(19) DANMARK

(10) **DK/EP 2686554 T3**

Patent- og Varemærkestyrelsen

(12) Oversættelse af europæisk patentskrift

(51) Int.Cl.: ***F 04 B 35/04 (2006.01)***

(45) Oversættelsen bekendtgjort den: **2015-10-12**

(80) Dato for Den Europæiske Patentmyndigheds bekendtgørelse om meddelelse af patentet: **2015-07-08**

(86) Europæisk ansøgning nr.: **12719245.8**

(86) Europæisk indleveringsdag: **2012-03-15**

(87) Den europæiske ansøgnings publiceringsdag: **2014-01-22**

(86) International ansøgning nr.: **BR2012000066**

(87) Internationalt publikationsnr.: **WO2012122615**

(30) Prioritet: **2011-03-15 BR PI1101094**

(84) Designerede stater: **AL AT BE BG CH CY CZ DE DK EE ES FI FR GB GR HR HU IE IS IT LI LT LU LV MC MK MT NL NO PL PT RO RS SE SI SK SM TR**

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(54) Benævnelse: **System til styring af en resonant lineær kompressor, fremgangsmåde til styring af en resonant lineær kompressor og en resonant lineær kompressor**

(56) Fremdragne publikationer:
EP-A2- 1 607 631
DE-A1- 10 314 007
US-A1- 2003 026 703
US-A1- 2003 175 125
US-A1- 2005 031 470
US-A1- 2007 241 698

DESCRIPTION

[0001] The present invention relates to an actuation system for a resonant linear compressor, applied to cooling systems, the latter being particularly designed to operate at the electromechanical resonance of said compressor, so that the system will be capable of raising the maximum power supplied by the linear actuator, in conditions of overload of said cooling system.

[0002] Additionally, the present invention relates to an actuating method for a resonant linear compressor, the operation steps of which enable one to actuate the equipment at the electromechanical resonance frequency, as well as to control the actuation thereof in overload condition.

[0003] Finally, the present invention relates to a resonant linear compressor provided with an actuating system as proposed in the presently claimed object.

Description of the Prior Art

[0004] The known alternating-piston compressors operate to the effect of generating a pressure to compress the gas inside a cylinder, employing an axial movement of the piston, so that the gas on the low-pressure side, called also suction pressure or evaporation pressure, will get into the cylinder through the suction valve.

[0005] The gas is then compressed within the cylinder by the piston movement and, after being compressed, it comes out of the cylinder through the discharge valve to the high-pressure valve, called also discharge pressure or condensation.

[0006] In the case of resonant linear compressors, the piston is actuated by a linear actuator that is formed by a support and magnets, which may be actuated by one or more coils. Such a linear compressor further comprises one or more springs, which connect the movable part (piston, support and magnets) to the fixed part, the latter being formed by the cylinder, stator, coil, head and structure. The movable parts and the springs form the resonant assembly of the compressor.

[0007] Said resonant assembly, actuated by the linear motor, has the function of developing a linear alternating motion, causing the movement of the piston inside the cylinder to exert an action of compressing the gas admitted by the suction valve, until it can be discharged through the discharge valve to the high-pressure side.

[0008] The operation range of the linear compressor is regulated by the balance of the power generated by the motor with the power consumed by the compression mechanism, besides the losses generated in this process. Ion order to achieve maximum thermodynamic efficiency and maximum cooling capacity, it is necessary for the maximum displacement of the piston to approach as much as possible the stroke end, thus reducing the dead gas volume in the compression process.

[0009] To make the process feasible, it becomes necessary for the piston stroke to be known in great accuracy, so as to present the risk of impact of the piston at the stroke end with the equipment head. This impact might generate loss of efficiency of the apparatus of even break of the compressor, in addition to generating acoustic noise.

[0010] Thus, the greater the error in estimating/measuring the piston position, the greater the safety coefficient required between the maximum displacement and the stroke end, in order to operate the compressor in safety, which leads to loss of performance of the product.

[0011] On the other hand, if it is necessary to reduce the cooling capacity of the compressor due to less need of the cooling system, it is possible to reduce the maximum operation piston stroke, reducing the power supplied to the compressor, and thus it is possible to control the cooling capacity of the compressor, obtaining a variable capacity.

[0012] An additional and quite important characteristic ion the operation of resonant linear compressors is their actuation frequency.

[0013] In general, resonant compressors are designed to function at the resonance frequency of the so-called mass/spring system, a condition in which the efficiency is maximum and wherein the mass considered is given by the sum of the mass of the movable part (piston, support and magnets), and the equivalent spring (K_T) is taken from the sum of the resonant spring of the system (K_{ML}), plus the gas spring generated by the compression force of the gas (K_G), which has a behavior similar to a non-linear variable spring, and that depends upon the evaporation and condensation pressures of the cooling system, as well as upon the gas used in said system.

[0014] Some solutions of the prior art try to solve the problem of actuation frequency of resonant compressors for certain operation conditions, as well be set forth hereinafter.

[0015] Document WO 00079671A1 uses detection of counter electromotive force (CEMF) of the motor to adjust the resonance frequency, but this technique has the disadvantage that it needs a minimum time without current to detect crossing by zero of the CEMF, thus impairing the maximum power supplied and the efficiency by distortion in the wave form of the current.

[0016] In turn, patent US5,897,296 discloses a control with position sensor and frequency control to minimize the current. This solution is similar to those already available in the prior art and has the disadvantage one has to disturb the system periodically for adjustment of the actuation frequency, which may impair greatly the performance of the final product.

[0017] Patent US 6,832,898 describes a control of the operation frequency by the maximum of power for a constant current. This technique employs the same principle of the preceding patent, and to it has the same disadvantage of disturbing the system constantly.

[0018] All the above solutions, in addition to those disclosed by documents US 5,980,211, KR0237562 and KR0176909, have the main objective of actuating the compressor at the resonance frequency of the mechanical system, regardless of the frequency adjustment method and, in this condition, the relationship between the displacement and the current is maximum (or velocity and current).

[0019] Although the efficiency is maximum at the mechanical resonance frequency, the feed voltage is not at the optimum point, that is, the relationship between the displacement and the feed voltage is not maximum at this frequency. So, depending on the design of the actuator and the load condition of the cooling system/and the compressor, the system may be limited by the maximum voltage which the control system can supply, limiting the maximum power of the system, or making the response time very long to lower the internal temperature of the cooling system, which may impair the preservation of the foods within the system.

[0020] A solution for this overload problem is the oversize of the linear actuator, which raises the cost and reduces the efficiency of the system in nominal condition.

[0021] On the basis of the foregoing, the present invention foresees a system and a method for actuating a piston of a resonant linear compressor, designed for supplying maximum power to the equipment in conditions of overload of the cooling system, reducing costs and raising the efficiency of the compressor it its nominal operation condition.

[0022] The document US 2003/175125 is seen as being the closest prior art and discloses the features of the preamble of claim 1.

Objectives of the Invention

[0023] A first objective of the present invention is to propose an actuation system for a resonant linear compressor, which should be capable of actuating the compressor at its electromechanical resonance frequency, so as to provide maximum power to the equipment in conditions of overload of a cooling system.

[0024] A second objective of the present invention is to provide an actuation system for a resonant linear compressor, so that it will contribute significantly to better preservation of the foods stored in the refrigerator, by raising the maximum power supplied to the equipment compressor.

[0025] A third objective of the present invention is to reduce the manufacture cost of the resonant linear compressor by optimizing the size of its linear actuator.

[0026] A further objective of the present invention consists in optimizing the efficiency of the actuator in nominal operation condition, on the basis of the improvement obtained in the sizing thereof.

[0027] Finally, another objective of the present invention is to provide a substantially more simplified solution with respect to the prior techniques for production thereof on industrial scale.

Brief Description of the Invention

[0028] The objectives of the present invention are achieved by providing an actuation system for a resonant linear compressor, the resonant linear compressor being an integral part of a cooling circuit, the resonant linear compressor comprising at least one cylinder, at least one head, at least one electric motor and at least one spring, the cylinder housing a piston operatively, the actuation system comprising at least one electronic control of actuation of the electric motor, the electronic actuation control comprising at least one control circuit and at least one actuation circuit, which are associated to each other, the electronic actuation control being electrically associated to the electric motor of the linear compressor, the actuation system being configured to detect at least one overload condition of the linear compressor, through at least one electric magnitude measured, or estimated, by the electronic actuation control, and adjust, from a overload control mode, the actuation frequency of the electric motor to an electromechanical resonance frequency or at an intermediate frequency between the mechanical resonance and the electromechanical resonance.

[0029] The objectives of the present invention are further achieved by providing an actuation method for a resonant linear compressor, the resonant linear compressor comprising at least one electric motor, the electric motor being actuated by a frequency inverter, the actuation method comprising the following steps:

1. a) measuring or estimating, at every operation cycle of the resonant linear compressor, an actuation or operation frequency, a maximum displacement of the piston of the resonant linear compressor and/or the displacement phase of the piston stroke and/or the velocity phase of the piston and/or the current phase;
2. b) companng the maximum displacement of the piston with a maximum reference displacement, and calculating a displacement error;
3. c) calculating an operation feed voltage value of the electric motor from a operation feed voltage value of a preceding cycle and the displacement error obtained at the preceding step (s);
4. d) comparing the operation feed voltage value of the electric motor calculated at the preceding step with a maximum feed voltage value;
5. e) if the operation feed voltage value calculated at the step "c" is lower than or equal to the maximum feed voltage value, then deactivate the overload control mode of the electric control and decrease the actuation frequency down to a mechanical resonance frequency value; and returning to step a),
6. f) if the operation feed voltage value calculated at the step "c" is higher than the maximum feed voltage value, then activate the overload control mode and increase the actuation frequency up to an electromechanical resonance frequency.

