United States Patent

Schoen

[15] 3,663,346

[45] May 16, 1972

[54] **HONEYCOMB CORE STRUCTURES OF MINIMAL SURFACE TUBULE SECTIONS**

[72] Inventor: **Alan H. Schoen,** West Concord, Mass.

[73] Assignee: **The United States of America as represented by the Administrator of the National Aeronautics and Space Administration**

[22] Filed: **July 22, 1970**

[21] Appl. No.: **57,253**

[52] **U.S. Cl.** **161/68,** 52/80, 52/DIG. 10, 161/7, 161/127

[51] **Int. Cl.** **B32b 3/12**

[58] **Field of Search** .... 156/197; 161/68, 69, 127; 29/455 LM; 52/61 S, 80, 615; 285/150; 287/54 A, 54 B

[56] **References Cited**

UNITED STATES PATENTS

3,227,598	1/1966	Robb	161/68
3,525,663	8/1970	Hale	161/68
3,360,883	1/1968	Glanzer	38/18 A X
3,131,789	5/1964	Watter	29/455 LM X
3,137,602	6/1964	Lincoln	161/69 X
3,391,511	7/1968	Harris et al.	161/69 X
3,568,381	3/1971	Hale	52/575 X

Primary Examiner—John T. Goolkasian
Assistant Examiner—Henry F. Epstein
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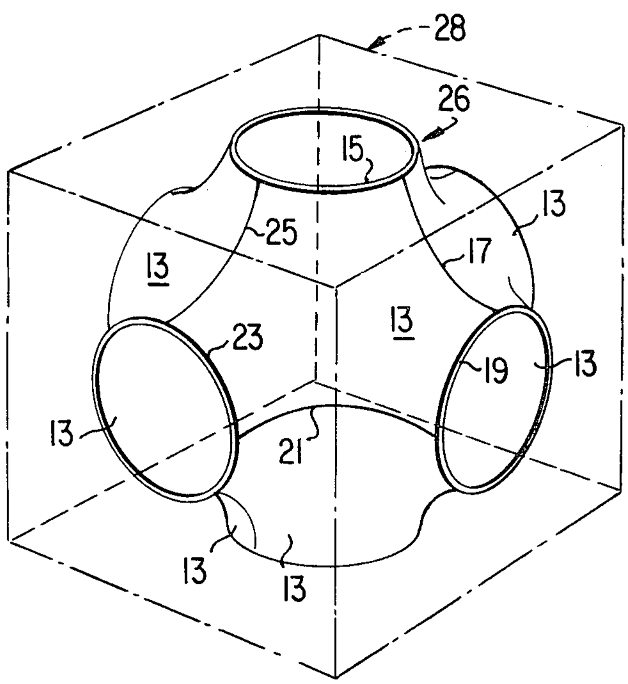
[57] **ABSTRACT**

Honeycomb cores formed of tubule sections bounded orthogonally by plane facings are described. The tubule sections are defined as being formed of minimal surface elements that orthogonally intersect all of the surfaces of an imaginary kaleidoscopic cell at least once. In other words, the tubule sections are broken into elements for definition purposes. The elements are defined as minimal surface elements, i.e., elements that have a mean curvature that is equal to zero at all points on their surface. These elements are further defined inside of an imaginary kaleidoscopic cell in that they orthogonally intersect all surfaces of an imaginary kaleidoscopic cell at least once. Moreover, the tubule sections are smoothly interconnected to form honeycomb core structures that have no internal discontinuities.

16 Claims, 33 Drawing Figures



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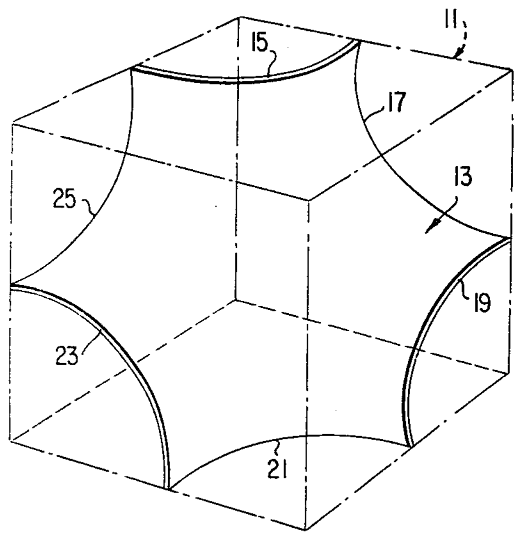


FIG. 1

FIG. 2

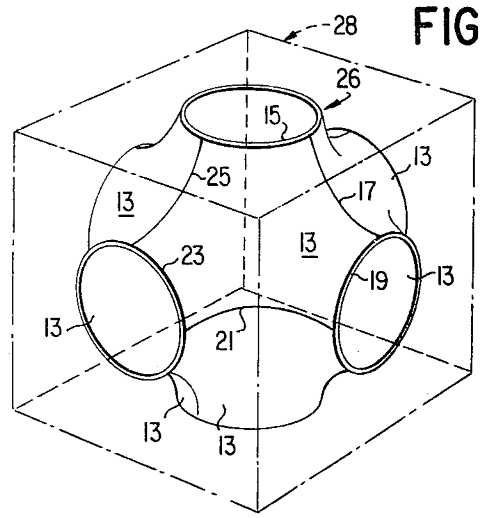
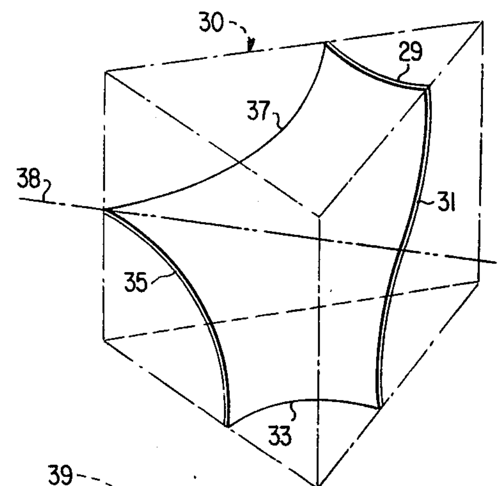


FIG. 4



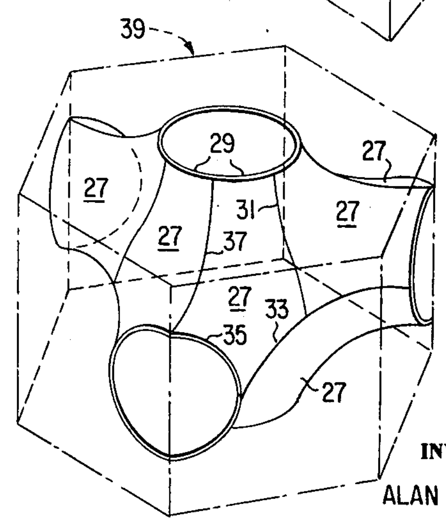


FIG. 5

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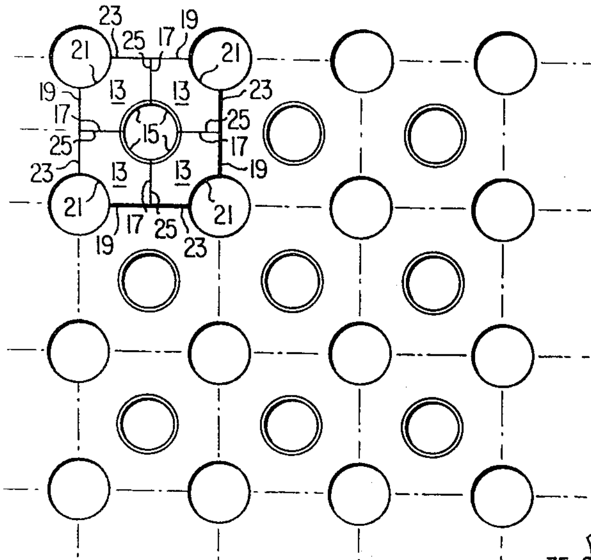

