

(No Model.)

W. F. C. MORSELL.

SECTIONAL MODEL FOR STEREOMETRIC REPRESENTATION.

No. 595,782.

Patented Dec. 21, 1897.

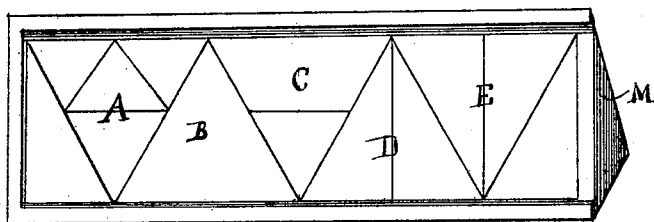


Fig. 1.

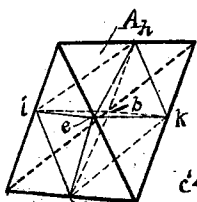


Fig. 2.

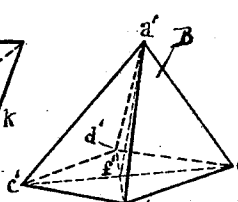


Fig. 3.

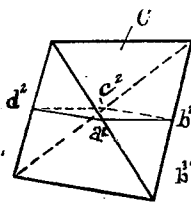


Fig. 4.

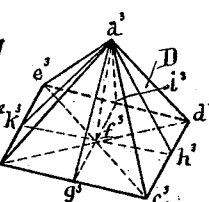


Fig. 5.

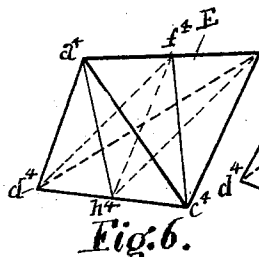


Fig. 6.

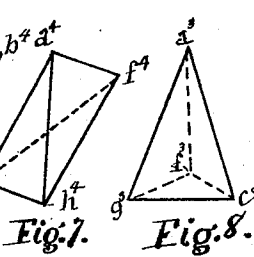


Fig. 7.

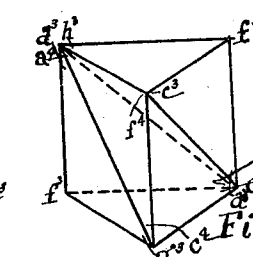


Fig. 8.

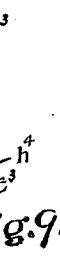


Fig. 9.

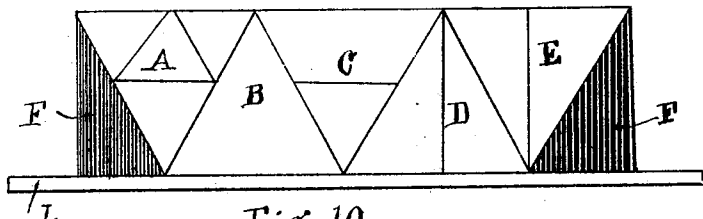


Fig. 10.

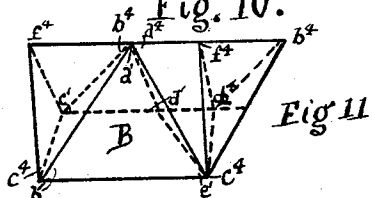


Fig. 11.

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SECTIONAL MODEL FOR STEREOMETRIC REPRESENTATION.

SPECIFICATION forming part of Letters Patent No. 595,782, dated December 21, 1897.

Application filed December 31, 1895. Serial No. 573,982. (No model.)

To all whom it may concern:

Be it known that I, WILLIAM F. C. MORSELL, a citizen of the United States, residing in Philadelphia, (Germantown,) in the county of Philadelphia, in the State of Pennsylvania, have invented a new and useful Improvement in Sectional Models for Stereometric Representation, of which the following is a clear and sufficient specification.

