July 29, 1941. L. BATCHELDER 2,250,461

ELECTRICAL NETWORK

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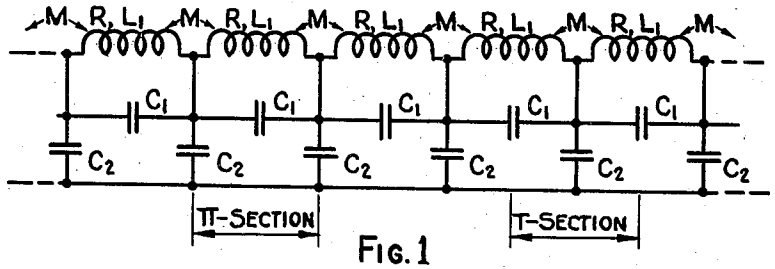

FIG. 1

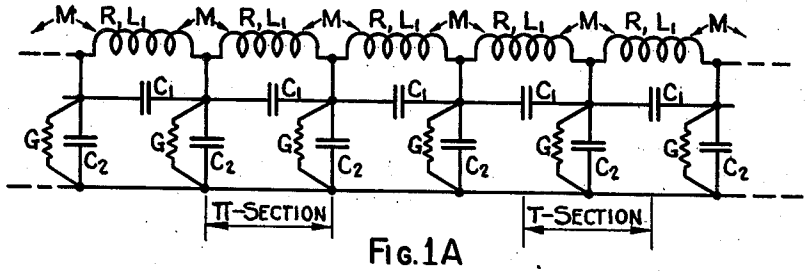

FIG. 1A

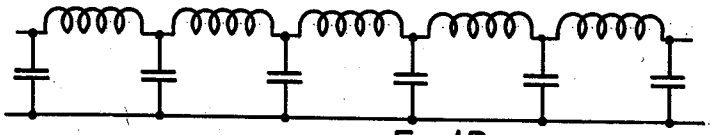

FIG. 1B

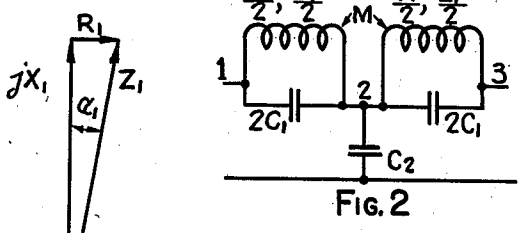

FIG. 2

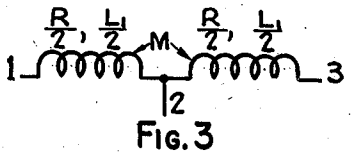

FIG. 3

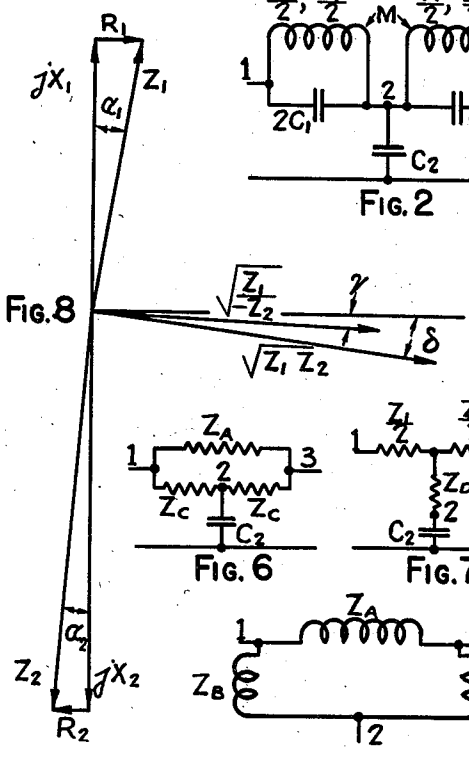

