

Sept. 13, 1966

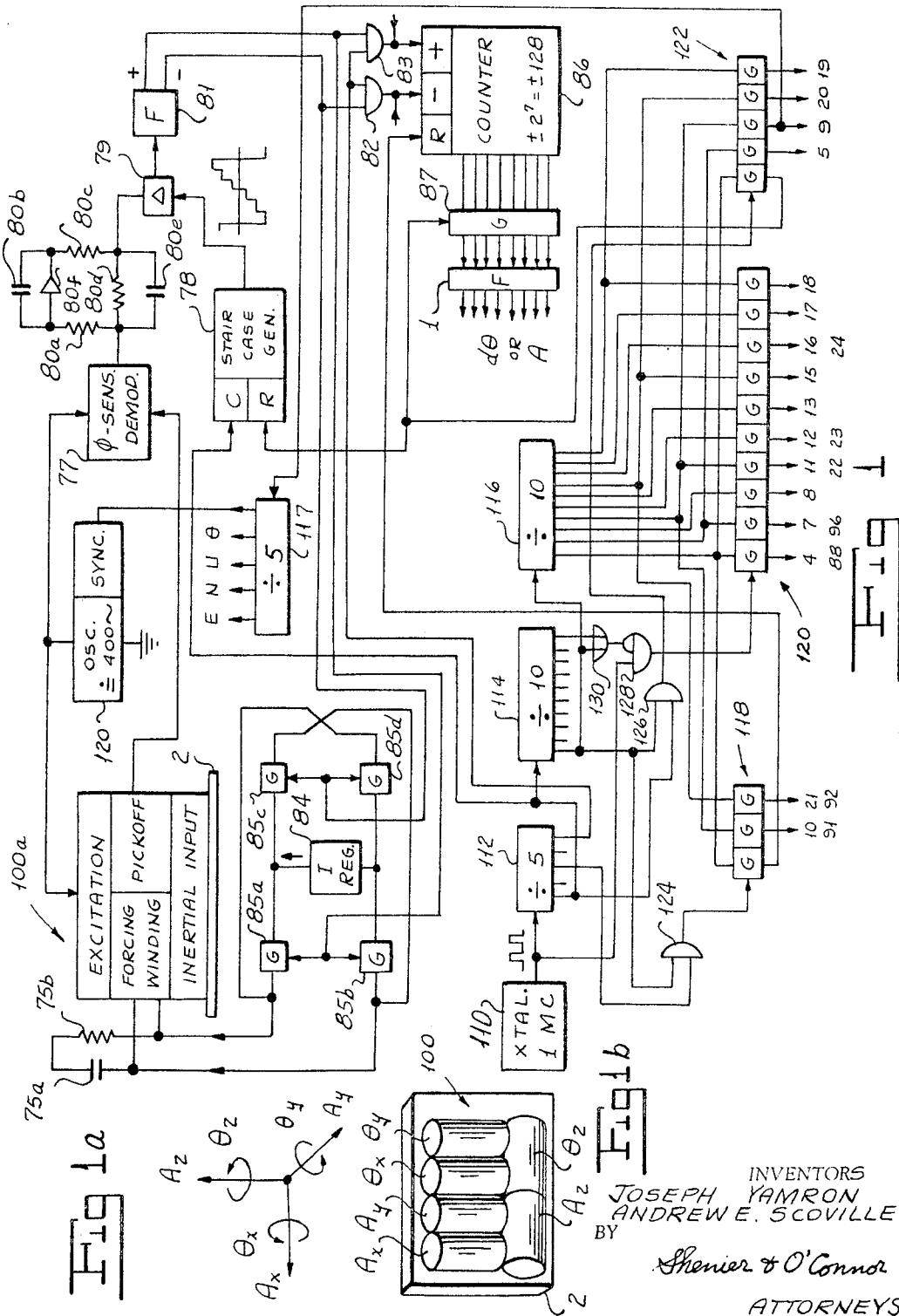
J. YAMRON ET AL

3,272,972

RANDOM ORIENTATION INERTIAL SYSTEM

Filed Jan. 15, 1962

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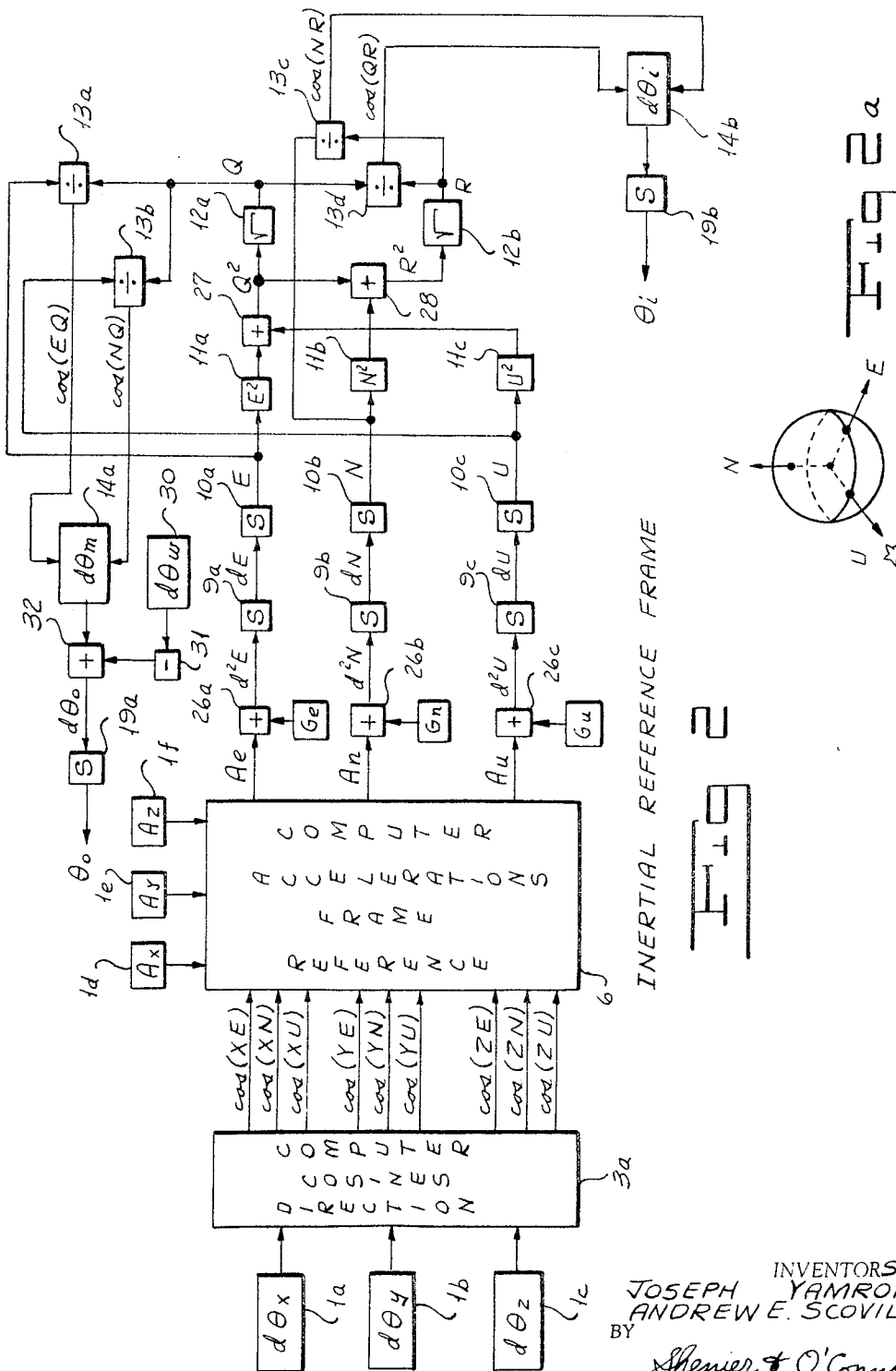
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8 Sheets-Sheet 2



