United States Patent [19]

Donohue

[11] **3,762,448**

[45] **Oct. 2, 1973**

[54] **THICK WALLED PRESSURE VESSEL**

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[73] Assignee: **Struthers Scientific and International Corporation**, New York, N.Y.

[22] Filed: **Oct. 13, 1970**

[21] Appl. No.: **80,511**

Related U.S. Application Data

[62] Division of Ser. No. 691,090, Nov. 22, 1967, abandoned.

[30] **Foreign Application Priority Data**

Nov. 24, 1966 Great Britain 52,653/66
Jan. 20, 1967 Great Britain 3,178/67

[52] **U.S. Cl.** .... **138/171**, 29/446, 29/475, 29/497.5, 113/120 S, 220/3, 220/DIG. 29

[51] **Int. Cl.** **B23p 11/02**

[58] **Field of Search** .... 113/120 S; 29/477.7, 29/477, 477.3, 475, 497.5, 446; 138/171; 220/3, DIG. 29

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Primary Examiner—Henry T. Klinksiek
Attorney—William A. Drucker

[57] **ABSTRACT**

A thick walled pressure vessel having a tubular wall is prestressed by bending to resist extreme external pressures, the bending being applied by the external pressure being resisted. The tubular or cylindrical wall contains internal axial cuts which divide the wall into axial sectors with faying surfaces therebetween so that the application of external pressure decreases the curvature of the sectors to prestress them.

4 Claims, 9 Drawing Figures

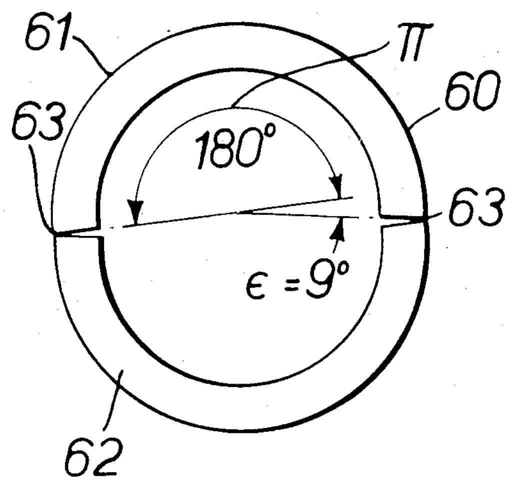
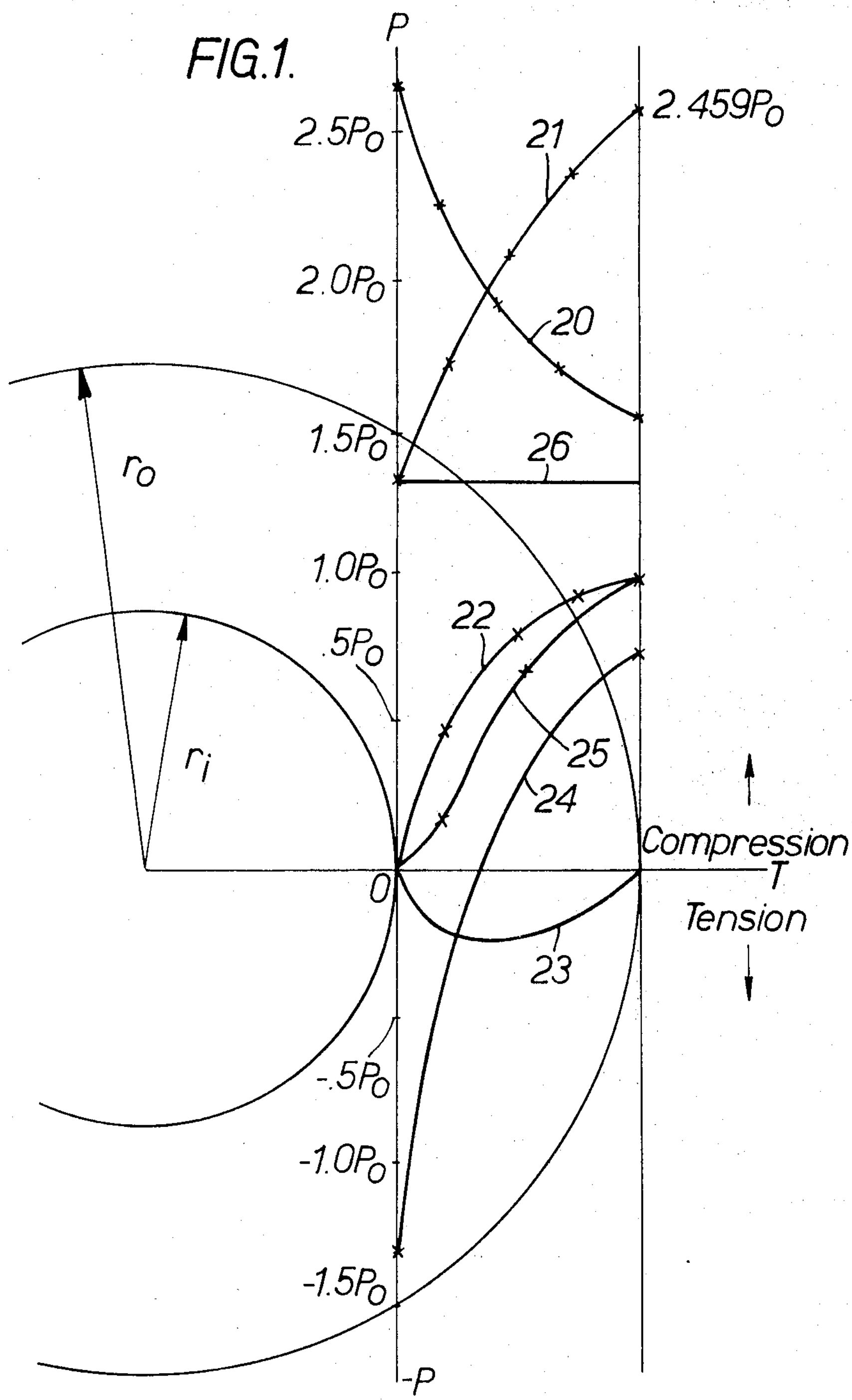