FIG. 8

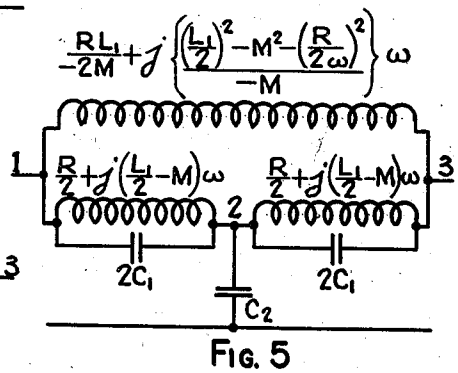

FIG. 5

FIG. 6

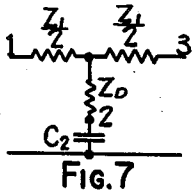

FIG. 7

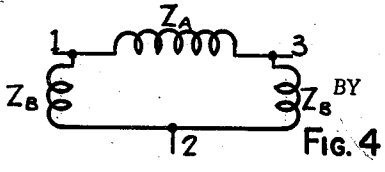

FIG. 4

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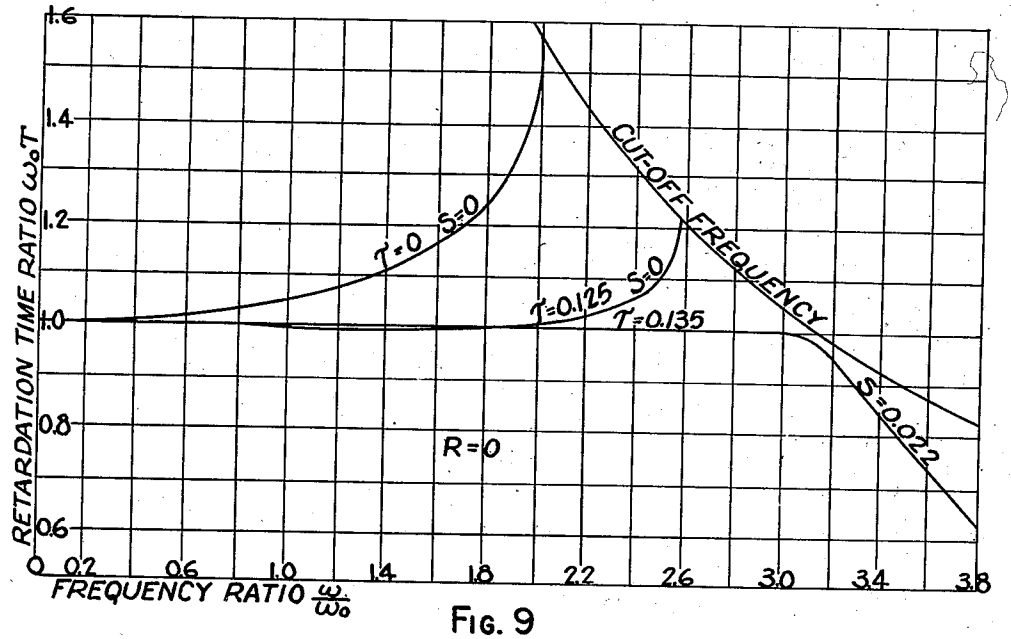

FIG. 9

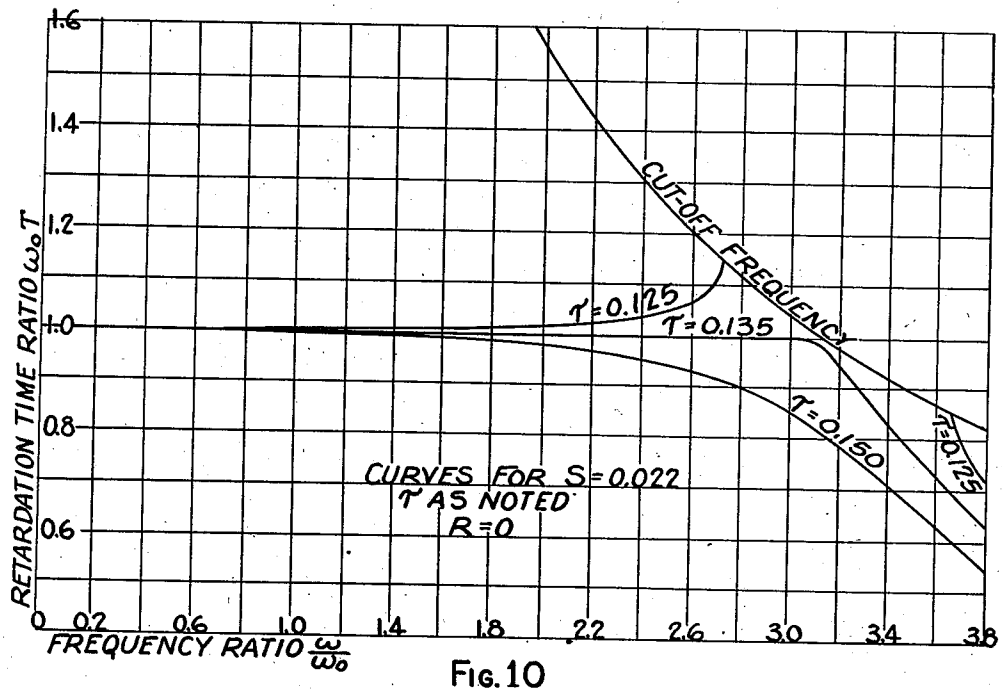

FIG. 10

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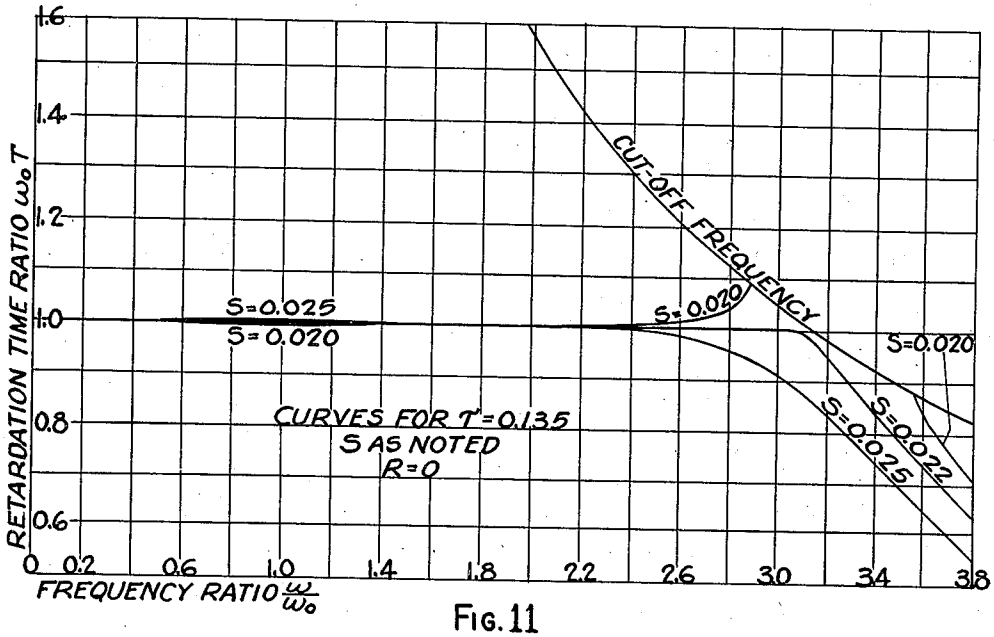

