Jan. 15, 1963 J. P. FORD 3,073,524

COMPENSATOR FOR SYSTEM OF PLURAL DEGREES OF FREEDOM

Filed May 25, 1959 5 Sheets-Sheet 1

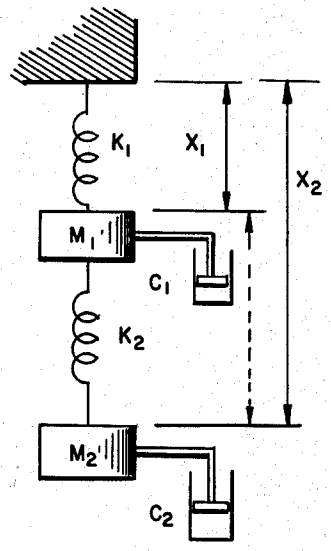


FIG. 8

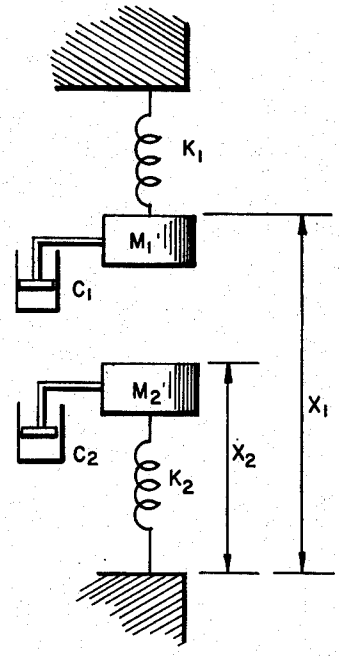


FIG. 7

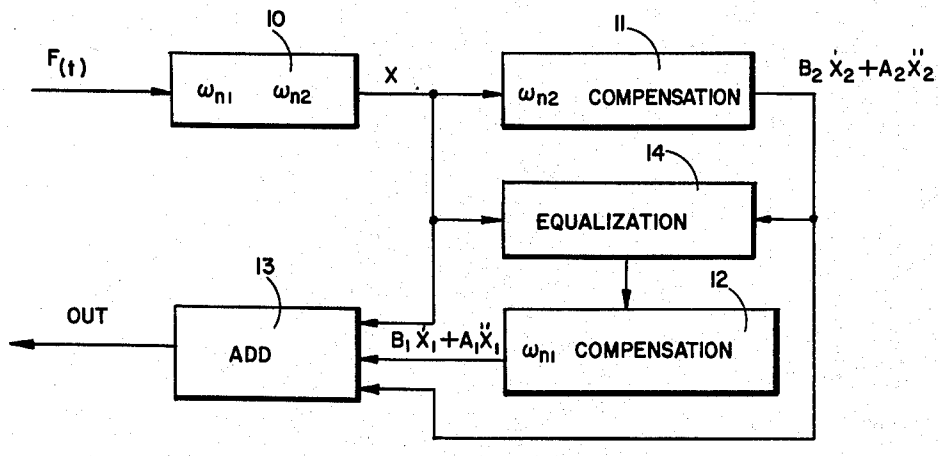


FIG. 1

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COMPENSATOR FOR SYSTEM OF PLURAL DEGREES OF FREEDOM

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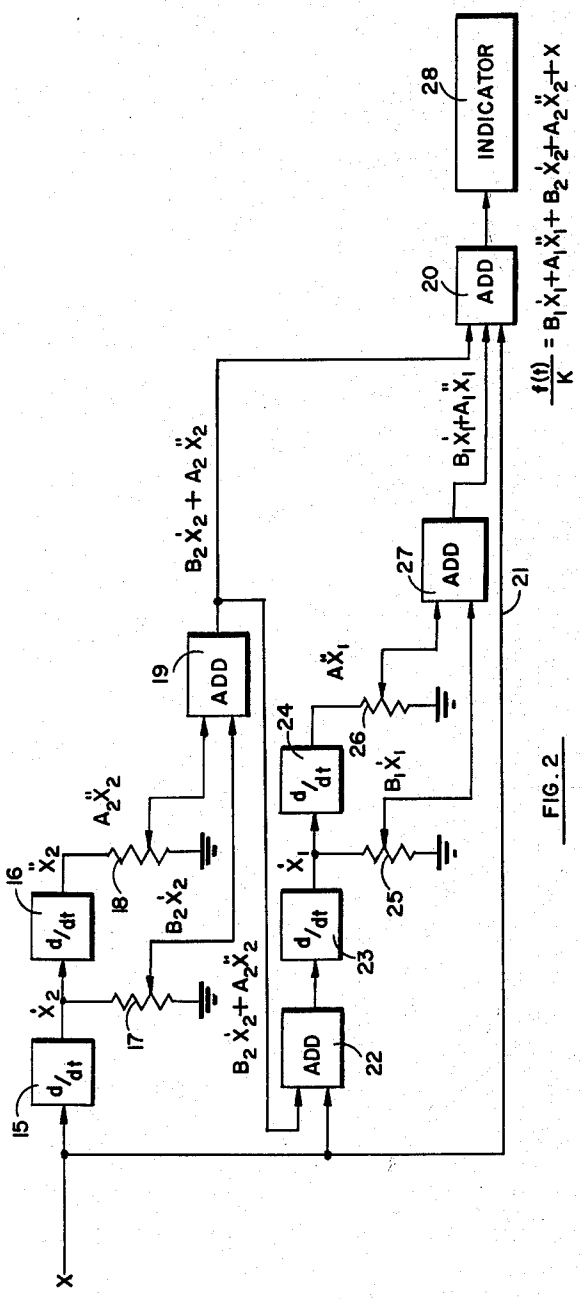