FIG.1.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26
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T
Compression
Tension

FIG.2.

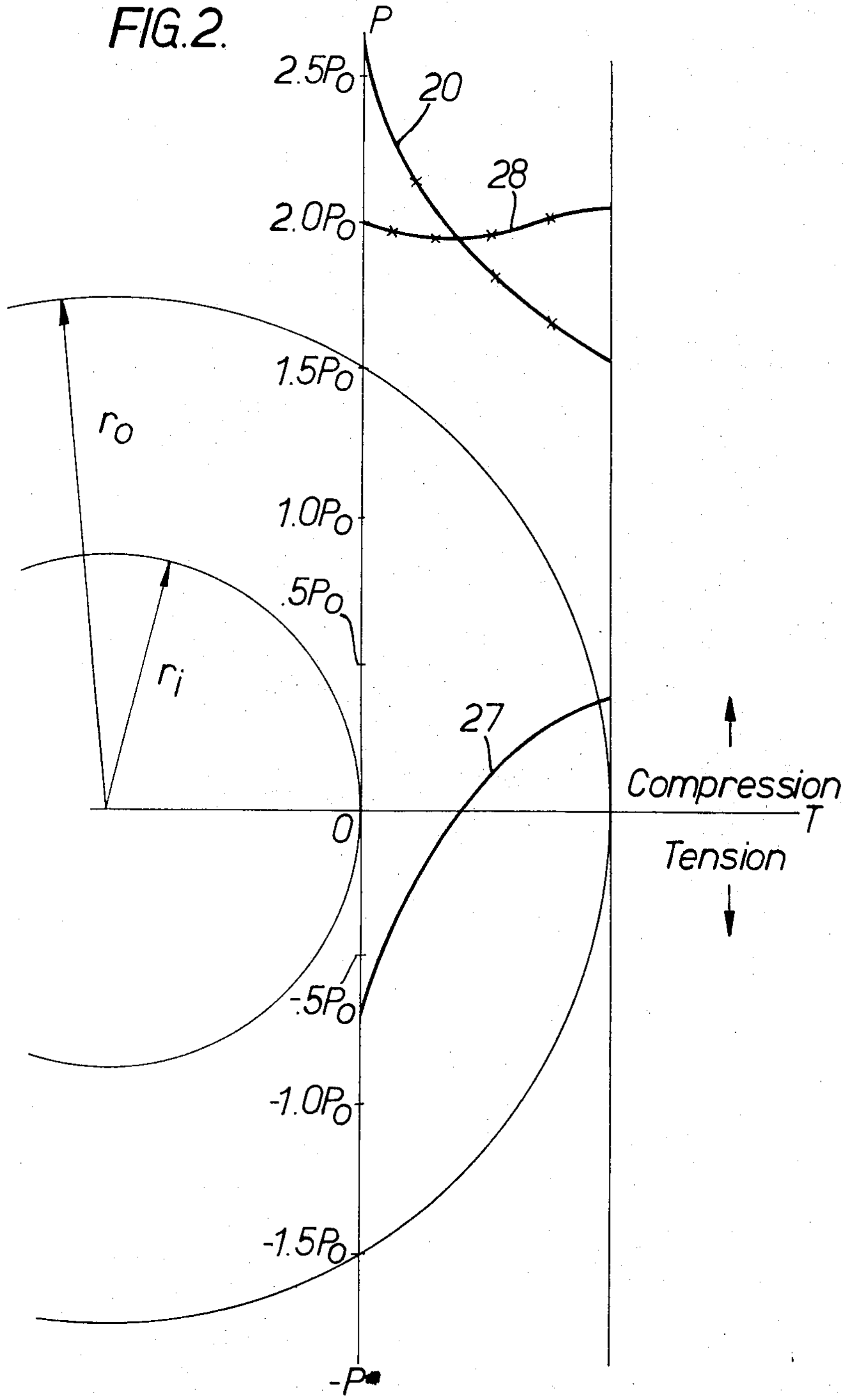
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FIG. 3.

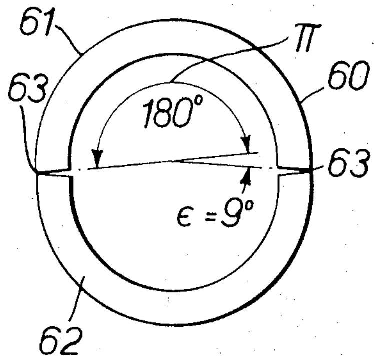


FIG. 4.

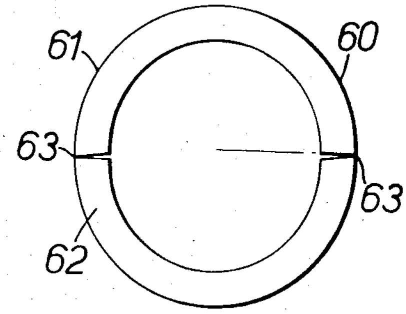


FIG. 5.

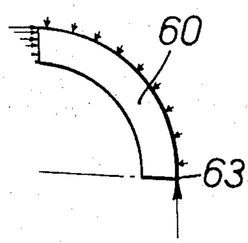


FIG. 6.

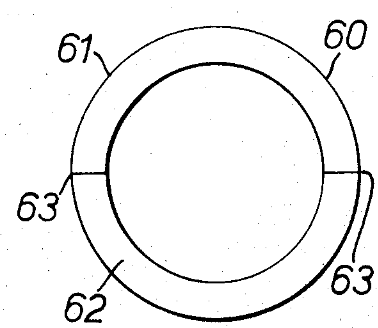


FIG. 7.

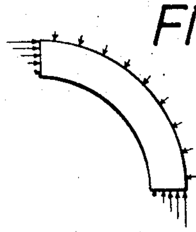


FIG. 8.

