United States Patent [19]

Nagoshi

[11] 3,905,563

[45] Sept. 16, 1975

[54] **SYSTEM FOR CONTROLLING A MISSILE MOTION IN THE HOMING MODE**

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[73] Assignee: **Fuji Heavy Industries Ltd.**, Tokyo, Japan

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[21] Appl. No.: **354,386**

[30] **Foreign Application Priority Data**

Sept. 28, 1972 Japan 47-97345

[52] **U.S. Cl.** **244/3.15**

[51] **Int. Cl.** **F42b 15/02;** F41g 7/00

[58] **Field of Search** 244/3.15

[56] **References Cited**

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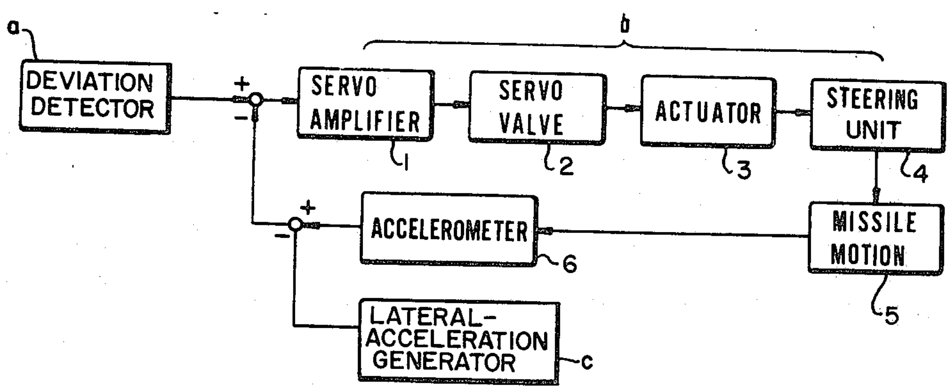
Assistant Examiner—C. T. Jordan
Attorney, Agent, or Firm—Ernest G. Montague; Karl F. Ross; Herbert Dubno

[57] **ABSTRACT**

A system for controlling a missile motion in the homing mode comprises means for designating by an error signal a direction in which a missile is guided in the homing mode, and means for compounding with the error signal a predetermined biasing signal imparting acceleration in one or two dimensions normal to a direction of the missile derivation. The biasing signal causes shells from anti-aircraft guns and anti-missile missiles to increase the miss-distance to the missile, within the allowable maneuverability of the missile, whereby the missile evades defensive action and finally zeroes in on a target.

8 Claims, 16 Drawing Figures



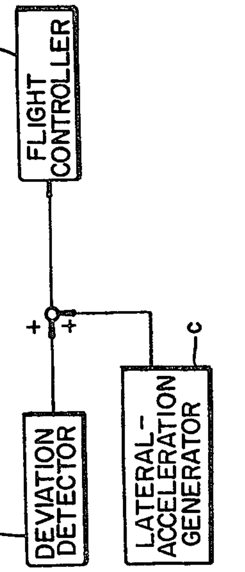


FIG. 1

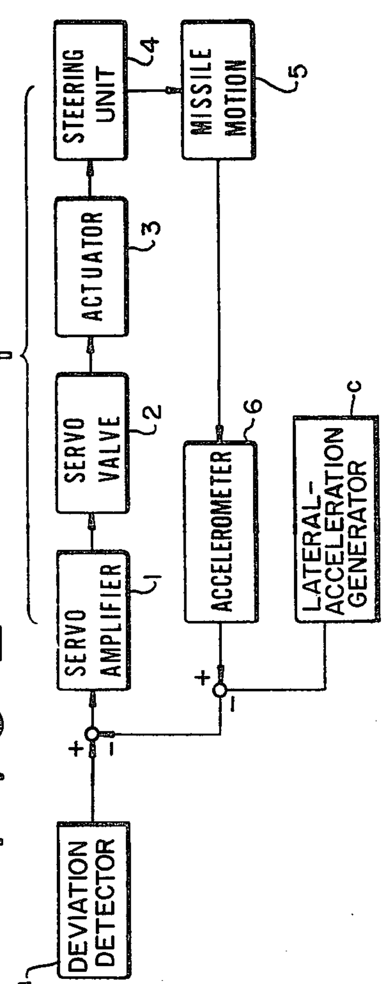


FIG. 2

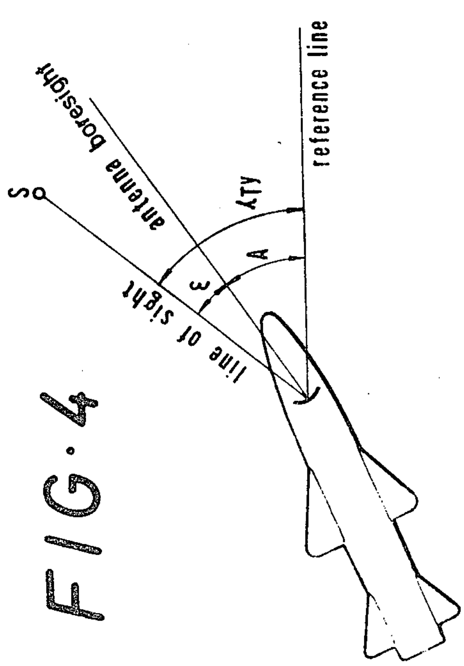


FIG. 4

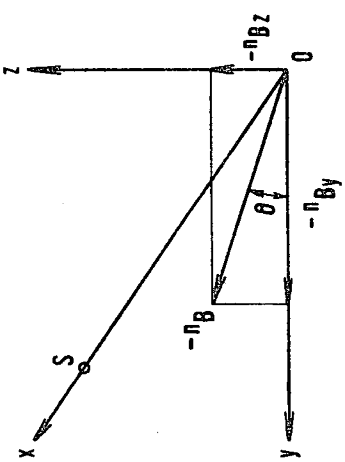


FIG. 6

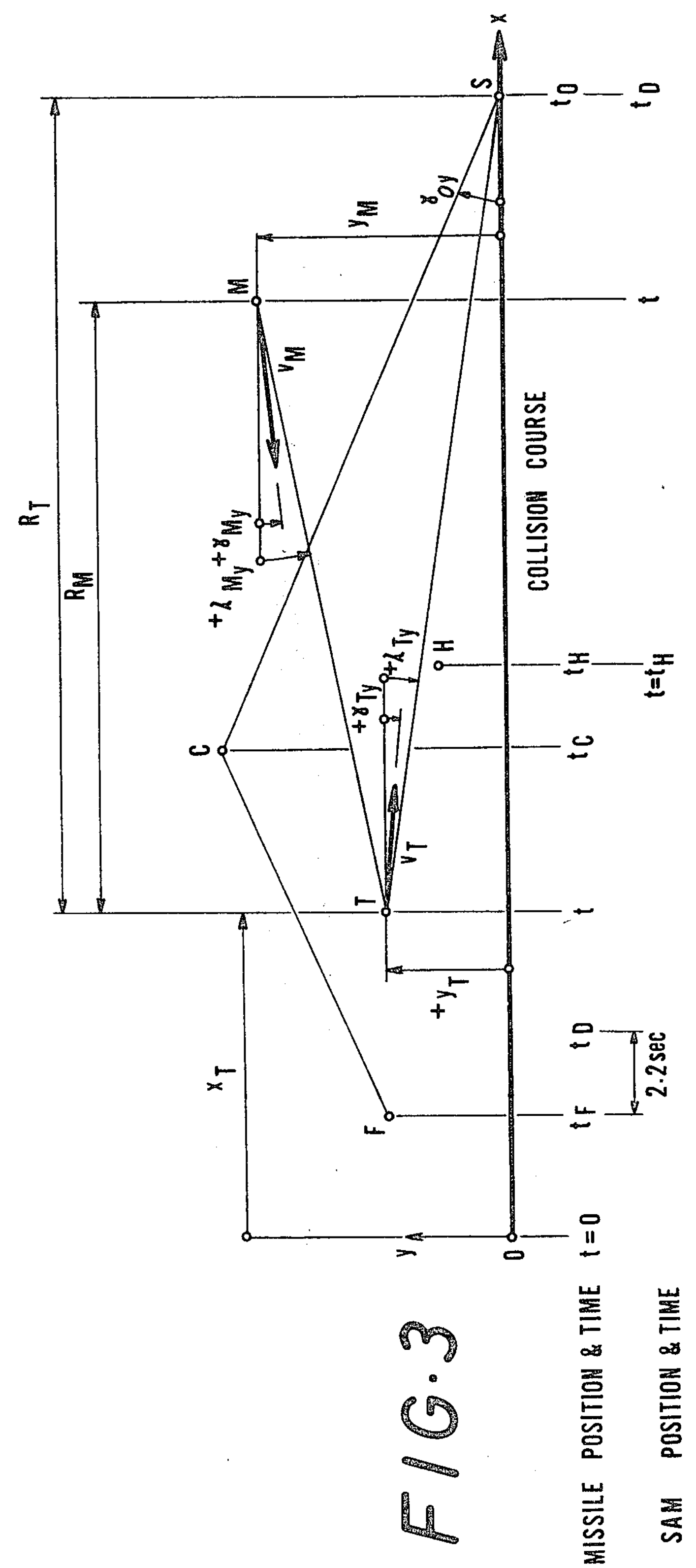

x
S
t_0
t_D
γ_{0y}
y_M
M
t
v_M
COLLISION COURSE
R_T
R_M
$+\lambda_{My} + \gamma_{My}$
H
$+\lambda_{Ty}$
$+\gamma_{Ty}$
t_H
$t=t_H$
C
t_C
v_T
T
t
$+y_T$
x_T
t_D
2.2sec
F
t_F
y
0
t=0
MISSILE POSITION & TIME
SAM POSITION & TIME

