



(86) **Date de dépôt PCT/PCT Filing Date:** 2008/11/13
 (87) **Date publication PCT/PCT Publication Date:** 2009/06/25
 (45) **Date de délivrance/Issue Date:** 2017/01/17
 (85) **Entrée phase nationale/National Entry:** 2010/05/07
 (86) **N° demande PCT/PCT Application No.:** US 2008/083431
 (87) **N° publication PCT/PCT Publication No.:** 2009/079123
 (30) **Priorité/Priority:** 2007/12/18 (US61/008,048)

(51) **Cl.Int./Int.Cl. E21B 49/00** (2006.01),
E21B 43/00 (2006.01)
 (72) **Inventeurs/Inventors:**
KIM, CHUL-SUNG, US;
DOBIN, MARK W., US
 (73) **Propriétaire/Owner:**
EXXONMOBIL UPSTREAM RESEARCH COMPANY,
US
 (74) **Agent:** BORDEN LADNER GERVAIS LLP

(54) **Titre : DETERMINATION D'ARCHITECTURE DE CONNECTIVITE DANS DES DONNEES HETEROGENES 2D ET 3D**
 (54) **Title: DETERMINING CONNECTIVITY ARCHITECTURE IN 2-D AND 3-D HETEROGENEOUS DATA**

(57) **Abrégé/Abstract:**

A method is disclosed for determining the connectivity architecture of a hydrocarbon reservoir in terms of locally optimal paths between selected source points, e.g. wells. In one embodiment of the invention, a fast-marching method (133) is used to compute the distance field (or the time of arrival field) from N selected source points in a heterogeneous media, i.e. in a non-uniform velocity field. This is done by propagating N labeled (132) fronts simultaneously from N objects. Then, a method (134) is disclosed for detecting Voronoi curves or Voronoi surfaces, where fronts of differing labels meet each other. Then, saddle points are found on the Voronoi curves or surfaces (135), and each saddle point is used to determine a locally optimal path (136) between a pair of equidistant (from the saddle point), closest (to the saddle point) source points.

(12) INTERNATIONAL APPLICATION PUBLISHED UNDER THE PATENT COOPERATION TREATY (PCT)

(19) World Intellectual Property Organization
International Bureau



(43) International Publication Date
25 June 2009 (25.06.2009)

PCT

(10) International Publication Number
WO 2009/079123 A1

(51) International Patent Classification:
G01N 15/08 (2006.01)

(21) International Application Number:

PCT/US2008/083431

(22) International Filing Date:

13 November 2008 (13.11.2008)

(25) Filing Language:

English

(26) Publication Language:

English

(30) Priority Data:

61/008,048 18 December 2007 (18.12.2007) US

(71) Applicant (for all designated States except US): **EXXON-MOBIL UPSTREAM RESEARCH COMPANY** [US/US]; CORP-URC-SW-359, P.O. Box 2189, Houston, TX 77252-2189 (US).

(72) Inventors; and

(75) Inventors/Applicants (for US only): **KIM, Chul-Sung** [US/US]; 8411 Hunters Creek Dr., Houston, TX 77024 (US). **DOBIN, Mark, W.** [US/US]; 19 Clarion Ridge, The Woodlands, TX 77382 (US).

(74) Agents: **SHANLEY, Matthew, T.** et al.; ExxonMobil Upstream Research Company, CORP-URC-SW-359, P.O. Box 2189, Houston, TX 77252-2189 (US).

(81) Designated States (unless otherwise indicated, for every kind of national protection available): AE, AG, AL, AM, AO, AT, AU, AZ, BA, BB, BG, BH, BR, BW, BY, BZ, CA, CH, CN, CO, CR, CU, CZ, DE, DK, DM, DO, DZ, EC, EE, EG, ES, FI, GB, GD, GE, GH, GM, GT, HN, HR, HU, ID, IL, IN, IS, JP, KE, KG, KM, KN, KP, KR, KZ, LA, LC, LK, LR, LS, LT, LU, LY, MA, MD, ME, MG, MK, MN, MW, MX, MY, MZ, NA, NG, NI, NO, NZ, OM, PG, PH, PL, PT, RO, RS, RU, SC, SD, SE, SG, SK, SL, SM, ST, SV, SY, TJ, TM, TN, TR, TT, TZ, UA, UG, US, UZ, VC, VN, ZA, ZM, ZW.

(84) Designated States (unless otherwise indicated, for every kind of regional protection available): ARIPO (BW, GH, GM, KE, LS, MW, MZ, NA, SD, SL, SZ, TZ, UG, ZM, ZW), Eurasian (AM, AZ, BY, KG, KZ, MD, RU, TJ, TM), European (AT, BE, BG, CH, CY, CZ, DE, DK, EE, ES, FI, FR, GB, GR, HR, HU, IE, IS, IT, LT, LU, LV, MC, MT, NL, NO, PL, PT, RO, SE, SI, SK, TR), OAPI (BF, BJ, CF, CG, CI, CM, GA, GN, GQ, GW, ML, MR, NE, SN, TD, TG).

Declarations under Rule 4.17:

- as to applicant's entitlement to apply for and be granted a patent (Rule 4.17(ii))
- as to the applicant's entitlement to claim the priority of the earlier application (Rule 4.17(iii))

Published:

- with international search report

(54) Title: DETERMINING CONNECTIVITY ARCHITECTURE IN 2-D AND 3-D HETEROGENEOUS DATA

(57) Abstract: A method is disclosed for determining the connectivity architecture of a hydrocarbon reservoir in terms of locally optimal paths between selected source points, e.g. wells. In one embodiment of the invention, a fast-marching method (133) is used to compute the distance field (or the time of arrival field) from N selected source points in a heterogeneous media, i.e. in a non-uniform velocity field. This is done by propagating N labeled (132) fronts simultaneously from N objects. Then, a method (134) is disclosed for detecting Voronoi curves or Voronoi surfaces, where fronts of differing labels meet each other. Then, saddle points are found on the Voronoi curves or surfaces (135), and each saddle point is used to determine a locally optimal path (136) between a pair of equidistant (from the saddle point), closest (to the saddle point) source points.



WO 2009/079123 A1

DETERMINING CONNECTIVITY ARCHITECTURE IN 2-D and 3-D
HETEROGENEOUS DATA

CROSS-REFERENCE TO RELATED APPLICATION

5 [0001] This application claims the benefit of U.S. Provisional Patent Application 61/008,048 filed 18-Dec-2007 entitled, Determining Connectivity Architecture In 2-D and 3-D Heterogeneous Data.

FIELD OF THE INVENTION

10 [0002] This invention relates generally to the field of geophysical prospecting, including reservoir delineation. More particularly, the invention is a method for determining locally optimal connected paths that characterize connectivity architecture of multiple objects in two-dimensional and three-dimensional data. This method could be applied to data models that describe subsurface geology or subsurface hydrocarbon reservoirs. Typical geologic data
15 volumes include seismic and seismic derived data volume, geologic model data volume, and reservoir simulation model data volume.

SUMMARY OF THE INVENTION

[0003] In one general aspect, a method for assessing connectivity between two or more
20 objects in a hydrocarbon reservoir in order to manage development of the reservoir includes (a) specifying a data volume of data elements on a discrete two or three-dimensional grid, said data representing a selected characteristic of the hydrocarbon reservoir at each cell in the grid; (b) specifying location of at least two objects in the data volume; (c) determining all
25 Voronoi curves (2-D) or surfaces (3-D) in the data volume for the at least two objects as propagation seeds, said Voronoi curves or surfaces defining where fronts started simultaneously from each object meet, wherein front propagation speed at each cell location is a function of the data element at that cell; (d) locating all saddle points on the Voronoi curves/surfaces; (e) for each saddle point, finding a locally optimal path between two objects
30 nearest to the saddle point by finding optimal paths between the saddle point and the two objects; and (f) assessing connectivity of the at least two objects based on the locally optimal paths connecting them.

[0004] Implementations of this aspect may include one or more of the following features. For example, the data volume may be a seismic attribute data volume. The data volume may be heterogeneous, meaning the data elements in all cells do not have the same value. The objects may consist of existing or proposed production or injection wells. The Voronoi curve/surface may be determined using a front propagation speed function based on the seismic attribute. The Voronoi curve/surface may be determined from a distance field representing front propagation distance from each of a corresponding pair of nearest objects. The fronts may be propagated to generate the distance field by solving an Eikonal equation using the front propagation speed function. The Eikonal equation may be solved by a fast marching method. The Voronoi curve/surface may be determined by detecting top of ridges of the distance field. Each saddle point may be distinguished from neighboring points on the Voronoi curve/surface by having locally minimum front arrival times. The locally optimal path may be found by tracing backward through the distance field from the saddle point to each of the two nearest objects. The reservoir's connectivity architecture may be inferred from the locally optimal paths. The Voronoi curves/surfaces may be determined by assigning a unique label to each seed object, then labeling cells in the grid to track progress of the propagating fronts. The reservoir's connectivity architecture may be used to manage development of the reservoir. The selected characteristic of the hydrocarbon reservoir may be porosity or transmissibility. The reservoir may be developed to produce hydrocarbons based at least in part on the connectivity assessment. Hydrocarbons may then be produced from the reservoir.

[0005] In another general aspect, a method for producing hydrocarbons from a subsurface reservoir includes (a) obtaining an assessment of connectivity of different parts of the reservoir, said connectivity assessment having been made by steps comprising: (i) specifying a data volume of data elements on a discrete two or three-dimensional grid, said data representing a selected characteristic of the subsurface hydrocarbon reservoir at each cell in the grid; (ii) specifying location of at least two objects in the data volume; (iii) determining all Voronoi curves (2-D) or surfaces (3-D) in the data volume for the at least two objects as propagation seeds, said Voronoi curves or surfaces defining where fronts started simultaneously from each object meet, wherein front propagation speed at each cell location is a function of the data element at that cell; (iv) locating all saddle points on the Voronoi curves/surfaces; (v) for each saddle point, finding a locally optimal path between two objects nearest to the saddle point by finding optimal paths between the saddle point and the two objects; and (vi) assessing connectivity of the at least two objects based on the locally

optimal paths connecting them; (b) relating each of the at least two objects to different parts of the reservoir; and (c) developing the reservoir to produce hydrocarbons based at least in part on the connectivity assessment.

[0006] In some embodiments of the invention, fronts are propagated using a fast marching method. In some embodiments of the invention, the objects are wells in a hydrocarbon reservoir, the selected characteristic of the hydrocarbon reservoir is porosity or transmissibility or a seismic attribute, the data volume is heterogeneous, and reservoir connectivity is assessed by determining all locally optimal paths between the two or more objects.

