig. 15. If T is the number of possible group of output symbol weights then for values of L between 1 and T, list ROW_LIST(L) contains all received groups of output symbols of weight WT_VAL(L), where 1=WT_VAL(1) < WT_VAL(2) < WT_VAL(3) < … < WT_VAL(T) and WT_VAL(T) is the maximum weight of any group of output symbols. Then, the rest of the operation of the decoder shown in Fig. 15 is exactly the same as for the decoder shown in Fig. 13, except that groups of output symbols are processed in order of increasing weight, as shown in step 1520 .

Normally, K + A output symbols will suffice to recover all input symbols. However, some sets groups of output symbols containing K + A output symbols in all might not suffice to recover all input symbols. In such cases, batches of G additional output symbols are received in groups and then processed until all input symbols are recovered. A good setting for G is $\sqrt{K}$.

Figs. 16-19 shows a snapshot of an example run of the process described in Fig. 15. In this example, the group size is two for all groups, and four groups of output symbols have been received with associates indicated as shown by the arrowed lines in Fig. 16. Initially, groups of output symbols with key 23 and values (A,D), key 159 and values (RS_1(A,E,F), RS_2(A,E,F)), key 33 and values (B,C) and key 835 and values (RS_1(C,F,G,H), RS_2(C,F,G,H)) (the first and third group of output symbols have weight two and the value function is the identity function, the second and fourth groups have weights three and four respectively, and the value function is the Reed-Solomon value function) are received and stored in GOSDS 505 as shown in Fig. 17. The row number in GOSDS 505 is stored in ROW_LIST in the row corresponding to the weight of the output symbol, as shown in Fig. 18. The groups of output symbols of weight two are in row 0 and in row 2 of GOSDS 505. Thus, ROW_LIST(0), which corresponds to groups of output symbols of weight WT_VAL(0) = 2, contains row numbers 0 and 2, as shown in Fig. 18. Similarly, WT_VAL(1) = 3 and ROW_LIST(1) contains 1 and WT_VAL(2) = 4 and ROW_LIST(2) contains 3.

At this point in the process, the first two groups of output symbols in order of increasing weight have been processed, the third group of output symbols in row 1 of GOSDS 505 has been processed by receive organizer 520 and this third group of output symbols is just about to be processed by recovery processor 525. Groups of output symbols in rows 0 and 2 have already been added to the schedule to eventually recover input symbols in positions 0, 3, 1 and 2, respectively. The group of output symbols in row 3 of GOSDS 505 has three associates at positions 5, 6 and 7 that have not as yet been recovered, and thus there are links from positions 5, 6 and 7 in ISDS 510 back to row 3 in GOSDS 505. The group of output symbols in row 1 of GOSDS 505 has three associates in positions 0, 4, and 5. The associate in positions 0 has already been marked as recovered, and thus there is no link from it back to row 1 (it caused the weight of this group of output symbols to be reduced from 3 to 2, which will trigger the recovery of the remaining input symbols in positions 4, 5, 6 and 7 once recovery processor 525 is executed). The associates in positions 4 and 5 have not been recovered, and thus the receive organizer 520 added a link from position 4 and from position 5 in ISDS 510 to row 1 in GOSDS 505. This is all shown in Fig. 17. Thus, at this point in the process, a total of only five links from input symbols back to groups of output symbols which have them as associates are in use. This compares favorably with the straightforward implementation that uses a link from every input symbol to every group of output symbols having it as an associate. In this example, there are eleven possible such links.

In general, the savings in storage space for links is dramatically reduced when using the process described in Figs. 14 and 15 over the process described in Fig. 13, e.g., the savings in space is typically a factor of 10 to 15 in link space when the number of input symbols is 50,000. The reason for this reduction is that smaller weight groups of output symbols are more likely to recover input symbols at the beginning of the process then at the end, and heavier weight groups of output symbols are much more likely to recover groups of output symbols at the end of the process then at the beginning. Thus, it makes sense to process the groups of output symbols in order of increasing weight. A further advantage of the process described in Figs. 14 and 15 over Fig. 13 is that the decoding is typically 30% to 40% faster. This is because the smaller weight groups of output symbols are more likely to be used to recover input symbols than the heavier weight groups of output symbols (since the smaller weight groups of output symbols are considered first), and the cost of recovering a particular input symbol directly depends on the weight of the group of output symbols used to recover it.

Fig. 19 shows a complete execution schedule for this example, which when executed will recover all eight input symbols based on the four groups of output symbols of group size two.

Selecting a Weight Distribution

An important optimization is to design the coding process so that an input file can be fully reconstructed with as few groups of output symbols as possible. This optimization is helpful where the transmission time and bandwidth is costly or limited, or where an input file must be decoded quickly, without waiting for additional groups of output symbols. Typically, the sufficient number of output symbols needed to reconstruct an input file is slightly larger than the number of input symbols in the original input file (assuming the same size input and output symbols). It can be shown that an arbitrary input file cannot be recovered when fewer bits have been received than are in the input file. Therefore, a perfect coding scheme will allow recovery of an input file from any set of groups of output symbols encoding the same number of bits as the input file, and one measure of coding efficiency is how few extra bits are needed under expected conditions.

In the decoder shown in Fig. 5, the maximum efficiency is obtained when recovery processor 525 recovers the last unknown input symbol after the decoder has received exactly K output symbols. If more than K output symbols have been received by the decoder by the time all the input symbols can be recovered, then groups of output symbols would have been received that were not needed or used for recovery of the input file. While maximum efficiency is nice, targeting for it should be tempered by the risk that DSS 515 will be empty before reconstruction is complete. In other words, at maximum efficiency, the size of the decodable set hits zero just as reconstruction ends, but encoding/decoding should be arranged so that there is no more than a small probability that the size of the decodable set would hit zero before the end of the reconstruction using K + A output symbols, so that additional sets of G output symbols contained in groups are not needed.

This point is illustrated in Fig. 20. That figure shows a plot of the size of a decodable set versus the number of input symbols reconstructed where the decoder is operating with K/b groups of output symbols, each group of size b, for an ideal distribution described below. In this example, A = 0, i.e., the number of output symbols received in groups to decode all the K input symbols is the minimum possible number (assuming input and output symbols are the same size). It should be understood that the plot may vary for any given function for determining weights and associates, and would also vary depending on which particular groups of output symbols are received. In that plot, the expected size of the decodable set size is one at first and remains one throughout the recovery process. Thus, in expected behavior, there is always one group of output symbols in the decodable set that can be used to recover the next b input symbols. Fig. 20 also shows an example of the actual behavior of the ideal distribution. Notice that in this actual run the decodable set is empty before recovery is completed, i.e., after only two of the 100,000 input symbols are recovered. This actual behavior of the ideal distribution is typical, i.e., for the ideal distribution random fluctuations almost always empty the decodable set before all input symbols are recovered, and this is the reason that a more robust distribution is needed as described below.

Efficiency is improved by limiting the number of times group of output symbols in the decodable set has strictly less than its group size unknown input symbols as associates when it is used to reconstruct values of associates. This can be accomplished by suitable selection of the function for generating W(I).

Thus, while it is possible to completely recover an input file with any desired degree of certainty, by receiving enough groups of output symbols, it is preferable to design a group chain reaction coding communications system such that there is a high probability of recovering the K input symbols comprising a complete input file with as few as K + A output symbols (assume the same size for input symbols and output symbols) for some small value of A. The minimum value for A is zero, and can be achieved in some coding schemes, such as when using a standard Reed-Solomon coding where each output symbol depends on all K input symbols, but this leads to slow encoding and decoding times. By accepting some small nonzero value for A, an improved communications system can be obtained.

Small values of A can be achieved in group chain reaction coding by using the appropriate distributions to determine the group sizes, the weight distribution for groups of output symbols, i.e., the distribution of W(I) over all I, and the distribution of associates over the groups of output symbols, i.e., the memberships of AL(I) over all I. It should be emphasized that while the decoding process can be applied regardless of the group sizes, the weight distribution and the distribution on the choice of the associates, the preferred embodiments will use group sizes, weight distributions and distributions on the choice of associates specifically chosen for near optimal performance. In fact, many distributions will perform well, as small variations in the chosen distribution may lead to only small changes in performance.

The methodology for determining the distributions in one preferred embodiment will now be described. In this embodiment, the group size of all groups of output symbols is the same value b for a fixed positive integer b. The actual weight distributions used are based on an ideal mathematical distribution. In the ideal weight distribution, the weights W(I) are chosen according to an "ideal" probability distribution. The smallest weight is b and the largest weight is K, where K is the number of input symbols. In the ideal distribution, a weight equal to a value of *i* is chosen with the following probability p:

for i=b:	p=b/K; and
for i=b+1,...,K:	p=b/(i(i-1)).

Once a weight W(I) chosen, a list AL(I) of W(I) associated input symbols are chosen independently and uniformly at random (or pseudorandomly, if needed), making sure that all chosen associates are distinct. Thus, the first associate is randomly selected among the K input symbols, each having a probability of 1/K of being selected. The second associate (if W>1) is then randomly selected among the remaining K-1 symbols. The weight probability distribution shown above has the property that if the system behaved exactly as expected, exactly K/b groups of output symbols would be sufficient to decode and recover all input symbols. This expected behavior for the ideal distribution is shown in Fig. 20 by the solid line. However, because of the random nature of selection of the weights and the associates, and because an arbitrary set of groups of output symbols are used in the decoding process, the process will not always behave that way. An example of an actual behavior for the ideal distribution is shown in Fig. 20 by the dotted line. Hence, the ideal weight distribution must be modified somewhat in practice.

Generally, the best parameters for a given setting can be found by computer simulation. However, a simple variation on the ideal weight distribution is an effective option. In this simple variation, the ideal weight distribution is modified slightly by increasing the probability of groups of output symbols of weight b and of high weight groups of output symbols, to reduce the chance of the decodable set emptying before all K input symbols are recovered. The extra supply of weight b groups of output symbols decreases the chance that the process will run out of weight at most b groups of output symbols (i.e., empty the decodable set) until the recovery process is near the end of the recovery of the input symbols. The extra supply of high weight groups of output symbols increases the chance that, near the end of the recovery process, for each yet unrecovered input symbol there will be at least one group of output symbols that will have that input symbol as an associate and at most b-1 other as yet unrecovered associates.

More specifically, the modified weight distribution is as follows:

for i=b:	p=n·b·R1/K;
for i=b+1,..., (K/R2 - 1):	p=n·b/(i(i-1)(1-iR2/K)); and
for i=K/R2,...,K:	p=n·HW(i)

where K is the number of input symbols, R1 and R2 are tunable parameters and n is a normalization factor used so that the p values all sum to one.

The calculation of HW(*i*) and sample values for R1 and R2 are shown in detail in Appendix A. There, the C++ symbols nStartRippleSize, nRippleTargetSize and nSymbols correspond with R1, R2 and K, respectively, in the above equations.

This modified distribution is similar to the ideal mathematical weight distribution, with more groups of output symbols of weight b and of higher weight and the distribution rescaled accordingly. As shown in the modified distribution, R1 determines the initial fraction of weight b groups of output symbols as well as determining the multiple of the increased probability of weight b symbols and R2 determines the boundary between the "higher" weights and the "not so high" weights.

Good choices for R1 and R2 generally depend on K/b and can be determined empirically. For example, having R1 equal to 1.4 times the square root of K/b and R2 equal to 2 plus 2.1 times the fourth root of K/b works well in practice. Thus, for K = 4000 and b = 5, setting R1 = 39 and R2 = 13 works well; when K is 64000 and b = 2, setting R1 = 250 and R2 = 30 works well. More detailed calculations of R1 and R2 are shown in Appendix A. Fig. 21 shows that the expected behavior of this distribution leaves the decodable set moderately large throughout the recovery process so that under actual runs the random fluctuations from the expected behavior is unlikely to empty the decodable set before all input symbols are recovered. Proper settings for the weight distribution ensure that under any loss conditions reception of a minimal number of output symbols enables recovery of all input symbols with high probability.

Although the reconstruction processes described above is similar to the one used for Tornado codes, the different process used to build the code leads to extremely different effects. In particular, as described earlier, the memory required for group chain reaction encoding is significantly less than for Tornado codes, and the ease of use of group chain reaction codes in diverse situations far exceeds that of Tornado codes, at possibly the expense of some speed. The mathematical details underlying the processes are described in more detail below.

Properties of Some Group Chain Reaction Codes

The number of groups of output symbols generated and sent through the channel is not limited with group chain reaction coding as with other coding schemes, since keys need not have a one-to-one correspondence with input symbols and the number of different values of I is not limited to some small constant fraction of the number of input symbols. Therefore, it is likely that even if the decodable set goes empty before the input file is reconstructed, the decoding process will not fail, since the decoder can gather as many more groups of output symbols as needed to get at least one more group of output symbols of weight its group size. When that group of output symbols of weight its group size is received, it populates the decodable set and, by the chain reaction effect, might cause reduction of previously received groups of output symbols down to weight at most their group size so that they can, in turn, be used to reconstruct input symbols.

In most of the examples described above, the input and groups of output symbols encode for the same number of bits and each group of output symbols is placed in one packet (a packet being a unit of transport that is either received in its entirety or lost in its entirety). In some embodiments, the communications system is modified so that each packet contains several groups of output symbols. The size of an output symbol value is then set to a size determined by the size of the input symbol values in the initial splitting of the file into input symbols, based on a number of factors including the (possibly varying) group size. The decoding process would remain essentially unchanged, except that groups of output symbols would arrive in bunches as each packet was received.

The setting of input symbol and output symbol sizes is usually dictated by the size of the file and the communication system over which the groups of output symbols are to be transmitted. For example, if a communication system groups bits of data into packets of a defined size or groups bits in other ways, the design of symbol sizes begins with the packet or grouping size. From there, a designer would determine how many groups of output symbols will be carried in one packet or group and that together with the (possibly varying) group size determines the output symbol size. For simplicity, the designer would likely set the input symbol size equal to the output symbol size, but if the input data makes a different input symbol size more convenient, it can be used.

Another factor in determining the input symbol size is to choose the input symbol size so that the number of input symbols, K, is large enough to keep the reception overhead minimal. For example, K = 10,000 leads to an average reception overhead of 5% to 10% with moderate fluctuations, whereas K = 80,000 leads to an average reception overhead of 1% to 2% with very little fluctuation. As an example, in one test comprising 1,000,000 trials with K = 80,000, the reception overhead never exceeded 4%.

The above-described encoding process produces a stream of packets containing groups of output symbols based on the original file. The groups of output symbols hold an encoded form of the file, or more succinctly the encoded file. Each group of output symbols in the stream is generated independently of all other groups of output symbols, and there is no lower or upper bound on the number of groups of output symbols that can be created. A key is associated with each group of output symbols. That key, and some contents of the input file, determines the values for the group of output symbols. Consecutively generated groups of output symbols need not have consecutive keys, and in some applications it would be preferable to randomly generate the sequence of keys, or pseudorandomly generate the sequence.

Group chain reaction decoding has a property that if the original file can be split into K equal-sized input symbols and each output symbol value is the same length as an input symbol value, then the file can be recovered from K + A output symbols on average, where A is small compared to K. For example, A might be 500 for K = 20,000. Since the particular groups of output symbols are generated in a random or pseudorandom order, and the loss of particular groups of output symbols in transit is assumed arbitrary, some small variance exists in the actual number of groups of output symbols needed to recover the input file. In some cases, where a particular collection packets containing K + A output symbols are not enough to decode the entire input file, the input file is still recoverable if the receiver can gather more packets from one or more sources of packets.

Because the number of groups of output symbols is only limited by the resolution of I, well more than K + A groups of output symbols should be able to be generated. For example, if I is a 32-bit number, four billion different groups of output symbols could be generated, whereas the file could comprise K = 50,000 input symbols. In practice, only a small number of those four billion groups of output symbols would be generated and transmitted and it is a near certainty that an input file can be recovered with a very small fraction of the possible groups of output symbols and an excellent probability that the input file can be recovered with slightly more than K output symbols (assuming that the input symbol size is the same as the output symbol size).

The average number of arithmetic operations required to produce each group of output symbols is proportional to log K and so the total number of arithmetic operations required to decode and recover the input file is proportional to K log K. As shown above, an efficient decoding process exists that uses only slightly more memory than the memory required to store the input file (typically around 15% more). The above numbers show significant reductions in operations and storage compared with previously known coding techniques.