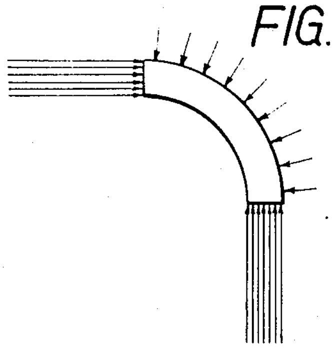
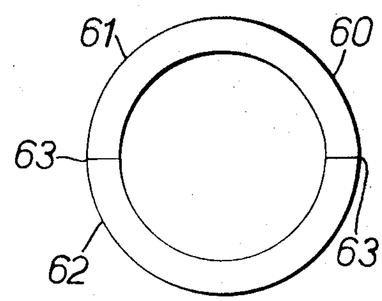


FIG. 9



THICK WALLED PRESSURE VESSEL

CROSS REFERENCES TO RELATED APPLICATIONS

This application is a division of patent application Ser. No. 691,090 filed Nov. 22, 1967, now abandoned.

BACKGROUND OF THE INVENTION

It is well known that when a normally unstressed thick-walled cylinder is subjected to high pressure on either the outside or the inside, the circumferential stress at the inside of the wall is greater than that at the outside by the amount of the pressure applied. It is, therefore, common in producing such cylinders that are to be subject to a pressure that is a significant fraction of the allowable stress, to trap into the wall a beneficial prestress which will be more or less cancel out the difference in stress between the outside and inside and thereby allow a higher average stress. This has been done by shrinking one or more cylinders together, winding with wire or tape under tension, applying controlled pressure so that a portion of the wall is stressed beyond its yield point, and by controlled quenching from the metal's plastic temperature. This disclosure describes a novel method of using bending to produce the desired results, and points out that its use has advantages not only in internally pressurized vessels, but even more particularly in externally pressurized vessels, where the first two methods cannot be used and the third is often impractical.

SUMMARY OF THE INVENTION

With regard to external pressure, where the desired prestress is circumferential tension at the inner wall and compression at the outer, with a consequent radial tension in the wall, winding is obviously impractical and concentric cylinders will have initial clearance, making them highly susceptible to buckling. With the bending method, however, all final stresses are compressive; and it can be shown that, by suitably calculating the planes of the faying surfaces, the prestresses can be applied simultaneously with the pressure. In fact, the paradoxical but sound conclusion can be reached that a vessel, lightly but suitably sealed, composed of two semi-cylindrical halves, can be stronger than a continuous cylindrical vessel of the same dimensions.

BRIEF DESCRIPTION OF THE DRAWING

FIGS. **1** and **2** are diagrams showing the internal and external radii of a thick walled pressure vessel with radial, circumferential, and axial stresses graphed as a function of cylinder wall thickness;

FIGS. **3, 4, 6** and **9** are end views of two sectors of a cylinder having internal faying surfaces so that the cylinder will be prestressed by bending as it is subjected to external pressure, FIGS. **3, 4, 6** and **9** showing the effect of progressively increasing external pressure on the cylinder; and

FIGS. **5, 7** and **8** show stress and pressure forces acting on a half of each of the sectors of FIGS. **4, 6** and **9** respectively.

DESCRIPTION OF THE PREFERRED EMBODIMENTS

To evaluate the desired prestress in a thick walled cylinder subjected to outside or inside pressure, a choice must be made from several theories of tube failure: maximum stress, strain energy, maximum shear, etc. In the case of the closed tubular vessel under external pressure only, there are no tensile stresses involved. Failure must occur at the inner surface as this is the only free surface. Also, since the stresses involved are compression in both axial and circumferential directions, and failure by direct compression is inconceivable, failure must be by shear. For the most efficient condition, the shear stress should be equal in all directions, or the axial and the circumferential compression at the inner surface should be equal, giving a resultant shearing stress equal to one-half the compressive stress and at 45° to any radius.