Brief Description of the Drawings

[0030] The present invention will now be described in greater details with reference to the attached drawings, in which:

- figure 1 represents a schematic view of a resonant linear compressor;
- figure 2 illustrates a schematic view of the mechanical model of the resonant linear compressor employed in the present invention;
- figure 3 illustrates a schematic view of the electric model of the resonant linear compressor of the present invention;
- figure 4 shows a graph of the position of the poles of the electric, mechanical and complete system, according to the teachings of the present invention;
- figure 5 illustrates a Bode diagram for the displacement of the mechanical system;
- figure 6 shows a Bode diagram for the velocity of the mechanical system;
- figure 7 illustrates a Bode diagram of the current of the complete electromechanical system of the present invention;
- figure 8 illustrates a Bode diagram of the displacement of the complete electromechanical system, according to the teachings of the invention;
- figure 9 illustrates a Bode diagram of the velocity of the complete electromechanical system of the present invention;
- figure 10 represents a simplified block diagram of the control with a sensor;
- figure 11 illustrates a block diagram of the control and of the inverter with a sensor;
- figure 12 shows a simplified block diagram of the control without sensor;

- figure 13 shows a block diagram of the control and inverter without sensor;
- figure 14 shows first flow chart capable of detecting the overload mode in a normal control proposal;
- figure 15 shows second flow chart intended for detection of the overload mode in a second normal control proposal;
- figure 16 shows an overload-control flow chart for maximum displacement;
- figure 17 shows an overload-control flow chart for the adjustment of the velocity phase;
- figure 18 shows an overload-control flow chart for the adjustment of the displacement phase; and
- figure 19 shows an overload-control flow chart for minimum current shift.

Detailed Description of the Figures

[0031] Figure 1 shows a schematic view of a resonant linear compressor 50, object of the present invention.
model of the linear compressor 50, such a mechanical model being defined on the basis of equation 1 below, and said electric model being defined from equation 2.
partir da equação 2.

$$m \cdot \frac{d^2 d(t)}{dt^2} = F_{MT}(i(t)) - F_{ML}(d(t)) - F_{AM}(v(t)) - F_G(d(t)) \qquad (1)$$

wherein:

$F_{MT}(i(t)) = K_{MT} \cdot i(t)$ - motor force [N];

$F_{ML}(d(t)) = K_{ML} \cdot d(t)$ - spring force [N];

$F_{AM}(v(t)) = K_{AM} \cdot v(t)$ - damping force [N];

$F_G(d(t))$ - force of gas pressure in the cylinder [N];

K_{MT} - motor constant

K_{ML} - spring constant

K_{AM} - damping constant

m - mass of the moveable par

v(t) - piston velocity

d(t) - piston displacement

i(t) - motor current

$$V_{ENT}(t) = V_R(i(t)) + V_L(i(t)) + V_{MT}(v(t)) \qquad (2)$$

Wherein:

$V_R(i(t)) = R \cdot i(t)$- resistance voltage [V];

$$V_L(i(t)) = L \cdot \frac{di(t)}{dt}$$

- inductor voltage [V];

$V_{MT}(v(ti = K_{MT} \cdot v(t)$- voltage induced in the motor or CEMF [V];

$V_{ENT}(t)$ - feed voltage [V];

R - electric resistance of the motor

L - motor inductance.

[0032] It should be pointed out that, the gas pressure force ($F_G(d(t))$) is variable with the suction and discharge pressures, with the non-linear piston displacement, with the other forces in the mechanical equation they are all linear, just as all the voltages in the electric equation. In order to obtain the complete model of the system, it is possible to replace the pressure force by the effects which it causes in the system, which are power consumption and variation in the resonance frequency.

[0033] The power consumption may be modeled by an equivalent damping and the variation in the resonance frequency by an equivalent spring.

[0034] Thus, the equation (1) above may be rewritten as follows:

$$m \cdot \frac{d^2(t)}{dt^2} = K_{MT} \cdot i(t) - (K_{ML} + K_{MLEq}) \cdot d(t) - (K_{AM} + K_{AMEq}) \cdot v(t) \quad (3)$$

or

$$m \cdot \frac{d^2(t)}{dt^2} = K_{MT} \cdot i(t) - K_{MLT} \cdot d(t) - K_{AMT} \cdot v(t) \quad (4)$$

Wherein:

K_{MLEq} - equivalent spring coefficient

K_{AMEq} - equivalent damping coefficient

$K_{MLT} = K_{ML} + K_{MLEq}$ - total spring coefficient

$K_{AMT} = K_{AM} + K_{AMEq}$ - total damping coefficient

[0035] Applying the Laplace transform to the equations (2) and (4), one can obtain the equation (5) below, which represents the electric equation at the minimum of the frequency and the mechanical equations (6) and (7), which represent, respectively the function of transfer between displacement and velocity with the current.

$$I(s) = \frac{V_{ENT}(s) - K_{MT}.V(s)}{L.s + R} \quad (5)$$

$$\frac{D(s)}{I(s)} = \frac{K_{MT}}{m.s^2 + K_{AMT}.s + K_{MLT}} \quad (6)$$

$$\frac{V(s)}{I(s)} = \frac{K_{MT}.s}{m.s^2 + K_{AMT}.s + K_{MLT}} \quad (7)$$

[0036] The equation (8) below represents the characteristic equation of the electric system, so that the equation (9) represents the characteristic equation of the mechanical system. The poles of this equation define the mechanical resonance frequency, region where the relationship between displacement/current, or velocity/current, is maximum, and therefore with maximum efficiency as well, just as described ion other solutions of the prior art.

$$EC_E = L.s + R \quad (8)$$

$$EC_M = m.s^2 + K_{AMT}.s + K_{MLT} \quad (9)$$

[0037] Working out mathematically the equations (5) to (9), one can obtain the equations (10), (11) and (12), which represent, respectively, the function of transfer of the current, of the displacement and of the velocity of the piston of the compressor 50, as a function of the input voltage, for the complete electromechanical system, according to the teachings of the present invention:

$$\frac{I(s)}{V_{ENT}(s)} = \frac{EC_M}{EC_M \cdot EC_E + K_{MT}{}^2.s} \quad (10)$$

$$\frac{D(s)}{V_{ENT}(s)} = \frac{K_{MT}}{EC_M.EC_E + K_{MT}{}^2.s} \quad (11)$$

$$\frac{V(s)}{V_{ENT}(s)} = \frac{K_{MT}.s}{EC_M.EC_E + K_{MT}^2.s} \quad (12)$$

[0038] One may further define the equation (13) or (14) below, as the characteristic equation of the electromechanical system designed in the present invention:

$$EC_S = EC_M.EC_E + K_{MT}^2.s \quad (13)$$

or:

$$EC_S = m.L.s^3 + (K_{AMT}.L + m.R)s^2 + (K_{MLT}.L + K_{AMT}.R + K_{MT}^2).s + K_{MLT} \cdot R \quad (14)$$

[0039] The pair of complex poles of the characteristic equation of the electromechanical system above defines the electromechanical resonance frequency, the region in which one has greater relation between current, the displacement and the velocity with the input voltage. Therefore, this is a region where it is possible to obtain maximum power of the resonant linear compressor, as proposed in the present invention.

[0040] For a better understanding of the characteristics of the actuation system and method proposed, which will be described in greater details later, one presents the values in Table 1 below, which define the coefficients of a resonant linear compressor, designed to operate at a mechanical resonant frequency of 50 Hz, for a nominal load of 50 W.

Table 1 - Coefficients of the resonant linear compressor

Coefficient	Value	Unit
R	12.9	Ω
L	0.75	H
K_{MT}	70	V.s/m or N/A
K_{MLT}	81029.5	N/m
K_{AMT}	10	N.s/m
m	0.821	Kg

[0041] Calculating the poles of the electric system and mechanical system in isolation, and of the complete electromechanical system, one will visualize the alteration in the system poles, according to Table2 below, and also from figure 4.

[0042] The mechanical resonance frequency is given by the module of the pair of complex poles of the characteristic equation of the mechanical system (314.2 rad/s or 50 Hz). The electromechanical resonance frequency is given by the module of the pair of complex poles of the characteristic equation of the electromagnetic system (326.6 rad/s or 51.97 Hz).

Table 2 - Poles of the electric, mechanical and electromechanical system

Poles		
System	Real	Complex
Electric	17.2	-
Mechanical	-	6.09±3141j
Electromechanical	-15.9	6.73±326.5j

[0043] In Bode diagrams of the transfer function of displacement and velocity, for the mechanical system, such as shown in figures 5 and 6, one can observe that, at the mechanical resonance frequency, the gain is maximum. In this case, the phase between the displacement with the current is of -90 degrees (displacement and current are in quadrature), and the phase of the velocity with the current is zero degree (velocity and current are in phase).