For example, standard Reed-Solomon codes are a standard code for communications applications. With standard Reed-Solomon codes, the input file is split into K input symbols, as with group chain reaction coding, but the K input symbols in Reed-Solomon codes are encoded into N output symbols, where N is typically fixed before the encoding process begins. This contrasts with the present invention, which allows for an indeterminate number of groups of output symbols.

One advantage of having an indeterminate number of groups of output symbols is that if a recipient misses more groups of output symbols than expected, either due to a poor channel or due to the recipient beginning after some groups of output symbols have already passed it by, the recipient can just listen for a little longer and pick up more groups of output symbols. Another advantage is that since the recipient may be gathering groups of output symbols produced from multiple encoders, each encoder may have to provide only a small fraction of the groups of output symbols needed to decode the input symbols and the number of groups of output symbols from one encoder may depend on how many encoders are supplying the recipient with groups of output symbols.

Standard Reed-Solomon codes also require substantially more time than chain reaction codes, for both encoding and decoding. For example, the number of arithmetic operations required to produce each output symbol with standard Reed-Solomon is proportional to K. The number of arithmetic operations required for decoding standard Reed-Solomon codes depends on which output symbols arrive at the recipient, but generally the number of such operations is proportional to (N-K)·K. Hence, in practice, acceptable values of K and N are very small, on the order of tens and possibly up to small hundreds. For example, Cross-Interleaved Reed-Solomon codes are used on compact disks (CDs) and CD-ROMs. For CDs, one standard code uses K = 24 and N = 28 and another standard code uses K = 28 and N = 32. For CD-ROMs, one standard code uses K = 24 and N = 26 and another standard code uses K = 43 and N = 45. Standard Reed-Solomon codes used for satellite transmission of MPEG files (MPEG is a file format for video and audio streams) use K = 188 and N = 204; generally these large values require specialized hardware.

Faster implementations of standard Reed-Solomon codes are known to exist which allow encoding in time cK *log* K and decoding in time c' K (*log* K)^2, but c and c' are prohibitively large constants that make these implementations slower than other implementations of standard Reed-Solomon codes for all but very large values of K, i.e., the efficiency crossover point is for values of K in the thousands or tens of thousands. Thus, for values of K below the crossover point, the other implementations of standard Reed-Solomon codes are faster. Although the faster implementations are faster than the other implementations at values of K above the crossover point, the faster implementations are slower at those values of K than group chain reaction codes, by orders of magnitude.

Because of the speed limitations, only small values of K and N are generally feasible for standard Reed-Solomon codes. Consequently, their use on large files requires that the files be split into many subfiles and each subfile separately coded. Such splitting decreases the effectiveness of the codes to protect against packet loss in transmission.

One feature of standard Reed-Solomon codes is that any K distinct output symbols can be used by the recipient to decode the input file. It is provable that at least K output symbols are required to decode an arbitrary input file, and hence standard Reed-Solomon codes are optimal in this regard since K is also the maximum number of output symbols needed to decode the input file. Group chain reaction coding, in contrast, generally requires K + A output symbols, where A is small compared to a suitably chosen K, or slightly more than K output symbols in total. In the network applications described previously, this disadvantage of possibly needing A additional symbols is greatly overshadowed by the speed advantage and the ability to seamlessly handle larger files.

Variations of the Basic Communication System

Embodiments of communication systems according to the present invention for a single channel have been described above in detail. The elements of those embodiments can be extended to advantageously use more than one channel.

Figs. 22-23 show systems between two computers incorporating a communications system such as that shown in Fig. 1. The first example (Fig. 22) has a sender computer 2200 sending an input file 2210 to a recipient computer 2220 over a network 2230. The second example (Fig. 23) has a sender computer 2300 broadcasting an input file 2310 to recipient computers 2320 (only one is shown) over a wireless channel 2330. Instead of network 2330, any other physical communications medium, such as the wires of the Internet, might be used. Wireless channel 2330 might be a wireless radio channel, a pager link, a satellite link, an infrared link or the like. The configuration shown in Fig. 23 might also be used with wired or unwired media when one sender is sending an input file to many recipients, when the recipient is obtaining the input file from many senders, or when many recipients are obtaining the input file from many senders.

As should be apparent upon reading the above description, the group chain reaction coding scheme described above can be used to send a file over a lossy transmission medium, such as a computer network, the Internet, a mobile wireless network or a satellite network, as a stream of packetized data with desirable properties. One such desirable property is that, once a decoding agent receives any set of sufficiently many packets from the stream, it can reconstruct the original file extremely quickly. Group chain reaction coding is also useful in situations where many agents are involved, such as when one transmitting agent is sending the file to multiple receiving agents in a multicast or broadcast setting.

The group chain reaction coding scheme is also useful in situations where multiple transmitting agents are sending the same file to multiple receiving agents. This allows for improved robustness against localized failure of the network infrastructure, allows receiving agents to seamlessly change from receiving packets from one transmitting agent to another, and allows receiving agents to speed up their download by receiving from more than one transmitting agent at the same time.

In one aspect, the group chain reaction coding process described above performs the digital equivalent of a holographic image, where each part of a transmission contains an image of the transmitted file. If the file is a megabyte file, a user can just tap into a transmitted stream to obtain any arbitrary megabyte's worth of data (plus some extra overhead) and decode the original megabyte file from that megabyte.

Because group chain reaction coding works with a random selection of data from the transmitted stream, downloads do not need to be scheduled or coherent. Consider the advantages of video-on-demand with group chain reaction coding. A particular video could be broadcast as a continuous stream on a particular channel without coordination between the receiver and the transmitter. The receiver simply tunes into a broadcast channel for a video of interest and captures enough data to reconstruct the original video, without having to figure out when the transmission started or how to get copies of lost portions of the broadcast.

These concepts are illustrated in Figs. 24-25. Fig. 24 illustrates an arrangement wherein one receiver 2402 receives data from three transmitters 2404 (individually denoted "A", "B" and "C") over three channels 2406. This arrangement can be used to triple the bandwidth available to the receiver or to deal with transmitters not being available long enough to obtain an entire file from any one transmitter. As indicated, each transmitter 2404 sends a stream of values, S(I). Each S(I) value represents a key I and a group of output symbol values B(I) = B_1(I),…,B_b(I), the use of which is explained above. For example, the value $S(n_A + i_A n'_A)$ is the key with value $n_A + i_A n'_A$ followed by the group of output symbols with values $B(n_A + i_A n'_A)$, where n_A and n'_A are randomly chosen numbers associated with transmitter 2404(A), and i_A is the sequence number of a packet that is sent from transmitter 2404(A) . The sequence of keys from one transmitter is preferably distinct from the sequence of keys from the other transmitters, so that the transmitters are not duplicating efforts. This is illustrated in Fig. 24 by the fact that the key sequence used at each transmitter is a function of information that is particular to that transmitter, e.g. the key sequence for transmitter 2404(A) depends on n_A and n'_A.

Note that transmitters 2404 do not need to be synchronized or coordinated in order not to duplicate efforts. In fact, without coordination, each transmitter is likely to send a sequence of keys that are largely distinct (i.e., $n_A + i_A n'_A \neq n_B + i_B n'_B \neq n_C + i_C n'_C$, where i_A, i_B, i_C is the sequence number of a packet transmitted by satellite SAT A, SAT B, SAT C, respectively, and where n_A, n'_A, n_B, n'_B , n_C, n'_C are randomly chosen numbers associated with SAT A, SAT B, SAT C, respectively.).

Since a random selection of K/b + A groups of output symbols, maybe with a few bunches of G extra groups of output symbols, can be used to recover an input file, the uncoordinated transmissions are additive instead of duplicative.

This "information additivity" is illustrated again in Fig. 25. There, copies of one input file 2502 are provided to a plurality of transmitters 2504 (two of which are shown in the figure). Transmitters 2504 independently transmit groups of output symbols generated from the contents of input file 2502 over channels 2506 to receivers 2508. If each transmitter uses a distinct set of keys for symbol generation, the streams are likely to be independent and additive (i.e., they add to the information pool used to recover input symbols) rather than duplicative. Each transmitter of the two shown might need to only transmit (K/b + A)/2 groups of output symbols before the receiver's decoder is able to recover the entire input file.

Using two receivers and two transmitters, the total amount of information received by a receiver unit 2510 can be as much as four times the information available over one channel 2506. The amount of information might be less than four times the single channel information if, for example, the transmitters broadcast the same data to both receivers. In that case, the amount of information at receiver unit 2510 is at least double the rate that can be achieved by sending the original file directly, especially if data is lost in the channel. Note that, even if the transmitters broadcast only one signal, but the receivers are in view at different times, there is an advantage to having more than one receiver listening to each transmitter. In Fig. 25, receiver unit 2510 performs the functions similar to the functions of receiver 150, decoder 155, key regenerator 160 and input file reassembler 165 shown in Fig. 1.

In some embodiments, input file 2502 is encoded in one computing device having two encoders so that the computing device can provide one output for one transmitter and another output for the other transmitter. Other variations of these examples should be apparent upon review of this disclosure.

It is to be understood that the coding apparatus and methods described herein may also be used in other communication situations and are not limited to communications networks such as the Internet. For example, compact disk technology also uses erasure and error-correcting codes to handle the problem of scratched disks and would benefit from the use of group chain reaction codes in storing information thereon. As another example, satellite systems may use erasure codes in order to trade off power requirements for transmission, purposefully allowing for more errors by reducing power and group chain reaction coding would be useful in that application. Also, erasure codes have been used to develop RAID (redundant arrays of independent disks) systems for reliability of information storage. The current invention may therefore prove useful in other applications such as the above examples, where codes are used to handle the problems of potentially lossy or erroneous data.

In some preferred embodiments, sets of instructions (or software) to perform the communication processes described above are provided to two or more multi-purpose computing machines that communicate over a possibly lossy communications medium. The number of machines may range from one sender and one recipient to any number of machines sending and/or receiving. The communications medium connecting the machines may be wired, optical, wireless, or the like. The above-described communications systems have many uses, which should be apparent from this description.

The above description is illustrative and not restrictive. Many variations of the invention will become apparent to those of skill in the art upon review of this disclosure. The scope of the invention should, therefore, be determined not with reference to the above description, but instead should be determined with reference to the appended claims along with their full scope of equivalents.

APPENDIX A. SOURCE CODE LISTINGS

1. DFSolitonDistribution.cpp

```
//////////////////////////////////////////////////////////////////////
// DFSolitonDistribution.cpp:
//      implementation of the CDFSolitonDistribution class.
//
// The probability distribution of the weights of groups of output
// symbols is computed in the constructor and then is referred to by
// the Key Decoder when it decides what weight and pool are associated
// with a key.
//////////////////////////////////////////////////////////////////////

#include "DFSolitonDistribution.h"
#include "DFRandomBits.h"
#include <math.h>

//////////////////////////////////////////////////////////////////////
// Construction/Destruction
//////////////////////////////////////////////////////////////////////

CDFSolitonDistribution::CDFSolitonDistribution(
int nSymbols,
int nArity,
int nRippleTargetSize,
int nStartRippleSize)
{
   m_nSegments    = nSegments;

   // Unless the values of R and S are specified, use the constants
   // from the .h file with the formulas based on the fourth root
   // and the square root of the number of symbols.

   m_nRippleTargetSize = nRippleTargetSize;

   if (!m_nRippleTargetSize)
      m_nRippleTargetSize =
         int(knRAdd + kdRFactor * sqrt(sqrt(m_nSymbols/nArity)));

   m_nStartRippleSize = nStartRippleSize;

      if (!m_nStartRippleSize)
      m_nStartRippleSize = int(kdSFactor * sqrt(m_nSymbols/nArity));

   ///////////////////////////////////////////////////////////
   // Compute parameters of the Soliton distribution:
   //
   // This is the modified Robust distribution with tapered
   // density at weights N/R, 2N/R,....N
   ///////////////////////////////////////////////////////////

   m_nModulus = 0x1 << knPrecision ;

   // For the previous variation of the Robust distribution, use
   // "-1" as the value
```

```
m_nRobustKinds= m_nSymbols/m_nRippleTargetSize - 1 - m_nArity;
if (m_nRobustKinds < 1)                 m_nRobustKinds = 1;
if (m_nRobustKinds > m_nSymbols)        m_nRobustKinds = m_nSymbols;

// In the previous variation of the Robust distribution,
// m_nTailKinds is 0
m_nTailKinds = 1;
for (int d = m_nRippleTargetSize; d > 1; d /= 2)
   m_nTailKinds++;    // becomes log_2(RippleTargetSize) + 1

m_nKinds = m_nRobustKinds + m_nTailKinds;
if (m_nKinds > m_nSymbols)
{
   m_nTailKinds = m_nSymbols - m_nRobustKinds;
   m_nKinds = m_nSymbols;
}

m_anKindWeight              = new int[m_nKinds];
m_anKindCumulativeDensity   = new int[m_nKinds];

// adKindFraction is the un-normalized distribution
double* adKindFraction      = new double[m_nKinds];

// Weight m_nArity output symbols:
adKindFraction[0] =
         double(nArity*m_nStartRippleSize)/double(m_nSymbols);
m_anKindWeight[0] = m_nArity;

// Densities according to the Robust Soliton distribution:
for (int i=1; i < m_nRobustKinds; i++)
{
int nWt = i + m_nArity;

adKindFraction[i] = adKindFraction[i-1] +
   double(nArity)/(double(nWt) * double(nWt-1) *
   (1.0 - (double((nWt)*m_nRippleTargetSize)/double(m_nSymbols))));

m_anKindWeight[i] = nWt;
}

int nRPower = 1;
int j;
for (i = 0; i < m_nTailKinds; i++)
   nRPower *= 2; //nRPower is next power of 2 > m_nRippleTargetSize

// Densities for the tapered tail at the end of the distribution:
// The weights go down from m_nSegments by a factor of 2 each time
// and the density is inversely proportional to the weight.

// j runs from 2 up to nRPower
for (i=m_nRobustKinds, j=2; i < m_nKinds; i++, j*=2)
{
   adKindFraction[i] = adKindFraction[i-1]
      + kdTFactor/(m_nArity*m_nArity) *
         double(nRPower)/double(j*m_nSymbols);
   m_anKindWeight[i] = (j*m_nSymbols)/nRPower;
}

// Normalize to m_nModulus
```

```
    for (i=0; i < m_nKinds; i++)
        m_anKindCumulativeDensity[i] = int(adKindFraction[i] *
                             double(m_nModulus)/adKindFraction[m_nKinds-1]);

    delete[] adKindFraction;

    // Calculate maximum and average weight
    m_nMaxWeight = 0;
    for (i=0; i < m_nKinds; i++)
        if (m_anKindWeight[i] > m_nMaxWeight)
                             m_nMaxWeight = m_anKindWeight[i];

    ComputeAverageWeight();
}

CDFSolitonDistribution::~CDFSolitonDistribution()
{
    if (m_anKindWeight)              delete[] m_anKindWeight;
    if (m_anKindCumulativeDensity)   delete[] m_anKindCumulativeDensity;
}

void CDFSolitonDistribution::ComputeAverageWeight()
{
    int i ;
    double dTemp ;
    dTemp = double(m_anKindWeight[0]) *
                double(m_anKindCumulativeDensity[0]) ;
    for (i=1; i < m_nKinds; i++)
        dTemp += double(m_anKindWeight[i]) *
                    double(m_anKindCumulativeDensity[i] -
                           m_anKindCumulativeDensity[i-1]) ;
    m_dAverageWeight = (dTemp/m_nModulus) + 1;
}
```

