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(54) Titre : PROCEDE D'ESTIMATION DU SITE D'UN PROJECTILE BALISTIQUE
(54) Title: METHOD FOR ESTIMATING THE ELEVATION ANGLE OF A BALLISTIC PROJECTILE

(57) Abrégé/Abstract:
The present invention relates to the implementation of a method making it possible to determine in an accurate manner, the elevation of a projectile along a ballistic trajectory by means of the information provided by a conventional Doppler surveillance radar. The method according to the invention consists, on the basis of the quantities (A) and (B) representing respectively the first derivative and the second derivative with respect to time of the Doppler speed d of the projectile, in calculating firstly the estimate (C) of the value of the radial component Γ_ρ of the acceleration of the projectile, then in calculating on the basis of d, (A) and (C) the estimate (D) of the speed V of the projectile, then finally in calculating on the basis of d and (D) the estimate $\hat{E}$ of its angle of elevation E. The method according to the invention applies in particular to the protecting of sensitive zones from the firing of ballistic projectiles.

ABSTRACT

The present invention relates to the implementation of a method enabling precise determination of the elevation of a projectile following a ballistic trajectory by means of information provided by a conventional Doppler surveillance radar.

The method according to the invention consists in calculating first the estimate $\hat{\Gamma}_\rho$ of the value of the radial component Γ_ρ of the acceleration of the projectile from the quantities $\dot{d}$ and $\ddot{d}$, respectively representing the first derivative and the second derivative with respect to time of the Doppler velocity d of the projectile, then calculating the estimate $\hat{V}$ of the speed V of the projectile from d, $\dot{d}$ and $\hat{\Gamma}_\rho$, and finally calculating the estimate $\hat{E}$ of its angle of elevation E from d et $\hat{V}$.

The method according to the invention applies in particular to the protection of sensitive areas against the firing of ballistic projectiles.

Figure 3

METHOD FOR ESTIMATING THE ELEVATION ANGLE OF A BALLISTIC PROJECTILE

The present invention relates to the general field of protecting areas against the firing of ballistic projectiles. It relates in particular to the determination of the trajectory of a mortar shell heading towards a protected area. More particularly still, it relates to the implementation of a method enabling precise determination of the elevation of such a projectile that is detected and tracked by a conventional Doppler surveillance radar.

When it is wished to protect a given area against the firing of artillery ammunition and in particular against the firing of mortar shells, it is known practice to install a surveillance radar. A conventional type of surveillance consists notably in employing one or more short-range surveillance radars, with a range of around ten km for example. It is generally preferable to use radars operating in the UHF band, a frequency band which, compared to higher frequency bands, L band or S band for example, has the advantage of increasing the radar cross section or RCS of objects illuminated by the emitted radar wave and therefore the probability of detecting these objects.

In addition, in order to effectively protect the area considered, and in particular individuals and assets that are in the area, it is important that, as soon as a projectile entering the area is detected, it is possible to know the trajectory and, in particular, the origin and the point of impact. In general, the trajectory of the projectile is reconstructed using position measurements carried out successively over time by the radar (radial distance and angular position in terms of azimuth and elevation angle). In order to effectively reconstruct this trajectory, it is obviously necessary to have available measurements of sufficient precision. The points of departure and of impact of the projectile are then determined to a precision described by uncertainty ellipses, the size of which is a direct function of the precision of measurements carried out.

In the case of a low-altitude surveillance radar, a category to which the radars used to carry out such surveillance generally belong, a particular problem arises, a problem linked with the reflections of the radar wave from

the ground or from obstacles located on the ground, in particular buildings. These spurious reflections create interference with the radar wave, interference which is known to have the effect of impairing the precision of measurements carried out, in particular the precision of elevation (or elevation angle) measurements carried out by the radar. The impairments are all the greater when the measured position is situated at a lower altitude. In particular they make the measurements carried out on the projectile close to the time it leaves the mortar, or more generally the launcher, imprecise or even unusable.

These impairments are, in addition, all the greater when the frequency band used by the radar is lower. That is why, in an environment encouraging multiple reflections, due to the fact, for example, that the ground has little relief and constitutes a surface that strongly reflects the radar waves or alternatively due to the presence of large obstacles, one is often led to use radars operating in higher frequency bands, for example microwave bands such as L band, that are less sensitive to the phenomenon. In so doing, the benefit of a larger RCS is lost and the probability of detection for a comparable emission power is reduced.

One aim of the invention is to solve this measurement precision problem and to propose a solution making it possible to use a conventional Doppler surveillance radar operating in the UHF band to carry out measurements of the position of a ballistic projectile moving at low altitude with a precision making it possible to reconstruct the trajectory of this projectile from the measurements carried out.