[0044] Additionally, one observes from the diagrams of figures 7, 8 and 9, represent, respectively, the Bode diagrams of the transfer functions of the current, the displacement of the velocity, as a function of the input voltage, which, at the electromechanical resonance frequency, the gain is maximum, according to the teachings of the present invention.

[0045] Moreover, it is possible to observe, in figure 7, that, in the mechanical resonance frequency, the value of the current is minimum, for which reason the efficiency is maximum. At the middle point between the mechanical resonance frequency and the electromechanical resonance frequency, the power factor of the linear actuator is maximum, since the phase of the current has the shortest delay.

[0046] The electromechanical resonance frequency is always above the mechanical resonance frequency, and at the electromechanical frequency the phase between the displacement and the input voltage is around -176 degrees, and the phase between the velocity and the input voltage is around -86 degrees, for the data presented in Table 1 above. The greater the difference between the real pole and the module of the pair of complex poles of the electromechanical system, the shift of the displacement and of the velocity will tend to -180 degrees and -90 degrees, respectively.

[0047] In the face of the foregoing, one proposes the present invention for the main purpose of supplying maximum power to the resonant linear compressor 50, for conditions of overload of the cooling system.

[0048] Such a system takes into account that the linear compressor 50-comprises at least one cylinder 2, at least one had 3, at least one electric motor and at least one spring, so that the cylinder 2 houses operatively a piston 1. Figure 1 shows said compressor 50 and its constituent parts.

[0049] As far as the electronic composition is concerned, it is possible to note, on the basis of figures 10 - 13, the main characteristics of the present actuation system. Such a system comprises at least one electronic actuation control 20 of the electric motor, this electronic actuation control 20 being provided with at least one control circuit 24 and at least one actuation circuit 26, associated electrically with each other.

[0050] The same figures show that the electronic actuation control 20 is electronically associated to the electric motor of the linear compressor 50, this electronic control 20 being composed of rectifying element, inverter (inverting bridge) and digital processor

[0051] A quite relevant characteristic of the presently claimed invention as compared with the prior techniques refers to the fact that the actuation system is particularly configured to detect at least one overload condition of the linear compressor (50), through at least one electric magnitude measured or estimated by the electronic actuation control 20, and to adjust, from a overload control mode, the actuation frequency of the electric motor to an electromechanical resonance frequency.

[0052] The electric magnitude measured or estimated is given by a actuating piston velocity value V_p, or still by a piston displacement value d_p. the actuation electronic control 20 is capable of actuating, according to the teachings of the invention, the electric motor of the compressor 50 with a PWM senoidal voltage starting from an amplitude and a controlled range.

[0053] As already mentioned before, the present invention has the central objective of detecting a condition of overload of the linear compressor 50, under conditions in which it is necessary to adjust the actuation frequency of said electric motor, in a determined operation mode in overload, in order to achieve the desired control of the cooling system in situations of high demand.

[0054] One first way to control the motor of the compressor 50 in this condition is illustrated in figure 16. Figures 14 and 15 shows two flow charts oriented to detect the overload mode in two different proposals of normal control. In this case, the overload control mode is configured to adjust the actuation frequency of the electric motor by taking as a basis a piston displacement value de ((t)), or $D_{MAX}[K]$, with respect to the maximum reference displacement D_{REF}. One observes that the function F illustrated in figure 14 (see second block A[k]=F(A[k-1],Ed[k]) may be a control P, PI or PID.

[0055] In a second mode, as shown in figure 17, the overload control is configured to adjust the actuation frequency of the electric motor by taking as a basis a velocity phase ϕv of the motor of the compressor 50m, with respect to a reference velocity ϕREF.

[0056] A third way to adjust the actuation frequency of the compressor 50 is shown in figure 18. In this case, the overload control mode is configured to adjust the actuation frequency of the electric motor by taking as basis a value of the displacement phase ϕ_d of the motor of the compressor, with respect to the reference displacement phase ϕ_{dREF}

[0057] Additionally, figure 19 shows an alternative way of adjusting the actuation frequency of said compressor 50. This is a way of controlling overload, configured to adjust the actuation frequency of the electric motor taking, as a basis, a minimum current phase value ϕc.

[0058] With regard to the above-described adjustment modes, they are given by the difference in phase between the piston displacement value ($d_e(t)$) and an input voltage phase (V_{int}.) preferably around -176 degrees (for the compressor defined by the parameters of Table 1). On the other hand, the adjustment of actuation frequency is given starting from the difference between the velocity phase value ϕv and an input voltage phase value Vint, preferably around -86 degrees (for the compressor defined by the parameters of Table 1).

[0059] The present invention has, as an innovatory and differentiated characteristic over the prior art, a set of steps capable of adjusting the actuation frequency of the compressor 50 in an efficient and quite simplified manner for the overload control mode foreseen. Such a methodology takes into account the fact that said compressor comprises at least one electric motor, the latter being actuated by a frequency inverter. Said method comprises essentially the following steps:

a-) measuring and estimating, at every operation cycle T_R of the resonant linear compressor 50, an actuation frequency F_R, a maximum piston displacement $d_e(t)$ of the resonant linear compressor 50, and/or the piston displacement phase ϕd and/or the piston velocity phase ϕv and/or the current phase ϕc;

b-) comparing the maximum piston displacement $d_e((t)$ with a maximum reference displacement D_{REF}, and calculating a displacement error Err;

c-) calculating an operation feed voltage value $_{Am\text{-}pop}$ of the electric motor, from an operation feed voltage value of previous cycle and of the displacement error Err obtained in the preceding step (s);

d-) comparing the operation feed voltage value A_{mpop} of the electric motor calculated at the preceding step with a maximum feed voltage value Amax;

e-) if the operation feed voltage value A_{mpop} calculated at step "c" is lower than or equal to the maximum feed voltage value A_{max}, then deactivate an overload control mode of the electric motor and decrease the actuation frequency F_R down to a mechanical resonance frequency; and returning to step a-);

f-) if the operation feed voltage value A_{mpop} calculated at step "c" is higher than the maximum feed voltage value A_{max}, then activate the overload control mode and increase the actuation frequency F_R up to an electromechanical resonance frequency.

[0060] As to the first overload control mode, as illustrated in figure 16, one can state that it further comprises the following step:

n) comparing the maximum piston displacement de(t) with a maximum piston displacement of a cycle $d_e(t-1)$ preceding the operation cycle T_R;

o) if the maximum piston displacement de(t) is higher than the piston displacement of the preceding cycle de(t), then comparing the actuation frequency F_R with the actuation frequency of the preceding cycle $F_{R((t-1)}$;

p) if the actuation frequency F_R is higher than the actuation frequency of preceding cycle $R_{F(t-1)}$, then increasing the actuation frequency F_R by a frequency delta value T_f and returning to step a);

q) if the actuation frequency F_R is not higher than the actuation frequency of the preceding cycle $F_{R(t-1)}$, then decreasing the actuation frequency F_R by a frequency delta value T_f and returning to step a);

r) if the maximum piston displacement $d_e(t)$ is not greater than the maximum piston displacement of preceding cycle $d_e(t-1)$, then comparing the actuation frequency F_R with an actuation frequency of preceding cycle F R(t-1);

s) if the actuation frequency F_R is lower than that actuation frequency of preceding cycle $F_{R(t-1)}$, then increasing the actuation frequency F_R by a frequency delta value T_f and returning to step a);

t) if the actuation frequency F_R is not lower than the actuation frequency of preceding cycle $F_{R(t-1)}$, then decreasing the actuation frequency F_R by a frequency delta value T_f and returning to step a).

[0061] It should be pointed out that steps "n" to "t" define an overload control mode for a maximum piston displacement value of the compressor 50.

[0062] For the second overload control mode, as shown in figure 17, the following steps are foreseen:

n) calculating a velocity phase ϕv of the piston of the compressor 50;

o) comparing the velocity phase ϕv, calculated at the preceding step, with a reference velocity phase value ϕ_{VREF};

p) if the velocity phase ϕv is higher than the reference velocity phase ϕ_{VREF}, then increase the actuation frequency F_R by a frequency delta value T_f and returning to step a);

q) if the velocity phase ϕv is not higher than the reference velocity phase ϕV_{VREF}, then decrease the actuation frequency F_R by a frequency delta value T_f and returning to step a).

for this second control mode, steps "n" to "q" define an overload control mode of the compressor 50 for an adjustment of reference velocity phase around -90 degrees (-86 for the compressor defined by the parameters of Table 1).

[0063] A third way to adjust the actuation frequency, according to the teachings of the present invention, and as illustrated in figure 18, comprises the following steps:

n) calculating a piston displacement phase ϕ_d of the compressor 50;

o) comparing the displacement phase ϕ_d calculated at the preceding step with a reference displacement phase value Φ_{DREF};

p) if the displacement phase ϕd is higher than the reference displacement phase ϕ_{DREF}, then increase the actuation frequency F_R by a frequency delta value T_f and returning to step a);

q) if the displacement phase ϕd is not higher than the reference displacement phase ϕ_{DREF}, then decrease the actuation frequency F_R by a frequency delta value T_f and returning to step a).

[0064] The last steps "n" to "q" above define an overload control mode of the compressor 50 for an adjustment of reference displacement phase around -180 (-176 degrees for the compressor defined by the parameters of table 1).