2. DFSolitonDistribution.h

```
///////////////////////////////////////////////////////////////////////
// DFSolitonDistribution.h:
//           interface for the CDFSolitonDistribution class.
//
// Note: The class CDFDominoKeyDecoder computes the weight and pool of
// neighbors for a given output symbol key. It refers to this class for
// the probability distribution for the weights and it refers to the
// CDFRandomBits class for the random integers used to make the choices.
//
///////////////////////////////////////////////////////////////////////

#include "DFRandomBits.h"
//class CDFRandomBits;

///////////////////////////////////////////////////////////////////////
//
// Constants used in the calculation of the distribution
//
///////////////////////////////////////////////////////////////////////

// The arithmetic precision of the distribution in bits.
// The density of the distribution is scaled to 2power this value.
const int       knPrecision     = 28;

// Constants relating to the calculation of R, the "ripple target size".
// This value is the expected ripple size in the "Robust Soliton
// Distribution".  R gets the value kdRFactor * 4th root N + knRadd
// where N is the number of input symbols (segments).
const double  kdRFactor = 2.1;
const int     knRAdd    = 2;

// S is the expected size of the ripple at the start of decoding,
// meaning the expected number of weight 1 output symbols when N symbols
// are received.
// S = kdSFactor * sqrt(N)
const double  kdSFactor = 1.4;

// The tail of the distribution is given higher density at degrees N,
// N/2,... down to N/R. The distribution density is inversely
// proportional to the symbol weight with kdTFactor as a constant of
// proportionality. The resulting distribution is still scaled according
// to the precision above.

// A prior value of 1.6 was used, but we increased it to 1.8 to cut
// down on "outliers", for which a long time is spent waiting for one
// last symbol.  We then increased it all the way up to 2.4 to take care
// of one outlier (in a million) that happened with 40000 symbols.
const double  kdTFactor = 2.4;

class CDFSolitonDistribution
{
   friend class CDFDominoKeyDecoder;

public:
    ////////////////////////////////////////////////////////////////////////
    //Output Symbol Key and Input Symbol Position
    //
    // This class defines the types TKey and TPos standing for Input
    // Symbol Position (previously called "Index").
    // Other classes that need these types should refer to these below.
    ////////////////////////////////////////////////////////////////////////

    // typedef unsigned int    TKey;
    typedef unsigned _int32 TKey;
    typedef int    TPos;

CDFSolitonDistribution(
                int nSymbols,
                int nArity           = 1,
                int nRippleTargetSize = 0,
                int nStartRippleSize  = 0);

    virtual ~CDFSolitonDistribution();

    inline double dAverageWeight()  { return m_dAverageWeight;     };
    inline int    nMaxWeight()      { return m_nMaxWeight;         };
    inline int    nParameterR()     { return m_nRippleTargetSize; };
    inline int    nParameterS()     { return m_nStartRippleSize;  };

    // The probability distribution comprises an array of kinds where
    // each kind corresponds to a weight (previously referred to as
    // degree) for output symbols and a density (probability) associated
    // with that weight.

    inline int    nKinds()          { return m_nKinds;      };

private:

    double m_dAverageWeight;
    void   ComputeAverageWeight();
    int    m_nRippleTargetSize;
    int    m_nStartRippleSize;

    // m_nKinds is the size of the array that holds the probability
    // distribution.  It is the number of different weights of output
    // symbols that are possible.
    int    m_nKinds;

    // The following are the number of kinds from the truncated robust
    // soliton distribution and the number of kinds due to the tapered
    // tail distribution.
    int    m_nRobustKinds;
    int    m_nTailKinds;

    int    m_nModulus; // 2 to the precision

    int*   m_anKindWeight; // the weight corresponding to a kind
    int*   m_anKindCumulativeDensity; // probability density of a kind

    int    m_nSymbols;
    int    m_nArity;
    int    m_nMaxWeight;
};
```

WHAT IS CLAIMED IS:

1. A method of generating a group of output symbols, a group being one or more output symbols wherein each output symbol is selected from an output alphabet and the group is such that an input file, comprising an ordered plurality of input symbols each selected from an input alphabet, is recoverable from a set of such groups, the method comprising the steps of:

obtaining a key I for the group, wherein the key is selected from a key alphabet and the number of possible keys in the key alphabet is much larger than the number of input symbols in the input file;

calculating, according to a predetermined function of I, a list AL(I) for the group, wherein AL(I) is an indication of W(I) associated input symbols associated with the group, and wherein weights W are positive integers that vary between at least two values and are greater than one for at least one value of I; and

generating an output symbol value for each output symbol in the group from a predetermined function of the associated input symbols indicated by AL(I).

2. The method of claim 1, wherein the step of obtaining key I comprises a step of calculating key I according to a random function or pseudorandom function.

3. The method of claim 1, wherein the step of calculating comprises a step of calculating W(I) according to a random function or pseudorandom function of I.

4. The method of claim 1, wherein the step of calculating comprises a step of calculating AL(I) according to a random function or pseudorandom function of I.

5. The method of claim 1, wherein the step of calculating comprises a step of calculating a number, G(I), of output symbols in the group where G(I) is a positive integer, said number, G(I), calculated according to a random function or pseudorandom function of I.

6. The method of claim 1, wherein the step of calculating comprises the steps of:

calculating, according to a predetermined function of I and a probability distribution, a weight W(I), wherein the probability distribution is over at least two positive integers, at least one of which is greater than one;

calculating a list entry for list AL(I); and

repeating the step of calculating a list entry for list AL(I) until W(I) list entries are calculated.

7. The method of claim 6, wherein the step of calculating W(I) comprises a step of determining W(I) such that W approximates a predetermined distribution over the key alphabet.

8. The method of claim 7, wherein the predetermined distribution is a uniform distribution.

9. The method of claim 7, wherein the predetermined distribution is a bell curve distribution.

10. The method of claim 7, wherein the predetermined distribution is such that W=1 has a probability of 1/K, where K is the number of input symbols in the input file, and W=i has a probability of 1/i(i-1) for i=2,...K.

11. The method of claim 1, wherein the predetermined function of the associated input symbols indicated by AL(I) is determined according to a Reed-Solomon code.

12. The method of claim 1, wherein the input alphabet and the output alphabet are the same alphabet.

13. The method of claim 1, wherein the input alphabet comprises 2^{Mi} symbols and each input symbol encodes Mi bits and wherein the output alphabet comprises 2^{Mo} symbols and each group of output symbols encodes Mo bits.

14. The method of claim 1, wherein each subsequent key I is one greater than the preceding key.

15. A method of encoding a plurality of groups of output symbols, each according to claim 1, the method further comprising steps of:

generating key I for each of the groups of output symbols to be generated; and

outputting each of the generated groups of output symbols as an output sequence to be transmitted through a data erasure channel.

16. The method of claim 15, wherein each key I is selected independently of other selected keys.

17. The method of claim 1, wherein the step of calculating AL(I) comprises the steps of:

identifying the number K of input symbols in the input file, at least approximately and a weight W(I);

determining the smallest prime number P greater than or equal to K;

if P is greater than K, at least logically padding the input file with P-K padding input symbols;

generating a first integer X such that $1 \leq X < P$ and a second integer Y such that $0 \leq Y < P$; and

setting the J-th entry in AL(I) to $((Y + (J-1) \cdot X) \bmod P)$ for each J from 1 to W(I).

18. The method of claim 17, wherein the step of setting the J-th entry in AL(I) for each J comprises the steps of:

setting the first entry V[J=0] in an array V to Y;

setting the J-th entry V[J] in the array V to (V[J-1] + X) mod P) for each J from 1 to W(I) minus one; and

using the array V as the list AL(I).

19. A method of transmitting data from a source to a destination over a packet communication channel, comprising the steps of:

a) arranging the data to be transmitted as an ordered set of input symbols, each selected from an input alphabet and having a position in the data;

b) generating a plurality of groups of output symbols, wherein each output symbol is selected from an output alphabet, wherein each group of the plurality of groups is generated by the steps of:

selecting, from a key alphabet, a key I for the group being generated;

determining a weight, W(I), as a function of I, wherein weights W are positive integers that vary between at least two values and over the key alphabet and are greater than one for at least some keys in the key alphabet;

selecting W(I) of the input symbols according to a function of I, thus forming a list AL(I) of W(I) input symbols associated with the group;

determining a number, G(I), of output symbols in the group; and

calculating a value B(I) of each output symbol in the group from a predetermined value function of the associated W(I) input symbols;

c) packetizing at least one of the groups of output symbols of at least one of the plurality of groups into each of a plurality of packets;

d) transmitting the plurality of packets over the packet communication channel;

e) receiving at least some of the plurality of packets at the destination; and

f) decoding the data from the plurality of received packets.

20. The method of claim 19, wherein the step of decoding the data comprises the steps of:

1) processing each received group of output symbols by the steps of:

a) determining the key I for the group;

b) determining the weight W(I) for the group; and

c) determining the W(I) associated input symbols for the group;

2) determining if enough information is received to decode any input symbols; and

3) decoding input symbols that can be decoded from the information received.

21. The method of claim 20, wherein the step of determining the key I comprises a step of at least partially determining the key I from data supplied in packets received over the packet communication channel.

22. The method of claim 19, wherein the step of decoding the data comprises the step of:

1) processing each received group of output symbols by the steps of:

a) determining the weight W(I) for the received group;

b) determining the W(I) associated input symbols for the received group; and

c) storing the values B(I) of the output symbols of the group in a group-of-output-symbols table along with the weight W(I) and the positions of the W(I) associates for the received group;

2) receiving additional groups of output symbols and processing them according to step 1) and its substeps;

3) for each group of output symbols, GOS1, having a weight of at most a group size and not being denoted as a used up group of output symbols, performing the steps of:
   a) calculating input symbols for input symbol positions corresponding to GOS1;
   b) identifying connected groups of output symbols in the group-of-output-symbols table, wherein a connected group of output symbols is a group of output symbols that is a function of at least one input symbol processed in step 3)a);
   c) recalculating the connected groups of output symbols to be independent of the input symbols processed in step 3)a);
   d) decrementing the weights of the groups of output symbols recalculated in step 3)c) to reflect their independence of the input symbols processed in step 3)a); and
   e) denoting GOS1 as a used up output symbol; and

4) repeating steps 1) through 3) above until the ordered set of input symbols is recovered at the destination.

23. The method of claim 22, wherein the step of denoting is a step of assigning a weight of zero to the used up output symbol.

24. The method of claim 22, wherein the step of denoting comprises a step of removing the used up group of output symbols from the group-of-output-symbols table.

25. The method of claim 19, wherein the step of packetizing is a step of packetizing at least one group of output symbols from at least one group of a plurality of groups of output symbols into each packet, the method further comprising a step of using the position of a group of output symbols within a packet as a part of the key for the group of output symbols.

26. The method of claim 1, wherein the step of calculating comprises a step of calculating a number, G(I), of output symbols in the group where G(I) is a positive integer and is the same positive integer for all values of I.

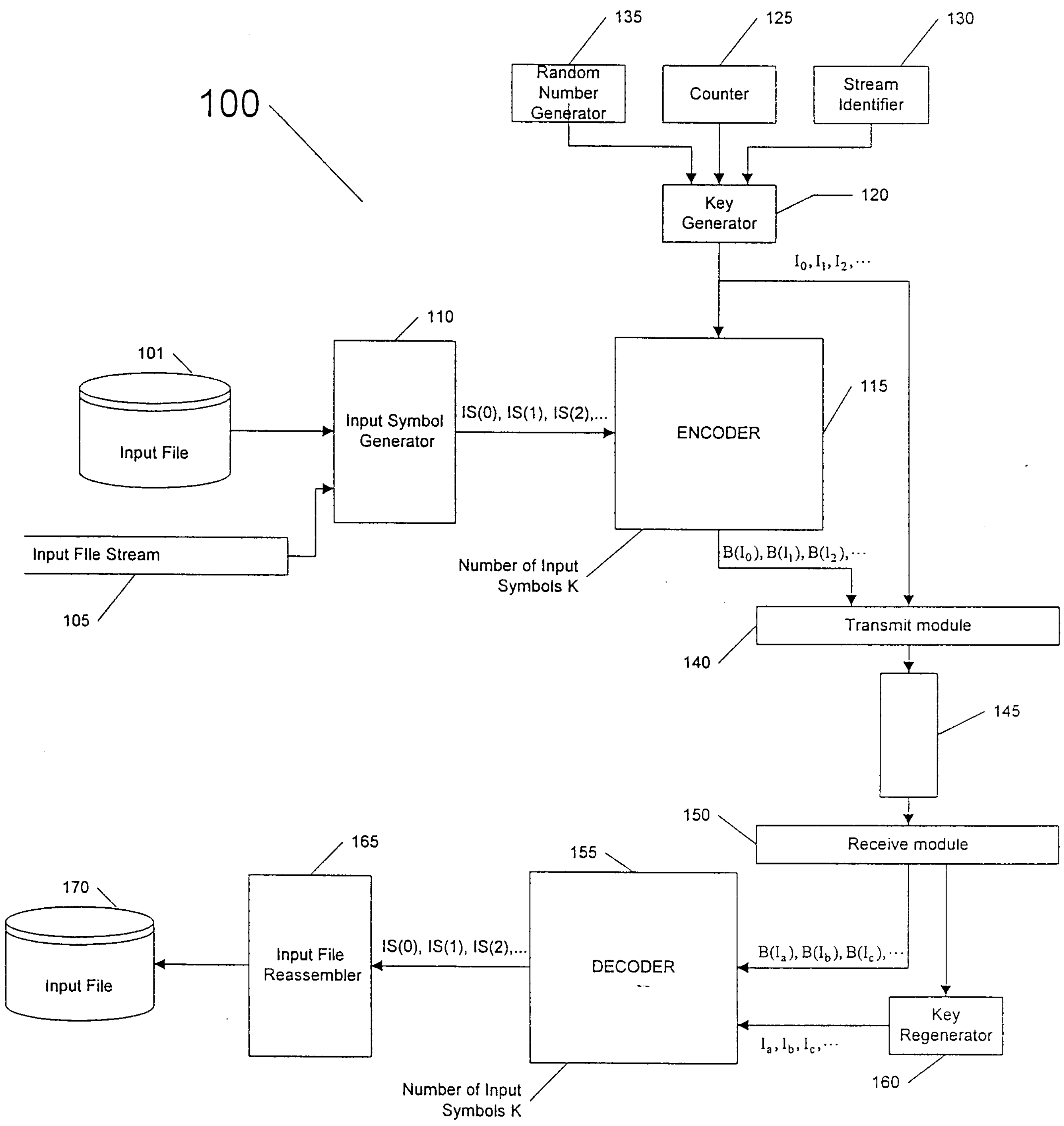

100
135
Random Number Generator
125
Counter
130
Stream Identifier
Key Generator
120
$I_0, I_1, I_2, \cdots$
110
101
Input File
Input Symbol Generator
IS(0), IS(1), IS(2),...
ENCODER
115
Input File Stream
105
Number of Input Symbols K
$B(I_0), B(I_1), B(I_2), \cdots$
Transmit module
140
145
150
Receive module
165
155
170
Input File
Input File Reassembler
IS(0), IS(1), IS(2),...
DECODER
$B(I_a), B(I_b), B(I_c), \cdots$
Key Regenerator
$I_a, I_b, I_c, \cdots$
160
Number of Input Symbols K

Figure 1

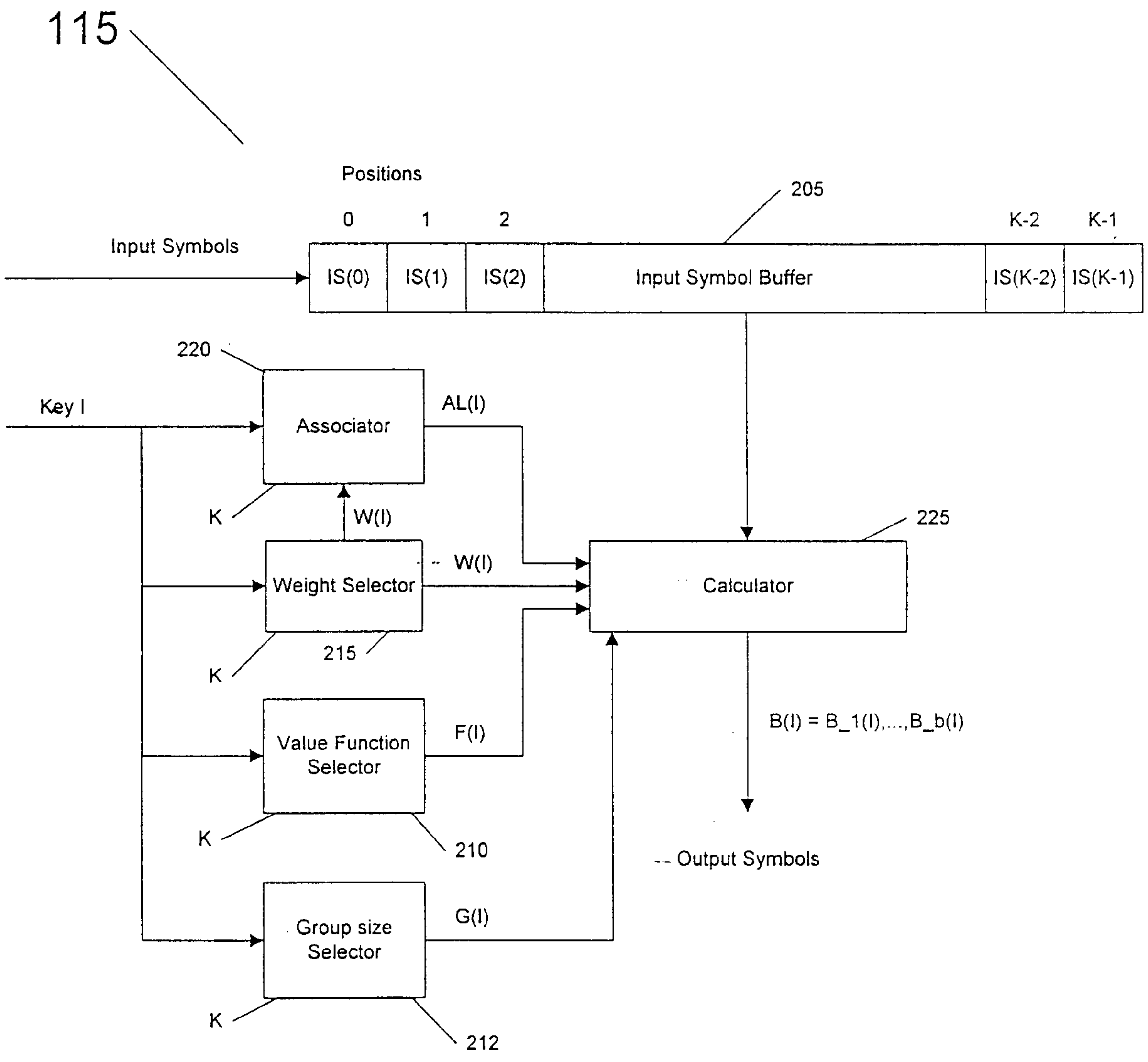

115
Positions
0
1
2
K-2
K-1
205
Input Symbols
IS(0)
IS(1)
IS(2)
Input Symbol Buffer
IS(K-2)
IS(K-1)
220
Key I
Associator
AL(I)
K
W(I)
Weight Selector
W(I)
225
Calculator
215
K
Value Function Selector
F(I)
B(I) = B_1(I),...,B_b(I)
K
210
Output Symbols
Group size Selector
G(I)
K
212

