United States Patent

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[54] ELECTRONIC ANGLE GENERATOR

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[73] Assignee: **The United States of America as represented by the Secretary of the Navy**

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[52] **U.S. Cl.** **235/186,** 235/189, 307/229

[51] **Int. Cl.** **G06g 7/22**

[58] **Field of Search** 235/186, 189, 197, 194; 328/160, 161, 142; 307/229, 230

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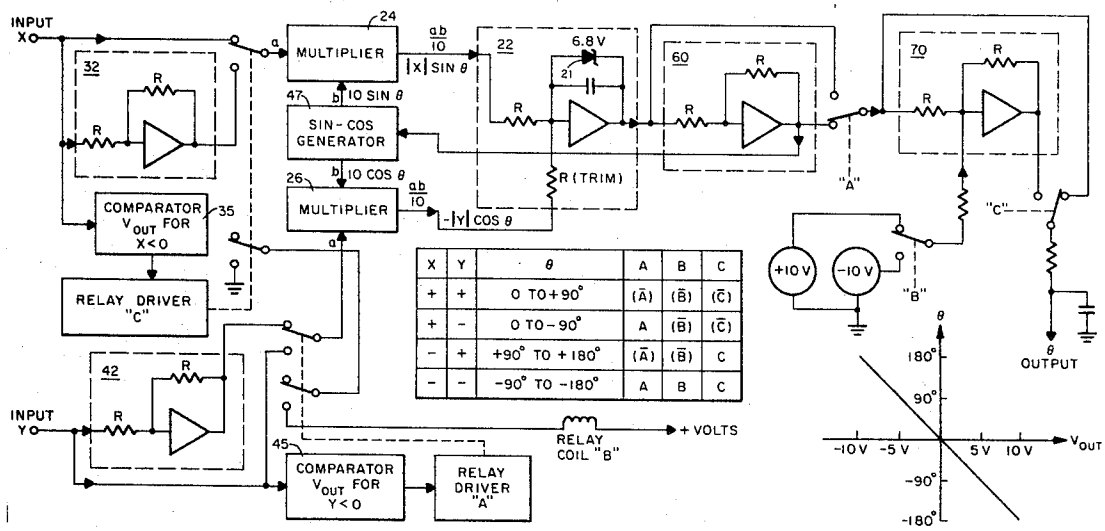
[57] **ABSTRACT**

An electronic angle generator for generating an output voltage proportional to an angle, the tangent of which is a ratio of the two input voltages to the generator, comprising multipliers and function generators utilized for solving the equation $X \sin \theta = Y \cos \theta$ by closed loop action (implicit computation).

2 Claims, 6 Drawing Figures



X	Y	θ	A	B	C
+	+	0 TO +90°	($\bar{A}$)	($\bar{B}$)	($\bar{C}$)
+	−	0 TO −90°	A	($\bar{B}$)	($\bar{C}$)
−	+	+90° TO +180°	($\bar{A}$)	($\bar{B}$)	C
−	−	−90° TO −180°	A	B	C