FIG. 2

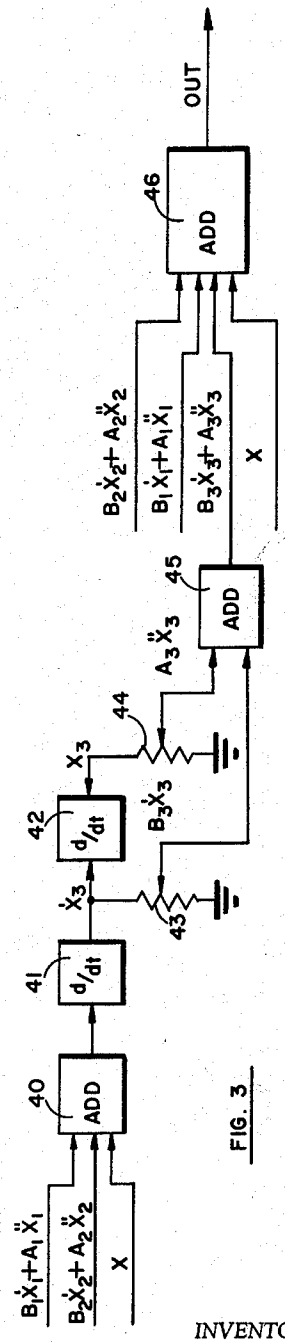


FIG. 3

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COMPENSATOR FOR SYSTEM OF PLURAL DEGREES OF FREEDOM

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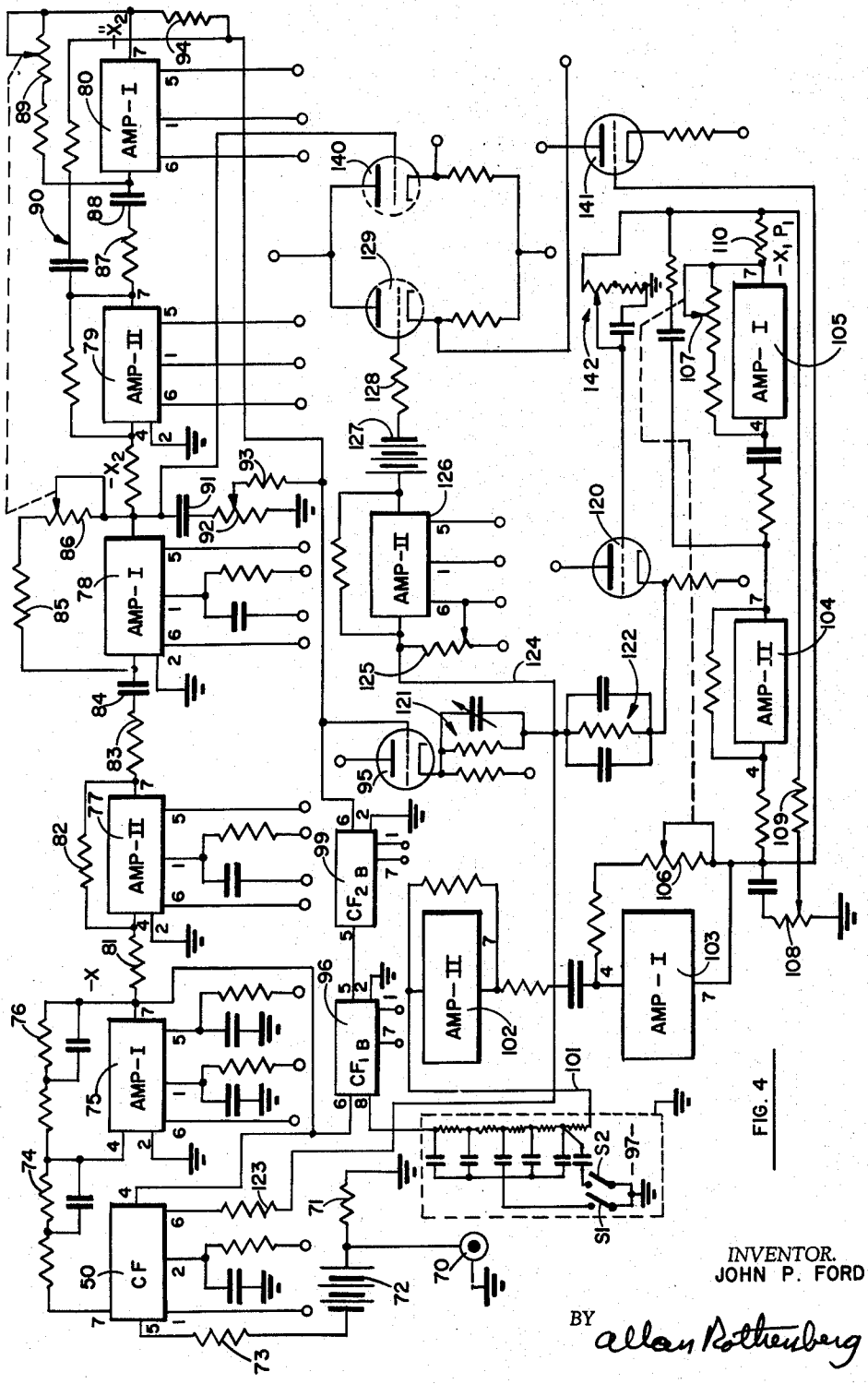


FIG. 4

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COMPENSATOR FOR SYSTEM OF PLURAL DEGREES OF FREEDOM

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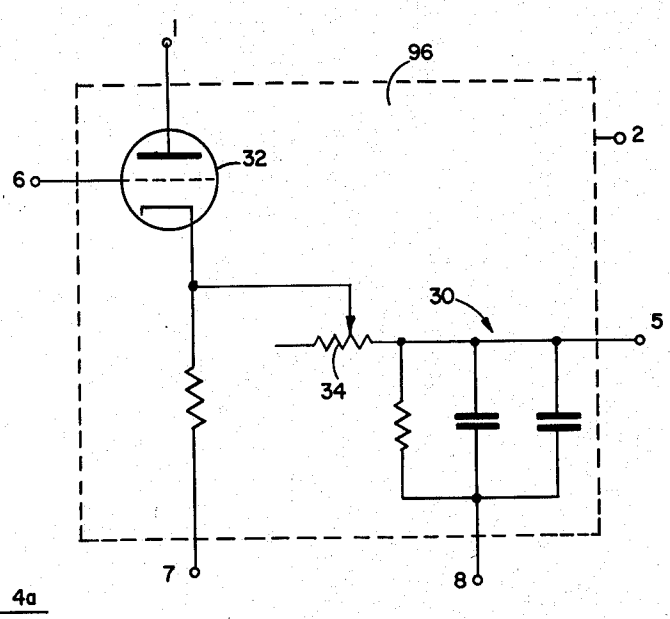


