

July 25, 1939.

H. A. WHEELER

2,167,137

TWO-TERMINAL BAND-PASS COUPLING SYSTEM

Filed April 22, 1938

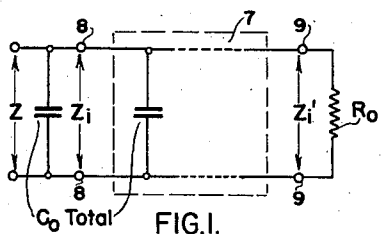


FIG. 1.

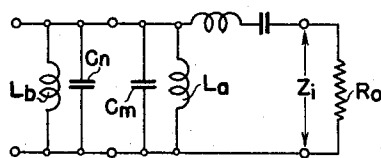


FIG. 3.

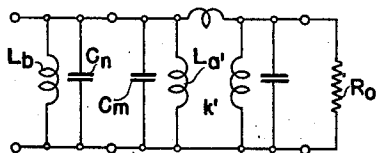


FIG. 4.

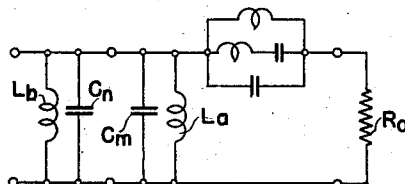


FIG. 5.

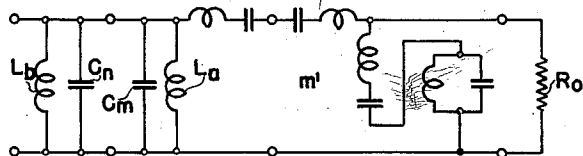


FIG. 6.

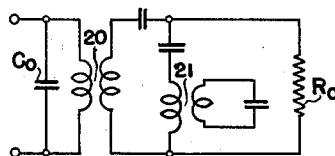


FIG. 6a.

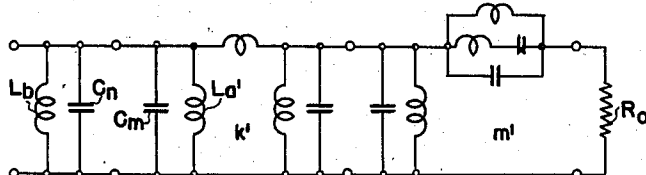


FIG. 7.

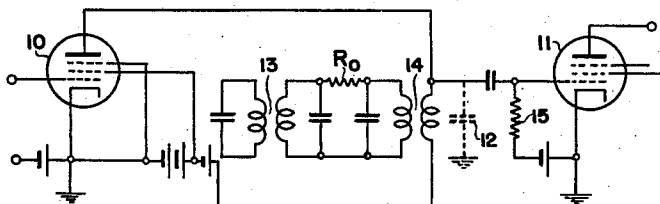


FIG. 8.

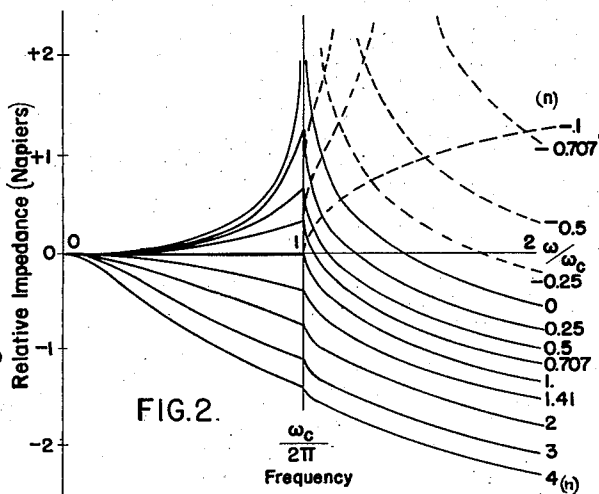


FIG. 2.