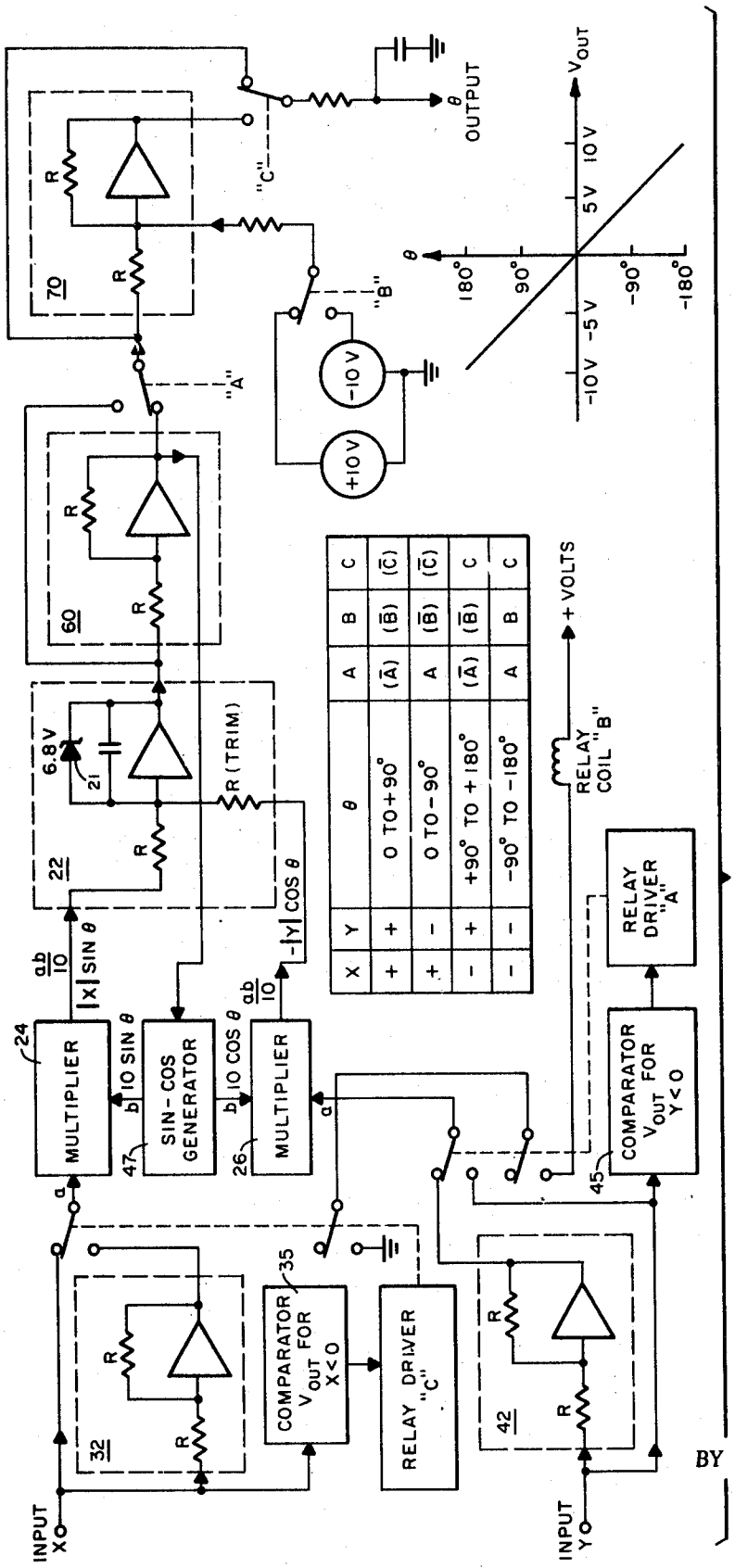


FIG. 1

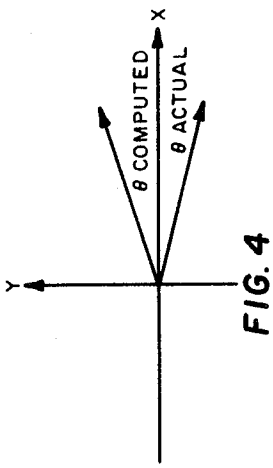


FIG. 4

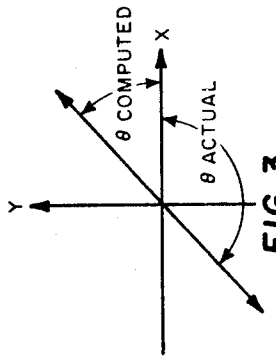


FIG. 3

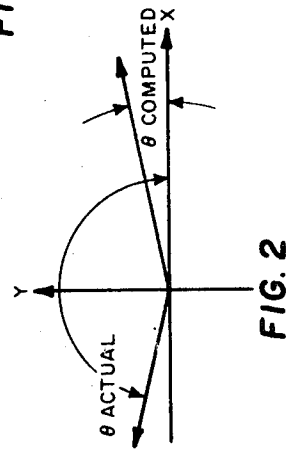


FIG. 2