FIG. 4a

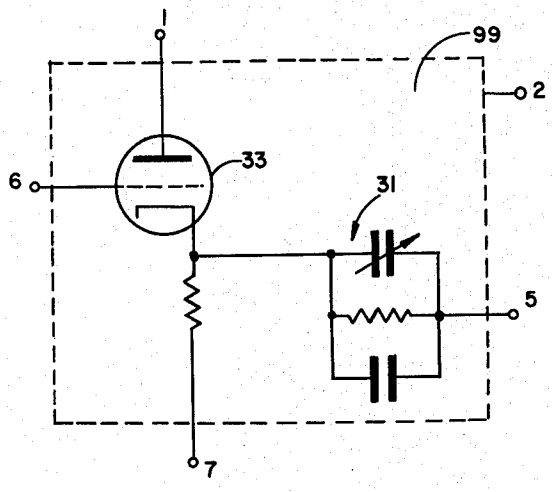


FIG. 4b

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COMPENSATOR FOR SYSTEM OF PLURAL DEGREES OF FREEDOM

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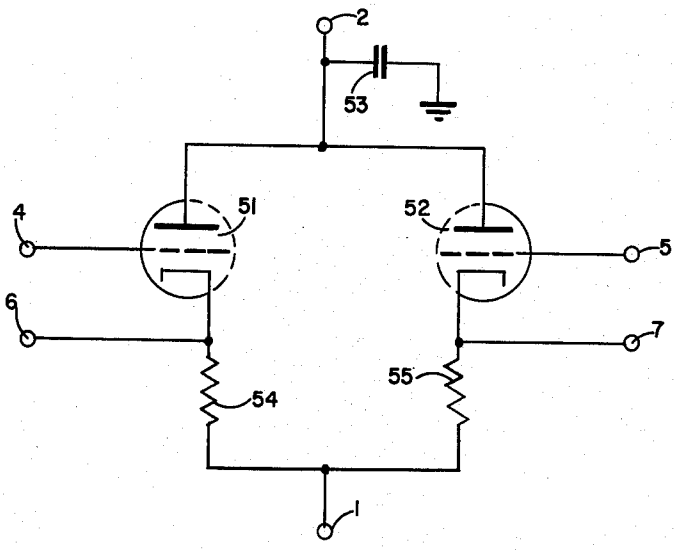


FIG. 5

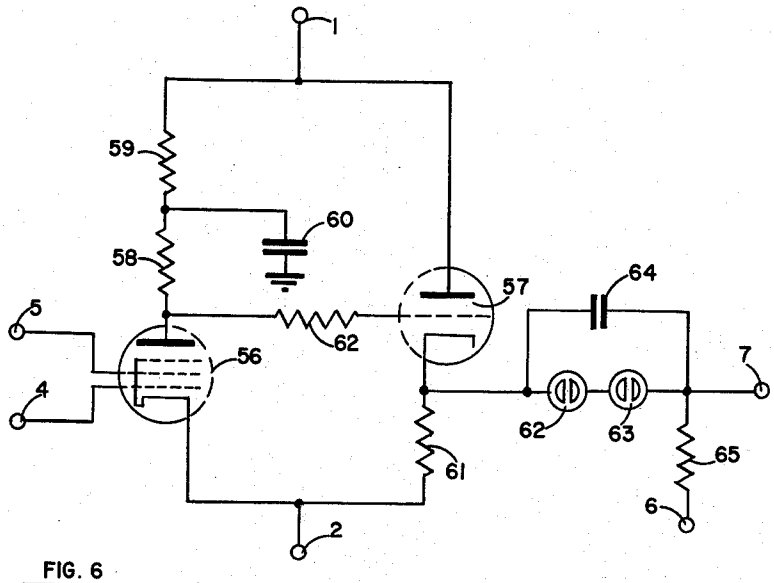


FIG. 6

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United States Patent Office

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3,073,524

COMPENSATOR FOR SYSTEM OF PLURAL DEGREES OF FREEDOM

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Filed May 25, 1959, Ser. No. 815,725
10 Claims. (Cl. 235—182)

This invention relates to electronic compensators, and more particularly concerns apparatus for facilitating the study of transient phenomena in or with dynamic systems. From one point of view this invention may be considered to be an analog computer described herein as used for compensation. It can also be used as a computing device to perform analysis of unknown linear systems.

The present invention comprises an improvement upon the apparatus described in a co-pending application, Serial No. 624,316 of T. W. Berwin et al., filed November 26, 1956, for Dyna-Electronic Compensator. As described in detail in said co-pending application, electromechanical pickups such as transducers of the capacitance, resistance bridge or other well-known types have a response which is limited by the characteristics of the inertia, compliance and damping which are peculiar to the pickup itself. The apparatus described in the co-pending application, while satisfactory for compensation for the inadequacies of systems of one degree of freedom, is severely limited in its use. This is so by reason of the fact that most commercially available transducers have more than one degree of freedom. When a second natural frequency (of a second degree of freedom) of the transducer to be compensated falls within the electrical bandpass of the prior apparatus, this second frequency will not be compensated. Furthermore, if such second frequency is higher than the first natural frequency for which the apparatus is adjusted, not only will this second frequency remain uncompensated, but it actually will be amplified due to the frequency characteristics of the compensating apparatus.

Accordingly, it is an object of this invention to provide compensation for a dynamic system (such as a transducer) having more than one degree of freedom.

In carrying out the invention in accordance with a preferred embodiment thereof, there is provided a pair of compensating networks, each of which is individually adjusted to a different one of two natural frequencies of two degrees of freedom of the system of which inadequacies are to be corrected. Since the second compensating network may operate to amplify the first natural frequency in addition to compensating for the second natural frequency, it is desirable to eliminate from the second network the dynamic effect of the first degree of freedom of the input dynamic system (transducer). To this end, the output of the first compensating network is combined with the input to the second compensating network for purposes of equalization whereby the signal which the second compensating network acts upon has removed therefrom the unwanted effects of the first natural frequency of the input system.

It is an object of this invention to provide equalization of distortion caused by a dynamic system having more than one degree of freedom.

It is another object to improve the measurement of transient phenomena.

A further object of this invention is to facilitate the study of dynamic system of plural degrees of freedom.

Still another object of this invention is the automatic compensation for inadequacies of electromechanical transducers.

A further object of the invention is the extension of frequency, range and transient response of electromechanical transducers having more than one degree of freedom.