10

BRIEF DESCRIPTION OF THE DRAWINGS

[0007] The present invention and its advantages will be better understood by referring to the following detailed description and the attached drawings in which:

[0008] Fig. 1 illustrates states (known, trial or far) of cells in a typical fast marching method;

15 [0009] Fig. 2 shows cell or grid indices in a 2D grid;

[0010] Fig. 3 illustrates three seeds (sources) in a 2D grid;

[0011] Fig. 4 shows an example of propagated labels from the three sources shown in Fig. 3;

[0012] Fig. 5A shows an example two-dimensional inhomogeneous medium with two barriers of zero propagation speed and two seed points;

20 [0013] Fig. 5B shows a computed distance field in gray scale, based on the seed points and propagation speed function of Fig. 5A, and also shows critical points (Voronoi curve) and critical saddle points;

[0014] Fig. 6 shows locally optimal paths for the example of Figs. 5A and 5B, in which optimal paths pass through two critical saddle points shown in black circles;

25 [0015] Fig. 7A shows an example two-dimensional reservoir where three wells are shown in black circles and reservoir quality is shown in gray scale;

[0016] Fig. 7B shows a distance field from the three well locations of Fig. 7A and the Voronoi curves;

30 [0017] Fig. 8 shows locally optimal paths in white lines for the example of Figs. 7A and 7B, in which optimal paths pass through four critical saddle points shown in black circles;

[0018] Fig. 9 shows an example two-dimensional seismic attribute displayed in 2D in gray scale, and sixteen detected seed points at locally maximum attribute values;

[0019] Fig. 10 shows distance fields from the sixteen seed points in gray scale for the example of Fig. 9, with Voronoi points (curve) shown in white cells and connected paths
5 between seed points shown in white lines;

[0020] Fig. 11 shows the same connected paths between seed points as in Fig. 10 but displayed against seismic attribute image and

[0021] Fig. 12 is a flowchart showing basic steps in one embodiment of the present inventive method.

10 [0022] The invention will be described in connection with its preferred embodiments. However, to the extent that the following detailed description is specific to a particular embodiment or a particular use of the invention, this is intended to be illustrative only, and is not to be construed as limiting the scope of the invention. On the contrary, it is intended to cover all alternatives, modifications and equivalents,

15

DETAILED DESCRIPTION OF EXAMPLE EMBODIMENTS

[0023] In one embodiment of the present invention, a fast-marching method is used to compute the distance field (or the time of arrival field) from N objects in a heterogeneous
20 media. This is done by propagating N labeled fronts simultaneously from N objects. Then, a method is disclosed for detecting Voronoi points or Voronoi surfaces, where fronts of differing labels meet each other. Then, a method is disclosed for determining saddle points among detected Voronoi points. Each saddle point is then used to determine one or more locally optimal paths between a pair of equidistant (from the saddle point), closest (to the
25 saddle point) objects.

[0024] Understanding reservoir connectivity is critical to the management of an oil or gas asset from exploration to abandonment. Connectivity assessment can greatly affect decisions made in all phases of asset's life cycle from optimally determining initial well locations to improving reservoir management decisions later in the field's life. Specifically, this invention
30 presents an efficient method of determining connectivity architecture of multiple objects in a heterogeneous geologic data volume, which describe porous geobodies such as sand deposits capable of being hydrocarbon reservoirs, scattered throughout a nonporous medium like

shale. The objects may include, but are not limited to, injection wells, production wells, and gas and water contacts or any other data points in hydrocarbon reservoirs. In the present invention, the basic components of connectivity architecture are the locally optimal paths between a pair of objects. For connectivity architecture of multiple objects, a method is provided for identifying pairs of objects that form Voronoi edges in two dimension or
5 Voronoi surfaces in three dimensions. Then, a method is provided for determining locally optimal paths that connect these identified pairs of objects.

[0025] A path is locally optimal if any neighboring path that deviates from this path has a larger distance. The limitation to paths within a specified *neighborhood* differentiates the
10 definition of a locally optimal path from that of the *globally* optimal path. Here, the distance of a path is measured by the travel time of a path. When speed at each location is the same (i.e. a homogeneous medium), there is only one locally optimal path (i.e, path of shortest distance) between two points or two objects. However, when speed is a function of location (i.e. a heterogeneous medium), there can be more than one locally optimal path.

[0026] The problem of determining an optimal path between two objects in a heterogeneous
15 medium, where speed of propagation is a function of a location, can be solved by many different approaches. One efficient method is that of using a fast marching method suggested by J. A. Sethian in *Level set methods and fast marching methods*, Cambridge University Press, 284-286 (1996). A more difficult problem is that of determining all locally optimal
20 paths between two objects in a heterogeneous media. The present inventors (PCT Patent Application Publication No. WO 2006/127151) disclose a method for determining N best paths and their quality between two objects. However, these N best paths are not guaranteed to be locally optimal in their path quality. The present invention describes a method for determining all locally optimal paths between two objects as well as among multiple objects.

[0027] A few publications discuss the use of Voronoi diagram for a medium in which the
25 distance metric changes at cell locations, i.e. for a heterogeneous medium. Nishida and Sugihara (“FEM-like fast marching method for the computation of the boat-sail distance and the associated Voronoi diagram,” paper available for download at
<http://citeseer.ist.psu.edu/647402.html>) use a modified fast-marching method, called a FEM-
30 like fast marching method, to compute Boat-Sail distance and associated Voronoi diagram, in which the boat sail distance with a constant boat speed is affected by an arbitrary continuous flow field. In the description of their method, their technique is designed to find a closest harbor (or a closest point) from a current boat position and the optimal path to that harbor

among N possible harbors. They did not describe any method for finding locally optimal paths among multiple objects where boat speed changes at cell level (heterogeneous media).

[0028] Sethian briefly mentions (p. 268) that, for a Euclidian distance metric, Voronoi diagrams can be constructed by using a fast-marching method. Kimmel and Sethian (“Fast voronoi diagrams and offsets on triangulated surfaces,” *Proceedings, AFA Conference on Curves and Surfaces*, Saint-Malo, France (July, 1999)) also present a fast marching based method for computing Voronoi diagrams on triangulated manifolds, again with Euclidian metric. But neither reference discusses locally optimal paths in heterogeneous media.

[0029] David Frankel addresses the subject of reservoir connectivity and geologic models in “Characterizing Connectivity in Reservoir Models Using Paths of Least Resistance, international patent publication number WO 2005/033739 A2,” which uses “shortest path” algorithm, or “Dijkstra’s method,” to determine an optimal path. This method is restricted to determining an optimal path between two objects. Moreover, an optimal path thus obtained appears to be less accurate than an optimal path obtainable by using a method such as that disclosed in the previously referenced Patent Publication No. WO 2006/127151 or the method suggested by Sethian in his book. This is because Dijkstra’s method solves a discrete network problem while a method such as Sethian’s tries to approximate a continuous solution to the problem, thereby avoiding the grid effect of Dijkstra’s method, i.e. Dijkstra’s solution is affected by the orientation of the grids. In addition, Frankel does not disclose any method for determining locally optimal paths that describe connectivity architecture among multiple objects.

[0030] There is thus a need for a method for determining connectivity architecture of multiple objects in structured or unstructured three dimensional grid volumes of heterogeneous attributes, which corresponds to finding alternative flow paths among multiple wells or between geologic objects in a hydrocarbon reservoir with heterogeneous attributes or heterogeneous permeability. The present invention satisfies this need.

Distance field computation with fast marching method

[0031] The present invention takes an approach of measuring the connected quality or the distance between two objects in a geologic model as the time needed to propagate a front from one object to the other. Here, the speed of the propagation of the front is proportional to the attributes of the cells in a geologic data volume. The basic computational module is a numerical method that computes propagating interface from an initial interface expanding outward, where the speed or the transmissibility is always positive.

[0032] The equation describing the front propagation is:

$$|\nabla T| F = 1 \quad (1)$$

$$T(x, y) = 0 \text{ on } \Gamma(t = 0), \quad (1a)$$

where $\Gamma(t = 0)$ is the initial location of the interface at time $t = 0$,

$$\text{Front} = \Gamma(t) = \{ (x, y) \mid T(x, y) = t \},$$

$T(x, y)$ is the time of arrival at a cell located at (x, y) , and

$F(x, y)$ is the speed of the propagation at a cell (x, y) .

A numerical method is employed to compute $T(x, y)$ at all cell locations given the initial condition specified in equation (1a) and the speed $F(x, y)$ at all (x, y) locations. The initial condition (1a) applies to the cells at seed locations. In this method, the speed F depends only on position and it becomes the "Eikonal" equation. A fast numerical method known as a fast-marching method may be used to solve equation (1) above in a three-dimensional space. However, the invention is not limited to the fast-marching method. Any method that solves the "Eikonal" equation or computes a distance field, such as for example a less accurate method known as Dijkstra's method ("A Note on Two Problems in Connection with Graphs," *Numerische Mathematic* **1**, 269-271 (1959)), may be employed.

Methods for determining Voronoi points

[0033] In heterogeneous media, where the distance metric at each cell or the speed of propagation at each cell changes, each segment of the Voronoi diagram is no longer a linear line; instead, it becomes a curve. A method will now be described for determining Voronoi

points or critical points that forms Voronoi non-linear surface in 3 dimensions or Voronoi curves in two dimensions This is done by propagating N labeled fronts from N seed points and detecting Voronoi points or Voronoi cells. In this document, this procedure will be called a *labeled* fast marching method.

5 [0034] In a normal fast marching method, all cells belong to one of the three categories: Known, Trial, and Far. Figure 1 shows a state of propagation (or arrival time computation) during an iterative process of a typical fast marching method. Here, Known cells (darker shading) are cells for which the arrival times have been already determined and accepted. Trial cells (dotted squares) are a band of cells surrounding the Known cells, which have
10 computed but not yet accepted arrival times. Far cells (white) are the remaining cells, for which the arrival times are yet to be computed. In an immediately following iteration of a fast marching method, a Trial cell with an earliest arrival time is accepted to become a Known cell. Then, the time of arrival of its four neighbors (or six neighbors in 3D-space) are updated according to equation (1) above. On a 2D Cartesian grid, Sethian (p. 86 of his book) showed
15 that the time of arrival at cell (x_i, y_j) can be computed or updated by solving the equation below.

$$\max((t_{i,j} - t_{i-1,j}), (t_{i,j} - t_{i+1,j}), 0)^2 + \max((t_{i,j} - t_{i,j-1}), (t_{i,j} - t_{i,j+1}), 0)^2 = (1/f_{i,j})^2 \quad (2)$$

where $t_{i,j}$ is $T(x_i, y_j)$ and $f_{i,j}$ is the speed F at (x_i, y_j) .

There is more than one known method of solving the equation (2) for $t_{i,j}$ given $t_{i-1,j}$, $t_{i+1,j}$,
20 $t_{i,j+1}$, $t_{i,j-1}$, and $f_{i,j}$. In the present invention, a preferred method is the following. With reference to cell indices in a two-dimensional grid as shown in Fig. 2, $t_{i-1,j}$ is the newly accepted arrival time at cell (i-1,j) and $t_{i,j}$ is being updated.

- (i) Let t_i be $\min(t_{i-1,j}, t_{i+1,j})$.
- (ii) Let t_j be $\min(t_{i,j-1}, t_{i,j+1})$.
- 25 (iii) If $t_i + (1/f_{i,j})$ is less than t_j , then $t_{i,j} = t_i + (1/f_{i,j})$.
- (iv) Otherwise, $t_{i,j} = (t_i + t_j + \sqrt{2(1/f_{i,j})^2 - (t_i - t_j)^2})/2$

[0035] In a labeled fast marching method of one embodiment of this invention, the process begins with two or more seeds or sources from which simultaneous fast marching starts. Figure 3 shows an example of three sources. Here, it may be noted that a source can be

composed of more than one cell. Initially, as shown in step **131** of the flowchart of Fig. 12, cells in each source are assigned arrival time of zero. Then in step **132**, cells in each source are assigned labels that are unique to each source. In Fig. 3, the three sources are labeled as **1**, **2**, and **3**. In a labeled fast marching method, as in step **133** in the flowchart of Fig. 12, these labels are propagated along with the arrival time computation described in equation (2) following a detailed arrival time computation procedure (i) through (iv) above. Specifically, propagations of labels may be performed as follows:

(a) Initially, all the cells have no labels except at the sources.