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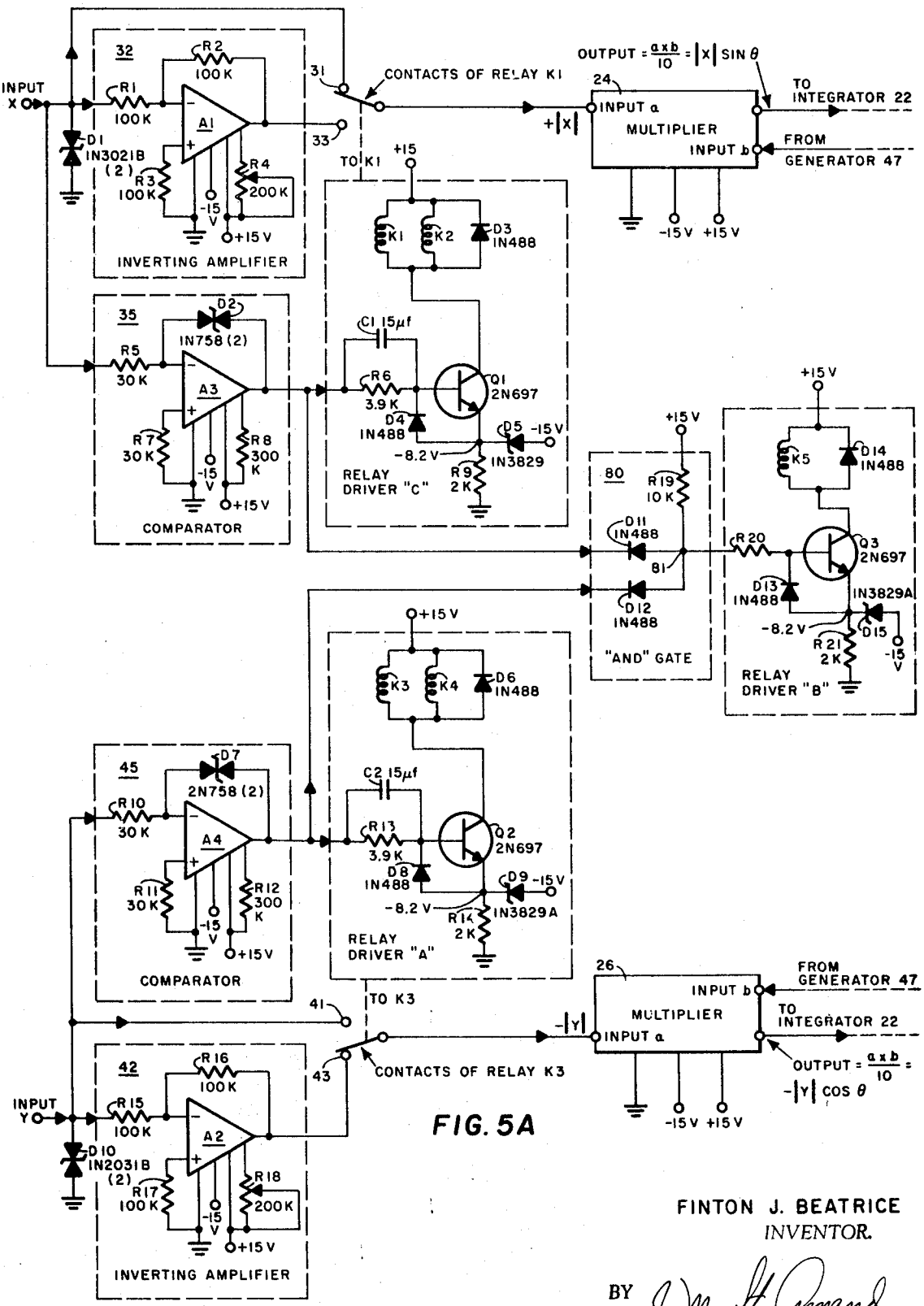

FIG. 5A

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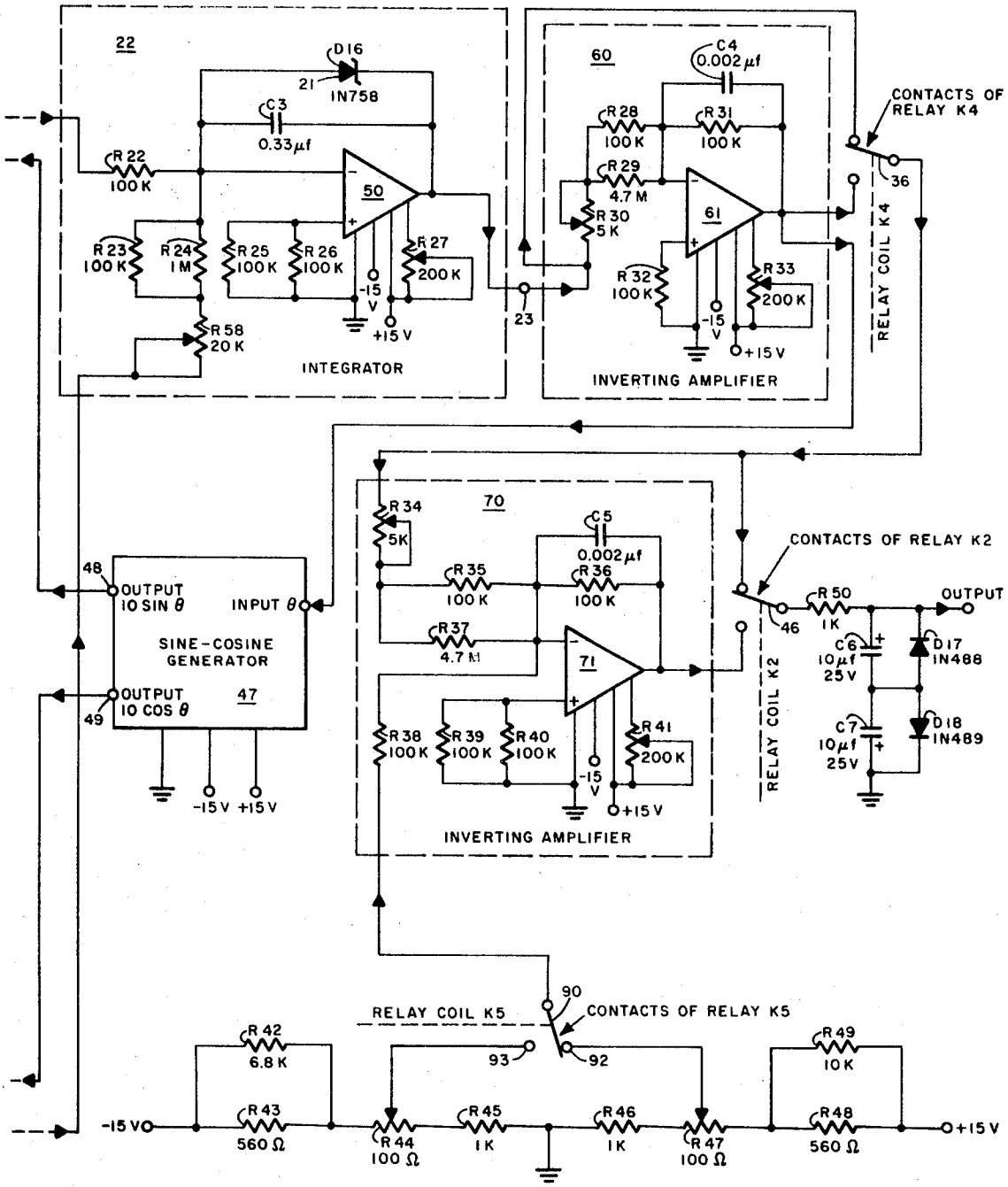

