

June 23, 1970

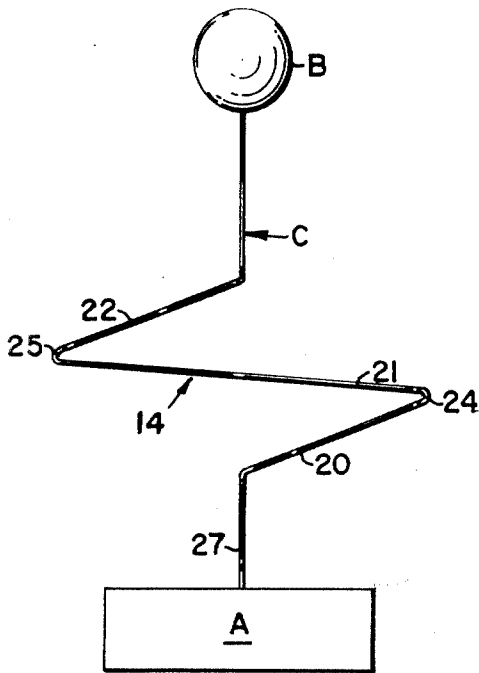
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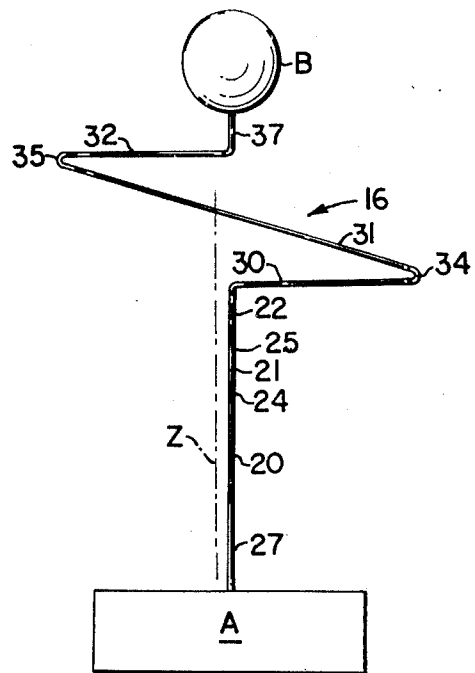
TOY HAVING LISSAJOUS VIBRATORY MOTION

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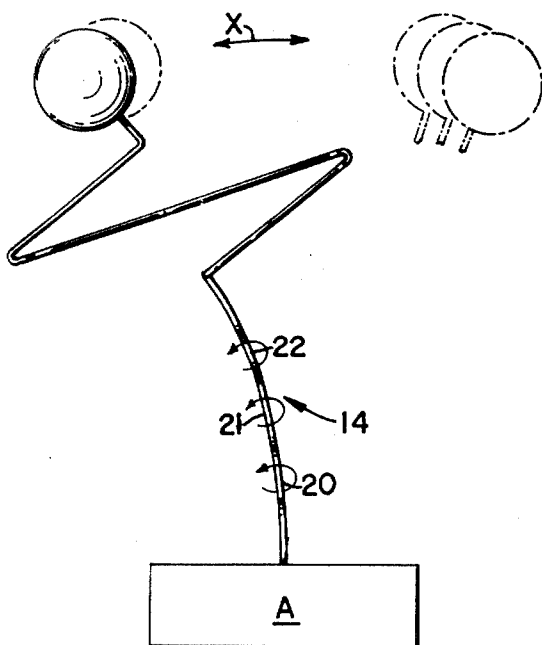
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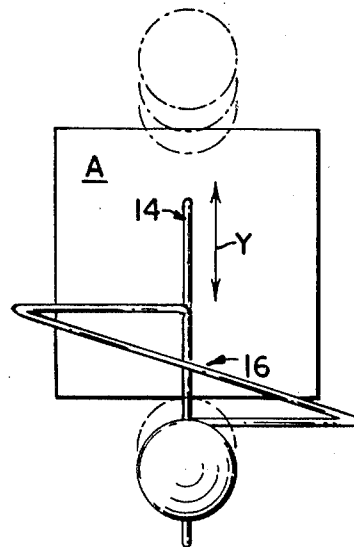
FIG\_1