FIG. 11

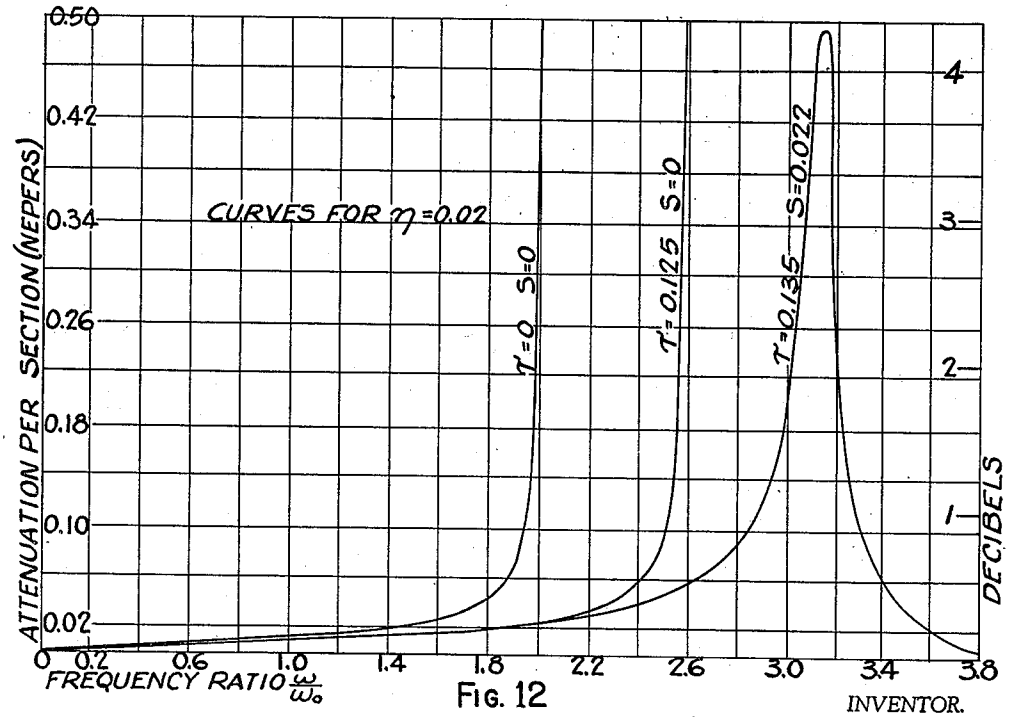

FIG. 12

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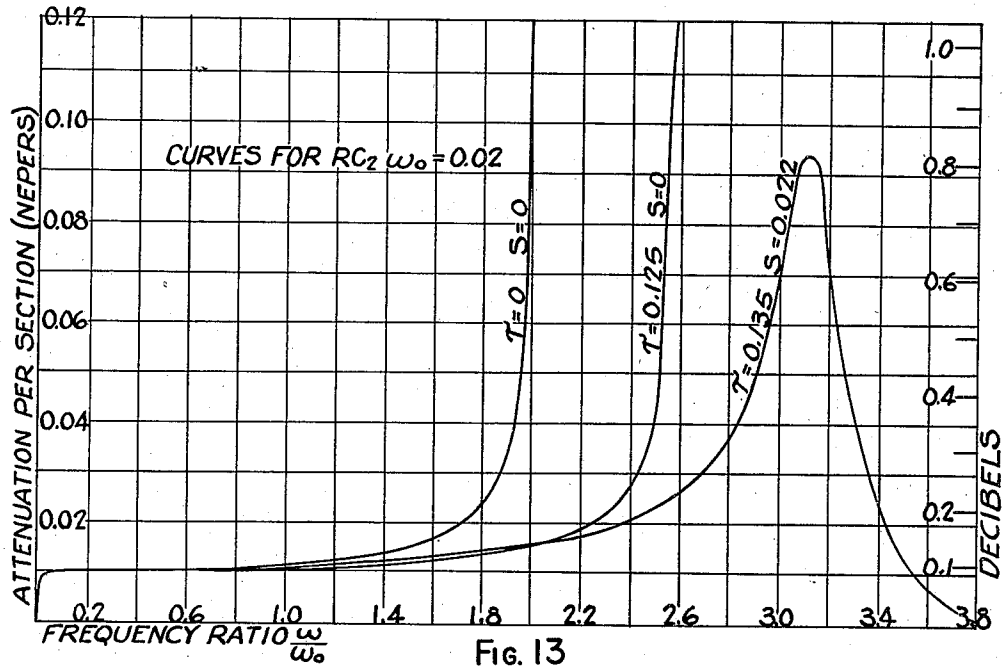

FIG. 13

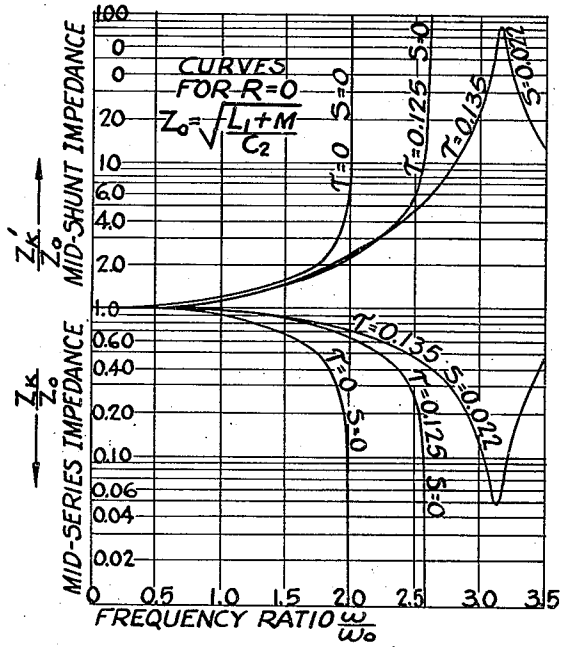

FIG. 14

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Patented July 29, 1941 2,250,461

UNITED STATES PATENT OFFICE

2,250,461

ELECTRICAL NETWORK

Laurence Batchelder, Cambridge, Mass., assignor to Submarine Signal Company, Boston, Mass., a corporation of Maine

Application May 28, 1938, Serial No. 210,795

9 Claims. (Cl. 178—44)

The present invention relates to electric retardation networks and in particular to those which are used in electric compensators for the determination of the direction of a source of wave energy.

