May 15, 1928. 1,669,919

N. TRBOJEVICH

METHOD OF GENERATING HELICAL GEARS

Filed July 21, 1924 4 Sheets-Sheet 1

Fig. 3.

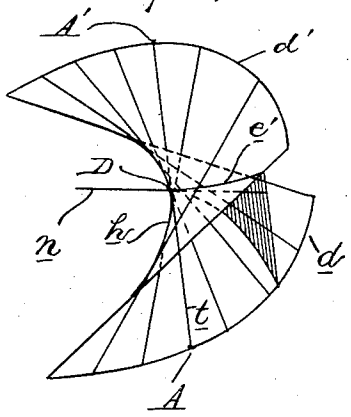


Fig. 4.

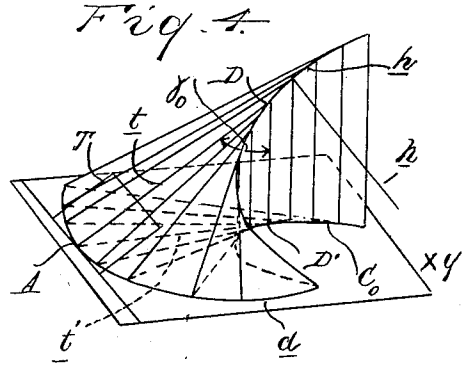


Fig. 2.

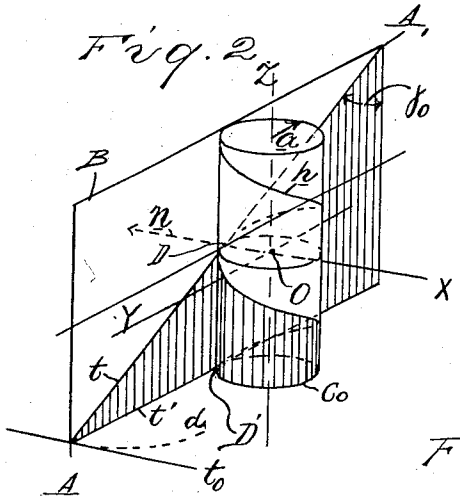


Fig. 5.

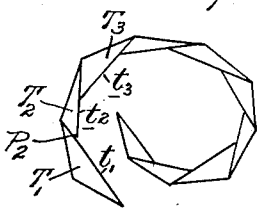
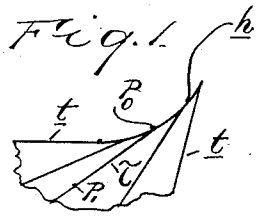


Fig. 1.



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METHOD OF GENERATING HELICAL GEARS

Filed July 21, 1924

4 Sheets-Sheet 2

Fig. 7.