FIG. 3

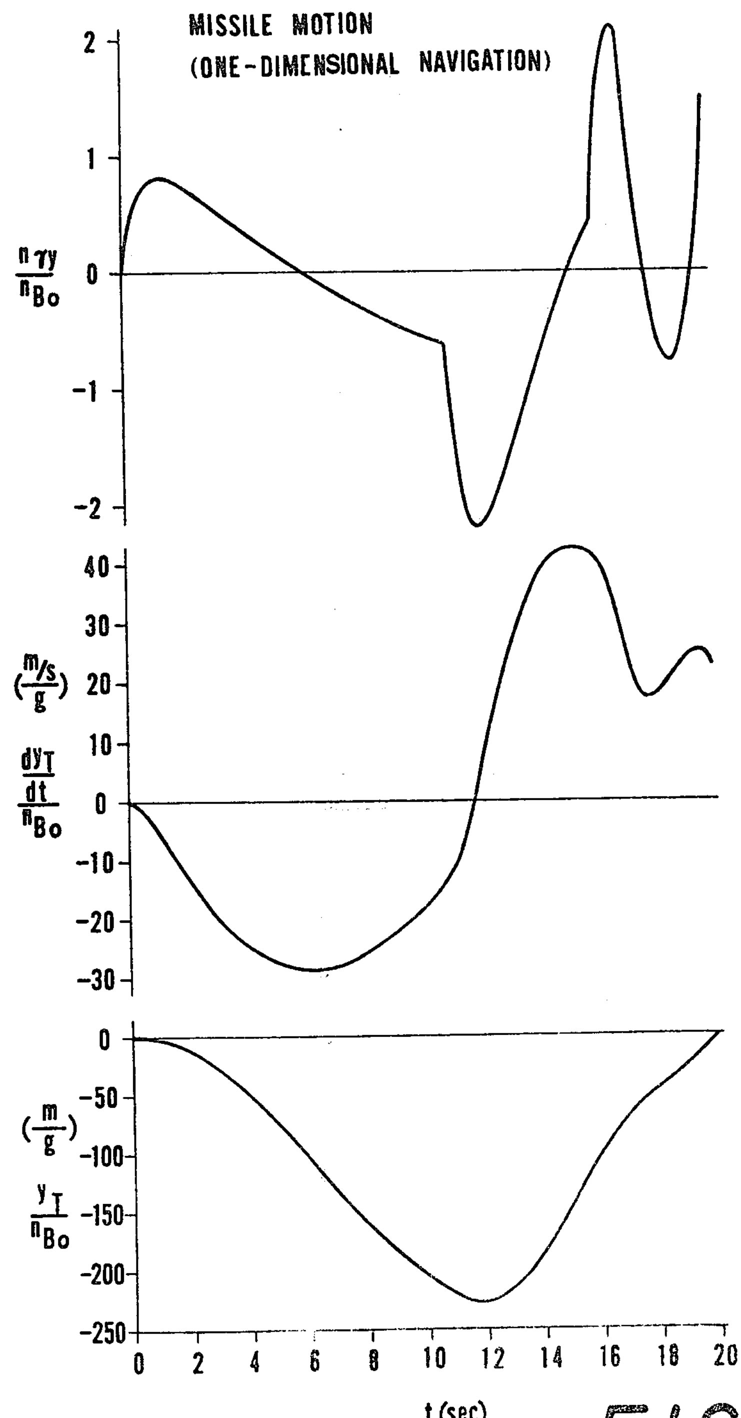

MISSILE MOTION
(ONE-DIMENSIONAL NAVIGATION)
$\frac{n_{\gamma y}}{n_{Bo}}$
$(\frac{m/s}{g})$
$\frac{dy_T/dt}{n_{Bo}}$
$(\frac{m}{g})$
$\frac{y_T}{n_{Bo}}$
t (sec)

FIG. 5

MISSILE FLIGHT PATH
(TWO-DIMENSIONAL NAVIGATION)

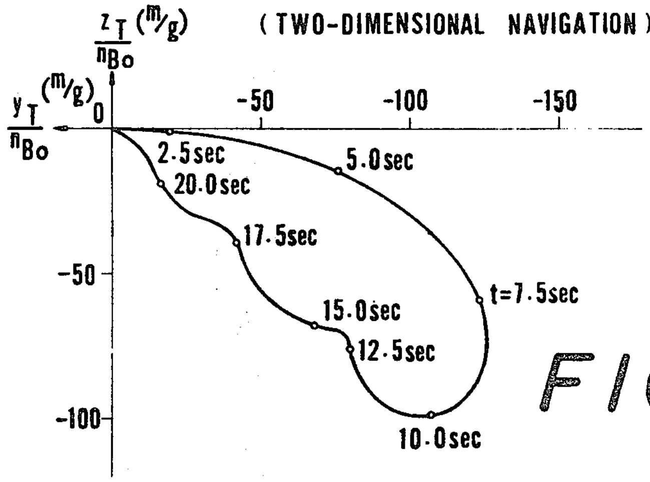


FIG. 7

MISSILE ACCELERATION
(TWO-DIMENSIONAL NAVIGATION)

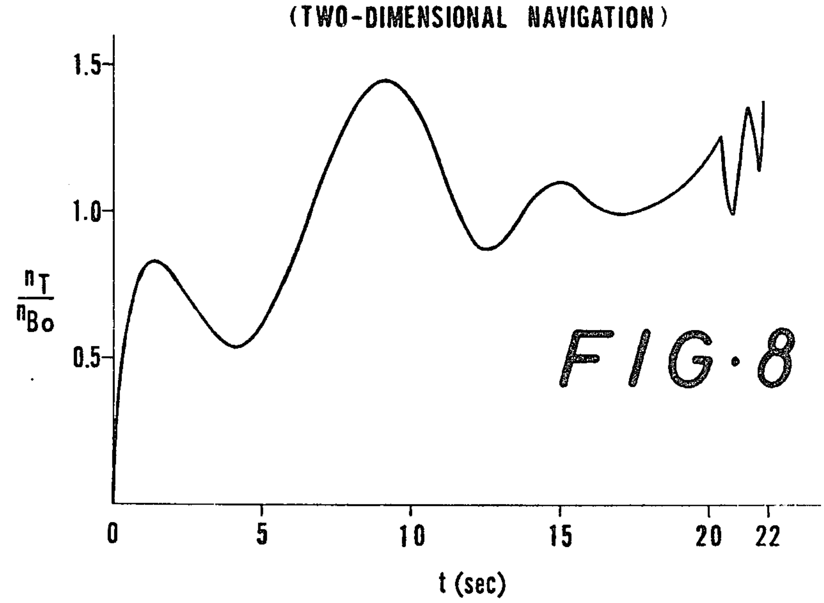


FIG. 8

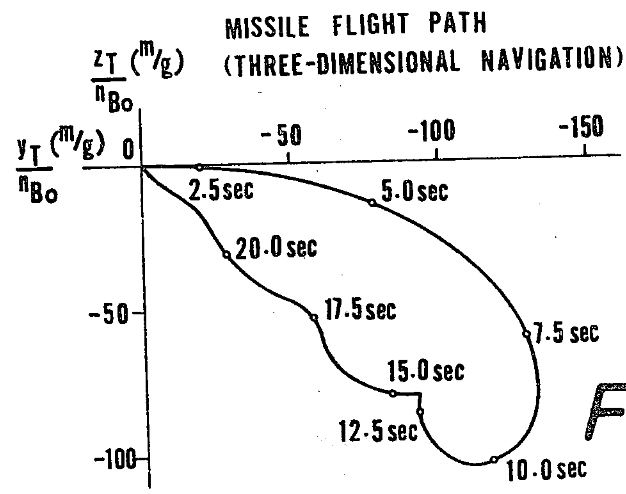

MISSILE FLIGHT PATH
(THREE-DIMENSIONAL NAVIGATION)
zT/nBo (m/g)
yT/nBo (m/g)
0
-50
-100
-150
-50
-100
2.5 sec
5.0 sec
7.5 sec
10.0 sec
12.5 sec
15.0 sec
17.5 sec
20.0 sec