FIG. 5B

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ELECTRONIC ANGLE GENERATOR

The invention herein described may be manufactured and used by or for the Government of the United States of America for governmental purposes without the payment of any royalties thereon or therefor.

This disclosure relates to an analog circuit arrangement which, as shown in FIG. 1, provides an output voltage proportional to an angle, the tangent of which is a ratio of the two input voltages X and Y. Multipliers and function generators are utilized for solving the equation $X \sin \theta = Y \cos \theta$ by closed loop action. The problems of ambiguity and instability, typical of conventional implicit computation approaches, are eliminated by constraining the closed loop which generates the angle θ always to operate in the first quadrant where the sine and cosine have opposite slopes and where feedback is negative. Although the resultant angle is generated in the first quadrant, the actual voltage output is made proportional to θ over all four quadrants by a linear transformation. When the tangent becomes large, a zenner diode serves as a protective device by limiting the magnitude of the signal.

The normal application of this device is to generate the resultant angle of a vector which is the sum of two orthogonal components (the two input signals).

The conventional electromechanical solution to such a problem is to use a resolver-servomotor approach. Such solution is limited in its speed of response and has an ambiguity in the mechanical angle which satisfies the control system.

The usual electronic equivalent of the resolver suffers from the ambiguity of solving the equation $X \sin \theta = Y \cos \theta$, and also from loop instability when the inputs suddenly change polarity.

The novelty of the invention disclosed herein is that it forces a nonambiguous solution and operates stably in all four quadrants.

The scale factor is $\pm$ 10 volts to represent $\pm$ 180°. The static accuracy of the angle generator holds the error within a band of $\pm$ 2.5° over its 360° range. Insofar as dynamic errors are concerned, the computational accuracy and the speed of response of the unit are limited by the characteristics of the commercial multipliers and sine-cosine generator in the computing loops. Because multipliers are used, dynamic performance will be degraded by any substantial reduction of the input signals. However, for signal frequencies up to 1 Hertz, and input amplitudes no less than 20 percent of the maximum, the dynamic error is less than 6 percent of full scale, except for a small region near the discontinuity at $\pm$ 180°.

Dynamic errors are minimal, since the input signals to be applied to the angle generator are the time-averaged outputs from instruments that are essentially narrow band wave analyzers. Therefore, the input components to the angle generator are low frequency signals normally scaled by the usual sensitivity adjustments of a frequency analyzer to produce as close to full range signals as possible.

The technique demonstrated by this invention is an electronic approach to the problem which eliminates the problems of ambiguity and instability which are typical of conventional implicit computation approaches.

Other objects and many of the attendant advantages of this invention will become readily appreciated as the same becomes better understood by reference to the following detailed description when considered in connection with the accompanying drawings wherein:

FIG. 1 is a block diagram of an embodiment of the electronic angle generator of this invention.

FIG. 2, 3 and 4 illustrate angle generation for various examples.

FIGS. 5A and 5B comprise a circuit schematic diagram showing an implementation of the block diagram of FIG. 1 with detailed subcircuits.

The electronic angle generator described herein generates the angle θ_{xy}, where the following definitions hold:

$$\theta_{xy}(f) = \tan^{-1} \frac{Q_{xy}(f)}{C_{xy}(f)}$$

where C_{xy} and Q_{xy} are the coincident and quadrature components of the cross-spectral density function G_{xy}

$$G_{xy} = C_{xy} + jQ_{xy}$$

C_{xy} and Q_{xy} are voltage signals between $\pm$ 10 volts with time averaging between 0.1 to 10 seconds.

The object is to generate an output voltage which is proportional to the angle whose tangent is the ratio of two input voltages. These voltages are orthogonal input voltages by definition, as stated above.