[0065] In turn, figure 19 shows a fourth way of adjusting the actuation frequency of the electric motor, consisting of the following steps:

n) calculating a current phase ϕc of the compressor 50;

o) comparing the current phase ϕc calculated at the preceding step with a current phase value ϕc-1 preceding the operation cycle TR;

p) if the current phase ϕc is higher than the previous cycle current phase value ϕc-1, then comparing the actuation frequency F_R with a previous cycle actuation frequency $F_R(t-1)$;

q) if the actuation frequency F_R is higher than the previous cycle actuation frequency $F_R(t-1)$, then increase the actuation frequency F_R by a frequency delta value T_f and returning to step a);

r) if the actuation frequency F_R is not higher than the previous cycle actuation frequency $F_R(-1)$, then decrease the actuation frequency F_R by a frequency delta value T_f and returning to step a);

s) if the current phase value ϕc is not higher than the previous cycle current phase value ϕc-1, then comparing the actuation frequency F_R with a previous cycle actuation frequency $F_R(t-1)$;

t) if the actuation frequency F_R is lower than the previous cycle actuation frequency $F_R(t-1)$, then increase the actuation frequency F_R by a frequency delta value Tf and returning to step a);

u) if the actuation frequency Fr is not lower than the previous cycle actuation frequency $F_R(t-1)$, then decrease the actuation frequency F_R by a frequency delta value Tf and returning to step a);

for steps "n" and "u" above, one defines an overload control mode of the compressor 50 for a minimum current shift.

[0066] It should be pointed out that, as the piston displacement reaches the maximum reference value and reaches the resonance frequency again, the present system and method are configured to come out of the overload control.

[0067] On the other hand, the present invention foresees a resonant linear compressor 50 provided with the presently designed actuation system and with the actuation method as defined in the claimed object.

[0068] Finally, one can state that the actuation system and method for a resonant linear compressor 50 as described above achieve their objectives inasmuch as it is possible to increase the maximum power supplied to said compressor ion conditions of high load or overload for the same equipment design.

[0069] Moreover, it should be pointed out that the present invention enables better preservation of the foods of the cooling equipment by increasing the maximum power supplied to said compressor. Further, it is possible, on the bases of the teachings of the invention, to reduce manufacture costs of the final product, as well as to increase the efficiency of the compressor 50 in its nominal operation condition, taking into account a better sizing of its linear actuator.

[0070] A preferred example of embodiment having been described, one should understand that the scope of the present invention embraces other possible variations, being limited only by the contents of the accompanying claims, which include the possible equivalents.

REFERENCES CITED IN THE DESCRIPTION

Patent documents cited in the description

- WO00079671A1 [0015]
- US5897296A [0016]
- US6832898B [0017]
- US5980211A [0018]
- KR0237562 [0018]
- KR0176909 [0018]
- US2003175125A [0022]

Patentkrav

1. System til styring af en resonant lineær kompressor (50), hvor den resonante lineære kompressor (50) er en integreret del af et kølekredsløb, hvor den resonante lineære kompressor (50) omfatter mindst én cylinder (2), mindst ét hoved (3), mindst én elektrisk motor mindst én fjeder, hvor cylinderen funktionelt (2) indeholder et stempel (1),
hvor styresystemet er kendetegnet ved at omfatte mindst én elektronisk styringskontrol (20) til styring af den elektriske motor, hvor den elektroniske styringskontrol (20) omfatter mindst ét styrekredsløb (24) og mindst ét styrekredsløb (26), som er indbyrdes forbundet,
hvor den elektroniske styringskontrol (20) er elektronisk forbundet med den elektriske motor af den lineære kompressor (50),
hvor styresystem er konfigureret til at detektere mindst én overbelastningstilstand af en lineær kompressor (50), gennem mindst én elektrisk størrelse målt eller vurderet af den elektroniske styringskontrol (20), og til fra en overbelastningskontrolmodus at regulere styringsfrekvensen af den elektriske motor til en elektromekanisk resonansfrekvens.

2. Styresystem ifølge krav 1, kendetegnet ved, at den målte eller vurderede elektriske størrelse gives af en stempelhastighedsværdi (V_p).

3. Styresystem ifølge krav 1, kendetegnet ved, at den målte eller vurderede elektriske størrelse gives af en stempelforskydningsværdi (V_p).

4. Styresystem ifølge krav 1, kendetegnet ved, at overbelastningskontrollen konfigureres til at regulere styringsfrekvensen af den elektriske motor ved som udgangspunkt at anvende stempelforskydningsværdien ($d_e(t)$) i forhold til den maksimale forskydning (D_{REF}).

5. Styresystem ifølge krav 1, kendetegnet ved, at overbelastningsmodusen konfigureres til at regulere styringsfrekvensen af den elektriske motor ved som udgangspunkt at anvende hastighedsfaseværdien (ϕv) af motoren af kompressoren (50) i forhold til en referencehastighedsfase (Φ_{REF}).

6. Styresystem ifølge krav 1, kendetegnet ved, at overbelastningsmodusen konfigureres til at regulere styringsfrekvensen af den elektriske motor ved som udgangspunkt at anvende forskydningsfaseværdien (ϕ_d) af motoren af kompressoren (50) i forhold til en referenceforskydningsfase (ϕ_{dREF}).

7. Styresystem ifølge krav 1, kendetegnet ved, at overbelastningsmodus konfigureres til at regulere styringsfrekvensen af den elektriske motor ved som udgangspunkt at anvende en minimal strømfaseværdi (ϕ_c).

8. Styresystem ifølge krav 6, kendetegnet ved, at reguleringen af styringsfrekvensen gives med udgangspunkt i en faseforskel mellem stempelforskydningsværdien ($d_e(t)$) og en indgangsspændings faseværdi (V_{int}) omkring -180 grader.

9. Styresystem ifølge krav 5, kendetegnet ved, at reguleringen af styringsfrekvensen gives med udgangspunkt i en faseforskel mellem hastighedsfaseværdien (ϕ_v) og en indgangsspændingsfaseværdi (V_{int}) omkring -90 grader.

10. Styrefremgangsmåde til en resonant lineær kompressor (50), hvor den resonante lineære kompressor (50) omfatter mindst én elektrisk motor, hvor den elektriske motor styres af en frekvensvekselretter, hvor styrefremgangsmåden er kendetegnet ved at omfatte de følgende trin med:
a-) at måle eller vurdere, ved hver driftscyklus (T_R) af den resonante lineære kompressor (50), en styringsfrekvens (F_R), en maksimal stempelforskydning ($d_e(t)$) af den resonante lineære kompressor (50) og/eller stempelforskydningsfasen (ϕ_d) og/eller stempelhastighedsfasen (ϕ_v) og/eller strømfase (ϕ_c).
b-) at sammenligne den maksimale stempelforskydning (de(t)) med en maksimal referenceforskydning (D_{REF}), og beregne en forskydningsfejl (Err),
c-) at beregne en driftsforsyningsspændingsværdi (A_{mpop}) af den elektriske motor, fra en driftsforsyningsspændingssværdi af den forudgående cyklus og af forskydningsfejlen (Err) opnået ved det eller de forudgående trin;
d-) at sammenligne driftsforsyningsspændingsværdien (A_{mpop}) af den elektriske motor beregnet ved det forudgående trin med en maksimal forsyningsspændingsværdi (A_{max});
e-) hvis driftsforsyningsspændingsværdien (A_{mpop}) beregnet ved trin "c" er mindre end eller lig med den maksimale forsyningsspændingsværdi (A_{max}),

så deaktivere en overbelastningskontrolmodus af den elektriske motor og nedsætte styringsfrekvensen (F_R) til en mekanisk resonansfrekvensværdi, og vende tilbage til trin a);

f-) hvis driftsforsyningsspændingsværdien (A_{mpop}) beregnet ved trin "c" er højere end den maksimale forsyningsspændingsværdi (A_{max}), så aktivere overbelastningskontrolmodusen og forhøje styringsfrekvensen (F_R) til en elektromekanisk resonansfrekvens.

11. Styrefremgangsmåde ifølge krav 10, kendetegnet ved, at overbelastningskontrolmodusen endvidere omfatter de følgende trin med:

g) at sammenligne den maksimale stempelforskydning ($d_e(t)$) med en stempelforskydningsværdi af en cyklus ($d_e(t-1)$), som går forud for tidsrummet af driftscyklus (T_R);

h) hvis den maksimale stempelforskydning (de(t)) er større end stempelforskydningen af den forudgående cyklus (de(t-1)), så sammenligne styringsfrekvensen (F_R) med en driftsfrekvens af den forudgående cyklus ($F_R(t-1)$;

i) hvis styringsfrekvensen (F_R) er højere end styringsfrekvensen af den forudgående cyklus ($F_R(t-1)$), så forhøje styringsfrekvensen (F_R) med en frekvens deltaværdi (T_f) og vende tilbage til trin a);

j) hvis styringsfrekvensen (F_R) ikke er højere end styringsfrekvensen af den forudgående cyklus ($F_R(t-1)$), så nedsætte styringsfrekvensen (F_R) med en frekvensdeltaværdi (T_f) og vende tilbage til trin a);

k) hvis den maksimale stempelforskydning ($d_e(t)$) ikke er større end den maksimale stempelforskydning af den forudgående cyklus ($d_e(t-1)$), så sammenligne styringsfrekvensen (F_R) med styringsfrekvensen af den forudgående cyklus ($F_R(t-1)$);

l) hvis styringsfrekvensen (F_R) er lavere end styringsfrekvensen af den forudgående cyklus ($F_R(t-1)$), så forhøje styringsfrekvensen (F_R) med en frekvensdeltaværdi (Tf) og vende tilbage til trin a);

m) hvis styringsfrekvensen (F_R) ikke er højere end styringsfrekvensen af den forudgående cyklus ($F_R(t-1)$), så nedsætte styringsfrekvensen (F_R) med en frekvensdeltaværdi (T_f) og vende tilbage til trin a).