An object of the present invention is to provide an electric retardation network more free from phase distortion because it has a flatter characteristic of retardation than in networks heretofore available.

Further objects of the invention are to provide a retardation network having lower and more uniform attenuation, resulting in less amplitude distortion; more nearly constant mid-series iterative impedance; and higher relative cut-off frequency permitting either a higher actual cut-off or a greater time lag per section resulting in fewer coils and capacitors. The invention also involves a closer spacing of coils. There is, therefore, an appreciable saving in the physical dimensions of a given line.

Moreover, by means of the present invention such phase distortion as is due to losses can be corrected for.

The simple prototype retardation line consists of a ladder network having inductances in series and capacitances in shunt. An improved line of this type, hereinafter called the "simple mutual type," employs mutual inductance between the adjacent series inductances equal to 12½% of the inductance of the individual series elements.

The present invention consists of a similar ladder network with mutual inductance between adjacent series coils. However, I use a somewhat larger value of mutual than was used heretofore, namely approximately 13½%. Furthermore, in addition to the capacitances which are shunted across the line, I provide capacitances in shunt to each inductance. These have a value of approximately 2.2% of the first-named capacitances.

My invention may best be understood from the following discussion in connection with the accompanying drawings in which—

Fig. 1 is a diagram of a network in accordance with my invention;

Fig. 1*a* is a modification of Fig. 1;

Fig. 1*b* represents the known prototype low-pass filter, or when mutual inductance of about 12½% is present between the coils, it represents the "simple mutual type" of line;

Fig. 2 is a T-section of the network shown in Fig. 1;

Figs. 3 and 4 are network structures equivalent to a portion of the structure shown in Fig. 2;

Fig. 5 is a T-section equivalent to Fig. 2;

Figs. 6 and 7 are structures equivalent to Fig. 5;

Fig. 8 is a vector diagram representing the impedance relation in the network of Fig. 1 with reference to the propagation constant;

Fig. 9 shows curves of retardation time ratio with respect to frequency ratio for the network of Fig. 1, and Fig. 1*b* with and without mutual;

Fig. 10 shows similar curves for the network of Fig. 1 with capacitance $C_1=0.022\ C_2$ and varying values of mutual inductance;

Fig. 11 shows similar curves for the network of Fig. 1 with mutual inductance of 13½% and varying values of capacitance C_1;

Fig. 12 shows curves of the attenuation per section with respect to frequency ratio for networks according to Fig. 1, and Fig. 1*b* with and without mutual;

Fig. 13 shows curves similar to those of Fig. 12 but with a constant resistance; and

Fig. 14 shows curves of mid-series and mid-shunt impedances for networks according to Fig. 1, and Fig. 1*b* with and without mutual.

The retardation line structure in accordance with the present invention is shown in Fig. 1. The π section by itself can not be realized because of the mutual inductance between adjacent coils. The T section is as shown in Fig. 2.

The two coils with their mutual inductance are redrawn in Fig. 3. Now the networks of Figs. 3 and 4 are equivalent if

$$Z_A=\frac{RL_1}{-2M}+j\left(\frac{\left(\frac{L_1}{2}\right)^2-M^2-\left(\frac{R}{2\omega}\right)^2}{-M}\right)\omega \qquad (1)$$

$$Z_B=\frac{R}{2}+j\left(\frac{L_1}{2}-M\right)\omega \qquad (2)$$

The sign of M is taken as positive when the coils L_1 are connected in series aiding. This means that the resistance and ordinarily the reactance of Z_A will be negative.

We may now replace the inductances of Fig. 2 by their equivalent network as given by Fig. 4, and redraw the equivalent T section as in Fig. 5. This structure is then seen to be of the bridged T type, and is equivalent to Fig. 6 where Z_A is given by Equation 1 and Z_c by (3):

$$Z_c=\frac{1}{2}\frac{R+j((L_1-2M)\omega[1+(L_1-2M)C_1\omega^2]-R^2C_1\omega)}{(RC_1\omega)^2+(1-(L_1-2M)C_1\omega^2)^2} \qquad (3)$$

The bridged T structure of Fig. 6 can be converted into the simple structure of Fig. 7, by finding the values of

$$\frac{Z_1}{2}$$

and Z_D such that the T and π networks between terminals 1, 2, 3 shall be equivalent.

The conditions of equivalence between the structures of Figs. 6 and 7 are

$$\frac{Z_1}{2}\equiv\frac{R_1}{2}+j\frac{X_1}{2}=\frac{Z_AZ_c}{Z_A+2Z_c} \quad (4)$$

$$Z_D\equiv R_D+jX_D=\frac{Z_c^2}{Z_A+2Z_c} \quad (5)$$

or

$$\frac{R_1}{2}=\frac{R_c(R_A^2+X_A^2)+2R_A(R_c^2+X_c^2)}{(R_A+2R_c)^2+(X_A+2X_c)^2} \quad (6)$$

$$\frac{X_1}{2}=\frac{X_c(R_A^2+X_A^2)+2X_A(R_c^2+X_c^2)}{(R_A+2R_c)^2+(X_A+2X_c)^2} \quad (7)$$

$$R_D=\frac{(R_c^2-X_c^2)(R_A+2R_c)+2R_cX_c(X_A+2X_c)}{(R_A+2R_c)^2+(X_A+2X_c)^2} \quad (8)$$

$$X_D=\frac{2R_cX_c(R_A+2R_c)-(R_c^2-X_c^2)(X_A+2X_c)}{(R_A+2R_c)^2+(X_A+2X_c)^2} \quad (9)$$

where from (1) and (3)

$$R_A=\frac{RL_1}{-2M} \quad (10)$$

$$X_A=\left(\frac{\left(\frac{L_1}{2}\right)^2-M^2-\left(\frac{R}{2\omega}\right)^2}{-M}\right)\omega \quad (11)$$

$$R_c=\frac{1}{2}\frac{R}{(RC_1\omega)^2+(1-(L_1-2M)C_1\omega^2)^2} \quad (12)$$

$$X_c=\frac{1}{2}\frac{(L_1-2M)\omega(1-(L_1-2M)C_1\omega^2)-R^2C_1\omega}{(RC_1\omega)^2+(1-(L_1-2M)C_1\omega^2)^2} \quad (13)$$