These and other objects will become apparent from the following description taken in connection with the accompanying drawings in which

FIG. 1 is a functional diagram illustrating the principles of this invention;

FIG. 2 comprises a functional block diagram showing the step-by-step mathematical operations;

FIG. 3 is a functional block diagram illustrating the extension of the principles of this invention to compensation of a system having more than two degrees of freedom;

FIG. 4 is a circuit diagram of an embodiment of the invention;

FIGS. 4*a* and 4*b* illustrate details of cathode follower plug-in units of the circuit of FIG. 4;

FIGS. 5 and 6 illustrate details of certain components of the circuit of FIG. 4;

And FIGS. 7 and 8 comprise diagrammatic illustrations depicting the characteristics of typical systems for which compensation may be afforded by this invention.

Throughout the drawings, like reference numerals refer to like parts.

Most commercially available transducers and many other types of dynamic transfer systems are inherently characterized by having two degrees of freedom of which one has a relatively low frequency and a relatively high amplitude and of which the other has a relatively high frequency and small amplitude. These frequencies may be designated as ω_{n1} for the relatively low natural frequency and ω_{n2} for the relatively high natural frequency of the system. In response to a step input such as shock or pressure wave applied to such a system, there will be provided an output which is distorted in accordance with each of the degrees of freedom of the system. Thus, a first component of the output distortion may be caused by the inertia, compliance and damping ratio of the first degree of freedom and a second component of distortion may be caused by the inertia, compliance and damping ratio of the second degree of freedom. As illustrated in FIG. 1, a dynamic system such as transducer **10** having two degrees of freedom will provide an electrical signal output X in response to a sharply rising step input $F(t)$. The output X includes components X_1 and X_2 respectively indicative of the amplitude of vibration due to each degree of freedom. As will be shown hereinafter, the input $F(t)$ is related to the output X by the expression

$$F(t)=A_1\ddot{X}_1+B_1\dot{X}_1+A_2\ddot{X}_2+B_2\dot{X}_2+X \qquad (1)$$

and

$$F(t)=\frac{f(t)}{K}$$

where:

K=Spring Constant

In most physical measuring systems the value of K is determined during static calibration. Since it is embodied in the calibration and is therefore present in the solution, it can be stated that the direct recorded output of the compensator is $F(t)$. In Equation 1, A_1 and A_2 are indicative of the first and second natural frequencies (inertia and compliance) respectively, and B_1 and B_2 are respectively indicative of the damping ratios of the first and second degrees of freedom respectively, $\dot{X}$ and $\ddot{X}$ denoting respectively the first and second derivatives of X. In accordance with the invention, there is provided a first compensating network **11** which is adjusted to compensate for one natural frequency ω_{n2} (the higher frequency, for example) substantially as described in the above-mentioned co-pending application. The output of compensa-

tion network **11** thus comprises a signal indicative of the quantity $B_2\dot{X}_2+A_2\ddot{X}_2$. The output X of transducer **10** is also applied to a second compensation network **12** which is adjusted for the second degree of freedom of natural frequency ω_{n1} (the lower natural frequency). The operation of this compensation network is similar to that of circuit **11** and provides as its output a signal indicative of the quantity $B_1\dot{X}_1+A_1\ddot{X}_1$. The outputs of compensating networks **11** and **12** are combined with the transducer output X in a summing network **13** to provide an output $F(t)$ which comprises the solution of Equation 1.

Since the compensation networks **11** and **12** will amplify signals at the frequency for which the opposite compensator network is adjusted and additionally will amplify higher frequency signals, it is necessary to eliminate the dynamic effects of the higher ω_{n2} frequency from the ω_{n1} compensating network. To this end, there is provided an equalization network **14** interposed between the output of transducer **10** and the input of the low frequency compensation network **12**. The equalization network **14** accomplishes its desired function by combining the output of the transducer with the output of the compensation network **11** in accordance with principles which will be detailed hereinafter.

The solution of Equation 1 is accomplished as illustrated in FIG. 2. The transducer output X is fed to a first differentiating network **15**, providing an output $\dot{X}_2$ which is fed to a second differentiating network **16** providing the second derivative $\ddot{X}_2$. Potentiometers or multipliers **17** or **18** are respectively coupled to the outputs of differentiators **15** and **16** and individually adjusted to multiply signals $\dot{X}_2$ and $\ddot{X}_2$ by predetermined constants B_2 and A_2 respectively. The multiplying constants B_2 and A_2 are respectively proportional to the damping ratio and to the inertia and compliance of the second degree of freedom of the system which provides the distorted output X. These constants may be determined by procedures well known in the art. For example, by a study of the transducer output and appropriate mathematical computation the transducer characteristics may be obtained. Alternatively, as described in the above-mentioned co-pending application, a known excitation may be applied to the transducer and the several potentiometers may be adjusted by a trial and error procedure until the output is a true reproduction of the transducer input whereby the potentiometer settings will then yield the value of the desired constants. Adjustment of potentiometer **18**, together with adjustable tuning circuits to be described below, thus effect adjustment of the first compensating channel to the natural frequency (ω_{n2}) of one degree of freedom of the dynamic input system **13**.

The outputs of the multipliers **17** and **18** are added in a combining or summing network **19** which provides one input to another summing or combining network **20**. A second input to summing network **20** is provided via lead **21** from the transducer output X. The output of summing network **19**, $B_2\dot{X}_2+A_2\ddot{X}_2$ is also fed as one input to a summing or equalization network **22** which receives as a second input thereto the signal X from the transducer output. The combining of the output of summing network **19** with the transducer output provides the desired equalization of the compensation for the second degree of freedom compensating channel by removing from the input thereto the dynamic effect of the higher natural frequency. This second channel is adjusted to the second natural frequency (ω_{n1}) by its multiplying potentiometer and adjustable tuning circuits to be described below. The output of network **22**, which comprises the transducer output equalized for the high frequency distortion component is fed to a differentiating circuit **23** providing as its output a signal indicative of $\dot{X}_1$ which in turn is fed to differentiating network **24** which provides as its output a signal indication $\ddot{X}_1$. The outputs of differentiators **23** and **24** are applied respectively to potentiometers **25** and **26** which introduce the multiplying factors B_1 and A_1 to provide signals respectively indicative of $B_1\dot{X}_1$ and $A_1\ddot{X}_1$. The outputs of multipliers **25** and **26** are combined in summing network **27** to provide an output signal indicative of the quantity $B_1\dot{X}_1+A_1\ddot{X}_1$ which is fed as the third input to summing network **20**. The latter thus provides as its output a signal proportional to $F(t)$ as defined in Equation 1 which may be fed to any suitable utilization circuit such as an indicator or recorder **28**.