To this end, the subject of the invention is a method for determining the angle of elevation of a projectile following a ballistic trajectory, characterized in that it comprises the following steps :

- a first step of estimating, at the time considered, the values of the quantities $\dot{d}$ and $\ddot{d}$ respectively representing the derivative and the second derivative with respect to time of the Doppler velocity d of the target ;

- a second step of estimating the radial component of the acceleration Γ_ρ of the projectile, this radial component being defined by the following approximation relation :

$$\ddot{d} \approx -2 \cdot \frac{d\Gamma_\rho}{dt}$$

- a third step of estimating the value of the velocity V of the projectile at the time considered, the estimated velocity being defined by the following relation :

$$\hat{V} = \left(d^2 - \dot{d} \cdot R - \hat{\Gamma}_\rho \cdot R\right)^{1/2}$$

- a fourth step of estimating the value of the estimation E of the elevation of the projectile at the time considered, this elevation being estimated by integration as a function of time from the following relation :

$$\frac{d\hat{E}}{dt} = \frac{1}{R} \cdot \left(\hat{V}^2 - d^2\right)^{1/2}$$

According to a variant implementation, the method according to the invention furthermore comprises, between the third and fourth steps, an intermediate step of estimating the value of the attitude angle presented by the projectile at the time considered, the attitude angle being defined by the following relation :

$$d = V \cdot \cos\psi$$

the estimation of the attitude angle ψ presented by the projectile making it possible to plot the value of the radar cross section presented by the projectile.

According to another variant implementation, the method according to the invention furthermore comprises a complementary step of calculating the precision σ_V^2 of the estimation of the velocity of the projectile and the precision σ_E^2 of the estimation of the angle of elevation, σ_V^2 and σ_E^2 being respectively defined by the following relations :

$$\sigma_V^2 \approx \frac{1}{4 \cdot \hat{V}^2} \cdot \left[4 \cdot d^2 \cdot \sigma_d^2 + K \cdot \frac{R^2 \cdot \sigma_d^2}{\Delta t^2} + \sigma_R^2 \cdot \left(\dot{d}^2 + \hat{\Gamma}_\rho^{\,2}\right)\right],$$

$$\sigma_E^2 = \frac{\Delta t^2}{2 \cdot R^2 \cdot \left(\hat{V}^2 - d^2\right)} \cdot \left[\left(\frac{\hat{V}^2 - d^2}{R} \right)^2 \cdot \sigma_R^2 + \hat{V}^2 \cdot \sigma_V^2 + \dot{d}^2 \cdot \sigma_d^2 \right]$$

in which the following quantities respectively represent :

- d, the Doppler velocity of the projectile,
- σ_d, the precision of d,
- R, the radial distance measured,
- σ_R, the precision of R,
- Δt, measurement refresh time.

The features and advantages of the invention will be better appreciated through the following description, which description draws on the appended figures, which figures present :

- figure 1, the coverage diagram of a UHF surveillance radar ;
- figure 2, a schematic illustration of the ballistic trajectory followed by the projectiles to which the invention more particularly applies ;
- figure 3, a flow chart of the method according to the invention in principle ;
- figures 4 and 5, curves demonstrating the conformity of estimates of the angle of elevation made by the method according to the invention with the ballistic trajectory model.

Figure 1 is considered first, which presents an example of a coverage diagram for a UHF (Ultra High Frequency) radar of the type often used for carrying out surveillance of the airspace surrounding an area to be protected and for detecting and tracking the trajectory of mortar shells. This diagram, marked by the curve 11, shows how the range of the radar considered varies as a function of the elevation angle. As can be observed in the figure, and as previously said, the elevation (elevation angle) coverage diagram shows large variations in the range of the radar as a function of the elevation, notably in relation to low elevation angles, typically between 0 and 20°.

Thus in the example of the figure, it is observed that the radar has a maximum range for very low elevation angles, with a maximum for elevation angles, elevations, extending from 1.5° to 4° or alternatively for elevations with a value situated around 8°, and a shorter range for the elevation angles between 4° and 8° and for elevation angles between 10° and 20°.

This range dispersion as a function of the elevation, which affects radars operating at UHF and which is due notably to the strong influence of multiple reflections of the signal and in particular of reflections from the ground, results in detection losses or even in late detection of projectiles having a particular angle of elevation corresponding to an area of short radar range. Thus, for a radar having a coverage diagram such as that of figure 1, a projectile approaching the radar with an angle of elevation of around 4° will be detected only from a relatively short distance, of around 7 km, while the same projectile approaching with an angle of elevation of 8° will for its part be detected earlier at a distance greater than 10 km. In addition, insofar as a projectile generally follows a trajectory that causes it to approach its objective with a variable angle, it is possible for an initially detected projectile to no longer be the object of detection for a given time lapse and consequently for the threat that it represents to be ignored.

Figure 2 represents a typical ballistic trajectory 21 such as, for example, a mortar shell may follow. This shell is fired in the direction of the target to be reached, situated for example in the area under surveillance by the radar, with a large firing angle α, typically around 50° to 75°. It then follows a ballistic trajectory determined mainly by its own physical characteristics, its initial acceleration, and by the firing angle.

As the type of trajectory followed by the projectile is known, it is possible to reconstruct the trajectory of the projectile and even to determine with a given precision its point of departure 24 and the probable point of impact 25, provided that it is possible to know its position 22 at various times.