The equations will now become prohibitively cumbersome unless we make some simplifying assumptions. As will be seen later, we are concerned with values of the elements in the following ranges:

$$0<M<0.15L_1 \quad (14)$$

$$0<C_1<0.025C_2 \quad (15)$$

$$0<\omega\sqrt{(L_1+2M)C_2}<4 \quad (16)$$

whence

$$0.6<1-(L_1-2M)C_1\omega^2<1 \quad (17)$$

Now we shall assume that R is so small that within the above ranges we may consider that

$$\eta^2<<0.385 \quad (18)$$

where as an abbreviation

$$\eta=\frac{R}{L_1\omega} \quad (19)$$

It then follows that

$$(RC_1\omega)^2<<1-(L_1-2M)C_1\omega^2 \quad (20)$$

$$R^2C_1<<(L_1-2M)(1-(L_1-2M)C_1\omega^2) \quad (21)$$

$$\left(\frac{R}{2\omega}\right)^2<<\left(\frac{L_1}{2}\right)^2-M^2 \quad (22)$$

Introducing the results (20), (21), (22) into Equations 11, 12, 13 we obtain much simplified expressions for the elements of the bridged T. They are:

$$R_A=\frac{RL_1}{-2M} \quad (10)$$

$$X_A=\frac{\left(\frac{L_1}{2}\right)^2-M^2}{-M}\omega=\frac{(L_1-2M)(L_1+2M)\omega}{-4M} \quad (23)$$

$$R_c=\frac{1}{2}\frac{R}{(1-(L_1-2M)C_1\omega^2)^2} \quad (24)$$

$$X_c=\frac{1}{2}\frac{(L_1-2M)\omega}{1-(L_1-2M)C_1\omega^2} \quad (25)$$

We may also write in consequence of (18)

$$R_A^2<<X_A^2 \quad (26)$$

$$R_c^2<<X_c^2 \quad (27)$$

so that (6), (7), (8), (9) become

$$\frac{R_1}{2}=\frac{R_cX_A^2+2R_AX_c^2}{(X_A+2X_c)^2} \quad (28)$$

$$\frac{X_1}{2}=\frac{X_AX_c}{X_A+2X_c} \quad (29)$$

$$R_D=-X_c\frac{X_c(R_A+2R_c)-2R_c(X_A+2X_c)}{(X_A+2X_c)^2} \quad (30)$$

$$X_D=+X_c\frac{2R_c(R_A+2R_c)+X_c(X_A+2X_c)}{(X_A+2X_c)^2} \quad (31)$$

At this point it is convenient to introduce the following abbreviations:

$$H\equiv1-(L_1-2M)C_1\omega^2 \quad (32)$$

$$I\equiv1-L_1C_1\omega^2 \quad (33)$$

$$J\equiv1-(L_1+2M)C_1\omega^2 \quad (34)$$

$$\tau\equiv\frac{M}{L_1} \quad (35)$$

Let us further assume that R is so small that

$$\eta^2<<\frac{H}{2}(1-2\tau)^2\left(\frac{\frac{1}{H}-\frac{1+2\tau}{4\tau}}{\frac{1}{H^2}-\frac{1}{2\tau}}\right) \quad (36)$$

This is an even more stringent restriction than (18) because from (14) and (17) we find that

$$0.090<\frac{H}{2}(1-2\tau)^2\left(\frac{\frac{1}{H}-\frac{1+2\tau}{4\tau}}{\frac{1}{H^2}-\frac{1}{2\tau}}\right)<0.250 \quad (37)$$

In view of (36) we may simplify (31) to the form

$$X_D=\frac{X_c^2}{X_A+2X_c} \quad (38)$$

Now, if we introduce Equations 10, 23, 24, 25 and also 32, 33, 34, 35 into Equations 28, 29, 30 and 38, we obtain the following expressions:

$$R_1=\frac{R}{J^2} \quad (39)$$

$$X_1=\frac{(L_1+2M)\omega}{J} \quad (40)$$

$$R_D=-R\frac{2MC_1\omega^2I}{H^2J^2} \quad (41)$$

$$X_D=-\frac{M\omega}{HJ} \quad (42)$$

Adding the impedance of the condenser C_2 to Z_D we obtain the total impedance Z_2 of the shunt arm:

$$Z_2=R_2+jX_2 \quad (43)$$

Assuming that C_2 is a pure capacitance, we have

$$R_2=-\frac{R2MC_1\omega^2I}{H^2J^2} \quad (44)$$

$$X_2=-\left(\frac{M\omega}{HJ}+\frac{1}{C_2\omega}\right) \quad (45)$$

The propagation constant $P=A+jB$ of the section is found from the relation:

$$\frac{1}{2}\sqrt{\frac{Z_1}{Z_2}}=\sinh\left(\frac{A}{2}+j\frac{B}{2}\right) \quad (46)$$

And when the dissipation is small we may write (46) in the form

$$\sqrt{\frac{Z_1}{-Z_2}}=2\sin\frac{B}{2}-jA\cos\frac{B}{2} \quad (47)$$

Now, we may evaluate

$$\sqrt{\frac{Z_1}{-Z_2}}$$

quite simply for the case of small dissipation. The relations are illustrated in Fig. 8 from which it is evident that

$$\gamma=\frac{\alpha_1-\alpha_2}{2} \quad (48)$$

$$\delta=\frac{\alpha_1+\alpha_2}{2} \quad (49)$$

Assuming that sines and tangents of small angles are equal to the angles themselves, and that the cosines are equal to unity, we have

$$\sqrt{\frac{Z_1}{-Z_2}}=\sqrt{\frac{X_1}{-X_2}}(1-j\gamma) \quad (50)$$

Whence

$$\sin\frac{B}{2}=\frac{1}{2}\sqrt{\frac{X_1}{-X_2}} \quad (51)$$

$$A=\frac{\gamma\sqrt{\frac{X_1}{-X_2}}}{\cos\frac{B}{2}}=2\gamma\tan\frac{B}{2} \quad (52)$$

