Dec. 23, 1941. C. R. BURROWS 2,267,268

HIGH FREQUENCY TRANSMISSION SYSTEM

Filed March 3, 1938 2 Sheets-Sheet 1

FIG. 1

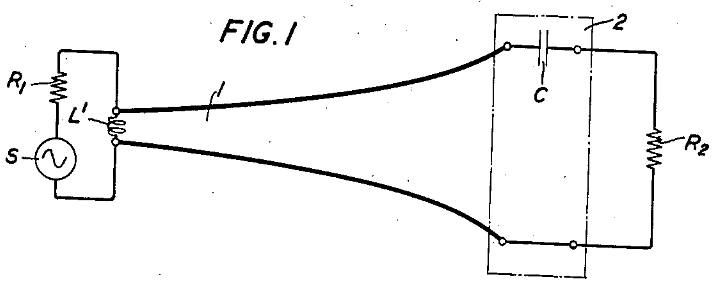


FIG. 2

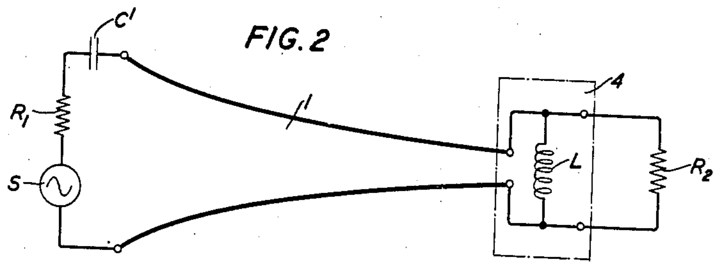


FIG. 3

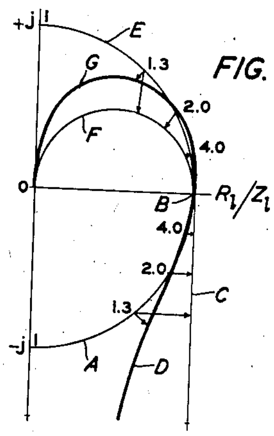


FIG. 4

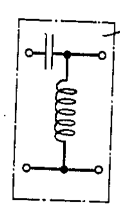


FIG. 5

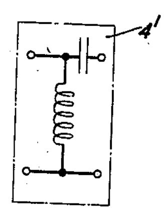
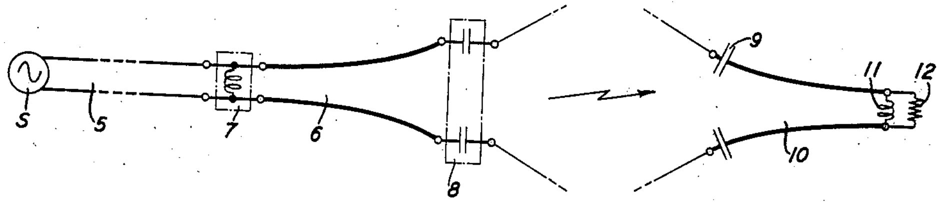


FIG. 6



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HIGH FREQUENCY TRANSMISSION SYSTEM

Filed March 3, 1938 2 Sheets-Sheet 2

FIG. 7

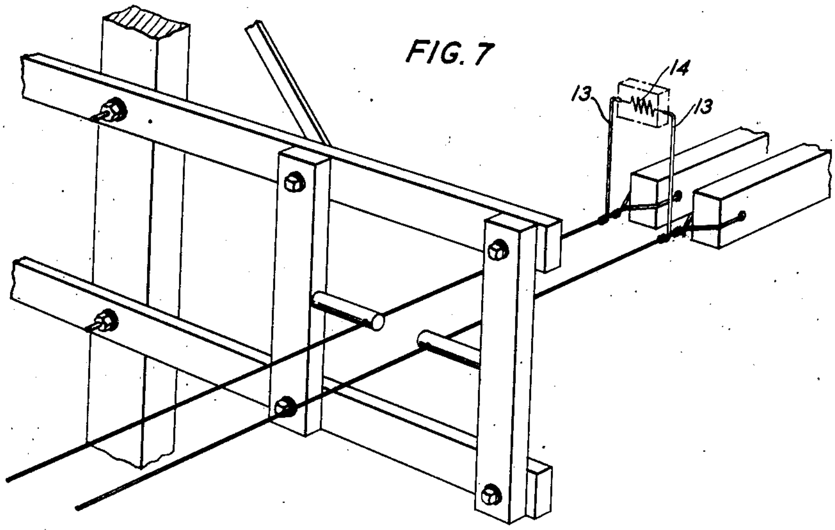


FIG. 8

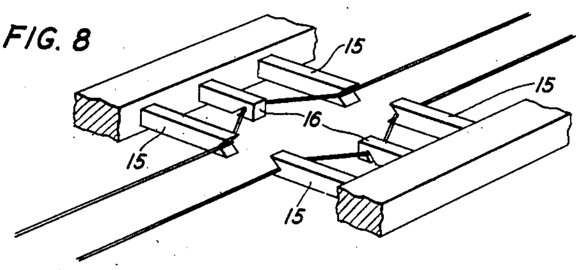
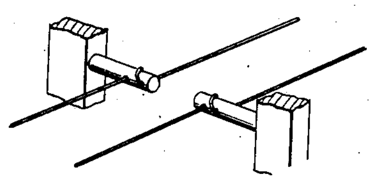