(b) If arrival time at a cell (i, j) is updated from one cell, as in step (iii) above, the labels at cell at (i, j) are updated by combining the label(s) of the cell (i, j) and the label(s) of the cell from which the arrival time was computed.

(c) Labels from two cells are combined as follows:

If two cells have a common label or common labels, the common label(s) becomes the resulting label. If two cells do not have at least one common label, the union of the labels from two cells becomes the resulting label.

(d) When the arrival time at cell (i, j) is computed from two cells, as in step (iv) above, the label(s) at (i, j) is combined with the label(s) of one of the two cells by using the label combining method in (c) above. Then the resulting labels are further combined with the label(s) of the other of the two cells.

[0036] Figure 4 shows an example of labels thus assigned for the three sources in Fig. 3. Here, the cells with two labels, for example label 1 and label 2, indicate that the arrival times at these cells are computed from two cells of different labels: in turn, it means that two fronts, which started from different seed points, are meeting each other at this cell. Therefore, the cells with two labels are equidistant from the two sources and are identified as Voronoi points (step **134** in the flow chart of Fig. 12). By similar reasoning, the cells with three labels are equidistant from three sources: Voronoi vertex. In addition, any cell that abuts a cell of different label is detected as a Voronoi point because the boundary between these two cells is the place where two propagating fronts from different sources meet each other. In this document, these Voronoi points or Voronoi cells in heterogeneous media are called critical points or critical cells.

[0037] The method of propagating labels along with time of arrival computation is not essential to the present inventive method. It is one way of tracking the simultaneous

propagation of fronts from multiple sources in an inhomogeneous medium, and identifying where the fronts meet. If labels are used, schemes other than the one suggested above may be devised.

[0038] An example two-dimensional space in Fig. 5A is considered an inhomogeneous media because the speed of propagation is 1 unit in the gray area while the speed of propagation is zero in the black areas **23** and **24**. In Fig. 5A, the two seed points are **22** located at upper-right and **21** at bottom-left corners. Figure 5B shows the distance field computed by using a fast marching method by propagating fronts from the two seed points simultaneously. The fronts from seed point **21** must propagate around the barriers **23** and **24** in order to meet the fronts from seed point **22**. In Fig. 5B, darker shades of the gray scale represent early arrival time (shorter distance) from either of the seed points. The white cells along the diagonal curve **25** (from top left to bottom right) are detected as critical points, where two fronts with differing labels meet each other. Thus, curve **25** is the Voronoi curve for the two seed points **21** and **22** in a medium with the heterogeneous propagation speed distribution shown in Fig. 5A.

Methods for determining critical saddle points

[0039] A saddle point of a distance field can be detected as a point where multiplication of two principal curvatures is negative or Gaussian curvature is negative. Since this calculus based method tends to be sensitive to numerical inaccuracies, the following saddle point detection method is preferred in this invention. As discussed in the previous section, critical points are detected (step **135** in the flow chart of Fig. 12) where two propagating fronts of different labels meet each other. Therefore, one principal curvature at a critical point is negative. Then, a critical saddle point can be detected as a critical point that has locally minimum arrival time among neighboring critical points: the other curvature is negative. In summary, a critical saddle point occurs where two fronts of different labels meet each other for the first time in its neighbor when a labeled fast marching progresses. Thus, for terminology clarification, a saddle point on a Voronoi curve, i.e. a curve of critical points, is for brevity sometimes referred to herein as a critical saddle point. From an implementation point of view, whenever a critical point is detected, an implementation algorithm can check its M -neighbors and test whether there exists any detected critical point with its arrival time less than the arrival time of the newly detected critical point. The term M -neighbors of a given cell means all of those cells that are no more than a pre-determined distance M away

from the given cell. Here, the distance metric may include, but is not limited to, Euclidean (crow path) as well as city-block cell distances. In Fig. 5B, the points or cells at **B** and **C** are critical saddle points detected by implementing the proposed method with $M = 3$.

5 *Locally optimal paths among objects*

[0040] A locally optimal path between two objects (or seed points), S1 and S2, is defined as a path that is shorter than any other path in its spatial vicinity. Since a critical saddle point has the smallest arrival time among critical points in its vicinity, a combination of an optimal path between a critical point C and S1 and an optimal path connecting the critical saddle point C
10 and S2 becomes a locally optimal path connecting S1 and S2. Determining locally optimal paths in this manner is step **136** of Fig. 12.

[0041] An optimal path from a critical saddle point to a seed point (or an object), for example seed point S1, is found by tracing backward following the propagation time (or the distance field) of the front that propagated from seed point S1. This may be done by solving an
15 ordinary differential equation as suggested at pages 284-286 in the previously cited book by Sethian. The other part of the locally optimal path is found the same way using the propagation time associated with the other seed. The combined path is a locally optimal path between the two seeds (or objects). Figure 6 shows two locally optimal paths that pass through the critical saddle points **B** and **C** for the example heterogeneous media in Fig. 6A.
20 Even though the method of determining locally optimal paths was described for the case of two seed points in two dimensions, the same method applies for determining locally optimal paths among multiple objects and in three dimensions.

[0042] Figure 7A is an example two-dimensional reservoir where three wells are located at black circles and the reservoir quality at each cell is shown in gray scale: lighter shades of
25 gray corresponding to higher transmissibility or higher reservoir quality. Figure 7B shows the distance field from the three wells (now white circles) in gray scale and Voronoi points or critical points indicated in white. Figure 8 shows four locally optimal paths that pass through four critical saddle points (black circles) for the example heterogeneous media in Fig. 7A.

30 *Voronoi curves and surfaces and zone of influence*

[0043] As stated above, Fig. 7B shows the distance field from three wells in gray scale and the Voronoi curves indicated in white. (The black areas in Fig. 7B are areas where the speed

of propagation or transmissibility is zero.) Since distance or arrival time to a cell is a measure of connectivity to that cell from a closest source, the Voronoi curves in Fig. 7B separate wells by their zones of influence, which is sometimes called the drainage pattern in an oil field. In three-dimensional data, the method will produce Voronoi surfaces instead of
5 Voronoi curves. Therefore, Voronoi surfaces obtained in this invention can be used to identify zones of reservoir that are influenced by specific wells.

Application to determining connectivity architecture of seismic data volume

[0044] The methods described so far were developed to determine connectivity architecture
10 of multiple objects in a three dimensional data volume. The objects include, but are not limited to, existing or proposed injection and/or production wells in addition to any geologically definable objects such as gas-water contact surfaces. However, it has been found that the same method can be used to determine the structural skeleton of two-dimensional and three-dimensional data volumes of seismic attributes where no object is
15 predetermined. A good example of a seismic attribute volume is an impedance volume, which is derived from the original seismic amplitude data volume. (This is acoustic impedance, or the product of density and velocity.) The oil industry uses a variety of such “derived” seismic volumes, which are generally called seismic attribute volumes. A certain attribute volume may fairly well represent porosity or transmissibility or other characteristics
20 of a hydrocarbon reservoir. In this case, one would like to have some skeletal representation of a reservoir that could provide visual and deeper insights into the connectivity architecture of a reservoir, especially for a visualization of a three-dimensional reservoir architecture in a seismic attribute volume.

[0045] Given a seismic attribute volume, initial seed points are determined by detecting
25 locally maximum (or minimum) attributes in a seismic attribute volume, where a high (low) attribute value is desired to constitute the skeleton. Figure 9 shows an example two dimensional seismic attribute data and sixteen detected seed points, depicted as black triangles. In this particular example, cells having locally maximum attribute values were identified as objects (seeds). Then, a speed function is assigned to allow fast propagation
30 speed at cells with high (low) attribute values and low speed of propagation at cells with low (high) attribute values. In Fig. 9, high attribute values are in lighter gray scale representing higher propagation speed. Then, the present inventive method is used to determine connectivity architecture of seed points: propagating fronts from all the detected seed points

simultaneously, detecting critical saddle points, and computing optimal paths that pass through critical saddle points. Figure 10 shows (1) the distance field from sixteen seed points in gray scale, (2) critical (or Voronoi) points at cells in white color (3) locally optimal paths among seed points in white lines, where each path passes through a detected critical saddle point. In this example, the present inventive method determined connectivity architecture of 16 objects by using only 14 locally optimal paths. These 14 paths connect object pairs that are closest to each critical saddle point. Therefore, the present inventive method, by detecting critical saddle points, determines pairs of objects that participate in constructing connective architecture of multiple objects. If one used a method of determining connectivity architecture by constructing optimal paths between all combinatorial pairs of 16 objects, the resulting connectivity architecture would be crowded with paths connecting **120** (16 times 15 divided by 2) different pairs of objects. Figure 11 shows the resulting seismic skeletal structure in white lines on a seismic attribute image, which fairly well represents or determines the connectivity structure of the seismic attribute image.

[0046] For illustrative purposes, the invention has been described using examples of determining connectivity architecture of relatively small number of objects or seed points in a two-dimensional space. However, the complexity of the problem and the value of this invention increase rapidly as the number of objects and degree of heterogeneity increases. A good application would be the problem of investigating connectivity architecture of 100 wells in a three-dimensional geologic model.

[0047] The present inventive method is considered to be an alternative or competing method to the method of analyzing reservoir connectivity described in the previously referenced WO 2006/127151. At least some of the steps in the present invention would typically be performed on a computer, i.e., the invention is computer-implemented in preferred embodiments. The connectivity information as well as Voronoi curves and surfaces may be outputted or stored in computer memory or other data storage devices as a final, real-world, concrete, tangible step.