It will be readily appreciated that the principles of this invention can be extended to dynamic systems or transducers having more than two degrees of freedom by expanding the system to include additional compensating channels each adjusted to compensate for distortion of a still lower natural frequency of such additional degree of freedom. Each such additional channel would be provided with a summing or equalizing network at the input thereof to remove the dynamic effects of all higher natural frequencies which will not be compensated by lower frequency compensating channels. Referring to FIG. 3, there is shown a third compensating channel which may be utilized together with two channels of FIG. 2 if the dynamic input system has a third degree of freedom of a third natural frequency ω_{n3} which is lower than either of the other two natural frequencies. The structure of FIG. 2 is modified by providing the additional circuitry as illustrated and connected in FIG. 3. Thus, the input to the third compensating channel would be provided by means of a network **40** receiving as the inputs thereto, a signal X from the transducer output, a signal $B_2X_2+A_2\dot{X}_2$ from the output summing network **19** of FIG. 2 and a signal $B_1\dot{X}_1+A_1\dot{X}_1$ from the output of summing network **27** of FIG. 2. This third compensating channel is similarly adjusted to the third natural frequency of the third degree of freedom by adjustment of one of its multiplying potentiometers and suitable adjustable tuning circuits. The output of network **40** is fed to a first differentiating circuit **41** providing at its output a signal indicative of $\dot{X}_3$ which is in turn differentiated in circuit **42** to provide the second derivative $\ddot{X}_3$. The outputs of differentiators **41** and **42** are applied respectively to potentiometers **43** and **44** which introduce the multiplying factors B_3 and A_3 to provide signals respectively indicative of $B_3\dot{X}_3$ and $A_3\ddot{X}_3$. The outputs of multipliers **43** and **44** are combined in summing network **45** to provide an output signal indicative of the quantity

$$B_3\dot{X}_3+A_3\ddot{X}_3$$

which is fed as the fourth input to an output summing network **46** having three other inputs thereto from the transducer output, from the output of summing network **27** and from the output of summing network **19**. The summing network **46** may be provided as a substitute for, or in addition to, network **20** of FIG. 2. Thus, it will be seen that any reasonable number of degrees of freedom may be handled in accordance with the principles of this invention.

The input system used in the description of the invention is an electromechanical transducer where, by substitution methods, the mechanical system is duplicated by electrical equivalent. Electrical systems comprising inductances, capacitances and resistances of unknown dynamical behavior can also be equalized or analyzed by the invention.

The circuit details of the compensator of FIG. 2 are illustrated in FIG. 4 as comprising a dual cathode follower **50** and a number of substantially identical amplifiers which may be of the plug-in type. These plug-in components are illustrated in FIGS. 5 and 6. A conventional plug-in type cathode follower as shown in FIG. 5 may comprise twin triode sections **51** and **52** having the plates thereof connected in common to plug-in terminal **2** and

alternating-current grounded through capacitor 53. The cathodes are connected to terminal 1 through cathode resistors 54 and 55. Input terminals are provided at terminals 4 and 5 while outputs are provided at terminals 6 and 7.

A typical amplifier, as illustrated in FIG. 6, may comprise in one envelope a pentode 56 and a triode 57. The triode plate is connected to plug-in terminal 1 while the pentode plate is connected to this terminal through resistors 58 and 59 and bypassed to ground for alternating current through capacitor 60. Two types of amplifiers are used and designated as Type I and Type II, differing solely in the value of capacitor 60 which may be on the order of 5 micromicrofarads for Type I and 82 micromicrofarads for Type II. The pentode cathode is directly connected to plug-in terminal 2 while the triode cathode is connected to this terminal through resistor 61. Screen grid supply is provided for the pentode at terminal 5 while the amplifier input is provided at terminal 4. The pentode output at its plate is applied via resistor 62 to the grid of the triode 57 which is connected as a cathode follower. The triode cathode is connected through a pair of voltage regulating gas tubes 62, 63 and capacitor 64 and resistor 65 to plug-in terminal 6 while the amplifier output appears at terminal 7 which is connected to the junction of the gas tubes and resistor 65.

As illustrated in FIG. 4, the input to the compensating apparatus from the output of the transducer or other dynamic system to be compensated is applied at input terminal 70 across grounded resistor 71 through a precision biasing voltage source 72 and resistor 73 to input terminal 5 of the plug-in dual cathode follower 50 which is detailed in FIG. 5. One cathode follower output at terminal 7 thereof is applied through a variable resistance capacitance network 74 to input terminal 4 of a Type I amplifier 75 of which the details are shown in FIG. 6. Amplifier 75 has terminal 2 grounded, terminal 1 capacitively bypassed to ground and resistance-coupled to a plate supply such as 250 volts, terminal 5 capacitively bypassed to ground and resistance-coupled to a screen grid supply such as +125 volts, and terminal 6 connected to a negative supply such as —75 volts. The output of amplifier 75 at terminal 7 thereof is fed back to its input for purposes of stability via resistance capacitance network 76. The networks 74 and 76 may be made of variable resistance and capacitance values as desired for purposes of effecting adjustment of the gain of the system.

The high frequency (ω_{n2}) compensation channel includes amplifiers 77, 78, 79 and 80. The output of amplifier 75 at terminal 7 is fed through resistor 81 to the input of amplifier 77 which is stabilized by a feedback resistor 82. The output of amplifier 77 which is of Type II is fed through resistor 83 and capacitor 84 to Type I amplifier 78 having a feedback resistor 85. This amplifier provides the first differentiation in the first channel. A variable feedback resistor 86 is provided in the feedback circuit of amplifier 78. The output of amplifier 78 is fed through a Type II amplifier 79 from whence it is fed through resistor 87 and capacitor 88 to Type I amplifier 80 which provide the second differentiation. Amplifier 80 has variable resistor 89 provided in its feedback circuit which introduces the multiplying constant A_2 either by itself or together with potentiometer 86 with which the potentiometer 89 may be ganged as illustrated. A series resistance capacitance circuit 90 may be resistance-coupled to the output of amplifier 80 and coupled to the output of amplifier 79 and made variable if so desired in order to control the range of input signals which the system can handle.