FIG. 3

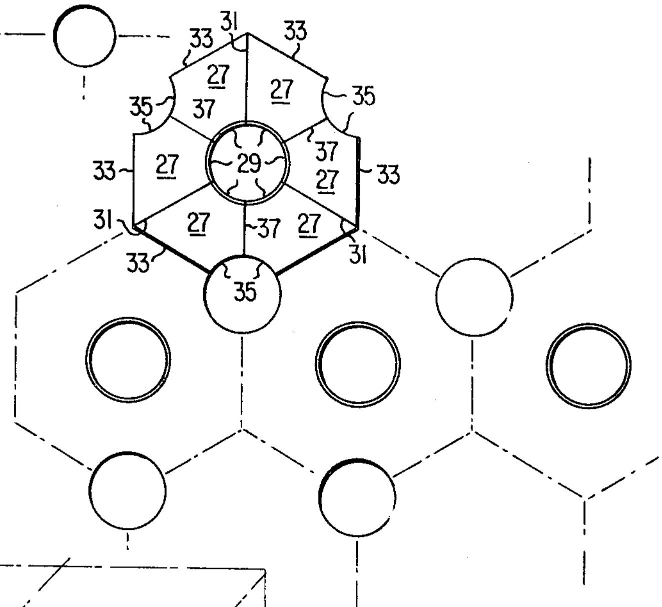

FIG. 6

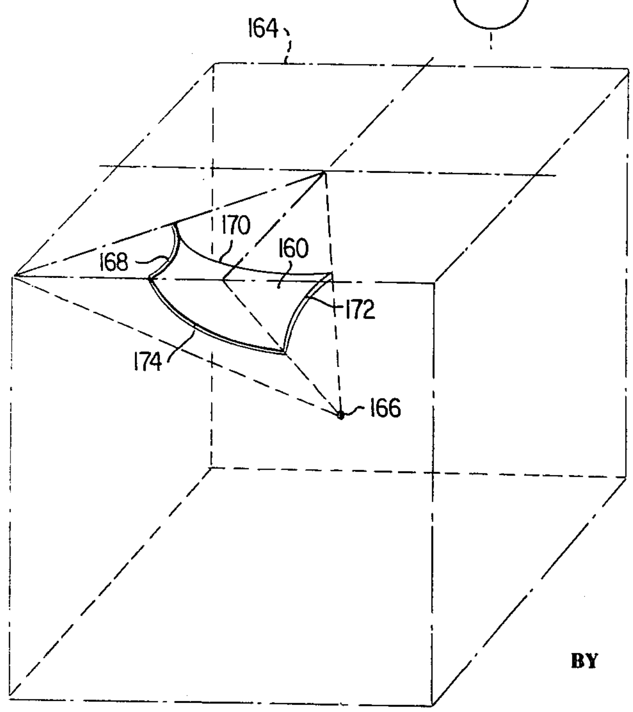

FIG. 26

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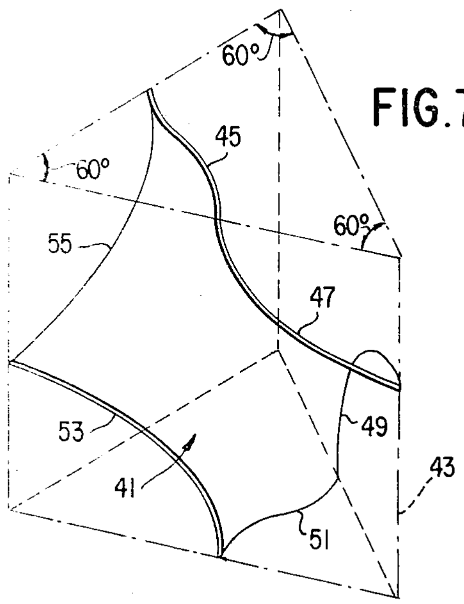

FIG.7

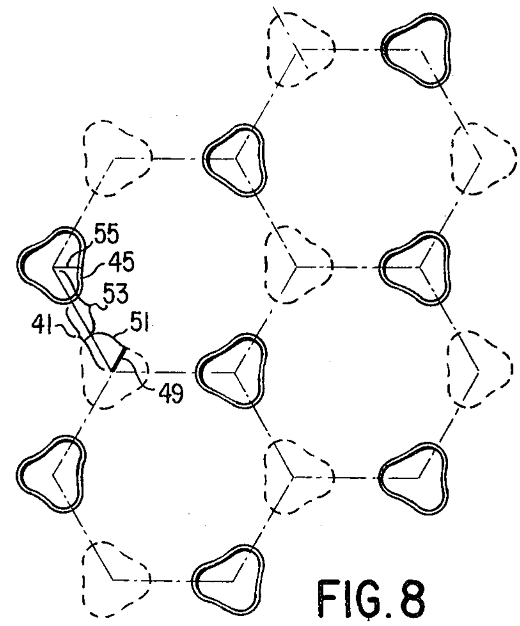

FIG.8

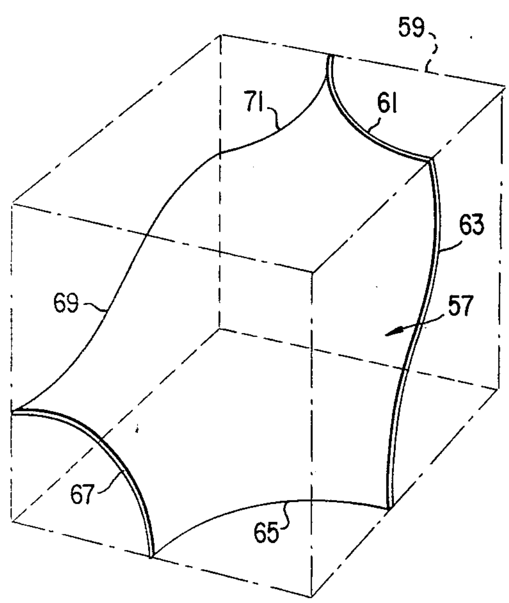

FIG.9

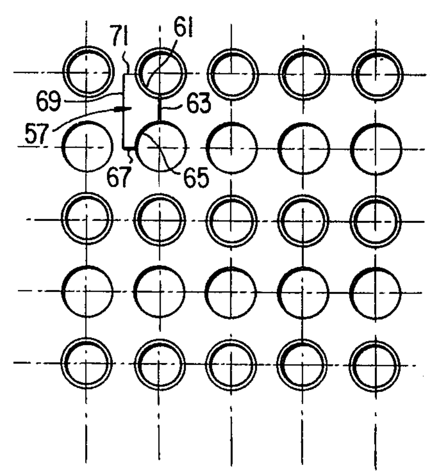

FIG.10

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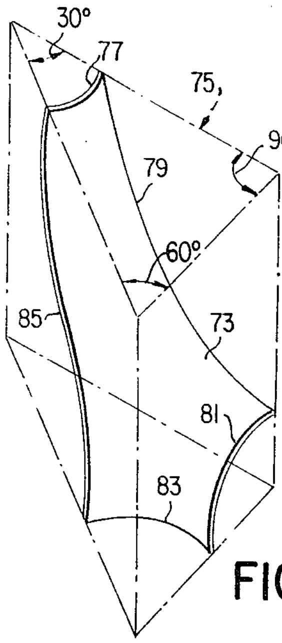


FIG. 11

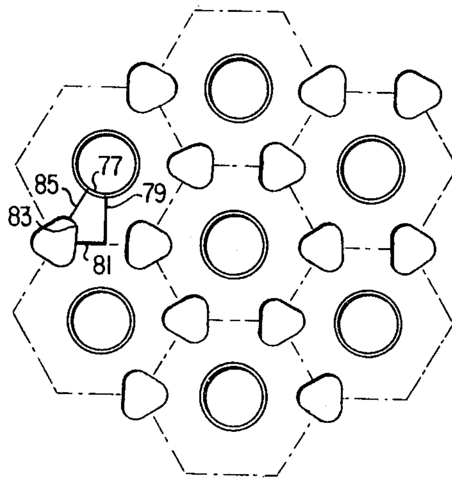
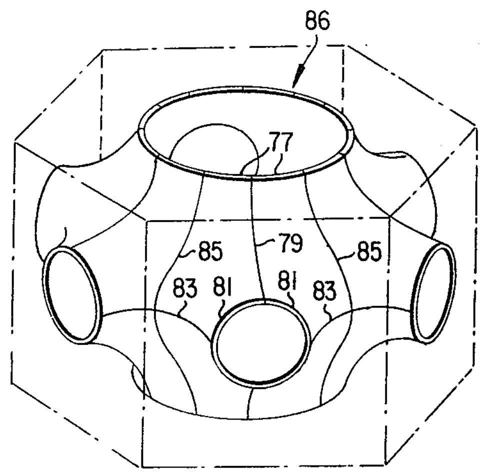


FIG. 13



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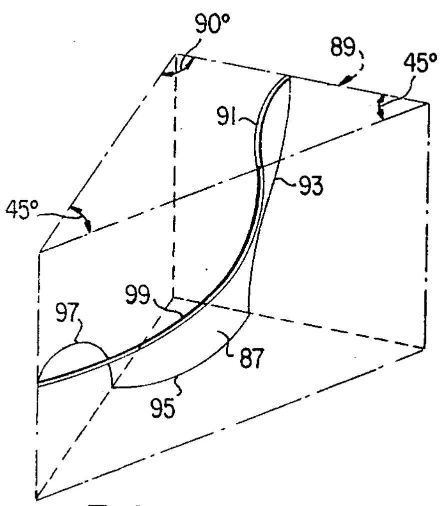