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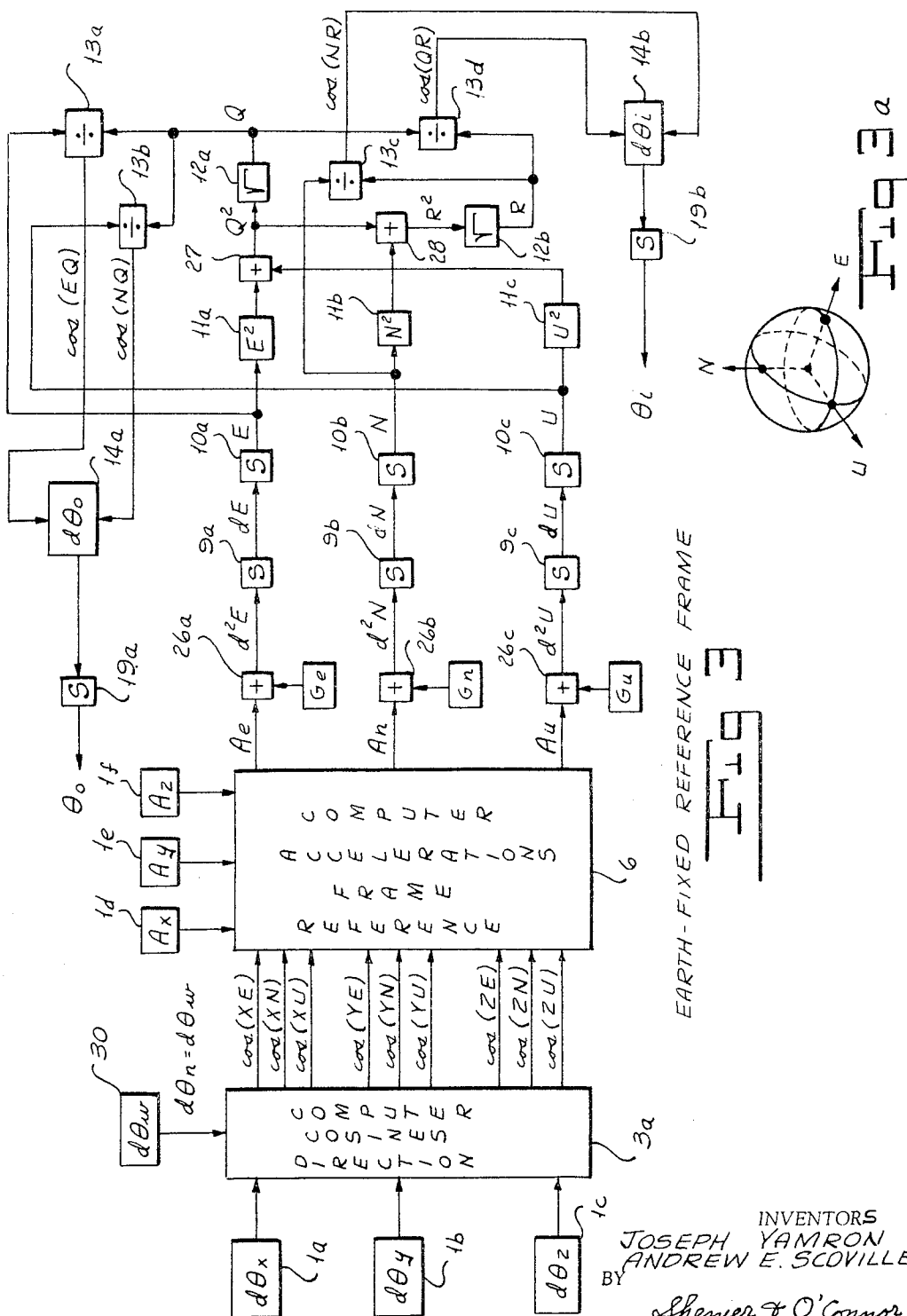
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EARTH-FIXED REFERENCE FRAME

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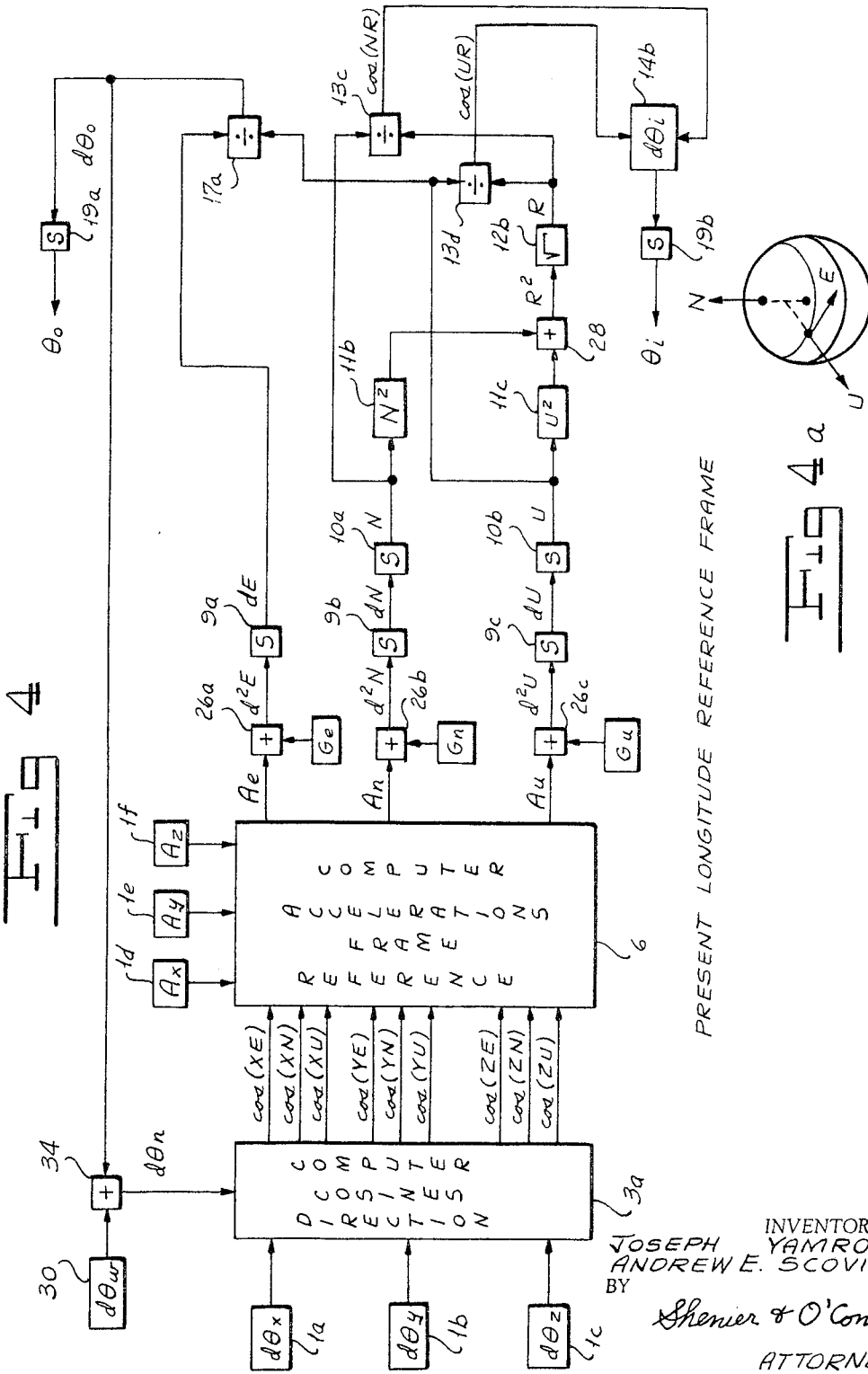
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8 Sheets-Sheet 4



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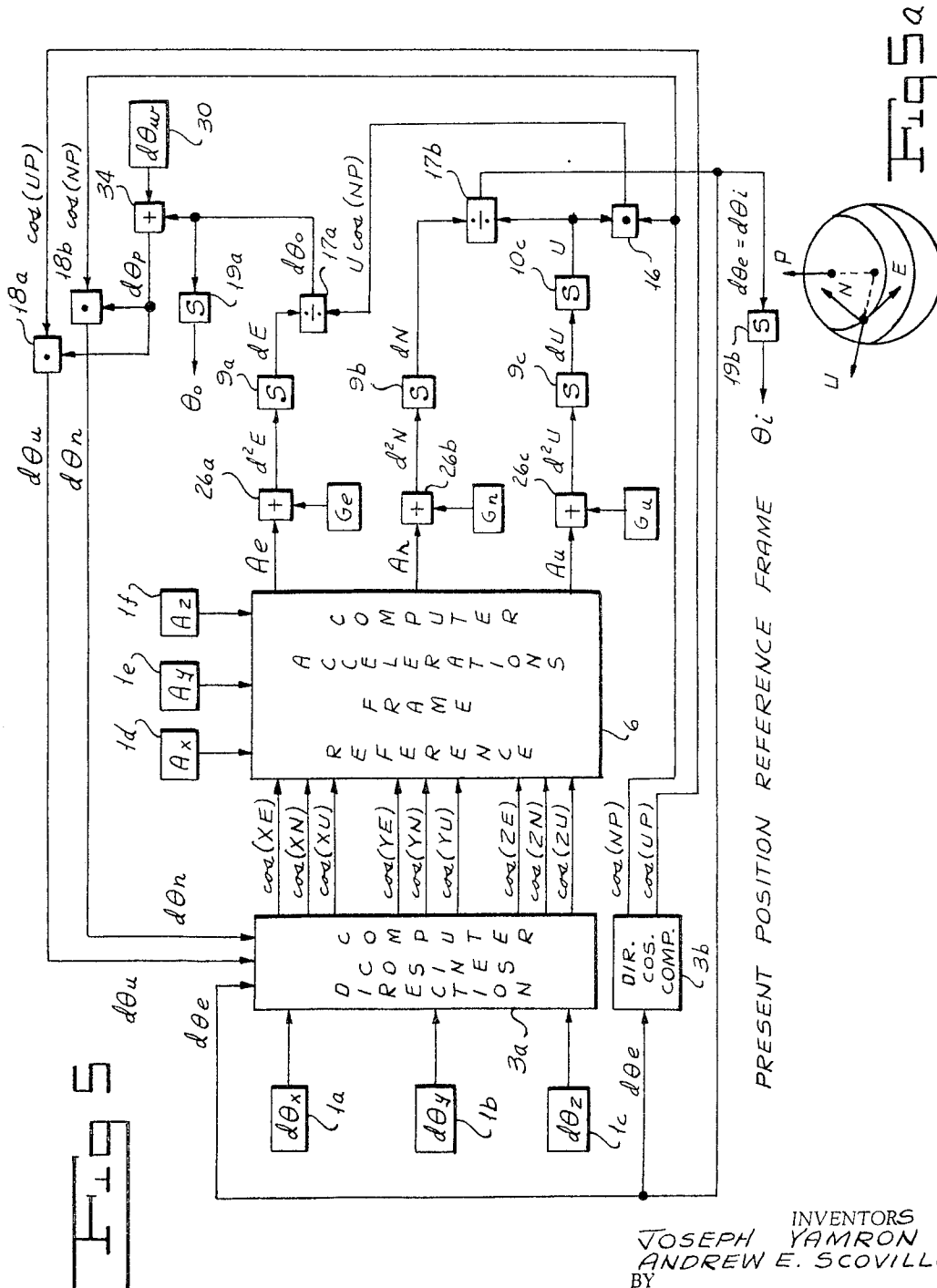
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RANDOM ORIENTATION INERTIAL SYSTEM