An electromechanical solution to the problem is to use a resolver-servomotor approach. However, an electronic approach was taken to avoid the problems associated with mechanical devices because of frequency response associated with time lag. The present technique uses multipliers and function generators to solve the equation

$$X \sin \theta = Y \cos \theta$$

by closed loop action (or implicit computation). A new technique has been developed to eliminate the problems of ambiguity and instability which are typical of the conventional implicit computation approaches.

This scheme restricts computation to first quadrant angles, and uses logical interrelations of the input polarities to determine suitable transformations which make the output voltage proportional to the desired angle over all four quadrants.

In the diagrams and circuit descriptions of FIG. 1, consider that $C_{xy} = X$, that $Q_{xy} = Y$, and that the angle

$$\theta = \tan^{-1} \left| \frac{Y}{X} \right| = \theta_{xy}.$$

The general problems of instruments which solve a sine-cosine equation by implicit computation are stability and ambiguity.

The generation of a voltage proportional to the angle whose tangent is (Y/X) is typically done by forcing the difference $(X \sin \theta - Y \cos \theta)$ to become arbitrarily small. However, it can be seen that two angles exist, θ and $(\theta + 180°)$, for which $(X \sin \theta - Y \cos \theta)$ vanishes for a given X and Y.

In the solid state hardware used to illustrate the present device, the voltage representing angle does not repeat as it does in a mechanical resolver. The voltage is a linear function of angle, with -10 volts representing $+180°$, and $+10$ volts representing $-180°$. The sign of the slopes of the sine and cosine are not consistent over the four possible quadrants. Hence, a fixed choice of polarity around the two loops, which forces $X \sin \theta = Y \cos \theta$ in one quadrant, can result in positive feedback and saturation of electronic amplifiers for other polarities of X and Y.

This invention avoids these problems, and is block diagrammed in FIG. 1. The closed loop which generates the angle θ is constrained to always operate in the first quadrant, where the sine and cosine have opposite slopes and the feedback is negative.

Although the computing loop is always solving

$$\theta = \tan^{-1} \left| \frac{Y}{X} \right|,$$

and therefore generating the angle in the first quadrant, the actual output voltage is made proportional to θ over all four quadrants by three linear transformations, as demonstrated in the following examples:

Second Quadrant (see FIG. 2). Consider the case when the inputs are $X=-$, $Y=+$. Note in FIG. 2 that $\theta_{ACTUAL} = 180° - \theta_{COMPUTED} = -(-180° + \theta$ computed). Therefore, since $-180° = +10$ volts, a positive 10 volt signal is added to the computed θ signal (which is negative), and the result is inverted to produce the output which is proportional to actual θ in the second quadrant. Relay driver "C" is used.

Third Quadrant (see FIG. 3). Consider $X=-$, $Y=-$. Note in FIG. 3 that $\theta_{ACTUAL} = -(180° - \theta_{COMPUTED})$. Thus, if $-\theta$, produced by actuating relay driver "A," is added to -10 volts (a $+180°$ equivalent), and the result is inverted using the inverting amplifier 70 and relay driver "C," the output is proportional to

actual θ. Relay driver "B" provides the −10 volt signal when both X and Y are negative.

Fourth Quadrant (see FIG. 4). Consider when the inputs are X=+, Y=−. *The output angle should be in the fourth quadrant. Note in FIG. 4 that it is of the same magnitude as the computed angle, but of opposite polarity. A logic arrangement senses the condition of* X=+, Y=−, and supplies the inversion of the output signal, by relay driver "A."

A zener diode **21** (e.g., 6.8 volts) will be noted across the amplifier in integrator **22** in the computing loop. This eliminates the possibility of saturation for the indeterminite case X=0, Y=0. Multipliers **24** and **26** used have the nonideal characteristic of producing some low output when one input is zero and the other input is a large voltage. Regardless of the relative polarity of the net signal to integrator **22**, the zener diode **21**, by both zener breakdown and normal diode action, limits the voltage output during the indeterminate case, and the recovery is rapid for any succeeding nonzero input to either X or Y.

Fig. **5** is a circuit diagram which implements the block diagram of FIG. **1**. The circuit shown uses relays and is limited in frequency response due to the nature of the particular electronic multipliers and function generators involved. The technique, however, is not limited to such switching means and multipliers. Thus, it has no significant limitation on potential frequency response.

The two input voltages, marked X and Y in FIGS. **1** and **5**, are direct voltages (i.e.—not carrier signals) with magnitudes of between −10 volts and +10 volts. The circuit output voltage is proportional to the angle whose tangent is the ratio of input voltages (Y divided by X). The scale factor of the output is linear, with minus **10** volts equaling plus 180°, and plus 10 volts equaling minus 180°.

FIG. **5** has dashed lines defining groups of components which perform specific functions. Several of these are repetitive, and so in the functional description each repetitive subcircuit will be examined down to the component level only once. In general, the subcircuits of FIG. **5** correspond to the functional boxes shown on FIG. **1**.