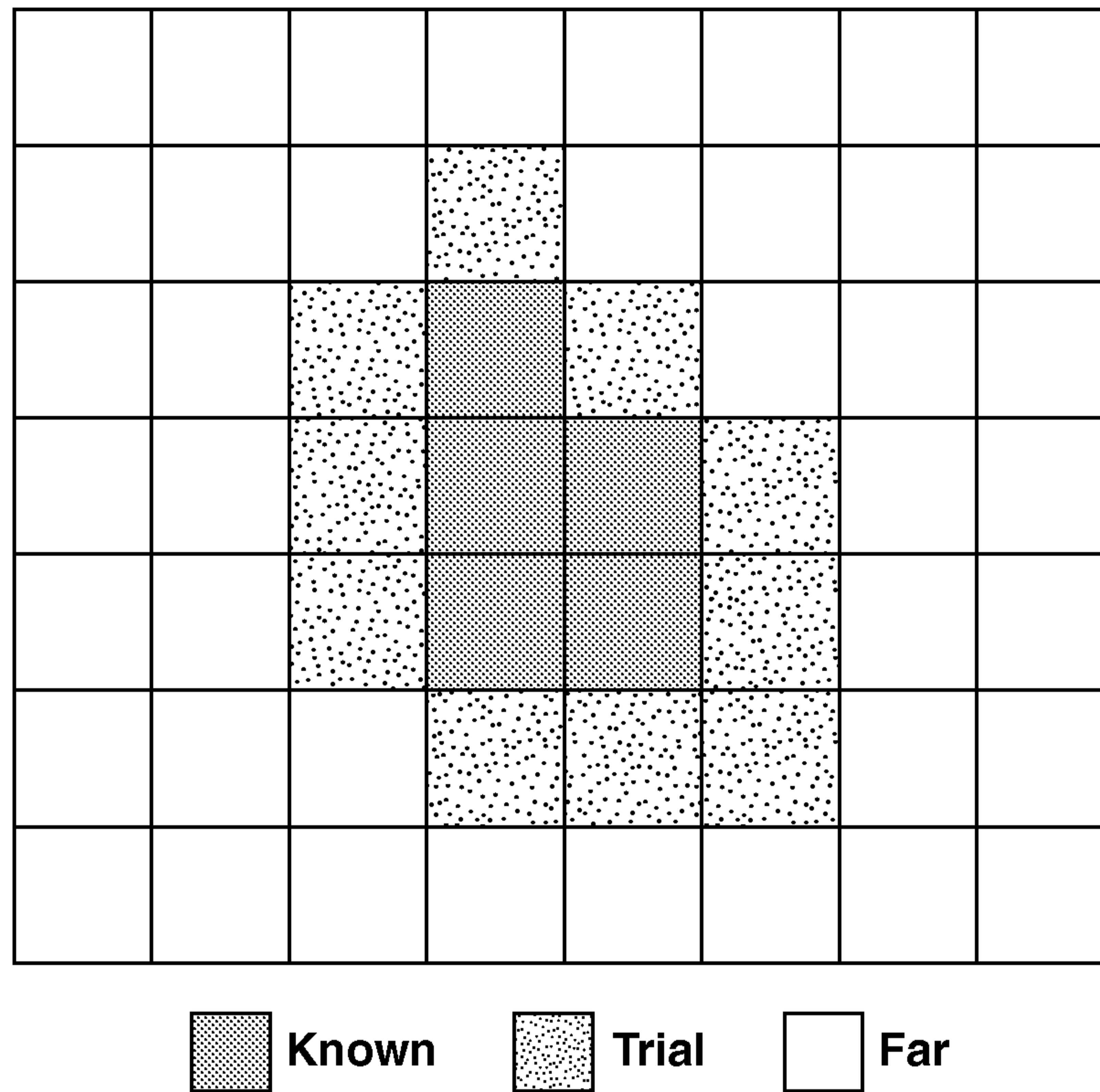
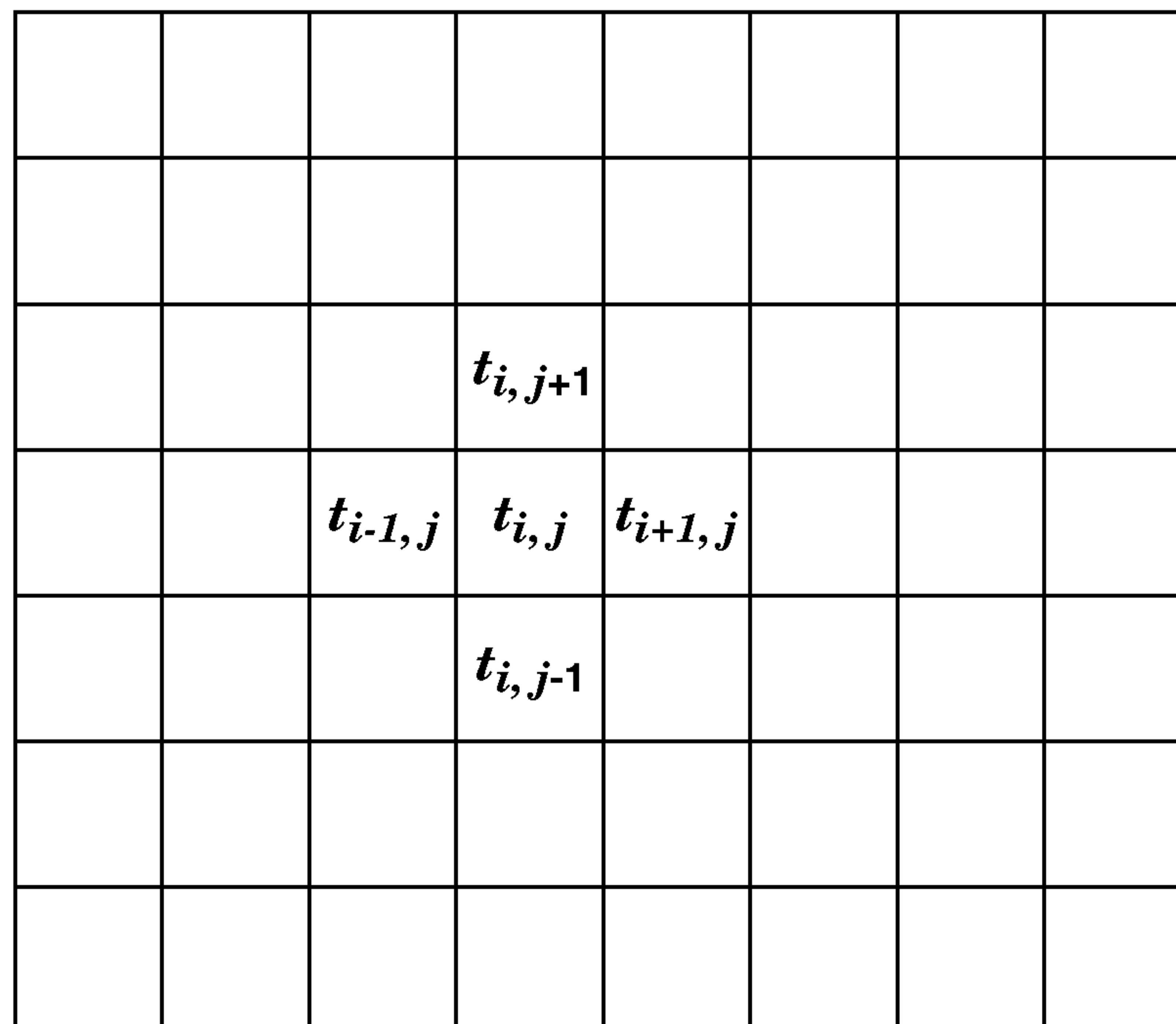
[0048] The foregoing application is directed to particular embodiments of the present invention for the purpose of illustrating it. It will be apparent, however, to one skilled in the art, that many modifications and variations to the embodiments described herein are possible. All such modifications and variations are intended to be within the scope of the present invention, as defined in the appended claims.

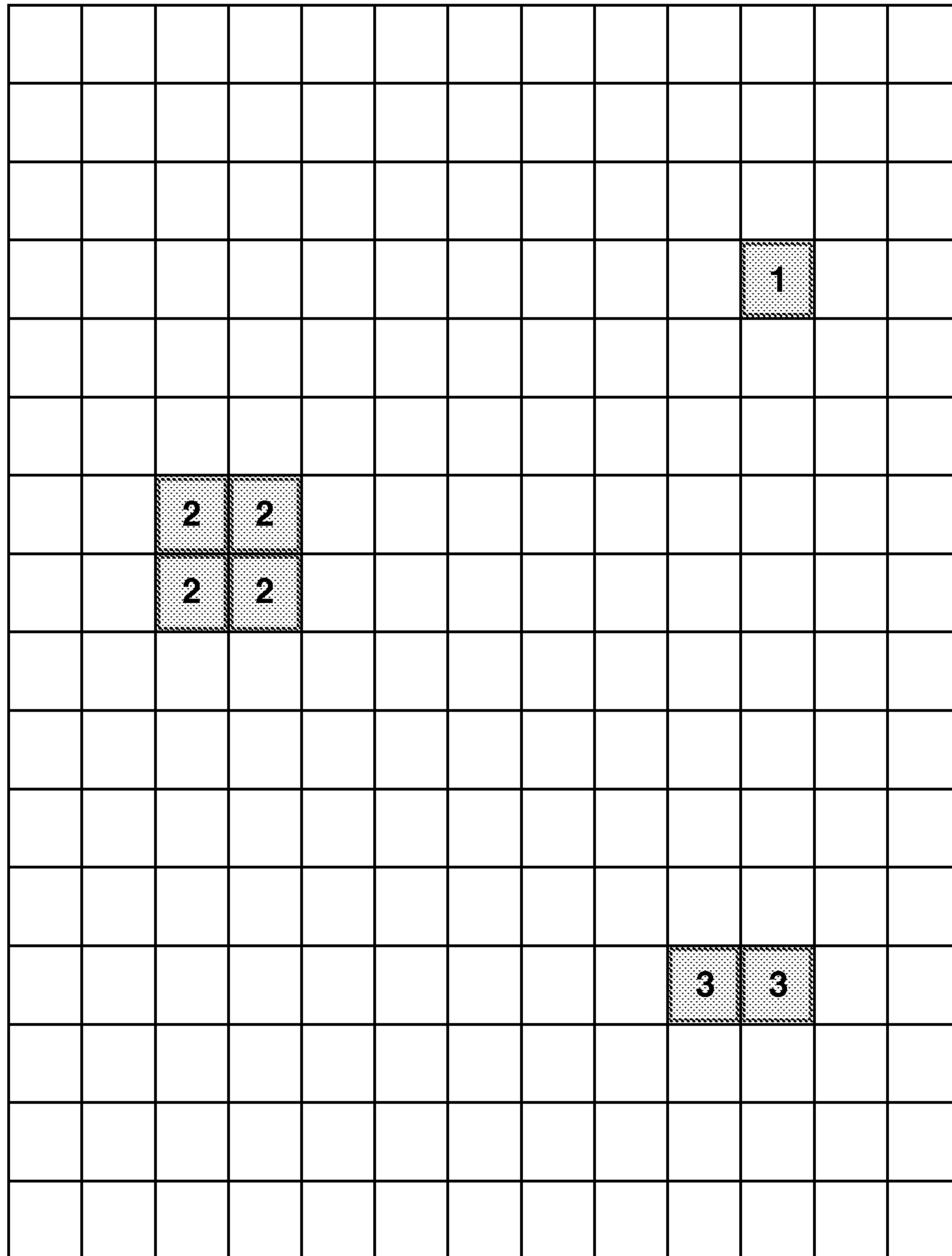
CLAIMS:

1. A method for managing development of a hydrocarbon reservoir by assessing connectivity between two or more objects in the hydrocarbon reservoir, comprising:
 - (a) specifying a data volume of data elements on a discrete two or three-dimensional grid, said data representing a selected characteristic of the hydrocarbon reservoir at each cell in the grid;
 - (b) specifying location of at least two objects in the data volume;
 - (c) determining all Voronoi curves (2-D) or surfaces (3-D) in the data volume for the at least two objects as propagation seeds, said Voronoi curves or surfaces defining where fronts started simultaneously from each object meet, wherein front propagation speed at each cell location is a function of the data element at that cell;
 - (d) locating all saddle points on the Voronoi curves/surfaces;
 - (e) for each saddle point, finding a locally optimal path between two objects nearest to the saddle point by finding optimal paths between the saddle point and the two objects;
 - (f) assessing connectivity of the at least two objects based on the locally optimal paths connecting them; and
 - (g) managing the development of the reservoir in response to the assessed connectivity.
2. The method of claim 1, wherein the data volume is a seismic attribute data volume.
3. The method of claim 1, wherein the data volume is heterogeneous, meaning the data elements in all cells do not have the same value.
4. The method of claim 1, wherein the objects consist of existing or proposed production or injection wells.
5. The method of claim 2, wherein the Voronoi curve/surface is determined using a front propagation speed function based on the seismic attribute.