Figure 2

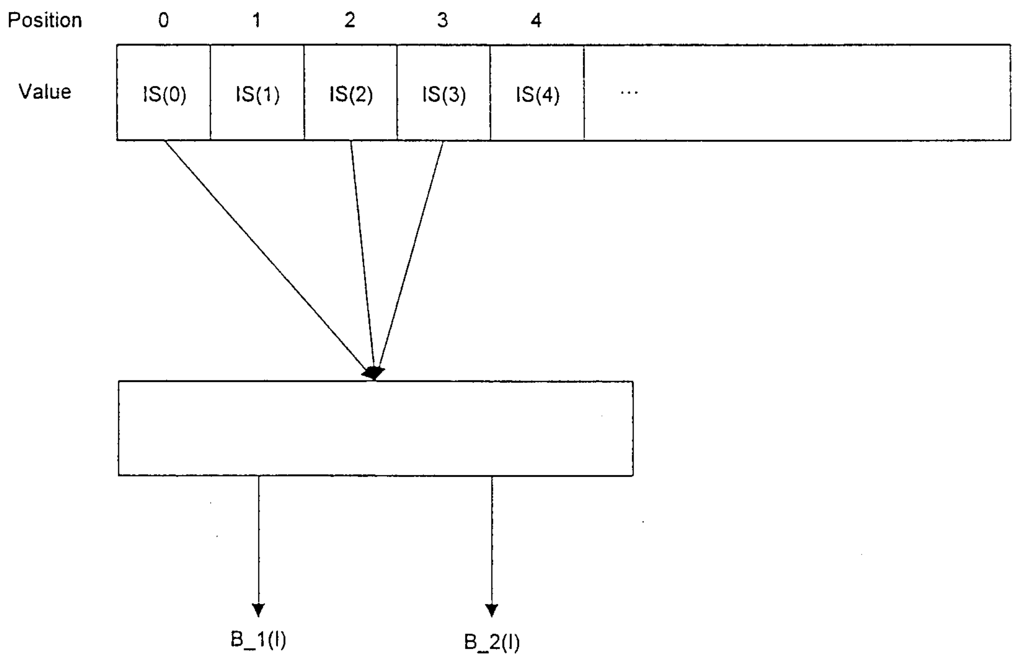


Figure 3

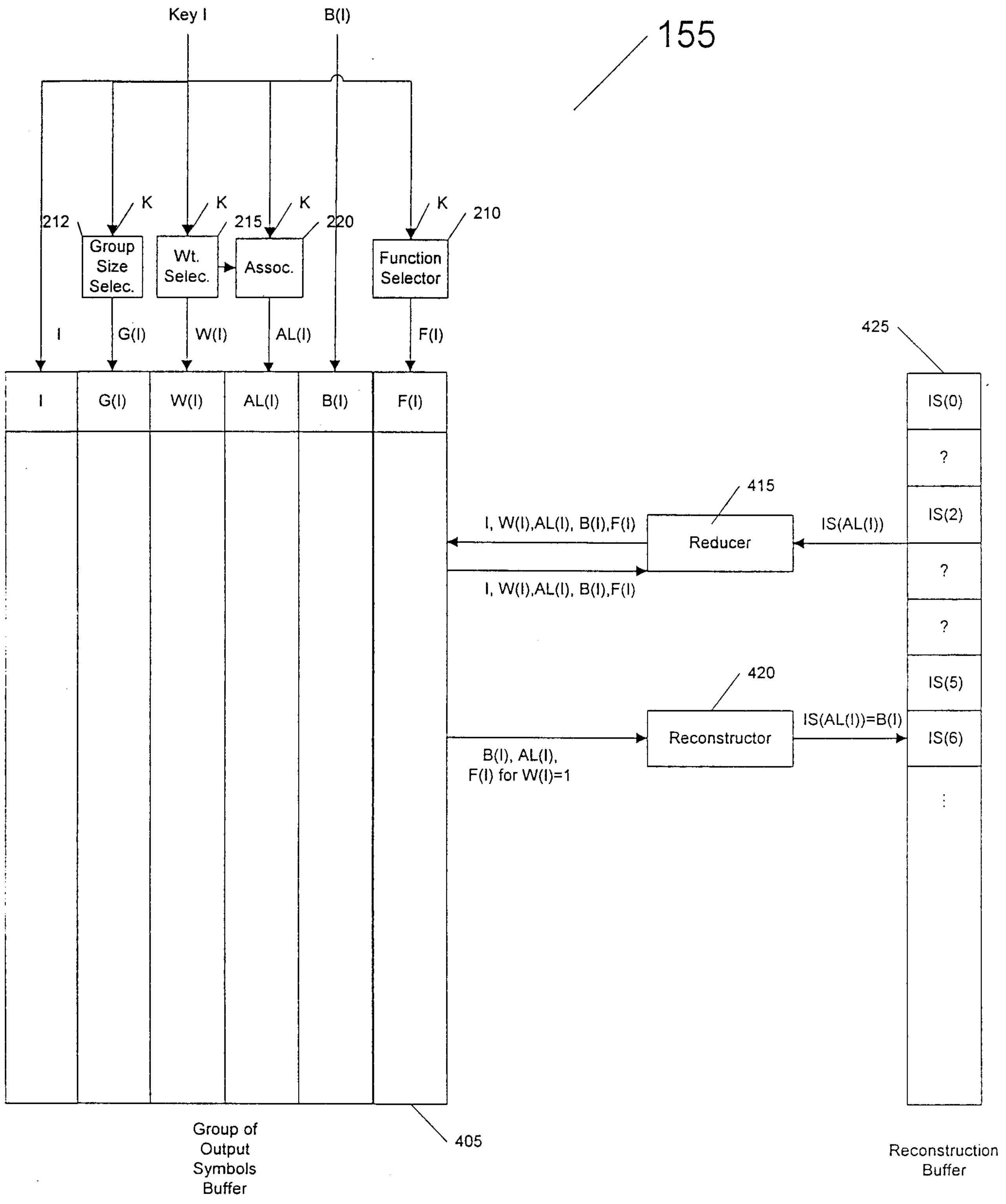

Key I
B(I)
155
212
K
Group Size Selec.
K
Wt. Selec.
215
K
Assoc.
220
K
Function Selector
210
I
G(I)
W(I)
AL(I)
F(I)
I
G(I)
W(I)
AL(I)
B(I)
F(I)
425
IS(0)
?
IS(2)
?
?
IS(5)
IS(6)
415
I, W(I),AL(I), B(I),F(I)
Reducer
IS(AL(I))
I, W(I),AL(I), B(I),F(I)
420
Reconstructor
IS(AL(I))=B(I)
B(I), AL(I), F(I) for W(I)=1
Group of Output Symbols Buffer
405
Reconstruction Buffer

Figure 4

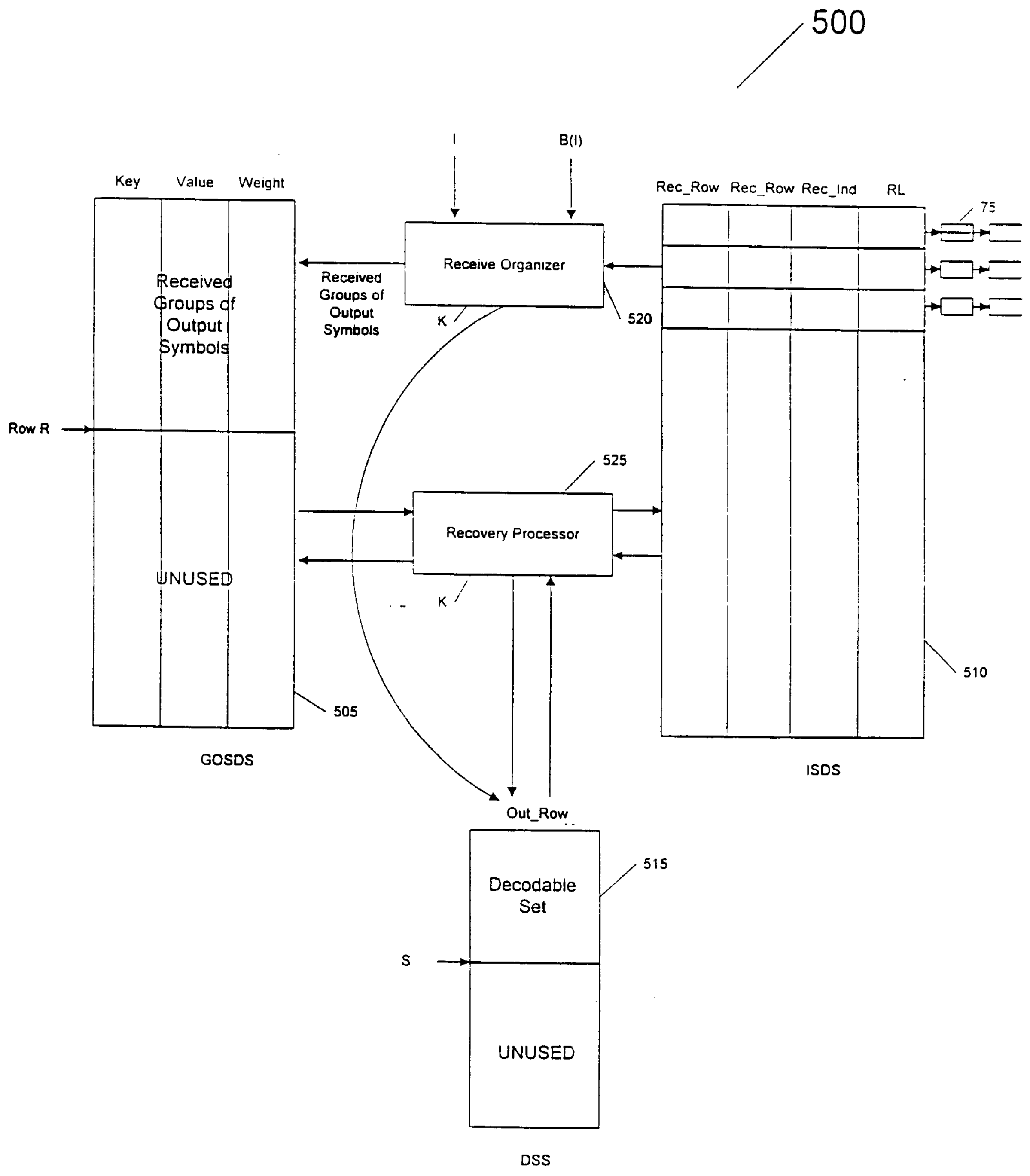

500
I
B(I)
Key
Value
Weight
Received Groups of Output Symbols
Receive Organizer
Received Groups of Output Symbols
K
520
Rec_Row
Rec_Row
Rec_Ind
RL
75
Row R
UNUSED
525
Recovery Processor
K
510
505
GOSDS
ISDS
Out_Row
515
Decodable Set
S
UNUSED
DSS