The input signal X branches out into three paths: it goes directly to the normally closed contact **31** operated by relay K-1 of relay driver "C;" it goes through a unity gain inverting amplifier **32** and appears at the normally open contact **33** of relay K-1; it goes to a comparator circuit which energizes relay driver circuit "C" when X goes negative. One of the two switching functions carried out by relay driver "C" is to energize relay K-1.

It is apparent then that the signal on the swinger of relay K-1, which becomes the "a" input to Multiplier **24**, is a positive voltage equal to the magnitude of X, for X either positive or negative.

Let us consider the circuit details of the three subcircuits mentioned. Resistors R1 and R2, each 100 K ohms, set the unity gain for plug-in operational amplifier A1. Amplifier A1 operates on plus and minus 15 volt power supplies. Potentiometer R4, 200 K ohms, is a zero output trim resistor for this amplifier. Resistor R3 is the impedance balancing resistor for the noninverting input of amplifier A1. Diode D1 is actually two back-to-back zener diodes (IN3021B, **11** volts +5 percent) which are used to limit any input transients which might significantly exceed the rated plus to −10 volt input range.

Comparator circuit **35** uses amplifier A3. This amplifier is similar to A1, has a differential gain of 100,000 volts/volt, and by having diode D2 (two back-to-back IN758 zener diodes) as its feedback element, it performs as a comparator circuit. Resistor R5 feeds in the X signal. Resistor R7 by going to ground provides 0 volts as the reference level for the comparator action. Thus for X signal positive the amplifier A3 output goes negative until clamped by one of the feedback zener diodes D2 across it. The amplifier output feeds relay driver circuit "C." Relay driver "C" is composed basically of transistor Q1 and its associated components. The common power supply voltages for the instrument are plus and minus 15 volts. The relays used are **24** volts relays, for example. Thus relay driven transistor Q1 has its emitter biased to −8.2 volts by diode D5 and resistor R9—the diode D5 reduces the −15 volt supply by 6.8 volts. The desired logic decision for X inputs of negative polarity requires the equivalent of a double pole double throw switch. This is performed by relays K1 and K2. Diode D3 is a voltage transient suppression diode across the inductive load of relay coils K1 and K2. Diode D4 provides reverse voltage protection for the base of transistor Q1. Resistor R6 limits the steady state base current to transistor Q1 and capacitor C1 provides a speed-up action in turning on transistor Q1.

Inverting amplifier **42** with amplifier A2 and associated resistors R15, R16, R17 and R18 operates in a similar manner to inverting amplifier **32** described. Comparator **45** with amplifier A4, back-to-back zener diodes D7, and resistors R10, R11 and R12 operates like comparator **35**. Relay driver "A" with relay coils K3 & K4, diodes D6, D8 & D9, transistor Q2, capacitor C2 and resistors R13 and R14 also operates like relay driver "C."

Relay coil K2 operates relay switch swinger arm **36** and relay coil K4 operates relay switch swinger arm **46**.

The input signal Y branches out into three paths, also: it goes directly to the normally open contact **41** operated by relay K3 of relay driver "A;" it goes through a unity gain inverting amplifier **42** and appears at the normally closed contact **43** of relay K3; it goes to a comparator circuit **45** which energizes relay driven circuit "A" when Y goes negative. One of the two switching functions carried out by relay driver "A" is to energize relay K3.

It is apparent then that the signal on the swinger of relay K3, which becomes the "a" input to Multiplier **26**, is a negative voltage equal to the magnitude of Y, for Y either positive or negative.

The implicit computation loop solves the equation $X \sin \theta - Y \cos \theta = 0$. It is mechanized by using the two multipliers **24** and **26**, a sin-cos generator **47** amplifier **50** with capacitor C3 feedback plus associated resistors in integrator **22**, and an inverting amplifier **60** with its associated resistors R28, R29, R30, R31, R32, R33 and capacitor C4.

The commercial multipliers **24** and **26** each have two inputs, i.e., "a" and "b." Each have an output equal to $(a \times b)/10$. The sin-cos generator **47** has the characteristic of accepting an input signal with a range from −10 to +10 volts representing the angle θ varying from plus 180° to minus 180°. The two outputs **48** and **49** of sine-cosine generator **47** are scaled to equal 10 sin θ and 10 cos θ respectively. Since "a" input to multiplier **24** is always $+X$, and the "a" input to multiplier **26** is always $-Y$, the computing loop will always be stable, i.e., it has no polarity reversals as X and Y go through all permutations of polarity. If the output of multiplier **24** is $X \sin \theta$, and the output of multiplier **26** is $-Y \cos \theta$, the sum of these two terms will be forced arbitrarily close to zero by the following closed loop action. These two signals are summed in integrator **22** by resistor R22 and the combination R23, R24, and R58. The sum is input to integrating amplifier **50** which functions as an inverting integrator due to capacitor C3. Since the computing loop is constrained to operate with one fixed set of signal polarities, the integrator **22** output at **23** will also always have one polarity for normal computing. For the indeterminate case of X=0 and Y=0, the imperfections of the commercial multipliers **24** and **26** may result in some small multiplier output signal for zero input. Such residual multiplier signals could cause computing loop instability for some permutations of polarity between them. The zener diode **21** across the integrating capacitor C3 eliminates this possibility of saturation for the indeterminate case. The maximum voltage developed by integrator **22** in normal computing is plus 5 volts (less than the zener voltage of 6.8 volts for zener diode **21**. If residual voltages whose sum is positive are fed to integrator **22**, the output level is clamped at about −0.5 volts by zener diode **21** providing diode action. Integrator **22** output at **23** drives inverting unity gain amplifier **60**, formed by operational amplifier **61** and its associated resistors R28, R29, R30,