6. The method of claim 1, wherein the Voronoi curve/surface is determined from a distance field representing front propagation distance from each of a corresponding pair of nearest objects.
7. The method of claim 6, wherein the fronts are propagated to generate the distance field by solving an Eikonal equation using the front propagation speed function.
8. The method of claim 7, wherein the Eikonal equation is solved by a fast marching method.
9. The method of claim 6, wherein the Voronoi curve/surface is determined by detecting top of ridges of the distance field.
10. The method of claim 1, wherein each saddle point is distinguished from neighboring points on the Voronoi curve/surface by having locally minimum front arrival times.
11. The method of claim 6, wherein the locally optimal path is found by tracing backward through the distance field from the saddle point to each of the two nearest objects.
12. The method of claim 1, further comprising inferring the reservoir's connectivity architecture from the locally optimal paths.
13. The method of claim 1, wherein Voronoi curves/surfaces are determined by assigning a unique label to each seed object, then labeling cells in the grid to track progress of the propagating fronts.
14. The method of claim 12, further comprising using the reservoir's connectivity architecture to manage development of the reservoir.

15. The method of claim 1, wherein the selected characteristic of the hydrocarbon reservoir is porosity or transmissibility.
16. The method of claim 1, wherein step (g) comprises:
developing the reservoir to produce hydrocarbons based at least in part on the connectivity assessment; and
producing hydrocarbons from the reservoir.
17. A method for producing hydrocarbons from a subsurface reservoir, comprising:
(a) obtaining an assessment of connectivity of different parts of the reservoir, said connectivity assessment having been made by steps comprising:
(i) specifying a data volume of data elements on a discrete two or three-dimensional grid, said data representing a selected characteristic of the subsurface hydrocarbon reservoir at each cell in the grid;
(ii) specifying location of at least two objects in the data volume;
(iii) determining all Voronoi curves (2-D) or surfaces (3-D) in the data volume for the at least two objects as propagation seeds, said Voronoi curves or surfaces defining where fronts started simultaneously from each object meet, wherein front propagation speed at each cell location is a function of the data element at that cell;
(iv) locating all saddle points on the Voronoi curves/surfaces;
(v) for each saddle point, finding a locally optimal path between two objects nearest to the saddle point by finding optimal paths between the saddle point and the two objects; and
(vi) assessing connectivity of the at least two objects based on the locally optimal paths connecting them;
(b) relating each of the at least two objects to different parts of the reservoir; and
(c) developing the reservoir to produce hydrocarbons based at least in part on the connectivity assessment.