Figure 5

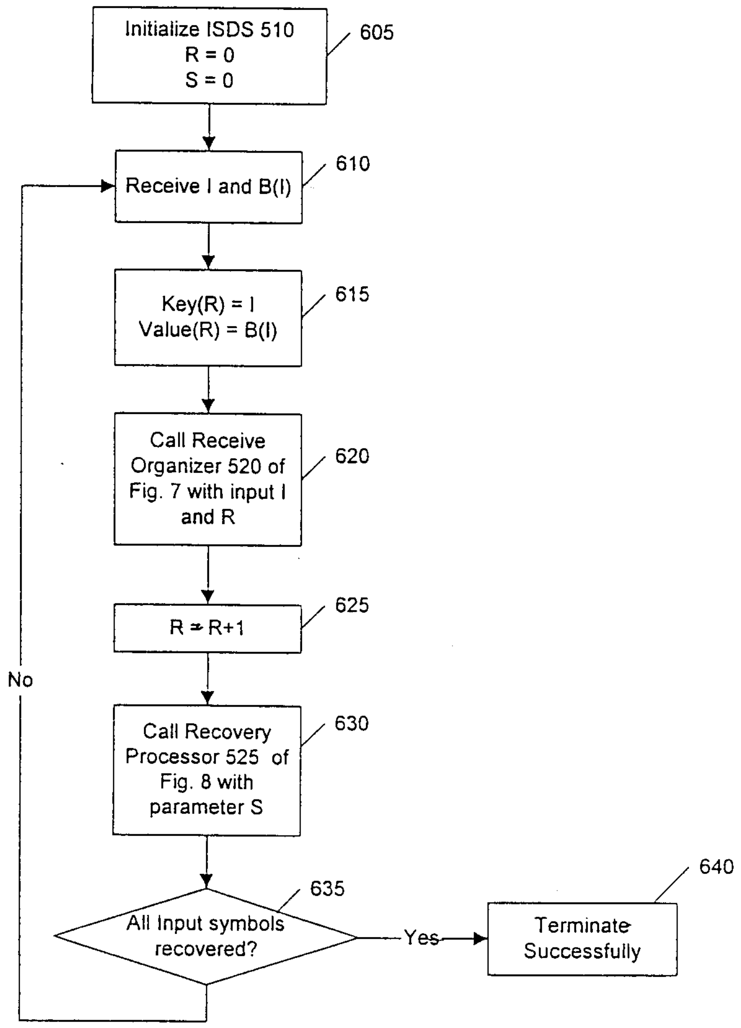
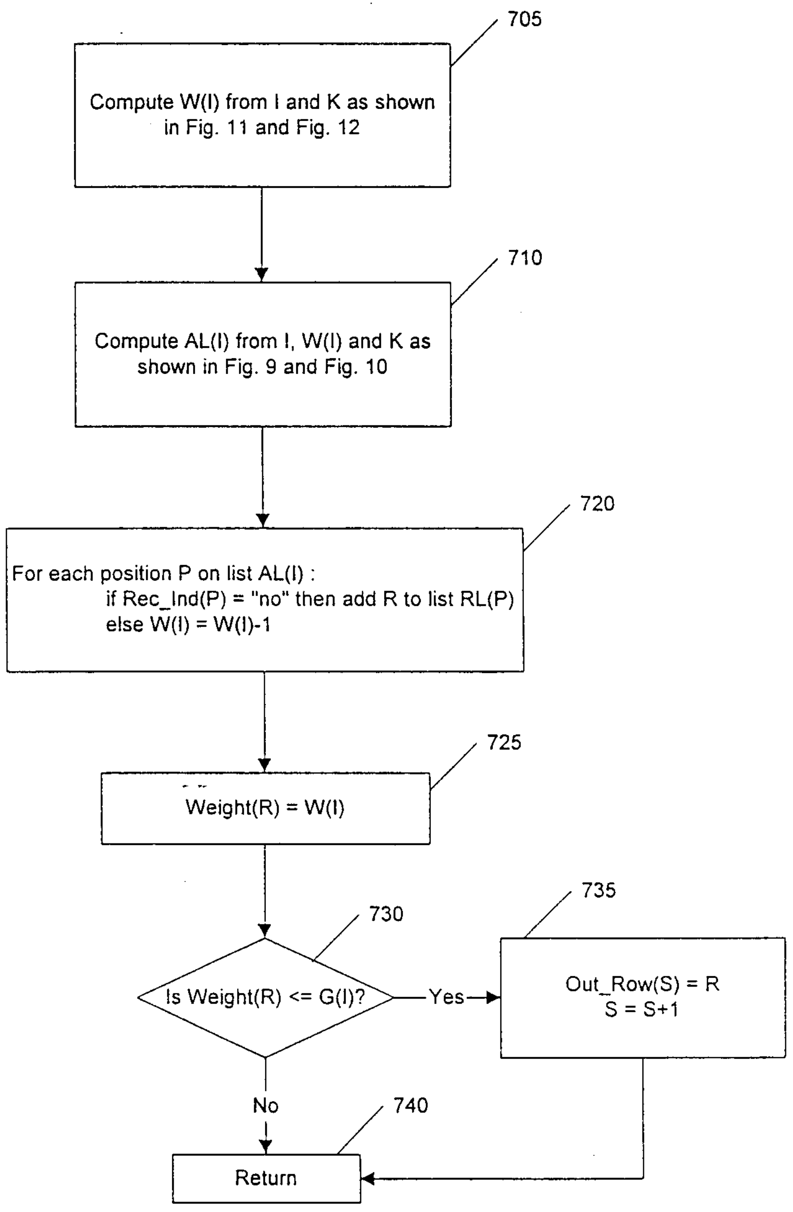
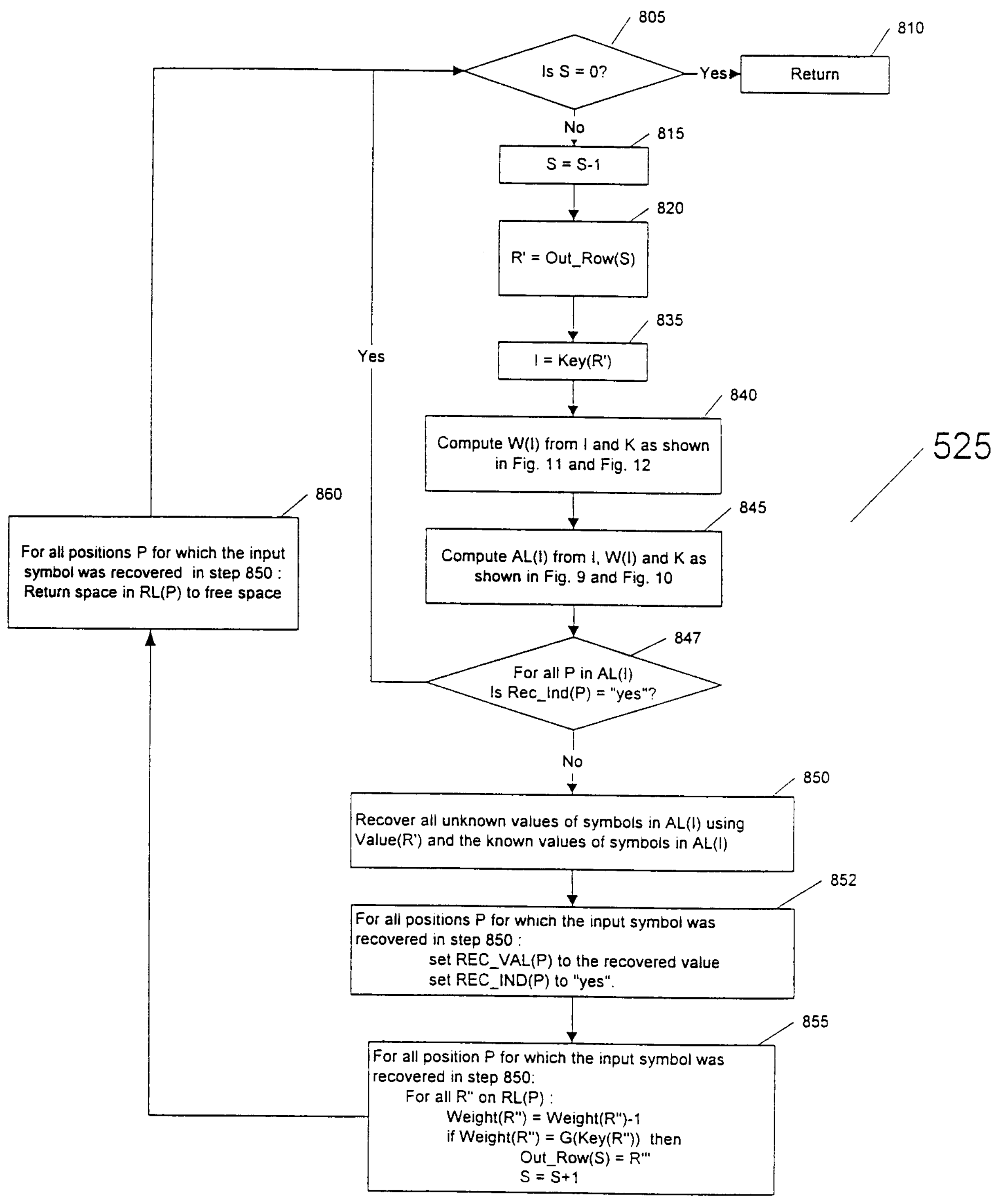

Initialize ISDS 510
R = 0
S = 0
605
Receive I and B(I)
610
Key(R) = I
Value(R) = B(I)
615
Call Receive Organizer 520 of Fig. 7 with input I and R
620
R = R+1
625
Call Recovery Processor 525 of Fig. 8 with parameter S
630
All Input symbols recovered?
635
Yes
No
Terminate Successfully
640

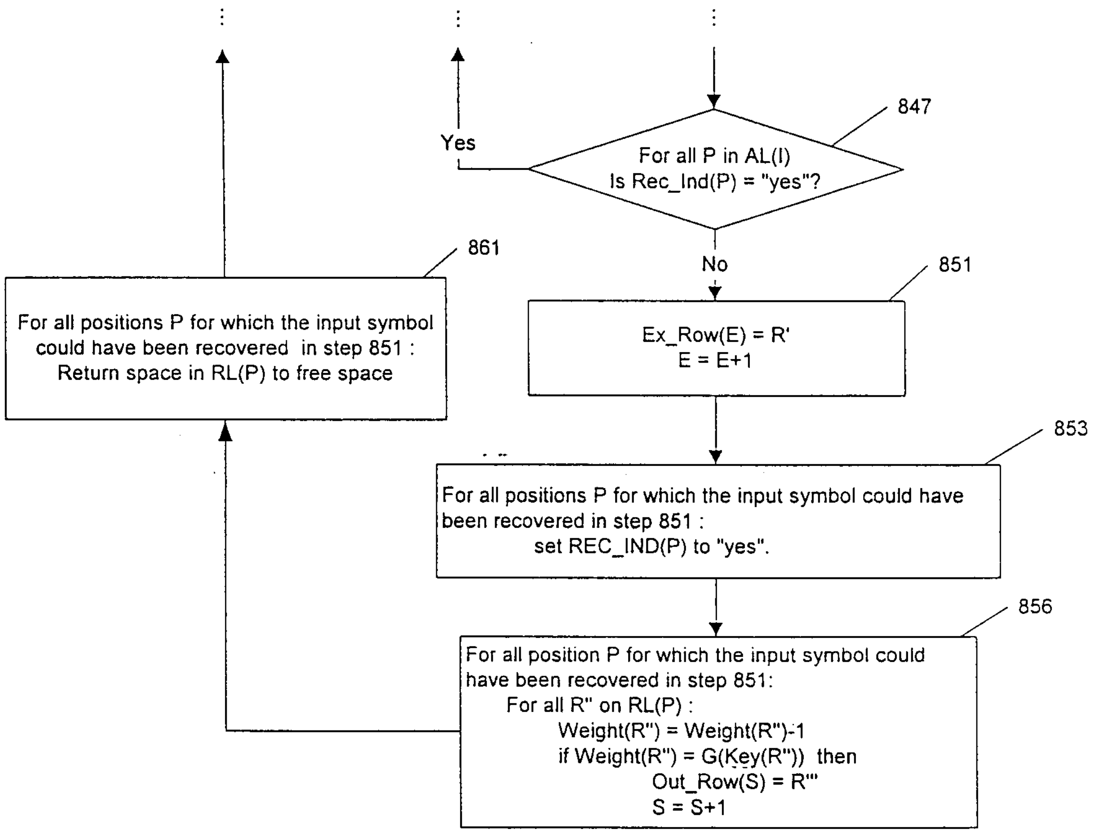
Figure 6


705
Compute W(I) from I and K as shown in Fig. 11 and Fig. 12
710
Compute AL(I) from I, W(I) and K as shown in Fig. 9 and Fig. 10
720
For each position P on list AL(I) :
if Rec_Ind(P) = "no" then add R to list RL(P)
else W(I) = W(I)-1
725
Weight(R) = W(I)
730
Is Weight(R) <= G(I)?
Yes
735
Out_Row(S) = R
S = S+1
No
740
Return

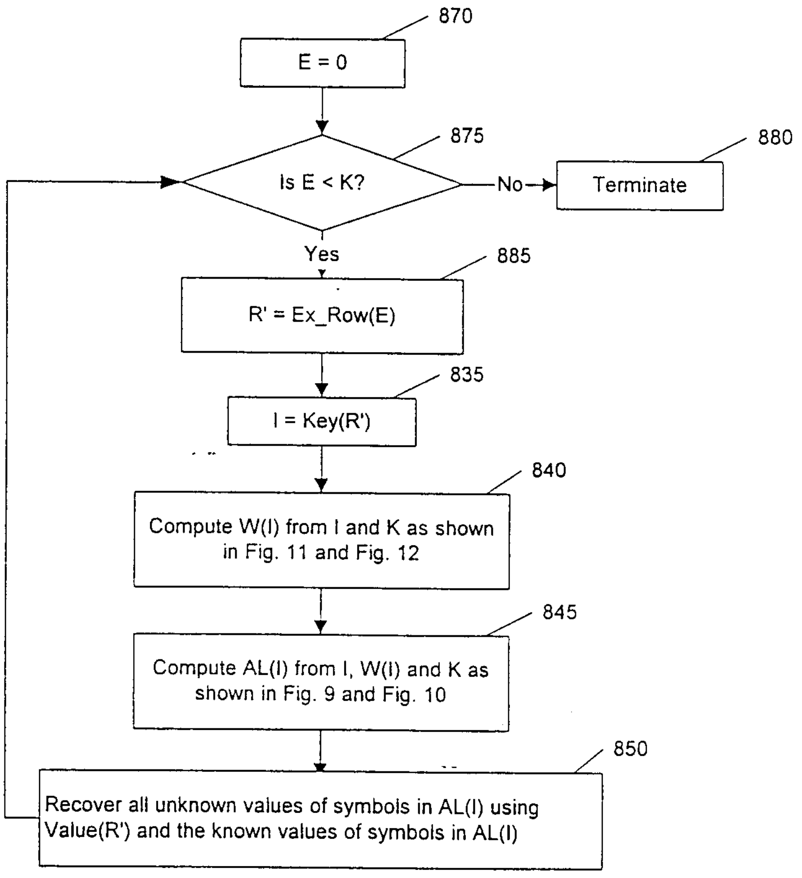
Receive Organizer 520 Flow Chart
Figure 7


805
Is S = 0?
Yes
810
Return
No
815
S = S-1
820
R' = Out_Row(S)
835
I = Key(R')
840
Compute W(I) from I and K as shown in Fig. 11 and Fig. 12
845
Compute AL(I) from I, W(I) and K as shown in Fig. 9 and Fig. 10
847
For all P in AL(I) Is Rec_Ind(P) = "yes"?
Yes
No
850
Recover all unknown values of symbols in AL(I) using Value(R') and the known values of symbols in AL(I)
852
For all positions P for which the input symbol was recovered in step 850 : set REC_VAL(P) to the recovered value set REC_IND(P) to "yes".
855
For all position P for which the input symbol was recovered in step 850: For all R" on RL(P) : Weight(R") = Weight(R")-1 if Weight(R") = G(Key(R")) then Out_Row(S) = R''' S = S+1
860
For all positions P for which the input symbol was recovered in step 850 : Return space in RL(P) to free space
525

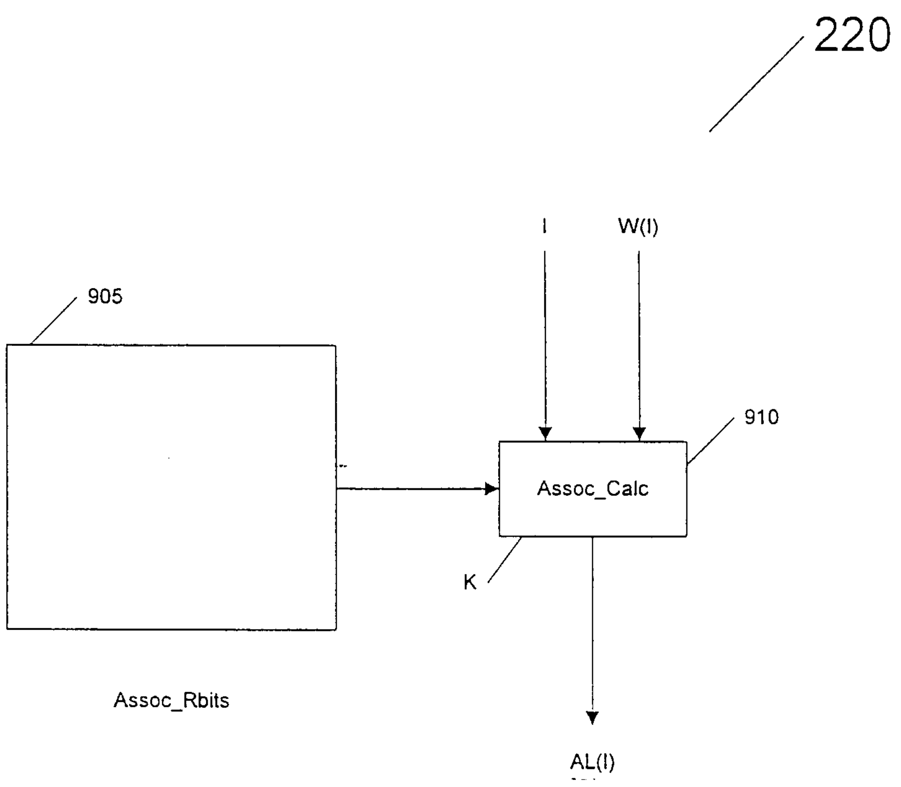
Recovery Processor Flow Chart
Figure 8(a)


847
For all P in AL(I)
Is Rec_Ind(P) = "yes"?
Yes
No
851
Ex_Row(E) = R'
E = E+1
853
For all positions P for which the input symbol could have been recovered in step 851 :
set REC_IND(P) to "yes".
856
For all position P for which the input symbol could have been recovered in step 851:
For all R" on RL(P) :
Weight(R") = Weight(R")-1
if Weight(R") = G(Key(R")) then
Out_Row(S) = R'"
S = S+1
861
For all positions P for which the input symbol could have been recovered in step 851 :
Return space in RL(P) to free space