The output of the first differentiating amplifier 78 is fed through capacitor 91 to multiplying potentiometer 92 which introduces the multiplying constant B_2. The outputs of potentiometer 92 and of the second differentiating amplifier 80 are combined in a summing network including resistors 93 and 94 and fed to the input of a cathode follower 95 at the cathode of which appears the signal indicative of the quantity $B_2\dot{X}_2+A_2\ddot{X}_2$.

For the second comensating channel the output of amplifier 75, at terminal 7 thereof, is fed to an equalizing or summing network which includes portions of a cathode follower plug-in unit 96, portions of a cathode follower plug-in unit 99 and a selectively variable low-pass filter network 97. As illustrated in FIG. 4*a*, plug-in unit 96 includes a triode 32, having a control grid input terminal 6 and terminals 1 and 7 for respectively connecting the plate and cathode to a suitable source of supply. Plug-in unit 99, illustrated in FIG. 4*b*, similarly comprises a triode 33 having a control grid input terminal 6 and plug-in terminals 1 and 7 connected to a voltage supply source. Built into the units 96 and 99 are portions of the summing, or equalizing, network. In unit 96 the cathode output is applied to a variable resistor 34 of the parallel resistance capacitance network 30 having an output at plug-in terminal 8. Networks 30 also has a plug-in terminal 5 connected with plug-in terminal 5 of unit 99. In the latter, the cathode output is fed through the variable parallel resistance capacitance circuit 31 to the unit output terminal 5. The equalizing network comprising circuits 30 and 31 is adjusted to compensate for phase shift and signal level of the composite input to the second compensating channel. The first input to the equalizing network for the second channel is obtained at terminal 6 of plug-in unit 96. The second input to the composite (equalizing) network 30, 31 is obtained at terminal 6 of plug-in unit 99 from the output of the first channel at the junction of resistors 93 and 94. Thus, the signal X from terminal 7 of amplifier 75 is fed as one input to the equalizing network 30, 31 via triode 32 and the output of the first channel is fed as a second input to the equalizing network via triode 33. By this means the dynamic effect of the higher frequency component is removed in the equalizing network 30, 31 of which the output at terminal 8 of the unit 96 is applied to the low-pass filter 97.

The filter 97 comprises a plurality of resistors and capacitors, connected as illustrated, and a pair of separately operable single-pole, single-throw switches *s*–1 and *s*–2 which are utilized to provide adjustable frequency cutoff in three discrete steps depending upon the collective positions of the two stitches. This operation is provided for the purpose of attenuating the higher frequency inputs ot the second channel when ω_{n1} diminishes with respect to ω_{n2}. The output of low-pass filter 97 on lead 101 is applied to the second compensating channel comprising Type II amplifier 102, Type I amplifier 103, Type II amplifier 104, and Type I amplifier 105, all constructed and arranged as are the similar amplifiers 77, 78, 79 and 80 of the first channel. Plug-in terminals 1, 2, 5, and 6 of amplifier 79, 80, 102, 103, 104, and 105 are all connected as indicated in connection with amplifiers 77 and 78. The differentiating amplifiers 103 and 105 include the variable feedback resistors 106 and 107 respectively which may be ganged as illustrated to introduce the multiplying factor A_1. The multiplying factor B_1 is introduced by the potentiometer 108 which is capacitively coupled to the output of amplifier 103. The output of potentiometer 108 is combined with the output of amplifier 105 in a summing network comprising resistors 109 and 110 to provide a signal indicative of the quantity $B_1\dot{X}_1+A_1\ddot{X}_1$ which is fed to the input of a cathode follower 120 through the adjustable low pass filter and signal multiplier control 142.

The output summing network which combines the outputs of the two compensating channels with the transducer output includes a first adjustable resistance capacitance network 121 having an input from the output of the first channel (obtained at the cathode of cathode follower 95) and a second variable resistance capacitance network 122 having an input from the output of the second channel (obtained at the cathode of cathode follower 120). The third input to this final summing network is obtained

from resistor 123 which is coupled to the output of operational amplifier 75 via terminals 4 and 6 of the second section of the plug-in dual cathode follower unit 50. The output of this final summing network on lead 124 is fed across the variable resistor 125 to the input of a Type II amplifier 126 and thence through a precision voltage source 127 and resistor 128 to the input of a cathode follower 129 at the cathode of which appears the desired output

$$B_1\dot{X}_1+A_1\ddot{X}_1+B_2\dot{X}_2+A_2\ddot{X}_2+X$$

which is proportional to the transducer input $F(t)$.

Since it is also desirable to study the first derivatives of the two frequency components, there are provided additional output cathode followers 140 and 141 having inputs respectively connected to the output of the first differentiating amplifier 78 of the first channel and to the first differeniating amplifier 103 of the second channel.

The parallel resistance capacitance circuits 30, 31, 97, 121, and 122, are adjusted to effect tuning of the individual channels to the respective natural frequencies. Thus, circuits 121 and 31 will be adjusted for the higher natural frequency ω_{n2} and circuits 122 and 30 will be adjusted for the lower natural frequency ω_{n1}. The attenuation introduced by these circuits may also be varied as desired in order to adjust the magnitude of the equalization (at the output of network 97) or to adjust the proportions of the signals which are combined to obtain the final output.

The above-described embodiment of invention has been successfully operated over a frequency range of 1000 cycles per second to 100,000 cycles per second.

Equation 1 which is solved by the described embodiment of this invention, may be shown to define either of two physical configurations of dynamic systems illustrated in FIGS. 7 and 8 respectively. It has been found that many dynamic systems and most, if not all, commercially available transducer pickups of two degrees of freedom may be represented by one or the other of the mechanical systems schematically depicted in FIGS. 7 and 8. In FIG. 7 is illustrated a dynamic system comprised of two independent spring mass systems independently damped. The masses (inertia) are indicated by M_1 and M_2, the spring constants (compliance) by K_1 and K_2 and damping (friction) by C_1 and C_2, respectively. It is assumed that $K_1 \approx K_2$ which is the case in transducers. If P_1 and P_2 be constants giving the portion of $F(t)$ which is effective in exciting vibrations in M_1 and M_2 respectively the two equations of motion of the system of FIG. 7 can be written and added to yield

$$M_1\ddot{X}+M_2\ddot{X}_2+C_1\dot{X}_1+C_2\dot{X}_2+K_1X_1 +K_2X_2=P_1F(t)+P_2F(t) \quad (2)$$

where $X=X_1+X_2$. Since X decreases as K increases for a given excitation, it may be assumed that $P_3K_1X_1+K_2X_2$, where P_3 is a constant. Equation 2 may be then rewritten as

$$M_1\ddot{X}_1+M_2\ddot{X}_2+C_1\dot{X}_1+C_2\dot{X}_2+(1 +P_3)K_1X_1=(P_1+P_2)F(t) \quad (3)$$