In practice, the trajectory of a projectile entering an area protected by a surveillance radar can generally be reconstructed from information about the position of the projectile relative to the radar, which information is periodically updated by the radar. In general, the information used is the radial distance R_i separating the projectile from the radar at a measurement time i, the azimuth A_i (not shown in the figure) and the angle of elevation E_i at which the projectile is detected. Consequently, as the radar is situated at a known point 23, it is possible to determine whether the trajectory of the projectile presents any danger for the projected area. Conventionally, the azimuth and the angle of elevation are measured by known methods of

angular error measurement which for each type of angle, azimuth and elevation, involve the formation of a channel called the "sum channel" and of a channel called the "difference channel". However, in order to obtain precise measurements and hence a precise determination of the position of the projectile at the instant in question, it is important for the signals received to have little interference. Yet experiment and practice show that this condition is sometimes poorly met, in particular when a radar operating in the UHF frequency band is employed to carry out surveillance. This is because operation in the UHF band makes the radar more sensitive, in particular, to interference due to multiple reflections of the signal, interference which more specifically impairs the measurement of the angle of elevation, so that a conventional angular error measurement proves to be unsatisfactory as a means of knowing precisely the angle of elevation of the projectile and hence its position. For this reason, in order to determine the angle of elevation, the method according to the invention employs an alternative method to the conventional error measurement calculation.

Figure 3 provides an overview of the various steps implemented by the method according to the invention in order to determine the angle of elevation of the projectile at a given time.

As figure 3 illustrates, the method according to the invention is applied to the information about the velocity d obtained from signal samples s(j) corresponding to periodic measurements of the signal s(t) carried out by the radar in the direction considered. Thus, if T represents the measurement refresh time, it is possible to write $S_j = S(j) = S(t_0+jT)$ where t_0 represents an arbitrary time origin. The velocity information d is here obtained conventionally, for example by spectral analysis of the signal by FFT over N points and interpolation of Doppler filters that have provided a significant response. As previously described, the method according to the invention comprises several steps which make it possible, starting with these input data, to obtain a precise estimate of the angle of elevation. These various steps are described in the remainder of the description.

The first step 31 consists in estimating, from the values of d obtained, the values of functions $\dot{d}$ and $\ddot{d}$ defined by :

$$\dot{d} = \frac{d(d)}{dt},$$

$$\ddot{d} = \frac{d^2(d)}{dt^2}$$

According to the invention, $\dot{d}$ and $\ddot{d}$ may be estimated by any known method from successive measurements of d, in particular by means of predictive estimators based on Taylor expansions, or alternatively by means of estimators with a structure based on a linear combination of Legendre polynomials. These methods, not set out here, thus make it possible, for example, to express the vector of values (d_j, $\dot{d}_j$, $\ddot{d}_j$) by the following relation:

$$\begin{bmatrix} d_j \\ \dot{d}_j \\ \ddot{d}_j \end{bmatrix} = M \cdot \begin{bmatrix} d_j \\ d_{j-1} \\ d_{j-2} \\ d_{j-3} \end{bmatrix} \qquad [1],$$

with :

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 11/6 & 3 & -3/2 & 1/3 \\ 1 & -5/2 & 2 & -1/2 \end{bmatrix} \qquad [2]$$

The second step then consists in estimating the value $\Gamma_\rho(j)$ of the radial component of the acceleration of the projectile.

Studies by the applicant have shown, as figure 4 illustrates, that over limited time intervals the acceleration $\Gamma_\rho(j)$ of the projectile is related to the quantity $\ddot{d}$ by the following relation :

$$\ddot{d} \approx -2 \cdot \frac{d\Gamma_\rho}{dt} \qquad [3]$$

Consequently, Γ_ρ can advantageously be estimated from the following relation :

$$\Gamma_\rho(t) \approx -\frac{1}{2} \cdot \int \ddot{d} \cdot dt + \Gamma_{\rho 0} \qquad [4]$$

This advantageous result is made use of in the course of step 32 in order to make an estimate of the value $\Gamma_\rho(j)$ of acceleration for a sample j, this estimation then being defined by the following recurrence relation :

$$\hat{\Gamma}_\rho(j) = \hat{\Gamma}_\rho(j-1) - \frac{\ddot{d}(j)\cdot\Delta t}{2} \quad for\ j \geq 2$$

and [5]

$$\hat{\Gamma}_\rho(1) = \Gamma_{\rho 0}$$

in which Δt represents the sampling period and $\Gamma_{\rho 0}$ the initial value of the radial component of the acceleration of the projectile.

In the case of a mortar shell, this radial component is dependent notably on the firing angle (i.e. angle of the sight), on the initial speed, on the mass, and on the ballistic coefficient of the projectile.

The similarity of the curves 41 and 42, which respectively represent a model of the derivative of the radial component of the acceleration of the projectile against time and the corresponding curve obtained from successive estimations of this function obtained by means of the method according to the invention, illustrates that this approximation is well founded in the case of a mortar shell.

The third step 33 then consists in estimating the actual speed of the projectile, or more precisely the magnitude of the velocity vector. According to the invention, the estimate $\hat{V}(j)$ of the speed of the projectile at a given measurement time is made from corresponding values of d(j), $\dot{d}(j)$ and $\hat{\Gamma}_\rho(j)$.