The formulae for stresses in a thick cylinder are

Internal Pressure External Pressure

$$s_c = -p_i \frac{r_i^2}{r_o^2 - r_i^2}\left(\frac{r_o^2}{r^2}+1\right) = p_o \frac{r_o^2}{r_o^2 - r_i^2}\left(\frac{r_i^2}{r^2}+1\right)$$

$$s_r = p_i \frac{r_i^2}{r_o^2 - r_i^2}\left(\frac{r_o^2}{r^2}-1\right) = p_o - \frac{r_o^2}{r_o^2 - r_i^2}\left(\frac{r_i^2}{r^2}-1\right)$$

where

s_c = circumferential stress

s_r = radial stress

s_a = axial stress for the case of a vessel closed at the ends

p = pressure

r = radius

and subscripts i and o denote "inside" and "outside," respectively. Pressure and compressive stress are considered positive and tensile stress negative.

Further

Internal Pressure		External Pressure
$s_a = -p_i\,[r_i^2/(r_o^2 - r_i^2)]$	=	$p_o\,[r_o^2/(r_o^2 - r_i^2)]$
$s_{ci} = -p_i\,[(r_o^2 + r_i^2)/(r_o^2 - r_i^2)]$	=	$2p_o\,[r_o^2/(r_o^2 - r_i^2)]$
$s_{co} = -2p_i\,[r_i^2/(r_o^2 - r_i^2)]$	=	$p_o\,[(r_o^2 = r_i^2)/(r_o^2 - r_i^2)]$
$p_o = 0$; $s_{Ri} = -p_i$		$p_i = 0$; $s_{Ro} = p_o$
$s_{Ro} = 0$		$s_{Ri} = 0$

For stresses due to the bending of a curved beam, we will not use the approximation of Winkler, but the more accurate formula of Guest (see Case, J., "Strength of Materials," Longmans, Green, and Co., 1925; sec.285 ff., for Winkler, and sec.309 for Guest). A slight modification of Guest's formulae give:

$$s_R = 2D\left[\frac{r_o^2\,7n\,r_o - r_i^2\,7n r_i - (r_o^2 - r_i^2)\,7n r - r_i^2 \frac{r_o^2}{r^2}\,7n \frac{r_o}{r_i}}{r_o^2 - r_i^2}\right]$$

$$s_p = 2D\left[\frac{r_i^2\,7n\,r_o - r_i^2\,7n r_i - (r_o^2 - r_i^2)(1 + 7n r) + \frac{r_i^2 r_o^2}{r}\,7n \frac{r_o}{r_i}}{r_o^2 - r_i^2}\right]$$

and, in particular,

$$s_{pi} = 2D\left[\frac{2r_i^2\,7n\,r_o/r_i - r_o^2 + r_i^2}{r_o^2 - r_i^2}\right]$$

where,

s_R = radial stress due to bending

s_p = circumferential stress due to bending

D = an arbitrary constant

For the case of external pressure on a vessel with closed ends, we will set up a prestress s_{pi} by bending such that

$s_{ci}+s_{pi}=s_a$ or $s_{pi}=-p_o(r_o^2)/(r_o^2-r_i^2)$

The equations above for bending stresses in a curved beam may be solved for D and the compensating stresses which may be obtained by bending may be calculated. As shown in FIG. **1**, a section through a cylinder has been drawn with an internal radius r_i and an external radius r_o so that $r_o/r_i=2$. Superimposed on the horizontal radius T of the cylinder is a graph with a horizontal axis OT and a positive vertical axis OP and a negative vertical axis O–P.

In FIG. **1**, line **20** is a graph of the uncompensated circumferential stress in terms of a given outside pressure p_o plotted on axis OP against cylinder wall thickness plotted on axis OT. Line **24** is a similar graph of the circumferential prestress set up in the cylinder wall by bending to decrease its curvature. Line **21** is a graph of the combined circumferential stresses which result in the cylinder when prestressed by bending and subjected to an outside pressure p_o. Line **21** indicates that a cylinder prestressed by bending may have an inside circumferential stress of about one half that of an unprestressed cylinder. While the outside stress of a prestressed cylinder is shown to be almost $2.5p_o$, there is no place for the metal to flow. It is to be noted that even axial flow will be resisted by the axial stress indicated by the graph line **26**.