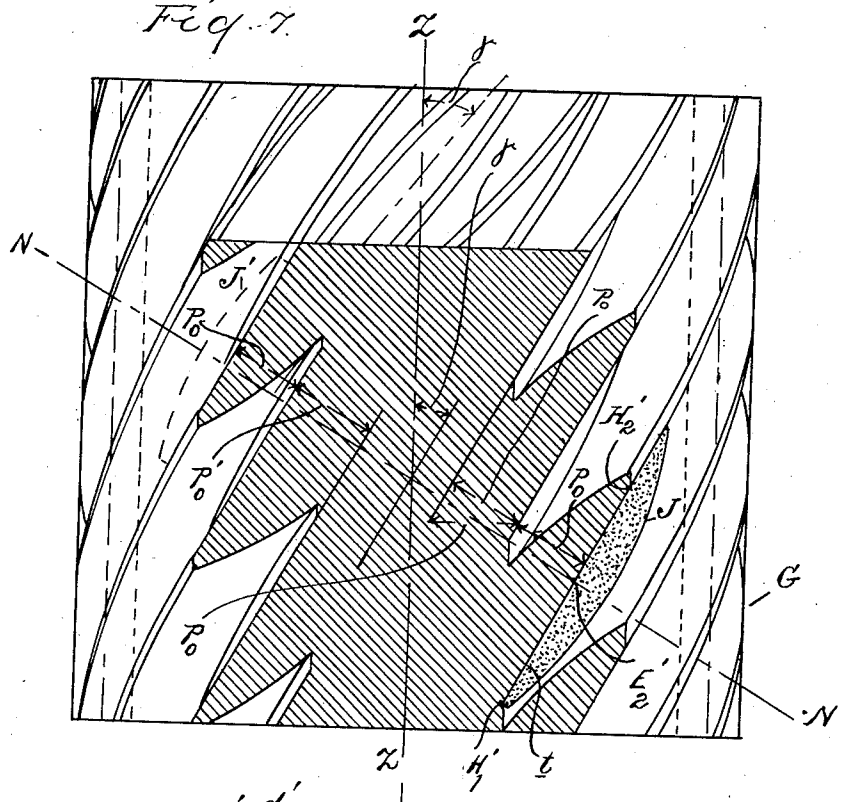
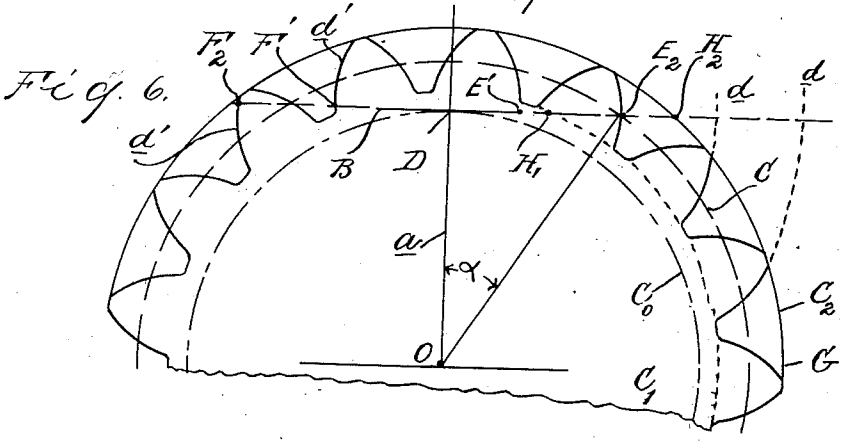


Fig. 6.



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1,669,919

METHOD OF GENERATING HELICAL GEARS

Filed July 21, 1924

4 Sheets-Sheet 3

Fig. 12.

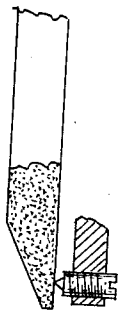


Fig. 9.

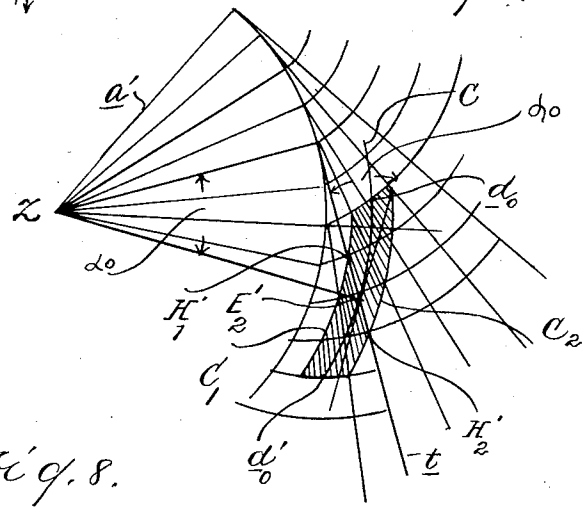
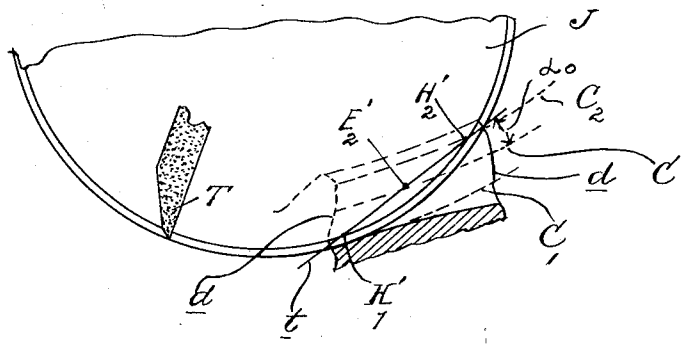


Fig. 8.