The mid-series iterative impedance of the network is given by

$$Z_K=\sqrt{Z_1Z_2}\sqrt{1+\frac{Z_1}{4Z_2}}=\sqrt{Z_1Z_2}\cosh\frac{P}{2} \quad (53)$$

While the corresponding mid-shunt impedance is

$$Z_k'=\frac{\sqrt{Z_1Z_2}}{\sqrt{1+\frac{Z_1}{4Z_2}}}=\frac{\sqrt{Z_1Z_2}}{\cosh\frac{P}{2}} \quad (54)$$

It should be noted that Z_k' has no real significance when mutual inductance is present since a true mid-shunt termination can not be physically attained. It is of interest, however, to examine the behaviour of Z_k' as well as of Z_k.

For the case of small dissipation we may write

$$\cosh\frac{P}{2}=\cos\frac{B}{2}+j\frac{A}{2}\sin\frac{B}{2} \quad (55)$$

Now from Fig. 8 it is evident that

$$\sqrt{Z_1Z_2}=\sqrt{-X_1X_2}(1-j\delta) \quad (56)$$

Whence by (51), (52) and (55)

$$Z_K=\sqrt{-X_1X_2}\cos\frac{B}{2}(1-j\delta)\left(1+j\frac{A}{2}\tan\frac{B}{2}\right) \quad (57)$$

or

$$Z_K=\sqrt{-X_1X_2}\cos\frac{B}{2}\left(1+\delta\gamma\tan^2\frac{B}{2}+j\left(\gamma\tan^2\frac{B}{2}-\delta\right)\right)$$

Similarly

$$Z'_K=\frac{\sqrt{-X_1X_2}}{\cos\frac{B}{2}}\times\frac{1-\delta\gamma\tan^2\frac{B}{2}-j\left(\gamma\tan^2\frac{B}{2}+\delta\right)}{1+\gamma^2\tan^4\frac{B}{2}} \quad (58)$$

where by (48) and (49)

$$\gamma=\frac{R_1}{2X_1}-\frac{R_2}{2X_2} \quad (59)$$

$$\delta=\frac{R_1}{2X_1}+\frac{R_2}{2X_2} \quad (60)$$

Equations 46 to 60 inclusive are entirely general and express the propagation constant and iterative impedances of any recurrent ladder structure with small dissipation. They may be applied to the particular network of Fig. 1 by introducing the values of R_1, X_1, R_2 and X_2 given by Equations 39, 40, 44 and 45, and hence the characteristics of the network may be computed.

Specific design for constant retardation

From the point of view of electric compensator design the most important property is the time delay which is given implicitly by Equation 51. This will be considered first because of its importance and because functions of the phase shift B enter into the simplest expressions for attenuation and iterative impedance.

From Equations 32, 33, 40 and 45 we find

$$\frac{X_1}{-X_2}=\frac{(L_1+2M)C_2\omega^2}{1-(L_1+2M)C_1\omega^2+\frac{MC_2\omega^2}{1-(L_1-2M)C_1\omega^2}} \quad (61)$$

Let us now introduce the following abbreviations:

$$S\equiv\frac{C_1}{C_2} \quad (62)$$

$$\omega_0\equiv\frac{1}{\sqrt{(L_1+2M)C_2}} \quad (63)$$

$$D\equiv(L_1+2M)C_1\omega^2=S\left(\frac{\omega}{\omega_0}\right)^2 \quad (64)$$

$$E\equiv(L_1-2M)C_1\omega^2=\frac{1-2\tau}{1+2\tau}S\left(\frac{\omega}{\omega_0}\right)^2 \quad (65)$$

$$F\equiv MC_2\omega^2=\frac{\tau}{1+2\tau}\left(\frac{\omega}{\omega_0}\right)^2 \quad (66)$$

where

$$\tau=\frac{M}{L_1} \quad (35)$$

Now, by substituting (61) into (51) and using the new abbreviations we have

$$\sin\frac{B}{2}=\frac{\frac{\omega}{\omega_0}}{\sqrt{1-D+\frac{F}{1-E}}} \quad (67)$$

The phase shift B may be computed from Equation 67 and the corresponding retardation time obtained from (68):

$$T=\frac{B}{\omega} \quad (68)$$

It is now necessary to find the values of τ and of S which will make T most nearly constant over a wide frequency range. I have found that for $\tau=0.135$ and $S=0.022$

$$0.999<\omega_0T<1.001$$

where

$$0<\frac{\omega}{\omega_0}<3$$

This is shown by a curve in Fig. 9, where for comparison I have also shown a curve for the prototype low-pass filter section ($\tau=0$ and $S=0$) and a curve for the simple mutual type of retardation line ($\tau=0.125$ and $S=0$).

It is evident that the cut-off frequency is reached when $\sin B/2=1$ or $B=\pi$. The maximum phase shift possible in a ladder section is π which corresponds to a retardation time of one-half period. Therefore, in obtaining a curve of T which is flat up to $\omega/\omega_0=3$ I have covered $3/\pi=95.5\%$ of the maximum possible frequency range.

In order to illustrate the effect of variation of τ or S on the retardation characteristic I have plotted in Figs. 10 and 11 curves of T for different values of τ and of S, respectively. It will be seen that an increase of τ reduces T at all frequencies, although the higher the frequency the higher the effect. If we hold τ constant and increase S, we reduce T at the higher frequencies but we increase it at lower frequencies. Thus, in Fig. 11, the curves all cross at about $\omega/\omega_0=2$. Roughly we may conclude that τ affects the slope while S affects the curvature of the retardation characteristic.