My invented device is mainly intended for the visible concrete stereometric representation of the dissection and composition of geometrical figures mainly with reference to their volume, so that a child can build into different well-known shapes blocks which have a fixed volume relation to a cube taken as the unit of volume, and so determine the volume relations of these shapes between each other and the cube.

In carrying out my invention I make use of blocks of various shapes and dissected in various manners into smaller blocks, which I will describe as I proceed.

Figure 1 is the view of a block having the shape of a prism contained in a closely-fitting box, the block being divided into a series of blocks, as will be described. Figs. 2, 3, 4, 5, and 6 show dissections of the blocks A, B, C, D, and E, respectively, into smaller blocks. Fig. 7 is a block which forms one of the two foundation forms from which are built the various forms and which I call the "quadrate;" and Fig. 8 is the other of these two blocks, which I call the "octet." Fig. 9 is a quarter of a square showing how the square is built up from octet and quadrate blocks, and Fig. 10 is a view of a modification of the dissected block shown in the box in Fig. 1. Fig. 11 is a view of an illustration of comparative volume that can be made with the blocks.

The letters in Figs. 2 to 9, inclusive, are used, as in geometrical works, to designate points, not parts, and the lines in these figures do not follow the usual rules of shading for mechanical drawings, but a heavy line has been used to designate the exterior edges of the blocks A, B, C, D, and E. These lines are full where the edges would appear in the block and are dotted where they would not. The lighter lines designate the edge of the smaller block which make up the larger (except where such edge coincides with the edge

of the larger) and are full where the lines of division would appear on the blocks and dotted where they would not.

In the box M, (which has the interfacial angle at the top approximately seventy-one degrees and those at the bottom approximately fifty-four degrees thirty minutes each,) which is preferably provided with a sliding cover, I place the prism-shaped block, made up of the blocks A, B, C, D, and E, which are a series of alternately regular tetrahedral-shaped blocks and pentahedral pyramidal-shaped blocks, with two supporting-blocks—one at either end of the series. The inter-facial angles of this prism-shaped block are at the top substantially seventy-one degrees and at the bottom fifty-four degrees thirty minutes each.

The block A has the shape of a regular tetrahedron, and its analysis is shown in Fig. 2. The faces of it are all equilateral triangles. This block A is dissected in a manner well known in crystallography—viz., by dividing it on the planes hkb , icb , cke , and hie , (see Fig. 2,) each of these points h , k , b , c , e , and i being the middle point of one of the edges of the block. These dissected off pieces are a series of tetrahedral-shaped blocks and leave behind a double regular pentahedral pyramid, which can be divided into two equal pyramidal-shaped blocks by cutting it on the plane $ickh$. This dissection of this block is not new, as it is well known in crystallography. The edges of this block A are equal in length to a diagonal of a face of a cube the volume of which is equal to the volume of a portion of the prism-block equal to the width at the base. This block is shown in Fig. 11.

B (see Fig. 3) is a block of the form of a regular pentahedral pyramid. It is dissected by dividing it on planes $a'b'd'$ and $a'e'e'$, forming the four blocks $a'b'f'c'$, $a'b'f'e'$, $a'e'f'd'$, and $a'd'f'e'$, which, if placed upon the sides of block A, will form a cubical-shaped block, of which the edges of the tetrahedral-shaped block will form the diagonals across its faces. It may therefore be said that the pentahedral block B is formed from the corners left after dissecting out from the cubical block the tetrahedral block A. This cube, as will below appear, is the "unit-cube" block.

The tetrahedral block C (see Fig. 4) is divided in two by cutting it on the line of the plane $a^2 b^2 c^2 d^2$. The two blocks thus made are intended to be identical in volume and shape.

The block D is shown dissected in Fig. 5. This division is made on the planes $a^3 b^3 d^3$ and $a^3 e^3 c^3$, again on the planes $a^3 g^3 i^3$ and $a^3 h^3 k^3$, the points h^3, g^3, k^3 , and i^3 being the middle points of the edges $c^3 d^3, b^3 c^3, b^3 e^3$, and $e^3 d^3$, respectively. This division divides the block D into eight smaller blocks, each being theoretically equal in volume to each of the others, and consequently each block is one-eighth the volume of block D.