FIG. 9

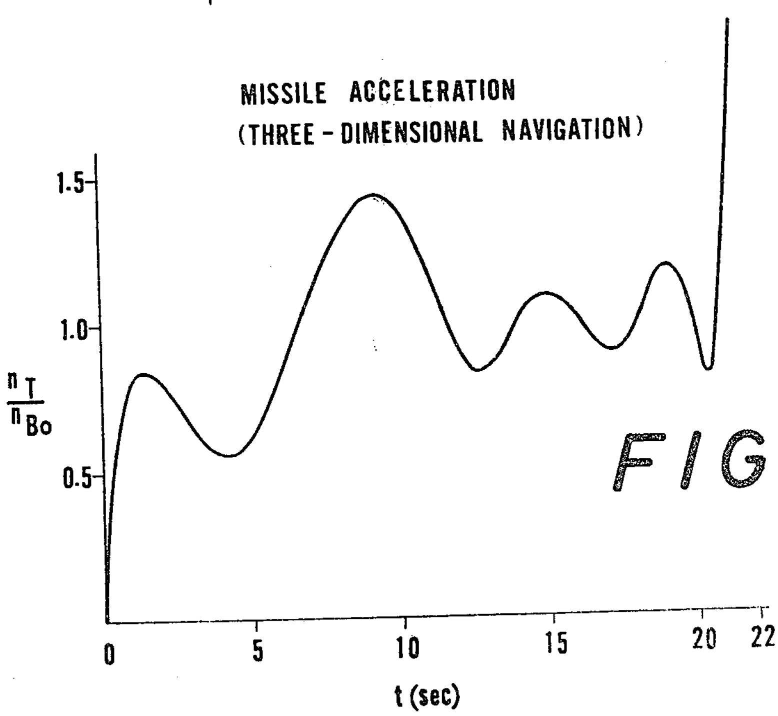

MISSILE ACCELERATION
(THREE-DIMENSIONAL NAVIGATION)
1.5
1.0
0.5
nT/nBo
0
5
10
15
20
22
t (sec)

FIG. 10

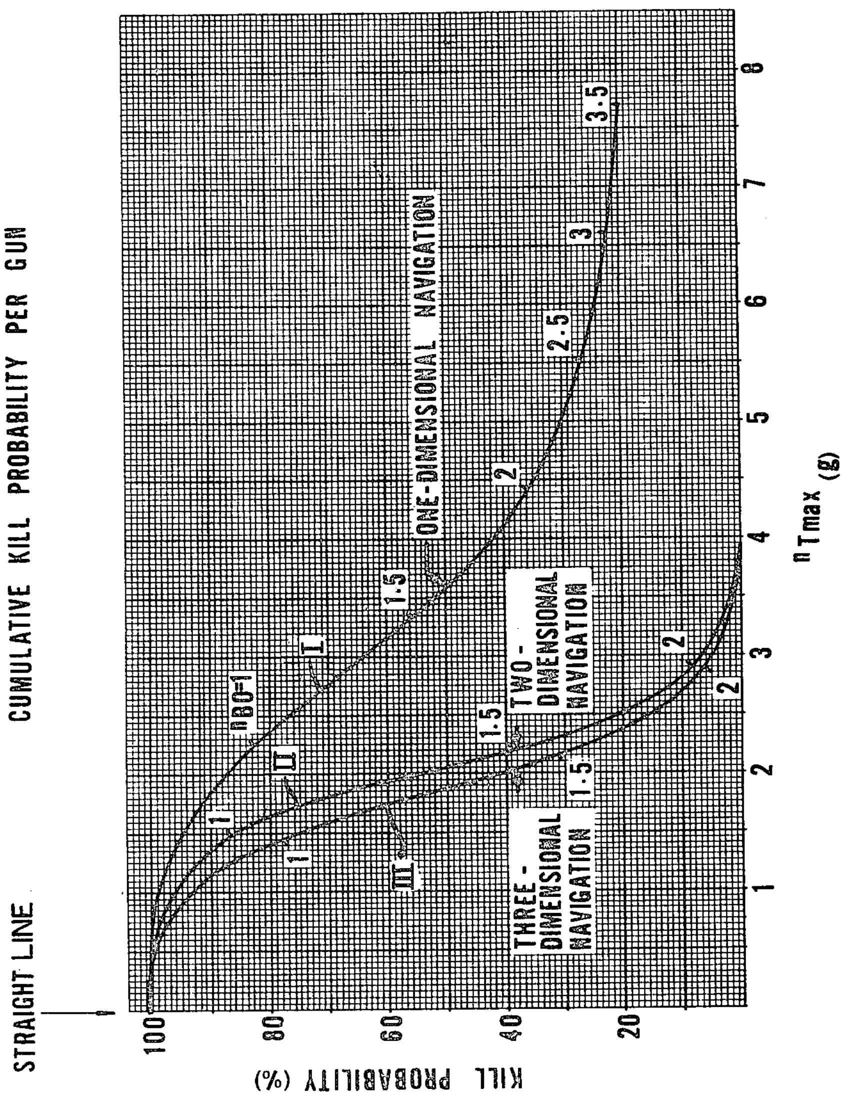

CUMULATIVE KILL PROBABILITY PER GUN
STRAIGHT LINE
KILL PROBABILITY (%)
100
80
60
40
20
1
2
3
4
5
6
7
8
$n_{T max}$ (g)
ONE-DIMENSIONAL NAVIGATION
TWO-DIMENSIONAL NAVIGATION
THREE-DIMENSIONAL NAVIGATION
n_{BO}=1
I
II
III
1.5
2
2.5
3
3.5

FIG. 11

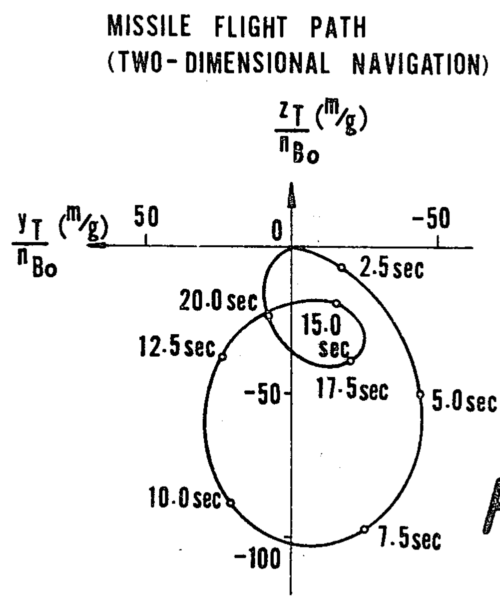

MISSILE FLIGHT PATH
(TWO-DIMENSIONAL NAVIGATION)
zT/nBo (m/g)
yT/nBo (m/g)
50
0
-50
2.5 sec
20.0 sec
15.0 sec
12.5 sec
-50
17.5 sec
5.0 sec
10.0 sec
7.5 sec
-100

FIG.12

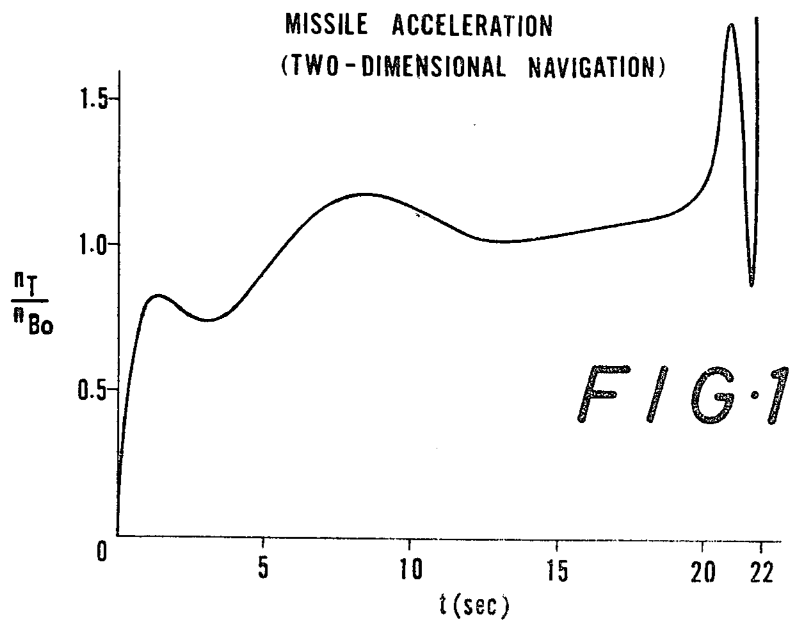

MISSILE ACCELERATION
(TWO-DIMENSIONAL NAVIGATION)
1.5
1.0
nT/nBo
0.5
0
5
10
15
20
22
t (sec)

FIG.13

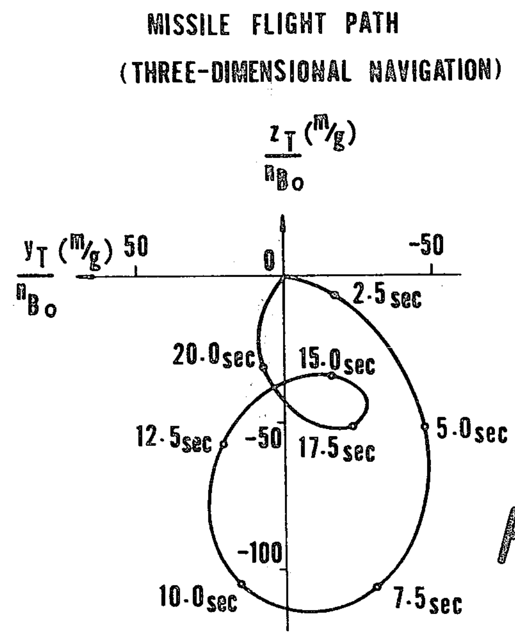

MISSILE FLIGHT PATH
(THREE-DIMENSIONAL NAVIGATION)
zT (m/g) / nBo
yT (m/g) / nBo
50
0
-50
2.5 sec
20.0 sec
15.0 sec
5.0 sec
12.5 sec
-50
17.5 sec
-100
10.0 sec
7.5 sec