The speed of the projectile is then estimated using the following relation :

$$\hat{V}(j) = \left(d^2(j) - \dot{d}(j)\cdot R(j) - \hat{\Gamma}_\rho(j)\cdot R(j)\right)^{1/2} \quad [6]$$

The fourth step 34 consists finally in estimating the value of the angle of elevation at a given measurement time t ($t= t_0+jT$) as a function of the estimate $\hat{V}(j)$ of speed made in the preceding step. The angle of elevation of the projectile is then estimated from the following relation :

$$\frac{d\hat{E}}{dt} = \frac{1}{R} \cdot \left(\hat{V}^2 - d^2\right)^{1/2} \qquad [7]$$

Such that, according to the invention, the angle of elevation of the projectile is estimated using the following relation :

$$\hat{E}(j) = \hat{E}(j-1) + \frac{1}{R(j)}\left(\hat{V}^2(j) - d^2(j)\right)^{1/2} \cdot \Delta t \quad \text{for } j \geq 2$$

and

$$\hat{E}(1) = E_0 \qquad [8]$$

in which Δt represents the sampling period and E_0 the initial value of the elevation of the projectile, fixed so as to take account of, notably, the firing angle of the projectile and the relief conditions of the area covered. In the case of a mortar shell, E_0 can be fixed at a value of the order of a few degrees, 2 degrees for example.

At the end of these four steps, an estimate of the angle of elevation E is finally obtained for each measurement carried out by the radar. As the curves 51 and 52 of figure 5 illustrate, which respectively represent a model of the variation over time of the angle of elevation of a mortar shell and the corresponding curve obtained from successive positions of the projectile estimated by means of the method according to the invention, the estimate made closely coincides over a defined interval, the divergence of the curves appearing only later when the probable trajectory of the projectile is known. Thus, the estimate made using the method according to the invention can therefore be seen to be a valid estimate of the angle of elevation, and the method according to the invention therefore advantageously offers a valid

alternative to a conventional measurement of the angle of elevation by angle error measurement.

As soon as a set of measurements is available, periodically separated in time, of the radial distance R, of the azimuth A and of the angle of elevation E, it is possible to reconstruct the probable trajectory of a projectile that has no means of guidance or propulsion of its own, as is the case with mortar shells. Likewise, it is possible to determine with a given precision the point of departure of the projectile and the position of the point of impact. This precision obviously depends on the precision with which the various parameters are estimated at each measurement. A method such as that described hereinabove, insofar as it makes it possible to improve the estimation of the angle of elevation relative to known methods while employing simple calculations is therefore of great advantage.

According to an advantageous variant implementation, the method according to the invention may also comprise a step 35, parallel to the step 34, which makes it possible to determine the precision of the estimation of the angle of elevation obtained at the end of step 34.

The processing carried out in the course of step 35, shown in dotted lines in figure 3, consists in calculating the standard deviation of the estimate of the angle of elevation from the values measured or estimated during the preceding steps, successively using the following relations which define the precision of the speed estimate and the precision of estimation of the angle of elevation :

$$\sigma_V^2 \approx \frac{1}{4 \cdot \hat{V}^2} \cdot \left[4 \cdot d^2 \cdot \sigma_d^2 + K \cdot \frac{R^2 \cdot \sigma_{\dot{d}}^2}{\Delta t^2} + \sigma_R^2 (\dot{d}^2 + \hat{\Gamma}_\rho^2) \right] \qquad [9]$$

where σ_V represents the precision of the estimate of the speed of the projectile,

$$\sigma_E^2 \approx \frac{\Delta t^2}{2 \cdot R^2 \cdot \left(\hat{V}^2 - d^2 \right)} \cdot \left[\left(\frac{\left(\hat{V}^2 - d^2 \right)}{R} \right)^2 \cdot \sigma_d^2 + \hat{V}^2 \cdot \sigma_V^2 + d^2 \cdot \sigma_d^2 \right] \qquad [10]$$

where σ_E represents the precision of the estimate of the angle of elevation of the projectile, in which relations the following variables respectively represent :

- σ_d, the precision of the determination of the Doppler velocity d,
- σ_R, the precision of the measurement of the measured radial distance,
- K, a coefficient that reflects the sensitivity of the precision that can be achieved when estimating the speed of the projectile at the acceleration given to this projectile. The coefficient K depends in particular on the method used for making the estimates of $\dot{d}$ and $\ddot{d}$.

According to a second variant implementation, which can furthermore be combined with the preceding variant, the method according to the invention may also comprise a complementary step 36 during which the value $\psi(j)$ of the aspect angle ψ associated with the projectile is estimated. Knowing the value of this angle ψ makes it in particular possible advantageously to plot the value of the radar cross section presented by the projectile at the time considered.

Step 36, represented by dotted lines in the illustration of figure 3, consists in estimating the value of the aspect angle $\psi(j)$ defined by the following relation :

$$d(j) = V(j) \cdot \cos\psi(j) \qquad [11]$$

where V represents the modulus of the velocity vector of the projectile at the time considered and d the measured Doppler velocity.