FIG\_2



FIG\_3



FIG\_4

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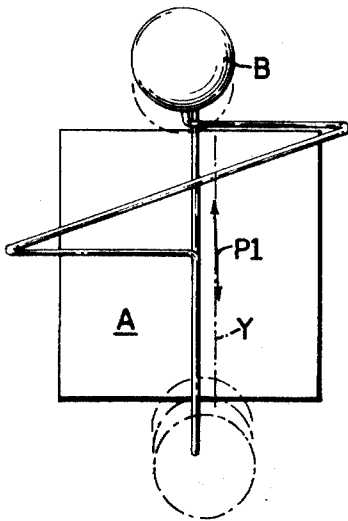
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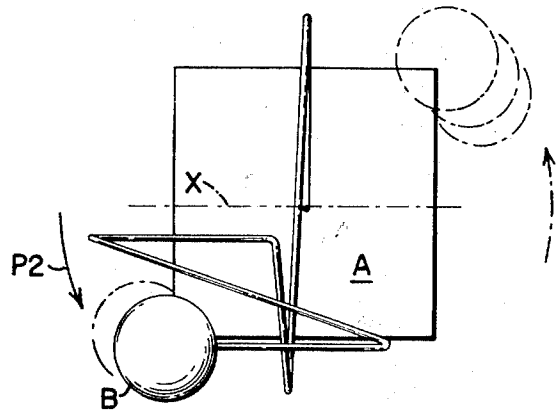
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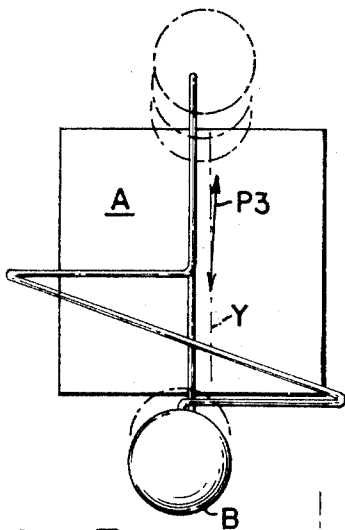
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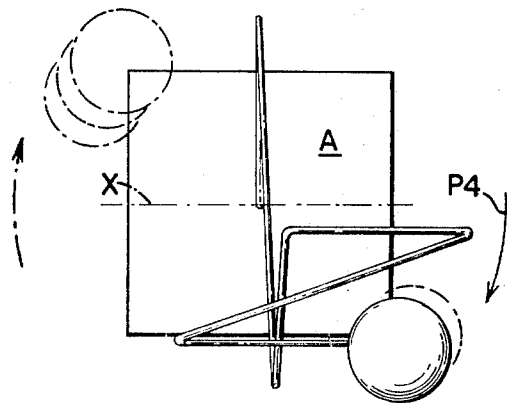
FIG\_5a



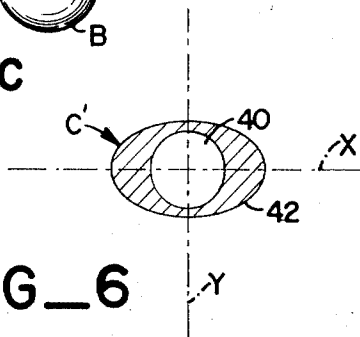
FIG\_5b



FIG\_5c



FIG\_5d



FIG\_6

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**TOY HAVING LISSAJOUS VIBRATORY MOTION**  
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10 Claims

## ABSTRACT OF THE DISCLOSURE

A toy including a mass mounted on the nonsupported end of an elastic cantilever for vibrational movement. The cantilever, typically constructed from a piece of wire, has a plurality of coplanar zigzags disposing segments of the wire substantially normal to the cantilever length. These segments provide substantially independent torsion and bending forces acting at right angles one to the other. The mass, when displaced and set in motion at the nonsupported end of a cantilever, undergoes vibrational motion deflected by the independent restoring forces and consequently traces Lissajous figures along its vibratory path.