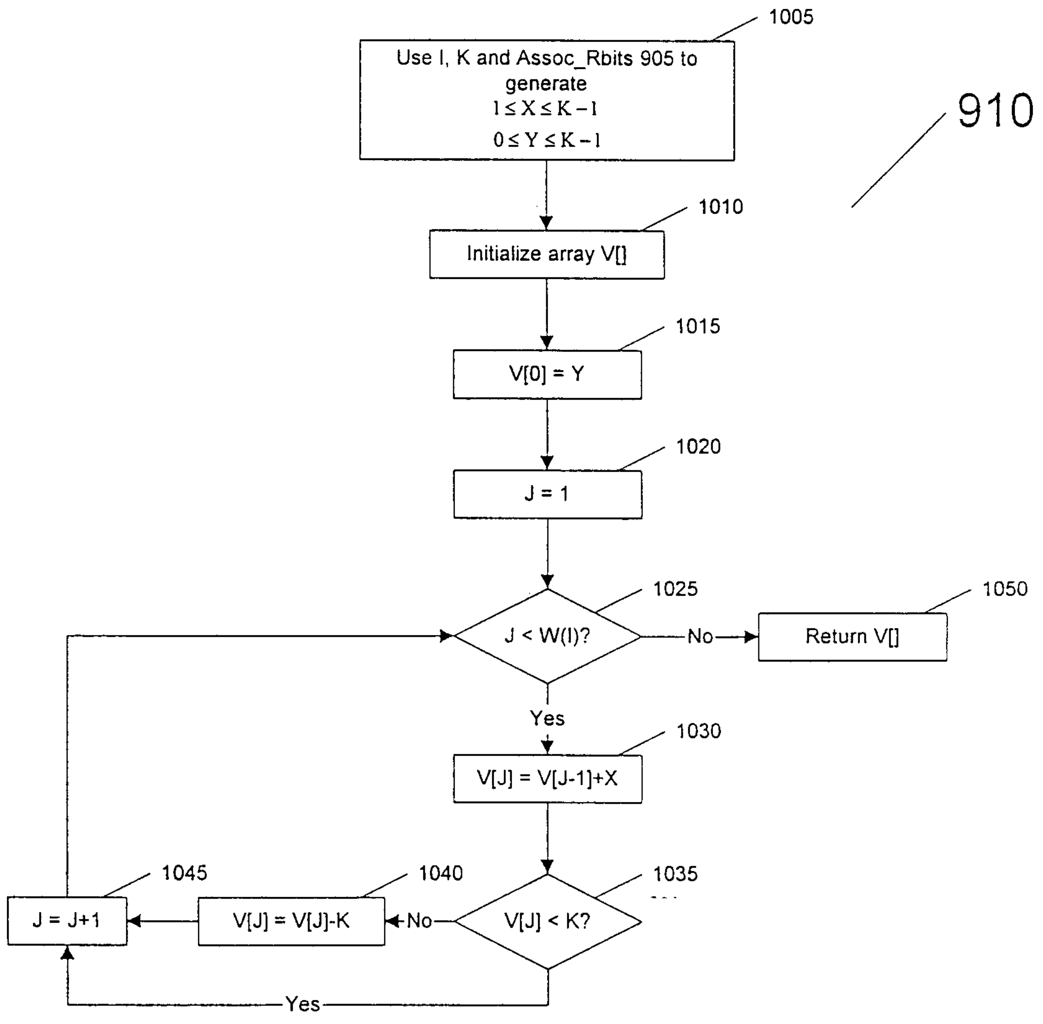
Prepare Execution Schedule
Figure 8(b)


870
E = 0
875
Is E < K?
No
880
Terminate
Yes
885
R' = Ex_Row(E)
835
I = Key(R')
840
Compute W(I) from I and K as shown in Fig. 11 and Fig. 12
845
Compute AL(I) from I, W(I) and K as shown in Fig. 9 and Fig. 10
850
Recover all unknown values of symbols in AL(I) using Value(R') and the known values of symbols in AL(I)

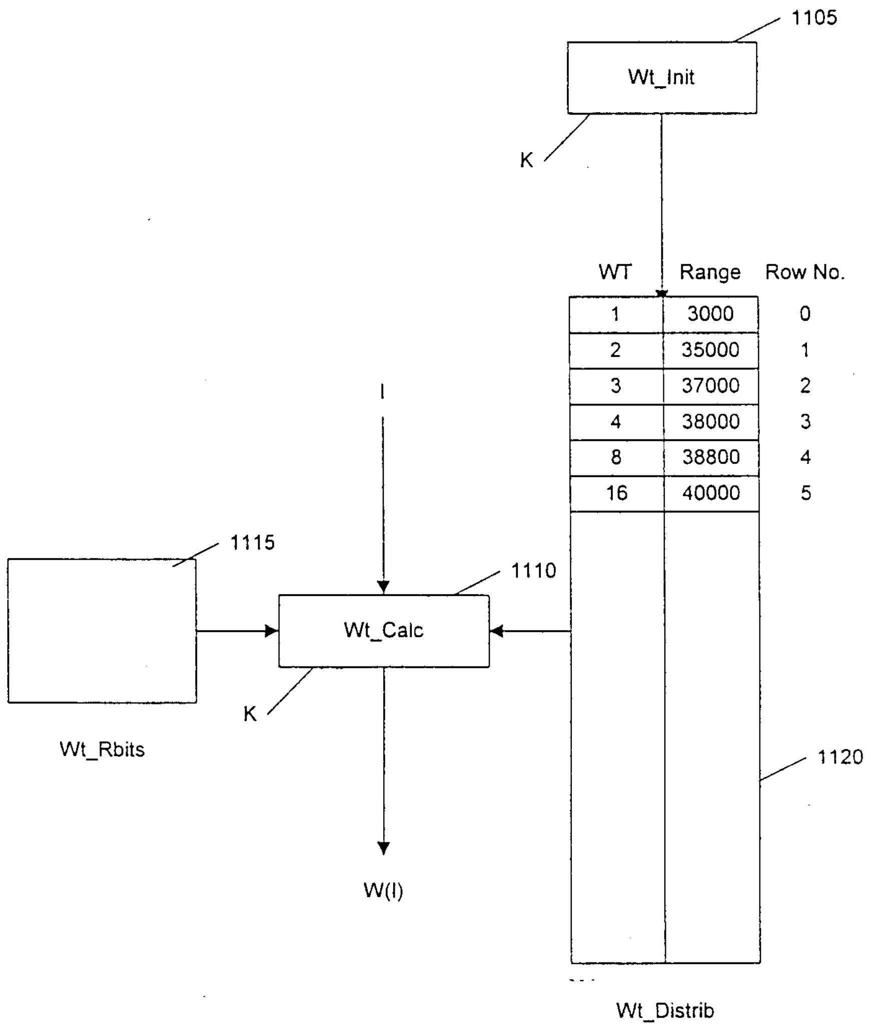
Process Execution Schedule
Figure 8(c)



Associator
Figure 9

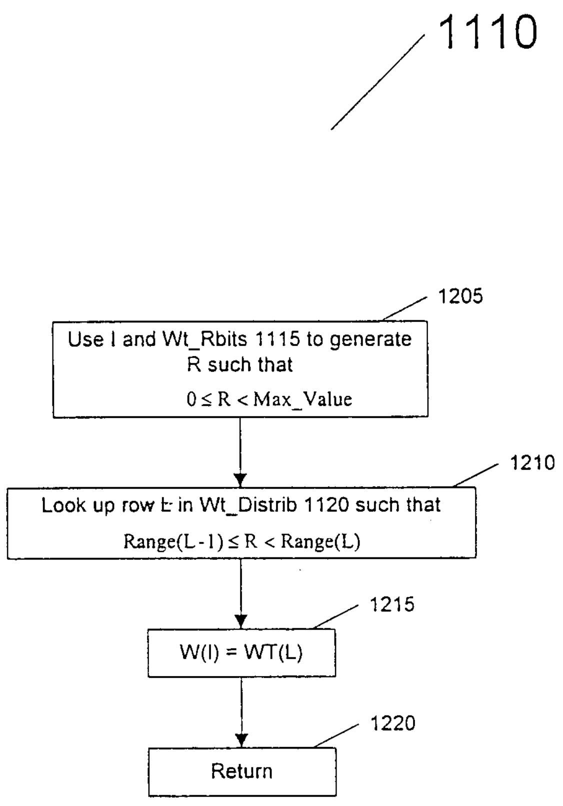


Assoc_Calc
Figure 10

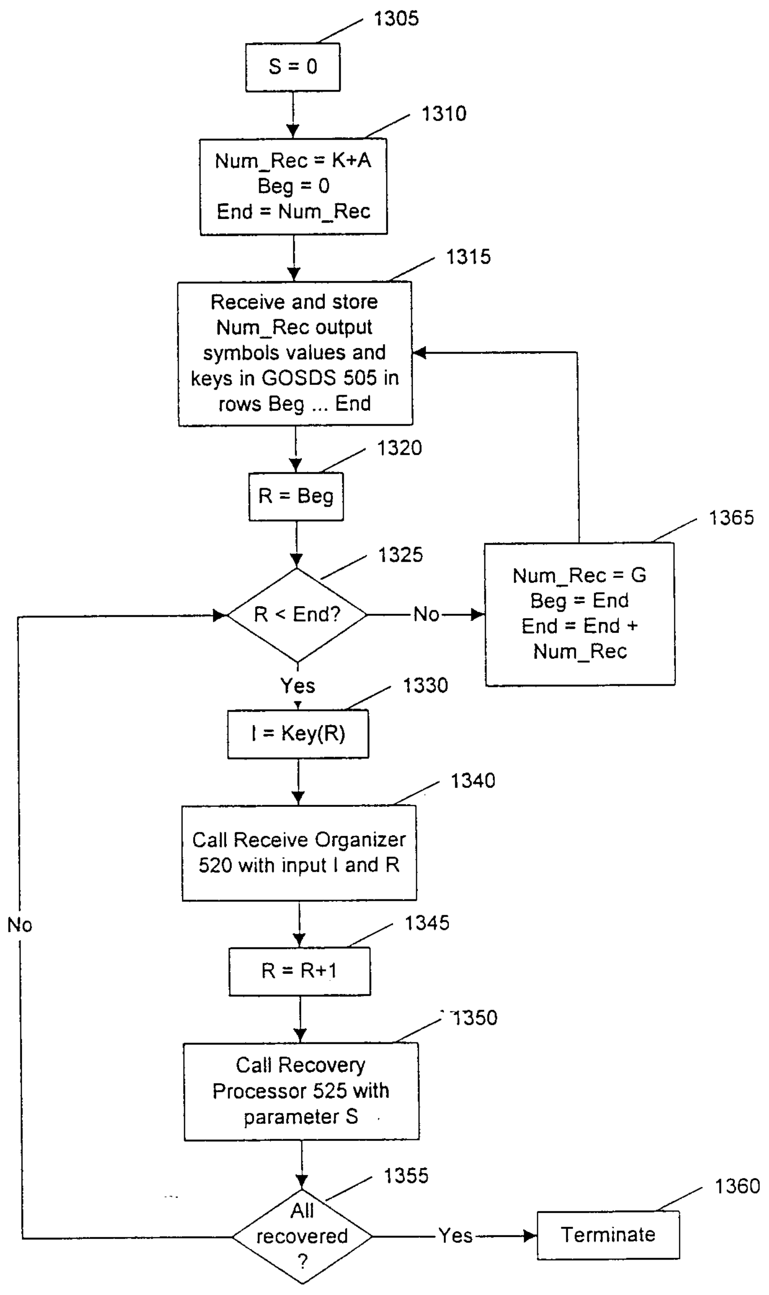


WT	Range	Row No.
1	3000	0
2	35000	1
3	37000	2
4	38000	3
8	38800	4
16	40000	5

Figure 11


1110
1205
Use I and Wt_Rbits 1115 to generate R such that
$0 \leq R < \text{Max\_Value}$
1210
Look up row L in Wt_Distrib 1120 such that
$\text{Range}(L-1) \leq R < \text{Range}(L)$
1215
W(I) = WT(L)
1220
Return

Wt_Calc
Figure 12



Lazy Decoder
Figure 13

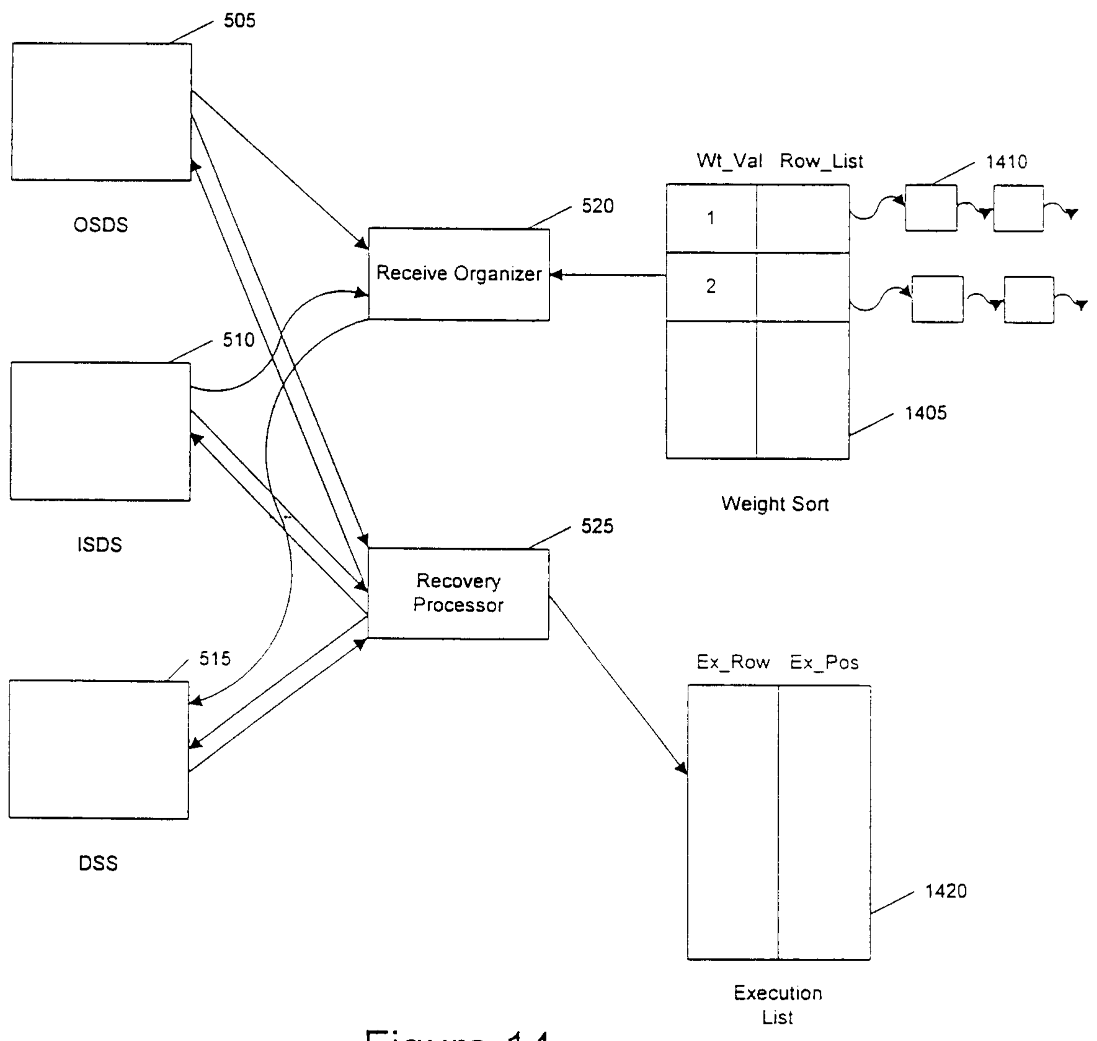

505
OSDS
510
ISDS
515
DSS
520
Receive Organizer
525
Recovery Processor
Wt_Val
Row_List
1410
1
2
1405
Weight Sort
Ex_Row
Ex_Pos
1420
Execution List