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8 Sheets-Sheet 5



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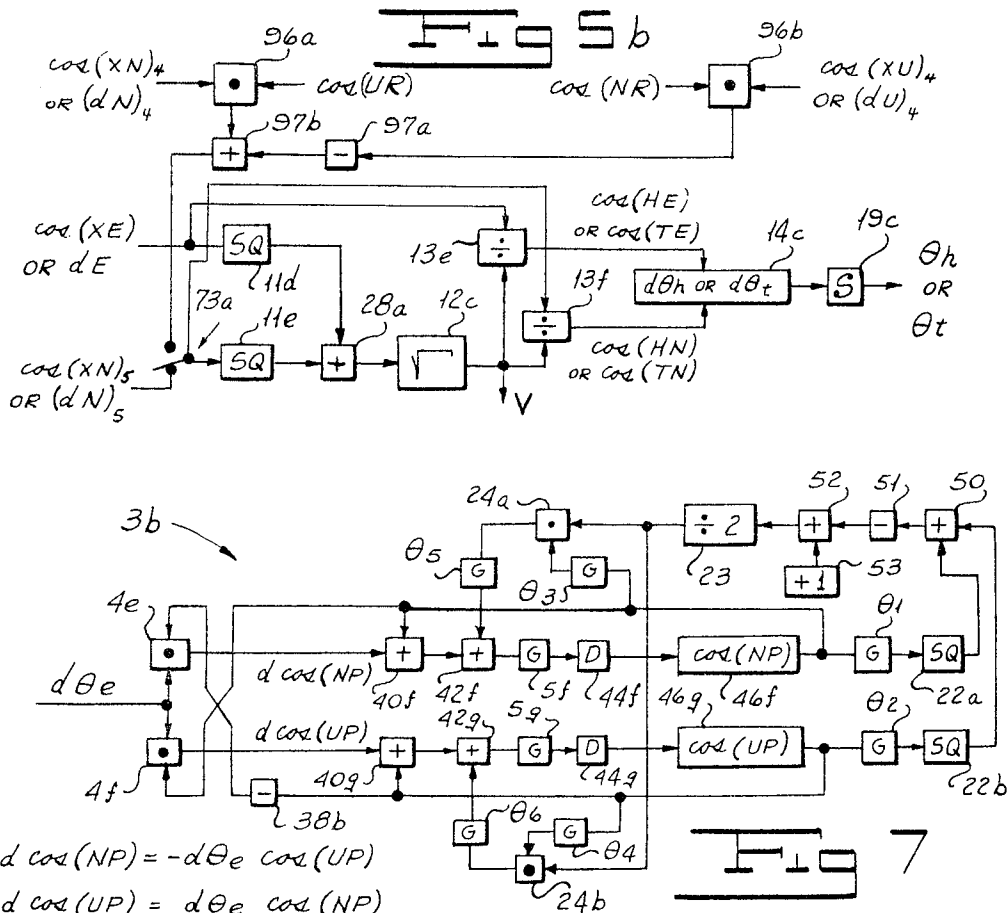
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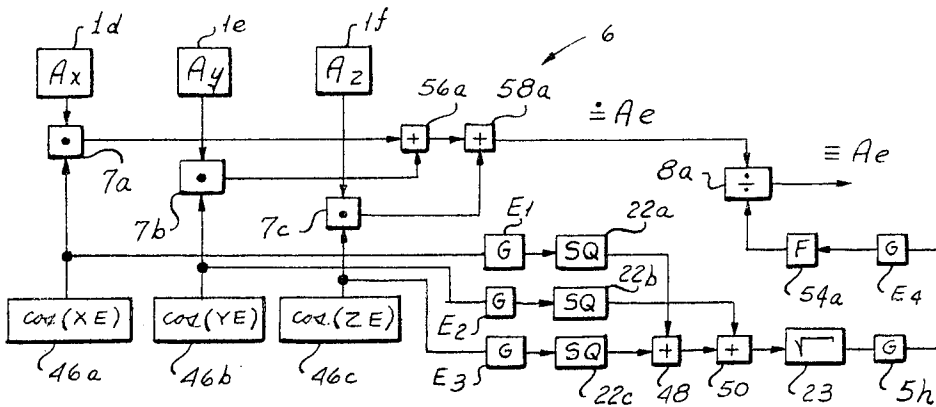
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$$d \cos(NP) = -d\theta_e \cos(UP)$$

$$d \cos(UP) = d\theta_e \cos(NP)$$



$$A_e = A_x \cos(XE) + A_y \cos(YE) + A_z \cos(ZE)$$

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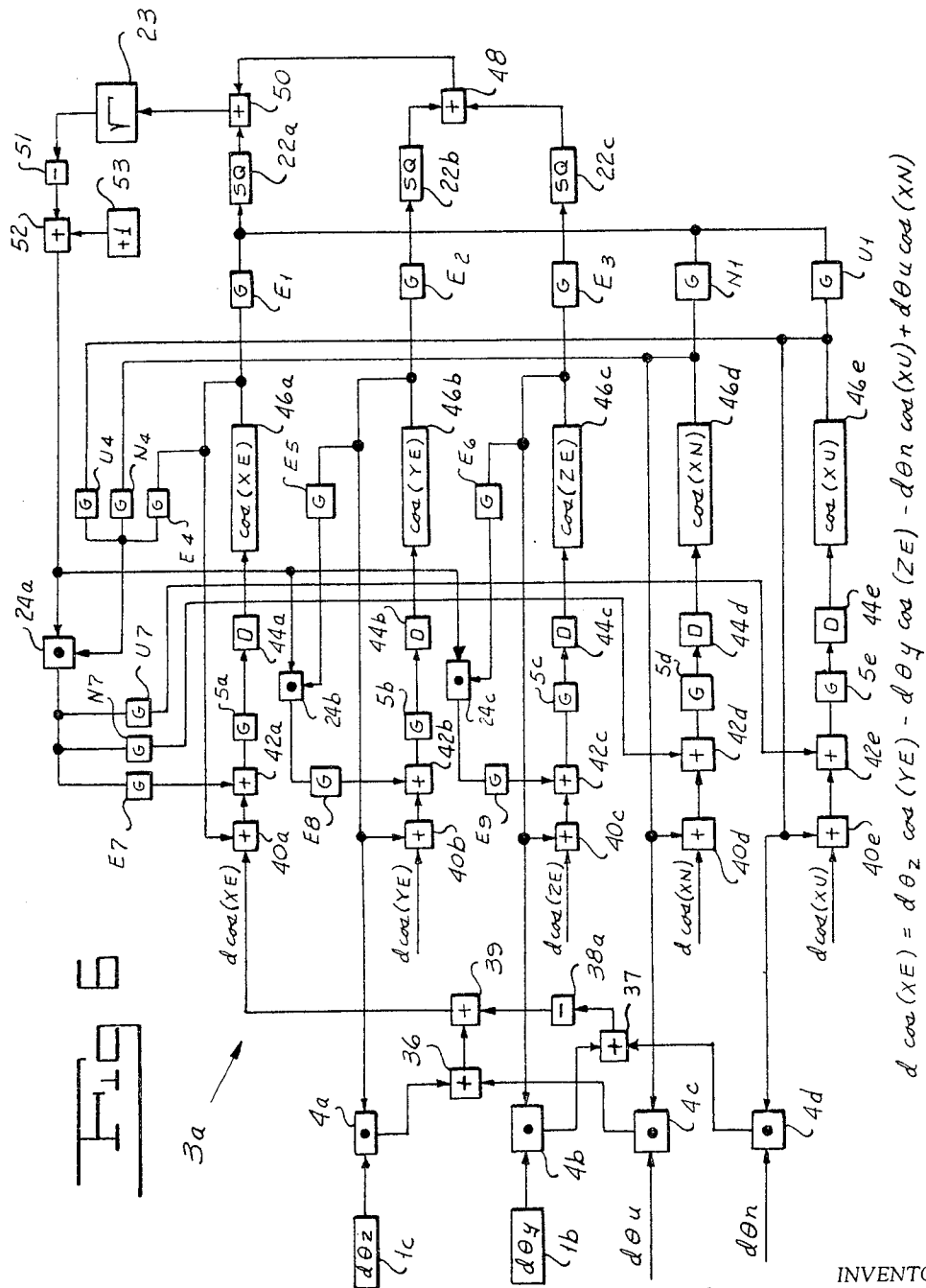
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RANDOM ORIENTATION INERTIAL SYSTEM