It will appear by an examination of the block D and its subdivisions that four of the smaller blocks are identical theoretically each to each and that the other four are likewise identical each to each, and that all of the blocks are bounded by an equal number of equal faces, and that, placing two equal faces of two non-identical blocks in coincidence, the like faces will and the entire two figures of the two blocks will be symmetrical with each other, taking the plane of the coincident faces as the plane of symmetry. As the position of these two kinds of smaller blocks in the larger block may be said to "mate" each other, I designate them as "right" and "left" mates. One of these parts I have shown in Fig. 8. I designate these smaller blocks as "octets," as being each one-eighth of the pyramid D. The faces are each right-angled triangles, the right angles of two of them being at g and two at f .

The dissection of block E is shown in Fig. 6, and is as follows: The middle points f^4 and h^4 of two non-adjointing edges are taken and planes passed through each of them and the edge opposite them. On these planes the block is cut, forming the blocks $b^4 c^4 h^4 f^4$, $a^4 h^4 f^4 c^4$, $a^4 f^4 h^4 d^4$, and $b^4 f^4 h^4 d^4$. Each of these smaller blocks is identical in volume with each of the others, and two of them identical in shape, theoretically, ($b^4 f^4 h^4 c^4$ and $a^4 f^4 d^4 h^4$), and symmetrical with the other two, which are also identical with each other. I speak of these blocks as being "right" and "left" mates for the same reason as explained with reference to the octet blocks. The blocks dissected from block E, being each one-quarter of the block E, I call a "quadrate." I have illustrated one of them in Fig. 7, and it will be readily observed that its faces are all right-angled triangles, two faces being right angled at f and two at h . It will be noticed also that when one of these faces is fitted to one of the faces of one of the octet blocks the opposite face of the quadrate block will fit the corresponding face of the mate of the octet. The octet blocks and the quadrate blocks are all of the same volume, one-twelfth of the "unit-square," and the length of the one side adjacent the right angle of each triangular face of the quadrate block is to the other side adjacent the right angle as 1 is to $\sqrt{3}$.

In Fig. 9 is shown the way in which a unit-square is built up. Two mated octet blocks are placed so that the edges of each that extend from the apex of pyramidal block D to one of the corners of the base are adjacent and the faces, including this edge and the perpendicular to the base from the apex of block D, in the same plane. Then I select a quadrate block that will fit these octet blocks, forming the triangular prism $abcdef$, in which $adfe$ and $abce$ are two octet blocks and $afec$ is the quadrate. By placing four of these together a cube is formed of twelve pieces, each one-twelfth of the unit-cube. This cube is evidently the same in volume as the cube mentioned as formed of the blocks A and B, as that is built out of a block having the shape of a regular tetrahedron and a block having the form of a regular pentahedral pyramid, each having, respectively, the form and volume of the blocks D and E, from which the parts to build the cube, one-quarter of which is shown in Fig. 9, are obtained. Both are then unit-cubes.

By leaving the portion $f^4 c^4 b^4 a^4$ of block E against one side of block B or block D and $f^4 c^4 b^4 d^4$ against the other (see Fig. 11) a prism-shaped block is produced. As this is built of the blocks that form the unit-cube it is evidently equal in volume to it.

Among the many uses to which my invented device can be applied may be mentioned, first, showing the volumes of oblique solids in relation to the unit-cube taken as a unit—for instance, the volume of each octet and quadrate is one-twelfth of the unit-cube, as it takes eight of them to build the regular pentahedral pyramid and four to build the regular tetrahedron having the same altitude. Therefore the first has twice the volume of the second. Again, as the volume of the regular pentahedral pyramid is two-thirds of the unit-cube and the octet blocks that are built into it require eight of their faces to form the base, while four of these faces form the base of the unit-cube and the altitude is the altitude of the same block in each case, it is clear that the volume of the pyramid is two-thirds of one-half its base area by its altitude (the square having one-half the base area that the prism has) or one-third of the product of its base area by its altitude.