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1,669,919

METHOD OF GENERATING HELICAL GEARS

Filed July 21, 1924

4 Sheets-Sheet 4

Fig. 11.

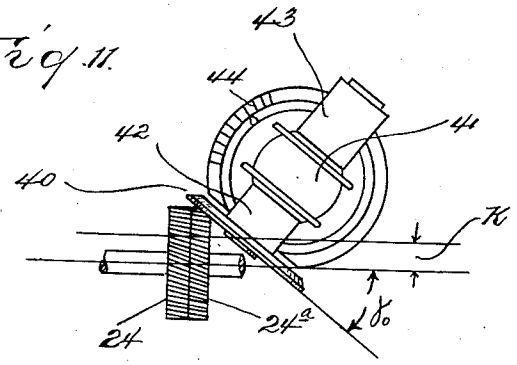
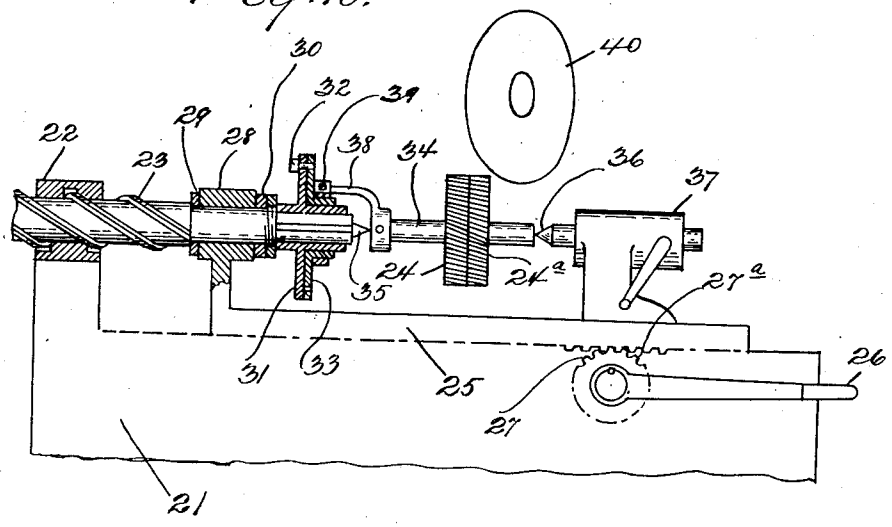


Fig. 10.



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UNITED STATES PATENT OFFICE.

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METHOD OF GENERATING HELICAL GEARS.

Application filed July 21, 1924. Serial No. 727,324.

The invention relates to a novel method of gear generating which is particularly applicable to grinding of helical gears. The advantages of the process are that it is extremely simple and also theoretically accurate, as true involute helical surfaces may now be produced by the combination of a plane disk wheel and a screw motion of the blank alone. In that respect the method is unique, as in no other gear grinding process known in the art is it possible to generate involute surfaces with a plane wheel without rolling the blank in a transverse plane against said wheel. In my method the rolling element is wholly absent; however, only gears having a considerable helical angle of teeth may be produced, the lowest practicable limit (on account of the size of the grinding wheel) being about 20 degrees. This does not materially detract from the value of the invention, as first, the helical gears now generally employed usually have a greater helical angle than 20 degrees, and second, owing to the peculiar result of this invention, helical gears may now be ground more accurately and economically than straight toothed gears, and it is not unreasonable to expect that the straight teeth in gears will be gradually replaced by helical teeth in the future.