FIG. 14

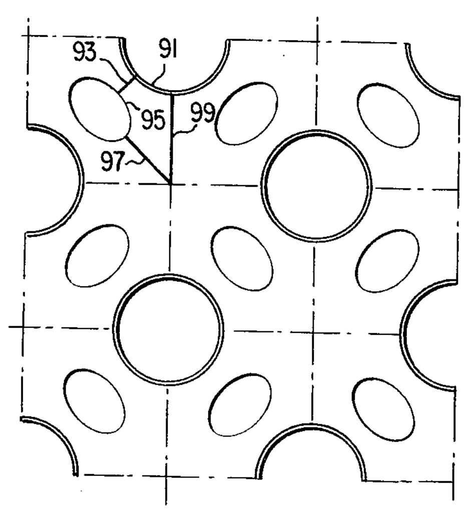

FIG. 15

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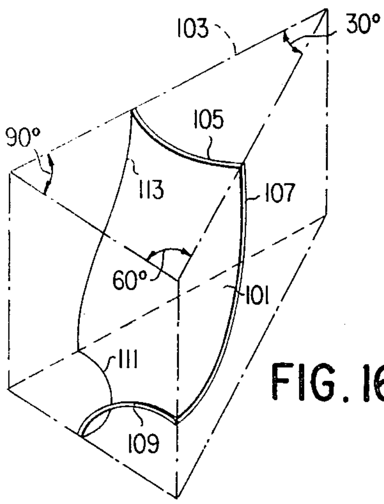

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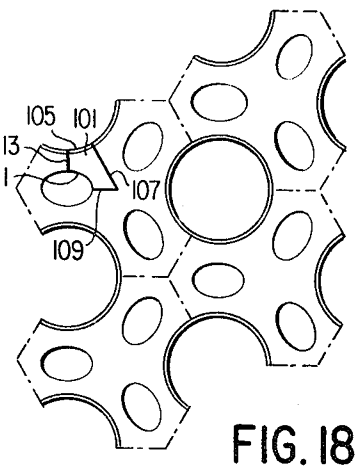

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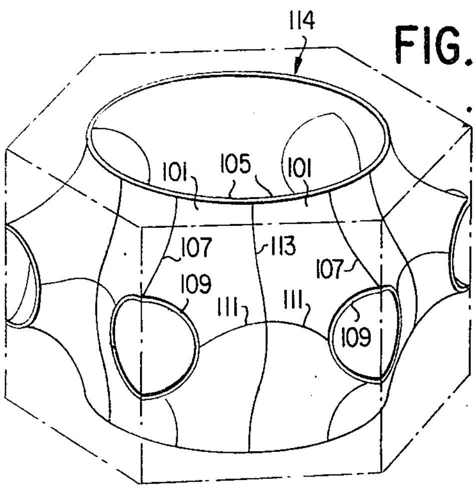
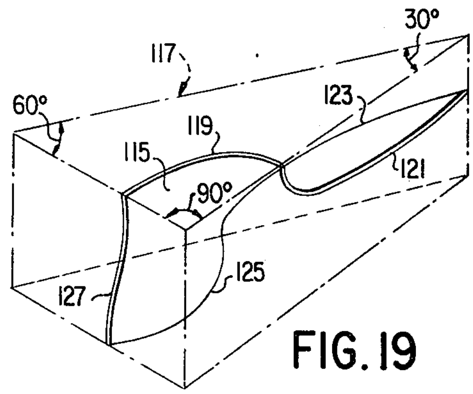
FIG. 17





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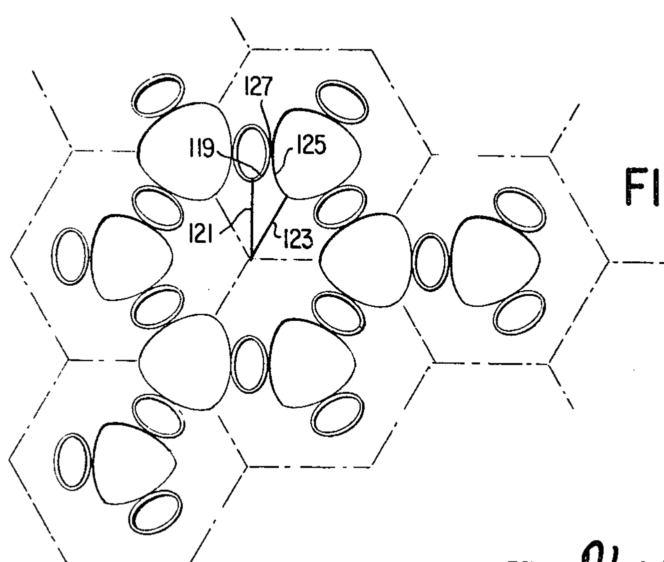

FIG. 20

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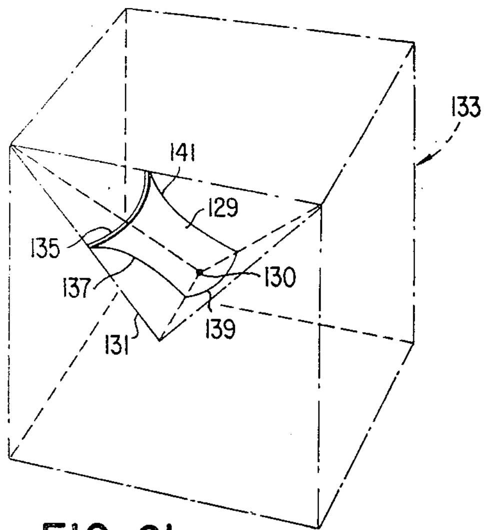
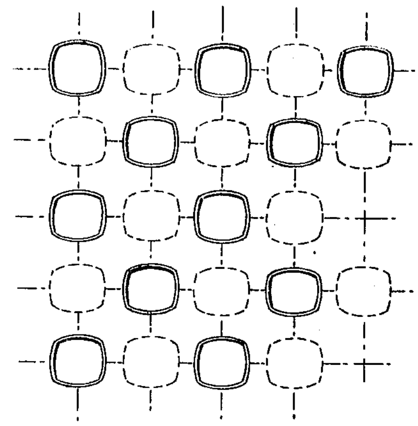
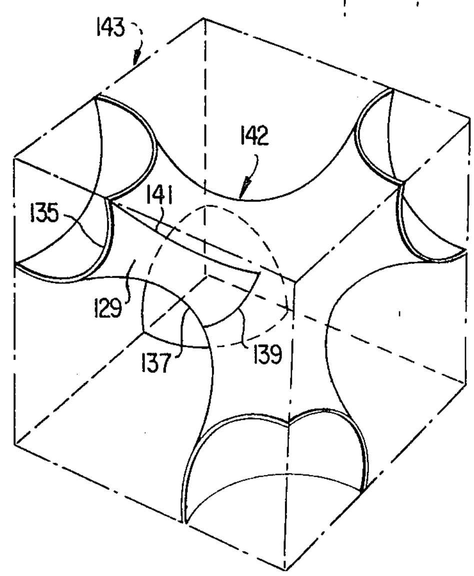


FIG. 21

FIG. 23





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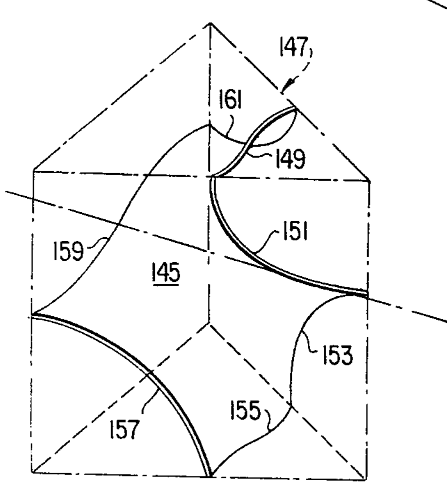

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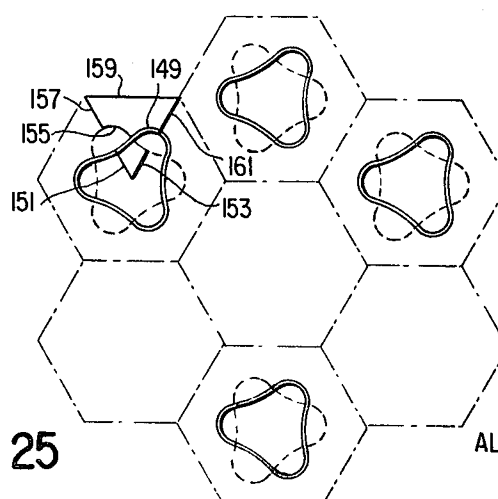