12. Styresystem ifølge krav 11, kendetegnet ved, at trinnene "g" til "m" definerer en overbelastningskontrolmodus for en maksimal stempelforskydning

af kompressoren (50).

13. Styrefremgangsmåde ifølge krav 10, kendetegnet ved, at den endvidere omfatter de følgende trin med:
n) at beregne hastighedsfasen (ϕ_V) af stemplet af kompressoren (50);
o) sammenligne hastighedsfasen (ϕ_V) af stemplet af kompressoren (50) med en referencehastighedsfaseværdi (ϕ_{VREF});
p) hvis hastighedsfasen (ϕv) er højere end referencehastighedsfasen (ϕ_{VREF}), så forøge styringsfrekvensen (F_R) med en frekvensdeltaværdi (T_f) og vende tilbage til a);
q) hvis hastighedsfasen (ϕ_V) ikke er højere end referencehastighedsfasen (ϕ_{VREF}), så nedsætte styringsfrekvensen (F_R) med en frekvensdeltaværdi (T_f) og vende tilbage til trin a).

14. Styrefremgangsmåde ifølge krav 13, kendetegnet ved, at trinnene "n" til "q" definerer en overbelastningskontrolmodus af kompressoren (50) til en regulering af frekvenshastighedsfasen omkring -90 grader.

15. Styrefremgangsmåde ifølge krav 10, kendetegnet ved, at den endvidere omfatter de følgende trin med:
n) at beregne forskydningsfasen (ϕ_V) af stemplet af kompressoren (50);
o) at sammenligne forskydningsfasen (ϕd) beregnet ved det forudgående trin med en referenceforskydningsfaseværdien (ϕ_{DREF});
p) hvis forskydningsfasen (ϕ_d) er større end referenceforskydningsfasen (ϕ_{DREF}), så forøge styringsfrekvensen (F_R) med en frekvensdeltaværdi (Tr) og vende tilbage til trin a);
q) hvis forskydningsfasen (ϕ_d) ikke er større end referenceforskydningsfasen (ϕ_{DREF}), så nedsætte styringsfrekvensen (F_R) med en frekvensdeltaværdi (T_f) og vende tilbage til trin a).

16. Styrefremgangsmåde ifølge krav 15, kendetegnet ved, at trinnene "n" til "q" definerer en overbelastningskontrolmodus af kompressoren (50) til en regulering af frekvenshastighedsfasen omkring -180 grader.

17. Styrefremgangsmåde ifølge krav 10, kendetegnet ved, at overbelastningskontrolmodusen endvidere omfatter de følgende trin med:

n) at beregne en strømfase (ϕ_c) af kompressoren (50);

o) at sammenligne strømfasen (ϕ_c), der er beregnet ved det forudgående trin med en strømfaseværdi af en cyklus (ϕ_c-1), som går forud for et tidsrum af driftscyklussen (T_R);

p) hvis strømfasen (ϕ_c) er højere end strømfaseværdien af den forudgående cyklus (ϕ_c-1), så sammenligne styringsfrekvensen (F_R) med en styringsfrekvens af den forudgående cyklus (F_R(t-1));

q) hvis styringsfrekvensen (F_R) er højere end styringsfrekvensen af den forudgående cyklus (F_R(t-1)), så forhøje styringsfrekvensen (F_R) med en frekvensdeltaværdi (T_f) og vende tilbage til trin a);

r) hvis styringsfrekvensen (F_R) ikke er højere end styringsfrekvensen af den forudgående cyklus (F_R(t-1)), så nedsætte styringsfrekvensen (F_R) med en frekvensdeltaværdi (T_f) og vende tilbage til trin a);

s) hvis strømfasen (ϕ_c) ikke er højere end strømfaseværdien af den forudgående cyklus (ϕc-1), så sammenligne styringsfrekvensen (F_R) med en styringsfrekvens af den forudgående cyklus (F_R(t-1));

t) hvis styringsfrekvensen (F_R) er lavere end styringsfrekvensen af den forudgående cyklus (F_R(t-1)), så forhøje styringsfrekvensen (F_R) med en frekvensdeltaværdi (T_f) og vende tilbage til trin a);

u) hvis styringsfrekvensen (F_R) ikke er lavere end styringsfrekvensen af den forudgående cyklus (F_R(t-1)), så nedsætte styringsfrekvensen (F_R) med en frekvensdeltaværdi (T_f) og vende tilbage til trin a).

18. Styrefremgangsmåde ifølge krav 17, kendetegnet ved, at trinnene "n" til "u" definerer en overbelastningskontrolmodus af kompressoren (50) for en minimal strømforskydning.

19. Resonant lineær kompressor (50), kendetegnet ved at omfatte et styresystem som defineret i kravene 1 til 9, og en styrefremgangsmåde som defineret i kravene 10 til 18.

DRAWINGS

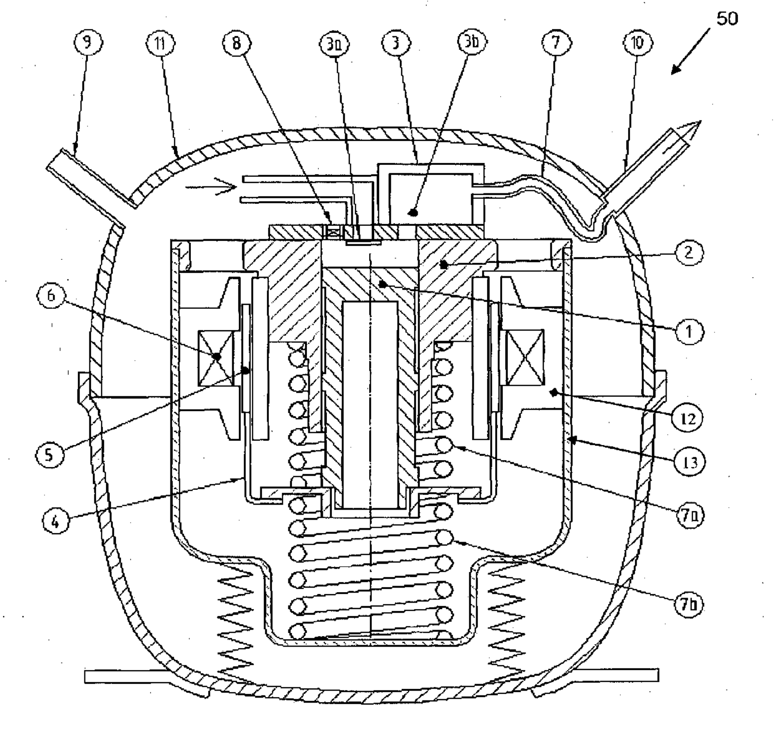

FIG. 1

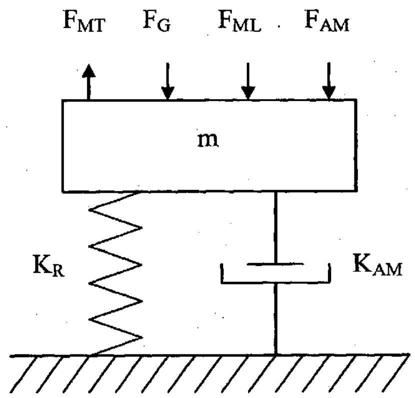

FIG. 2

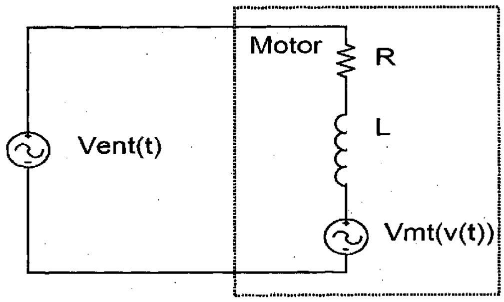

FIG. 3

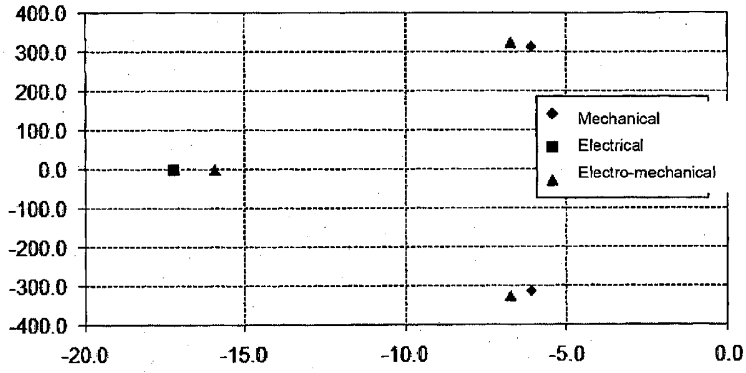
400.0
300.0
200.0
100.0
0.0
-100.0
-200.0
-300.0
-400.0
-20.0
-15.0
-10.0
-5.0
0.0
Mechanical
Electrical
Electro-mechanical