CLAIMS

1. A method for determining the angle of elevation of a projectile following a ballistic trajectory, characterized in that it comprises the following steps :

- a first step of estimating, at the time considered, the values of the quantities $\dot{d}$ and $\ddot{d}$ respectively representing the first derivative and the second derivative with respect to time of the Doppler velocity d of the projectile ;

- a second step of estimating the radial component of the acceleration Γ_ρ of the projectile, this radial component being defined by the following approximation relation :

$$\ddot{d} \approx -2 \cdot \frac{d\Gamma_\rho}{dt}$$

- a third step of estimating the value of the velocity V of the projectile at the time considered, the estimated velocity being defined by the following relation :

$$\hat{V} = \left(d^2 - \dot{d} \cdot R - \hat{\Gamma}_\rho \cdot R\right)^{1/2}$$

- a fourth step of estimating the value of the estimation E of the elevation of the projectile at the time considered, this elevation being estimated by integration as a function of time from the following relation :

$$\frac{d\hat{E}}{dt} = \frac{1}{R} \cdot \left(\hat{V}^2 - d^2\right)^{1/2}$$

2. The method as claimed in claim 1, characterized in that it furthermore comprises, between the third and fourth steps, an intermediate step of estimating the value of the attitude angle presented by the projectile at the time considered, the attitude angle being defined by the following relation :

$$d = V \cdot \cos\psi$$

the estimation of the attitude angle ψ presented by the projectile making it possible to plot the value of the radar cross section presented by the projectile.

3. The method as claimed in claim 1, characterized in that it furthermore comprises a complementary step of calculating the precision σ_V^2 of the estimation of the velocity of the projectile and the precision σ_E^2 of the estimation of the angle of elevation, σ_V^2 and σ_E^2 being respectively defined by the following relations :

$$\sigma_V^2 \approx \frac{1}{4 \cdot \hat{V}^2} \cdot \left[4 \cdot d^2 \cdot \sigma_d^2 + K \cdot \frac{R^2 \cdot \sigma_d^2}{\Delta t^2} + \sigma_R^2 \cdot \left(\dot{d}^2 + \hat{\Gamma}_\rho^{\,2} \right) \right],$$

$$\sigma_E^2 = \frac{\Delta t^2}{2 \cdot R^2 \cdot \left(\hat{V}^2 - d^2 \right)} \cdot \left[\left(\frac{\hat{V}^2 - d^2}{R} \right)^2 \cdot \sigma_R^2 + \hat{V}^2 \cdot \sigma_V^2 + \dot{d}^2 \cdot \sigma_d^2 \right]$$

in which the following quantities respectively represent :

- d, the Doppler velocity of the projectile,
- σ_d, the precision of d,
- R, the radial distance measured,
- σ_R, the precision of R,
- Δt, measurement refresh time.