INVENTOR
HAROLD A. WHEELER

BY *Lawrence B. Odds*
ATTORNEY

UNITED STATES PATENT OFFICE

2,167,137

TWO-TERMINAL BAND-PASS COUPLING
SYSTEMHarold A. Wheeler, Great Neck, N. Y., assignor
to Hazeltine Corporation, a corporation of
Delaware

Application April 22, 1938, Serial No. 203,598

8 Claims. (Cl. 178-44)

This invention relates generally to band-pass coupling system and particularly to two-terminal coupling systems including a pair of terminals between which there is substantial capacitance and between which it is desired to build up an impedance which is the maximum that can be maintained therebetween and which is substantially uniform over a wide band or wide bands of frequencies.

In many coupling systems it is desirable to build up a substantially uniform impedance having the largest possible value over a wide band of frequencies between terminals across which there is substantial capacitance. For example, in the design of band-pass vacuum-tube amplifiers to pass a wide band of frequencies, it is desirable to build up across the inherent capacitance of the tube circuits to be coupled the maximum impedance that can be maintained substantially uniform over the operating frequency band of the amplifier. Such impedance characteristics are procured in the coupling systems of the amplifier and the maximum value of the impedance which can be maintained over a given frequency band is limited by the inherent capacitance of the tube circuits to be coupled. Prior art circuit arrangements designed for this purpose have fallen far short of the desired results.

It is an object of the invention, therefore, to provide a band-pass coupling system comprising a pair of terminals between which there is substantial capacitance wherein a maximum mean value of impedance is maintained between the terminals of the coupling system over a wide band of frequencies.

It is another object of the invention to provide a band-pass coupling system suitable for use between two successive tubes of a vacuum-tube amplifier and having maximum impedance over a wide frequency band.

In accordance with the invention, the band-pass two-terminal signal-transferring system for operation over a wide range of frequencies comprises substantial capacitance across its terminals tending to limit the response of the system over its range. A dead-end filter is coupled to the terminals having a predetermined image impedance over its range, the filter comprising only a part of the capacitance associated with the terminals as a terminal mid-shunt element. A terminating resistor is provided for the dead end of the filter, the filter comprising an impedance termination proportioned substantially to match the image impedance of the filter with the resistor over the range. The reactive constants of

the filter are so proportioned relative to the capacitance associated with the terminals and the operating frequency range that the impedance across the terminals is substantially uniform over the range and substantially the maximum that can be maintained thereacross over the range.

Other modifications and applications of the dead-end filter of the present invention are described and claimed in applicant's copending applications Serial No. 203,595, Serial No. 203,596, and Serial No. 203,597, filed concurrently with the present application while a general circuit arrangement for maintaining uniform impedance over a wide range of frequencies across a pair of terminals having inherent reactance associated therewith and tending to limit the response of the system over such range is disclosed and broadly claimed in applicant's copending application Serial No. 161,017, filed August 26, 1937, all assigned to the same assignee as the present application.

The novel features believed to be characteristic of the invention are set forth with particularity in the appended claims. The invention itself, however, both as to its organization and method of operation, together with further objects and advantages thereof, will best be understood by reference to the specification taken in connection with the accompanying drawing, in which Fig. 1 is a simplified or equivalent circuit diagram of a band-pass filter embodying the invention, in order to explain the general theory of the invention; Fig. 2 is a graph illustrating certain of the operating characteristics of the circuit of Fig. 1; and Figs. 3-8, inclusive, are circuit diagrams illustrating different embodiments of two-terminal band-pass coupling systems incorporating the invention.

The principles of the theoretical relations underlying the invention are described most simply by reference to a nondissipative wave filter of the constant- k type. This filter may be assumed to have an infinite number of sections or to be terminated with its iterative impedance to give the same effect. The input impedance of such a filter is uniform over the pass bands if the input termination is full-series or full-shunt as distinguished from the usual mid-series or mid-shunt. The input impedance is the iterative impedance measured in series with a full-series arm or in parallel with a full-shunt arm, as distinguished from the conventional image impedance measured at mid-series or mid-shunt. This input impedance may be regarded as a two-terminal coupling impedance, the remainder of the

filter serving merely as a dead-end supplementary network utilized to secure the desired uniform impedance.