FIG. 9



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UNITED STATES PATENT OFFICE

2,267,268

HIGH FREQUENCY TRANSMISSION SYSTEM

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Application March 3, 1938, Serial No. 193,660

4 Claims. (Cl. 178—44)

This invention relates to the art of communicating by electromagnetic waves and more particularly to the design and application of transmission line systems for the transmission of ultra-high frequency waves.

An object of the invention is to improve the electrical characteristics of a transmission line adapted for waves occupying or distributed over a wide range of frequencies and more especially to reduce impedance irregularities within and at the ends of such a line.

Another object of the invention is to improve the impedance match between a tapered transmission line and an impedance element connected thereto, and in particular to effect such an impedance match over a wide range of frequencies.

Another and more specific object of the invention is to provide a substantially reflectionless impedance transformer comprising an exponential transmission line, the input and output impedances of which are substantially purely resistive over a wide frequency range the lower limit of which approaches the cut-off frequency of the line.

Still another object of the invention is to improve the impedance characteristics and energy dissipating capacity of a transmission line, the conductors of which are of resistive material.

A problem that is repeatedly encountered in the design of communication systems is that of efficiently coupling together two transmission elements having unlike impedances, and the problem is ordinarily the more acute the wider the frequency range over which the system is to be operated. As a means for coupling such elements there has been suggested heretofore a transmission structure the surge impedance of which varies progressively from one terminal of the structure to the other from an impedance value approximating that of the circuit element to be connected at that terminal to a different value approximating the impedance of the circuit element to be connected at the other terminal. Typical of the structures that have been suggested is a transmission line the conductors of which are spaced at progressively increasing or decreasing distances apart, the rate of progression being such that the surge impedance varies exponentially with distance along the line.

It is a matter of experimental fact that a fairly good impedance matching coupling can be realized with a tapered transmission line of the kind described if the frequency of operation is exceedingly high or if the line is exceedingly long and the rate of change of impedance is exceedingly small, but that at lower frequencies or with a more rapid rate of impedance change serious operating difficulties are encountered. More particularly, it is found that reflection arises at the terminals, so that not all of the power available at the one terminal is transmitted to the other and the per cent of power transmitted becomes an irregular function of frequency.

Applicant has found that these reflection losses and the aforementioned disadvantages may be substantially reduced by modification of the characteristics at the terminals of the line. Applicant proposed also to compensate for intermediate impedance irregularities that are practically unavoidable in the mechanical construction of the line. In describing the invention and typical embodiments thereof, reference will be made to the accompanying drawings, in which:

Figs. 1, 2, 4 and 5 illustrate exponential lines and networks therefor in accordance with the invention;

Fig. 3 shows certain impedance characteristics thereof;

Fig. 6 shows a system incorporating various features of the invention; and

Figs. 7, 8 and 9 illustrate various novel means for compensating stray capacitance in a transmission line.

Applicant's invention is of such nature that it can best be considered from a mathematical point of view. In the following mathematical treatment it is assumed that the impedance transformer comprises a line of the exponential type, although tapered transmission lines in which the impedance follows some other law can be treated in the same manner. By an exponential line is meant one in which the surge impedance varies along the line in accordance with an exponential function of distance, as will appear hereinbelow.

The differential equations of a transmission line are:

$$yv = -\frac{di}{dx} \quad (1)$$

$$zi = -\frac{dv}{dx} \quad (2)$$

where

y=the shunt admittance per unit length of line,
z=the series impedance per unit length of line,
v=the potential difference between the wires,
i=the current in one wire, and
x=the distance from the beginning of the line.