This invention relates to a vibrating toy and more particularly to a cantilever supported mass which traces Lissajous figures along its vibratory path.

Mechanical devices which trace Lissajous figures along their vibratory path have heretofore been known. These devices, however, have comprised complex pendulums or other mechanisms, which have included specialized hinges and drives to provide the substantially independent sinusoidal forces necessary for the generation of the Lissajous figures.

In contradistinction to these heretofore known devices, this invention provides a simple vibratory mechanism capable of readily tracing Lissajous figures along its vibratory path. Accordingly, an elastic cantilever is provided having a mass mounted for vibrational motion at the free or nonsupported end. The elastic cantilever has provided therein substantially independent restoring forces, each restoring force acting along an axis intersecting the axis of the other restoring force. When the mass is displaced and set in motion at an acute or obtuse angle to the axes of the cantilever's restoring forces, it is subjected to the simultaneous interaction of both restoring forces, tracing Lissajous figures along its vibratory path.

A further object of this invention is to provide a cantilever constructed from a piece of elastic wire which has substantially independent restoring forces along mutually perpendicular axes. According to this aspect a wire having differing elastic restoring forces in bending and torsion is provided with a plurality of zigzag segments. These zigzag segments, are all disposed substantially horizontally to the longitudinal axis of the cantilever within a common plane including the longitudinal axis of the cantilever. When the mounted mass vibrates within the plane of the zigzag, the horizontal segments of the wire are subjected to pure bending forces, giving the vibrating mass a first sinusoidal motion. When the mounted mass is vibrated normally to the plane of the zigzag, the horizontal segments of the wire are subjected substantially to pure torsion forces, giving the vibratory mass a second sinusoidal motion normal to the first sinusoidal motion. By displacing or setting the mass in motion at acute or obtuse angles from the common plane of the zigzag segments, the mass vibrates along a path which traces Lissajous figures.

A further object of this invention is to provide the mass undergoing vibration with a vibratory frequency wherein the Lissajous pattern of its vibration can be readily observed. Accordingly, the mass and cantilever are

preselected to provide constant vibratory oscillations at near but not equal rates along each of the intersecting axes. These near but not equal rates are each in a range between 10-100 cycles per minute. When the mass is displaced and set in motion, it traces a Lissajous pattern which can be plainly seen.

A further object of this invention is to provide a cantilever constructed of wire, in which the zigzag bends impart maximum dependence of their respective independent restoring forces in bending and torsion. Accordingly, the elastic wire cantilever has its horizontal zigzag segments close to the supported end. When the vibrating mass is set in motion, maximum elastic deflection of the wire cantilever occurs adjoining its supported end, accentuating the substantially independent restoring forces of the wire and emphasizing the Lissajous vibratory path of the mass.

Other objects, features and advantages of the present invention will be more apparent after referring to the following specification and attached drawings in which:

FIG. 1 is a front elevation of the vibratory toy showing its lower zigzag segments in the plane of the figure;

FIG. 2 is a side elevation of the vibratory toy taken at right angles to the view of FIG. 1 showing the upper zigzag segment in the plane of the figure;

FIG. 3 is a side elevation of the vibratory toy vibrating so that its lower zigzag segments are in the stable or torsion mode;

FIG. 4 is a plan view of the vibratory toy vibrating so that the lower zigzag segments are in the unstable or bending mode;

FIG. 5a through 5d show plan views of the vibratory toy undergoing sequential phases of vibratory movement tracing Lissajous figures in which:

5a shows a substantially linear phase of vibratory motion of the mass at a small acute angle counterclockwise from the plane of the lower zigzag segments.

5b shows a substantially circular phase of vibratory motion of the mass in a counterclockwise direction about the cantilever base,

5c shows a second substantially linear phase of vibratory motion of the mass at a small acute angle clockwise from the plane of the lower zigzag segments, and

5d shows a substantially circular phase of vibratory motion of the mass in a clockwise direction about the cantilever base; and

FIG. 6 illustrates an alternate cross-sectional construction of a cantilever useful for the practice of this invention.