FIG. 4

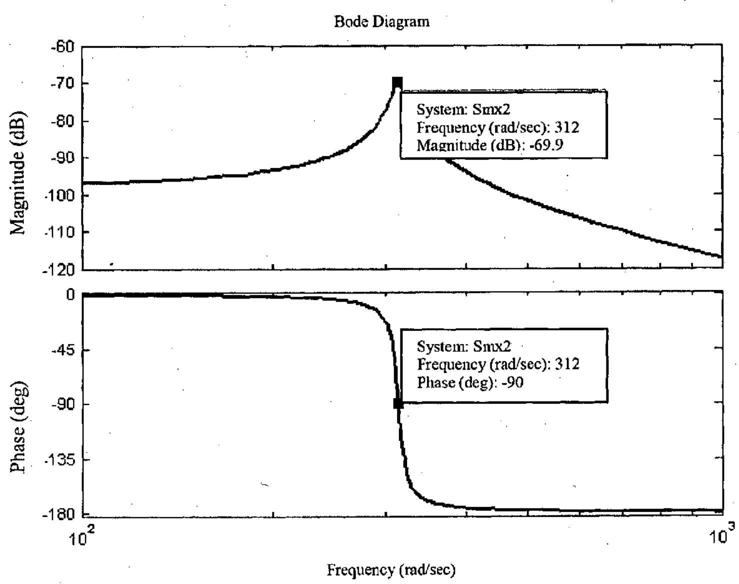
Bode Diagram
Magnitude (dB)
-60
-70
-80
-90
-100
-110
-120
System: Smx2
Frequency (rad/sec): 312
Magnitude (dB): -69.9
Phase (deg)
0
-45
-90
-135
-180
System: Smx2
Frequency (rad/sec): 312
Phase (deg): -90
10^2
10^3
Frequency (rad/sec)

FIG. 5

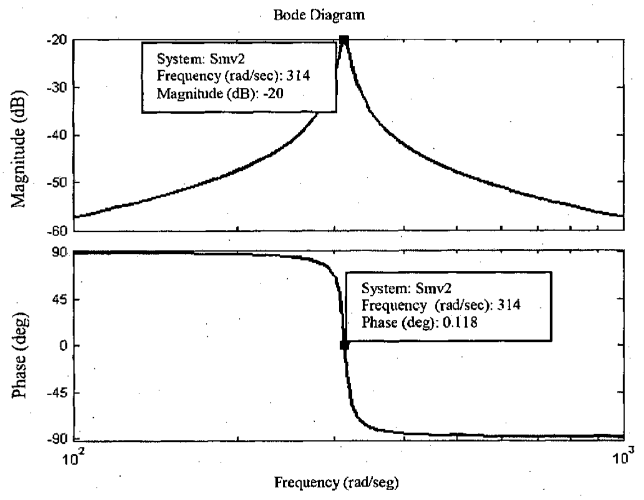
Bode Diagram
System: Smv2
Frequency (rad/sec): 314
Magnitude (dB): -20
System: Smv2
Frequency (rad/sec): 314
Phase (deg): 0.118
Magnitude (dB)
Phase (deg)
Frequency (rad/seg)

FIG. 6

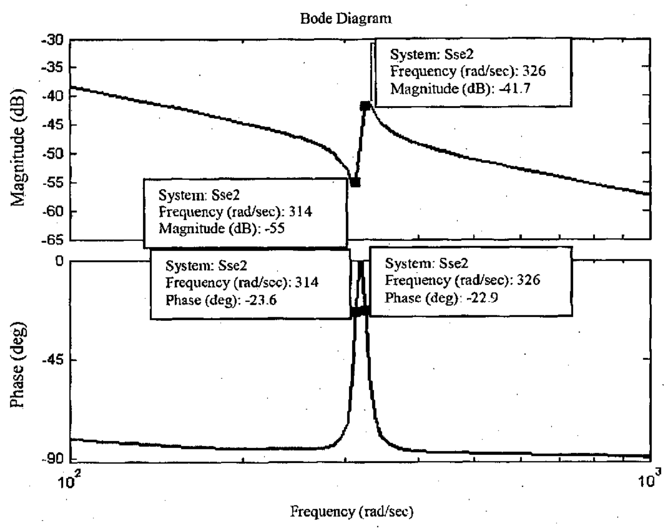
Bode Diagram
System: Sse2
Frequency (rad/sec): 326
Magnitude (dB): -41.7
System: Sse2
Frequency (rad/sec): 314
Magnitude (dB): -55
System: Sse2
Frequency (rad/sec): 314
Phase (deg): -23.6
System: Sse2
Frequency (rad/sec): 326
Phase (deg): -22.9
Magnitude (dB)
Phase (deg)
Frequency (rad/sec)

FIG. 7

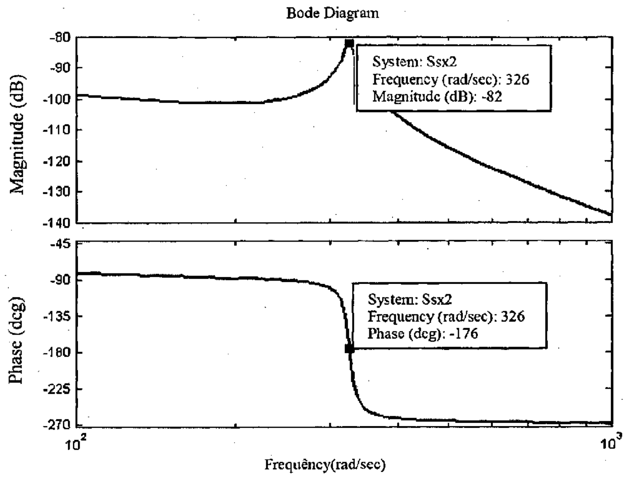
Bode Diagram
Magnitude (dB)
-80
-90
-100
-110
-120
-130
-140
System: Ssx2
Frequency (rad/sec): 326
Magnitude (dB): -82
Phase (deg)
-45
-90
-135
-180
-225
-270
System: Ssx2
Frequency (rad/sec): 326
Phase (deg): -176
10^2
10^3
Frequência(rad/sec)

FIG. 8

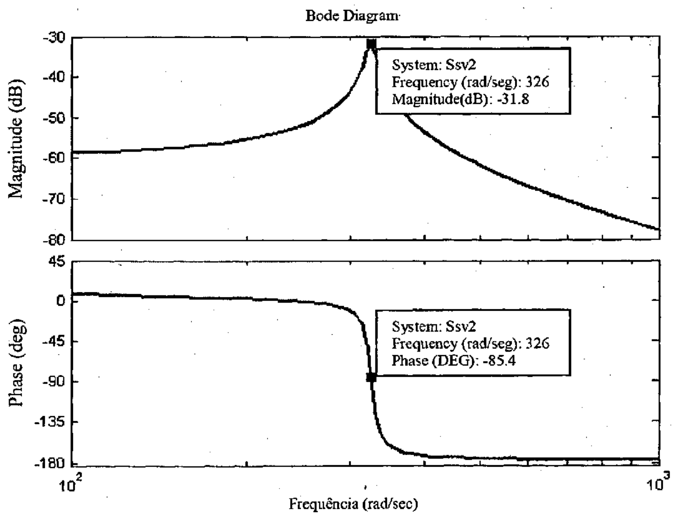
Bode Diagram
Magnitude (dB)
-30
-40
-50
-60
-70
-80
System: Ssv2
Frequency (rad/seg): 326
Magnitude(dB): -31.8
Phase (deg)
45
0
-45
-90
-135
-180
System: Ssv2
Frequency (rad/seg): 326
Phase (DEG): -85.4
10^2
10^3
Frequência (rad/sec)

FIG. 9

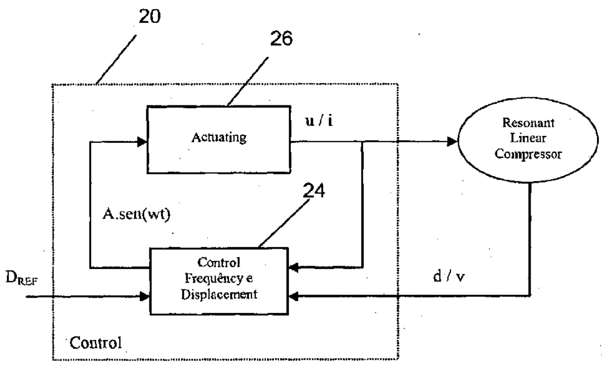
20
26
Actuating
u / i
Resonant
Linear
Compressor
24
A.sen(wt)
Control
Frequência e
Displacement
D_{REF}
d / v
Control

FIG. 10

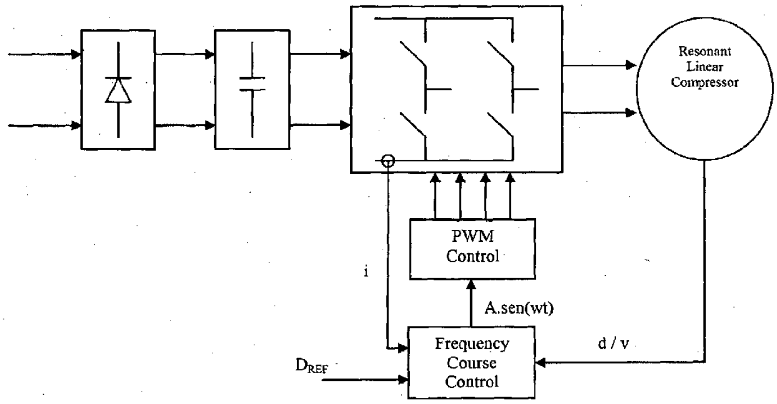
Resonant
Linear
Compressor
PWM
Control
i
A.sen(wt)
Frequency
Course
Control
D_{REF}
d / v