R31, R32 and R33, which serves a double purpose. It provides the correct signal polarity to the sin-cos generator for stable computing loop operation, and is used in the transformations to provide the correct output angle. Since integrator **22** continues to change its output level until the $X \sin \theta$ signal equals the $-Y \cos \theta$ signal, and the integrator output (inverted) drives the sin-cos generator **47**, the value at the input of sin-cos generator **47** must be proportional to the angle θ, whose tangent is Y over X.

Inverting amplifier **70** with operational amplifier **71** and associated circuitry including resistors R35, R36, R37, R38, R39, R40 and R41 is similar to inverting amplifier **60**. Inverting amplifiers **60** and **70** are slightly different from the earlier described inverting amplifiers **32** and **42**. Operational amplifiers **61** and **71** have a wider frequency response and faster transient recovery time than amplifiers A1 and A2 used in inverting amplifiers **32** and **42**, respectively. The effective input resistances formed by resistors R28, R29, and potentiometer R30 in inverting amplifier **60** and R35, R37, and potentiometer R34 in inverting amplifier **70**, are trimmable for calibration purposes. Small rolloff capacitors C4 and C5 are used across the feedback resistors R31 and R36, respectively, because of the greater frequency response of amplifiers **61** and **71**.

This circuit restricts computation to first quadrant angles and uses logical interrelations of the input signal polarities to determine suitable transformations which make the output voltage proportional to the desired angle over all four quadrants. See FIG. **1** for the tabulation of which relays are energized for the permutations of input signal polarities. Details of the reasoning for the transformations used have been previously discussed.

In FIG. **5** it can be seen that resistor **R19** and diodes **D11** and **D12** form an "AND" gate circuit **80** to energize relay driver "B" for the condition that both X and Y input signals are negative. If either input is positive, its respective comparator **35** or **45**, i.e., the output of amplifiers A3 and A4, will be at negative 10 volts, and either diodes **D11** or **D12** will be conducting. Thus the junction **81** of diodes **D11** and **D12**, resistors **R19**, and **R20** will be about minus 9.5 volts, keeping relay driver "B" off. When both input signals to gate **80** are negative, both amplifiers A3 and A4 outputs will be at plus 10 volts and diodes **D11** and **D12** will be cutoff, since relay driver "B" will come on as the base voltage of transistor **Q3** becomes more positive than the −8.2 volts at the Q3 emitter. With relay driver "B" on, the voltage at the junction **81** is approximately minus 5 volts. Relay coil **K5** of relay driver "B" operates to move switch swinger arm **90** from its normal position contacting contact **92** to contact **93**. Diodes **D13**, **D14** and **D16**, and transistor Q3 operate like similar ones in relay drivers "A" and "C."

The resistance dividers formed by resistors **R42** through **R49** provide two trimmable voltages of approximately plus and minus 10 volts. These are required for the transformations when the input X is negative. Resistor **R50**, capacitors **C6** and **C7**, diodes **D17** and **D18** comprise a filter on the output signal which is used to minimize switching transients. Since the output is bipolar (plus 10 to minus 10 volts), diode **D17** is needed to protect capacitor **C6** for negative output voltages. Similarly, diode **D18** is needed to protect capacitor **C7** for positive output voltages. Both capacitors **C6** and **C7** are polarized solid tantulum capacitors.

What is claimed is:

1. An electronic angle generator system for generating an output voltage proportional to an angle, the tangent of which is a ratio of two input voltages to the generator, comprising:

a. a first and a second input channel means to which input voltages X and Y are applied, respectively,

b. a closed computing loop circuit means for generating an output voltage proportional to an angle θ by solving the equation

$$\theta = \tan^{-1} \left| \frac{Y}{X} \right|,$$

the outputs of said first and second input channel means being applied to said computing loop circuit means,

c. said first input channel means comprising

1. a first input terminal to which the first input voltage X is applied,

2. said first input terminal being connected to: the normally closed contact of a first switch means, a first channel voltage inverting means whose output is connected to the normally open contact of said first switch means, and a first comparator circuit means whose output is connected to a first switch operating means which operates to switch said first switch means from its normally closed contact to its normally open contact when the input voltage X to said first channel input terminal goes negative,