In the drawings:

Figs. 1 to 5 are geometrical diagrams explanatory to the theory of the process. These diagrams have been borrowed with a few changes from the quoted work by Scheffers.

Fig. 6 is a plan view of a common helical gear.

Fig. 7 shows the plan elevation of the same gear in intersection with the rectifying plane.

Fig. 8 is a view of Fig. 7 taken in the normal plane N—N.

Fig. 9 is the plane development of the tooth surface shown in Figure 8.

Fig. 10 is the front elevation of a gear grinding machine operating according to the new method.

Fig. 11 is a detail showing the plan view of the grinder head.

Fig. 12 is a diagram showing the method of trimming the plane face of the grinder used in this method.

The principle upon which this method of helical gear grinding is based is wholly mathematical and relates to the theory of developable twisted surfaces. It has long been known in the higher mathematics that if a series of tangents are drawn to a twisted curve, they form the so called tangential surface of the curve, which surface has the unique property among all other twisted or warped surfaces that it is developable into a plane without tearing or crumpling. In particular, I have found that the tangential surface to a circular helix is identical with the tooth surface of the well known involute helical gear tooth.

This fact may be mathematically proved as it will now be briefly indicated. Let us first write down the equations of a helix in a parametric form, selecting the length of arc s as the parameter. This leads to three equations in x, y, z of the following form:

$$\begin{aligned} x &= f_1(s) \\ y &= f_2(s) \\ z &= f_3(s) \end{aligned} \quad (1)$$

where f_1, f_2, and f_3 denote three functions of s.

It is now evident that the tangent to the helix, expressed by equations (1) may be defined as follows:

$$\begin{aligned} X &= f_1(s) + \tau f'_1(s) \\ Y &= f_2(s) + \tau f'_2(s) \\ Z &= f_3(s) + \tau f'_3(s) \end{aligned} \quad (2)$$

where X Y Z are the current coordinates of the tangent to the helix, τ is a parameter denoting the length of the tangent, from P_0 to P_1 (see Figure 1), and the prime indexes in the quantities $f'_1(s)$ etc. denote a differentiation of x, y, z, with respect to s.

Let us now assume that the helix h (Figure 2) is wrapped upon the cylinder of a radius a (the base cylinder of the gear); that the helical angle is equal to γ_0 and that an X Y Z coordinate system is so selected that the axis Z coincides with the axis of

the base cylinder and of the base h, the plane X Y being perpendicular to said axis. In that case the equations (2) assume the following form:

$$X = a \cos \frac{s \sin \gamma_0}{a} - \tau \sin \gamma_0 \sin \frac{s \sin \gamma_0}{a}$$

$$Y = a \sin \frac{s \sin \gamma_0}{a} + \tau \sin \gamma_0 \cos \frac{s \sin \tau_0}{a} \quad (3)$$

$$Z = s \cos \gamma_0 + \tau \cos \gamma_0$$

(The equations (3), except for the difference in notation, may be found in Geo. Scheffers, Differential Geometry, vol. 1, page 359.)

(The equations (3) furnish a mathematical proof that the surface therein defined is identical with the common involute helical tooth surface). Because, the common helical tooth surface is defined as being always an involute of a circle in any plane section perpendicular to the gear axis, and a circular helix in any cylindrical section concentric with said axis. It is readily seen that the surface (3) satisfies both above conditions. First, a plane Z=O is perpendicular to the Z axis, and by substituting that value in the last of the equations (3) we have

$$\tau = -s \quad (4)$$

If we now replace the last found value of τ in the first two equations, the locus is undoubtedly an involute drawn from the base circle a. On the other hand, a cylindrical section is obtained of the surface (3) if the value

$$\tau = \text{const} \quad (5)$$

is substituted. The result is a circular helix, drawn upon a cylinder of a radius