Any such filter of finite total band width can be arranged to include directly across its full-shunt arm, a capacitance of the value

$$C = 2/R\Delta\omega \quad (1)$$

in which R is the mid-band image impedance and $\Delta\omega$ is 2π times the total width of the pass bands. The uniform full-shunt iterative impedance across the capacitance C has the magnitude

$$R = 2/C\Delta\omega \quad (2)$$

This relationship expresses the theoretically maximum value of impedance that can be maintained across the capacitance C , throughout frequency bands of total width $\Delta\omega$. In the case of a simple low-pass filter, the value of R is twice the reactance of the capacitance C at the cutoff frequency.

The reciprocal problem of building up impedance across a shunt capacitance is building up the admittance through an inductance. A filter can be arranged to include directly in series with all the other elements of its series arm, an inductance of the value

$$L = 2R'/\Delta\omega \quad (3)$$

The uniform full-series iterative impedance through the inductance L has the magnitude

$$R' = L\Delta\omega/2 \quad (4)$$

This relationship expresses the theoretically minimum value of impedance that can be maintained through an inductance L over the frequency bands of total width $\Delta\omega$. In the case of a simple low-pass filter, this value is half the reactance of inductance L at the cutoff frequency. The specification will not be generalized to include derivations of expressions relating to maintaining a given value of impedance through an inductance, because all the relations hereinafter given can be applied to this problem by the analogy which appears in the preceding formulae.

The image impedance of a filter of the type under discussion is purely resistive in the pass bands, though not uniform. On the other hand, the uniform iterative impedance has a substantial phase angle which is a lagging phase angle as referred to the impedance across a capacitance or the admittance through an inductance. The phase characteristic is that of a half-section of a constant- k filter. A more general analysis is given hereinafter with reference to a simple low-pass filter, without any loss of generality in the foregoing concepts. Referring now to the drawing, Fig. 1 shows the basic filter circuit in simplified form, the filter, per se, being represented schematically at 7 and having input terminals 8 and output terminals 9. The input termination C_m of filter 7 is a mid-shunt element and the input image impedance Z_i follows the constant- k characteristic. The output termination of filter 7 is either mid-series or mid-shunt and its image impedance Z_i' preferably follows an m -derived characteristic to match closely the output load resistance R_o . In developing the theoretical relations, this impedance matching is assumed to be exact in the pass bands, since any required degree of approximation is possible by means of multiple m -derivations.

For the sake of generality and clarity, the total capacitance C_o across the input terminals 8 may have any value and is divided in two parts,

C_m comprising the mid-shunt capacitance termination of the filter, and C_n the external added capacitance, thus, only a part of the total capacitance across the terminals 8 is included in the filter as a mid-shunt element. The input impedance Z_i is, therefore, across the total shunt capacitance, $C_o = C_m + C_n$. This impedance is to be built up with the aid of a dead-end filter acting merely as a passive auxiliary network.

The shunt capacitance C_m in the filter is the mid-shunt element of a terminal half-section. This half-section may be of m -derived type ($m < 1$) or of constant- k type ($m = 1$) since either type is available in a form whose mid-shunt termination presents the desired constant- k image impedance Z_i across shunt capacitance C_m . In such a half-section, the value of the mid-shunt capacitance is

$$C_m = m/R_{ocw} \quad (5)$$

in which ω_c is $2\pi f_c$, where f_c is the cutoff frequency of the low-pass filter. The external shunt capacitance C_n can have any value as determined by the choice of the parameter n in the formula,

$$C_n = n/R_{ocw} \quad (6)$$

Therefore, the total shunt capacitance is

$$C_o = C_m + C_n = (m+n)/R_{ocw} \quad (7)$$

The factor $(m+n)$ may have any positive value. One of the parts may be negative, since the parts do not have to exist separately. Negative C_n merely means that the total capacitance C_o is less than the mid-shunt capacitance C_m of the filter. Equation 7 may also be written as

$$\omega_c = (m+n)/C_o R_o \quad (8)$$

$$R_o = (m+n)/C_{ocw} \quad (9)$$

$$R_o C_{ocw} = m+n \quad (10)$$

The image impedance Z_i depends only on R_o and ω_c , not on C_m and m , because it always has the constant- k characteristic

$$Z_i = \frac{R_o}{\sqrt{1 - (\omega/\omega_c)^2}} \quad (11)$$

The image impedance is resistive in the pass band, capacitive in the attenuation band, and infinite at the cutoff frequency. It is convenient to use the parameter $x = \omega/\omega_c$ to denote the relative frequency in subsequent expressions. The relative impedance of C_n and Z_i in parallel is

$$\frac{Z}{R_o} = \frac{1}{R_o/Z_i + j\omega C_n R_o} = \frac{1}{\sqrt{1-x^2} + jxn} \quad (12)$$