FIG. 14

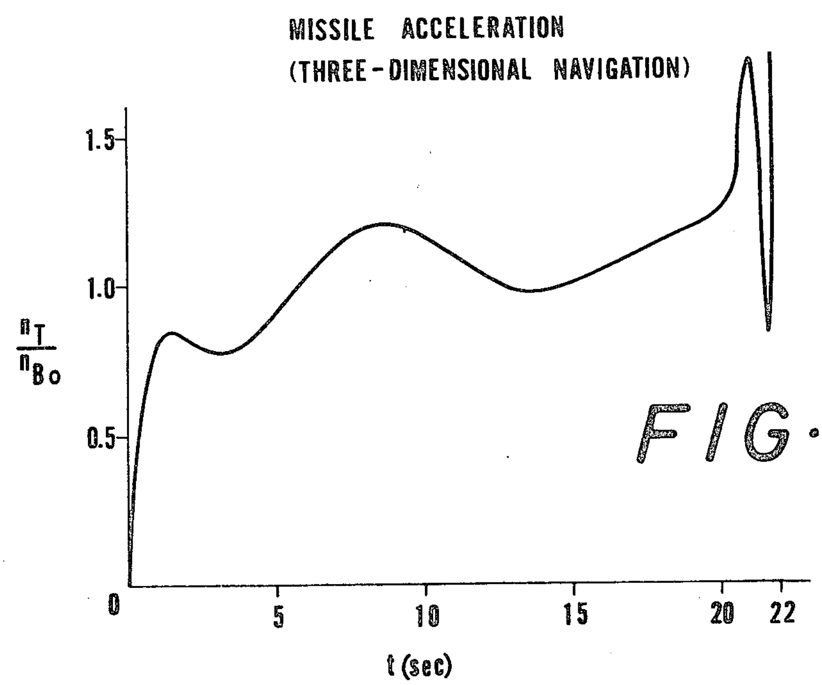

MISSILE ACCELERATION
(THREE-DIMENSIONAL NAVIGATION)
1.5
1.0
0.5
nT / nBo
0
5
10
15
20
22
t (sec)

FIG. 15

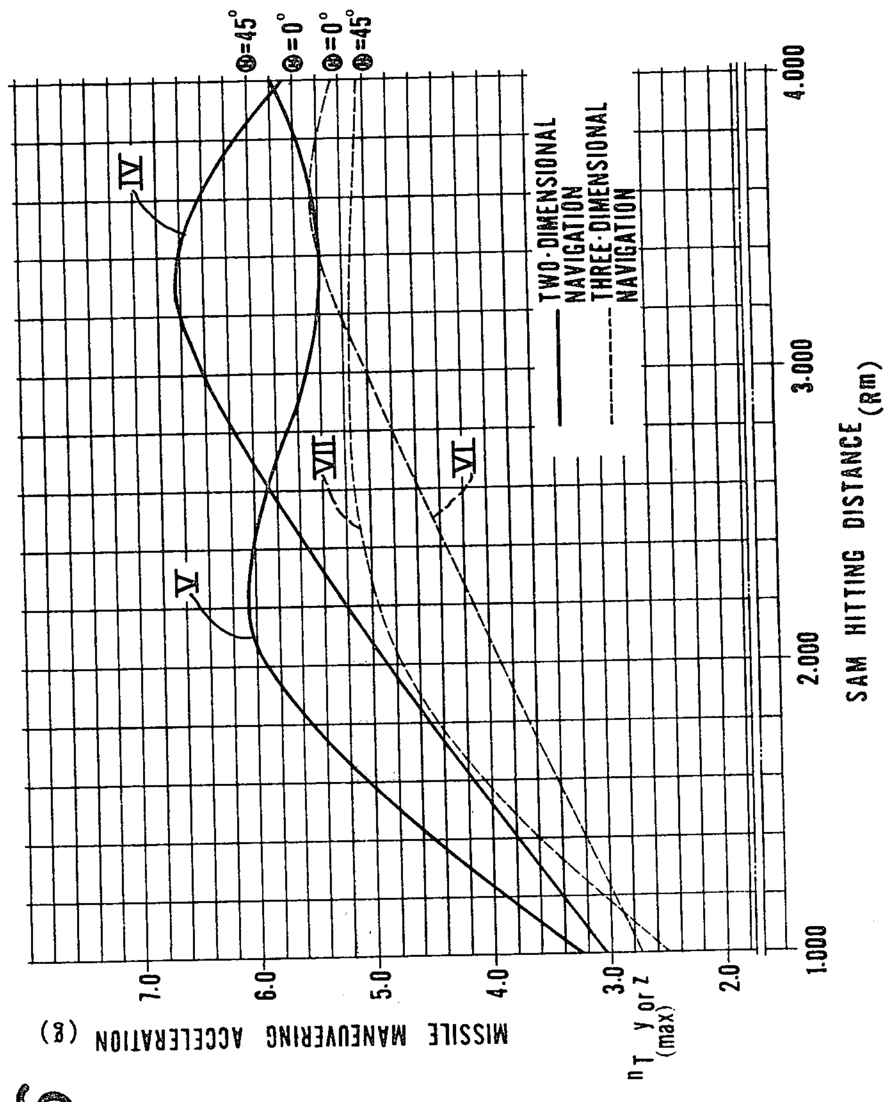

MISSILE MANEUVERING ACCELERATION (g)
n_{T} y or z (max)
7.0
6.0
5.0
4.0
3.0
2.0
1.000
2.000
3.000
4.000
SAM HITTING DISTANCE (Rm)
Θ=45°
Θ=0°
Θ=0°
Θ=45°
IV
V
VI
VII
TWO-DIMENSIONAL NAVIGATION
THREE-DIMENSIONAL NAVIGATION

FIG. 16

SYSTEM FOR CONTROLLING A MISSILE MOTION IN THE HOMING MODE

The present invention relates to a control system for guiding a missile in homing mode to a target, with the missile evading defensive action by anti-aircraft guns and anti-missile missiles.

A missile is generally controlled in the homing mode in a manner tending to reduce the error angle of the missile flight path to the target at the initial stage of derivation, thereby nearly maintaining a constant-bearing course. Consequently, the missile flight path straightens gradually (unless the motion of a target necessitates considerable maneuvering of the missle) so that an intercept computer makes an exact forecast of the future missile position enabling effective defense with anti-aircraft guns or anti-missile missiles.

Conventional missile-control systems impart additional acceleration and deceleration along the missile's course with the intention of reducing a kill probability of the anti-aircraft guns or anti-missile missiles. Nevertheless, such control is hardly available to evade defensive action under actual conditions where the enemy's weapons are often situated near the target. Since the effective intercepting time of shots of the anti-aircraft guns corresponds to the final period of the missile derivation in which the error angle to the target should be quickly eliminated, it is very difficult to evade such defensive action except by replacing the missile homing mode in the final period of derivation by a conventional programming mode that results in deflecting the missile from the target on account of the unavoidable accumulation of the missile divation error.

As object of the present invention is to provide means in such a system for so improving the control of missile navigation as to enable its effective evasion of defensive action all along the flight path.

I achieve this goal by adding a lateral acceleration bias in one or two dimensions to a error signal due to the missile deviation, with continued guidance of the missile flight toward the target for eventual collision therewith.

Thus, my present system includes aboard an in-flight homing missile such conventional components as sighting means for continuously determining the location of a line of sight from the missile to the target, detector means responsive to the sighting means for generating the aforementioned error signal as a function of a deviation of the missile flight path from the line of sight, steering means for changing the course of the missile, and control means therefor connected to the detector means for delivering to the steering means a corrective signal derived from the error signal. In accordance with the present improvement, I further provide biasing means for superimposing upon this corrective signal a lateral-acceleration signal effective in a direction perpendicular to an invariable reference direction generally parallel to the flight path, such as the axis (X) of a set of three orthogonal coordinate axes X, Y, Z. As described in another expression hereinafter, a differential signal (in which this lateral-acceleration signal is subtracted from the output of the feedback path involved in steering diminishes the error signal in the input of the control means so that its effect upon the steering means tends to change the existing flight path.

With two-dimensional navigation, two lateral-acceleration signals are generated which are effective in directions parallel to axes Y and Z. In this case, to achieve the desired flight-path changing effect, these two signals must of course be superimposed upon respective Y and Z components of the error signal.

The various features and advantages of the present invention will become more clearly apparent from the following description given with reference to the accompanying drawing in which:

FIG. **1** is a diagram illustrating in principle a control system embodying the present invention;

FIG. **2** is a diagram illustrating details of the control system;

FIG. **3** is a geometrically exaggerated diagram of the missile deviation;

FIG. **4** is a plot of angular parameters in a tracking system;

FIG. **5** is a set of graphs of the missile motion determined by one-dimensional bias in the direction of deviation;

FIG. **6** is a vector diagram of the acceleration bias for the missile maneuvering;

FIG. **7** is an orthogonal coordinate system indicating the missile deviation with two-dimensional bias;

FIG. **8** is a graph of the missile acceleration required for a bias according to FIG. **7**;

FIG. **9** is an orthogonal coordinate system indicating the missile deviation with three-dimensional bias;

FIG. **10** is a graph of the missile acceleration required for a bias according to FIG. **9**;

FIG. **11** is a graph of total kill probabilities in terms of lateral acceleration;

FIG. **12** is an orthogonal coordinate system indicating the missile deviation with two-dimension bias;

FIG. **13** is a graph of the missile acceleration required for a bias according to FIG. **12**;

FIG. **14** is an orthogonal coordinate system indicating the missile deviation with three-dimensional bias;

FIG. **15** is a graph of the missile acceleration required for a bias according to FIG. **14**; and

FIG. **16** is a graph of the missile maneuvering acceleration necessary to keep a definite miss-distance from a surface-to-air missile.