Combining Equations 1 and 2 and inserting the values of y and z that are appropriate for an exponential line, viz:

$$z=z_x=z_0e^{\delta x} \quad (3)$$

$$y=y_x=y_0e^{-\delta x} \quad (4)$$

where z_0 and y_0 are the series impedance and shunt admittance respectively at the beginning of the line where $x=0$ and δ is the rate of taper, the following equation may be derived:

$$\frac{d^2v}{dx^2}-\delta\frac{dv}{dx}-y_0z_0v=0 \quad (5)$$

The general solution of Equation 5 can then be shown to be:

$$v_x=Ae^{-\left(\Gamma-\frac{\delta}{2}\right)x}+Be^{+\left(\Gamma+\frac{\delta}{2}\right)x}=Ae^{-\left(\Gamma-\frac{\delta}{2}\right)x}\left[1+\frac{B}{A}e^{2\Gamma x}\right] \quad (6)$$

where

$$\Gamma=\sqrt{z_0y_0+\delta^2/4}$$

Substitution of Equation 6 in Equation 2 gives:

$$i_x=\frac{A}{Z_0}\frac{\Gamma-\frac{\delta}{2}}{\gamma}e^{-\left(\Gamma+\frac{\delta}{2}\right)x}-\frac{B}{Z_0}\frac{\Gamma+\frac{\delta}{2}}{\gamma}e^{+\left(\Gamma-\frac{\delta}{2}\right)x}=\frac{A}{Z_0}\frac{z-\frac{\delta}{2}}{\gamma}e^{-\left(\Gamma+\frac{\delta}{2}\right)x}\left[1-\frac{B}{A}\frac{\Gamma+\frac{\delta}{2}}{\Gamma-\frac{\delta}{2}}e^{2\Gamma x}\right] \quad (7)$$

when the following relations are considered.

(A) $$Z_x=\sqrt{z/y}=Z_0e^{\delta x}$$

is the surge impedance of the exponential line at the point x, which is equal to the characteristic impedance Z_0 of the uniform line that has the same distributed constants as the exponential line has at the point x;

(B) $$\gamma=\sqrt{zy}=\sqrt{z_0y_0}$$

is the propagation constant of the uniform line that has the same distributed constants as the exponential line; and

(C) $$\Gamma=\sqrt{\gamma^2+\delta^2/4}=\alpha+j\beta$$

is the transfer constant of the exponential line. $+\Gamma\gamma$ and $+\Gamma$ refer to the values of the indicated roots that are in the first quadrant.

The propagation constant is $\Gamma-\delta/2$ for voltage waves traveling in the positive x direction and $\Gamma+\delta/2$ for voltage waves traveling in the negative x direction. For current waves the corresponding propagation constants are $\Gamma+\delta/2$ and $\Gamma-\delta/2$. Borrowing the terminology of wave filters, Γ is the transfer constant and δ is the impedance transformation constant. $\delta/2$ is the voltage transformation constant and $-\delta/2$ is the current transformation constant. The real and imaginary parts of Γ, viz., α and β, are the attenuation and phase constants, respectively.

The remaining equations may be written in clearer form if we define

$$j\nu=-\frac{\delta}{2\gamma} \quad (8)$$

so that

$$\Gamma=\gamma\sqrt{1-\nu^2} \quad (9)$$

where the indicated root is in the fourth quadrant. For a substantially lossless line, ν is real and inversely proportional to the frequency. When ν is less than unity the radical in Equation 9 is real and the transfer constant Γ is a pure imaginary. The transfer constant is then less than that for a uniform line by the factor

$$\sqrt{1-\nu^2}$$

so that both phase velocity and wave-length are larger for the exponential line than for the uniform line by the reciprocal of this factor. When ν is greater than unity the radical is a pure imaginary and the propagation constant is real. Hence, the exponential line is a high pass filter such that its cut-off frequency, f_1, is that frequency for which $\nu=1$.

If the line is terminated at $x=l$ with an impedance $\mathbf{Z}_l=v_l/i_l$, the ratio of the reflected to direct wave is found to be

$$\frac{B}{A}=-\frac{1-(\mathbf{Z}_l/Z_l)(\sqrt{1-\nu^2}+j\nu)}{1+(\mathbf{Z}_l/Z_l)(\sqrt{1-\nu^2}-j\nu)}e^{-2\Gamma l} \quad (10)$$

where the coefficient of the exponential is the voltage reflection coefficient.

The desired condition is that there be no reflection at the far end of the line, that is, where $x=l$, and this condition obtains when the numerator of Equation 10 is zero, that is, when the terminating impedance conforms with the equation:

$$\mathbf{Z}_l=Z_l/(\sqrt{1-\nu^2}+j\nu) \quad (11)$$

This is the magnitude of the forward-looking characteristic impedance at $x=l$. For frequencies above cut-off it is approximately equal to the surge impedance at the corresponding terminal but differs from it in phase. As the frequency is increased ν becomes a small quantity so that this phase difference approaches zero. Hence, as noted hereinbefore, for frequencies so high that ν is practically zero an exponential line may be terminated with the same impedance that would terminate a uniform line having the same distributed constants as the exponential line has at this point.