It must be remembered that these curves are true only when $\eta^2<<0.09$ (see Equations 36 and 37) and that in practice this condition can not always be met. It can never be met at very low frequencies and T will rise sharply as ω approaches zero. This distortion of low frequencies (where $\omega/\omega_0<0.3$) is usually of no consequence, but if desired it can be made very small by introducing across C_2 a leak of conductance G where

$$G=\frac{RC_2}{L_1+2M} \tag{69}$$

A network containing this conductance element is shown in Fig. 1*a*. At the higher frequencies Equation 67 and the curves computed from it may be considered correct if η is sufficiently small. If η is too large, some distortion will be introduced.

If the dissipation is so large that (67) is not true, the correct equation would be so complicated as to render the labor of computation prohibitive. The problem is further complicated because R varies with frequency in such a way that neither R nor η can be considered constant over any very large range. There is also some error introduced by assuming that the distributed capacity of L_1 may be lumped and included in C_1. In such a case the optimum values of τ and S for minimum distortion should be determined by trial and error, using laboratory measurements rather than numerical computations. The curves of Figs. 10 and 11 would serve as a guide for this work.

Attenuation of the specific design

Let us now consider the attenuation constant A which is expressed most conveniently by (52)

$$A=2\gamma \tan \frac{B}{2} \tag{52}$$

Now we know from (59) that

$$2\gamma=\frac{R}{J(L_1+2M)\omega}-\frac{2RIMC_1\omega^2}{HJM\omega+\frac{H^2J^2}{C_2\omega}}$$

which after simplification becomes

$$2\gamma=\frac{\eta}{1+2\tau}\left(\frac{1-K}{1-D}\right) \tag{70}$$

Similarly, from (60) it follows that

$$2\delta=\frac{\eta}{1+2\tau}\left(\frac{1+K}{1-D}\right) \tag{71}$$

where for abbreviation:

$$K\equiv\frac{2\frac{I}{H}S\left(\frac{\omega}{\omega_0}\right)^2}{1+\frac{HJ(1+2\tau)}{\tau}\left(\frac{\omega_0}{\omega}\right)^2}=\frac{2D\left(1-\frac{SF}{\tau}\right)}{1+\frac{(1-D)(1-E)}{F}} \tag{72}$$

From these equations I have computed A for the case of $\tau=0.135$ and $S=0.022$ for one particular value of dissipation $\eta=0.02$. This is shown by a curve in Fig. 12, where for comparison are given computed curves for the simple mutual type of line ($\tau=0.125$ and $S=0$) and also for the prototype low-pass filter ($\tau=0$ and $S=0$). At low frequencies the three curves are much alike, and in each case we find the approximate relation

$$A=\frac{1}{2}\frac{R}{(L_1+2M)\omega_0}=\frac{1}{2}RC_2\omega_0 \tag{73}$$

Equation 73 shows clearly how mutual inductance reduces attenuation, and the reduction is apparent in the curves.

The assumption that η is constant requires that R be proportional to frequency, and consequently $R=0$ at $\omega=0$. This is, of course, impossible, so I have plotted in Fig. 13 curves of attenuation computed for the same three types of line but with R constant. These curves also agree with Equation 73 at low frequencies. The reduction of attenuation by mutual inductance does not appear in Fig. 13 because if $RC_2\omega_0$ is kept constant, R increases when τ increases. The curves of Fig. 13 drop to zero when $\omega=0$. If the leaks of Equation 69 had been included, these curves would be flat all the way to zero frequency.

On account of skin effect and possibly other losses we know that R can not be constant, but must increase with frequency. Consequently Fig. 13 also does not conform to the practical case. The correct curves, however, would be intermediate between those of Figs. 12 and 13. These figures serve mainly to show how the additions of mutual inductance and then of capacitance C_1 successively raise the cut-off frequency. Here, Equation 52 does not hold, for, when dissipation is present, the attenuation is never infinite although it does rise to a very high value when $S=0$.

For the specific design it is interesting to note that the cut-off frequency is approached but never reached at $\omega/\omega_0=\pi$. This is apparent from Fig. 11 where if S is sufficiently large, the curve of time lag does not meet the cut-off envelope, although at $S=0.022$, the two curves come very close to each other. At the corresponding frequency we find in Figs. 12 and 13 a sharp peak of attenuation.

Iterative impedance of the specific design

Fig. 14 gives curves of Z_K and Z_K' for the specific design, and for comparison, the corresponding curves for the simple mutual type and for the prototype low-pass filter. All these curves have been computed for the non-dissipative case where Z_K and Z_K' are pure resistances below the cut-off frequency. When small dissipation is present, it will modify slightly the curves (Fig. 14) of the resistive component and will also introduce a small reactive component in Z_K and Z_K'. If the loss occurs in the series arm, i. e. in L_1, the reactance will be positive in Z_K and negative in Z_K'.

Fig. 14 shows again how the additions of mutual inductance and then of capacitance C_1 successively raise the cut-off frequency. For the prototype low-pass filter the cut-off occurs at $\omega/\omega_0=2.0$, while for the simple mutual type of line it is at $\omega/\omega_0=2.58$, but when $\tau=0.135$ and $S=0.022$ the cut-off is approached but never reached at $\omega/\omega_0=\pi$.

Of the three pairs of curves in Fig. 14, only in the case of the prototype are Z_K and Z_K' exactly reciprocal. This is because the prototype is the only one of the three which is a "constant-K" structure. In the other structures the product

Z_1Z_2 is a function of frequency. The lack of symmetry between Z_K and Z_K' is most pronounced for the specific design ($\tau=0.135$ and $S=0.022$), and for this case the curve of mid-series impedance Z_K is the most constant of all six curves. This is fortunate, for the mid-shunt termination can not be attained correctly on account of the mutual inductance.