1/9**FIG. 1****FIG. 2**

2/9**FIG. 3**

3/9

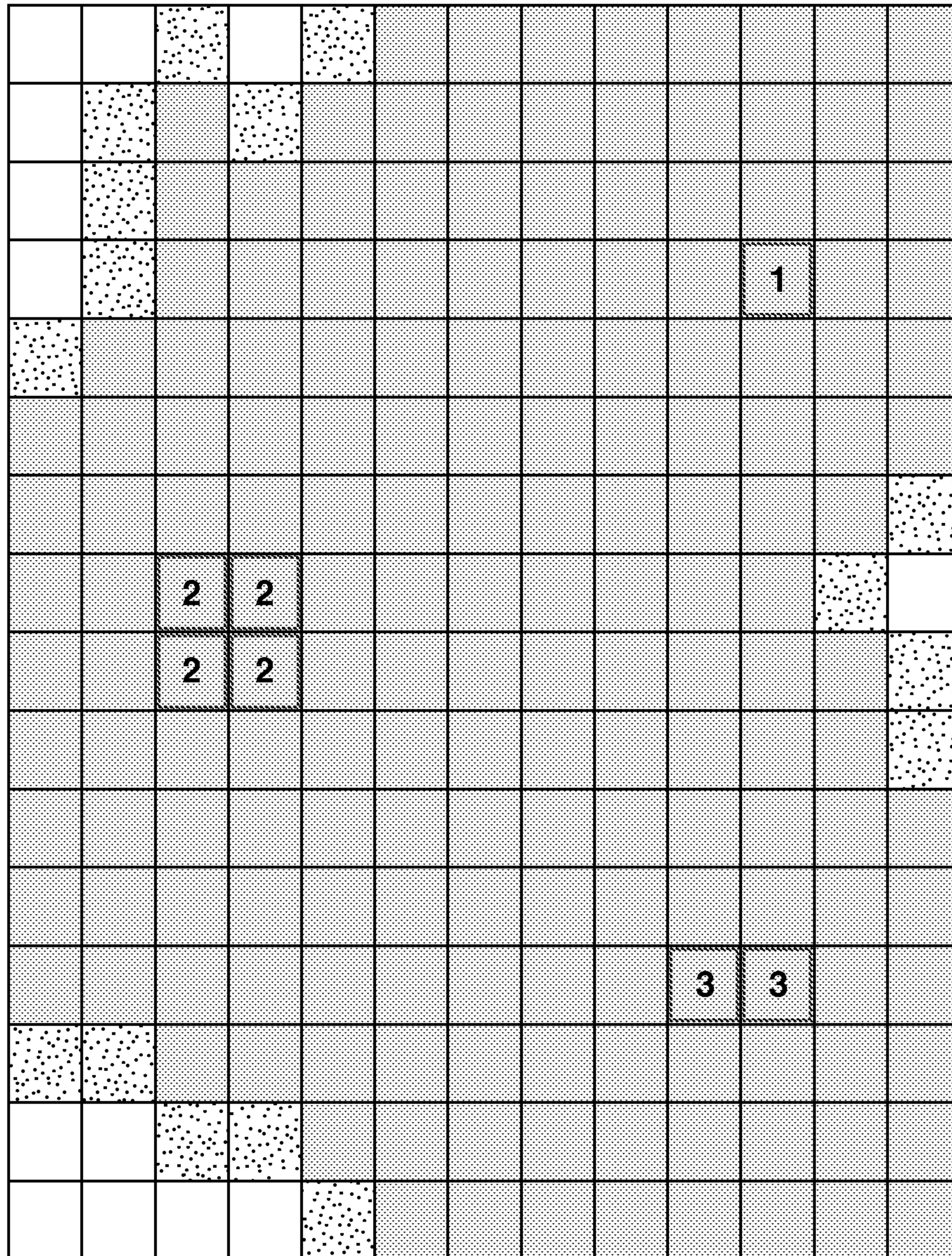


FIG. 4

4/9

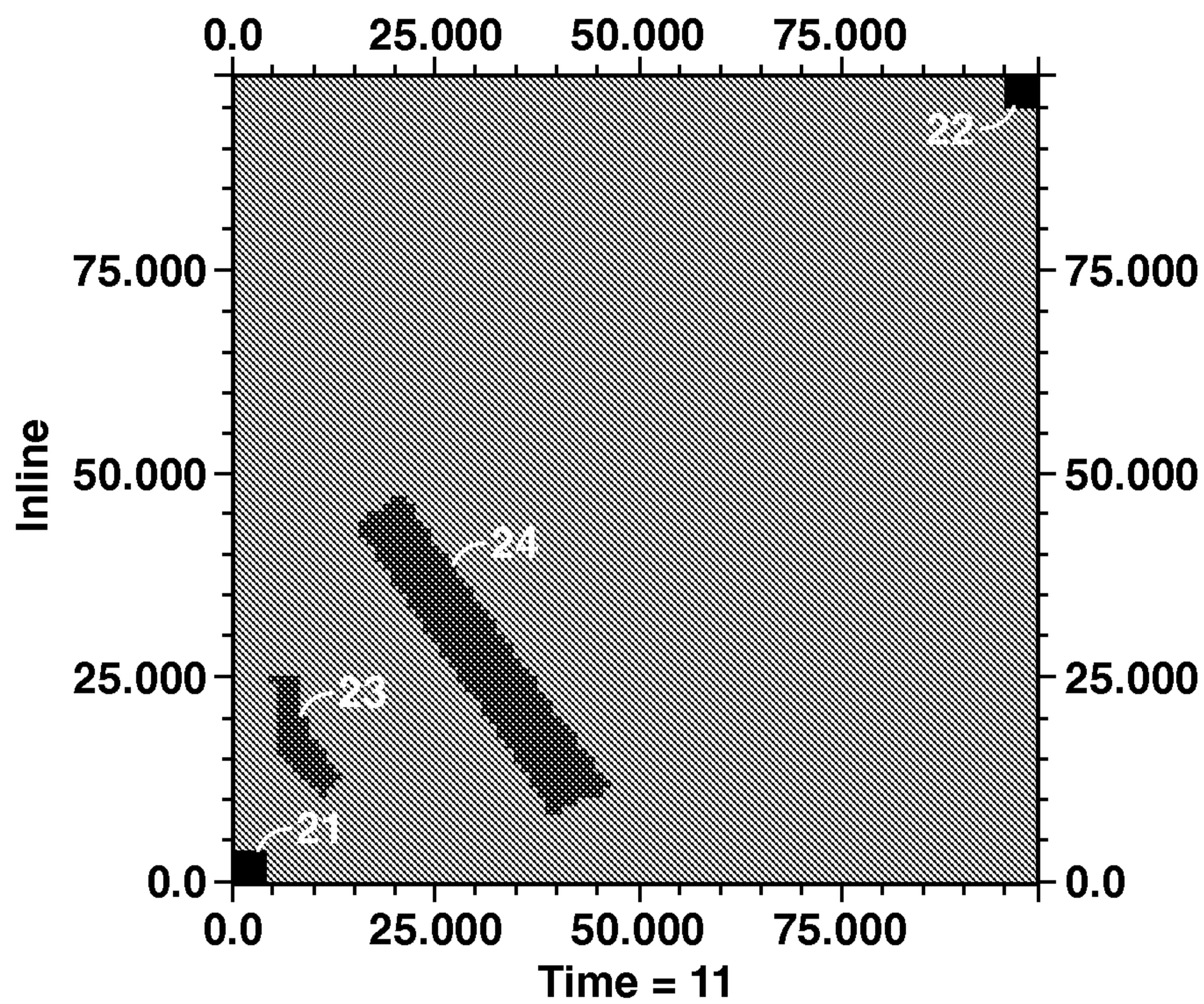


FIG. 5A

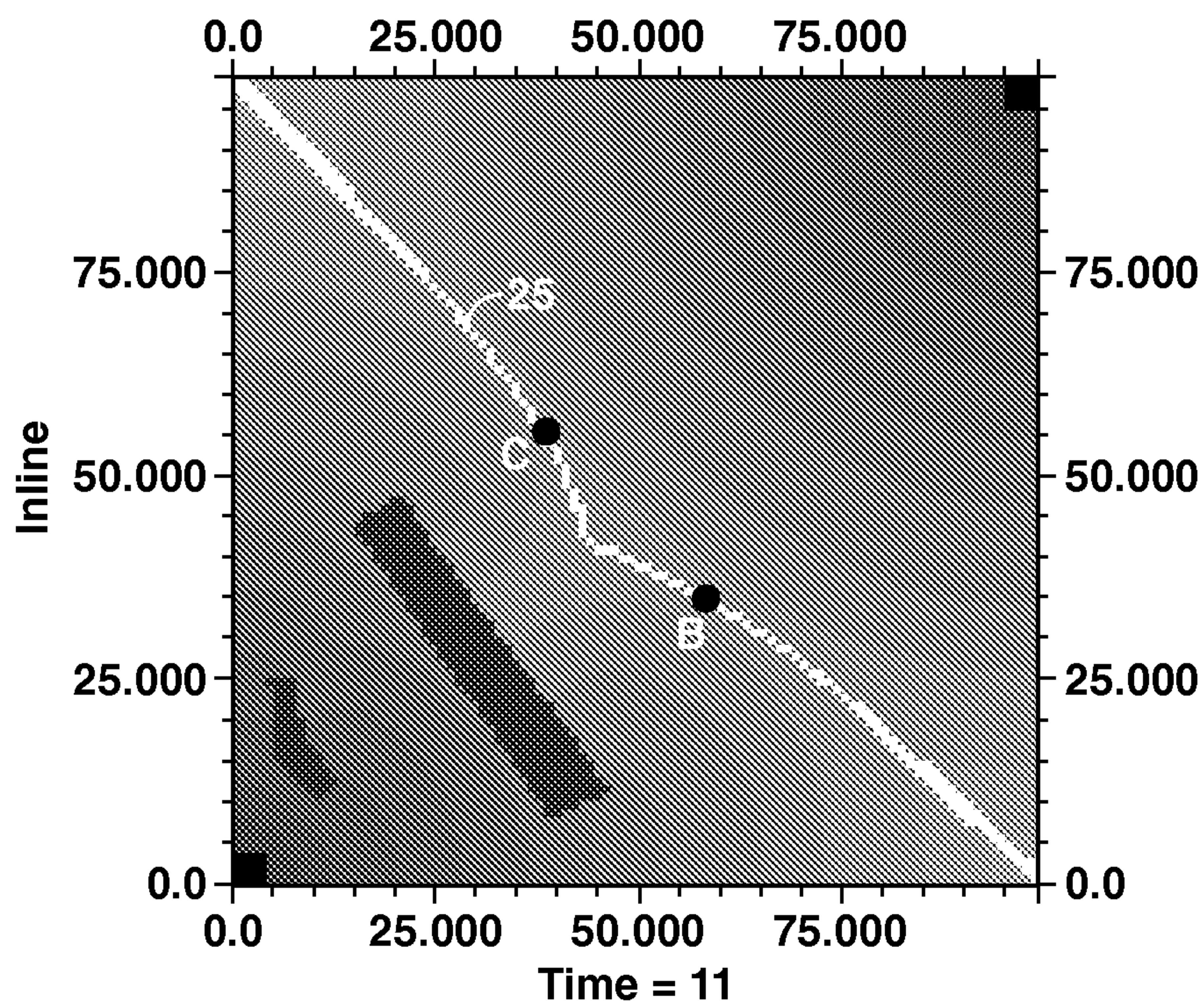


FIG. 5B

5/9