In a control system according to my present invention there is illustrated in block form in FIG. **1** and error signal supplied from a missile-deviation detector *a*, which signal is not applied immediately to a control device *b* but is compounded with a lateral-acceleration bias from a generator *c* so that the compound signal is applied to the control device *b* as a control signal.

FIG. **2** shows details of such a control system in which the control device *b* includes a servo amplifier **1**, a servo valve **2** and an actuator **3** for a steering unit **4**, serially arranged in a forward path, as well as a feedback path from steering unit **4** to the input of amplifier **1** comprising aerodynamic motion transfer function unit **5** working into an accelerometer **6**. Thus, a signal fed back from the output of the accelerometer **6** diminishes the bias signal from the output of generator *c* to produce a differential signal. A control signal is obtained by deducting the differential signal from the error signal supplied by the deviation detector *a*. Instead of the feedback signal from the output of the accelerometer, as shown, the feedback signal could come from the output of a rate gyro, or the deflection angle of a control surface could be utilized for the same purpose.

To point out the advantage of the missile navigation obtained by the superimposition of an acceleration bias in accordance with my invention, some test results will be reported hereinafter in cases with one- and two-dimensional acceleration bias as well as, in some instances, an acceleration bias in a third dimension, i.e. in the direction of the missile bearing course, added to the error signal in a control system designed to guide the missile to a stationary target by proportional navigation in the homing mode.

1. The Missile Motion

The X-axis of the missile coordinate system extends along a theoretical missile-collision course as shown in FIG. **3**, wherein the homing trajectory of the missile leads from a starting position O to a pinpoint S of the target. Furthermore, the missile position T at a time t is defined by coordinates x_T, Y_T.

Proportional navigation requires that the change of the missile flight direction always be directly proportional to the angular rate of change of a line of sight which leads from the missile to the target (with a proportionality factor N_T).

$$\frac{d\gamma_{Ty}}{dt} = N_T \frac{d\lambda_{Ty}}{dt} \qquad (1)$$

where

γ_{Ty} = instantaneous missile flight-path angle, relative to a fixed reference line coinciding with the theoretical collision course

λ_{Ty} = instantaneous line-of-sight angle, relative to the fixed reference line

The missile turning rate is

$$\frac{d\Xi_{Ty}}{dt} = \frac{N_{Ty} g}{V_T} \qquad (1')$$

where

n_{Ty} = missile lateral acceleration, i.e. the acceleration directed along the Y-axis of the missile coordinate system but with a negative sign

g = gravitational acceleration

V_T = missile velocity

Now substitute

$\frac{d\gamma_{Ty}}{dt}$ from Eq. (1') into Eq. (1).

$$N_{Ty} = \frac{N_T \cdot V_T}{G} \cdot \frac{d\lambda_{Ty}}{dt} \qquad (2)$$

The line-of sight rate ($d\lambda_{Ty}/dt$) is measured by a tracking device which may be a radar, an optical system, or an infrared system. The tracking device points its antenna in the direction of the line of sight to the target, as accurately and as quickly as possible.

The principle of the measurement of ($d\lambda_{Ty}/dt$) by means of the tracking device will be explained in connection with FIG. **4**.

λ_{Ty} = line-of-sight angle relative to the fixed reference line

A = boresight angle of a tracking antenna relative to the same fixed reference line (if the tracking were perfect, the antenna boresight would coincide with the line of sight.)

ϵ = error angle, i.e. the angle corresponding to the difference between A and λ_{Ty}. For a perfect tracking system having no time lag, $\epsilon = 0$.

In the actual physical case, the antenna lags behind the instantaneous line of sight by an amount which is determined by the tracking time constant τ_T, so that we have

$$A = \frac{\lambda_{Ty}}{1 + \tau_T p} \quad \left(\text{where } p = \frac{d}{dt}\right)$$

Since the error signal ϵ multiplied by a constant gain factor k equals the lateral acceleration n_{Ty}, it may be written as

$$n_{Ty} = K\epsilon = \frac{K\tau_T p}{1 + \tau_T p} \lambda_{Ty} \qquad (3)$$

The right-hand side of Eq. (2) is equal to that of Eq. (3) with the exception of the factor ($1/1 + \tau_T p$), so that we may write:

$$k\tau_T = \frac{N_T \cdot V_T}{g}$$

Substitute this into Eq. (3),

$$n_{Ty} = \frac{N_T \cdot V_T}{g} \cdot \frac{p}{1 + \tau_T p} \lambda_{Ty} \qquad (4)$$

Eq. (4) is well known as a trajectory equation of the missile flight path in the homing mode.

In the control system according to this invention, the lateral-acceleration bias is added to the aforementioned error signal so as to produce a control signal. The lateral acceleration n_{Ty} for maneuvering given by the control signal has a time lag determined by a time constant τ_T, with reference to the acceleration bias n_{By}, so that Eq. (4) may be rewritten as

$$n_{Ty} = \frac{N_T \cdot V_T}{g} \cdot \frac{p}{1 + \tau_T p} \lambda_{Ty} + \frac{n_{By}}{1 + \tau_T p} \qquad (5)$$

with $n_{Ty} g = -\frac{d^2 y_T}{dt^2}$

(where y_T is a distance normal to the fixed reference line, i.e. measured on the Y-axis coordinate)

Substitute this equation into Eq. (5):

$$\frac{d^2 y_T}{dt^2} = -N_T V_T \frac{p}{1 + \tau_T p} \lambda_{Ty} - \frac{g n_{By}}{1 + \tau_T p} \qquad (5')$$

Transpose the denominator of the right-hand side of Eq. (5') to the left-hand side:

$$\tau_T \frac{d^3 y_T}{dt^3} + \frac{d^2 y_T}{dt^2} = -N_T V_T \frac{d\tau_{Ty}}{dt} - g n_{By} \qquad (6)$$

The instantaneous line-of-sight angle λ_{Ty} is given by:

$$\lambda_{Ty} = \frac{Y_T}{R_T} \qquad \text{(referring to FIG. 3)}$$

where R_T = remaining distance of the missile flight path

Differentiate that with respect to time:

$$\frac{dr_{Ty}}{dt} = \frac{1}{R_T} \cdot \frac{dy_T}{dt} + \frac{V_T}{R_T^2} y_T$$

$$\left(\frac{dR_T}{dt} = - V_T \quad \text{from the illustration of FIG. 3} \right)$$

Consequently, Eq. (6) is rewritten as

$$\tau_T \frac{d^3 y_T}{dt^3} + \frac{d^2 y_T}{dt^2} + N_T \frac{V_T}{R_T} \cdot \frac{dy_T}{dt} + N_T \left(\frac{V_T}{R_T} \right)^2 y_T = -g n_{By} \tag{7}$$

In FIG. **6**, the collision course coincides with the X-axis $\overline{OS}$, the Y-axis and Z-axis are perpendicular to the X-axis, and the acceleration bias $-n_{Bz}$ follows the Z-axis, so that an equation for the missile motion along the Z-axis is

$$\tau_T \frac{d^3 z_T}{dt^3} + \frac{d^2 z_T}{dt^2} + N_T \frac{V_T}{R_T} \cdot \frac{dz_T}{dt} + N_T \left(\frac{V_T}{R_T} \right)^2 z_T = -g n_{Bz} \tag{8}$$

(where z_T is a distance normal to the fixed reference line, i.e., measured on the Z-axis coordinate)

2. The Missile Acceleration

The change of the missile velocity is defined with one- and two-dimensional acceleration as follows:

for $t = 0$ $\quad V_T = V_1$
for $t = t_o$ $\quad V_T = V_2$
(t_o being the total homing flight time)

1. In case of one-dimensional acceleration:

The missile velocity at a time t is determined by

$$V_T = V_1 + (V_2 - V_1) \frac{t}{t_o}$$

Therefore, the remaining distance of the missile flight path is

$$R_T = \int_t^{t_o} V_T dt = V_1 (t_o - t) + \frac{V_2 - V_1}{2t_o} (t_o^2 - t^2) \quad \ldots\ldots \quad (9)$$

Substitute $t = t_H$ in Eq. (9):

(t_H is the hitting time of a shot from a defensive weapon.)

$$t_H^2 + \frac{2V_1 t_o}{V_2 - V_1} \cdot t_H - \frac{(V_1 + V_2) t_o^2 - 2(R_T)_{t=tH}\, t_o}{V_2 - V_1} = 0 \tag{10}$$

The hitting time is obtained from Eq. (10) which includes the hitting range $(R_T)_{t=t_H}$ from the target.