$$\gamma_0 = \sqrt{a^2 + \tau^2} \sin {}^2\gamma_0$$

Referring now again to Fig. 2, the generation of a helical tooth surface by means of a moving tangent along a helix will be understood. Let the plane B be tangent to the base cylinder of a radius a, which plane we shall term as the base plane of the gear (in mathematics the plane is known as the "rectifying plane.") A straight line t lying in the plane B is inclined with respect to the axis Z at an angle γ_0 (the base helix angle) and touches the cylinder at the point. If the plane B be now rolled without slipping upon the base cylinder always maintaining its parallel position with respect to the axis Z, the line t will describe a helix h upon the base cylinder, and it also will describe a twisted surface composed of two nappes about said cylinder. The plane sections of the generated surface (parallel to the plane XY) are all involutes such as d described by the point A. That the curve d is an involute is easily seen, because the projection t' of the tangent t rolls without slipping upon, and in the plane of the circle c_0. The involute described by the opposite point A' is also an involute similar to d except that it is of an opposite hand. This fact, as we shall see, is of practical importance because in a gear tooth, the opposite sides are both involutes, one being wound clockwise, and the other counterclockwise. Thus, by the simple expedient of offsetting the generating element from one side of the base helix to the other (from the point A to A') first one and then the other side of a tooth may be finished.

The tangent t_0 to the involute d (Fig. 2) is perpendicular to both lines t' and t, and is parallel to the line DO=n, the principal normal of the helix h. The plane composed of the three intersecting lines n, t and t_0 is the so called osculating plane of the base helix h, and is tangent to the generated helical surface along the entire length of the line t. Further, said plane is perpendicular to the plane B. This, again is of practical importance because in this method of gear grinding the plane of the grinding wheel always lies in the osculating plane T of the base helix and thus envelops or generates the surface by touching it along the generator t.

In Figure 3 the two nappes of the tangential surface are shown in perspective. The plane (n_1 t) osculates the helix h at D and is tangent to both nappes along the line AA'=t. Said plane crosses or intersects the surface at the point D, but it does not intersect said surface anywhere else. The normal plane erected at the point D to the base helix is perpendicular to both the osculating and the rectifying planes and intersects the surface in curves e and e' which curves although somewhat similar to, are not involutes.

Fig. 4 shows a portion of an involute helicoid in perspective. That diagram will be easily understood from what already has been disclosed. The osculating plane T is shown in full view in the form of a small parallelogram at the left lower side of the diagram. It is significant that all osculating planes, no matter at which point of a helical tooth surface they are drawn, intersect the axis of the gear at a constant angle γ_0 which angle, however, is different from the commonly known and understood pitch helix angle γ. This forms an important factor in my discovery, as I am the first to generate helical gears on the osculating plane principle.

The surface shown in Fig. 3 and Fig. 4 is developable into a plane. This fact is well established in mathematics, and is also more or less obvious from Fig. 5. Let the consecutive osculating planes (separated from each other only by an infinitesimal distance of angle) be T_1 T_2 T_3 etc. Accord-

ing to the supposition each of those planes contains the corresponding tangent or generator t_1 t_2 t_3, in their full length. On the other hand, the plane T_1 intersects the consecutive plane T_2 in the line t_1. Therefore, the lines t_1 and t_2 lying in the same plane T_2, intersect each other at P_2, and they may be developed into any desired plane without tearing or crumpling the surface by rotating the generator t_1 or t_2, or both, about the point of intersection P_2 until they coincide with the plane of development.

The practical application of the above theoretical considerations will now be shown. In Fig. 6 and Fig. 7 two views of a common helical gear (right handed) are shown. In the transverse section (Fig. 6) a number of similar equispaced teeth are arranged circumferentially along the pitch circle c, said teeth being bounded by the left hand involutes d at one side, and the right hand involutes d' on the other side. All involutes are drawn from the same base circle c_0 having a radius a. If we now denote the pitch radius $OE_2=r$, and the angle $DOE_2=\alpha$, (the transverse pressure angle), we have this fundamental relation:

$$a=\gamma \cos \alpha \quad (6)$$

Let us now intersect the gear G with the rectifying or the base plane B drawn through the point D. This plane intersects the involutes d at right angles at the points E_1 E_2 etc. and the involutes d' on the opposite side of the center line OD, also at right angles, at the points F_1 F_2 etc. Further, the spacing of the points $E_1 E_2 = E_2 E_3 = F_1 F_2 = F_2 F_3 = p_0$ is constant and is equal to the base pitch, its exact value being

$$p_0=p \cos \alpha \quad (7)$$

where p is the circular pitch of the gear (measured along the pitch circle c).