FIG. 25

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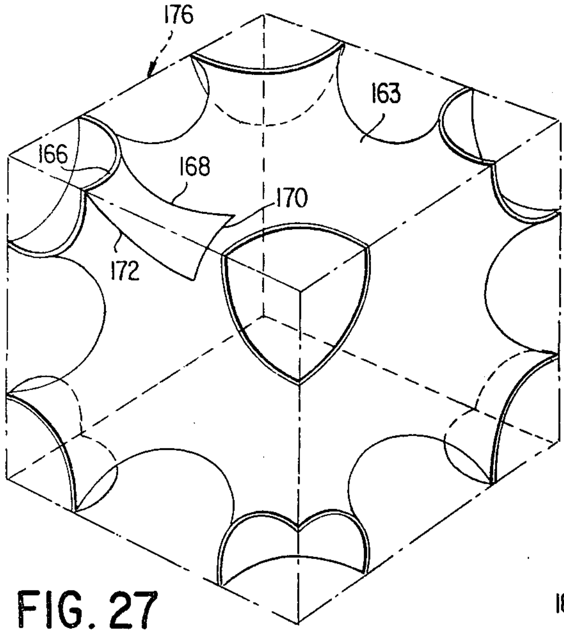

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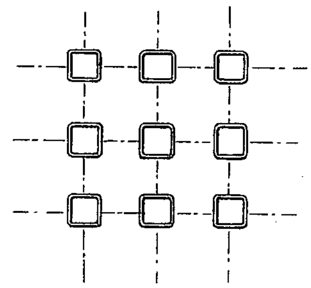
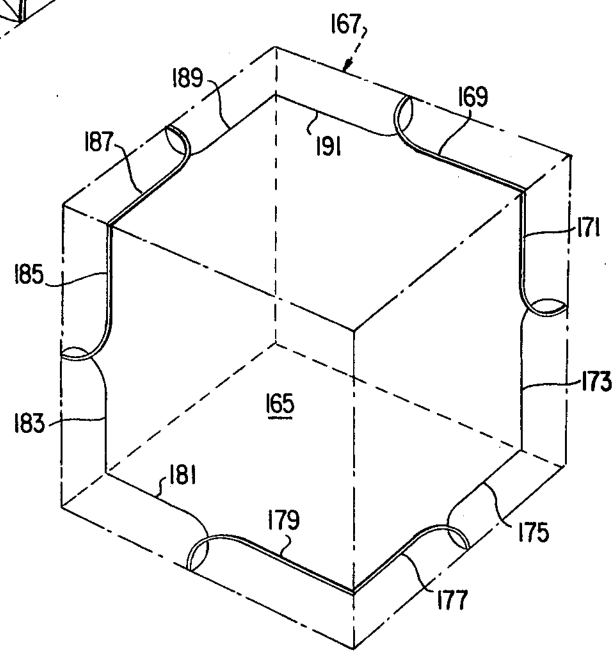
FIG. 28





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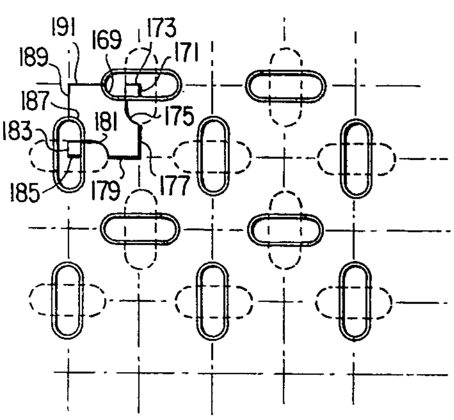

FIG. 30

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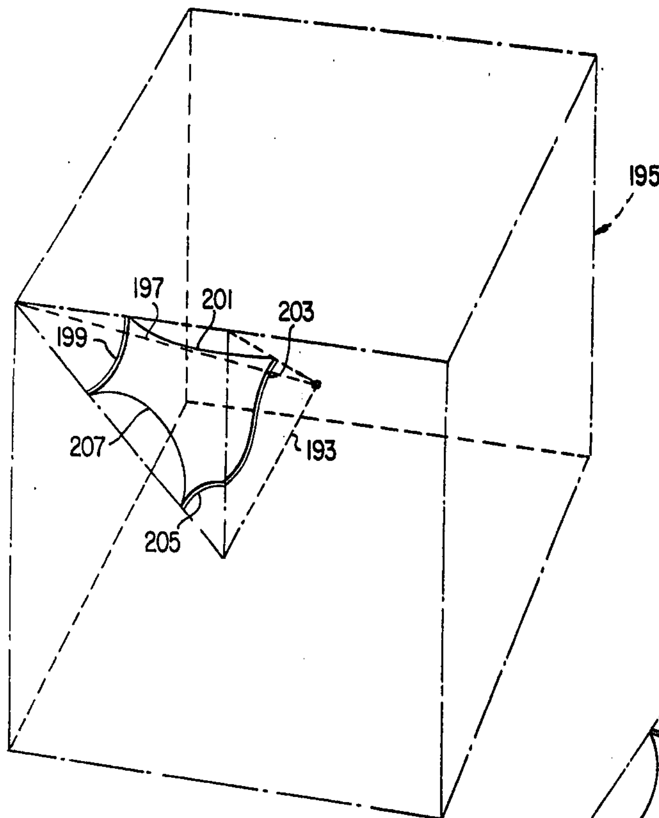
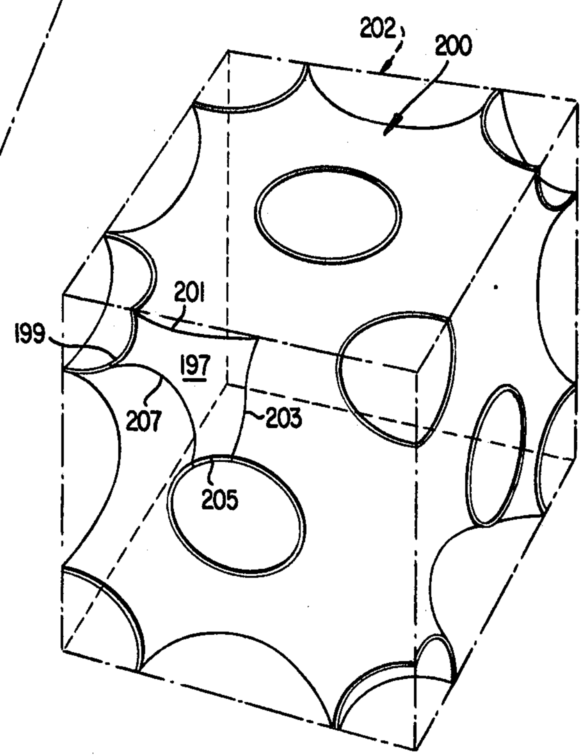


FIG. 31

FIG. 32



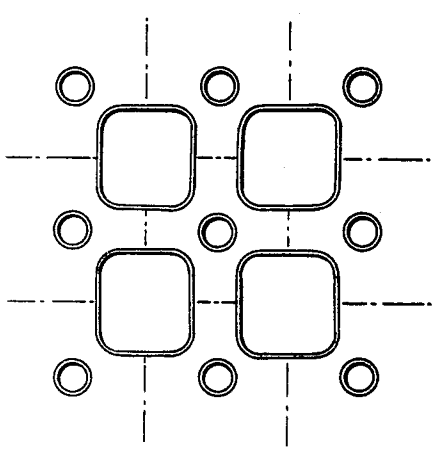


FIG. 33

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HONEYCOMB CORE STRUCTURES OF MINIMAL SURFACE TUBULE SECTIONS

ORIGIN OF THE INVENTION

The invention described herein was made by an employee of the United States Government and may be manufactured and used by or for the Government for governmental purposes without the payment of any royalties thereon or therefor.

BACKGROUND OF THE INVENTION

This invention relates to honeycomb cores and more particularly to honeycomb cores having low weight, and high strength and rigidity.

Various types of honeycomb cores have been proposed and are in use. One common use of honeycomb cores is in panels for use in construction as desk tops, furniture panels, walls, floors, etc. Honeycomb cores are also used in the construction of boxes for transporting various items. Honeycomb cores have also found use in other structures, such as spacecraft frames, air frames, heat exchangers and reinforced liquid and gas tanks of varying geometrical shapes. In general, honeycomb cores are used in these and other structures when it is desired to have a high strength-to-weight ratio in the resultant structure. The honeycomb cores uniquely provide this desired result because they are labyrinths that disperse the stresses applied to one of the boundary surfaces of the overall structure. Honeycomb cores have been formed of metal, plastic, pressed paper and other materials that provide a relatively rigid structure when honeycomb formed.

It will be appreciated from the foregoing brief description of honeycomb cores that the optimum honeycomb core will have a maximum amount of strength and rigidity for the least possible weight. While various attempts have been made to optimize the strength-to-weight ratio of honeycomb cores, they have not been entirely successful. Originally, honeycomb cores were formed from a plurality of corrugated sheets stacked in planes whereby hollows were formed in the overall honeycomb core structure. The major problem with honeycomb core structures of this nature is that discontinuities are created where the corrugated sheets intersect. These discontinuities create weak points making the honeycomb cores subject to rupture and shear, particularly along the planes defined by the intersections. More recently, honeycomb core structures have been formed of curved cellular members that eliminate many internal intersections. While these approaches have improved the strength-to-weight ratio of honeycomb core structures, they have not been entirely satisfactory because some discontinuities still exist within the overall core structures, particularly where the curves intersect.