This formula has a discontinuity at the cutoff frequency ($x=1$), where the Z_i term of the denominator changes from real to imaginary. It is seen that the form of the impedance characteristic depends only on

$$n = R_o C_n \omega_c \quad (13)$$

from Equation 6, and does not depend on

$$m = R_o C_m \omega_c \quad (14)$$

from Equation 5.

In the pass band ($x < 1$), Z/R_o is complex and its magnitude is

$$\left| \frac{Z}{R_o} \right| = \frac{1}{\sqrt{1+x^2(n^2-1)}} \quad (15)$$

If $n=1$, this is constant (unity). For greater values of n , it has the form of a simple resistor and capacitor in parallel, though the phase angle

is not the same. The impedance has a lagging phase angle

$$b = \tan^{-1} \frac{\pi x}{\sqrt{1-x^2}} \quad (16)$$

This is the same as the formula for a half-section m -type filter except that the m of the filter is replaced by the parameter n which determines the relative value of C_n . If $n=1$, it simplifies to

$$b = \sin^{-1} x \quad (17)$$

In the attenuation band ($x > 1$), Z/R_o is imaginary and its magnitude is

$$\left| \frac{Z}{R_o} \right| = \frac{1}{\sqrt{x^2 - 1 + x\pi}} \quad (18)$$

It has a lagging phase angle, $b = \pi/2$. These characteristics cannot be realized exactly because the output image impedance Z_i' cannot match R_o outside the pass band. As the attenuation increases, this failure has less effect on the input impedance. Therefore, any degree of approximation to the theoretical characteristics outside the pass band can be realized with a sufficient number of sections designed to secure adequate attenuation.

Fig. 2 shows the theoretical impedance characteristics for various values of the parameter n between -1 and $+4$. It is noted that the values ± 1 yield uniform impedance in the pass band. If the filter has a constant- k whole section on the input side, so that $m=1$, the values $+1$ and -1 for n correspond, respectively, to full-shunt and full-series termination. In the former case, the addition of C_n doubles the mid-shunt element, while in the latter case it cancels the mid-shunt element. The sign of the parameter n does not affect the magnitude of the impedance in the pass band, but it does determine the sign of the phase angle.

Since only positive self-reactance elements can be realized in a passive network, the value of m in the end half-section of a filter must be between 0 and $+1$. Since $m+n$ must be positive, the value of n must be between -1 and $+\infty$. Negative values of n make the impedance infinite at the frequency

$$x = \frac{1}{\sqrt{1-n^2}} \quad (19)$$

At higher frequencies, the phase angle is reversed in sign, $b = -\pi/2$. The product of two impedances with equal positive and negative values of n is

$$Z_+ Z_- = \frac{1}{1-x^2(1-n^2)} \quad (20)$$

This loses the discontinuity at the cutoff frequency. It is real, so the phase angle is zero below the frequency of infinite impedance and π at higher frequencies. This product is the same as that of a pair of like impedances with $n=0$ and with the cutoff frequency shifted to the frequency of infinite impedance. If $n = \pm 1$ in Equation 20, the product is unity so the impedances are mutually reciprocal.

Any of the impedance characteristics of Fig. 2 are theoretically obtainable across a maximum value of total shunt capacitance if $m=1$, n having the value identified with the characteristic curve. If m is less than one, the total capacitance directly across the input terminals is less by the amount added indirectly across the input terminals through the other arms of the filter. Therefore, a uniform impedance equal to R_o in

the pass band is obtainable across a maximum total capacitance, directly and indirectly in shunt, whose value is computed by letting $m=n=1$:

$$C_o = 2/R_o \omega \quad (21)$$

which corresponds to Equation 1. Any change of the total capacitance, by changing the value of n , causes an inverse change of the average impedance in the pass band, although its value R_o at zero frequency remains the same.