To eliminate reflection at lower frequencies, however, and this is one of the principal objects of the present invention, the terminating impedance must satisfy Equation 11 at those frequencies. Applicant has found that for the usual and important case of a line with negligible loss the terminating impedance required by Equation 11 may be approximated with a very simple network for frequencies in the pass band. In this case both $\mathbf{Z}_l$ and ν are real, and $\nu=\pm f_1/f$. The plus sign corresponds to a line the characteristic impedance of which increases with distance along the line, δ positive; the minus sign corresponds to a line the characteristic impedance of which decreases with distance, δ negative.

In the first case, that is, δ positive, where the far-end terminating network is at the high impedance end of the exponential line, the network takes the form of a capacitance connected in series with the resistive load.

In the second case, that is, δ negative, where the far-end terminating network is at the low impedance terminal, the network takes the form of an inductance in shunt relation to the resistive load.

In each case the reactance is such that at the cut-off frequency of the line it is equal in magnitude to the surge impedance of the exponential line at the end of the line at which the terminating network is located.

The first case is illustrated in Fig. 1 where the high impedance end of an exponential line 1 comprising an impedance step-up transformer is connected to the input terminals of the network **2** and the resistance load R_2 is connected across the output terminals of the network. Internally, the network comprises only a condenser C connected in series in one arm and a short circuit in the other. To preserve electrical symmetry the requisite capacitance can be attained by providing condensers in both arms of the network. At the low impedance end of the line is connected an input circuit comprising a source S of high frequency waves which presents to the line an impedance of R_1 ohms. The function of the shunt inductance L′ will be described hereinafter.

As a specific example of practice in accordance with Fig. 1 it may be supposed that the exponential line **1** is 15 meters long and that it is connected between an input circuit having an impedance R_1 of 300 ohms and an output circuit having an impedance R_2 of 600 ohms. The input circuit may comprise, for example, two 600-ohm open-wire transmission lines connected in parallel. In this example, then, the surge impedance at the low impedance end of the line, $Z_1=R_1=300$ ohms; the surge impedance at the high impedance end of the line, $Z_2=R_2=600$ ohms; the impedance transformation ratio

$$k=Z_2/Z_1=600/300=2$$

and the frequency f_0 at which the line is one wave-length long is $3\times10^8/15$ or 20 megacycles per second. The theoretical cut-off frequency f_1 is

$$f_1=\frac{\log_e k}{4\pi}f_0 \qquad (12)$$

or 1.10 megacycles per second. The required series capacitance C may then be determined from the relation

$$1/2\pi f_1 C=Z_2 \qquad (13)$$

$$C=1/2\pi f_1 Z_2 \qquad (14)$$

$$=240 \text{ micromicrofarads}$$

The second case is illustrated in Fig. 2 where the network **4** is interposed between the low impedance end of the exponential line **1** and the resistance load R_2. The network comprises only a shunt inductance L. Suppose that the exponential line considered in the last example is connected as an impedance step-down transformer between an input circuit having an impedance R_1 of 600 ohms and an output circuit having an impedance R_2 of 300 ohms. In this case the required shunt inductance L may be determined from the relation

$$2\pi f_1 L=Z_1 \qquad (15)$$

$$L=Z_1/2\pi f_1 \qquad (16)$$

$$=43.3 \text{ microhenries}$$

The function of the series capacitance C′ will be described hereinafter.

Fig. 3 is of interest in that it shows graphically the ideal terminating impedance Z_l in relation to the approximate terminations just described. In this figure, each point represents the terminating impedance at some frequency. The horizontal distance from the origin 0 represents the resistive component of the impedance and vertical distance represents the reactive component, inductive reactance being represented above the horizontal axis and capacitive reactance below. Curve A represents the ideal terminating impedance Z_l for an impedance step-up line. Point B represents the usual resistance termination, which coincides with the ideal only at infinite frequency. Curve C represents the resistance-capacitance termination of Fig. 1. The numbers along curve A give the ratio of the frequency f appropriate to the point in question to the cut-off frequency f_1. The arrows indicate the deviation of the termination from the ideal. Curves E and F are similarly applicable to the impedance step-down line and correspond respectively with curves A and C.

The deviation from the ideal at any frequency f is a function of the ratio of that frequency to the cut-off frequency f_1. For the usual resistance termination it is approximately proportional to (f_1/f). For the terminations illustrated in Figs. 1 and 2 it is of a lower order of magnitude, being approximately proportional to $(f_1/f)^2$.

It is worthy of note that the terminating networks as described with reference to Figs. 1 and 2 coincide with the ideal only at infinite frequency. By decreasing the value of resistance R_2 in the case illustrated in Fig. 1 it is possible to obtain the correct impedance at any one particular frequency above the cut-off frequency, and for the case illustrated in Fig. 2 there is a corresponding reciprocal resistance-inductance combination.