It is now evident that the helical teeth when intersected with the base plane will show a straight line of intersection on their sides, lying furthest from the center line OD. Because, as already illustrated in Fig. 2, the base plane B intersects the involutes d at right angles and contains the straight line generator t passing through the point A_1 in full. Said line t lies wholly on the tooth surface and is inclined with respect to the axis Z at an angle γ_0. In Fig. 7 the line $t=H_2' H_1'$ is shown in its true length and this length may be determined if the length $H_1 H_2$ (Fig. 6) intercepted in projection by the root circle c_1 and the top or outside circle c_2 of the gear is known, as well as the angle γ_0. Thus,

$$t=H'_1H'_2=\frac{H_1H_2}{\sin \gamma_0} \quad (8)$$

The normal base pitch P_0' (the distance of the osculating plane at two consecutive teeth) may also be determined:

$$p'_0=P_0 \cos \gamma_0 \quad (9)$$

Let now the lead of the helix (either of the pitch helix or the base helix) be denoted with L. Then the pitch helix angle is evidently:

$$\tan \gamma=\frac{2\gamma\tau}{L} \quad (10)$$

and the base helix angle:

$$\tan \gamma_0=\frac{2a\tau}{L} \quad (11)$$

or, from (6)

$$\tan \gamma_0=\tan \gamma \cos \alpha \quad (12)$$

The above equation (12) is of fundamental importance in this process as the grinding wheel J (Fig. 7) must be inclined to the angle γ_0 with respect to the gear axis.

Thus, the new method of gear grinding may now be defined. The line $t=H'_1 H'_2$, Fig. 7, is evidently the trace of the osculating plane in the plane of paper as that plane always contains the line t, and is perpendicular to the rectifying plane B (the plane of paper.) The line t enters, therefore, the tooth surface at the bottom of tooth at H'_1, crosses the pitch line at an acute angle α_0 at the pitch point E'_2 and emerges on the top of the tooth at H'_2. This condition is shown in detail in Fig. 8 which is a view of Fig. 7 taken in the normal plane N—N. The object now is to replace the osculating plane T with the grinding wheel J. This may be satisfactorily done providing the grinder J is of a sufficiently large diameter so as to include the whole length of the line $H'_1 H'_2$ as its chord, and is placed so with respect to the gear (offset to one side) that its outer circumference will clear the bottoms of the interdental spaces, and its conical rear side (as shown in section at the left of the Fig. 8) will not interfere with the adjacent tooth. In a practical example, I have ground timing gears used in a well known automobile, 66 teeth, 10 normal diametral pitch 25° helix angle, 14½° pressure angle, 1¼″ face, with a wheel of only 7½″ diameter. The required diameter of the wheel as is seen from the equation (8) rapidly increases however as the angle γ_0 decreases, and for $\gamma_0=0$ (spur gears) t=infinite. For that reason spur gears cannot be ground on this principle.