With reference to the figures, the toy comprises a support A having elastic cantilever C embedded therein with ball or mass B attached to the free end of the cantilever for vibratory movement about the support.

Support A can be constructed of a hardwood block of approximately 400 grams of weight with a base 4-5 inches square. This base has a width and weight sufficient to hold the cantilever and ball stationary while they undergo vibratory motion. Ball B is a plastic mass weighing approximately  $\frac{1}{8}$  that of the base or 45 grams.

Cantilever C comprises  $\frac{1}{16}$  inch tinned music wire bent so as to have a plurality of coplanar zigzag segments disposed normally to its longitudinal axis. Cantilever C is embedded within and supported by support A and extends vertically upward suspending mass B at its nonsupported upward extremity. Between support A and mass B cantilever C is here shown having a first set of planar zigzags 14 disposed in the cantilever adjacent support A and a second set of planar zigzags 16 disposed adjacent mass B in cantilever C.

Describing the zigzags, zigzags 14 comprise substantially horizontal cantilever segments 20, 21 and 22. These segments have located between their lengths opposed

3

acute bends 24 and 25, bend 24 being between segments 20 and 21 and bend 25 being between segments 21 and 22. A short vertical segment 27 extends between the lower portion of segment 20 and base A. As is apparent from the view of FIG. 2, segments 20-22, are all disposed in coplanar relation along a plane including the vertical axis of cantilever C.

Similar to first planar zigzags 14, second planar zigzags 16 include substantially horizontal segments 30, 31, and 32. Opposed bends 34 and 35 interconnect segments 30 and 31, and segments 31 and 32, respectively. A short vertical segment 37 is formed along cantilever C immediately overlying that point on support A where the cantilever is embedded at its supported end. As is apparent from FIGS. 1 and 2, the common plane of zigzags 14 is normal to the common plane of zigzags 16.

Having described the supported cantilever, the sinusoidal vibratory motion which is supported mass describes can now be set forth. For simplicity in understanding, such motion will only be described with respect to lower zigzags 14.

Considering FIG. 3, mass B is shown set in motion along a plane which is normal to that plane of lower planar zigzags 14. The apparatus can be considered for theoretical purposes to be a mass B oscillating about a point in space. The movement of the mass can be conveniently described using Cartesian coordinates taken along a plane normal to the view of FIG. 3. The vibratory motion of the mass is described by equating the dynamic forces acting on the mass to the static forces which act on the mass. Assuming that vibration in the mode shown in FIG. 3 occurs along the X axis (with  $X=0$  being equivalent to the rest position) the dynamic forces acting on the mass can be obtained from Newton's second law of motion:

$$F_1 = ma = m \frac{d^2x}{dt^2}$$

where  $F_1$  equals the force acting on the mass along the X axis,  $M$  is the mass of ball B,  $A$  is acceleration (expressed as the second derivative of X axis movement in the second part of the equation), and  $t$  is time.

The restoring forces acting on the mass can be obtained from Hooke's law and are equal to:

$$F_2 = K_x X$$

where  $K_x$  is the spring constant of the cantilever in the X axis (which spring constant is assumed to be substantially linear), and  $F_2$  is the static forces acting on the mass.

Since  $F_1$  and  $F_2$  are the only forces acting on the mass in the X direction, the two may be equated:

$$m \frac{d^2x}{dt^2} = -K_x X$$

and integrated to obtain the periodic motion of the mass as a function of time  $t$  to give the equation:

$$X = A_x \sin \left( \sqrt{\frac{K_x}{m}} t + Q_1 \right)$$

where  $A_x$  is the constant for the initial amplitude of vibration along the X axis and  $Q_1$  is the constant for the angular position of the mass at time zero. It is readily observed that this function is essentially a sinusoidal motion.

The spring constant  $K_x$  of the cantilever in its vibration about the X axis will include torsion imparted to substantially horizontal segments 20, 21 and 22. These horizontal segments will tend to elastically rotate rather than elastically bend as the mass vibrates from one extreme position relative to support A to the opposed extreme position relative to support A in the plane of FIG. 3.