Therefore, it is an object of this invention to provide new and improved honeycomb core structures.

It is a further object of this invention to provide honeycomb core structures that have maximum strength-to-weight ratios.

It is another object of this invention to provide new and improved honeycomb core structures that lack internal discontinuities, are light in weight, and have maximum strength-to-weight ratios.

It will be appreciated by those skilled in the art and others that one of the major problems of prior art honeycomb structures is that while they may provide maximum rupture strength in one direction (usually the direction that is orthogonal to the parallel planes defined by their outer surface), they have limited strength-to-weight ratios in other directions, such as shear strength along their discontinuity planes. Consequently, while many prior art honeycomb core structures can be utilized to form panels, they cannot be utilized in thicker structures, such as air frames, gas and liquid tanks and other similar structures wherein rupture forces may be applied in various directions. In other words, prior art honeycomb structures are not normally utilized to form relatively bulky structures because their primary strength-to-weight ratio lies in only one direction.

Consequently, it is a further object of this invention to provide new and improved honeycomb core structures that have maximum strength-to-weight ratios in various directions.

It is a still further object of this invention to provide new and improved honeycomb core structures that can be utilized to strengthen various types of geometrical structures by providing high strength-to-weight ratios in a plurality of directions.

SUMMARY OF THE INVENTION

In accordance with principles of this invention, honeycomb core structures formed of tubule sections bound orthogonally by plane facings are provided. The tubule sections are arrayed in an essentially infinite periodic manner. The tubule section intersections are smooth whereby the resultant honeycomb structures lack discontinuities. The tubule sections are formed of minimal surface elements that are defined within imaginary kaleidoscopic cells. By minimal surface is meant that the elements have mean curvature at all points on their surfaces that equal zero. In other words, the honeycomb structures formed in accordance with the invention can be described as a periodic array of smoothly interconnected tubule sections, each section being formed of elements that have minimal surfaces with the minimal surfaces being defined within imaginary kaleidoscopic cells.

In accordance with further principles of this invention, the tubule sections are arranged such that they form two parallel plane facings (boundary planes), thus defining a honeycomb sandwich. The tubule boundary curves at the parallel plane facings are smooth simple closed curves. In addition, each tubule boundary curve intersects the plane facings at an angle of exactly 90° at every point on the tubule boundary curve. This orthogonal intersection grants maximum strength and rigidity to the assembled honeycomb structure.

In accordance with other principles of this invention, the array of tubule sections results in honeycomb structures that have other boundary planes wherein the tubule boundary curves are smooth simple closed curves that also intersect these planes at exactly 90°. The additional boundary planes orthogonally intersect the parallel plane facings and intersect one another at predetermined angles.

It will be appreciated from the foregoing brief summary of the invention that honeycomb structures having optimum strength-to-weight ratios are provided by the invention. More specifically, the honeycomb core structures are based on minimal surface forms in doubly-curved surface configurations whereby the mechanism of membrane action is operative. This action leads to the maximum diffusion of applied loads throughout the entire structure away from the point of load application. Hence, maximum strength for a given weight is provided by the invention. In addition, because the tubule sections that make up the overall honeycomb structures are smoothly interconnected, there are no internal discontinuities in the honeycomb structure. Hence, the structurally weak points normally associated with such discontinuities are eliminated. Further, because the honeycomb core structures of the invention define boundary planes which they intersect at right angles, they can support large loads at those planes. That is, rather than axially project such loads toward an opposing plane in the manner of a column, the invention disperses the load throughout the core toward many points in the opposing plane as well as toward side or end planes. Also, while the inventive honeycomb cores can be used in panels, they can also be used to form more bulky structures. Moreover, while the cores to some extent define the shapes of such structures, they can also be used with other structural shapes, such as air frames or spheres, with a slight loss in strength due to intersection of angles other than 90°.

BRIEF DESCRIPTION OF THE DRAWINGS

The foregoing objects and many of the attendant advantages of this invention will become more readily appreciated as the same becomes better understood from the following detailed description when taken in conjunction with the accompanying drawings, wherein:

FIG. 1 is a perspective view of a minimal surface element of a tubule section defined by an imaginary rectangular parallelopiped;

FIG. 2 is a perspective view of a tubule section formed of minimal surface elements of the type illustrated in FIG. 1;

FIG. 3 is a top view of a honeycomb core structure formed of an array of tubule sections of the type illustrated in FIG. 2;

FIG. 4 is a perspective view of a minimal surface element of a tubule section defined by an imaginary equilateral triangular prism;

FIG. 5 is a perspective view of a tubule section formed of minimal surface elements of the type illustrated in FIG. 4;

FIG. 6 is a top view of a honeycomb core structure formed of an array of tubule sections of the type illustrated in FIG. 5;

FIG. 7 is a perspective view of an alternate embodiment of a minimal surface element of a tubule section defined by an imaginary equilateral triangular prism;

FIG. 8 is a top view of a honeycomb core structure formed of an array of tubule sections formed of minimal surface elements of the type illustrated in FIG. 7;

FIG. 9 is a perspective view of an alternate embodiment of a minimal surface element of a tubule section defined by an imaginary rectangular parallelopiped;

FIG. 10 is a top view of a honeycomb core structure formed of an array of tubule sections formed of minimal surface elements of the type illustrated in FIG. 9;

FIG. 11 is a perspective view of a minimal surface element of a tubule section defined by an imaginary 30°–60° triangular prism;

FIG. 12 is a perspective view of a tubule section formed of minimal surface elements of the type illustrated in FIG. 11;

FIG. 13 is a top view of a honeycomb core structure formed of an array of tubule sections of the type illustrated in FIG. 12;

FIG. 14 is a perspective view of a minimal surface element of a tubule section defined by an imaginary 45° triangular prism;

FIG. 15 is a top view of a honeycomb core structure formed of an array of tubule sections formed of minimal surface elements of the type illustrated in FIG. 14;

FIG. 16 is a perspective view of an alternate embodiment of a minimal surface element of a tubule section defined by an imaginary 30°–60° triangular prism;

FIG. 17 is a perspective view of a tubule section formed of minimal surface elements of the type illustrated in FIG. 16;

FIG. 18 is a top view of a honeycomb core structure formed of an array of tubule sections of the type illustrated in FIG. 17;

FIG. 19 is a perspective view of an alternate embodiment of a minimal surface element of a tubule section defined by an imaginary 30°–60° triangular prism;

FIG. 20 is a top view of a honeycomb core structure formed of an array of tubule sections formed of minimal surface elements of the type illustrated in FIG. 19;

FIG. 21 is a perspective view of a minimal surface element of a tubule section defined by an imaginary trirectangular tetrahedron;

FIG. 22 is a perspective view of a tubule section formed of minimal surface elements of the type illustrated in FIG. 21;

FIG. 23 is a top view of a honeycomb core structure formed of an array of tubule sections of the type illustrated in FIG. 22;

FIG. 24 is a perspective view of an alternate embodiment of a minimal surface element of a tubule section defined by an imaginary equilateral triangular prism;

FIG. 25 is a top view of a honeycomb core structure formed of an array of tubule sections formed of minimal surface elements of the type illustrated in FIG. 24;

FIG. 26 is a perspective view of a minimal surface element of a tubule section defined by an imaginary quadrirectangular tetrahedron;

FIG. 27 is a perspective view of a tubular section formed of minimal surface elements of the type illustrated in FIG. 26;

FIG. 28 is a top view of a honeycomb core structure formed of an array of tubule sections of the type illustrated in FIG. 27;

FIG. 29 is a perspective view of an alternate embodiment of a minimal surface element of a tubule section defined by an imaginary rectangular parallelopiped;

FIG. 30 is a top view of a honeycomb core structure formed of an array of tubule sections formed of minimal surface elements of the type illustrated in FIG. 29;

FIG. 31 is a perspective view of an alternate embodiment of a minimal surface element of a tubule section defined by an imaginary quadrirectangular tetrahedron;

FIG. 32 is a perspective view of a tubule section formed of minimal surface elements of the type illustrated in FIG. 31; and,

FIG. 33 is a top view of a honeycomb core structure formed of an array of tubule sections of the type illustrated in FIG. 32.

DESCRIPTION OF THE PREFERRED EMBODIMENTS

The following description describes a set of examples of distinct structural honeycomb structures formed in accordance with the invention. Each honeycomb structure can be described as an arrangement of smoothly interconnected tubule sections where the sections are based on the forms of periodic minimal surfaces. That is, each tubule section can be described as being formed of minimal surface elements and each minimal surface element can be defined in an imaginary kaleidoscopic cell. A minimal surface is herein defined as a surface wherein the mean curvature at any point on the surface is zero and the surface is not a plane.