Line **23** is a graph of the radial prestress produced by bending. Line **22** is a graph of the radial stress produced by an external pressure of p_o. Line **25** is a graph of the combined radial stresses in a prestressed cylinder subjected to an external pressure p_o. It is interesting to note that the radial stress produced by bending and neglected by the Winkler theory is of benefit in reducing the radial stress near r_i is a prestressed cylinder. This reduction of radial stress reduces the tendency for shear failure by radially inward flow.

If, however, it is desired to maintain a constant compressive stress across the tube wall, as called for by the maximum stress theory of failure, such a stress would be

$$s_K= p_o\ (r_o)/(r_o-r_i) =s_{ci}+s_{pi}$$

or

$$s_{pi}= -p_o\ (r_o)/(r_o-r_i)$$

This again may be solved for D and the stresses calculated.

FIG. **2** shows a cylinder and a graph similar to that shown in FIG. **1**. Line **20** again graphs the circumferential stress resulting from an outside pressure p_o. Line **27** graphs the circumferential stress in the cylinder resulting from bending so that the combined circumferential stresses in the cylinder will be approximately constant through its thickness when subjected to an external pressure p_o. Line **28** is a graph of the combined circumferential stresses in the cylinder which may thus be prestressed to approximate a constant compressive stress across its wall when subjected to an external pressure p_o.

To analyze longitudinal joints in a cylinder under hydrostatic external pressure, let us consider such a cylinder of indefinite length with prestress due only to bending. The stress pattern will be the same over any radial section because the bending moment across any such section is constant and independent of the central angle. The external pressure is constant, directed toward the center, and independent of the central angle. There is, therefore, no shear on any radial plane. Under full design pressure, there will be only compression on such a radial plane.

Therefore, if it is imagined making a diametral cut while maintaining the external pressure, there will be no change in shape or stress distribution, since such a change would have to be due to tension (separation of cut surfaces), shear (sliding of one cut surface on the other), or a change of shape of the mating surfaces without separation, which could only be caused by isolated irregularities in the stress pattern. While such may exist, they will be equally superimposed on the stressed and unstressed state and may, for this discussion, be neglected. With release of the external pressure, the two halves will separate along the cut, and the prestress due to bending will be released so that the halves will be free of stress. As the prestress was produced by a bending moment tending to open up the halves to π radians or 180°. the halves in the unstressed position will have a slightly greater angular measure, say $\pi + E$.

Since the angle at which I make the diametral cut makes no difference, a second diametral cut at an angle to the first should have not more effect, except that on release of pressure I will have four unstressed sectors. Any adjacent pair will then have a total angular measure of $\pi + E$. Also, when unstressed, adjacent pairs must fit tightly together, because any separation would indicate a stressed condition before the cut, contrary to the original hypothesis. The plane cuts originally made under pressure, therefore, remain plane in the released condition, and conversely.

FIGS. 3–9 show two sectors or halves **61** and **62** of a cylindrical shell **60** joined at **63**. The outer wall of each sector curves 5 percent more than 180° as shown in the Drawing. This would be in the order of 0.5 percent for a steel shell **60**. FIG. **3** shows the shell **60** with no external pressure on it. FIG. **6** shows the effect of sufficient external pressure to reduce the excess angle E to one-half the original.