Having now established the conditions for eliminating reflections at the far end of the line, it remains to consider the conditions at the input end of the line.

It can be shown that with an ideal far-end termination the input impedance of the exponential line is not purely resistive over the entire frequency range but that a reactive component is present. To eliminate such a reactive component is one of the objects of the invention.

Where the termination at the far end is of the kind described with reference to Figs. 1 and 2 the input impedance of the line may be shown to approximate that of a capacitance and resistance in series in the first case and that of an inductance and resistance in parallel in the second case. In accordance with a feature of the present invention the reactive component of the input impedance is minimized or substantially eliminated by means of an appropriate network at the input terminal of the line.

In the case of Fig. 1 the input network comprises simply a shunt inductance L′, as illustrated, whereas in the case of Fig. 2 it comprises a series capacitance C′. In both cases the reactance of the element at the cut-off frequency of the line should be equal in magnitude to the surge impedance of the line at the point where it is connected. In the numerical examples given with reference to Figs. 1 and 2, therefore, the inductance L′ is 43.3 microhenries and the capacitance C′ is 240 micromicrofarads. Thus, whether a given exponential line be used as a step-up transformer or as a step-down transformer, the same amount of shunt inductance is required at the low impedance end of the line and the same amount of series capacitance at the high impedance end. In other words, with the exponential line provided with the terminating and corrective networks described it may be considered as a four-terminal impedance transformer having approximately ideal characteristics over a frequency range extending upwards from some predetermined frequency lying above the cut-off frequency of the line.

A closer approximation to the ideal far-end termination can be obtained by a modification of the terminating network that includes the addition of an inductance across the load in the first case and the insertion of a capacitance in series with the load in the second case so that the inductance is in shunt relation to the series combination. These two terminating networks are illustrated in Figs. 4 and 5. That shown in Fig. 4 is adapted for the high impedance end of the line and the other for the low impedance end, and for a given line impedance level each is the electrical reciprocal of the other. With these networks it is possible to obtain exactly the required impedance at two frequencies, as for example, at twice the cut-off frequency and at infinite frequency. Curves D and G of Fig. 3 are applicable to Figs. 4 and 5, respectively, in the same manner that curves C and F are applicable to Figs. 1 and 2, respectively, for the specific case where the termination is designed for ideal impedance at infinite frequency and at twice the cut-off frequency. It will be apparent that the deviation from the ideal termination is small for all frequencies over 1.3 times the cut-off frequency and exceedingly small for all frequencies above twice the cut-off frequency.

The design of networks in accordance with Figs. 4 and 5 for any particular case may proceed as follows. Let C_a and L_a represent a capacitance and an inductance, respectively, the reactance of which at the cut-off frequency of the exponential line is equal in magnitude to the surge impedance of the line at its low impedance terminal. Let C_b and L_b represent reactances corresponding similarly to the surge impedance at the high impedance terminal. It is evident that $L_b=kL_a$ and that $C_a=kC_b$.

To design the network of Fig. 4 to give the correct termination at two frequencies, that is, to determine the proper values for the inductance L'_1 and the capacitance C'_1 at the high impedance end of the line, we have

$$Z=R_b\left[\frac{\rho}{\rho^2+\nu_1^2}+j\left(\frac{\nu_1}{\rho^2+\nu_1^2}-\nu_2\right)\right] \quad (17)$$

whereas the desired impedance is

$$R_b[\sqrt{1-\nu^2}-j\nu] \quad (18)$$

where the impedance of L'_1 is written $j\omega L'_1=jR_b/\nu_1$ and that of L_b is written $j\omega L_b=jR_b/\nu$ so that

$$L'_1=\frac{\nu}{\nu_1}L_b \quad (19)$$

and the impedance of C'_1 is written $1/j\omega C'_1=-jR_b\nu_2$ and that of C_b is written $1/j\omega C_b=-jR_b\nu$ so that

$$C'_1=\frac{\nu}{\nu_2}C_b \quad (20)$$

and the terminating resistance $R=R_b/\rho$. These two impedances are equal at infinite frequency if $\rho=1$. To make them equal at a frequency $1/\nu_*$ times the cut-off frequency they must be equal when $\nu=\nu_*$.

The real parts will be equal if

$$\sqrt{1-\nu_*^2}=\frac{1}{1+\nu_1^2} \text{ since } \rho=1 \quad (21)$$

This gives

$$\nu_1=\sqrt{\frac{1}{\sqrt{1-\nu_*^2}}-1} \quad (22)$$

The imaginary parts will be equal if

$$\nu_*=\nu_2-\frac{\nu_1}{1+\nu_1^2} \quad (23)$$

This gives

$$\nu_2=\nu_*+\frac{\nu_1}{1+\nu_1^2} \quad (24)$$

When the value of ν_* is chosen then both ν_1 and ν_2 can be calculated from the above formulae and hence L'_1 and C'_1.