FIG. 11

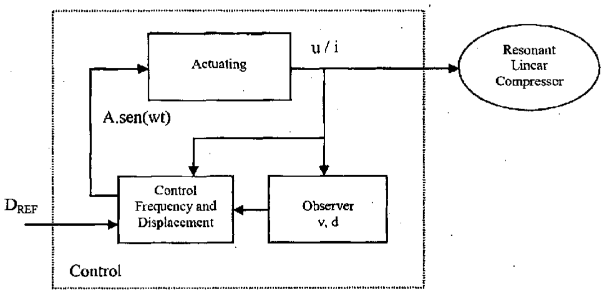
u / i
Actuating
Resonant
Linear
Compressor
A.sen(wt)
D_{REF}
Control
Frequency and
Displacement
Observer
v, d
Control

FIG. 12

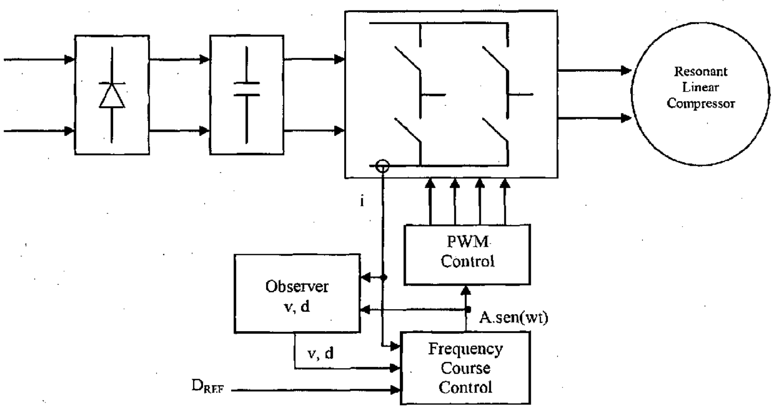
Resonant
Linear
Compressor
i
PWM
Control
Observer
v, d
A.sen(wt)
v, d
Frequency
Course
Control
D_{REF}

FIG. 13

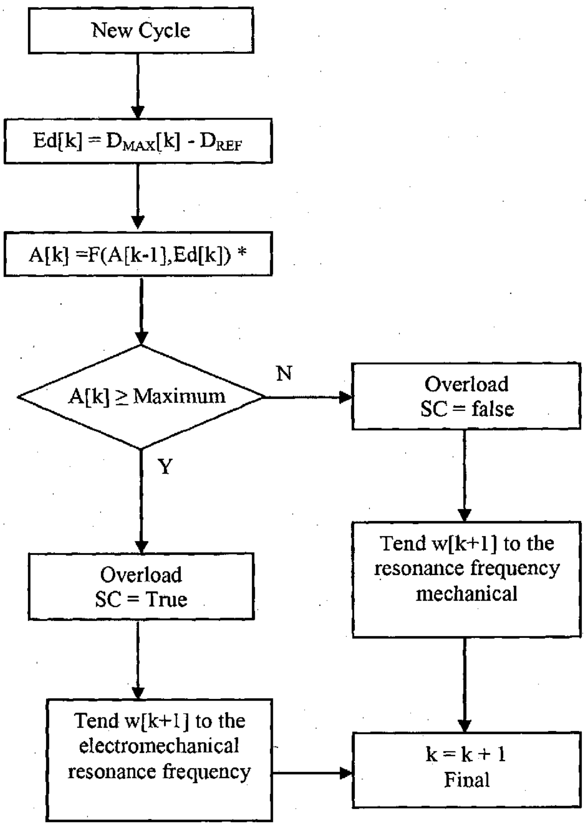
New Cycle
Ed[k] = D_MAX[k] - D_REF
A[k] =F(A[k-1],Ed[k]) *
A[k] ≥ Maximum
N
Overload
SC = false
Y
Overload
SC = True
Tend w[k+1] to the
resonance frequency
mechanical
Tend w[k+1] to the
electromechanical
resonance frequency
k = k + 1
Final

FIG. 14

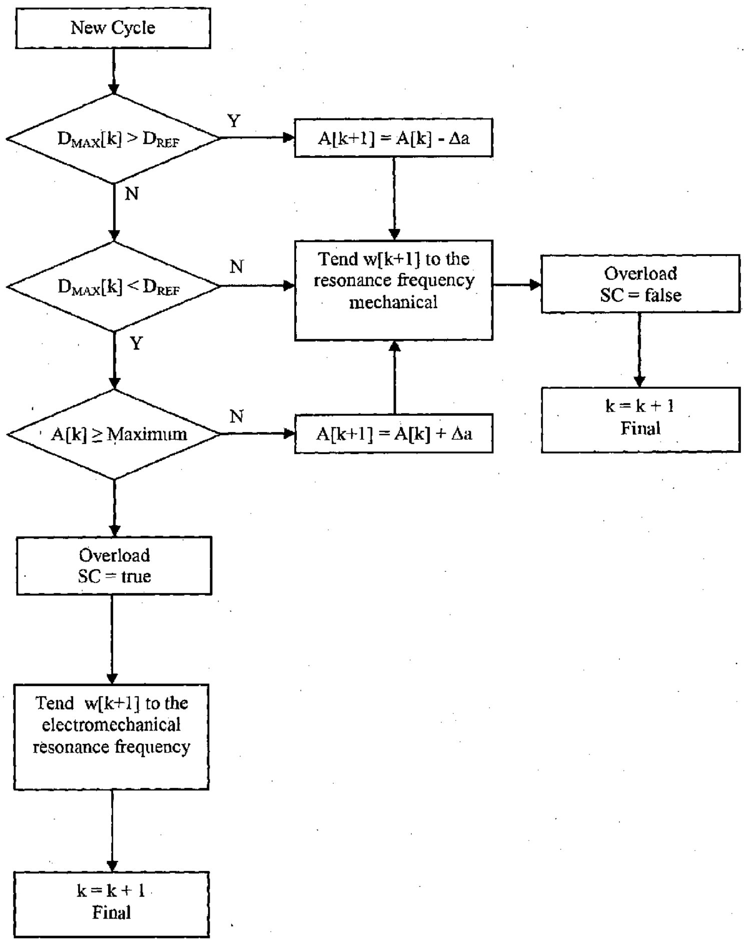
New Cycle
$D_{MAX}[k] > D_{REF}$
Y
N
$A[k+1] = A[k] - \Delta a$
$D_{MAX}[k] < D_{REF}$
N
Y
Tend w[k+1] to the resonance frequency mechanical
Overload SC = false
$A[k] \geq$ Maximum
N
$A[k+1] = A[k] + \Delta a$
k = k + 1 Final
Overload SC = true
Tend w[k+1] to the electromechanical resonance frequency
k = k + 1 Final

FIG. 15

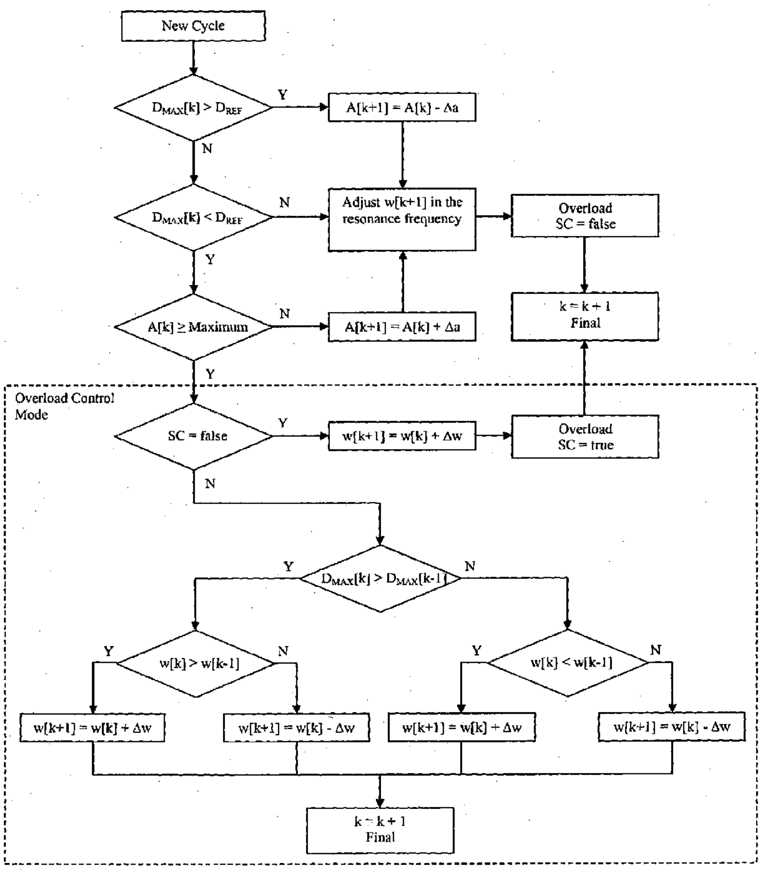
New Cycle
$D_{MAX}[k] > D_{REF}$
Y
N
A[k+1] = A[k] - Δa
$D_{MAX}[k] < D_{REF}$
N
Y
Adjust w[k+1] in the resonance frequency
Overload
SC = false
A[k] ≥ Maximum
N
Y
A[k+1] = A[k] + Δa
k = k + 1
Final
Overload Control Mode
SC = false
Y
N
w[k+1] = w[k] + Δw
Overload
SC = true
$D_{MAX}[k] > D_{MAX}[k-1]$
Y
N
w[k] > w[k-1]
w[k] < w[k-1]
w[k+1] = w[k] + Δw
w[k+1] = w[k] - Δw
w[k+1] = w[k] + Δw
w[k+1] = w[k] - Δw
k = k + 1
Final