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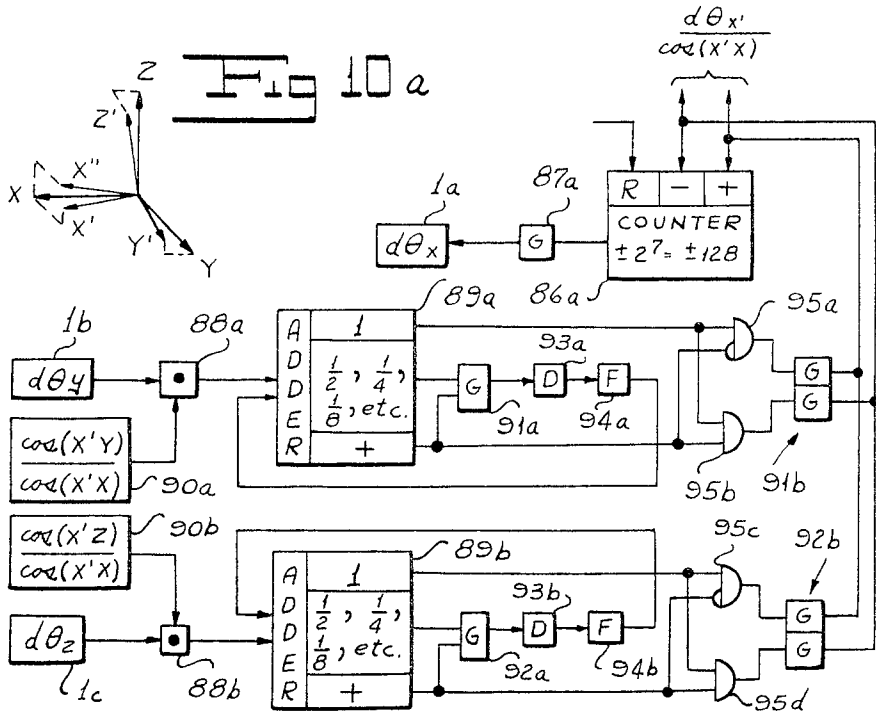
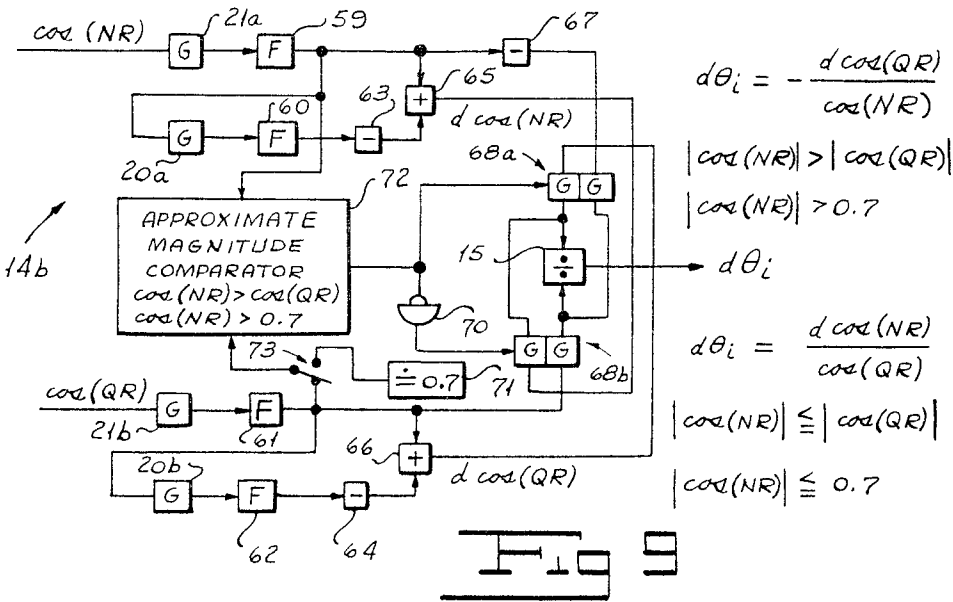
$$d\cos(XE) = d\theta_z \cos(YE) - d\theta_y \cos(ZE) - d\theta_n \cos(XU) + d\theta_u \cos(XN)$$

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$$d\theta_x = \frac{d\theta_{x'}}{\cos(x'x)} - d\theta_y \frac{\cos(x'y)}{\cos(x'x)} - d\theta_z \frac{\cos(x'z)}{\cos(x'x)}$$

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3,272,972

RANDOM ORIENTATION INERTIAL SYSTEM

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12 Claims. (Cl. 235—150.25)

Our invention relates to a random orientation inertial system, and more particularly, to an inertial system in which the orientation of accelerometers is permitted to vary relative to a given reference frame.

In the prior art accelerometers have been mechanically stabilized so that their axes are maintained fixed relative to some reference frame. This requires the use of gimbals and mechanical servomotors which increases the weight and volume of the inertial system and creates inaccuracies due to the extreme requirements of mechanical precision and rigidity.

We have invented an inertial system which need not employ gimbals and their associated mechanical servomotors. Our inertial system requires less weight and volume and is of greater mechanical simplicity than inertial systems of the prior art. In our inertial system the accelerometer axes are subjected, at least partially, to rotations of the base or vehicle. The co-pending application of Joseph Yamron, Serial No. 148,761, filed October 30, 1961, for "Stellar-Azimuth Inertial Guidance System" shows the combined application of a random orientation inertial system with a gimbaled or conventional stable platform system.

In a random orientation inertial system it is necessary to resolve the accelerometer outputs onto some reference frame. This requires the generation of the direction cosines for the transformation between the unstabilized accelerometer axes and the stabilized reference frame. In the computer art, direction cosines, or for that matter any of the trigonometric functions, have been provided by the use of extensive memory devices either alone or in combination with interpolator circuits. It will be appreciated that such memories are bulky and heavy. We have provided a direction cosine computer which is entirely arithmetic and which need not employ any trigonometric memories. In an inverse application of our direction cosines computer we may, by entirely arithmetic means, determine angles corresponding to direction cosines. Rotations of the accelerometer axes and the accelerations along such axes are digitized by a Feedback Integrating System as shown in the copending application of Charles B. Brahm, Serial No. 138,008, filed September 14, 1961, now Patent No. 3,192,371. In such feedback integrating system the forcing windings of the gyros and accelerometers are subjected to digitally controlled restoring pulses in a semi-proportional closed loop. In inertial systems of the prior art calibration and alignment is a tedious and time consuming procedure. In our inertial system the accelerometers and gyroscopes comprise a fixed unit which may be readily calibrated by arithmetic rather than mechanical procedures.

One object of our invention is to provide an inertial system having random oriented accelerometer axes which need not be mechanically stabilized.

A further object of our invention is to provide a random orientation inertial system employing single-degree-of-freedom gyroscopes which are digitally and semi-proportionally pulse torqued in a feedback integrating system.

A further object of our invention is to provide a random orientation inertial system in which accelerometer outputs are resolved onto a given reference frame by purely arithmetic direction cosine computations.

A further object of our invention is to provide a

random orientation inertial system in which latitude and longitude angles are determined by purely arithmetic computation from trigonometric functions of such angles.

A still further object of our invention is to provide a random orientation inertial system in which calibration and alignment of gyroscopes and accelerometers is accomplished by computation rather than mechanical adjustment.

Other and further objects of our invention will appear from the following description:

In general, our invention contemplates the provision of three single-degree-of-freedom gyroscopes and three accelerometers fixed to a member which is not fully stabilized and thus partakes, at least partially, of vehicle or base rotation. The accelerometers and gyros are subjected to proportional digital current pulses in a feedback integrating system. The outputs of the gyros and accelerometers are computationally corrected by factors determined during alignment procedure. The gyro outputs are coupled to a purely arithmetic computer which generates direction cosines. The direction cosines are then used to resolve the accelerometer outputs onto a given reference frame. These reference frame accelerations are then corrected for such extraneous effects as gravitational, centrifugal and Coriolis accelerations. The corrected reference frame accelerations are then double integrated to provide distances or angles relative to the reference frame. The quotients of various of these distances are determined to generate the direction cosines of latitude and longitude. By a purely arithmetic computation, the latitude and longitude angles are determined from such direction cosines.