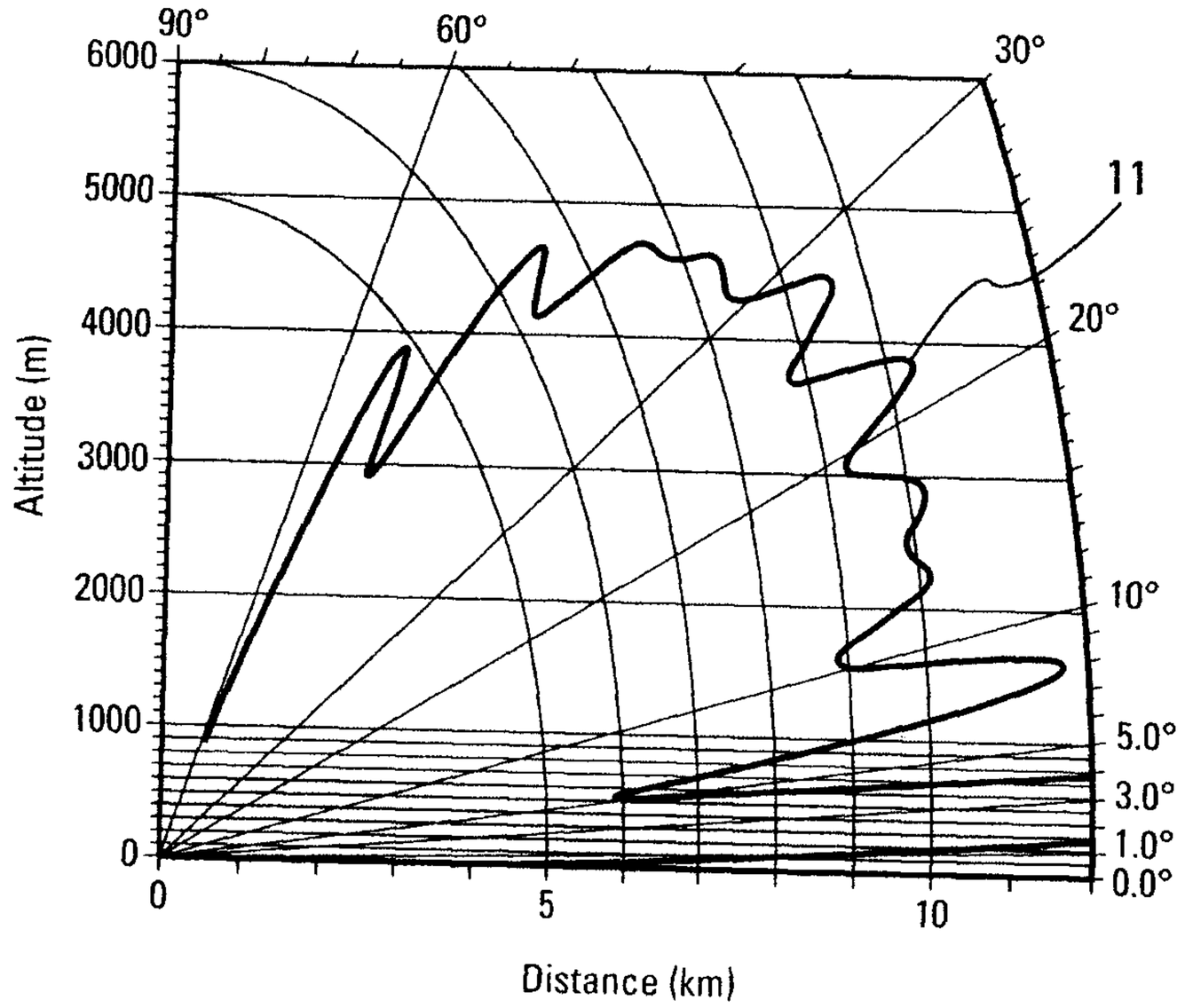

90°
60°
30°
6000
5000
4000
3000
2000
1000
0
Altitude (m)
11
20°
10°
5.0°
3.0°
1.0°
0.0°
0
5
10
Distance (km)