Suppose now that the grinder J (Figure 8) touches the gear tooth along the straight line $H'_1 H'_2$. The grinder is rapidly rotated on its axis in a fixed position, and the blank G, Fig. 7, is moved in a helical path, that is, rotated to the right about and translated along the axis Z in such a ratio that for each revolution about the axis the blank will

advance an axial distance L equal to the lead of helix. In this manner all the consecutive generators t lying on the same tooth surface come progressively into contact with the grinder, and as it might be described, develop themselves into the plane represented by the rotary grinder and are thus ground over. This will be better understood from Figure 9 which shows the conditions existing in Figure 8 when the quadrilateral tooth surface bounded by the helixes c_1 and c_2 and the two opposite involutes d is developed into a plane. In that case the helixes c_1 and c_2 become concentric circles, the two involutes d become the parallel involutes d_0 drawn from the base circle having a radius a' equal to the normal radius of curvature of the base helix h, that is:

$$a' = \frac{a}{\sin^2 \gamma_0} \quad (13)$$

and the straight line generators t all remain straight lines tangent to the new base circle a', and intersecting the pitch helix c at an acute angle α_0. When the blank G, Figure 7, is given a screw motion, in development (Figure 9) it will appear as if the development itself was rotated about the axis Z'. The grinding wheel, however, does not participate in this rotation, as its line of action H_1 H_2 (Figure 9) remains stationary with respect to said development and therefore describes a series of diverging straight lines, all tangent to the developed base circle a', upon the tooth surface d_0 c_1 d_0 c_2 as indicated by shaded area in Figure 9. It is seen that in this manner gears having any desired width of face may be correctly generated.

When one side of a tooth has thus been generated, the blank is indexed, that is disengaged from its driving arbor and turned through an angle corresponding to the spacing of teeth, and the process is so repeated until all teeth are ground on their one side. To grind the other side either the blank is removed from the arbor and reversed, or the grinder is reversed and moved to the position J' Figure 7, or again, two or more grinders may be mounted upon the same arbor, and if they are correctly spaced from each other, they will simultaneously grind the teeth upon their both sides.

A front elevation of a machine which may be used in connection with this method is shown in Figure 10, Figure 11 is a plan view showing the method of placing the grinder relative to the blank, and Figure 12 shows the method of wheel dressing by means of a trimming diamond. A machine of this kind to be operative must contain the following elements: Means for translating the blank in a helical path, means for indexing, means for angularly adjusting the grinder in a plane parallel to gear axis, means for offsetting the wheel, and means for trimming the wheel to a true plane surface perpendicular to the axis of rotation of said wheel.

Upon the base 21 (Figure 10) a stationary nut 22 is mounted engaging the master screw 23 having the exact lead L and the same hand of helix which it is desired to generate upon blanks 24 and 24a. A longitudinally slidable table 25 is mounted in suitable ways upon said base 21 and may be reciprocated by turning the hand lever 26 keyed to the pinion 27, said pinion engaging a rack 27a at the bottom of the slide 25. The master screw 23 is rotatably housed in the bearing 28 integral with the slide 25 and the end play is prevented by means of the thrust collars 29 and 30. At the right hand extremity of the master screw 23 a disk 31 is keyed, carrying the index pin 32. The index plate 33 having a number of equally spaced holes or notches corresponding to the number of teeth which it is desired to generate is rotatably mounted upon the boss integral with the disk 31, and may thus be disengaged from said disk by withdrawing the index pin 32. The blanks 24 and 24a, are mounted upon the arbor 34 suspended between the centers 35 and 36, the latter being housed in the tail stock 37. A dog 38 serves to transmit the rotation from the screw 23 to the blank arbor 34 and by virtue of the set screw 39 it also serves for imparting a fine rotative adjustment to the blanks relative to the wheel.

The grinder 40 (Figure 11) faces the gear blanks with its accurately trimmed plane surface, and is tilted with respect to the axis of the blank to the base helix angle γ_0 while its lowest point is at a distance K (in vertical projection) from said gear axis. The distance K may be determined either graphically or by calculation and corresponds to the distance of the point E'_2 from the Z axis (see Figure 7). The grinder 40 is mounted upon a suitable spindle which is driven by means of the pulley 41 and is housed in two suitable bearings 42 and 43. The base of the grinding head is rotatable in circular ways, and is adjustable relative to the dial 44 to any exact angle γ_0.

The plane side of the grinder 40 is periodically trimmed to its exact contour by reciprocating the trimmer diamond 45 across the face of wheel while the wheel is rapidly rotating, as will be understood from Figure 12.