In the pass band, the impedance near the cut-off frequency is determined mainly by the value of n , that is, by the external shunt capacitance C_n . The image impedance Z_i having less effect, the tolerance of mismatching between Z_i' and R_o at the far end of the filter becomes greater. Also this tolerance increases with n . The number of sections in the filter affects the number of peaks and valleys in the actual impedance curve of Z_i , while the phase characteristics affect their spacing along the frequency axis.

The general concepts utilized above to develop expressions relating to low-pass filters are generally valid for filters having any number of pass bands. The mid-shunt capacitance C_m may be supplemented by parallel branches to form an impedance network of the form required in a constant- k shunt arm to give the required pass bands. C_n also may be supplemented, and in this case the inside and outside shunt arms can still be merged into one. The generalized relation corresponding to Equation 10 is

$$R_o C_o \Delta \omega = m + n \quad (22)$$

in which $\Delta \omega$ is the total width of the pass bands. Uniform impedance of magnitude R_o is secured in all pass bands if $n=1$. It is secured with maximum shunt capacitance if also $m=1$. This expression, for uniform impedance, may be designated as the "figure of merit" of any particular network, its maximum value being two.

Fig. 3 shows the application of the general concepts to a simple band-pass filter. It is assumed that the impedance level is the same at both ends of the filter, otherwise both Z_i' and R_o are multiplied by a factor of transformation. The values of shunt capacitance are

$$C_m = \frac{m}{R_o(\omega_2 - \omega_1)}; C_n = \frac{n}{R_o(\omega_2 - \omega_1)} \quad (23)$$

in which ω_1 and ω_2 are 2π times the cutoff frequencies. The basic relation of Equation 22 is then

$$R_o C_o (\omega_2 - \omega_1) = m + n \quad (24)$$

Parallel inductances L_a and L_b are provided to resonate with C_m and C_n , respectively, at the mean resonant frequency of the band in order to provide the desired band-pass characteristics. The band-pass filter of Fig. 3 comprises, in the order named, a parallel-resonant circuit comprising inductance L_b and capacitance C_n , a constant- k half-section of which the parallel-resonant elements C_m and L_a comprise a mid-shunt element, and a terminating resistor R_o the resistance of which approximately matches the image impedance of the filter over the pass band.

Figs. 4-8, inclusive, relate to filters comprising additional or modified sections which are particularly useful practical embodiments of the invention.

The filter of Fig. 4 comprises, in the order named, a parallel-resonant circuit L_b , C_n , a modified constant- k full section k' of which elements C_m and L_a' comprise a mid-shunt element, and a terminating resistor R_o the resist-

ance of which approximately matches the image impedance of the filter over the pass band.

The filter of Fig. 5 comprises, in the order named, a parallel-connected inductance L_0 and capacitance C_n , an m -derived half-section of which the parallel connected inductance L_n and capacitance C_m form a mid-shunt element, and a terminating resistor R_0 the resistance of which approximately matches the image impedance of the filter over the pass band.

The filter of Fig. 6 comprises, in the order named, a parallel-connected inductance L_0 and capacitance C_n , a constant- k half-section of which capacitance C_m and inductance L_0 comprise the mid-shunt element, an m -derived half-section indicated as m' in the drawing, and a terminating resistor R_0 which very closely matches the image impedance of the dead-end of the filter over the pass band. The circuit of Fig. 6a is the full electrical equivalent of the circuit of Fig. 6 in which the inductive elements of Fig. 6 have been replaced by equivalent transformers 20 and 21, and in which elements C_n and C_m have been combined into the single element C_0 .

The circuit of Fig. 7 comprises, in the order named, a parallel resonant circuit L_0 , C_n , a modified constant- k filter section k' of which capacitance C_m and inductance L_n' comprise a mid-shunt element, an m -derived half-section indicated as m' on the drawing, and a terminating resistor R_0 which very closely matches the image impedance of the dead end of the filter over the pass band.