If

$$\nu_*=\frac{1}{2},\ \nu_1=\sqrt{\frac{2}{\sqrt{3}}-1}=0.394,\ \frac{\nu}{\nu_1}=1.27$$

$$\nu_2=\frac{1}{2}+\frac{\sqrt{3}}{2}\sqrt{\frac{2}{\sqrt{3}}-1}=0.841,\ \frac{\nu}{\nu_2}=0.595$$

To design the network of Fig. 5 to give the correct termination at two frequencies, that is, to determine the proper values for the inductance L'_2 and the capacitance C'_2 at the low impedance end of the line, we have

$$\frac{1}{Z}=\frac{1}{R_a}\left[\frac{\rho}{\rho^2+\nu_1^2}+j\left(\frac{\nu_1}{\rho^2+\nu_1^2}-\nu_2\right)\right] \quad (25)$$

whereas the desired admittance is

$$\frac{1}{R_a}[\sqrt{1-\nu^2}-j\nu] \quad (26)$$

where now

$$C'_2=\frac{\nu}{\nu_1}C_a \quad (27)$$

and

$$L'_2=\frac{\nu}{\nu_2}L_a \quad (28)$$

and the terminating resistance

$$R=\rho R_a \quad (29)$$

The same values of ρ_1, ν_1 and ν_2 will make this network equal to the ideal at infinite frequency and another frequency given by ν_* as for the network of Fig. 4.

Applying the foregoing design instructions to the specific numerical examples hereinbefore considered, where the terminal impedances are 300 ohms and 600 ohms, respectively, and the line is 15 meters long, it follows that at the high impedance end of the line the network comprises a shunt inductance

$L'_1=1.27\ L_b=110$ microhenries, and a series capacitance;

$C'_1=0.595\ C_b=143$ micromicrofarads, and at the low impedance end, a shunt inductance;

$L'_2=0.595\ L_a=25.8$ microhenries, and a series capacitance;

$C'_2=1.27\ C_a=610$ micromicrofarads.

In accordance with another feature of applicant's invention there is provided a resistance load capable of dissipating large amounts of power comprising a tapered transmission line the conductors of which are of iron, steel or other material having high resistivity. One of the principal objects associated with this feature of the invention is to provide a power dissipating load that appears as a substantially pure resistance over the operating frequency range. Another object is to reduce the length of line required to dissipate a given amount of power, or in another aspect to increase the efficiency with which the resistive material is utilized as a power dissipator. A more particular object is to provide an efficient far-end termination for a rhombic antenna.

It is well known that in the operation of rhombic antenna systems it is the practice to terminate the far end of the antenna, that is, the end farthest from the transmitter, in a resistor. This resistor is called on to dissipate approximately 2 or 3 decibels less than the input power applied to the antenna, and in some systems this power may amount to 50 kilowatts. Heretofore it has been proposed to replace the resistor with a uniform transmission line of high resistance conductors so that the energy flowing from the antenna and through the conductors is transformed into heat and radiated as such from the conductors. At the far end of the resistance line the power level is low so the line can be short-circuited or terminated without difficulty in a resistor of small power rating.

In the case of a uniform resistive line the minimum length is fixed by the maximum tolerable power dissipation per unit length of line, which in turn is dependent on the maximum tolerable current intensity. The current intensity is greatest at the input terminals of the line and diminishes fairly rapidly with distance along the line. It will be evident, therefore, that only the initial portion of the uniform line is loaded to the maximum current carrying and power dissipating capacity.

Considering now the dissipative exponential line from a mathematical standpoint, the attenuation and phase constants are given by

$$\alpha=\alpha_0\sqrt{1-\nu_0^2}\left[1+\frac{\nu_0^2}{1-\nu_0^2}+\cdot\quad\cdot\quad\cdot\right]\rightarrow \alpha_0\left(1+\frac{\nu_0^2}{2}+\cdot\quad\cdot\quad\cdot\right) \quad (30)$$

and

$$\beta=\beta_0\sqrt{1-\nu_0^2}\left[1-\frac{\nu_0^2\alpha_0^2/\beta_0^2}{1-\nu_0^2}\right]\rightarrow\beta_0\sqrt{1-\nu_0^2} \quad (31)$$

for frequencies above and not too near the cut-off frequency, where

$$\gamma=\alpha_0+j\beta_0 \quad (32)$$

is the propagation constant for the uniform line that has the same distributed constants as the exponential line has at the point where α and β are evaluated.

$$\nu_0=\frac{\delta}{2\beta_0} \quad (33)$$

is the cut-off frequency of the equivalent lossless line. Hence for lines in which the attenuation in nepers per wave-length is small compared with $\frac{1}{2}\pi$ the phase constant is the same as for a lossless line and for frequencies above twice the cut-off frequency the attenuation constant is approximately equal to $(1+\nu_0^2/2)$ times that for the equivalent uniform line.