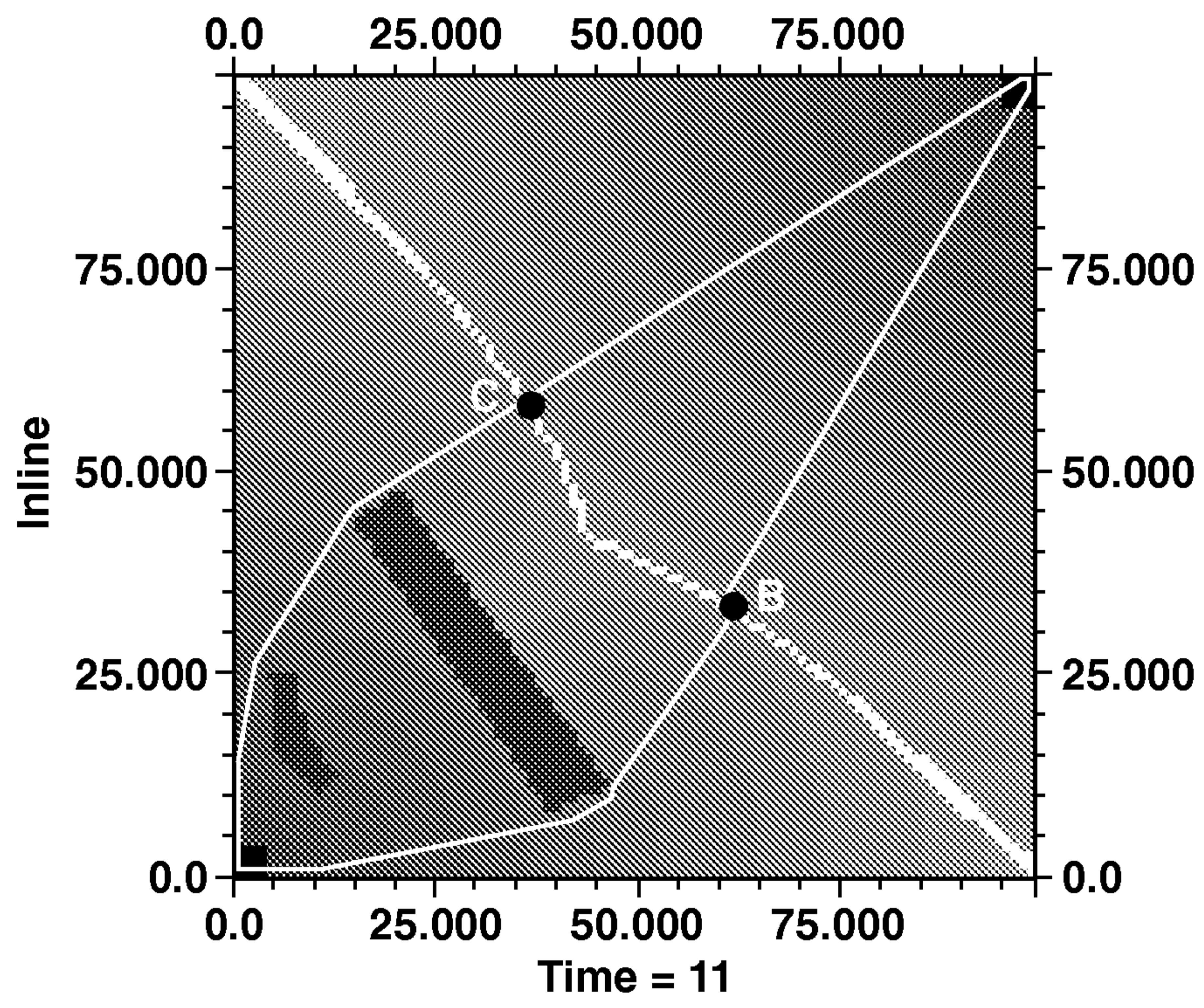


FIG. 6

6/9

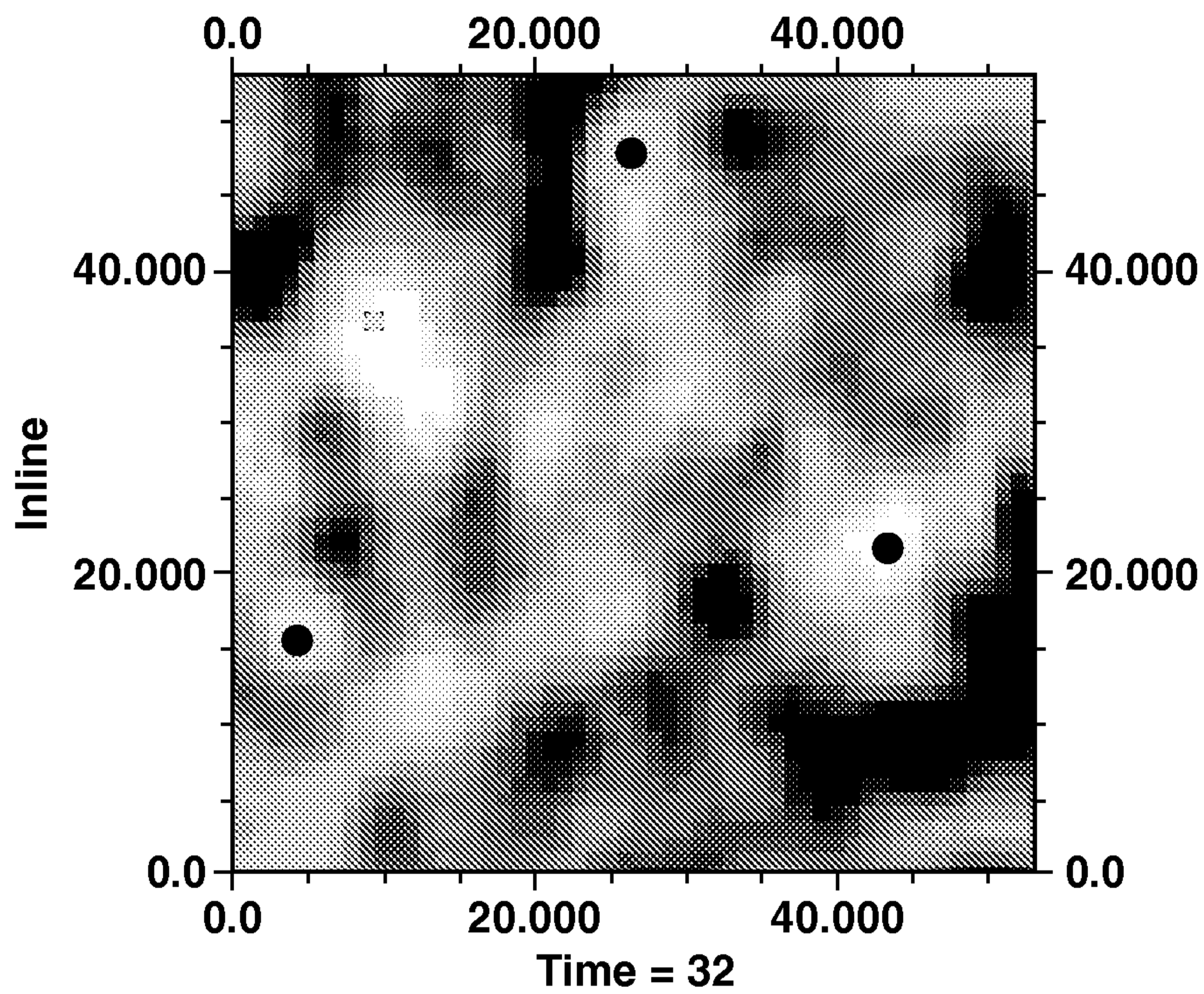


FIG. 7A

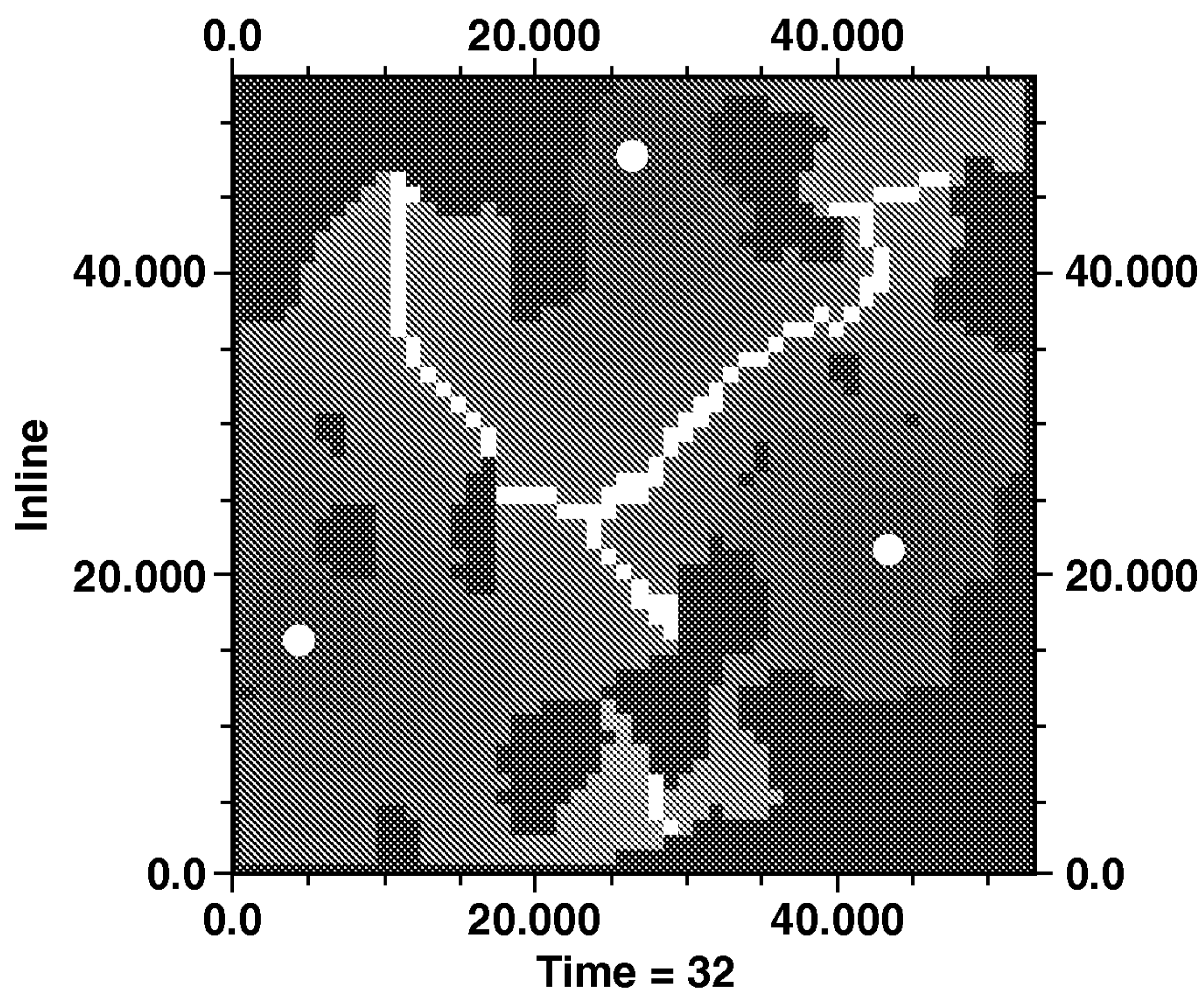


FIG. 7B

7/9

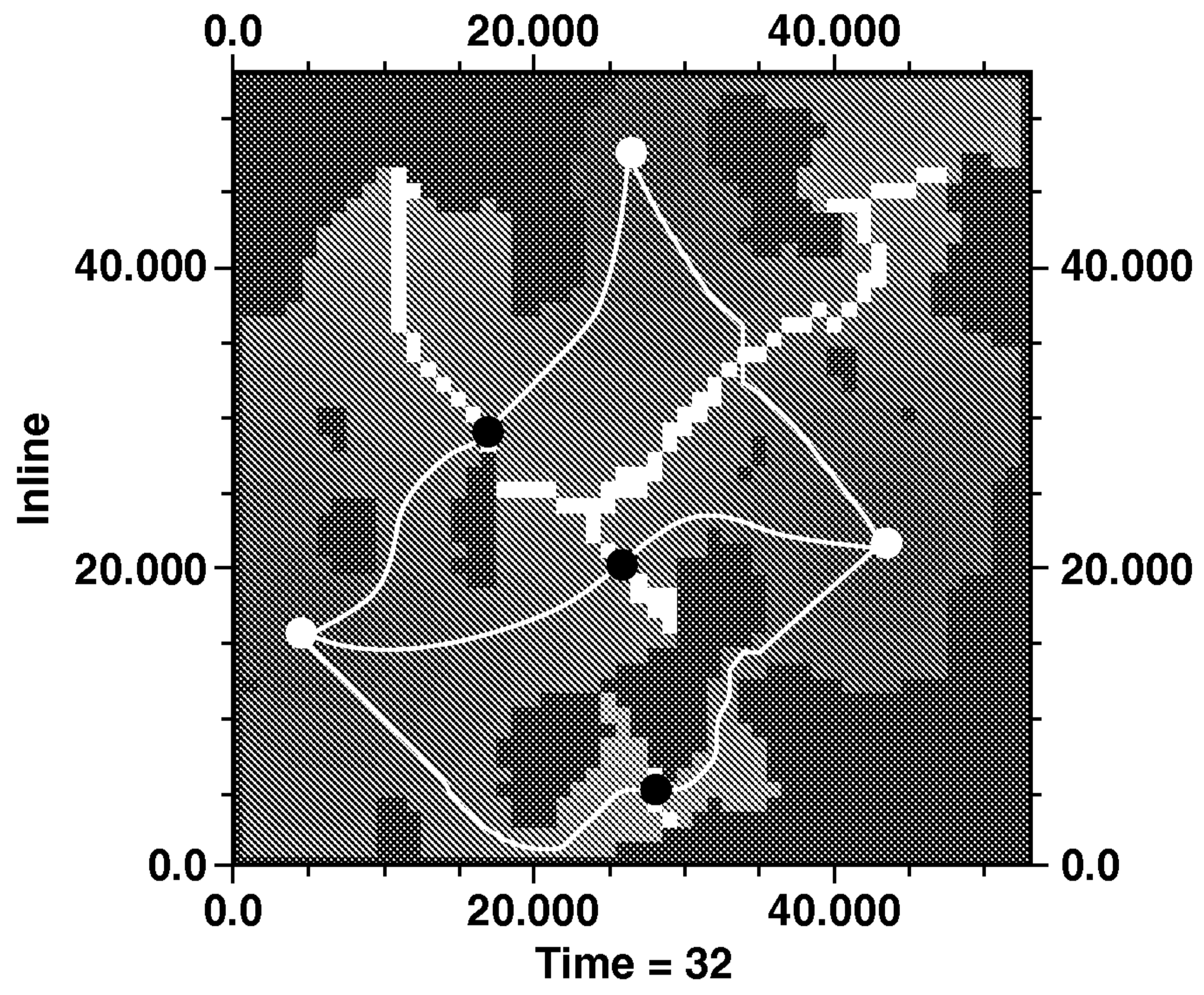


FIG. 8