The dead-end filter of the circuit of Fig. 8 is in all respects equivalent to that of Fig. 7 and is utilized as a band-pass coupling impedance between amplifier tubes 10 and 11. Capacitance elements C_n and C_m are combined in the capacitance 12 shown in dotted lines in the drawing. This capacitance 12 is comprised mainly of the inherent output capacitance of tube 10 and the input capacitance of tube 11. Transformers 13 and 14, utilized in the filter of Fig. 8, are the full electrical equivalents of the corresponding inductance elements utilized in Fig. 7, and may be more convenient and economical to build. Adjacent reactance elements of the same kind in the filter of Fig. 7 have been combined in the filter of Fig. 8. The position of the terminating resistor and adjacent mid-series filter elements in the filter has been interchanged. A grid leak 15 is provided for the input circuit of vacuum tube 11. The other elements of the circuit of Fig. 8 are conventional and its operation will readily be understood from the description of the filter of the invention given above, it being understood that by utilizing the dead-end filter of the present invention, the presence of the shunt capacitance 12 does not seriously reduce the response of the coupling system nor impair its uniformity over the pass band. On the contrary, there is built up across this capacitance a high impedance which is nearly uniform over the pass band, giving a correspondingly high and uniform response characteristic.

In the design of the dead-end filters of the invention, the preferred value of m in the m -derived sections is of the order of 0.6. Values of m between the limits of 0.5 to 0.7 result in a matching of the image impedance of the m -derived filter section with the terminating resistor R_0 at four points in a band-pass band. Filters having an m -derived termination with a value of m more than 0.7 cannot match the

terminal resistor R_0 at more than two points in a pass band filter. Each additional m -derivation in the filter section included in the filter termination makes it possible to match the image impedance with the terminal resistor at two additional points in the pass band.

While there have been described what are at present considered to be the preferred embodiments of this invention, it will be obvious to those skilled in the art that various changes and modifications may be made therein without departing from the invention, and it is, therefore, aimed in the appended claims to cover all such changes and modifications as fall within the true spirit and scope of the invention.

What is claimed is:

1. A band-pass two-terminal signal-translating system for operation over a wide range of frequencies comprising substantial capacitance across said terminals tending to limit the response of said system over said range, a dead-end filter having a predetermined image impedance over said range and coupled to said terminals, said filter comprising only a part of said capacitance as a terminal mid-shunt element, a terminating resistor for the dead end of said filter, said filter comprising an impedance termination proportioned substantially to match the image impedance of said filter with said resistor over said range, the reactive constants of said filter being so proportioned relative to said capacitance and the operating frequency range that the impedance across said terminals is substantially uniform over said range and substantially the maximum that can be maintained thereacross over said range.

2. A band-pass two-terminal signal-translating system for operation over a wide range of frequencies comprising substantial capacitance across said terminals tending to limit the response of said system over said range, a dead-end filter having a predetermined image impedance over said range and coupled to said terminals, said filter comprising only a part of said capacitance as a terminal mid-shunt element and an inductance in parallel with said capacitance effectively providing a circuit resonant at the mean frequency of said band, a terminating resistor for the dead end of said filter, said filter comprising an impedance termination proportioned substantially to match the image impedance of said filter with said resistor over said range, the reactive constants of said filter being so proportioned relative to said capacitance and the operating frequency range that the impedance across said terminals is substantially uniform over said range and substantially the maximum that can be maintained thereacross over said range.

3. A band-pass two-terminal signal-translating system for operation over a wide range of frequencies comprising substantial capacitance across said terminals tending to limit the response of said system over said range, a dead-end filter having a predetermined impedance over said range and coupled to said terminals, said filter comprising only a part of said capacitance as a terminal mid-shunt element, a section of said filter having a uniform image impedance at one end and an image impedance substantially matching the image impedance of another section of said filter at the other end, a terminating resistor having a value of resistance equal to said uniform impedance coupled to said one end of said section, the reactive constants of said filter

being so proportioned relative to said capacitance and the operating frequency range that the impedance across said terminals is substantially uniform over said range and substantially the maximum that can be maintained thereacross over said range.