In the accompanying drawings which form part of the instant specification and which are to be read in conjunction therewith, and in which like reference characters are used to indicate like parts in the various views:

FIGURE 1 is a schematic view of one of the inertial measuring devices with its associated proportional digital current pulsing feedback system in conjunction with timing circuitry.

FIGURES 1a and 1b are orthographic views which should be read together showing the orientation of the gyroscopes and the accelerometers and their associated axes.

FIGURE 2 is a schematic view of a system having an inertial reference frame.

FIGURE 2a is an orthographic view showing the inertial reference frame axes.

FIGURE 3 is a schematic view of a system having an earth-fixed reference frame.

FIGURE 3a is an orthographic view showing the earth-fixed reference frame axes.

FIGURE 4 is a schematic view of a system having a present longitude reference frame.

FIGURE 4a is an orthographic view of the present longitude reference frame axes.

FIGURE 5 is a schematic view of a system having a present position reference frame.

FIGURE 5a is an orthographic view of the present position reference frame axes.

FIGURE 5b is a schematic view showing the determination of heading and track in both FIGURES 4 and 5.

FIGURE 6 is a schematic view showing the generation of one of the direction cosines by purely arithmetic computation.

FIGURE 7 is a schematic view of the complete system for generating the additional direction cosines required in the present position reference frame of FIGURE 5.

FIGURE 8 is a schematic view showing the resolution of accelerometer outputs onto one of the reference frame axes.

FIGURE 9 is a schematic view showing the purely

arithmetic computation of latitude angle from the latitude direction cosines.

FIGURE 10 is a schematic view showing the correction of a gyro output by factors determined during calibration and alignment procedure.

FIGURE 10a is an orthographic view showing the relationship between the skewed instrument axes and the desired set of orthogonal axes.

Referring now more particularly to FIGURE 1b of the drawings, we have shown an inertial measurements unit comprising three single-degree-of-freedom gyroscopes and three accelerometers indicated generally by the reference numeral 100 which are secured to a member 2 which may be the frame of a vehicle.

Referring to FIGURES 1a and 1b, the sensitive axes of accelerometers A_x , A_y , and A_z of FIGURE 1b are aligned with the three corresponding orthogonal axes of FIGURE 1a. Similarly, the three input axes of single-degree-of-freedom gyroscopes θ_x , θ_y , and θ_z of FIGURE 1b are aligned with the three corresponding orthogonal axes shown in FIGURE 1a. Thus, the inertial measurements unit of FIGURE 1b is adapted to measure accelerations along and rotations about the three orthogonal axes shown in FIGURE 1a.

Referring now to FIGURE 1, we have shown one of the six inertial measuring devices indicated generally by the reference character 100a. Device 100a may be either a single-degree-of-freedom gyroscope or an accelerometer which is shown secured to the member 2 from which it derives its inertial input. Device 100a is supplied with excitation from an oscillator 120 which is tuned approximately to 400 cycles per second. The pick-off output of device 100a is coupled to a phase-sensitive demodulator 77 which receives a reference signal from oscillator 120. The output of demodulator 77 is coupled through a parallel circuit comprising a resistor 80d and a capacitor 80e to a first input of a high-gain differential amplifier 79. The output of phase-sensitive demodulator 77 is also coupled through a resistor 80a to the input of a high-gain, chopper-stabilized, direct-current amplifier 80f which is provided with a feedback capacitor 80b. The circuit combination comprising amplifier 80f, input resistor 80a, and feedback capacitor 80b form an integrator. The output of the electronic integrator is coupled through a resistor 80c to the first input of differential amplifier 79. We provide a precision, temperature-compensated, crystal-controlled one megacycle pulse generator 110 which may conveniently provide pulses of one-half microsecond duration. The output of the one megacycle pulse generator 110 is coupled to a pulse divider circuit 112 which may comprise a five-element ring counter and reduces the pulse frequency by a factor of five. Pulse divider circuit 112 is provided with five output terminals. The first output terminal of pulse divider 112 is coupled to a further pulse divider 114 which comprises a ten-element ring counter and further divides the number of pulses by a factor of ten. Pulse divider circuit 114 is provided with ten output terminals. The first output terminal of circuit 114 is coupled to subsequent pulse divider 116 which again divides the number of pulses by a factor of ten. Circuit 116 sequentially provides outputs at its ten output terminals. The first output of pulse divider 112 is also coupled to the counting input of a staircase generator 78. Staircase generator 78 provides 100 equal voltage steps each of five microseconds duration which are symmetrical about ground potential, the first fifty steps being negative and the second fifty steps being positive. The output of staircase generator 78 is coupled to the second input of differential amplifier 79. The first, second, fourth, seventh, and tenth outputs of divider circuit 116 are coupled to a plurality of simultaneously actuated gates indicated generally by the reference numeral 122. The first outputs of each of pulse dividers 112 and 114 are coupled to an AND circuit 126, the output of which

actuates gates 122. The output of the first of gates 122 is coupled to the retrace input of staircase generator 78. The output of differential amplifier 79 drives a flip flop 81. Flip flop 81 provides a negative output when the first input of differential amplifier 79 is more positive than its second input from staircase generator 78. Flip flop 81 provides a positive output when the first input of differential amplifier 79 is more negative than the second input from staircase generator 78. The positive output of flip flop 81 is coupled to one input of an AND circuit 83 and to the control input of gates 85a and 85b. The negative output of flip flop 81 is coupled to one input of an AND circuit 82 and to the control input of each of gates 85c and 85d. The fifth output of pulse divider 112 is coupled to the other input of each of AND circuits 82 and 83. The output of AND circuit 82 is coupled to the negative counting input of a binary coded counter 86 which is adapted to count to at least ± 100 and thus may count to $\pm 2^7$ which is equal to ± 128 . The output of AND circuit 83 is connected to the positive counting input of counter 86. Further signals are coupled to the positive and negative inputs of counter 86 as will be described in detail in conjunction with FIGURES 10 and 10a. The eight outputs of counter 86 are coupled through a gate 87 to flip-flop storage register 1. The eight outputs of register 1 represent either an incremental angular change $d\theta$ about or the acceleration A along one of the orthogonal axes shown in FIGURE 1a. Gate 87 is actuated by the output of the first of gates 122. We have provided a current regulator 84 which is adapted to be coupled either by positive gates 85a and 85b or by negative gates 85c and 85d to the forcing winding of device 100a. Gates 85a through 85d comprise a reversing switch for determining the polarity of current through the forcing winding of device 100a. The forcing winding of device 100a is shunted by a series circuit comprising a capacitor 75a and a resistor 75b. The resistance value of resistor 75b should be equal to the total resistance seen when looking into the forcing winding terminals. The capacitive time constant of the compensating circuit comprising components 75a and 75b should be equal to the inductive time constant of the forcing winding circuit. The first, fourth, and seventh outputs of pulse divider 116 are coupled to a plurality of simultaneously actuated gates indicated generally by the reference numeral 118. The third output of pulse divider 112 and the first output of pulse divider 114 are coupled to an AND circuit 124 the output of which actuates gates 118. The output of the first of gates 118 is coupled to a reset input of counter 86 which causes the output thereof to revert to zero. The ten outputs of pulse divider 116 are coupled to a plurality of simultaneously actuated gates indicated generally by the reference numeral 120. The first and tenth outputs of pulse divider 114 are coupled to an OR circuit 130. The output of OR circuit 130 is coupled to the inhibit input of an AND circuit 128. The output of pulse generator 110 is coupled to the other input of AND circuit 128. The output of AND circuit 128 actuates gates 120. The output of the third of gates 122 is coupled to a pulse divider circuit 117 which divides the number of input pulses by a factor of five. The fifth output of pulse divider 117 is connected to the synchronizing input of oscillator 120 thereby to maintain its output precisely at a frequency of 400 cycles per second. If device 100a is an accelerometer then its excitation input is provided only for the pick-off. If device 100a is a single-degree-of-freedom gyroscope then the excitation input is used not only for the pick-off but also for driving the gyro rotor. The outputs of components 118, 120, 122, and 117 are further employed as will appear hereinafter.