Figure 14

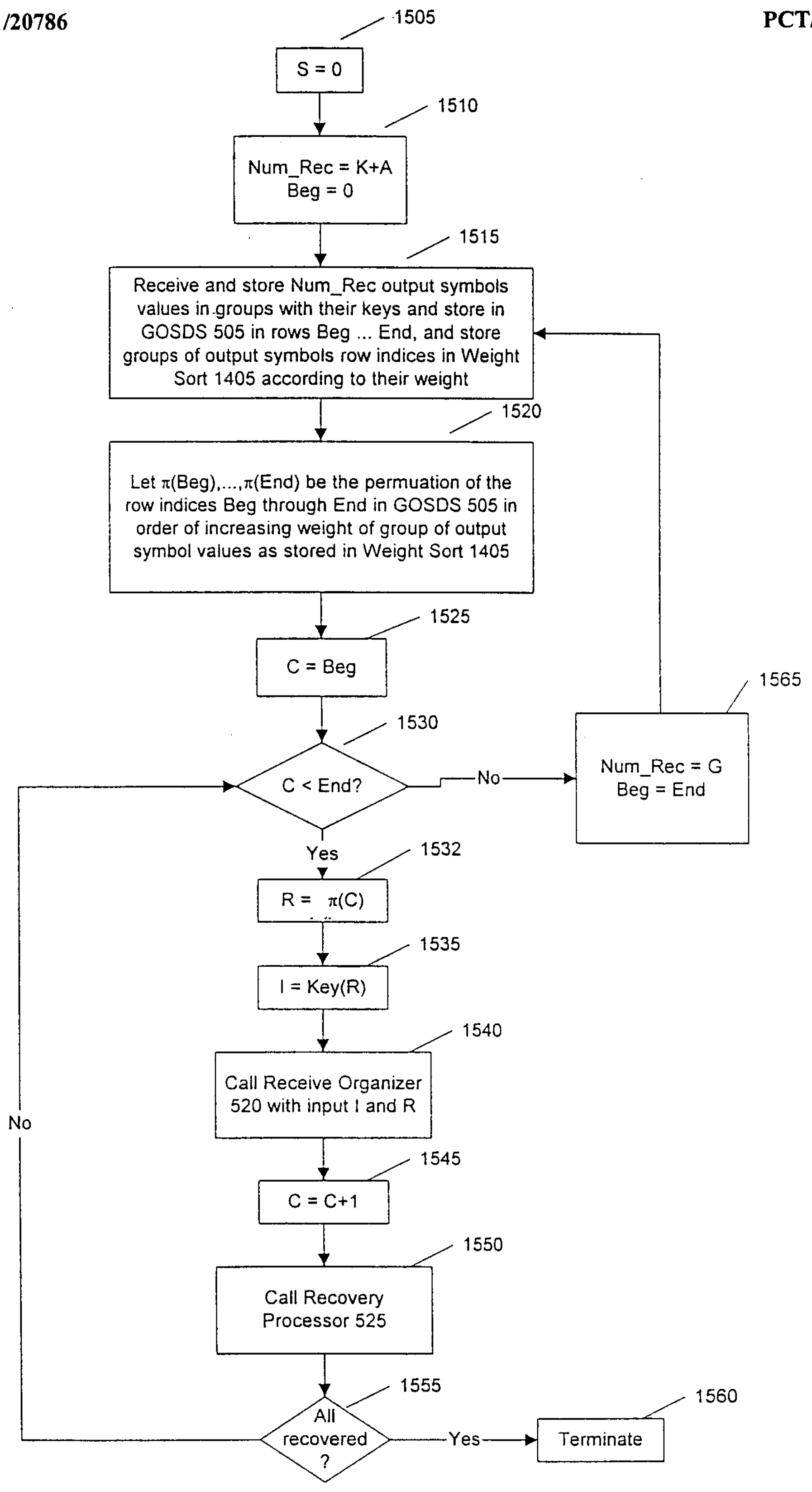

1505
S = 0
1510
Num_Rec = K+A
Beg = 0
1515
Receive and store Num_Rec output symbols values in groups with their keys and store in GOSDS 505 in rows Beg ... End, and store groups of output symbols row indices in Weight Sort 1405 according to their weight
1520
Let π(Beg),...,π(End) be the permuation of the row indices Beg through End in GOSDS 505 in order of increasing weight of group of output symbol values as stored in Weight Sort 1405
1525
C = Beg
1530
C < End?
No
1565
Num_Rec = G
Beg = End
Yes
1532
R = π(C)
1535
I = Key(R)
1540
Call Receive Organizer 520 with input I and R
No
1545
C = C+1
1550
Call Recovery Processor 525
1555
All recovered ?
Yes
1560
Terminate

Figure 15

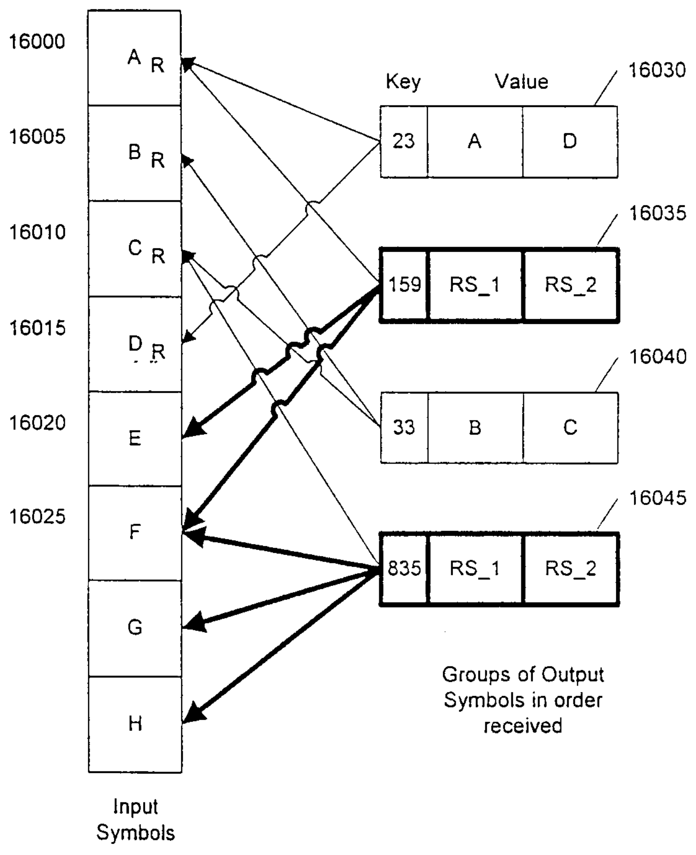
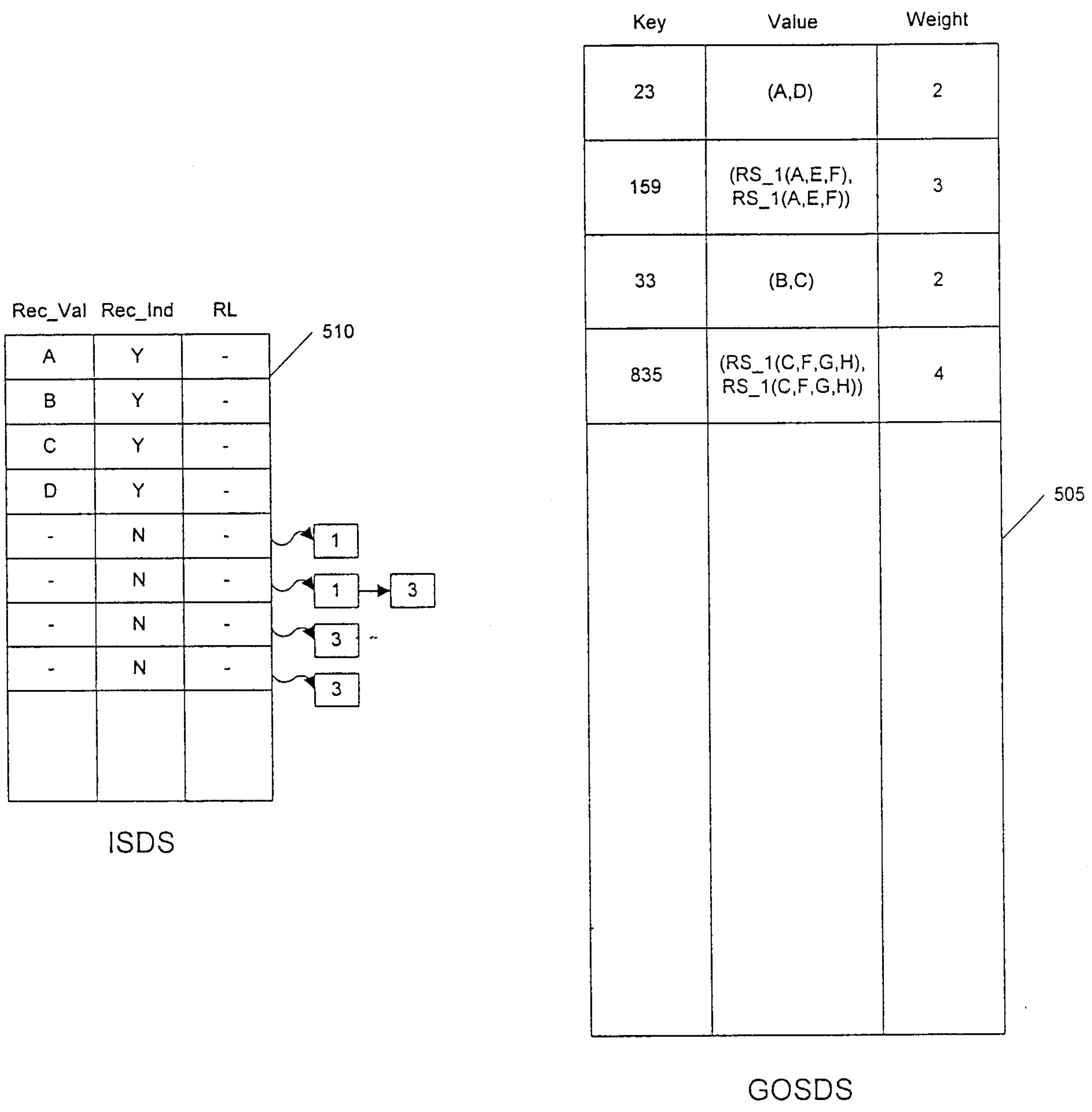

16000
16005
16010
16015
16020
16025
A R
B R
C R
D R
E
F
G
H
Input Symbols
Key
Value
16030
23
A
D
16035
159
RS_1
RS_2
16040
33
B
C
16045
835
RS_1
RS_2
Groups of Output Symbols in order received

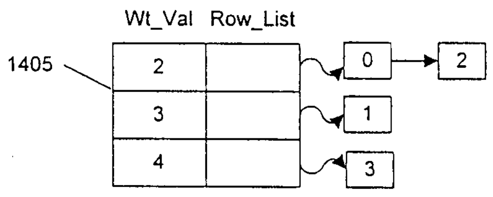
Figure 16



Rec_Val	Rec_Ind	RL
A	Y	-
B	Y	-
C	Y	-
D	Y	-
-	N	-
-	N	-
-	N	-
-	N	-

Key	Value	Weight
23	(A,D)	2
159	(RS_1(A,E,F), RS_1(A,E,F))	3
33	(B,C)	2
835	(RS_1(C,F,G,H), RS_1(C,F,G,H))	4

Figure 17



Weight Sort

Figure 18

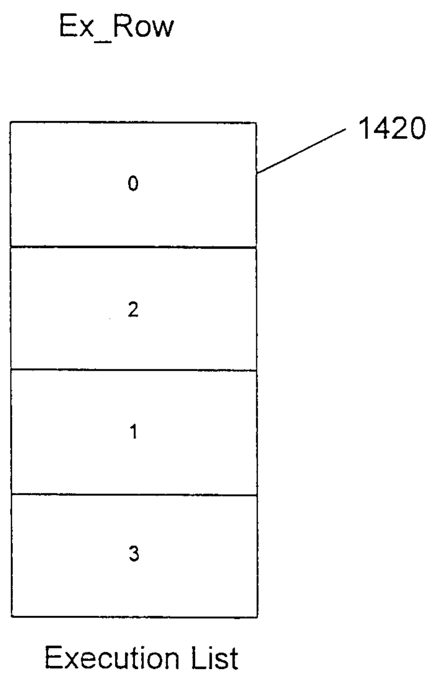
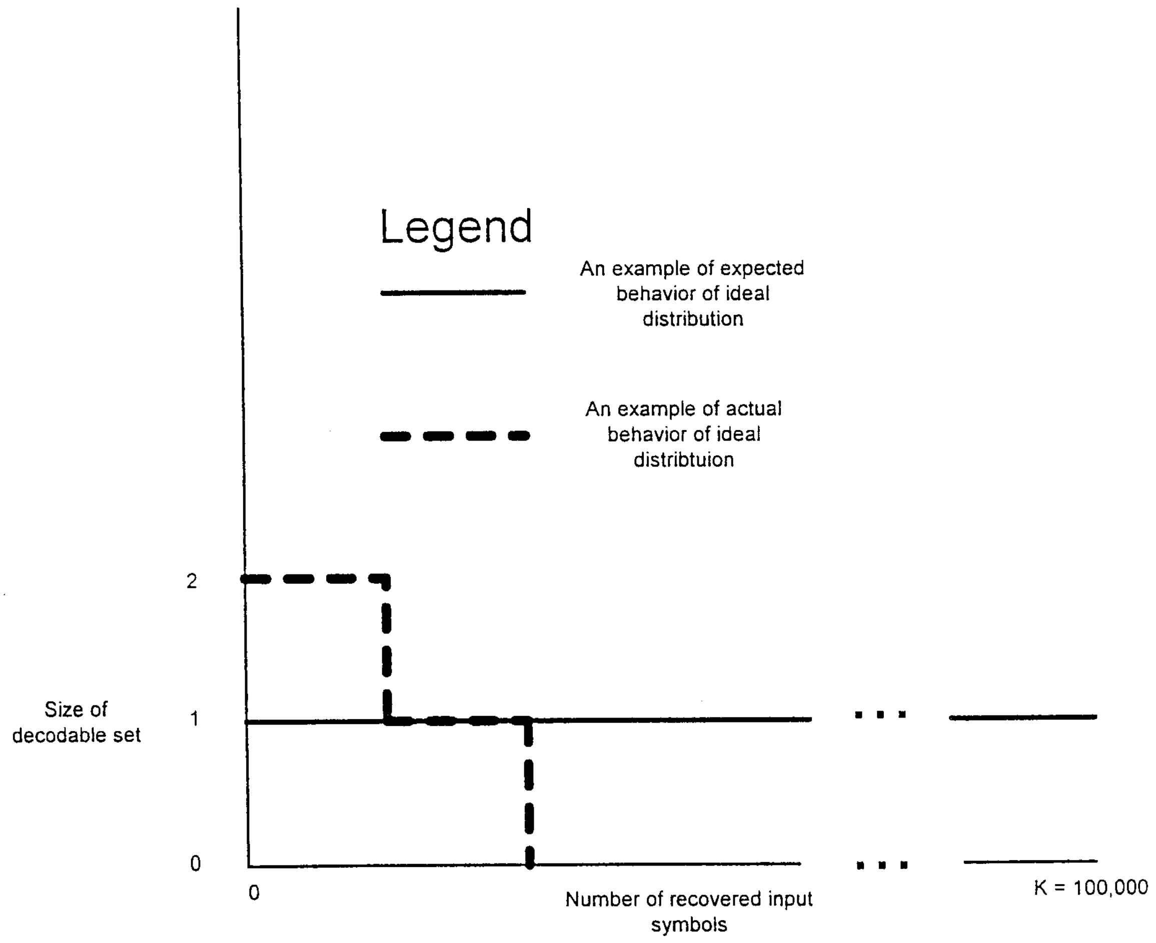


Figure 19



Legend
An example of expected behavior of ideal distribution
An example of actual behavior of ideal distribtuion
2
1
0
Size of decodable set
0
Number of recovered input symbols
K = 100,000