2. In case of two-dimensional acceleration:

The missile velocity at a time t is determined by:

$$V_T = V_1 + (V_2 - V_1) \left(\frac{t}{t_o} \right)^2 \tag{11}$$

Therefore, the remaining distance of the missile flight path is

$$R_T = \int_t^{t_o} V_T dt = V_1 (t_o - t) + \frac{V_2 - V_1}{3t_o^2} (t_o^3 - t^3) \quad \ldots\ldots \quad (12)$$

Substitute $t = t_H$ in Eq. (11):

$$t_H^3 + \frac{3V_1 t_o^2}{V_2 - V_1} t_H - \frac{(2V_1 + V_2) t_o^3 - 3(R_T)_{t=tH} \cdot t_o^2}{V_2 - V_1} = 0 \tag{13}$$

The hitting time is obtained from Eq. (13) which again includes the hitting range $(R_T)_{t=t}$ from the target.

3. Determination of Position and Motion Data of the Missile

From information obtained by a defending FCS (Fire-Control System), the missile position and time indicated in FIG. **3** by reference characters F and t_F are determined as follows:

1. Anti-aircraft (Guns

The predicted time t_F, coinciding with the instant when a shell fired at S has covered a distance $(R_T)_{t=t_H}$, may be written as

$$t_F = t_H - 0.5\ (\text{sec}) - \frac{(R_T)_{t=t_H}}{V_s} \tag{14}$$

(V_s = average velocity of the shell)

but a delay from the calculation of the missile position and motion data to the firing of the shell amounts to 0.5 second in practice.

2. SAM (Surface-to-air missile)

$$t_F = t_H - 2.2\ (\text{sec}) - \frac{(R_T)_{t=t_H}}{V_m} \tag{15}$$

but, supposing that the SAM takes an acceleration of 20.4g until it reaches a velocity V_M = 2 mach (680m/s), a time required for acceleration is 680/**20.4g** = 3.4 (sec). The SAM velocity may be assumed as 2 mach by subtracting a dead time from the time required for acceleration, this dead time being 3.4 sec × ½ = 1.7 sec. Eq. (15) employs a time of 2.2 sec as mentioned above, which is given by the delay occurring between the calculation of the missile data and the SAM firing, added to the dead time, i.e.

$$0.5 + 1.7 = 2.2\ (\text{sec})$$

3. Determination of the Future Position of the Missile with Two-Dimensional Acceleration

The FCS predicts the future position of the missile pointed at a location C in FIG. **3.** Since the calculation of the FCS is based on position, velocity and acceleration data of the missile, the calculated time t_C does not coincide with the hitting time t_H (though the former coincides with the latter with one-dimensional acceleration).

The remaining distance for homing is calculated by FCS as

$$(R_T)_{t=t_C} = (R_T)_{t=t_F} - (V_T)_{t=t_F} \times (t_C - t_F) - \left(\frac{dV_T}{dt}\right)_{t=t_F} \times \frac{(t_C - t_F)^2}{2} \quad (16)$$

On the other hand, substitute $t=t_F$ into Eqs. (**12**) and (**11**).

$$(R_T)_{t=t} = V_1(t_o - t_F) + \frac{V_2 - V_1}{3t_o^2}(t_o^3 - t_F^3)$$

$$(V_T)_{t=t} = V_1 + \frac{V_2 - V_1}{t_o^2} t_F^2$$

and substitute $t=t_F$ into the Eq. (**11**) differentiated with respect to t:

$$\left(\frac{dV_T}{dt}\right)_{t=t_F} = \frac{2(V_2 - V_1)}{t_o^2} t_F$$

These equations together give us the following equation:

$$(R_T)_{t=t_C} = V_1(t_o - t_F) + \frac{V_2 - V_1}{3t_o^2}(t_o^3 - t_F^3) - \left(V_1 + \frac{V_2 - V_1}{t_o^2} t_F^2\right)(t_C - t_F) - \frac{V_2 - V_1}{t_o^2} t_F (t_C - t_F)^2 \quad (17)$$

1. Anti-aircraft Guns

A predicated hitting time of the shell is defined as in Eq. (14):

$$t_C = t_F + 0.5\ (\text{sec}) + \frac{(R_T)_{t=t_C}}{V_s} \quad (18)$$

2. Sam

A predicted hitting time of the SAM is defined as in Eq. (15).

$$t_C = t_F + 2.2\ (\text{sec}) + \frac{(R_T T)_{t=t_C}}{V_M} \quad (19)$$

The calculated time t_C is determined by solving Eqs. (17) and (18) for anti-aircraft guns and by solving Eqs. (17) and (19) for the SAM, in using the technique of successive approximation.

5. The Miss-Distance of a Missile-Intercepting Shell

On the assumption that the impact of the shell on the missile occurs when the shell and the missile simultaneously arrive at a point H whose projection on the X-axis represents the hitting time t_H, the miss-distance m may be written as

$$m = \sqrt{m_Y^2 + m_Z^2} \quad (20)$$

where

m_Y = component in the Y-axis direction

m_Z 32 component in the Z-axis direction

1. One-dimensional Acceleration

The Y-axis component is

$$m_Y = (y_T)_{t=t_H} - \left\{(y_T)_{t=t_F} + \left(\frac{dy_T}{dt}\right)_{t=t_F} \times (t_H - t_F) + \left(\frac{d^2y_T}{dt^2}\right)_{t=t_H} \times \frac{(t_H - t_F)^2}{2}\right\} \quad (21)$$

Similarly, the Z-axis component is

$$m_Z = (z_t)_{t=t_H} - \left\{(z_T)_{t=t_F} + \left(\frac{dz_T}{dt}\right)_{t=t_F} \times (t_H - t_F) + \left(\frac{d^2z_T}{dt^2}\right)_{t=t_H} \times \frac{(t_H - t_F)^2}{2}\right\} \quad (22)$$

The miss-distance will be obtained by substituting Eqs. (21) and (22) into Eq. (20).

2. Two-dimensional Acceleration

The displacement of the missile along the Y-axis predicted by FCS is

$$(y_T)_{t=t_C} = (y_T)_{t=t_F} + \left(\frac{dy_T}{dt}\right)_{t=t_F} \times (t_C - t_F) + \left(\frac{d^2y_T}{dt^2}\right)_{t=t_F} \times \frac{(t_C - t_F)^2}{2} \quad (23)$$

Supposing the flight path of the shell to be straight, the displacement of the shell along the Y-axis at the hitting time t_H is

$$(y_T)_{t=t_C} \times \frac{(R_T)_{t=t_H}}{(R_T)_{t=t_C}}$$

whence the miss-distance along the Y-axis may be written as

$$m_Y = (y_T)_{t=t_H} - (y_T)_{t=t_C} \times \frac{(R_T)_{t=t_H}}{(R_T)_{t=t_C}} \quad (24)$$

the missile displacement along the Z-axis is

$$(z_T)_{t=t_C} = (z_T)_{t=t_F} + \left(\frac{dz_T}{dt}\right)_{t=t_F} \times (t_C - t_F) + \left(\frac{d^2z_T}{dt^2}\right)_{t=t_F} \times \frac{(t_C - t_F)^2}{2} \quad (25)$$

The miss-distance along the Z-axis may be written as

$$m_z = (z_T)_{t=t_H} - (z_T)_{t=t_C} \times \frac{(R_T)_{t=t_H}}{(R_T)_{t=t_C}} \quad (26)$$

The miss-distance will be obtained by substituting Eqs. (24) and (26) in Eq. (20).

6. The SAM Navigation

The FCS predicts the factors of the missile in order to count the future position C of the missile aimed at by the SAM when the missile reaches a point F on the missile bearing course. The SAM starts along a collision course to the left of time t_D of 2.2 (sec) after the FCS prediction, this collision course being illustrated by the straight line SO in FIG. **3**.

On the assumption that the SAM is derived by proportional navigation (with a proportionality factor N_M), the lateral acceleration n_{Mu} of the SAM in the Y-axis direction is obtained as in Eq. (4):

$$n_{Mu} = \frac{N_M V_M}{g} \cdot \frac{p}{1 + \tau_M p} \lambda_{Mu} \tag{27}$$

where
- V_M = average velocity of the SAM
- τ_M = time constant of the SAM
- λ_{Mu} = aiming angle of the SAM relative to the reference line

(the lateral acceleration n_{Mu} having a negative sign)

$$\text{with } n_{Mu}\, g = -\frac{d^2 y_M}{dt^2}$$

(Y_M = displacement of the SAM in the Y-axis direction)

Substitute the foregoing equation into Eq. (27):

$$\frac{d^2 y_M}{dt^2} = -N_M V_M \frac{p}{1 + \tau_M p} \lambda_{Mu}$$

Transpose the denominator of the right-hand side of this equation to the left-hand side.

$$\tau_M \frac{d^3 y_M}{dt^3} + \frac{d^2 y_M}{dt^2} = -N_M V_M \frac{d\lambda_{Mu}}{dt} \tag{28}$$

Integrate both sides of Eq. (28):

$$\tau_M \frac{d^2 y_M}{dt^2} + \frac{d y_M}{dt} = -N_M V_M \lambda_{Mu} + C \tag{29}$$

where C is the constant of integration.