The operation of the circuit of FIGURE 1 is more fully described in the aforementioned copending application of Charles B. Brahm but will be here described briefly. With a "one" output from each of pulse dividers 112, 114, and 116, AND circuit 126 is enabled; and the

first of gates 122 produces a signal which actuates sampling gate 87 thereby setting flip-flop 1 in accordance with the output of counter 86 and which resets staircase generator 78 to its maximum negative voltage. Subsequently with a "three" output from circuit 112 and "one" outputs from each of circuits 114 and 116, AND circuit 124 is enabled; and the first of gates 118 produces a signal which resets counter 86 to zero. The negative output of staircase generator 78 causes the second input of differential amplifier 79 to become more negative than its first input thereby setting flip flop 81 negative. This negative setting of flip flop 81 enables AND circuit 82 and actuates gates 85c and 85d to cause a negative current flow through the forcing winding. The first output of pulse divider 112 causes staircase generator 78 to step at the rate of 200 kilocycles per second. The fifth output of pulse divider 112 actuates AND circuit 82 at the same rate pulsing counter 86 negatively. When the output of staircase generator 78 becomes more positive than the sum of the signals through resistors 80c and 80d and through capacitor 80e, differential amplifier 79 triggers flip flop 81 positive which disables AND circuit 82 and gates 85c and 85d and enables AND circuit 83 and gates 85a and 85b. Thus, the fifth outputs of divider 112 cause AND circuit 83 to pulse counter 86 positively. The positive setting of flip flop 81 produces a positive floor of current through the forcing winding. Pulses appear at the output of the first of gates 122 at a two kilocycle per second rate, resetting staircase generator 78 and actuating sampling gates 87. The flip flop 81 is thus set positive within a period after retrace which varies in discrete five-microsecond increments. At the end of a sampling period at which occurs a pulse from the first of gates 122, counter 86 provides an output in accordance with the difference between the number of negative and positive pulses. Correspondingly, the net current flow to the forcing winding is the difference between the time of negative current flow and the time of positive current flow, the absolute current value being maintained constant by regulator 84. The compensating circuit comprising components 75a and 75b prevents switching transients in the forcing winding from affecting the constant flow of current from regulator 84. Phase lead capacitor 80e stabilizes the feedback loop so that oscillations are limited to one increment. Electronic integrator 80f is provided to support the required signal at the first input of differential amplifier 79 for generating the necessary feedback signal while permitting the output of demodulator 77 and the pick-off to be precisely zero.

Referring now to FIGURE 2 of the drawings, we have shown a system having an inertial reference frame. The outputs of storage flip-flops 1a, 1b, and 1c, which provide the respective quantities $d\theta_x$, $d\theta_y$, and $d\theta_z$, are coupled to a direction cosine computer 3a which provides the output quantities $\cos(XE)$, $\cos(XN)$, $\cos(XU)$, $\cos(YE)$, $\cos(YN)$, $\cos(YU)$, $\cos(ZE)$, $\cos(ZN)$, and $\cos(ZU)$. The quantity $\cos(XE)$, for example, represents the cosine of the angle between the X measurements unit axis and the E reference frame axis.

It may be seen by referring to FIGURE 2a that the direction U is parallel to the equatorial plane of the earth and fixed relative to the stars. The direction E is parallel to the equatorial plane of the earth and fixed relative to the stars, being at right angles to the direction U. The direction N is parallel to the rotational axis of the earth.

The nine output direction cosines of computer 3a are coupled to a reference frame accelerations computer 6. The outputs of storage flip-flops 1d, 1e, and 1f, which respectively provide the quantities A_x , A_y , and A_z , are also coupled to accelerations computer 6. Accelerations computer 6 provides the output quantities A_e , A_n , and A_u , which represent not only craft accelerations along the reference frame axes, but also the components of gravitational acceleration along said axes. The quanti-

ties A_e , A_n , and A_u , are combined in respective adding circuits 26a, 26b, and 26c with the respective gravitational acceleration quantities G_e , G_n , and G_u , to provide the second derivatives, d^2E , d^2N , and d^2U , of distances along the reference frame axes.

For an inertial reference frame:

$$(1) \quad G_e = -kE/R^3$$

$$(2) \quad G_n = -kN/R^3$$

$$(3) \quad G_u = -kU/R^3$$

where R is the distance of the craft from the center of the earth, where k is the product of g and the square of the radius of the earth, and where g is the standard value of gravitational acceleration at the surface of the earth. The foregoing equations, of course, assume a spherical earth.

The outputs of adding circuits 26a, 26b, and 26c are coupled to respective integrators 9a, 9b, and 9c which provide the output quantities dE , dN , and dU representing velocities along the reference frame axes. The outputs of integrators 9a, 9b, and 9c are coupled to respective integrators 10a, 10b, and 10c which provide the output quantities E, N, and U representing distances along the reference frame axes. The outputs of integrators 10a, 10b, and 10c are coupled to respective squaring circuits 11a, 11b, and 11c. The outputs of squaring circuits 11a and 11c are coupled to an adder 27. The Q^2 output of adder 27 is coupled to a square root circuit 12a which provides the output quantity Q representing the distance of the projection of the position of the craft in the equatorial plane from the center of the earth. The output of adder 27 is combined with the output of squaring circuit 11b in an adder 28. The R^2 output of adder 28 is coupled to a square root circuit 12b which provides the quantity R representing the distance of the craft from the center of the earth. The N output of integrator 10b is divided by the R output of square root circuit 12b in circuit 13c to provide the quantity $\cos(NR)$. The Q output of square root circuit 12a is divided by the R output of square root circuit 12b in a circuit 13d to provide the quantity $\cos(QR)$. The E and U outputs of the integrators 10a and 10c are each divided by the Q output of square root circuit 12a in respective circuits 13a and 13b to provide the respective quantities $\cos(EQ)$ and $\cos(NQ)$. The outputs of dividing circuits 13c and 13d are coupled to a circuit 14b which arithmetically computes from the direction cosines the quantity $d\theta_1$ representing the incremental change in latitude angle. The output of circuit 14b is coupled to an integrator 19b which provides the latitude output θ_1 . The outputs of dividing circuits 13a and 13b are coupled to a circuit 14a which arithmetically computes the quantity $d\theta_m$, representing the incremental change in meridian angle for a non-rotating earth. This represents the incremental change in the local hour angle of the first point of Aries, which we shall term "sidereal longitude." The fixed quantity $d\theta_w$ which represents the rate of earth rotation is provided by circuit 30 and coupled to a circuit 31 which either changes the polarity of the sign of or generates the complement of the output of circuit 30. The output of circuit 31 is combined in an adding circuit 32 with the output of circuit 14a to provide the quantity $d\theta_o$ which represents the incremental change in earth longitude. The output of adding circuit 32 is impressed upon an integrator 19a which provides the earth longitude output θ_o .

The rate of earth rotation is approximately $15^\circ/\text{hr}$, which is approximately equal to 7.26×10^{-5} radian/sec. Since we employ 2,000 integration periods per second, the quantity $d\theta_w$ is approximately equal to 3.63×10^{-8} .

It will be appreciated that the computation of latitude θ_1 assumes a spherical earth. As will be appreciated by those ordinarily skilled in the art, we may correct the quantities G_e , G_n , and G_u by well known formulas

to compensate for the ellipsity of the earth's spheroid. As will be further appreciated by those ordinarily skilled in the art, we may compute the geodetic latitude from the geocentric latitude θ_1 by employing well known correction factors.

In operation of FIGURE 2 the inertial measurements unit provides the three $d\theta$ and the three A quantities in conjunction with the proportional pulse torquing circuit of FIGURE 1. The angular relationships between the X, Y, and Z axes of the inertial measurements unit and the inertial reference frame axes E, N, and U are provided by the direction cosines computer 3a. The accelerations computer 6 translates accelerations along the X, Y, and Z axes into accelerations along the E, N, and U axes. The apparent accelerations A_e , A_n , and A_u are then corrected by the gravitational quantities G_e , G_n , and G_u to provide the true accelerations d^2E , d^2N , and d^2U along the reference frame axes. These true accelerations are doubly integrated to provide the distances, E, N, and U along the inertial axes. Latitude and meridian direction cosines are then computed. Circuits 14a and 14b then provide the incremental latitude and meridian angles. The incremental latitude is integrated to provide latitude. The incremental meridian angle for a non-rotating earth is combined with earth rate to provide the incremental longitude angle. This is integrated to provide longitude.

Referring now to FIGURE 3, we have shown a system having an earth fixed reference frame. FIGURE 3 is similar to FIGURE 2. However, it will be noted that the rate of earth rotation $d\theta_w$ which is provided by circuit 30 is coupled to the direction cosines computer 3a and that the output of circuit 14a now inherently represents the incremental change in longitude angle $d\theta_0$ so that the output of circuit 14a may be directly coupled to integrator 19a without the necessity of subtracting therefrom the rate of earth rotation.

It may be seen by referring to FIGURE 3a that the direction U is parallel to the equatorial plane of the earth and fixed relative to some particular meridian of longitude. The direction E is parallel to the equatorial plane of the earth and fixed relative to a meridian spaced by 90° .

In an earth fixed reference frame, as in FIGURE 3, the acceleration corrections G_e , G_n , and G_u must include further terms to account for centrifugal and Coriolis accelerations. For an earth fixed reference frame:

$$(4) \quad G_e = -kE/R^3 + E(d\theta_w)^2 - d\theta_w dU$$

$$(5) \quad G_n = -kN/R^3$$

$$(6) \quad G_u = -kU/R^3 + U(d\theta_w)^2 + d\theta_w dE$$

These equations are readily calculated from the quantities available. The quantity R^3 is obtained by multiplying the R^2 output of adding circuit 28 by the R output of square root circuit 12b. The quantities E, N, and U are available from integrators 10a, 10b, and 10c. Preferably, however, G_n is computed by dividing the $\cos(NR) = N/R$ output of circuit 13c by the R^2 output of circuit 28. The quantities dU and dE are available from integrators 9b and 9c. The second term in each of Equations 4 and 6 represents centrifugal acceleration. The final term in each of Equations 4 and 6 represent Coriolis acceleration. It will be noted from Equation 5 that only gravitational acceleration need be computed for G_n .