3. a first multiplier means whose first of two inputs is connected to said first switch means, the voltage to the first input to said first multiplier means being a positive voltage for both positive or negative first channel voltage X inputs,

d. said second input channel means comprising

1. A second input terminal to which the second input voltage Y is applied,

2. said second input terminal being connected to: the normally open contact of a second switch means, a second channel voltage inverting means whose output is connected to the normally closed contact of said second switch means, and a second comparator circuit means whose output is connected to a second switch operating means which operates to switch said second switch means from its normally closed contact to its normally open contact when the input voltage Y to said second channel input terminal goes positive,

3. a second multiplier means whose first of two inputs is connected to said second switch means, the voltage to the first input to said second multiplier means being a negative voltage for both positive or negative second channel voltage Y inputs,

e. said closed computing loop circuit means solving said equation for the angle θ by restricting computation to first quadrant angles and using logic circuit means for logical interrelations of input polarities to determine transformations to make said output voltage proportional to said angle over all four quadrants.

2. An electronic angle generator system for generating an output voltage proportional to an angle θ, the tangent of which is a ratio of two input voltage X and Y to the generator, comprising:

a. first and second input channel means to which input voltages X and Y are applied, respectively,

b. a closed computing loop circuit means for generating an output voltage proportional to an angle θ by solving the equation

$$\theta = \tan^{-1} \left| \frac{Y}{X} \right|$$

by implicit computation restricting computation to first quadrant angles using logic circuit means for logical interrelations of input polarities to determine transformations to make said output voltage proportional to said angle over all four quadrants,

c. said first input channel means comprising:

1. a first input terminal to which the first input voltage X is applied,

2. said first input terminal being connected to: the normally closed contact of a first switch means, a first channel voltage inverting means whose output is connected to the normally open contact of said first switch means, and a first comparator circuit means whose output is connected to a first switch operating means which operates to switch said first switch means from its normally closed contact to its normally open contact when the input voltage X to said first channel input terminal goes negative,

3. a first multiplier means whose first of two inputs is connected to said first switch means, the voltage to the first input to said first multiplier means being a positive voltage for both positive or negative first channel voltage X inputs,

d. said second input channel means comprising:

1. a second input terminal to which the second input voltage Y is applied,
2. said second input terminal being connected to: the normally open contact of a second switch means, a second channel voltage inverting means whose output is connected to the normally closed contact of said second switch means, and a second comparator circuit means whose output is connected to a second switch operating means which operates to switch said second switch means from its normally closed contact to its normally open contact when the input voltage Y to said second channel input terminal goes positive,
3. a second multiplier means whose first of two inputs is connected to said second switch means, the voltage to the first input to said second multiplier means being a negative voltage for both positive or negative second channel voltage Y inputs,

e. said closed computing loop circuit means comprising:

1. an integrator means having first and second inputs,
2. a first loop voltage inverting means,
3. a second loop voltage inverting means,
4. an output terminal, and
5. a sine-cosine generator means whose input represents the angle θ which can vary from plus 180° and having first and second outputs representing the sine and cosine of the angle θ, respectively,

f. the output of said integrator means connected to the input of said first loop voltage inverting means and to the normally closed contact of a third switch means,

g. the output of said first loop voltage inverting means being connected to the normally open contact of said third switch means and to the input to said sine-cosine generator means, said first loop voltage inverting means output providing correct signal polarity to the input of said sine-cosine generator means for stable loop operation,

h. the first output of said sine-cosine generator means being connected to the second of the two inputs to said first multiplier means, and the second output of said sine-cosine generator means being connected to the second of the two inputs to said second multiplier means,

i. the output of said first multiplier means being directly proportional to the voltage X sine θ and connected to the first input to said integrator means,

j. the output of said second multiplier means being directly proportional to the voltage $-Y$ cosine θ and connected to the second input to said integrator means, the first and second inputs to said integrator means being summed by said integrator means,

k. said third switch means being connected to the input to said second loop voltage inverting means and to the normally closed contact of a fourth switch means, said third switch means also being operated by said second switch operating means which operates to switch said third switch means from its normally closed contact to its normally open contact when the input voltage Y to said second channel input terminal goes negative,

1. the output of said second loop voltage inverting means being connected to the normally open contact of said fourth switch means to provide correct angle output at said output terminal when said fourth switch means is switched from its normally closed contact to its normally open contact,

m. said fourth switch means being connected to said output terminal, said fourth switch means also being operated by said first switch operating means which operates to switch said fourth switch means from its normally closed contact to its normally open contact when the input voltage X to said first channel input terminal goes negative,

n. the output of said first loop voltage inverting means also providing a correct angle output at said output terminal when said third switch means is switched from its normally closed contact to its normally open contact,

o. said computing loop circuit solving said equation for the angle θ restricting computation to first quadrant angles using loop circuitry means for logical interrelations of input polarities to determine transformations to make said output voltage proportional to said angle over all four quadrants.

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