Again, without going into the details of an explanation, the faces of the blocks used to form a face of the unit-cube can be rebuilt into different forms. Again, it can be visibly shown that the diagonal of a face of a cube is the side of a square of double the area and that it is incommensurable with the sides of the first square, and hence that it represents the value $\sqrt{2}$ when the area of the first square is 1.

Some of the trigonometrical relations can be briefly indicated and the variations of the interfacial angles indicated and many other uses made of the blocks, which would suggest themselves to a skilled teacher.

Theoretically the correct angles for my model have been named, (except that seventy-one degrees has been given as the angle in place of seventy degrees thirty-two minutes, and fifty-four degrees thirty minutes in place of fifty-four degrees forty-four minutes;) but variations from them or from precise accuracy in the length of the sides may be made without interfering seriously with the operation of my device.

Fig. 10 shows my prism mounted upon a board L, so that a view of the blocks can be had on two sides. The blocks are held in place by the supports F F, glued or otherwise suitably secured to board L.

Having now described the principle of my invention and the best form of which I am now aware of embodying the same, what I claim, and desire to secure by Letters Patent, is—

1. A composite block having the shape of a prism with the interfacial angle at the edge substantially seventy-one degrees and the interfacial angles at the other two edges substantially fifty-four degrees thirty minutes each and consisting of a series of blocks alternately of the shape of a regular tetrahedron and a regular pentahedral prism, one of said tetrahedral-shaped blocks being subdivided into four substantially equal and identically-shaped blocks and one of said pentahedral-shaped blocks being divided into eight substantially equal and identically-shaped blocks each of said four blocks being equal in volume but of a different shape from the said eight blocks all substantially as described.

2. A composite block having the shape of a prism and having the interfacial angle at one edge substantially seventy-one degrees and the interfacial angles at the other edges fifty-four degrees thirty minutes each and consisting of a series of blocks alternately of the shape of a regular tetrahedron and of a regular pentahedral pyramid, all of said blocks having all of their edges of equal length and all of their faces of equal area, and the face angles formed by these edges all equal to sixty degrees, one of said tetrahedral-shaped

blocks being divided into four smaller blocks substantially identical in shape and volume each to each, and one of the pyramidal-shaped blocks being divided into eight smaller blocks substantially identical each to each in form and volume, each of the latter being equal in volume to each of the former though differing therefrom in shape substantially as described.

3. A tetrahedral block having all of its faces equal right-angled triangles, the length of one side about the right angle being in the ratio of 1 to the $\sqrt{3}$ to the other side about the right angle substantially as described.

4. A prism-shaped block having one interfacial angle substantially seventy-one degrees and the other interfacial angles fifty-four degrees thirty minutes each, and formed of a block having the shape of a regular tetrahedron and a block having the shape of a regular pentahedral pyramid lying adjacent each to each said tetrahedral-shaped block being divided into four smaller blocks on the line of planes at right angles with each other through one edge and the middle point of the opposite edge, and the pyramid-shaped block being divided into eight smaller blocks by division on planes passed at right angles with each other through the apex and opposite side edges and on planes passed at right angles with each other and through the apex and through the middle points of the opposite lower edges substantially as described.

5. The combination of three tetrahedral blocks, two of which are right and left mates, similar in all respects except right and left position and being the eighth part of a right pentahedral pyramid, and a third tetrahedral block having two faces which are adapted to fit two like faces of the pair when the three are placed together to form a right-angular triangular prism substantially as described.

In witness whereof I have hereunto set my hand this 11th day of December, 1895.

WILLIAM F. C. MORSELL.

Witnesses:

EDWIN L. BRADFORD,
R. A. MCPHERSON, Jr.