Referring to FIG. 4, vibration of the mass B in the plane of first planar zigzags 14 is illustrated. This motion

4

occurs along the Y axis (with  $Y=0$  being the rest position). Similar to the vibration formula previously derived, the vibrational motion of the mass in the Y axis will equal:

$$Y = A_y \sin \left( \sqrt{\frac{K_y}{m}} t + Q_2 \right)$$

where  $A_y$  is the original amplitude in the Y direction,  $K_y$  is the spring constant in the Y direction and  $Q_2$  is a constant for the angular displacement at time zero. It will be observed that lower zigzag 14 here is subjected to bending forces rather than the torsion forces previously illustrated in FIG. 3.

It has been found that the torsion spring constant  $K_x$  for a wire can be substantially less than the bending spring constant  $K_y$ . Accordingly, the forces experienced by the vibratory mass in the vibrational mode along the X axis of FIG. 3 are substantially less than those shown for the vibratory motion along the Y axis shown in FIG. 4. Typically, when the mass is vibrated normal to the plane of first zigzags 14, the mass B will easily remain in such motion along its X axis. When the mass is vibrated parallel to the plane of the lower zigzag or along its Y axis, it will only remain in such motion when the mass is set in motion precisely along the Y axis. Any vibrational movement imparted to the mass B slightly deviated from the Y axis will cause the mass B to seek a vibrational path of least resistance including motion along the X axis and consequently will result in a vibrational motion tracing Lissajous figures. This vibrational motion can best be described with reference to FIGS. 5a through 5d.

FIG. 5a assumes that the mass has been set in motion along path  $P_1$  inclined at a small counterclockwise acute angle relative to the axis Y. In such a vibratory motion the mass B will oscillate back and forth along path  $P_1$  for a short period of time and then seek a vibratory path  $P_2$  (shown in FIG. 5b) which includes vibration in the X axis.

As shown in FIG. 5b when vibratory movement including substantial movement in the X axis is attained, mass B will tend to follow a substantially circular oscillating path counterclockwise about support A. This substantially circular path will continue for a short period of time; thereafter, the forces acting on the mass will tend to return the mass to a vibrational path  $P_3$  substantially parallel to the Y axis shown in FIG. 5c.

With reference to FIG. 5c, the mass will approach a vibratory path  $P_3$  which adjoins the Y axis and is inclined clockwise to the Y axis at a small acute angle. Mass B will oscillate along this path for a short period of time and then seek a path  $P_4$  which again includes a vibratory motion in the X axis. Similar to path  $P_2$ , path  $P_4$  will be circular but will generally be substantially clockwise with respect to support A. The mass will vibrate along path  $P_4$  for a short period of time finally seeking a vibratory motion which then includes a path substantially similar to path  $P_1$  originally illustrated in FIG. 5a. The vibratory motion of FIGS. 5a through 5d will then be sequentially repeated.

The mass along its vibratory path will thus describe a plainly visible Lissajour figure. This figure will include sequential vibrations substantially parallel to the Y axis, vibrations circular about base A in a first direction, vibrations again substantially parallel to the Y axis, and finally an opposed circular path about base A. This pattern will be repeated so long as the forces of friction do not damp and arrest the motions of the mass B.

The vibratory motion of the mass has thus far been described with reference to the lower zigzag 14. Upper zigzag 16 complements such motion. As is apparent, this upper zigzag 16 has its stable mode in the Y axis, and its unstable mode in the X axis. While the upper zigzag 16 is not necessary for practice of this invention, it does tend

to impart additional independent restoring forces to the moving mass and complements the lower zigzag so as to give the vibrational toy a symmetrical appearance.

The specific form of bends in cantilever C used to produce the differing vibrational constants  $K_x$  (torsion) and  $K_y$  (bending) are not critical. Any bend which disposes a segment of the wire in a substantially horizontal disposition relative to the vertical axis of the cantilever C will tend to produce the differing torsion and bending elastic constants necessary for the generation of the described Lissajous patterns of motion. The bends here shown, however, are preferred to accent such motion.