The kaleidoscopic cells, of which there are only seven, contemplated by the invention have a specific definition. They are defined as the seven convex polyhedra which are fundamental regions of discrete groups generated by reflections. In this regard, reference is made to a text entitled "Regular Polytopes" by H. S. M. Coxeter, published by MacMillan, New York, 1963. For purposes of this disclosure, the seven cells are also defined as follows: (1) a rectangular parallelopiped — a right rectangular prism; (2) a tetragonal disphenoid — an isogonal, isohedral tetrahedron with isoceles triangular faces having a base to side ratio of $(2/3)\sqrt{3 \text{ to } 1}$; (3) a trirectangular tetrahedron — one-half of a tetragonal disphenoid obtained by sectioning a tetragonal disphenoid along a plane of mirror symmetry; (4) a quadrirectangular tetrahedron - one-half of a trirectangular tetrahedron obtained by sectioning a trirectangular tetrahedron along its plane of mirror symmetry; (5) an equilateral tri-angular prism — a right prism wherein the base faces form equi-lateral triangles; (6) a 45° triangular prism - a right prism wherein the base faces are 45° right triangles; and (7) a 30°–60° triangular prism — a right prism wherein the angles of the base faces are 30°, 60° and 90°.

The minimal surface elements of the invention also have the requirement that they intersect all faces of the imaginary kaleidoscopic cells in which they are defined at least once and at orthogonal angles. In short, each of the elements that make up a tubule section of the type hereinafter described has three requirements: (1) it must be a minimal surface element, i.e., the mean curvature of all points on its surface must equal zero and the surface must not be a plane; (2) it must be definable in an imaginary kaleidoscopic cell along its various edges, i.e., the curved edges of the element must define planes that intersect to form a kaleidoscopic cell; and, (3) it must have edges that intersect all faces of the kaleidoscopic cell at least once and at orthogonal angles.

Turning now to the embodiments of the invention illustrated in the drawings, FIG. 1 illustrates in phantom an imaginary rectangular parallelopiped 11. Located inside of the imaginary rectangular parallelopiped 11 is a minimal surface element 13. The imaginary rectangular parallelopied is, of course, one of the seven kaleidoscopic cells and is merely used herein to help define the minimal surface element 13. More specifically, the minimal surface element 13 has six curved edges, one of which intersects each of the six faces of the imaginary rectangular parallelopiped 11. That is, the upper edge 15 of the minimal surface element 13 (as viewed in FIG. 1) intersects the upper face of the imaginary rectangular parallelopiped 11. A first rear edge 17 intersects one of the rear faces of the imaginary rectangular parallelopiped. A first front edge 19 intersects one of the front faces of the imaginary rectangular parallelopiped and a bottom edge 21 intersects the bottom face of the imaginary rectangular parallelopiped. Similarly, a second front edge 23 intersects the other front face of the imaginary rectangular parallelopiped 11 and a second rear edge 25 intersects the other rear face of the imaginary rectangular parallelopiped.

In addition to the fact that one edge of the minimal surface 13 intersects each of the faces of the imaginary rectangular parallelopiped 11, it should also be noted that these edges at the point of intersection meet the faces orthogonally. That is, while there is a continuous curvature to the minimal surface 13 at all points, which curvature has a mean value at all points equal to zero, the curvatures are such that when they intersect the surfaces of the imaginary rectangular parallelopiped the intersections are orthogonal. In other words, the planes defined by the six curved edges intersect to form the imaginary rectangular parallelopiped 11.

FIG. 2 illustrates a tubule section 26 formed in accordance with the invention which comprises eight minimal surface elements of the type illustrated in FIG. 1 joined in a continuous curved manner. The minimal surface elements 13 are joined in such a manner that the overall tubule section 26 fits exactly inside of an imaginary rectangular parallelopiped 28 that encloses a volume that is exactly eight times the volume of the imaginary rectangular parallelopiped 11 illustrated in FIG. 1. In addition, as just stated, the eight minimal surface elements 13 are attached together in an image manner (related by reflection) such that a smooth continuous closed curve of approximately circular shape is formed in each of the faces of the imaginary rectangular parallelopiped 28. In other words, the resultant tubule section 26 has six outer closed curved ends which define six planes. These six planes intersect to form the imaginary rectangular parallelopiped 28.

By joining tubule sections 26 of the type illustrated in FIG. 2 together at the closed curved ends located in the six faces of the imaginary rectangular parallelopiped 28, a honeycomb core structure formed of tubule sections is formed. FIG. 3 is a top view of such an array of tubule sections 26. For ease of illustration, only the upper left hand corner of FIG. 3 specifically illustrates the four minimal surface elements 13 that fully define the overall honeycomb core array. The reference numbers utilized in the upper left hand corner of FIG. 3 are the same as those used in FIG. 1 so that the location of the various edges of the minimal surface elements is easily observable.

It should be noted that while FIG. 3 is designated as the top view of a honeycomb core structure formed in accordance with the invention utilizing tubule sections of the type illustrated in FIG. 2, it could just as easily be one of the side views defined by either of the side planes. It should also be noted that the honeycomb core structure illustrated in FIG. 3 is an essentially infinite periodic array, hence, the invention is an essentially infinite periodic minimal surface honeycomb core structure. The honeycomb core is infinite along the three principle axes of a three dimensional structure whereby the honeycomb core can be used in bulky as well as plane structures. That is, the honeycomb core structure of the invention has great resistance to side forces, as well as to top and bottom forces, particularly in the planes defined by the edges of the tubule sections. Hence, not only can the invention be utilized to provide a honeycomb core for panels, it can also be utilized to provide a honeycomb core for other structures, such as gas and liquid tanks and containers. Finally, it should be also noted that while the geometry of the tubule sections defines planes which define the "best" shape of any structure using the core, such shape is not absolute. That is, the tubule core structures of the invention, can be used in other types of geometric structure even though there may be some loss in strength at the surface of the structure due to other than 90° intersections.

FIG. 4 illustrates a minimal surface element 27 defined within an imaginary equilateral triangular prism 30. As illustrated in FIG. 4, an equilateral triangular prism has five faces. These five faces are intersected by the five edges 29, 31, 33, 35 and 37 of the minimal surface element 27. As with the minimal surface element 13 illustrated in FIG. 1, the mean curvature at all points on the surface of the minimal surface element 27 illustrated in FIG. 4 is zero. In addition, all surfaces of the imaginary equilateral triangular prism 30 are intersected by an edge of the minimal surface element 27. Further, all of the intersections are orthogonal. Moreover, the minimal surface element 27 is symmetrical about an axis of 2-fold symmetry 38.

FIG. 5 illustrates a tubule section formed of twelve minimal surface elements 27 of the type illustrated in FIG. 4. By joining the minimal surface elements in the image manner illustrated in FIG. 2 with regard to the minimal surface elements 13 illustrated in FIG. 1, an imaginary geometric shape 39 is defined. The imaginary geometric shape 39 illustrated in FIG. 5 has a regular hexagonal top and bottom connected by vertical sides that intersect the top and bottom orthogonally. Continuous closed curves intersect the top and bottom of the imaginary geometric structure. In addition, closed curves are formed at alternate intersections of the vertical sides of the geometric structure.

FIG. 6 is a top view of a periodic tubules, core array of tubule sections of the type illustrated in FIG. 5. The FIG. 6 array has a generally hexagonal periodicity. As with the array illustrated in FIG. 5, the array illustrated in FIG. 6 is essentially infinite in nature whereby it can be enclosed by any three dimensional body of any size, as desired. However, a planar top, bottom and sides is defined by the honeycomb core. That is, the array illustrated in FIG. 6 is a honeycomb structure that provides maximum strength for a minimum weight. However, because the strength is maximum along the axes defined by the tubules, the strength of the geometric structure enclosing the honeycomb core is maximum when the structure is defined by the array termination planes. As with FIG. 3, FIG. 6 includes a region having the reference numbers given to the minimal surface element 27 in FIG. 4.

FIG. 7 illustrates an alternate embodiment of a minimal surface element 41 defined inside of an imaginary equilateral triangular prism 43. The minimal surface element 41 illustrated in FIG. 7 is similar to the minimal surface element 27 illustrated in FIG. 4 in that it intersects all faces of the imaginary equilateral triangular prism 43 orthogonally; however, it differs in that it intersects one of the faces more than once. In other words, the minimal surface element 41 illustrated in FIG. 7 has six edges, 45, 47, 49, 51, 53 and 55, and edges 47 and 53 intersect the same face of the imaginary equilateral triangular prism 43.