As previously stated, the production of the machine shown in Figure 10 may be increased by mounting two or more grinders upon the grinder spindle, such as the grinders J and J' in Figure 7. It is also obvious that instead of grinders, milling cutters may be employed. It should be noted, however, that if several cutters are simultaneously employed, their outer diameters must be so selected and correlated that they will all cut

to the same depth in the gear blank. They also must be correctly spaced, the spacing depending upon the dimension p'_0, the number of teeth, helix angle, etc. of the blank, as shown in Figure 7.

What I claim as my invention is:—

1. The method of grinding helical gears consisting in arranging a plane disk grinder to contact with the gear tooth so that the straight line generator of the twisted helical tooth surface will appear as a chord fully lying in said grinder and lying in a plane parallel to the axes of both the cutter and the grinder and in imparting to the blank a relative movement such as to develop the successive generators progressively into the cutting plane of said grinder.

2. A method of generating involute helical gear teeth consisting in mounting a gear blank upon a rotary arbor, in selecting a rotary plane disk cutter whose cutting plane is perpendicular to its axis of rotation, in placing said cutter in a position coinciding with the osculating plane of the imaginary base helix of which the tooth surface is the tangential developable, in outwardly offsetting the cutter in said osculating plane until it contacts with the side of the helical tooth lying farthest from the center line of gear and until it includes as its chord the portion of the generating straight line extending obliquely across and from top to the bottom of said surface, in imparting to the blank a relative helical movement concentric with its axis and in continuing with said helical movement until the tooth is finished in the entire length of its face and at all its bearing points.

3. A method of generating helical teeth in a gear blank by means of a plane rotary disk cutter in which the position of the cutter relative to the blank is determined by first drawing a rectifying plane tangent to the imaginary base cylinder of the gear, in finding the intersection of said rectifying plane with the gear tooth which is a straight line, in placing the cutting plane of the cutter in a perpendicular position relative to the rectifying plane so that it will include the portion of the generating straight line lying on the gear tooth, as its chord, in rotating and translating the blank along its axis in a helical path until the successive straight line generators lying on the same tooth surface are all finished, and in finishing the rest of the gear teeth by indexing and repeating the helical movement.

4. A method of generating twisted surfaces which are tangential developables of a base curve consisting in selecting a plane rotary disk cutter, in aligning its cutting plane with the momentary osculating plane to the curve and offsetting said cutting plane in the osculating plane until the line to be generated appears as a chord drawn in the cutting circle, in imparting to the surface a relative twist so proportioned that the cutting plane always osculates the base curve, and in continuing said twisting movement until all straight lines lying appearing upon the surface are fully generated.

5. A method of generating helical gears consisting in mounting the gear blank upon the work arbor and two plane disk cutters upon the cutter arbor, the arrangement being such that the cutter arbor lies in the normal plane drawn to the base helix, and the plane side of each cutter faces the other cutter and the center line of the gear and coincides with the osculating planes drawn to the corresponding tooth surfaces acted upon, the line of contact in each case appearing as a chord drawn in the cutting circle of the corresponding cutter, in imparting to the blank a relative movement of translation along its axis and in a helical path, and in indexing the blank to repeat the cut at another portion of the circumference of the blank.

6. A method of generating twisted surfaces which are tangential developables of a base curve consisting in mounting the gear blank upon the work arbor and two plane disk cutters upon the cutter arbor, the arrangement being such that the cutter arbor lies in the normal plane drawn to the base helix, and the plane side of each cutter faces the other cutter and the center line of the gear and coincides with the osculating planes drawn to the corresponding tooth surfaces acted upon, the line of contact in each case appearing as a chord drawn in the cutting circle of the corresponding cutter, in imparting to the blank a relative twist so proportioned that the cutting planes always osculate the base curves and in indexing the blank to repeat the cut at another position of the circumference of the blank.

In testimony whereof I affix my signature.

NIKOLA TRBOJEVICH.