Provided the attenuation is not too large the current and voltage distribution will be the same as for a lossless line except for the additional power loss so that the equations for an exponential line may be used even though the distributed series resistance and shunt leakage do not vary exponentially with distance.

Suppose that the conductor size and resistance that will just dissipate the desired input power results in an attenuation constant α_∞ for a uniform transmission line. To a first approximation the conductors can carry the same current irrespective of the impedance level. The current wave will be given by

$$i=i_0e^{-\frac{\delta}{2}x-\alpha x} \quad (34)$$

except for a phase factor, where α is the attenuation constant. In order that the current will not increase,

$$\delta=-2\alpha \quad (35)$$

The actual attenuation "constant,"

$$\alpha_x=\alpha_{0x}\left(1+\frac{\nu_0^2}{2}\right)=\left(\frac{R}{2Z_x}+\frac{GZ_x}{2}\right)\left(1+\frac{\nu_0^2}{2}\right)$$

$$=\alpha_\infty\left(1+\frac{\nu_0^2}{2}\right)e^{2\alpha_\infty x}, (GZ_x<<R/Z_x) \quad (36)$$

will increase with distance down the line so that the current will decrease but not as rapidly as with a uniform line. The total attenuation in nepers is

$$\left(1+\frac{\nu_0^2}{2}\right)\int_{x=0}^{l}\alpha_\infty e^{2\alpha_\infty x}dx=\left(1+\frac{\nu_0^2}{2}\right)\left(\frac{e^{2\alpha_\infty l}-1}{2}\right) \quad (37)$$

Comparing the attenuation in decibels for a uniform line and for the tapered line for frequencies sufficiently above cut-off that $\nu_0^2/2$ may be neglected in comparison with unity, shows that at the point where the attenuation of the uniform line is 6 decibels the tapered line has an additional attenuation of 7 decibels above the uniform line or a total attenuation of more than twice. The current has been reduced to less than half. Here an improvement may be made by increasing the dissipation by either changing the wire size or resistivity of the conductor. A greater improvement would result from changing the resistivity because then the capacity for heat dissipation would be the same. Suppose, however, that one conductor material is to be used throughout and the dissipation capacity is proportional to the wire surface, then at this point the wire size could be reduced to one-half, thus doubling the attenuation factor. It is already four times that for the uniform line so this increases it to eight times. This gives 30 decibels attenuation in a length that would have less than 7 decibels attenuation if the line were uniform; or if 30 decibels attenuation were required the length of line could be reduced by a factor of about 4.4. Of course, the spacing is very close at the end of this line, but the line could be shorted at the end. This would approximately double the current at the end, but here again the current carrying capacity of the line is more than double the current traveling down the line. With the line shorted the reflected current would be 60 decibels down, which would not ordinarily seriously affect the input impedance. For the first 13 decibels of attenuation the impedance of the line would be free from changes due to changes in spacing resulting from wind, vibration, etc. When the spacing is small enough to be affected by wind, vibration, etc., the attenuation will be great enough to suppress these small irregularities.

The dissipative exponential line may be provided at its terminals with networks of the kind shown in Figs. 1, 2, 4 and 5 to minimize reactive components in its input impedances. The design of such networks is the same as for the non-dissipative exponential lines hereinbefore considered, hence illustrative examples would be superfluous.

Whereas in the exponential resistive line just described the current decreases somewhat with distance along the line, the current can be maintained substantially the same throughout the line if the impedance Z_x varies as the exponent of the exponent of the exponent, etc. For this case we have for the attenuation "constant,"

$$\alpha_x=\alpha_0\frac{Z_0}{Z_x} \quad (38)$$

and for the impedance transformation constant to give constant current,

$$\alpha=-2\alpha_x=-2\alpha_0\frac{Z_0}{Z_x}=-2\alpha_0e^{-\alpha x} \quad (39)$$

The impedance is then,

$$Z_x=Z_0e^{\alpha x}=Z_0e^{-2\alpha_0xe^{2\alpha_0xe^{2\alpha_0x}}} \text{ etc.} \quad (40)$$

and the total attenuation in nepers is

$$\int_{x=0} \alpha_0 e^{2\alpha_0 x e^{2\alpha_0 x \text{ etc.}}} dx = \alpha_0 l + (\alpha_0 l)^2 + \frac{(\alpha_0 l)^3}{2} + 5(\alpha_0 l)^4 + \ldots \quad (41)$$