Referring now to FIGURE 4, we have shown a system having a present longitude reference frame. In FIGURE 4 it will be noted that the direction cosines computer 3a is provided with an input $d\theta_n$ comprising the output of adding circuit 34 and hence the sum of the earth rotation rate input $d\theta_w$ from circuit 30 and the incremental change in earth longitude $d\theta_0$. This sum represents the incremental change in sidereal longitude; that is, the in-

cremental change in the local hour angle of the first point of Aries.

It will be seen from FIGURE 4a that the direction U is parallel to the equatorial plane and passes through the meridian of present craft longitude. Thus, the quantity U represents the distance of the craft from the earth's axis of rotation. The direction E is also parallel to the equatorial plane. In the present longitude reference frame we do not compute distances along the E axis, but, instead, compute only velocities dE along the E axis. The velocity dE is then employed to generate the incremental change in longitude $d\theta_0$ by dividing by U. As in FIGURES 2 and 3, the quantity N represents the distance of the craft from the equatorial plane.

For a present longitude reference frame as in FIGURE 4:

$$(7) \quad G_e = -d\theta_w dU$$

$$(8) \quad G_n = -kN/R^3$$

$$(9) \quad G_u = -kU/R^3 + U(d\theta_w)^2$$

It will be noted from Equation 7 that G_e contains only a Coriolis acceleration term. From Equation 8, G_n contains only a gravitational acceleration term. From Equation 9, G_u contains no Coriolis acceleration term.

The dE output of integrator 9a and the U output of integrator 10b are coupled to a dividing circuit 17a which computes the incremental changes in longitude $d\theta_0$. The output of dividing circuit 17a is coupled to adding circuit 34. The N^2 and U^2 outputs of circuits 11b and 11c are combined in an adding circuit 28. The R^2 output of adding circuit 28 is coupled to square root circuit 12b. It will be appreciated that in the present longitude reference frame the quantity R provided by the output of square root circuit 12b may be computed from the distances N and U without regard to the velocity dE . Again, R represents the distance of the craft from the center of the earth. In the present longitude reference frame the latitude θ_1 , as in FIGURES 2 and 3, is computed in circuit 14b from the direction cosines $\cos(NR)$ and $\cos(UR)$ which are provided by dividing circuits 13c and 13d. However, the incremental change in longitude $d\theta_0$ may be computed by dividing the velocity dE along the E axis by the quantity U which represents the distance of the craft from the earth's axis of rotation.

Referring now to FIGURE 5, we have shown a present position reference frame system. As can be seen in FIGURE 5a the quantity U represents the distance of the craft from the center of the earth. The direction N is in the plane of a meridian of longitude; and the direction E is in the plane of a parallel of latitude. In the present position reference frame we do not compute distances along either of the N or E axes, but, instead, compute only the velocities dN and dE . These velocities are then employed to generate the incremental changes in latitude and longitude by dividing by the quantity U.

For the present position reference frame:

$$(10) \quad G_e = d\theta_w(dN \cos \overline{UP} - dU \cos \overline{NP})$$

$$(11) \quad G_n = -d\theta_w d\theta_n U$$

$$(12) \quad G_u = -k/U^2 + U(\overline{d\theta_e}^2 + \overline{d\theta_n}^2)$$

From Equation 10, G_e contains only Coriolis acceleration terms. From Equation 11, G_n contains only a centrifugal acceleration term. From Equation 12, G_u contains no Coriolis acceleration term.

We provide an auxiliary direction cosines computer 3b which generates the quantities $\cos(NP)$ and $\cos(UP)$ in response to incremental angular inputs $d\theta_0$ representing angular velocity about the E axis. The outputs of the direction cosines computer 3b are in effect the sine and cosine of latitude. The dN output of integrator 9b is divided by the U output of integrator 10c in circuit 17c to provide the incremental change in latitude angle $d\theta_e = d\theta_1$. The U output of integrator 10c and the $\cos(NP)$ output of the auxiliary direction cosines computer

3b are coupled to a multiplying circuit 16 which provides the output, $U \cos(NP)$. This represents the distance of the craft from the earth's axis of rotation. The dE output of integrator 9a is divided by the output of multiplying circuit 16 in circuit 17a to provide the incremental change in longitude $d\theta_o$. The $d\theta_o$ output of dividing circuit 17a is coupled to integrator 19a which provides the longitude output θ_o . The $d\theta_o$ output is combined with the $d\theta_w$ earth-rate output of circuit 30 in an adding circuit 34 to provide the quantity $d\theta_p$ which represents total angular velocity about the earth's polar axis P shown in FIGURE 5a. The $d\theta_p$ output of adder 34 is multiplied by the $\cos(NP)$ and $\cos(UP)$ outputs of computer 3b in respective circuits 18a and 18b to provide the quantities $d\theta_u$ and $d\theta_n$. Thus the angular velocity about the earth's polar axis P is resolved into angular velocities about the N and U axes. The outputs of multiplying circuits 18a and 18b are coupled to the direction cosines computer 3a. The $d\theta_o = d\theta_1$ output of dividing circuit 17b is coupled not only to the auxiliary cosines computer 3b, but also to the cosine computer 3a. In FIGURE 5, as in FIGURE 4, we do not calculate the distance E but instead divide the velocity dE by the distance of the craft from the earth's polar axis of rotation to obtain the longitude angular velocity $d\theta_o$. In the present position reference frame of FIGURE 5, we do not compute the distance N, but instead divide the linear velocity dN by the distance of the craft from the center of the earth to obtain latitude angular velocity $d\theta_1 = d\theta_o$.

Let us refer now to FIGURE 7, which shows the auxiliary direction cosines computer 3b of FIGURE 5. The $d\theta_e$ input is coupled to multiplying circuits 4e and 4f. We provide storage registers 46f and 46g for the quantities $\cos(NP)$ and $\cos(UP)$. The output of storage register 46f is impressed on multiplier 4f. The output of register 46g is coupled to subtraction circuit 38b, which either changes the polarity of the sign of or generates the complement of the output of register 46g. The output of circuit 38b is impressed on multiplier 4e. The outputs of multipliers 4e and 4f represent the respective incremental changes in direction cosines, $d \cos(NP)$ and $d \cos(UP)$, and are coupled to respective adding circuits 40f and 40g. The outputs of registers 46f and 46g are respectively coupled to adders 40f and 40g. The outputs of adders 40f and 40g are coupled to correction adders 42f and 42g. The outputs of adders 42f and 42g are coupled through respective gates 5f and 5g and then through respective delay circuits 44f and 44g to the cosine storage registers 46f and 46g. Circuits 44f and 44g should each provide a time delay slightly greater than one microsecond. As will be pointed out in detail hereinafter, gates 5f and 5g will be actuated for a period of one microsecond and the delay provided by circuits 44f and 44g should exceed this period by a reasonable safety margin.

The circuit of FIGURE 7 mechanizes the following equations:

$$(13) \quad d \cos(UP) = d\theta_e \cos(NP)$$

$$(14) \quad d \cos(NP) = -d\theta_e \cos(UP)$$

The closed loop system thus mechanizes equations where the double integral of a quantity is equal to itself. This is a second order undamped differential equation where the forcing function is the input $d\theta_e$. As will be appreciated by those skilled in the art, the characteristic equation for such a system includes the sine and cosine functions. The integration is accomplished by adding circuits 40f and 40g which combine the outputs of registers 46f and 46g with the incremental outputs of multipliers 4e and 4f. Gates 5f and 5g in conjunction with delay circuits 44f and 44g produce a well-known shift-register action so that the cumulative summations in registers 46f and 46g are continuously updated.

In the circuit of FIGURE 7, small errors occur because the number of integration periods per second is not infinite and correspondingly $d\theta_e$ does not approach zero. Furthermore, regardless of the number of digital places

of significance, some round-off error will be present. Accordingly, the integrals in storage registers 46f and 46g will build up error for large changes in θ_e .

We have provided a correction circuit in FIGURE 7. The outputs of registers 46f and 46g are coupled through respective gates 01 and 02 to squaring circuits 22a and 22b. The outputs of these squaring circuits are combined in an adder 50. The output of adder 50 is coupled to a subtracting circuit 51 which either changes the polarity of the sign of or generates the complement of the output of adder 50. The output of circuit 51 is impressed upon one input of an adder 52. The other input of adder 52 is the fixed quantity, positive unity, provided by circuit 53. The output of adder 52 is coupled to a halver 23 which divides the output of circuit 52 by a factor of two. The output of halver 23 is coupled to each of multipliers 24a and 24b. The outputs of register 46f and 46g are coupled through respective gates 03 and 04 to multipliers 24a and 24b. The outputs of multipliers 24a and 24b are coupled through gates 05 and 06 to correction adders 42f and 42g respectively.