FIG. 8 is a top view of a honeycomb core array formed of tubule sections of minimal surface elements 41 of the type illustrated in FIG. 7. The intermediate tubule section figure between FIGS. 7 and 8 is not illustrated due to drawing difficulty; however, it can be constructed in the same manner as previously described with respect to FIGS. 2 and 5. In addition, only the various edges of one minimal surface element are illustrated in one portion of FIG. 8; however, it will be obvious that this illustration can be extended as in FIGS. 3 and 6. It should be noted that while the honeycomb array core illustrated in FIGS. 3 and 6 is formed of closed curves of approximately circular forms, the closed curves illustrated in FIG. 8 depart significantly from circular shape. Rather, the closed curves that are illustrated in FIG. 8 are in the form of three-leaf clovers. In essence, however, these curves are still simple closed curves.

FIG. 9 illustrates an alternative embodiment of a minimal surface element 57 formed inside of an imaginary rectangular parallelopiped 59. Again, the minimal surface element intersects all of the faces of the imaginary rectangular parallelopiped 59 at right angles. These intersections create six edges 61, 63, 65, 67, 69 and 71.

FIG. **10** is a top view of an array of tubule sections formed of minimal surface elements **57** of the type illustrated in FIG. **9**. Again, for ease of understanding, the minimal surface edges are numbered in one region of FIG. **10**. Moreover, due to drawing difficulties, the tubule section intermediate between FIGS. **9** and **10** is not illustrated; however, it can be formed, if desired.

FIG. **11** illustrates a minimal surface element **73** defined inside of an imaginary 30°–60° triangular prism **75**. The minimal surface element **73** illustrated in FIG. **11** intersects each of the five faces of the imaginary 30°–60° triangular prism **75** once, and all of the intersections are orthogonal. Hence, the minimal surface element **73** has five edges **77, 79, 81, 83**, and **85**.

FIG. **12** illustrates a tubule section **86** formed of 24 minimal surface elements of the type illustrated in FIG. **11**. The tubule section illustrated in FIG. **12** can be defined inside of an imaginary geometrical structure having a regular hexagonal top and bottom and sides that intersect the hexagonal top and bottom at right angles. This imaginary geometric structure has a closed curved tubule end in the center of each of its faces. For ease of understanding, two minimal surface elements **73** are defined in FIG. **12** by the edge reference numbers given to the minimal surface element in FIG. **11**.

FIG. **13** is a top view of a honeycomb core formed of an array of tubule sections of the type illustrated in FIG. **12**. It can be seen that FIG. **13** is a hexagonal periodic array which is infinite. In the center of each hexagon is an approximately circular section. The junctions of the hexagons are defined by rounded triangular-like closed curved regions. Again, the reference numbers for a single minimal surface element are shown in one portion of FIG. **13** so that the overall array can be more easily understood.

FIG. **14** illustrates a minimal surface element **87** defined inside of an imaginary 45° triangular prism **89**. The edges of the minimal surface element intersect each face of the right triangular prism **89** once. Each of these intersections is orthogonal. Hence, the minimal surface element **87** illustrated in FIG. **14** has five edges, **91, 93, 95, 97** and **99**.

FIG. **15** is a top view of a honeycomb structure formed of an array of tubule sections formed of minimal surface elements **87** of the type illustrated in FIG. **14**. In order to better understand the array relationship, a region utilizing the edge reference number designation given in FIG. **14** is illustrated in FIG. **15**. Again, an intermediate tubule section between FIGS. **14** and **15** is not shown because of the difficulty in illustrating such a section; however, it can be made.

FIG. **16** illustrates a minimal surface element **101** defined inside of an imaginary 30°-60° triangular prism **103**. An edge of the minimal surface element **101** intersects each of the faces of the imaginary right triangular prism **103** in an orthogonal manner only once. Hence, the minimal surface **101** illustrated in FIG. **16** has five edges **105, 107, 109, 111** and **113**.

FIG. **17** illustrates a tubule section **114** formed of twenty-four minimal surface elements **101** of the type illustrated in FIG. **16**. And, FIG. **18** is a top view of a honeycomb core structure formed of tubule sections of the type illustrated in FIG. **17**. FIG. **18** includes in one area the edge reference numbers given to the minimal surface element **101** in FIG. **16** so that the layout of the array can be more easily understood. It will be appreciated from viewing FIGS. **16, 17** and **18** that the periodicity of the array of tubule sections is generally hexagonal in nature.

FIG. **19** illustrates a minimal surface element **115** defined inside of an imaginary 30°–60° triangular prism **117**. As with the previously described embodiments, the minimal surface **115** intersects each of the faces of the imaginary 30°–60° triangular prism **117** once, and each intersection is orthogonal whereby the minimal surface element **115** has five edges **119, 121, 123, 125** and **127**.

FIG. **20** is a top view of a honeycomb core structure formed of tubule sections formed of an array of minimal surface elements **115** of the type illustrated in FIG. **19**. For ease of understanding the honeycomb core array illustrated in FIG. **20**, the reference numbers used to define the minimal surface element **115** in FIG. **19** are displayed in one region. As with previously described embodiments, the intermediate tubule section between FIGS. **19** and **20** is not illustrated due to the difficulty of illustrating it; however, it can be made.

FIG. **21** illustrates a minimal surface element **129** defined inside of an imaginary trirectangular tetrahedron **131**. In order to better illustrate the form of a trirectangular tetrahedron, it is illustrated in FIG. **21** formed inside of an imaginary cube **133**. In essence, a trirectangular tetrahedron may be defined as a geometric figure starting from the center **130** of the cube. The central point expands outwardly along three planes. The three planes are described as: (1) a plane that contains one of the edges of the cube; (2) a plane that contains a diagonal of a face of the cube adjacent to the edge; and, (3) a plane that contains the other diagonal of the same adjacent face of the cube. The trirectangular tetrahedron is enclosed by the thusly designated face of the cube. As can be seen from FIG. **21**, a trirectangular tetrahedron has four faces wherein the four edges **135, 137, 139** and **141** of the minimal surface element **129** lie.

FIG. **22** illustrates a tubule section **142** formed of minimal surface elements **129** of the type illustrated in FIG. **21**. The tubule section illustrated in FIG. **22** is defined inside of an imaginary cube **143**. FIG. **23** is a top view of honeycomb core structure formed of tubule sections of the type illustrated in FIG. **22**. Due to the difficulty of illustrating minimal surface elements in FIG. **23**, they are not illustrated therein. However, one is illustrated in FIG. **22**.

FIG. **24** illustrates a minimal surface element **145** defined inside of an imaginary equilateral triangular prism **147**. As can be seen from FIG. **24**, the minimal surface element **145** has two more edges than the number of faces in an imaginary equilateral triangular prism **147**. More specifically, the minimal surface **145** illustrated in FIG. **24** has seven edges **149, 151, 153, 155, 157, 159** and **161**. However, all of the edges including the two additional edges intersect the appropriate faces of the equilateral triangular prism orthogonally.

FIG. **25** is a top view of a honeycomb core structure formed of an array of tubule sections formed of minimal surfaces of the type illustrated in FIG. **24**. For ease of understanding the honeycomb core structure illustrated in FIG. **25**, the seven edges of the minimal surface element designated in FIG. **24** are illustrated in one region of FIG. **25**.

FIG. **26** illustrates a minimal surface element **160** defined inside of an imaginary quadrirectangular tetrahedron which is one-half of a trirectangular tetrahedron of the type illustrated in FIG. **21**. Hence, for ease of understanding the imaginary quadrirectangular tetrahedron is illustrated in FIG. **26** inside of an imaginary cube **164** as extending outwardly from the center **166** of the imaginary cube. As can be seen from FIG. **26**, a quadrirectangular tetrahedron has four faces wherein the edges **168, 170, 172** and **174** of the minimal surface element **160** lie.

FIG. **27** illustrates a tubule core section **163** formed of minimal surface elements **160** of the type illustrated in FIG. **26** defined inside of an imaginary rectangular parallelopiped **176**. FIG. **28** is a top view of an array of tubule core sections **163** of the type illustrated in FIG. **27**. Because of the difficulty in illustration, the minimal surface elements forming the honeycomb core structure illustrated in FIG. **28** are not shown. However, one such element is illustrated in FIG. **27**.

FIG. **29** illustrates a minimal surface element **165** defined inside of an imaginary rectangular parallelopiped **167** wherein the minimal surface element has two edges in each face. In other words, the minimal surface element **165** illustrated in FIG. **28** has twelve edges **169, 171, 173, 175, 177, 179, 181, 183 185, 187, 189** and **191**. All of the edges intersect the appropriate surface of the imaginary rectangular parallelopiped **167** orthogonally.

FIG. 30 is a top view of a honeycomb core structure formed of an array of tubule sections formed of minimal surface elements **165** of the type illustrated in FIG. **29**. For ease of understanding of the honeycomb core structure illustrated in FIG. **30**, the reference numerals given the minimal surface element in FIG. **29** are contained in one region.

FIG. **31** illustrates an alternate embodiment of a minimal surface element **197** defined inside of an imaginary quadrirectangular tetrahedron **193**. The imaginary quadrirectangular tetrahedron **193** is illustrated inside of an imaginary cube **195**. As can be seen from FIG. **31**, the minimal surface element **197** has four edges **199**, **201**, **203** and **205** which intersect the four faces of the imaginary quadrirectangular tetrahedron **193** orthogonally.