The equations of motion for the system of FIG. 8 are

$$M_1\ddot{X}_1+C_1\dot{X}_1+K_1X_1+K_2(X_1-X_2)=0 \text{ or } F(t) \quad (4)$$

$$M_2\ddot{X}_2+C_2\dot{X}_2-K_2(X_1-X_2)=F(t) \text{ or } 0 \quad (5)$$

Adding Equations 4 and 5 yields

$$M_1\ddot{X}_1+M_2\ddot{X}_2+C_1\dot{X}_1+C_2\dot{X}_2+K_1X_1=F(t) \quad (6)$$

The equation solved by the described embodiment of the invention

$$A_1\ddot{X}_1+B_1\dot{X}_1+A_2\ddot{X}_2+B_2\dot{X}_2+X=\frac{f(t)}{K}=F(t) \quad (1)$$

may be written as

$$\left(\frac{M_1}{K_1}\ddot{X}_1\right)+\left(\frac{C_1}{K_1}\dot{X}_1\right)+\left(\frac{M_2}{K_2}\ddot{X}_2\right)+\frac{C_2}{K_2}\dot{X}_2\Big)+X=F(t) \quad (7)$$

where

$$A_1=\frac{M_1}{K_1}, B_1=\frac{C_1}{K_1}, A_2=\frac{M_2}{K_2}, B_2=\frac{C_2}{K_2}$$

Writing the relation between constans K_1 and K_2 as $K_2=P_4K_1$, where P_4 is a constant, Equation 7 may be written as

$$\left(\frac{M_1}{K_1}\ddot{X}_1\right)+\left(\frac{C_1}{K_1}\dot{X}\right)+\frac{1}{P_4}\left[\left(\frac{M_2}{K_1}\ddot{X}_2\right)+\left(\frac{C_2}{K_1}\dot{X}_2\right)\right]+X =F(t) \quad (8)$$

The factor

$$\frac{1}{P_4}$$

relates the amplitudes of the outputs of the first compensating channel to the output of the second channel and is introduced by relatively attenuating the inputs to the final summing circuit through suitable adjustment of variable network 142.

In order to show the equivalence of Equations 3 and 6 (which describe the configurations of FIGS. 7 and 8 respectively) with Equation 8, it may be assumed that X_1 is much greater than X_2 whereby $X_1+X_2 \cong X \cong X_1$. This assumption is a fairly close approximation in view of the fact that one component of the two degrees of freedom system normally has a much higher frequency and a much lower amplitude than the other component. This assumption does not imply that $\dot{X}_1$ is much greater than $\dot{X}_2$ nor that $\ddot{X}_1$ is much greater than $\ddot{X}_2$. The first and second derivatives of X are proportional to ω and ω^2 respectively. Therefore, since the angular frequency of the lower amplitude component is in most transducers normally much higher than the angular frequency of the higher amplitude component, $\dot{X}_2$ may be equal to or higher than $\dot{X}_1$. The same holds true for higher derivatives.

Thus, it will be seen that Equation 1 which is solved by the described embodiment of the invention accurately defines systems of two degrees of freedom such as illustrated in FIGS. 7 and 8 whereby the invention may be utilized for accurate compensation of any dynamic system which has a transfer function that can be characterized by the linear differential equation

$$[M\ddot{X}_n(t)+C\dot{X}_n(t)+X_n(t)]=F(t) \quad (9)$$

Although the invention has been described and illustrated in detail, it is to be clearly understood that the same is by way of illustration and example only and is not to be taken by way of limitation, the spirit and scope of this invention being limited only by the terms of the appended claims.

I claim:

1. A compenstor for use with a dynamic system having two degrees of freedom of respectively first and second natural frequencies and having an output, said system having a transfer function including first and second components corresponding to said first and second frequencies, comprising in combination, means for providing an input signal indicative of said output, a first compensating circuit adjusted to said first frequency and responsive to said input signal, equalizing means responsive to said input signal and said circuit for removing the dynamic effect of said first frequency, a second compensating circuit adjusted to said second frequency and responsive to said equalizing means, said first and second circuits each including circuit means for providing a transfer function which is the reciprocal of a respective one of said system

transfer function components, and a combining network responsive to said input signal and both said compensating circuits.

2. Compensating apparatus for a dynamic system having at least two degrees of freedom and two natural frequencies comprising an input terminal; a first channel tuned to one of said frequencies and comprising a first differentiator having an input from said terminal, a second differentiator having an input from said first differentiator, first and second multipliers respectively having inputs from said first and second differentiators, a first summing network having inputs from said multipliers; a second summing and equalizing network having inputs from said first summing network and said input terminal; a second channel tuned to a second one of said frequencies and comprising a third differentiator having an input from said second summing and equalizing network, a fourth differentiator having an input from said third differentiator, third and fourth multipliers respectively having inputs from said third and fourth differentiators, and a third summing network having inputs from said third and fourth multipliers; and a fourth summing network having inputs from said first and third summing networks and from said input terminal.

3. Compensating apparatus for a dynamic system having at least two degrees of freedom and two natural frequencies comprising an input terminal; a first channel tuned to one of said frequencies and comprising a first differentiator having an input from said terminal, a second differentiator having an input from said first differentiator, first and second multipliers respectively having inputs from said first and second differentiators, a first summing network having inputs from said multipliers; a second summing and equalizing network having inputs from said first summing network and said input terminal; a second channel tuned to a second one of said frequencies and comprising a third differentiator having an input from said second summing and equalizing network, a fourth differentiator having an input from said third differentiator, third and fourth multipliers respectively having inputs from said third and fourth differentiators, and a third summing network having inputs from said first summing network, from said third and fourth multipliers and from said input terminal.