Figure 20

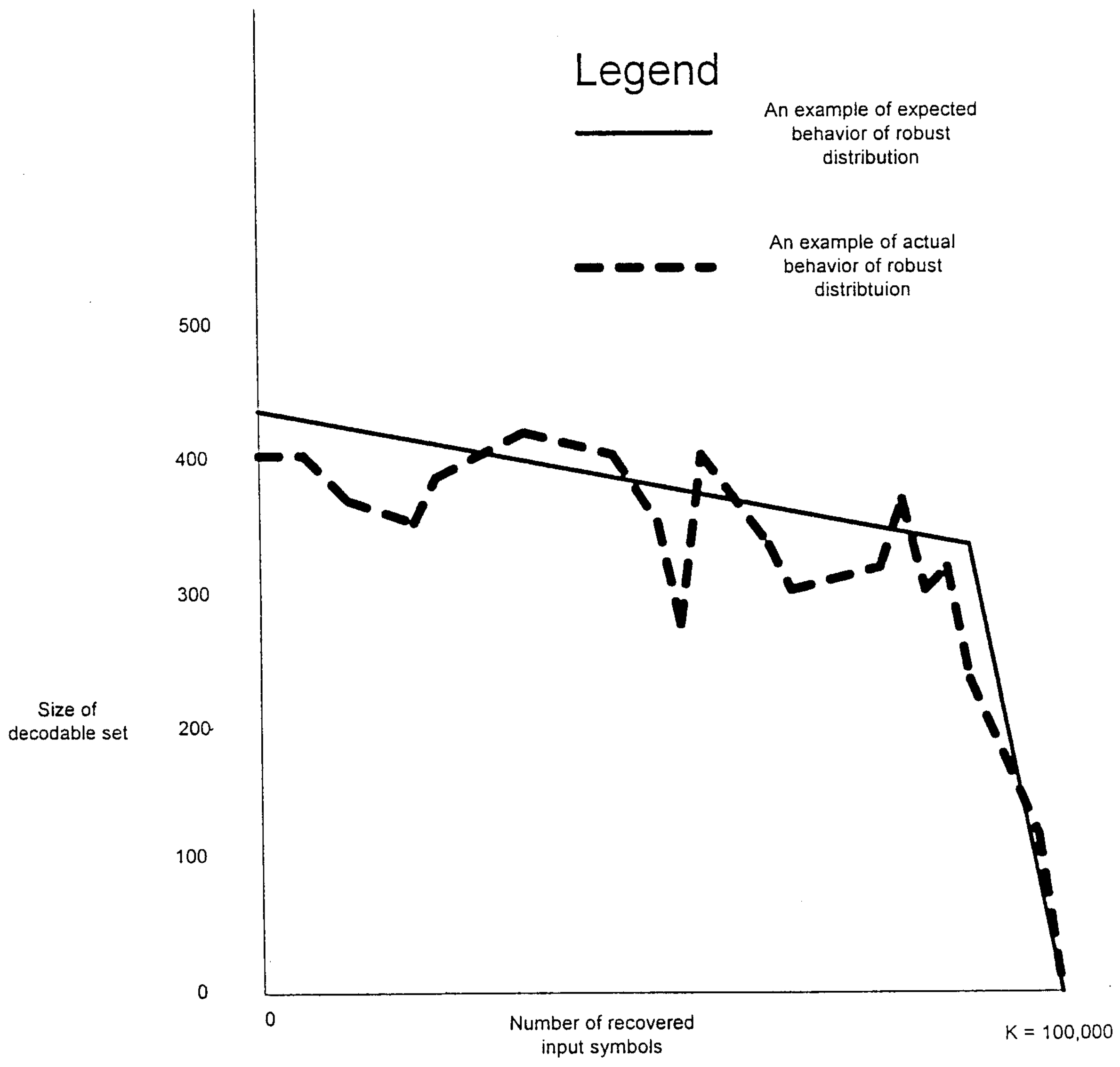

Legend
An example of expected behavior of robust distribution
An example of actual behavior of robust distribtuion
500
400
300
Size of decodable set
200
100
0
0
Number of recovered input symbols
K = 100,000

Figure 21

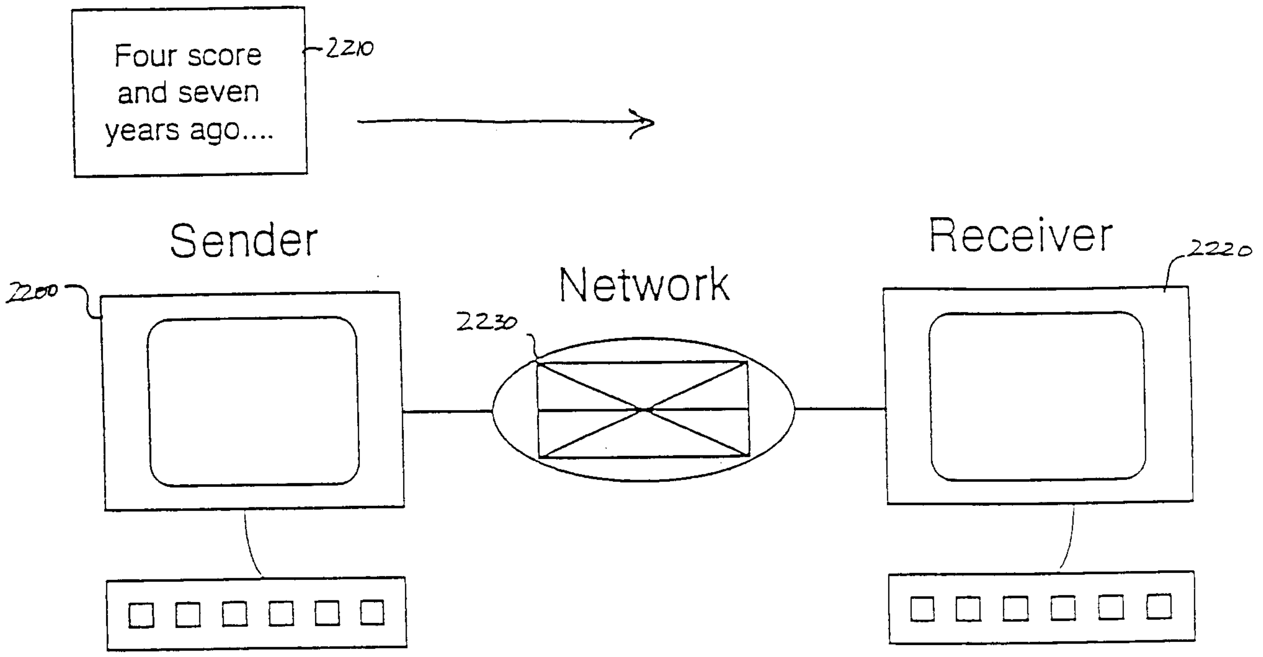

Four score and seven years ago....
2210
Sender
2200
Network
2230
Receiver
2220

FIG. 22

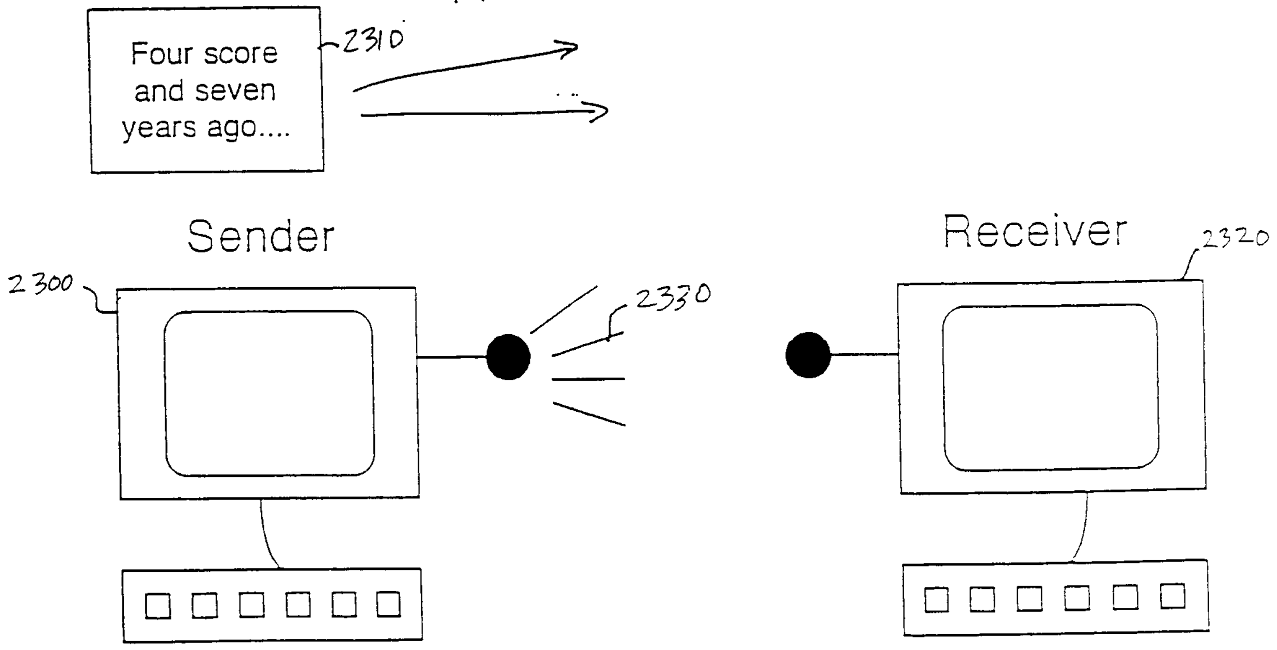

Four score and seven years ago....
2310
Sender
2300
2330
Receiver
2320

FIG. 23

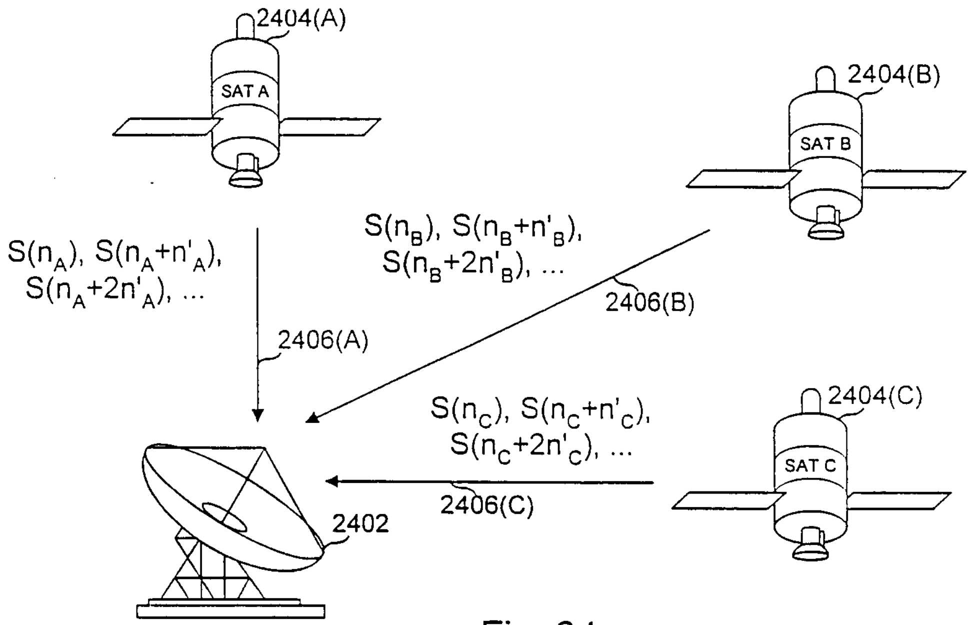

2404(A)
SAT A
2404(B)
SAT B
S(n_A), S(n_A+n'_A), S(n_A+2n'_A), ...
S(n_B), S(n_B+n'_B), S(n_B+2n'_B), ...
2406(B)
2406(A)
2404(C)
SAT C
S(n_C), S(n_C+n'_C), S(n_C+2n'_C), ...
2406(C)
2402

Fig. 24

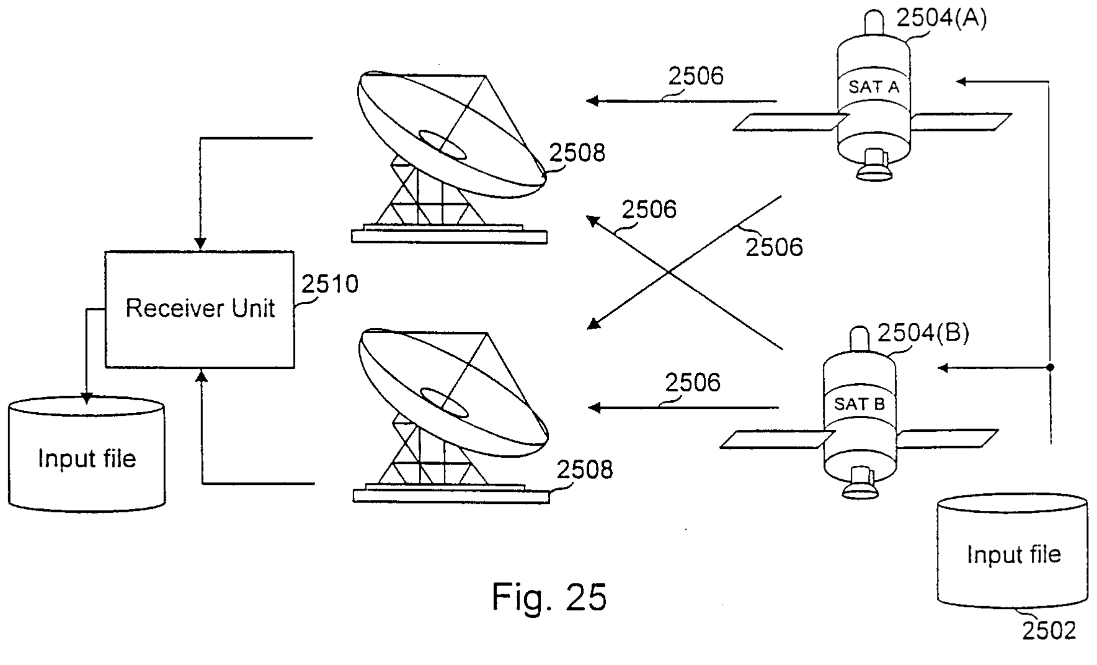

2504(A)
SAT A
2506
2508
2506
2506
Receiver Unit
2510
2504(B)
SAT B
2506
Input file
2508
Input file
2502

Fig. 25

INTERNATIONAL SEARCH REPORT

Inte ional Application No
PCT/US 00/25405

A. CLASSIFICATION OF SUBJECT MATTER
IPC 7 H03M13/47

According to International Patent Classification (IPC) or to both national classification and IPC

B. FIELDS SEARCHED

Minimum documentation searched (classification system followed by classification symbols)
IPC 7 H03M G11B

Documentation searched other than minimum documentation to the extent that such documents are included in the fields searched

Electronic data base consulted during the international search (name of data base and, where practical, search terms used)

EPO-Internal, WPI Data, INSPEC, COMPENDEX

C. DOCUMENTS CONSIDERED TO BE RELEVANT

Category °	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
X	WO 98 32231 A (QUALCOMM INC) 23 July 1998 (1998-07-23) abstract page 2, line 9 - line 35 page 4, line 16 -page 5, line 12 page 6, line 7 -page 7, line 10 page 7, line 27 -page 8, line 2 page 8, line 22 - line 33 page 9, line 4 -page 10, line 36 page 11, line 13 - line 34 figures 2-5,7 ---	1,11,13, 15, 19-21, 25,26

-/--

[X] Further documents are listed in the continuation of box C.

[X] Patent family members are listed in annex.

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"A" document defining the general state of the art which is not considered to be of particular relevance

"E" earlier document but published on or after the international filing date

"L" document which may throw doubts on priority claim(s) or which is cited to establish the publication date of another citation or other special reason (as specified)

"O" document referring to an oral disclosure, use, exhibition or other means

"P" document published prior to the international filing date but later than the priority date claimed

"T" later document published after the international filing date or priority date and not in conflict with the application but cited to understand the principle or theory underlying the invention

"X" document of particular relevance; the claimed invention cannot be considered novel or cannot be considered to involve an inventive step when the document is taken alone

"Y" document of particular relevance; the claimed invention cannot be considered to involve an inventive step when the document is combined with one or more other such documents, such combination being obvious to a person skilled in the art.

"&" document member of the same patent family

Date of the actual completion of the international search 23 February 2001	Date of mailing of the international search report 01/03/2001
Name and mailing address of the ISA European Patent Office, P.B. 5818 Patentlaan 2 NL – 2280 HV Rijswijk Tel. (+31-70) 340-2040, Tx. 31 651 epo nl, Fax: (+31-70) 340-3016	Authorized officer Barel-Faucheux, C

2

INTERNATIONAL SEARCH REPORT

Inte ional Application No
PCT/US 00/25405

C.(Continuation) DOCUMENTS CONSIDERED TO BE RELEVANT

Category °	Citation of document, with indication,where appropriate, of the relevant passages	Relevant to claim No.
X	PURSLEY M B ET AL: "VARIABLE-RATE CODING FOR METEOR-BURST COMMUNICATIONS" IEEE TRANSACTIONS ON COMMUNICATIONS,US,IEEE INC. NEW YORK, vol. 37, no. 11, 1 November 1989 (1989-11-01), pages 1105-1112, XP000074533 ISSN: 0090-6778 the whole document ---	1-7,13, 15, 19-21, 25,26
X	BIGLOO A M Y ET AL: "A ROBUST RATE-ADAPTIVE HYBRID ARQ SCHEME FOR FREQUENCY-HOPPED SPREAD-SPECTRUM MULTIPLE-ACCESS COMMUNICATION SYSTEMS" IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATIONS,US,IEEE INC. NEW YORK, vol. 12, no. 5, 1 June 1994 (1994-06-01), pages 917-924, XP000464977 ISSN: 0733-8716 the whole document ---	19-21,25
A	BYERS J W ET AL: "ACCESSING MULTIPLE MIRROR SITES IN PARALLEL: USING TORNADO CODES TOSPEED UO DOWNLOADS" PROCEEDINGS IEEE INFOCOM. THE CONFERENCE ON COMPUTER COMMUNICATIONS,US,NEW YORK, NY: IEEE, 21 March 1999 (1999-03-21), pages 275-283, XP000868811 ISBN: 0-7803-5418-4 the whole document ---	1-26
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