FIG. **32** illustrates a tubule section **200** formed of minimal surface elements **197** defined within an imaginary rectangular parallelopiped **201**. One of the minimal surface elements forming the tubule section illustrated in FIG. **32** is illustrated in one portion of FIG. **32**. FIG. **33** is a top view of a honeycomb core array formed of tubule sections of the type illustrated in FIG. **32** wherein a minimal surface element is not shown due to the difficulty of illustrating such an element in FIG. **33**.

It will be appreciated by those skilled in the art and others that the drawings and the foregoing description only illustrate and describe a limited number of the total number of honeycomb core structures that can be formed in accordance with the invention. While the illustrated and described structures are the preferred form of the invention, other forms fall within the scope thereof.

While the surface configuration of structures formed in accordance with the invention can be mathematically determined and described, a considerably less complicated and time-consuming method of determining the exact surface configuration can be used. More specifically, a kaleidoscopic cell of any of the types previously described is constructed out of plastic or some other transparent material. A soap film is constructed inside of the transparent kaleidoscopic cell and varied in position until the desired configuration is formed. The correct configuration is obtained by blowing and sucking air through a metal tube inserted into the transparent kaleidoscopic cell through a suitable hole cut out of a corner (or corners) of the cell. The configuration of the soap film is determined to be that of the desired minimal surface by observing when the film reaches the so-called stationary state, i. e., the state of unstable mechanical equilibrium. Because of the frictional drag between the soap film and the bounding faces of the enclosing transparent kaleidoscopic cell, the soap film remains stationary in this equilibrium position long enough for detailed measurements to be made of the configuration of the soap film. These measurements may be made using various optical techniques, such as the sighting, along orthogonal axes, of a reflected laser beam.

Whenever the symmetry of the minimal surface element implies that one or more straight line segments are contained in the surface of the element, such as along the 2-fold symmetry axis **38** illustrated in FIG. **4**, tightly stretched fine filaments are placed in the positions of these line segments. This operation transforms the unstable equilibrium of the soap film into a stable equilibrium and the film then remains absolutely stationary indefinitely. This situation is also illustrated in FIG. **1**. More specifically, when the enclosing rectangular parallelopiped **11** is a cube, the six-edged minimal surface element is in its most symmetrical form. At this point, all three pairs of diagonally opposite vertices of the minimal surface element **13** may be joined by straight line segments, all of which lie in the surface of the minimal surface element. By placing fine filaments along these lines, the minimal surface element remains absolutely stationary indefinitely.

As an alternate method for deriving the detailed configuration of any of the minimal surface elements considered herein, the following technique can be employed. This technique is, in fact, preferred because it will work for all of the minimal surface elements illustrated, whereas it has been found experimentally that the foregoing technique will not work well for all such minimal surface elements. In accordance with this technique a closed polygonal boundary, composed of straight line segments which are orthogonal, respectively, to the planes containing the successive (curved) edges of the desired minimal surface, is constructed from fine stretched filaments. This polygonal boundary is dipped into a stable soap solution to form a stable equilibrium minimal surface spanning the boundary. A laser beam is directed onto the film at a large number of points very near the polygonal boundary, around the entire boundary of the soap film. Measurements are made of the orientation of a line normal to the film at all of these points. Then by making use of the classical theory of Bonnet of the bending of simply connected minimal surfaces, a good approximation of the detailed shape of all of the curved edges of the desired minimal surface element is derived. When a final model of this derived boundary for the desired minimal surface element is constructed and dipped into a suitable soap solution, a stable equilibrium model of the desired minimal surface itself is obtained. From this point on, detailed optical measurements using sightings along orthogonal axes of laser-illuminated spots on the surface are employed, as described above, to determine the configuration of the desired minimal surface element.

The direct construction of a model of a given minimal surface element in the form of a soap film orthogonally bounded by the interior faces of an appropriate kaleidoscopic cell leads — in many of the cases described herein — to a detailed determination of the configuration of the minimal surface which does not depart significantly from the true mathematical form of the surface. Hence, for all intents and purposes, this form can be used to obtain an approximately minimal surface element. However, as stated above, the alternate method provides an independent means of determining the configuration of each minimal surface element and works for all of the minimal surface elements described herein.

It will be appreciated by those skilled in the art and others that among the seven kaleidoscopic cells, the three tetrahedrons have invariant proportions. However, any of the honeycomb core structures derived from the minimal surface elements defined in one of the three tetrahedral kaleidoscopic cells can also be obtained in a form of modified symmetry. The modified forms are obtained by suitable elongations or compressions of the forms described herein along one or more cubic axes of the tubule section. Such modifications are equivalent to a description of the specified honeycomb core structure which is based on a tubule section defined within a rectangular right prism, rather than a cube.

Each of the four prisms (including the rectangular parallelopiped which is a right rectangular prism) can be constructed with a variable height-to-width ratio. In the case of each prism, the maximum height-to-width ratio defined for that prism is specific to each given minimal surface element. In other words, a given minimal surface element exists, in its appropriate kaleidoscopic cell, only for height-to-width ratios up to and including some finite limiting value. In practice, the maximum height-to-width ratio is easily determined by detailed experimental investigation using the foregoing soap film techniques.

It will also be appreciated by those skilled in the art and others that the structures of the invention can be formed of many materials by various methods. For example, if honeycomb core structures formed in accordance with the invention are to be utilized in architectural building construction, the individual elements may be formed in the sizes of the minimal surface elements defined inside of a kaleidoscopic cell. These individual elements may be joined in the illustrated manner to form the resultant honeycomb core structure. In this case, the elements would be formed of reinforced concrete, for example, or of plastic, fiberglass, etc., with toothed or other edges for interconnecting purposes. Alternatively, if the honeycomb core is to be used in a smaller structure, such as a panel, heat exchanger or tank, it can be formed in composite layers of minimal surface elements. The various honeycomb core arrays illustrated as top views in the drawings show the boundary configurations of the layers or panels which can be joined to form a relatively bulky honeycomb core, if desired. If the honeycomb core is formed of plastic, injection molding techniques can be utilized. Alternatively, compression techniques can be utilized if the honeycomb core is to be formed of compressed paper or metal. Hence, various methods and materials can be utilized to form honeycomb core structures of the type heretofore described.

What is claimed is:

1. A honeycomb core structure comprising at least one tubule section, said tubule section comprising a plurality of approximately minimal surface elements formed in a smooth continuous manner.

2. A honeycomb core structure as claimed in claim 1 wherein said honeycomb core structure comprises an array of said tubule sections joined in a smooth continuous manner.

3. A honeycomb core structure as claimed in claim 2 wherein each of said plurality of minimal surface elements is definable within an imaginary kaleidoscopic cell.

4. A honeycomb core structure as claimed in claim 3 wherein each of said minimal surface elements intersect each surface of said kaleidoscopic cell at least once and each intersection is substantially orthogonal.

5. A honeycomb core structure as claimed in claim 3 wherein said imaginary kaleidoscopic cell is either a rectangular parallelopiped, a tetragonal disphenoid, a trirectangular tetrahedron, a quadrirectangular tetrahedron, an equilateral triangular prism, a 45° triangular prism, or a 30°–60° triangular prism.

6. A honeycomb core structure as claimed in claim 4 wherein each of said tubule sections terminates in a least one simple closed plane curve.

7. A honeycomb core structure as claimed in claim 4 wherein said honeycomb core structure intersects at least one boundary plane with all of said tubule sections that adjoin said boundary plane being substantially orthogonal therewith.

8. A honeycomb core structure as claimed in claim 7 wherein said all of said tubule sections that adjoin said boundary plane terminate in simple closed curves in said boundary plane.

9. A honeycomb core structure as claimed in claim 8 wherein said kaleidoscopic cell is a rectangular parallelopiped

10. A honeycomb core structure as claimed in claim 8 wherein said kaleidoscopic cell is a trirectangular tetrahedron.

11. A honeycomb core structure as claimed in claim 8 wherein said kaleidoscopic cell is a quadrirectangular tetrahedron.

12. A honeycomb core structure as claimed in claim 8 wherein said kaleidoscopic cell is an equilateral triangular prism.

13. A honeycomb core structure as claimed in claim 8 wherein said kaleidoscopic cell is a 45° triangular prism.

14. A honeycomb core structure as claimed in claim 8 wherein said kaleidoscopic cell is a 30°–60° triangular prism.

15. A honeycomb core structure formed of tubule sections selected from anyone of the tubule sections illustrated in FIGS. 2, 5, 12, 17, 22, 27 or 32.

16. A honeycomb core structure formed of tubule sections formed of minimal surface elements selected from anyone of the minimal surface elements illustrated in FIGS. 1, 4, 7, 9, 11, 14, 16, 19, 21, 24, 26, 29 or 31.

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