Insert the initial limiting conditions for $t = t_D$ into Eq. (29) for defining the constant:

$$C = \tau_M \left(\frac{d^2 y_M}{dt^2}\right)_{t=t_D} + \left(\frac{d y_M}{dt}\right)_{t=t_D} + N_M V_M (\lambda_{Mu})_{t=t_D}$$

(with $t_D = t_F + 2.2$ sec)

The illustration of FIG. 3 yields

$$\lambda_{Mu} = \frac{y_M - y_T}{R_M}$$

where R_M = remaining distance of the SAM homing flight path.

Substitute this into Eq. (29):

$$\tau_M \frac{d^2 y_M}{dt^2} + \frac{d y_M}{dt} + \frac{N_M V_M}{R_M} y_M = \frac{N_M V_M}{R_M} y_T + \tau_M \left(\frac{d^2 y_M}{dt^2}\right)_{t=t_D} + \left(\frac{d y_M}{dt}\right)_{t=t_D} + N_M V_M (\lambda_{Mu})_{t=t_D} \tag{30}$$

wherein, for the firing condition of the SAM,

$$\left(\frac{d^2 y_M}{dt^2}\right)_{t=t} = 0, \quad \left(\frac{d y_M}{dt}\right)_{t=t} = V_M \gamma_{ou},$$

$$(\lambda_{Mu})_{t=t} = -\frac{(y_T)_{t=t}}{(R_M)_{t=t}}$$

(where γ_{ou} = angle of the SAM in the XY-plane, when aiming at the future position C of the missile from the point S, relative to the fixed reference line).

Substitute these values into Eq. (30):

$$\tau_M \frac{d^2 y_M}{dt^2} + \frac{d y_M}{dt} + \frac{N_M V_M}{R_M} y_M = \frac{N_M V_M}{R_M} y_T + V_M \gamma_{ou} - N_M V_M \frac{(y_T)_{t=t_D}}{(R_M)_{t=t_D}} \tag{31}$$

FIG. 3 indicates that

$$\gamma_{ou} = \frac{(y_T)_{t=t_F} + \left(\frac{d y_T}{dt}\right)_{t=t_F} \times (t_C - t_F) + \left(\frac{d^2 y_T}{dt^2}\right)_{t=t_F} \times \frac{(t_C - t_F)^2}{2}}{V_M (t_C - t_D)} \tag{32}$$

Equations for the SAM are also derived in connection with the component in the direction of the Z-axis, i.e.:

$$\tau_M \frac{d^2 z_M}{dt^2} + \frac{d z_M}{dt} + \frac{N_M V_M}{R_M} z_M = \frac{N_M V_M}{R_M} z_T + V_M \gamma_{oz} - N_M V_M \frac{(z_T)_{t=t_D}}{(R_M)_{t=t_D}} \tag{33}$$

$$(z_T)_{t=t_F} \; \frac{(z_T)_{t=t_F} + \left(\frac{d z_T}{dt}\right)_{t=t_F} \times (t_C - t_F) + \left(\frac{d^2 z_T}{dt^2}\right)_{t=t_F} \times \frac{(t_C - t_F)^2}{2}}{V_M (t_C - t_D)} \tag{34}$$

where
- z_M = displacement of the SAM in the Z-axis direction,
- γ oz = angle of the SAM in the XZ-plane, when aiming at the future position C of the missile, from the

point S relative to the fixed reference line.

The next equations are derived from the illustration of FIG. 3.

$$t_D = t_H - \frac{(R_T)_{t=t_H}}{V_M} \qquad (35)$$

$$R_M = V_M(t_H - t) + R_T - (R_T)_{t=t_H} \qquad (36)$$

1. One-dimensional Accelerating

Substitute Eq. (9) into Eq. (36):

$$R_M = (V_1+V_M)(t_H-t) + \frac{V_2-V_1}{2t_o}(t_H^2-t^2) \qquad (37)$$

2. Two-dimensional Acceleration

Substitute Eq. (12) into Eq. (36):

$$R_M = (V_1+V_M)(t_H-t) + \frac{V_2-V_1}{3t_o^2}(t_H^3-t^3) \qquad (38)$$

7. The Miss-Distance of the "SAM" With Reference to the Missile

The miss-distance of the SAM to the missle is based on the unavoidable fact that as the calculated maneuvering acceleration of the SAM attains and surpasses the g-limit, the actual maneuvering acceleration reaches its saturation value.

Accordingly, the evaluation of the components of the SAM miss-distance in the Y-axis and Z-axis directions must be executed with the addition of factors of g-saturation and desaturation. As the final period of the SAM derivation involves the fixed g-saturation, the following equations are established:

$$m_Y = (y_T)_{t=t_H} - \left\{ (y_M)_{t=t_{sy}} + \left(\frac{dy_M}{dt}\right)_{t=t_{sy}} \times (t_H - t_{sy}) + \left(\frac{d^2y_M}{dt^2}\right)_{t=t_{sy}} \times \frac{(t_H-t_{sy})^2}{2} \right\} \qquad (39)$$

$$m_z = (z_T)_{t=t_H} - \left\{ (z_M)_{t=t_{sz}} + \left(\frac{dz_M}{dt}\right)_{t=t_{sz}} \times (t_H - t_{sz}) + \left(\frac{d^2z_M}{dt^2}\right)_{t=t_{sz}} \times \frac{(t_H-t_{sz})^2}{2} \right\} \qquad (40)$$

The miss-distance will be obtained by substituting m_Y and m_Z of Eqs. (39) and (40) into Eq. (20).

According to the foregoing, the SAM miss-distance is vectorially defined by Y-axis and Z-axis coordinates. Equations (31) and (33) are available to calculated the miss-distance in the event that the bearing courses of the missile and SAM are based on the same coordinate system; otherwise, the value of Y_T and z_T must be replaced by others obtained by coordinate conversion.

The preceding description explains the way of determining the miss-distance with three-dimensional deviation.

MISSILE NAVIGATION WITH TWO-DIMENSIONAL MANEUVERING

Since the velocity V_T is constant, Eq. (6) for the missile motion may be rewritten as follows by integrating its two sides:

$$\tau_T \frac{d^2y_T}{dt^2} + \frac{dy_T}{dt} = -N_T V_T \lambda_{Ty} - g\int_0^t n_{By}\,dt + c \;\ldots\ldots\; (41)$$

The integration constant C is defined by

$$\frac{d^2y_T}{dt^2} = \frac{dy_T}{dt} = \lambda_{Ty} = O,$$

going to zero in the limiting case t=0, with

$$\lambda_{Ty} = \frac{y_T}{R_T} = \frac{y_T}{V_t(t_o-t)}$$

Substitute this equation into Eq. (41):

$$\tau_T \frac{d^2y_T}{dt^2} + \frac{dy_T}{dt} + \frac{N_T}{t_o-t} y_T = -g\int_0^t n_{By}\,dt \;\ldots\ldots\ldots\; (42)$$

Simultaneously, the component in the Z-axis direction is given by:

$$\tau_T \frac{d^2z_T}{dt^2} + \frac{dz_T}{dt} + \frac{N_T}{t_o-t} z_T = -g\int_0^t n_{Bz}\,dt \;\ldots\ldots\ldots\; (43)$$

As will be apparent from FIG. 3, the hitting time is

$$t_H = t_o - \frac{(R_T)_{t=t_H}}{V_T} \qquad (44)$$

In order to determined the predicted time t_F, Eq. (14) can be used with anti-aircraft guns, and Eq. (15) with the SAM.

The miss-distance of the shell is derived from Eqs. (20), (21) and (22), and the SAM motion from the application of Eqs. (31), (32), (33), (34) and (35) as well as FIG. 3 indicates that

$$R_M = (V_M + V_T)(t_H - t) \qquad (45)$$

The miss-distance of the SAM with reference to the missile is derived from the application of Eqs. (20), (39) and (40).

MISSILE NAVIGATION WITH ONE-DIMENSIONAL MANEUVERING

When the missile is navigated in sea-skimming flight, its interception by the SAM is difficult so that it is only necessary to consider the use of anti-aircraft guns.

In this case, the missile motion is determined from Eq. (42), the hitting time from Eq. (44), the predicted time from Eq. (14) and the miss-distance of the shell from Eq. (21).

8. Example of Test Results Concerning Missile Deviation With One-Dimensional Maneuvering (to Evade Anti-Aircraft Gun)

The equations to be applied are based on the following numerical values.

	$V_T = 0.9$ mach	
	$N_T = 4$	
	$\tau_T = 0.5$ sec	
	$t_o = 20$ sec	
for:	$t = 0 \sim 11.0$ sec	$n_{By} = n_{Bo}$ (constant)
	$t = 11.0 \sim 16.0$	$n_{By} = -n_{Bo}$
	$t = 16.0 \sim 17.0$	$n_{By} = 1.2\ n_{Bo}$
	$t = 17.0 \sim 20.0$	$n_{By} = 0$

The missile motion is numerically plotted by the graphs of FIG. **5**, wherein

$n_{TY(max)}/n_{Bo} = 2.193$ and

$Y_T / n_{Bo} \rightarrow 0$ as the homing terminates (t=20 sec).

The low value of n_{TY}/n_{Bo} results in high accuracy of the final derivation of the missile.