Design particulars

For a particular application such as a compensator, a line can be made to meet three requirements: iterative impedance, attenuation, and either the cut-off frequency or the retardation per section. These last two are interdependent since $\omega_0 T=1$ throughout the range $0<\omega<3\omega_0$. Cut-off occurs at

$$\omega=\omega_0=\frac{3}{T}$$

and the cut-off frequency is:

$$f_c=\frac{3}{2\pi T} \quad (74)$$

Let us assume that Z_0 and T are specified, where

$$Z_0=\sqrt{\frac{L_1+2M}{C_2}} \quad (75)$$

and by (63)

$$T=\frac{1}{\omega_0}=\sqrt{(L_1+2M)C_2} \quad (76)$$

From (75) and (76) it follows at once that

$$C_2=\frac{T}{Z_0} \quad (77)$$

and

$$(L_1+2M)=Z_0T \quad (78)$$

Since we have chosen $S=0.022$, (62) gives

$$C_1=0.022\frac{T}{Z_0} \quad (79)$$

and since $\tau=0.135$, (35) gives

$$L_1=0.787Z_0T \quad (80)$$

and

$$M=0.1063Z_0T \quad (81)$$

The resonant frequency of the coil must be at least as high as f_s, where by (79), (80) and (74)

$$f_s=\frac{1}{2\pi\sqrt{L_1C_1}}=\frac{1.21}{T}=2.53f_c \quad (82)$$

This is necessary in order that the capacitance inherent in the coil shall not exceed the desired capacitance C_1. The actual capacitor connected at C_1 should have not the value given by (79), but rather the value necessary to tune L_1 to f_s.

If it is required to limit the attenuation to a definite value A, we find from (73) and (77) that R must be limited to

$$R=2Z_0A \quad (83)$$

where A is in nepers. As will be seen from Fig. 13, Equation 83 is true only at frequencies well below the cut-off.

In actual practice, especially in electric compensators, the additional capacitance element C_1 which my invention requires is relatively small, both electrically and physically, so that there is no real disadvantage in its use. This is particularly true in view of the advantages of my network, which are set out above in connection with the objects of the invention.

Having now described my invention, I claim:

1. An electric retardation line comprising a ladder network of recurrent sections having series inductive elements with mutual inductance between adjacent elements, and shunt capacitive elements and an additional capacitive element in shunt to each inductive element.

2. An electric retardation line comprising a ladder network of recurrent sections having series inductive elements with mutual inductance between adjacent elements, and shunt capacitive elements and an additional capacitive element in shunt to each inductive element, the said network having a substantially constant retardation time per section throughout the frequency range of zero to at least

$$\frac{3}{2\pi}$$

times the reciprocal of the retardation time per section in seconds.

3. An electric retardation line comprising a ladder type network of recurrent sections having inductive series elements each having an inductance of L_1 and a mutual inductance between adjacent elements of M, shunt capacitive elements each having a capacity of C_2, and additional capacitive elements each having a capacity of C_1 shunted across said inductive series elements, the values of M and C_1 being so adjusted that the retardation time per section is substantially a constant throughout the frequency range of zero to at least

$$\frac{3}{2\pi}$$

times the reciprocal of the retardation time per section in seconds.

4. An electric retardation line comprising inductive series elements and shunt capacitive elements, said inductive elements having a mutual inductance between adjacent elements of substantially 13.5% of the self inductance of one element, and a capacitive element in shunt to each inductive element, said second-named capacitive elements having a capacity of substantially 2.2% of the capacity of said first-named capacitive elements.

5. An electric retardation line comprising a ladder type network having inductive series elements each having an inductance of L_1 and a mutual inductance between adjacent elements of M, shunt capacitive elements each having a capacity of C_2, and additional capacitive elements each having a capacity of C_1 shunted across said inductive series elements, the value of M being $0.135L_1\pm x$ and the value of C_1 being $(0.022\pm y)C_2$, where the deviations x and y are zero for a no-loss line and are just sufficient to compensate for losses when such are present in order to make the retardation time per section substantially a constant throughout the desired frequency range.

6. A electric retardation line comprising a ladder type network having inductive series elements each having an inductance of L_1 and a mutual inductance between adjacent elements of M, shunt capacitive elements each having a capacity of C_2, and additional capacitive elements each having a capacity of C_1 shunted across said inductive series elements, the value of M being $0.135L_1\pm x$ and the value of C_1 being $(0.022\pm y)C_2$, where the deviations x and y are zero for a no-loss line and are just sufficient to compensate for losses when such are present in order to make the retardation time per section T substantially

a constant for all frequencies from a very low value to the frequency defined by

$$\frac{3}{2\pi T}$$

cycles per second.

7. An electric retardation line comprising a ladder network having series inductive elements with mutual inductance between adjacent elements and shunt capacitive elements, an additional capacitive element in shunt to each inductive element and a conductive element in shunt to each of said first-named capacitive elements.

8. An electric retardation line comprising a ladder type network having inductive series elements each having an inductance of L_1 and a resistance of R as well as a mutual inductance between adjacent elements of M, shunt capacitive elements each having a capacity of C_2, additional capacitive elements each having a capacity of C_1 shunted across each of said inductive elements and a conductive element having a conductance of G shunted across each of said second-named capacitive elements; where

$$M=0.135L_1$$

$$C_1=0.022C_2, \text{ and}$$

$$G=\frac{RC_2}{L_1+2M}$$

9. An electric retardation line comprising inductive series elements and shunt capacitive elements, said inductive elements having a mutual inductance between adjacent elements of substantially 13.5% of the self inductance of one element, and a capacitive element in shunt to each inductive element, said second-named capacitive elements having a capacity of substantially 2.2% of the capacity of said first-named capacitive elements minus the distributed capacity of the said inductive elements.

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