FIG. 16

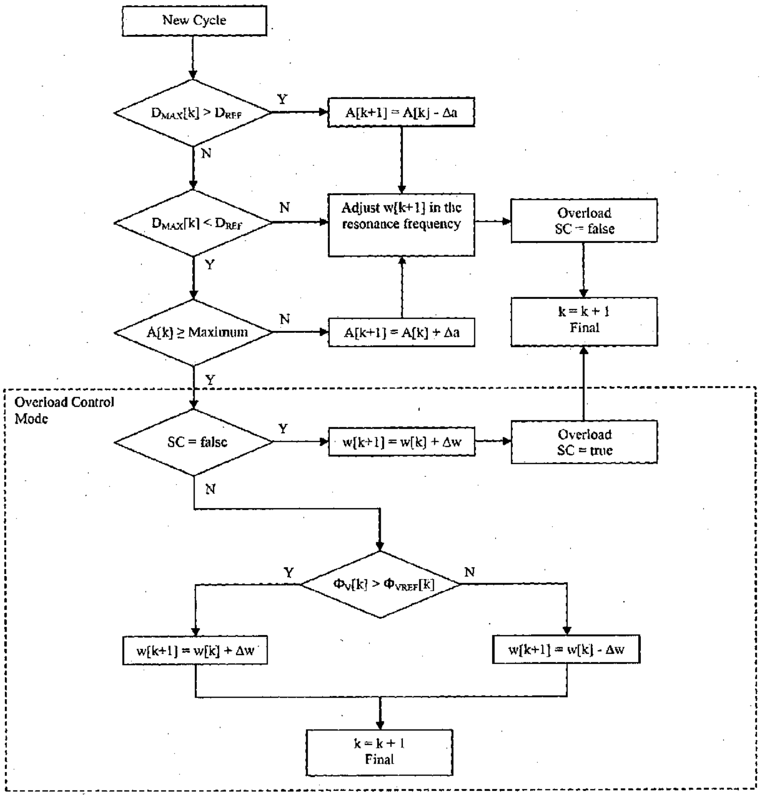
New Cycle
$D_{MAX}[k] > D_{REF}$
Y
N
$A[k+1] = A[k] - \Delta a$
$D_{MAX}[k] < D_{REF}$
Adjust w[k+1] in the resonance frequency
Overload
SC = false
A[k] ≥ Maximum
$A[k+1] = A[k] + \Delta a$
k = k + 1
Final
Overload Control Mode
SC = false
$w[k+1] = w[k] + \Delta w$
Overload
SC = true
$\Phi_V[k] > \Phi_{VREF}[k]$
$w[k+1] = w[k] + \Delta w$
$w[k+1] = w[k] - \Delta w$
k = k + 1
Final

FIG. 17

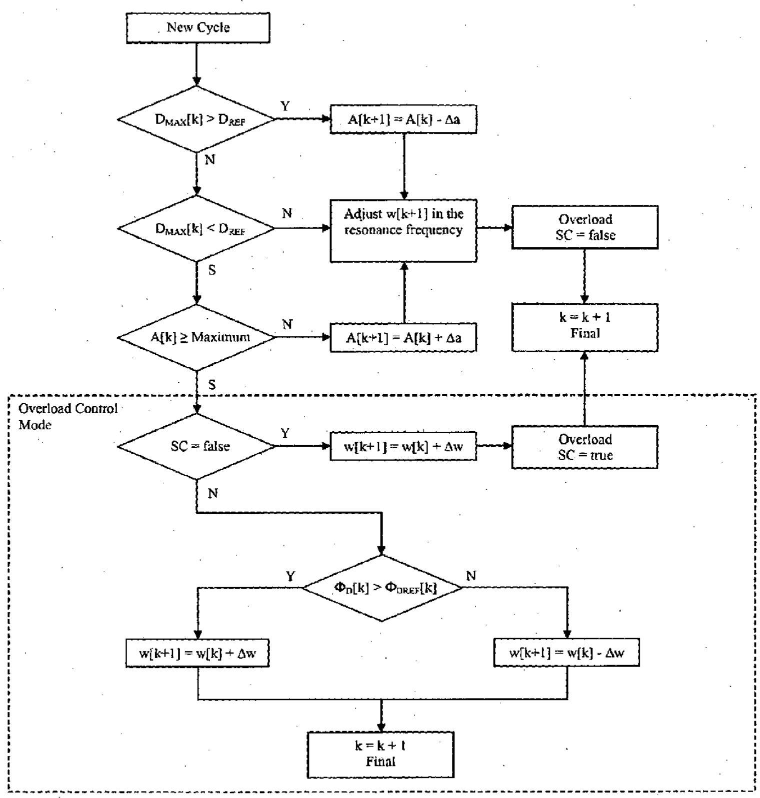
New Cycle
$D_{MAX}[k] > D_{REF}$
Y
N
$A[k+1] = A[k] - \Delta a$
$D_{MAX}[k] < D_{REF}$
N
S
Adjust w[k+1] in the resonance frequency
Overload
SC = false
$A[k] \geq$ Maximum
N
S
$A[k+1] = A[k] + \Delta a$
k = k + 1
Final
Overload Control Mode
SC = false
Y
N
$w[k+1] = w[k] + \Delta w$
Overload
SC = true
$\Phi_D[k] > \Phi_{DREF}[k]$
Y
N
$w[k+1] = w[k] + \Delta w$
$w[k+1] = w[k] - \Delta w$
k = k + 1
Final

FIG. 18

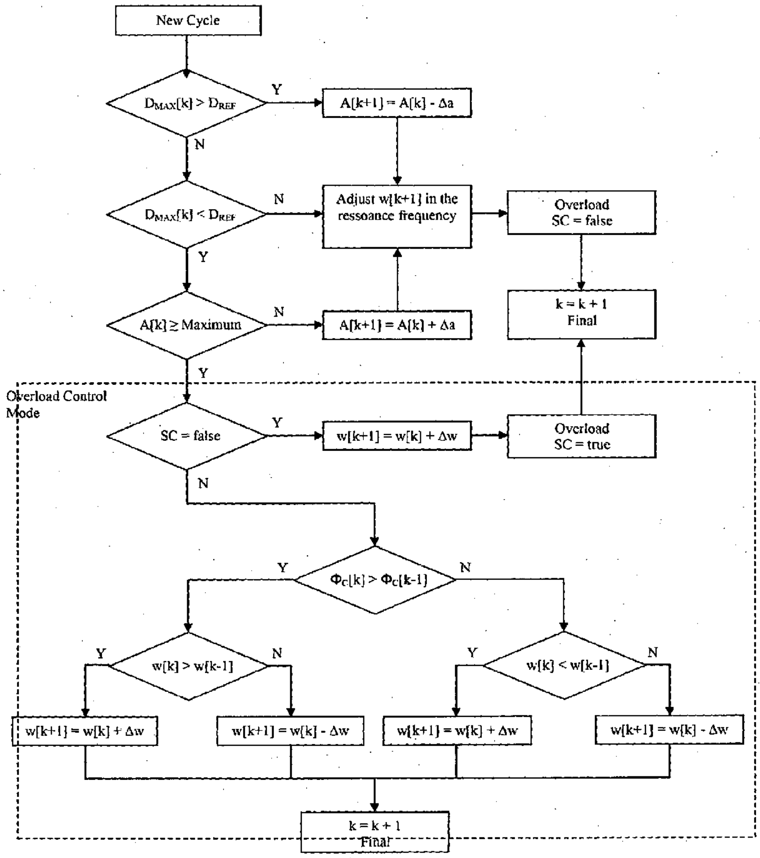
New Cycle
$D_{MAX}[k] > D_{REF}$
Y
N
$A[k+1] = A[k] - \Delta a$
$D_{MAX}[k] < D_{REF}$
N
Y
Adjust w[k+1] in the ressoance frequency
Overload
SC = false
A[k] ≥ Maximum
N
$A[k+1] = A[k] + \Delta a$
k = k + 1
Final
Y
Overload Control Mode
SC = false
Y
$w[k+1] = w[k] + \Delta w$
Overload
SC = true
N
$\Phi_C[k] > \Phi_C[k-1]$
Y
N
w[k] > w[k-1]
w[k] < w[k-1]
$w[k+1] = w[k] + \Delta w$
$w[k+1] = w[k] - \Delta w$
$w[k+1] = w[k] + \Delta w$
$w[k+1] = w[k] - \Delta w$
k = k + 1
Final

FIG. 19