In contradistinction thereto, the kill probability of the anti-aircraft guns is unsatisfactory to the defense as described hereinafter:

muzzle velocity = 1025 m/s

time of flight to the effective range of gun of 3Km = 3.8 sec

firing rate = 120 rounds/min

lethal radius of shell = 5m (with proximity fuse)

time from FCS predicting the missile position to firing of gun = 0.5 sec

gun dispersion = 0.00358 (1σ)

where R = distance measured from a firing position to a hitting position

σ = standard deviation

The equation of the predicted position of the missle is

$$y_t = y_{t=0} + \left(\frac{dy}{dt}\right)_{t=0} \times t + \left(\frac{d^2y}{dt^2}\right)_{t=0} \times \frac{t^2}{2}$$

In this example, the shells are successively launched at a firing rate within the hitting range (R) = 500 ~ 4,000m.

Let us assume that the missile is deviated in its sea-skimming flight and, though the explosion of shells concentrated in a region circumscribed by their lethal radius centered on the missile is theoretically effective to kill the missile, the explosion of some shells concentrated in the lower half of that region near the sea surface is too early on account of premature operation of the proximity fuse caused by the sea clutter so as to have no destructive effect on the missile.

Consequently, the cumulative kill probability per gun is shown in FIG. **11.** This diagnosis indicates the need for increasing the evading capability of the missile, because the kill probability of the shell with reference to a missile subjected to lateral acceleration pursuant to this invention is limited to a value of 41% ($n_{T(max)} \geqq 4.0$), according to a curve I, in comparison with the case of a homing missile of conventional navigation where the limit is 99.5% ($n_T = 0$).

9. Example of Test Results Concerning Missile Deviation With Two-Dimensional Maneuvering to Evade Anti-Aircraft GUN)

If we consider the acceleration bias n_B as swung into the YZ-plane of the orthogonal coordinate system shown in FIG. **6,** the missile motion may be plotted as illustrated in FIGS. **7** and **8** with the following numerical values:

for:

$t = 0 \sim 20.4$ sec, $d\theta/dt = 5.45$ o/sec.

$|n_B| = n_{Bo}$ (constant)

where

θ = angle due to lateral-acceleration bias ($-n_B$) at a time t, equaling 0 for $t = 0$

for:

$t = 20.4 \sim 22$ sec $n_B = 0$

$V_T = 0.9$ mach

$N_T = 4$

$\tau_T = 0.5$ sec

$t_o = 22$ sec

(with $n_T = \sqrt{n_{Ty}^2 + n_{Tz}^2}$)

The cumulative kill probability of the anti-aircraft gun is represented by a curve II in FIG. **11** on the assumption of conditions indentical with those of the first example, i.e. with the missile not deviated in a sea-skimming flight. Curve II indicates a reduction of the kill probability to a value of 6% ($n_{T(max)} \geqq 3.0$), i.e. the missile deviation with two-dimensional maneuvering acceleration produces an evading efficiency higher than that given by the missile deviation with one-dimensional maneuvering acceleration.

10. Example of Test Results Concerning Missile Deviation With Three-Dimensional Maneuvering (to Evade Anti-Aircraft Gun)

The variation of the missile speed conforms to the two-dimensional acceleration and is numerically defined as follows:

for: $t = 0 \sim 15$ sec $\quad V_T = 0.9$ mach

for: $t = 15 \sim 22$ sec $\quad V_T = 0.9 + 0.6 \times \left(\frac{t-15}{7}\right)^2$ mach

The other values are the same as in the second example so that the missile motion is apparent from FIGS. **9** and **10.**

The cumulative kill probability of the anti-aircraft gun is determined by another curve III of FIG. **11** under conditions identical with those of the second example. The kill probability of the gun is further reduced to a value of 4% ($n_{T(max)} \geqq 3.0$), i.e. the evading efficiency of the missile in this case is still higher than that given by the missile deviation with two-dimensional maneuvering acceleration.

11. Example of Test Results Concerning Missile Deviation With Two-Dimensional Maneuvering (to Evade Anti-Missile Missile MISSILE)

If we again consider the acceleration bias n_B as swung into the YZ-plane, the missile motion may be plotted as illustrated in FIGS. **12** and **13** with the following numerical values:

for:

$t = 0 \sim 20.4$ sec $d\theta/dt = 30$ o/sec.

$|n_B| = n_{Bo}$ (constant)

where

θ= angle due to lateral-acceleration bias ($-n_B$) at a time t, again equaling 0 for $t = 0$

for:

$t = 20.4 \sim 22$ sec $n_B = 0$

$V_T = 0.9$ mach

$N_T = 4$

$\tau_T = 0.5$ sec

$t_o = 22$ sec

The SAM parameters are determined as follows:

$V_M = 2.0$ mach

$N_M = 4$

$\tau_M = 0.5$ sec

maneuvering-acceleration limit of SAM = 14g (relative to both the Y-axis and the Z-axis)

hitting distance of the SAM = 1,000 ~ 4,000m

Under the above-mentioned conditions, the missile maneuvering accelerations along the Y-axis or the Z-axis are plotted in full lines in FIG. **16** for a miss-distance of 20m. Cruve IV represents the case where the missile and the SAM share the same set of coordinates ($\theta = 0°$), whereas curve V applies where their coordinates differ ($\theta = 45°$) with the Y-axis and the Z-axis of the missile include respective angles of 45° with the corresponding axes of the SAM.

Consequently, the required maximum maneuvering accleration of the missle is about

6.7g in the case of $\theta = 0°$

6.0g in the case of $\theta = 45°$

12. Example of Test Results Concerning Missile Deviation With Three-Dimensional Maneuvering (to Evade Anti-Missile Missile

The variation of the missile speed conforms to the one-dimensional acceleration and is numerically defined as follows:

for:

$t = 0–22$ sec

$V_T = 0.9 + 0.6 \times (t/22)$ mach

The other values are the same as in the fourth example so that the missile motion is apparent from FIGS. **14** and **15.**

In this case, the SAM parameters are determined under conditions similar to those of fourth example, the missile maneuvering accelerations along the Y-axis or the Z-axis being given by dotted lines in FIG. **16** and the required maximum maneuvering acceleration of the missile is about

5.5g in the case of $\theta = 0°$ (Curve VI)

5.2 g in the case of $\theta = 45°$ (Curve VII).

This result shows the advantage of three-dimensional acceleration over two-dimensional acceleration.

Though the numerical examples given above deal specifically with the navigation of a missle to a stationary target, the principles of my invention can be readily extended to situations where the target is in motion.

What is claimed is:

1. A system for guiding an in-flight homing missile to a target, comprising:

sighting means for tracking the location of a line of sight from the missile to the target;

detector means responsive to said sighting means for generating an error signal depending upon any deviation of the missile flight path from said line of sight;

steering means for changing the course of the missile;

control means for said steering means connected to said detector means for delivering a corrective signal derived from said error signal to said steering means; and

biasing means connected to said control means for superimposing upon said corrective signal to a lateral-acceleration signal effective in a direction perpendicular to an invariable reference direction generally parallel to said flight path.

2. A system as defined in claim **1,** further comprising circuit means for generating a feedback signal subtracted by said lateral-acceleration signal to obtain a differential signal which is from said error signal so as to obtain a control signal.

3. A system for guiding an in-flight homing missile to a target, comprising:

sighting means for tracking the location of a line of sight from the missile to the target;

detector means responsive to said sighting means for generating an error signal depending upon any deviation of the missile flight path from said line of sight;

steering means for changing the course of the missile;

control means for said steering means connected to said detector means for delivering a corrective signal derived from said error signal to said steering means; and

biasing means connected to said control means for superimposing upon said corrective signal a lateral-acceleration signal effective in a direction perpendicular to an invariable reference direction generally parallel to said flight path and further adding an ecceleration bias to the missile derivation direction.

4. A system as defined in claim **3,** further comprising circuit means for generating a feedback signal subtracted by said lateral-acceleration signal to obtain a differential signal which is deducted from said error signal so as to obtain a control signal.

5. A system for guiding an in-flight homing missile to a target, comprising:

sighting means for tracking the location of a line of sight from the missile to the target;

detector means responsive to said sighting means for generating an error signal depending upon any deviation of the missile flight path from said line of sight;

steering means for changing the course of the missile;

control means for said steering means connected to said detector means for delivering a corrective signal derived from said error signal to said steering means, said control means establishing a set of three coördinate axes including a first axis generally parallel to said flight path and two other axes orthogonal thereto; and

biasing means connected to said control means for superimposing upon said corrective signal two lateral-acceleration signals effective in directions parallel to said other axes.

6. A system as defined in claim **5,** further comprising circuit means for generating feedback signals substracted by said lateral-acceleration signals to obtain a differential signal which is deducted from said error signal so as to obtain a control signal.

7. A system for guiding an in-flight homing missile to a target, comprising:

sighting means for tracking the location of a line of sight from the missile to the target;

detector means responsive to said sighting means for generating an error signal depending upon any deviation of the missile flight path from said line of sight;

steering means for changing the course of the missile;

control means for said steering means connected to said detector means for delivering a corrective signal derived from said error signal to said steering means, said control means establishing a set of three coordinate axes including a first axis generally parallel to said flight path and two other axes orthogonal thereto; and

biasing means connected to said control means for superimposing upon said corrective signal two lateral-acceleration signals effective in directions parallel to said other axes and further adding in acceleration bias to the missile derivation direction.

8. A system as defined in claim **7,** further comprising circuit means for generating a feedback signal subtracted by said lateral-acceleration signal to obtain a differential signal which is deducted from said error signal so as to obtain a control signal.

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