Fig. 6 illustrates a typical radio transmission system embodying the various features of the invention hereinbefore described. The generator S represents a source of modulated waves occupying a wide frequency range, as for specific example, 4 to 20 megacycles per second, or it may be considered as representing a source of signals the frequency of which is changed from time to time within the frequency range mentioned. An open-wire transmission line **5**, which may be assumed to have a purely resistive characteristic impedance of 600 ohms, extends from the source S over a distance of perhaps several hundred feet to an exponential line **6** having an input impedance of 600 ohms and an output impedance of 800 ohms. At the terminals of the exponential line are connected reactive networks **7** and **8** of the kind described with reference to Fig. 1. An antenna of the well-known rhombic type having an approximately resistive input impedance of 800 ohms is connected to the high impedance end of the line **6**. The far end of the antenna is terminated in a dissipative exponential line **10** which is reactively compensated by series capacitance **9** and shunt inductance **11** as hereinbefore described. The line **10** is terminated at its distant end in a shunt resistor **12**. The resistance of which is equal in magnitude to the surge impedance of the line where it is connected, for example, 100 ohms.

In the practical design of exponential lines as hereinbefore described, and also in the design of uniform transmission lines, substantially lumped stray reactances are likely to be encountered which tend to cause reflection. Thus, at the insulators supporting or tensioning the conductors a stray shunt capacitance of considerable magnitude may be unavoidable. Similarly, where the line changes direction and at the ends of the line there may be an abrupt change in the surge impedance of the line at those points, a change that tends to introduce an irregularity in the impedance-frequency or attenuation-frequency characteristic of the line.

In accordance with a further feature of the present invention stray reactances of the kind described are compensated by introducing reactance of the opposite sign and of magnitude such that the surge impedance of the line at the point where the stray reactance appears is substantially the same as the surge impedance at other points in the same vicinity. The net effect of the stray reactance and its compensating reactance is that of a short section of line introduced at that point, or substantially so for all frequencies up to those for which the equivalent inserted length of line is an appreciable fraction of a wave-length.

Thus, for specific example, where a transmission line is terminated in a shunt resistor there is found to be greater shunt capacitance at the resistor, due to "end effect," than at other points. In such case the impedance irregularity that would otherwise result may be eliminated by increasing the lengths of the leads **13** from the ends of the conductors to the resistor **14**, as illustrated in Fig. 7. The leads **13** are of such length that the series inductance associated with the shunting connection bears the same relation to the shunt capacitance at that point as obtains at other points along the line in the same vicinity.

Likewise, at intermediate points along a high frequency transmission line, as for specific example in an exponential line in accordance with the present invention, the desired relation between distributed series inductance and shunt capacitance may be disturbed by an excess of shunt capacitance where, for example, the wires are supported by insulators. The resulting impedance irregularity may be minimized by introducing an additional series inductance of appropriate amount at such points. If Z_x represents the desired surge impedance at the point in question and C_o represents the excess of shunt capacitance then the inductance L_o to be inserted is $L_o = Z_x^2 C_o$ where L_o is in microhenries and C_1 in microfarads. A convenient arrangement for introducing the required inductance at any intermediate point along the line is illustrated in Fig. 8 where a pair of strain insulators **16** disposed between pairs of compression insulators **15** are utilized to increase the separation of the wires over a short length of the line. An alternative arrangement is shown in Fig. 9, where each wire of the transmission line is looped around its insulator to increase the inductance at that point. The amount of inductance to be added in either arrangement can best be determined by trial, i. e., by varying the separation of the insulators **16** in Fig. 8, and by changing the number and size of the loops in Fig. 9.

What is claimed is:

1. A resistive load capable of dissipating large amounts of radio frequency power comprising an unsheathed open-wire transmission line, said line comprising a plurality of wire conductors the effective resistivity of which is substantially equal to or greater than that of iron and the surge impedance of said line progressively decreasing from the high impedance end at such rate that on application of power to said high impedance end the current flow through said conductors and the power dissipation therefrom is substantially more nearly uniform than in a uniform transmission line whereby said line is capable of dissipating as much power as a substantially longer uniform line comprising conductors of the same size and material.

2. A combination in accordance with claim 1 in which said line is so proportioned that the radio frequency power dissipation per unit length is substantially constant throughout the length of said line.

3. A combination in accordance with claim 1 in which said line is exponentially tapered.

4. In a radio transmission system, a dummy load comprising an unsheathed open-wire transmission line having a plurality of line canductors the effective resistivity of which is substantially equal to or greater than that of iron, said line being so proportioned that the attenuation per unit length of line increases progressively and substantially from one end of said line to the other, and means for applying radio frequency power to be dissipated to that end of said line where the attenuation per unit length is lower, whereby the power dissipating capacity of said dummy load is substantially greater than that of a uniform line of the same length and material.

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