4. A band-pass two-terminal signal-translating system for operation over a wide range of frequencies comprising substantial capacitance across said terminals tending to limit the response of said system over said range, a dead-end filter having a predetermined image impedance over said range coupled to said terminals, said filter effectively comprising only a part of said capacitance as a terminal mid-shunt element, a terminal resistor for said dead-end filter, said filter effectively including an impedance termination comprising a modified constant- k section including two parallel-resonant shunt arms with an inductance series arm interposed therebetween, said termination being coupled and proportioned substantially to match the image impedance of said filter with said resistor over said range, the reactive constants of said filter being so proportioned relative to said capacitance and the operating frequency range that the impedance across said terminals is substantially uniform over said range and substantially the maximum that can be maintained thereacross over said range.

5. A band-pass two-terminal signal-translating system for operation over a wide range of frequencies comprising substantial capacitance across said terminals tending to limit the response of said system over said range, a dead-end band-pass filter having a predetermined image impedance over said range coupled to said terminals, said filter comprising only a part of said capacitance and an inductance as a terminal mid-shunt element, and an impedance termination for said filter effectively comprising a parallel-resonant circuit as a shunt arm and a series-resonant circuit and a parallel-resonant circuit coupled in parallel as a series arm, a terminal resistor for said impedance termination, said termination being coupled to the dead end of said filter and being proportioned substantially to match the image impedance of said filter with said resistor over said range, the reactive constants of said dead-end filter being so proportioned relative to said capacitance and the operating frequency range that the impedance across said terminals is substantially uniform over said range and substantially the maximum that can be maintained thereacross over said range.

6. A band-pass two-terminal signal-translating system for operation over a wide range of frequencies comprising substantial capacitance across said terminals tending to limit the response of said system over said band, a dead-end band-pass filter having a predetermined image impedance over said range coupled to said terminals, said filter effectively comprising only a part of said capacitance and an inductance in parallel therewith as a terminal mid-shunt element, an m -derived impedance termination effectively comprising a series-resonant series arm

and a shunt arm including a series-resonant circuit and a parallel-resonant circuit effectively in series, a terminal resistor for said impedance termination, said termination being coupled to the dead end of said filter and being proportioned substantially to match the image impedance of said filter with said resistor over said range, the reactive constants of said dead-end filter being so proportioned relative to said capacitance and the operating frequency range that the impedance across said terminals is substantially uniform over said range and substantially the maximum that can be maintained thereacross over said range.

7. A band-pass two-terminal signal-translating system for operation over a wide band of frequencies comprising substantial capacitance across said terminals tending to limit the response of said system over said range, an impedance network having a predetermined image impedance over said range coupled to said terminals, said impedance network comprising a first transformer having a primary winding tuned by a portion of said capacitance, a second transformer having a primary and a tuned secondary winding, two condensers connected in series with the primary of said second transformer across the secondary winding of said first transformer, a resistor coupled across the primary circuit of said second transformer and one of said two condensers, the reactive constants of said impedance network being so proportioned relative to the value of said resistor, said capacitance, and the operating frequency range that the impedance across said terminals is substantially uniform over said range and substantially the maximum that can be maintained thereacross over said range.

8. A band-pass two-terminal signal-translating system for operation over a wide range of frequencies comprising substantial capacitance across said terminals tending to limit the response of said system over said range, a dead-end filter having a predetermined image impedance over said range coupled to said terminals, said filter effectively comprising only a part of said capacitance and an inductance as a terminal mid-shunt element and a modified constant- k section including two parallel-resonant shunt arms and an inductive series arm, an m -derived impedance termination coupled to the dead end of said filter comprising a series arm including a series-resonant circuit and a parallel-resonant circuit connected in parallel, a terminal resistor, said impedance termination being coupled to the dead end of said filter and being proportioned substantially to match the image impedance of said filter with said resistor over said range, the reactive constants of said dead-end filter being so proportioned relative to said capacitance and the operating frequency range that the impedance across said terminals is substantially uniform over said range and substantially the maximum that can be maintained thereacross over said range.

HAROLD A. WHEELER.