Fig. 1

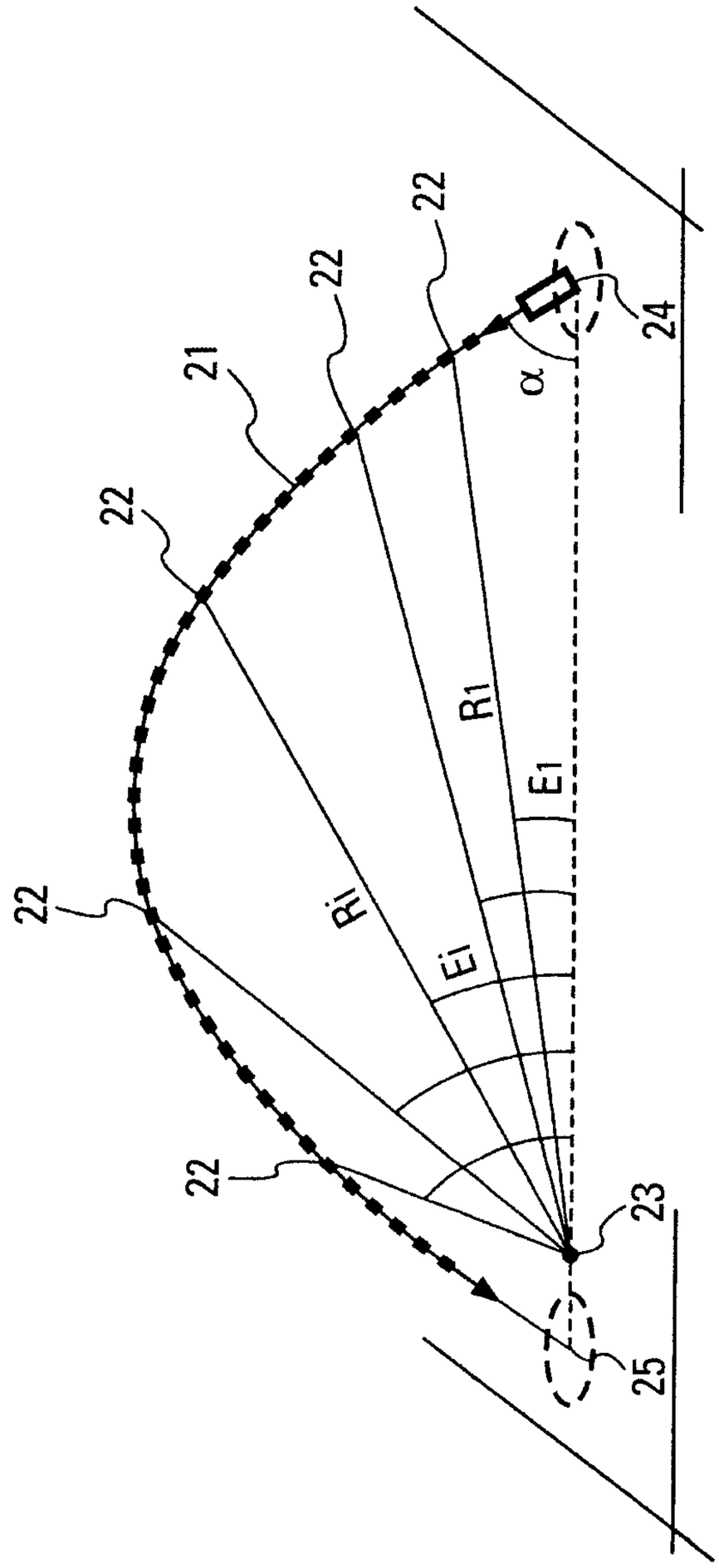


Fig. 2

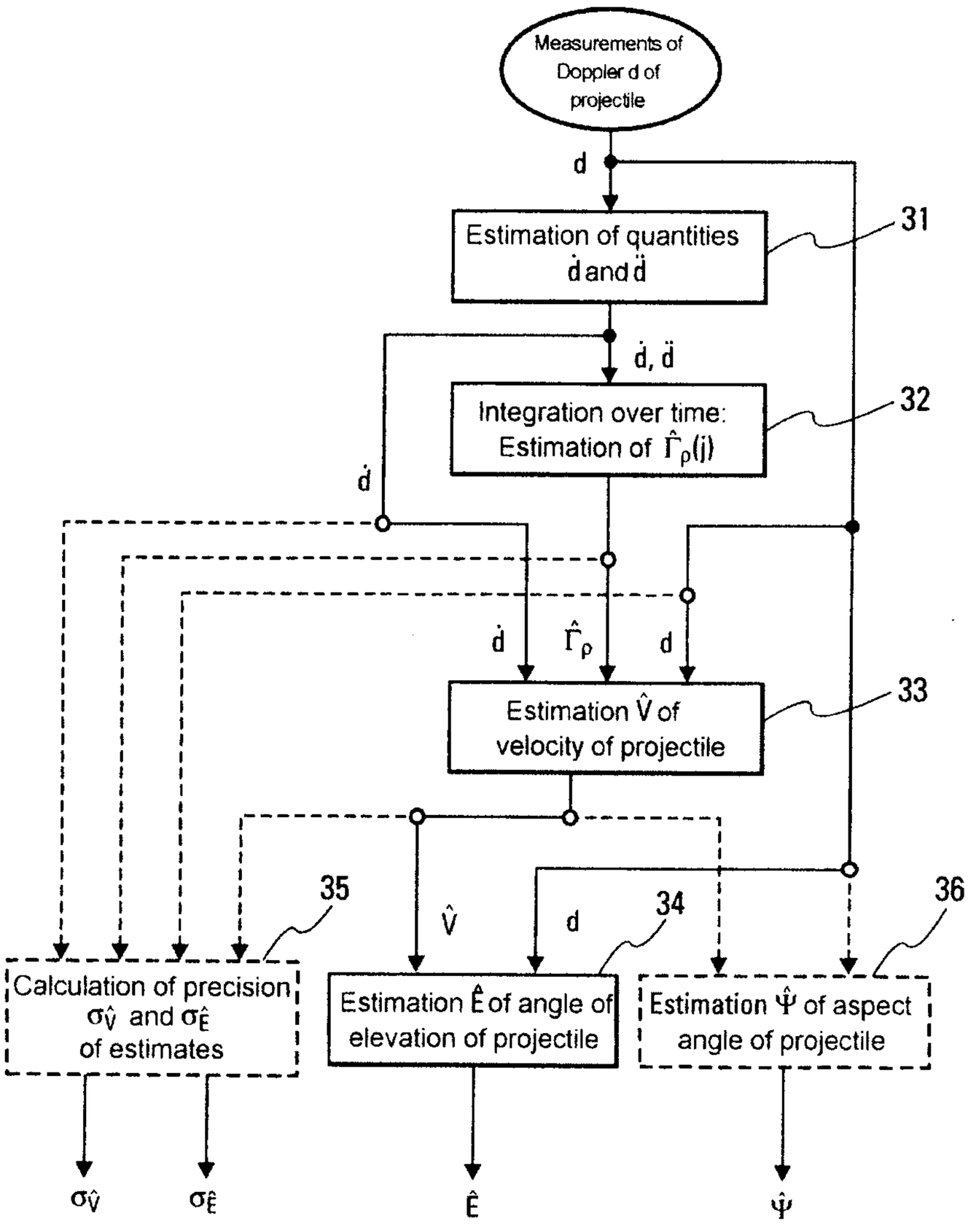

Measurements of Doppler d of projectile
d
Estimation of quantities $\dot{d}$ and $\ddot{d}$
31
$\dot{d}$, $\ddot{d}$
Integration over time: Estimation of $\hat{\Gamma}_\rho(j)$
32
$\dot{d}$
$\dot{d}$
$\hat{\Gamma}_\rho$
d
Estimation $\hat{V}$ of velocity of projectile
33
35
$\hat{V}$
d
34
36
Calculation of precision $\sigma_{\hat{V}}$ and $\sigma_{\hat{E}}$ of estimates
Estimation $\hat{E}$ of angle of elevation of projectile
Estimation $\hat{\Psi}$ of aspect angle of projectile
$\sigma_{\hat{V}}$
$\sigma_{\hat{E}}$
$\hat{E}$
$\hat{\Psi}$