It is possible to construct a cantilever C' which is vertical and without zigzags to accomplish the described vibratory motion. Such a cantilever C' is illustrated in the cross section of FIG. 6.

With reference to FIG. 6, a cantilever cross section is illustrated having a circular and hollow interior 40 surrounded by an elliptically shaped resilient mass 42. Since the resilient mass 42 has the bulk of its material disposed closer to the X axis than to the Y axis, forces acting upon a vibrating mass along the Y axis will tend to be less than forces acting upon a mass vibrating in the X axis. Unfortunately, resilient elements constructed in the configuration shown in FIG. 6 are very difficult to make; it is immediately realized that simple bending of circular tinned music wire according to the zigzags of the present invention is a greatly simplified construction.

It is known that cantilevers undergoing elastic deformation experience their greatest elastic deformation adjoining the supported end of the cantilever. Accordingly, it is preferred to dispose a planar zigzag immediately adjoining the support A. In this position, the differing elastic constants  $K_x$  of torsion and  $K_y$  of pure bending are accentuated. This accentuation results in creating the maximum differential between the resultant elastic forces in the X and Y axis.

As is apparent, vibration of mass B can also be attained by moving the mass upwardly or downwardly with respect to support A. This vibration compresses zigzags 14 and 16 and results in an additional vibrational motion imposed upon the mass in the Z axis (shown in FIG. 2). Unfortunately, in the embodiment of the invention shown here, vibrations in the Z axis are much more rapid than those occurring in the X and Y axis. These rapid vibrations make the visual observance of any Lissajous pattern occurring simultaneously in the X, Y and Z axis difficult.

It has been found, however, that if cantilever C is horizontally disposed, the vibratory motions along the Z axis can be made to approach the frequency of the vibrations occurring in the X and Y axis. This latter construction permits three-dimensional Lissajous figures to be traced by the vibratory motion of the mass B. In such an embodiment it is preferred that the wire of cantilever C be reduced in size and resiliency so that the periodic motions of the mass become much more gradual.

What is claimed is:

1. In a toy for providing vibrational movement of a mass about a base an elastic cantilever having a fixed end supported from said base and a free end mounted for swaying deflection about its fixed end; a mass mounted to the free end of the cantilever for vibratory motion to and from a position where said mass is at rest on the end of said cantilever; said cantilever including an uncoiled wire formed with at least one co-planar zig-zag.

2. The invention of claim 1 wherein said cantilever is supported vertically with said mass mounted to the upper end of said cantilever.

3. The invention of claim 1 wherein said elastic cantilever includes at least one segment of said wire disposed substantially normal to the longitudinal axis of said cantilever.

4. The invention of claim 1 wherein said co-planar zig-zag is disposed adjacent the supported end of said cantilever.

5. The invention of claim 1 wherein said co-planar zig-zag is within a plane that includes the longitudinal axis of said cantilever.

6. The invention of claim 5 wherein said cantilever includes a plurality of said zig-zag segments, said co-planar zig-zag segments each being formed in different planes.

7. In a toy for providing vibrational movement of a mass about a support, the combination with said support and mass comprising: an elastic cantilever having a fixed end and a free end attached to said support at one end and to said mass at the opposite end for supporting said mass at a rest position relative to said support; said elastic cantilever being a single uncoiled wire formed with at least one co-planar zig-zag.

8. The invention of claim 7 wherein said co-planar zig-zag includes a segment of said wire substantially normal to the axis of said cantilever and disposed adjacent the supported end of said cantilever.

9. The invention of claim 7 and wherein said elastic cantilever further includes a second co-planar zig-zag disposed in a plane including a longitudinal axis of said cantilever and intersecting the plane of a first co-planar zig-zag.

10. The invention of claim 7 wherein the plane of said second co-planar zig-zag is perpendicular to the plane of a first co-planar zig-zag.

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U.S. Cl. X.R.

33—27; 35—19, 30