8/9

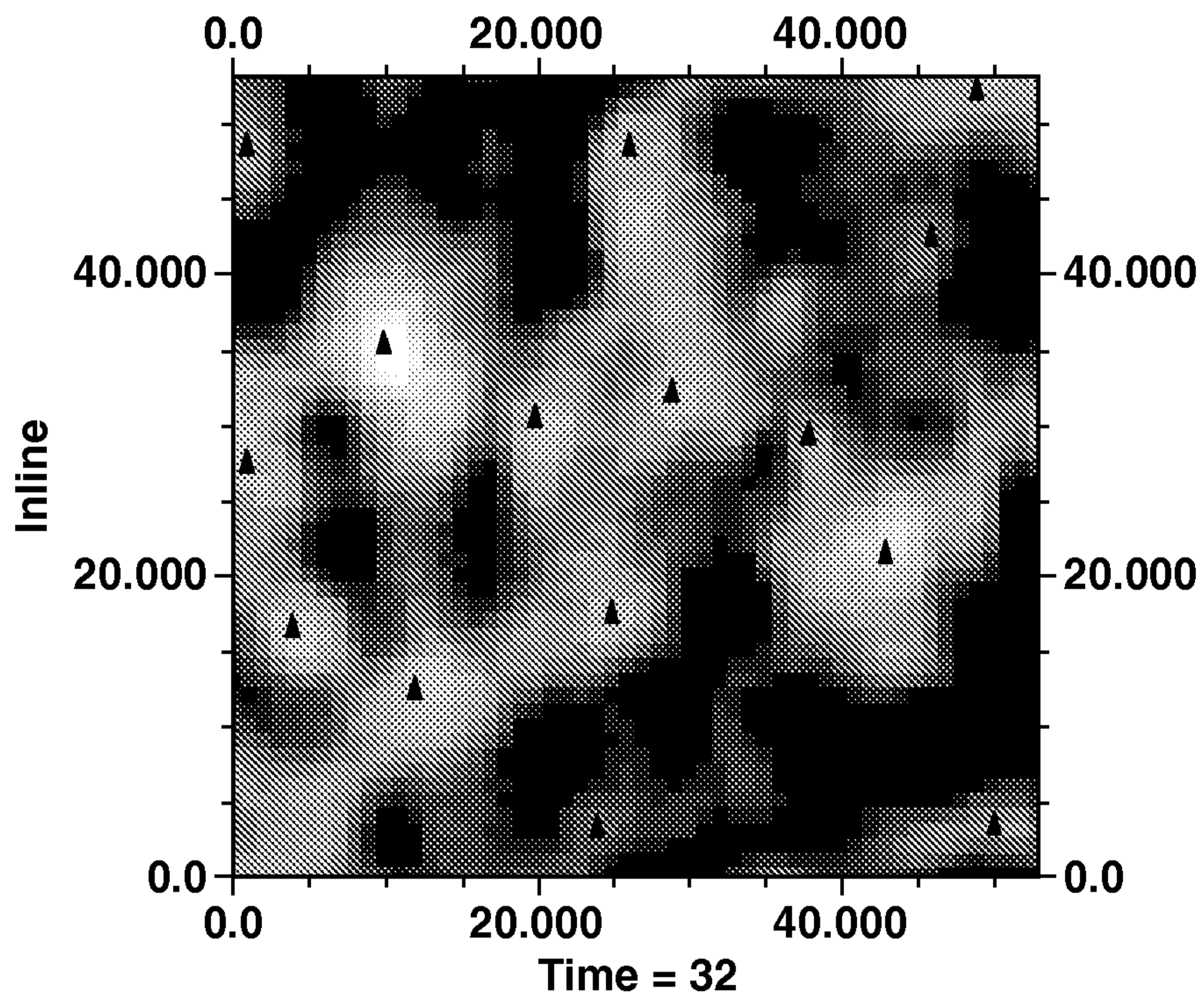


FIG. 9

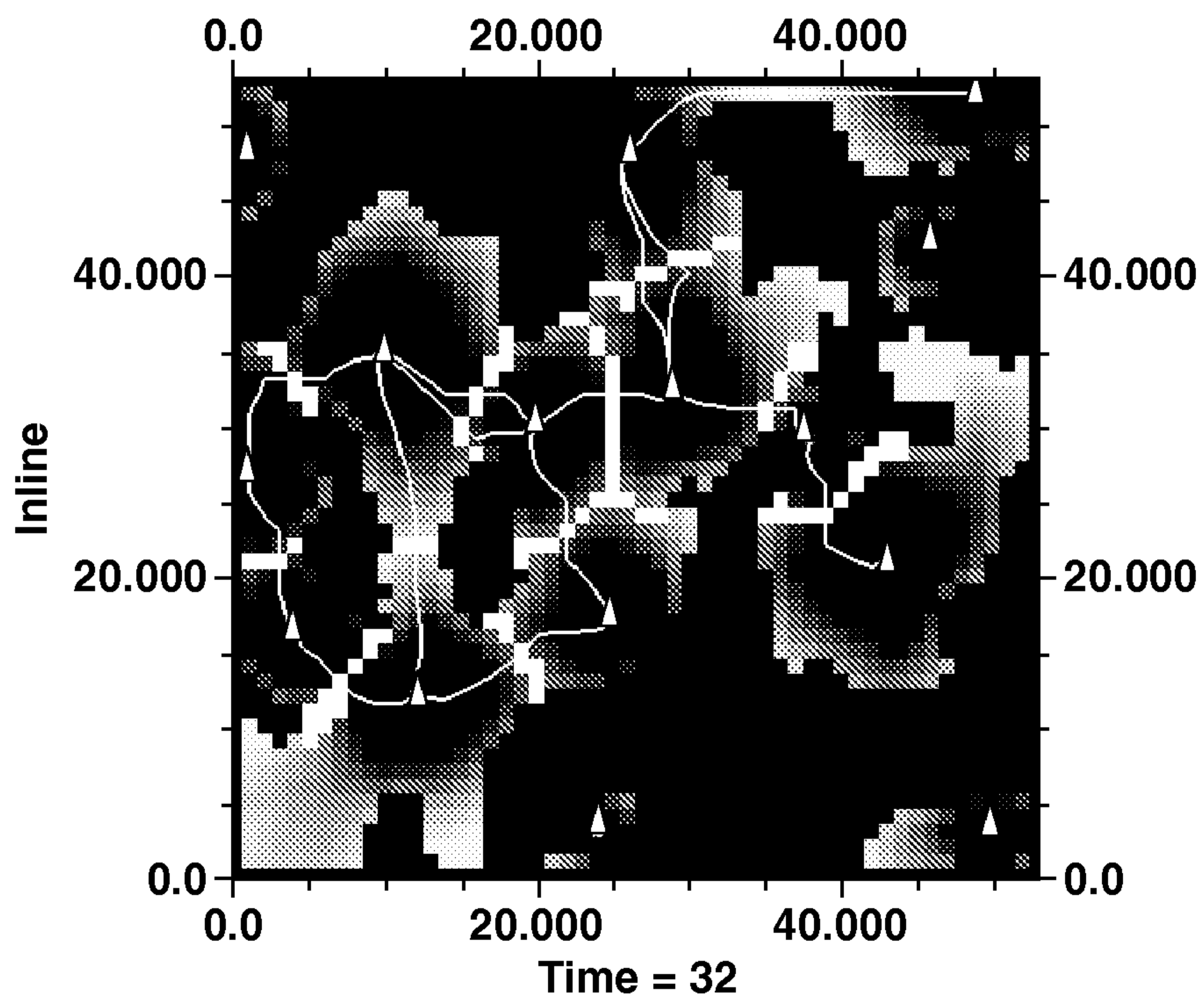
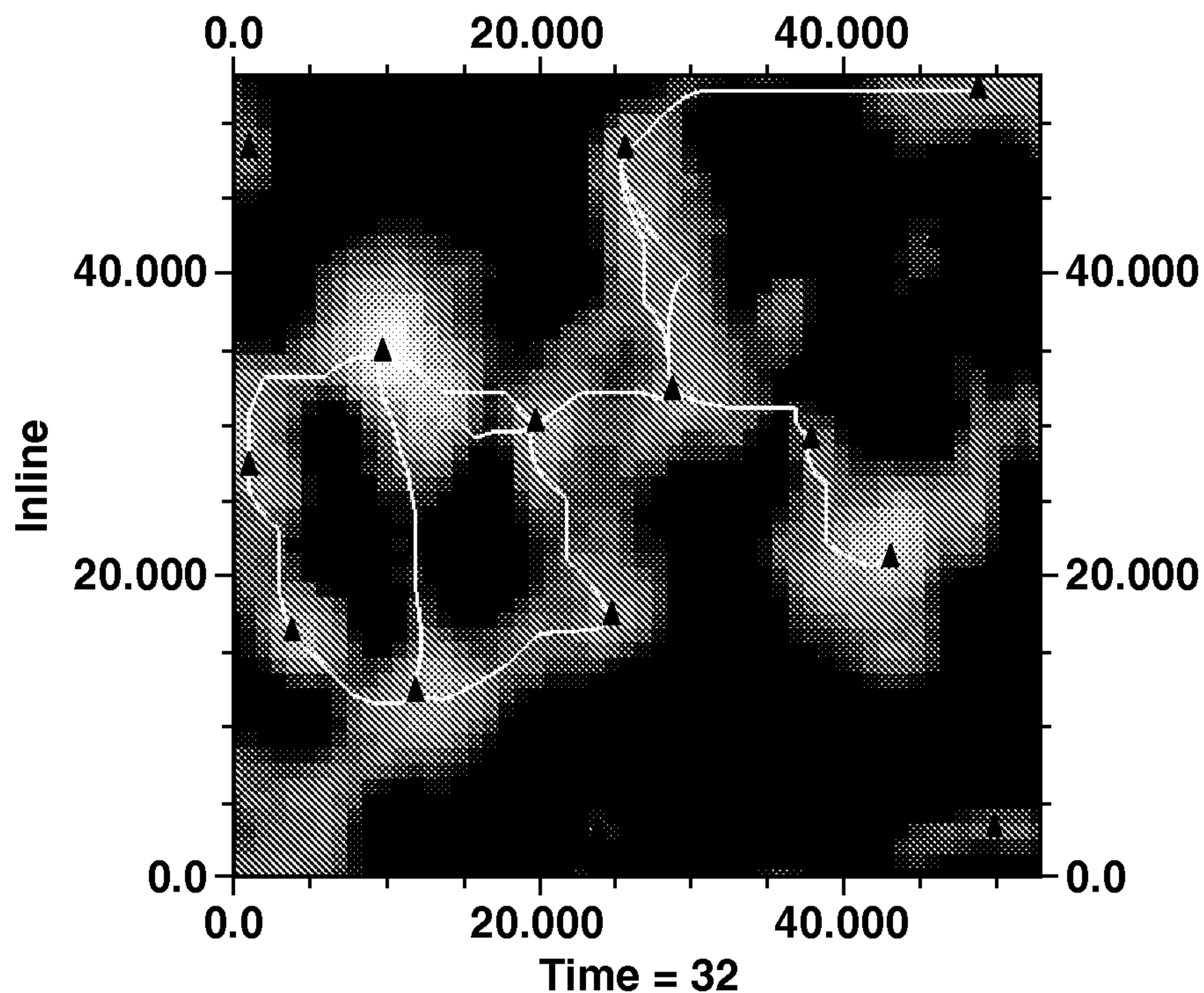
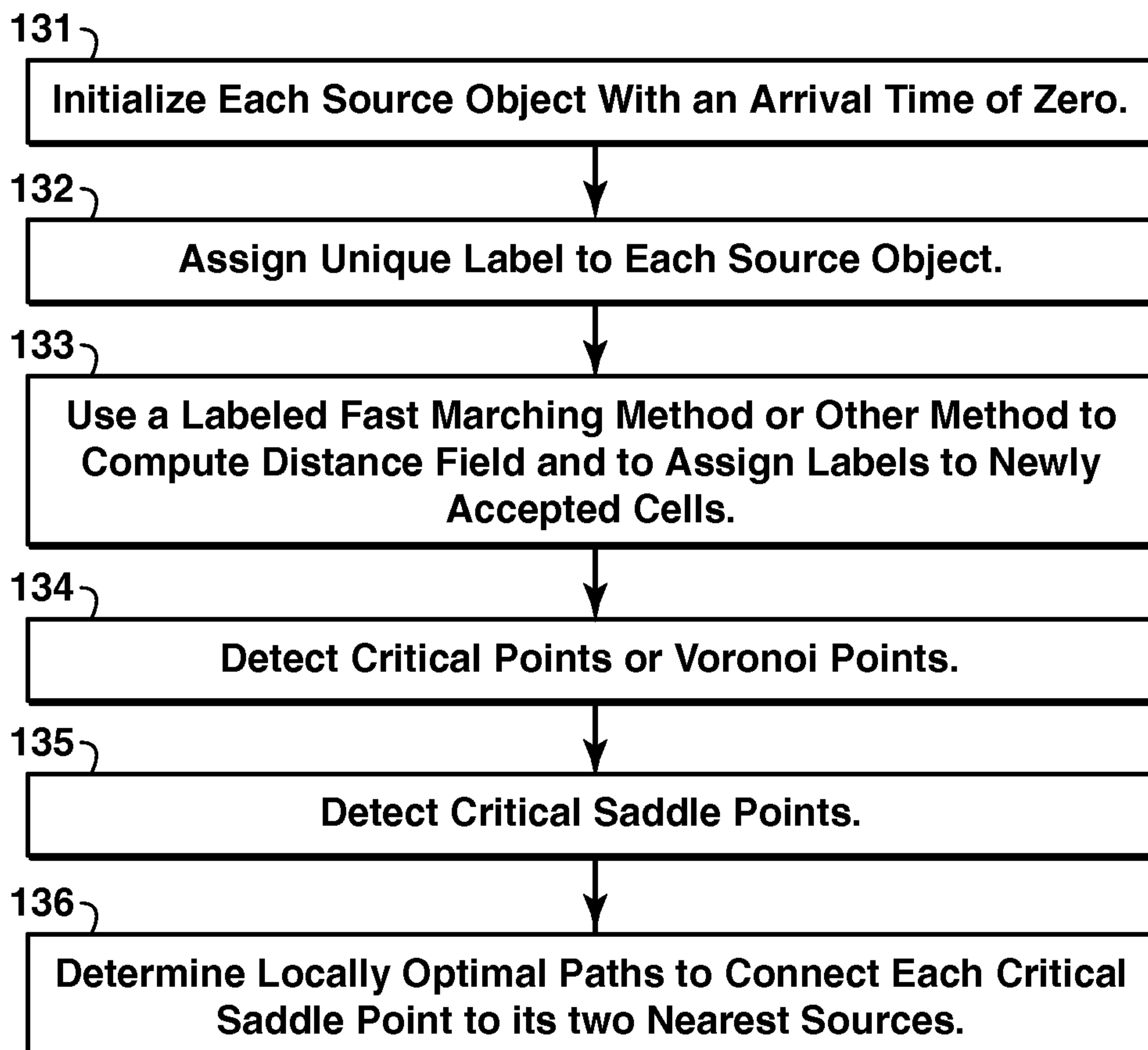


FIG. 10

9/9**FIG. 11****FIG. 12**