Fig. 3

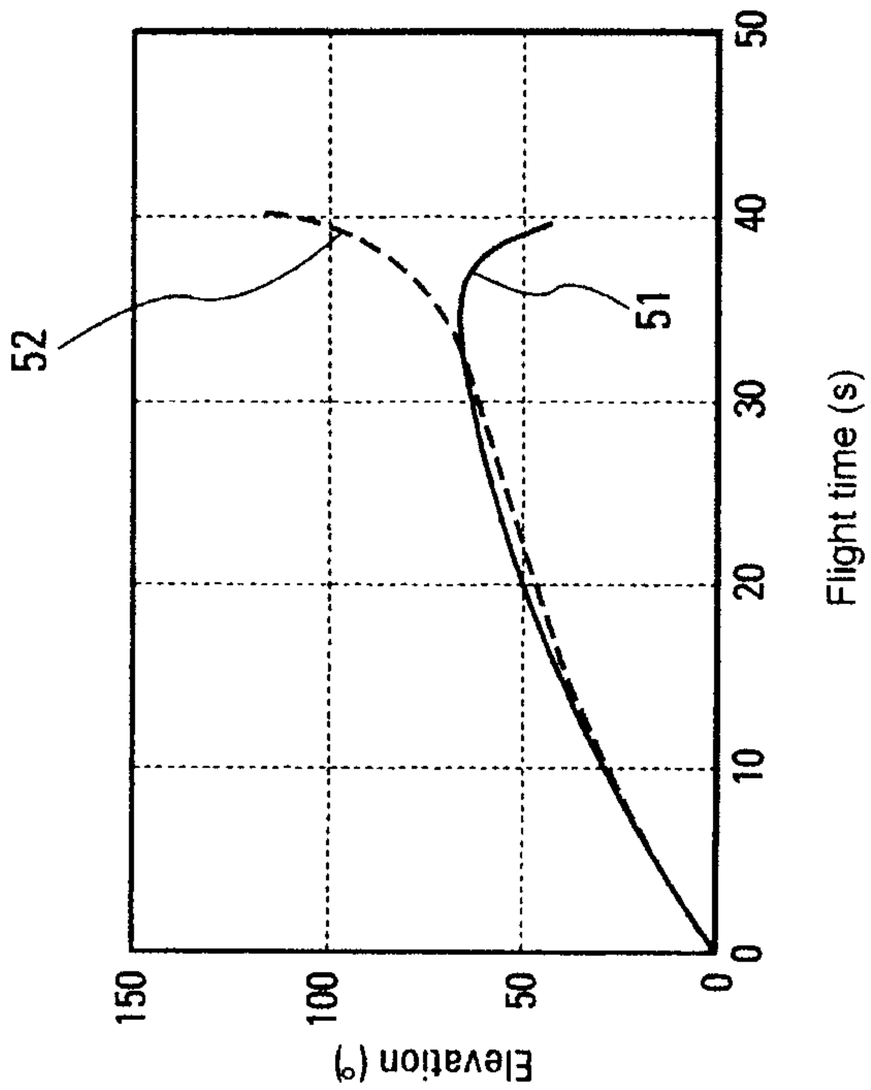

50
40
30
20
10
0
Flight time (s)
52
51
150
100
50
0
Elevation (°)

Fig. 5

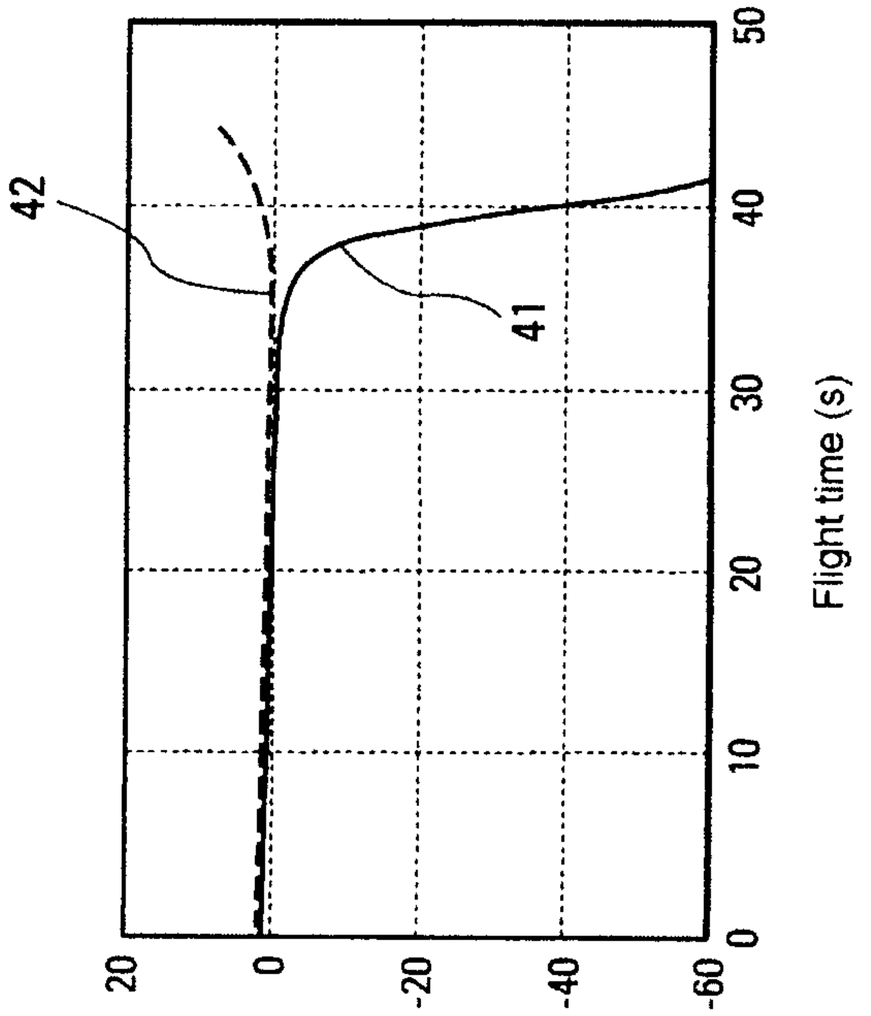

50
40
30
20
10
0
Flight time (s)
42
41
20
0
-20
-40
-60
d²(d)/dt²

Fig. 4