FIG. **5** shows the stresses and forces on on half the upper section. The downward component of the pressure p is resisted by an upward force $F=pr_o$ more or less concentrated at the outer corner **63**. While the horizontal component of the pressure plus the excess moment of the vertical force $M=Fr_o$ is resisted by an unsymmetrical pattern of stresses across the vertical cross section which range from a fairly high compression at the outside to a low tension on the inside. In FIG. **6**, the external pressure has increased to a value sufficient to close the mating surfaces completely. At this external pressure, the stress at the inside radius is zero around the circumference. Since there is zero stress, there must be zero strain, and the inside circumference is the same as in the unstressed state. However, as the total angular measure has decreased 5 percent, the inside radius has increased 5 percent. Now, neglecting radial strains, which are slight, the wall thickness has not changed. Therefore, while the outer radius has increased by the same amount, it has only increased by three-fourths as great a percent. Thus, since the decrease in angular measurement is the same throughout, there is a shortening or compressive strain of the outer circumference of 1¼ percent. FIG. 7 shows the stress and pressure pattern for this condition. It can be shown that the outer stress will be equal to the maximum design pressure and that the pressure to produce this stress will be $(r_o-r_i)/2r_o$ times this design pressure.

FIG. 9 shows the cylinder 60 under the maximum design pressure which has produced a uniform maximum design stress across the wall in accordance with the formulae hereinbefore given. It should be noted that between FIGS. 6 and 9, r_i has been compressed to its original value, causing a circumferential strain of 5 percent. r_o has also returned to its original value, causing an additional strain of 3¾ percent which, added to the strain already present in FIG. 7 makes the external strain equal to the internal.

Thus, it may be seen that longitudinal sectors of a pressure vessel joined only at their outer points of contact to provide a seal may be able to resist greater external pressures than a solid cylindrical pressure vessel of the same wall thickness. If desired, the faying surfaces of the sectors of such a pressure vessel may be forced together by suitable bending and then welded to prestress the vessel to resist greater external pressures. However, in most applications such as deep diving marine exploration devices, the bending and the prestress may be applied by the external pressure which is to be resisted.

While it is possible to apply the concepts of this invention to a spherical pressure vessel by making cuts in a thick walled sphere to divide the sphere into sectors with bases corresponding to regular polygons, the cuts would be made while the sphere was subjected to pressure. On release of the pressure, the sectors, being concave cones, could not fit perfectly tightly without distortion.

However, solid unprestressed spherical ends may be used on a cylindrical vessel with the same inside and outside radii as the stresses in the unprestressed ends of a spherical ended cylindrical vessel will generally be well under the stresses in the prestressed cylindrical portion. While this invention has been applied to cylindrical pressure vessels, it could equally well be applied to vessels with oval or other axial enclosing walls.

I claim:

1. In a thick walled pressure vessel, a tubular wall for resisting high external pressure, said wall containing internal axial cuts dividing the wall into axial sectors with faying surfaces therebetween, the faying surfaces being sealed at their outer edges at the outer surface of said wall, said sectors elastically bending to decrease their curvature under external pressure to prestress said sectors and thereby said wall of said pressure vessel.

2. The combination according to claim **1** wherein said faying surfaces are closed under external pressure.

3. The combination according to claim **1** wherein said tubular wall is cylindrical, and wherein said axial sectors when unprestressed have curvatures totalling more than 360° and are elastically bent by external pressure to close said faying surfaces and have curvatures then totalling substantially 360°.

4. In the process of forming a thick walled pressure vessel having a curved metal closing wall of an integral layer of material wherein the closing wall has axial sectors joined and sealed at their outer edges;

abutting the metal closing wall axial sectors leaving open faying surfaces therebetween,

and bending the closing wall to decrease its curvature to resist higher external pressures by applying ecternal pressure to close the faying surfaces.

* * * * *

UNITED STATES PATENT OFFICE
CERTIFICATE OF CORRECTION

Patent No. 3,762,448 Dated October 2, 1973

Inventor(s) John Donohue Howell

It is certified that error appears in the above-identified patent and that said Letters Patent are hereby corrected as shown below:

On the cover sheet item [75] "John Donohue, Howell, N.Y." should read -- John Donohue Howell, New City, N.Y. --.

Signed and sealed this 26th day of February 1974.

(SEAL)
Attest:

EDWARD M.FLETCHER,JR.
Attesting Officer

C. MARSHALL DANN
Commissioner of Patents