4. Computing apparatus for a dynamic system having at least two degrees of freedom and two natural frequencies comprising an input terminal; a first channel tuned to one of said frequencies and comprising a first differentiating amplifier having a variable feedback resistor and having an input from said terminal, a second differentiating amplifier having a variable feedback resistor and having an input from said first differentiating amplifier, a first potentiometer having an input from said first differentiating amplifier, a first summing network having inputs from said potentiometer and said second amplifier; a second summing and equalizing network having inputs from said first summing network and said terminal; a second channel tuned to a second one of said frequencies and comprising a third differentiating amplifier having a variable feedback resistor and having an input from said second summing network, a fourth differentiating amplifier having a variable feedback resistor and having an input from said third differentiator, a second potentiometer having an input from said third differentiating amplifier, a third summing network having inputs from said second potentiometer and said fourth amplifier, and a fourth summing network having inputs from said first and third summing networks and from said terminal.

5. For use with a dynamic signal transfer system having two degrees of freedom and which produces an output distorted in accordance with each said degree of freedom, a compensator comprising in combination, first compensating means having an input responsive to said output for correcting for distortion of said output caused by said first degree of freedom, second compensating means for correcting for distortion of said output caused by said second degree of freedom, said first and second compensating means including circuit means having a transfer function which is the reciprocal of a respective one of the components of the transfer function of said system corresponding to said first and second degrees of freedom, and equalizing means responsive to said output and connected between said compensating means for removing the dynamic effect of said first degree of freedom from said second compensating means.

6. For use with a dynamic signal transfer system having two degrees of freedom and which produces an output distorted in accordance with first and second system transfer function components corresponding to each said degree of freedom, a compensator comprising in combination, first compensating means having an input responsive to said output for correcting for distortion of said output caused by said first degree of freedom, second compensating means for correcting for distortion of said output caused by said second degree of freedom, said first and second compensating means each including means for providing a transfer function which is the reciprocal of a respective one of said components, equalizing means connected to said output and between said first and second compensating means for removing the dynamic effect of said first degree of freedom from said second compensating means, and means for combining the outputs of said first and second compensating means in a predetermined proportion.

7. Apparatus for reproducing the true waveform of a transient phenomenon comprising a transducer adapted to sense said phenomenon, said transducer having a first damping ratio, inertia and compliance indicative of a first degree of freedom thereof and providing a first transducer transfer function component, and having a second damping ratio, inertia and compliance indicative of a second degree of freedom thereof and providing a second transducer transfer function component, a first variable gain compensating circuit having an input from said transducer, means for adjusting the gain of said circuit in proportion to said first damping ratio, inertia and compliance, a first summing and equalizing network having inputs from said transducer and said compensating circuit, a second variable gain compensating circuit having an input from said summing network, means for adjusting the gain of said second compensating circuit in proportion to said second damping ratio, inertia and compliance, each said compensating circuit including means for providing a transfer function which is the reciprocal of a respective one of said transducer transfer function components, and a second summing network having inputs from both said compensating circuits and said transducer.

8. Apparatus for reproducing the true waveforms of a transient phenomenon comprising a transducer adapted to sense said phenomenon, said transducer having a first damping ratio, inertia and compliance indicative of a first degree of freedom thereof and providing a first transducer transfer function component, and having a second damping ratio, inertia and compliance indicative of a second degree of freedom thereof and providing a second transducer transfer function component, a first tunable variable gain compensating circuit having an input from said transducer, means for adjusting the gain of said circuit in proportion to said first damping ratio, inertia and compliance, means for tuning said circuit to the natural frequency of said first degree of freedom, a first summing and equalizing network having inputs from said transducer and said compensating circuit, a second tunable variable gain compensating circuit having an input from said summing network, means for adjusting the gain of said second compensating circuit in proportion to said second damping ratio, inertia and compliance, means for tuning said second circuit to the natural frequency of said second degree of freedom, each said compensating circuit including means for providing a transfer function which is the reciprocal of a respective one of said transducer trans-

fer function components, and a second summing network having inputs from both said compensating circuits and said transducer.

9. Compensating apparatus for a dynamic system having at least two degrees of freedom and two natural frequencies comprising means for receiving an input signal; a first channel tuned to one of said frequencies and comprising means for producing first and second derivatives of said signal, first summing means for combining predetermined portions of said derivatives; second summing and equalizing means for combining said combined derivatives with said input signal to remove dynamic effects of said one frequency and produce a second signal; a second channel tuned to a second one of said frequencies and comprising means for producing first and second derivatives of said second signal, and means for combining predetermined portions of said last-mentioned derivatives with said input signal and with said portions of said first-mentioned derivatives.

10. Compensating apparatus for a dynamic system having at least two degrees of freedom and two natural frequencies comprising means for receiving an input signal; a first channel tuned to one of said frequencies and comprising first differentiating means for producing first and second derivatives of said signal, first summing means for combining predetermined portions of said derivatives; second summing and equalizing means for combining said

combined derivatives with said input signal to remove dynamic effects of said one frequency and produce a second signal; a second channel tuned to a second one of said frequencies and comprising second differentiating means for producing first and second derivatives of said second signal; means for tuning said first and second differentiating means to said natural frequencies respectively, and means for combining predetermined portions of said last-mentioned derivatives with said input signal and with said portions of said first-mentioned derivatives.

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UNITED STATES PATENT OFFICE
CERTIFICATE OF CORRECTION

Patent No. 3,073,524 January 15, 1963

John P. Ford

It is hereby certified that error appears in the above numbered patent requiring correction and that the said Letters Patent should read as corrected below.

Column 4, line 34, for "$B_2X_2+A_2X_2$" read -- $B_2\dot{X}_2+A_2\dot{X}_2$ --; line 35, for "$B_1\dot{X}_1+A_1\dot{X}_1$" read -- $B_1\dot{X}_1+A_1\ddot{X}_1$ --; line 43, for "$\dot{X}_3$" read -- $\ddot{X}_3$ --; line 47, for "$A_3\dot{X}_3$" read -- $A_3\ddot{X}_3$ --; column 6, line 3, for "comensating" read -- compensating --; line 19, for "Networks" read -- Network --; line 44, for "stitches" read -- switches --; line 46, for "ot" read -- to --; column 8 line 11, for "constans" read -- constants --; line 61, for "compenstor" read -- compensator --.

Signed and sealed this 22nd day of October 1963.

(SEAL)
Attest:

ERNEST W. SWIDER
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EDWIN L. REYNOLDS
Acting Commissioner of Patents