The correction circuit of FIGURE 7 changes the multipliers of the quantities $\cos(NP)$ and $\cos(UP)$ in accordance with their existing ratio so that the resultant of the quadrature combination of the quantities is equal to unity. If the sum of the squares of the two cosines differs from unity by a small amount, then the square root of the sum of the squares of the two cosines differs from unity by approximately half such amount. In FIGURE 7 we have taken advantage of this approximation and employed a halver 23 rather than a square root circuit. The use of a square root circuit is shown in FIGURE 6. The output of halver 23 thus represents the approximate error by which the square root of the sum of the square of the two cosines differs from unity. The error output of halver 23 is used to correct the direction cosines in accordance with their existing relative magnitudes. The correction circuit of FIGURE 7 changes the length of the resultant vector defined by the sum of the two direction cosines without changing the angle of the vector. The approximation involved in extracting the square root is not detrimental to accuracy. For an iterative sequence of corrections, the error rapidly converges on zero.

Referring now to FIGURE 6, we have shown a direction cosines computer 3a of the type required in FIGURE 5. The $d\theta_x$ and $d\theta_y$ outputs of storage registers 1c and 1b are coupled to one input of multipliers 4a and 4b respectively. We provide storage registers 46a through 46e, which contain the respective quantities $\cos(XE)$, $\cos(YE)$, $\cos(ZE)$, $\cos(XN)$, and $\cos(XU)$. The outputs of registers 46b and 46c are coupled to the other input of multipliers 4a and 4b respectively. The quantities $d\theta_n$ and $d\theta_u$ are coupled to one input of each of multipliers 4d and 4c respectively. The outputs of registers 46d and 46e are coupled to the other input of each of multipliers 4c and 4d. The outputs of multipliers 4a and 4c are combined in an adder 36; and the outputs of multipliers 4b and 4d are combined in an adder 37. The output of adder 37 is operated on by a subtraction circuit 38a, the output of which is coupled to an adder 39. The output of adder 36 is also coupled to adder 39. The output of adder 39 represents the incremental change, $d \cos(XE)$, which should be accumulated in storage register 46a.

For base-motion isolation in each of FIGURES 2 through 5, the following equations apply:

$$(15) \quad d \cos(XE) = d\theta_x \cos(YE) - d\theta_y \cos(ZE)$$

$$(16) \quad d \cos(YE) = d\theta_x \cos(ZE) - d\theta_z \cos(XE)$$

$$(17) \quad d \cos(ZE) = d\theta_y \cos(XE) - d\theta_x \cos(YE)$$

$$(18) \quad d \cos(XN) = d\theta_x \cos(YN) - d\theta_y \cos(ZN)$$

$$(19) \quad d \cos(YN) = d\theta_x \cos(ZN) - d\theta_z \cos(XN)$$

$$(20) \quad d \cos(ZN) = d\theta_y \cos(XN) - d\theta_x \cos(YN)$$

$$(21) \quad d \cos(XU) = d\theta_x \cos(YU) - d\theta_y \cos(ZU)$$

$$(22) \quad d \cos(YU) = d\theta_x \cos(ZU) - d\theta_z \cos(XU)$$

$$(23) \quad d \cos(ZU) = d\theta_y \cos(XU) - d\theta_x \cos(YU)$$

In the inertial reference frame of FIGURE 2, the reference frame remains fixed relative to the stars. However, in FIGURES 3 through 5, the orientation of the reference frame is varied relative to inertial space; and the following equations apply:

$$\begin{aligned} (24) \quad d \cos (XE) &= d\theta_u \cos (XN) - d\theta_n \cos (XU) \\ (25) \quad d \cos (XN) &= d\theta_e \cos (XU) - d\theta_u \cos (XE) \\ (26) \quad d \cos (XU) &= d\theta_n \cos (XE) - d\theta_e \cos (XN) \\ (27) \quad d \cos (YE) &= d\theta_u \cos (YN) - d\theta_n \cos (YU) \\ (28) \quad d \cos (YN) &= d\theta_e \cos (YU) - d\theta_u \cos (YE) \\ (29) \quad d \cos (YU) &= d\theta_n \cos (YE) - d\theta_e \cos (YN) \\ (30) \quad d \cos (ZE) &= d\theta_u \cos (ZN) - d\theta_n \cos (ZU) \\ (31) \quad d \cos (ZN) &= d\theta_e \cos (ZU) - d\theta_u \cos (ZE) \\ (32) \quad d \cos (ZU) &= d\theta_n \cos (ZE) - d\theta_e \cos (ZN) \end{aligned}$$

Combining equations 15 and 24, we obtain

$$(33) \quad d \cos (XE) = d\theta_z \cos (YE) - d\theta_y \cos (ZE) - d\theta_n \cos (XU) + d\theta_u \cos (XN)$$

The circuit of FIGURE 6 mechanizes Equation 33. In order to integrate the increments $d \cos (XE)$ provided by adder 39 we coupled the output of adder 39 and of storage register 46a to an adder 40a. The output of adder 40a is coupled through a correction adder 42a, then through a gate 5a and a delay circuit 44a to the input of register 46a. In a similar manner, we derive the incremental quantities $d \cos (YE)$, $d \cos (ZE)$, $d \cos (XN)$, and $d \cos (XU)$, which are coupled to respective adders 40b through 40e. The outputs of storage registers 46b through 46e are coupled to the other input of each of adders 40b through 40e respectively. The outputs of adders 40b through 40e are coupled through respective correction adders 42b through 42e, then through respective gates 5b through 5e, and finally through respective delay networks 44b through 44e to the inputs of respective storage registers 46b through 46e. As in FIGURE 7, each of the delay networks 44a through 44e should provide a time lag somewhat longer than one microsecond.

For the present position reference frame of FIGURE 5, each of the incremental corrections in the nine direction cosines of computer 3a will involve the four terms of Equation 33. For the inertial reference frame of FIGURE 2, Equations 15 through 23 apply; and Equation 33 reduces to the first two terms. In both the earth fixed reference frame of FIGURE 3 and the present longitude reference frame of FIGURE 4, only the $d\theta_n$ input is coupled to computer 3a; and Equation 33 reduces to the first three terms.

The circuit of FIGURE 6 also includes means for correcting the magnitudes of the direction cosines while keeping their relative ratios of amplitudes constant, in order to compensate for errors in digital integration due to round-off. The outputs of registers 46a through 46c are coupled through respective gates E1 through E3 to respective squaring circuits 22a through 22c. The outputs of the squaring circuits 22b and 22c are combined in an adder 48, the output of which is combined with the output of squaring circuit 22a in another adder 50. The output of adder 50 is coupled to a circuit 23 which extracts the square root of the sum of the squares of the three direction cosines relating to the E axis. Nominally the square root of the sum of the three squares should be equal to unity. In FIGURE 6 the derivation of the error is exact. The output of square root circuit 23 is coupled to a subtraction circuit 51, the output of which is impressed upon an adder 52. The output of circuit 53, which provides the quantity, positive unity, is impressed on the other input of adder 52. The output of adder 52 represents the error which is to be apportioned among the three direction cosines in accordance with their existing magnitudes. The outputs of storage registers 46a through 46c are connected through respective gates E4 through E6 to one input of respective multipliers 24a through 24c. The output of adder 52 is connected to the other input of each of multipliers 24a through 24c. The output of multipliers 24a through 24c

are connected through respective gates E7 through E9 to the other inputs of correction adders 42a through 42c. Since, in FIGURE 6, square root circuit 23 provides an exact correction output from adder 52, the three direction cosines provided in registers 46a through 46c will be restored to their proper values after one correction operation. It will be recalled that in FIGURE 7 the approximation in deriving the square root by halver 23 requires a sequence of corrections so that the error converges on zero.

In the direction cosines computer 3a the direction cosines may be alternatively combined to produce a quantity which is nominally unity. We have shown in FIGURE 6 the combination of the quantities, $\cos (XE)$, $\cos (YE)$ and $\cos (ZE)$ relating to the E reference frame axis. However it will be appreciated that the alternate combination of the quantities, $\cos (XE)$, $\cos (XN)$, and $\cos (XU)$ relating to the X measurements unit axis, should also be unity. Accordingly we may employ either combination of cosines with equal advantage. As will subsequently appear, the direction cosines are corrected three at a time, with the correction circuit being time-shared. For the alternate combination, the second correction period would operate on the Y axis quantities, $\cos (YE)$, $\cos (YN)$, and $\cos (YU)$; and the third correction period would operate on the Z axis quantities, $\cos (ZE)$, $\cos (ZN)$, and $\cos (ZU)$. In accordance with the combination of direction cosines of FIGURE 6, the second correction period operates upon the N axis quantities, $\cos (XN)$, $\cos (YN)$, and $\cos (ZN)$; and the third correction period operates on the U axis quantities, $\cos (XU)$, $\cos (YU)$, and $\cos (ZU)$. The outputs of storage registers 46d and 46e are selectively coupled through gates N1 and U1 to squaring circuit 22a and are selectively coupled through gates N4 and U4 to the first input of multiplier 24a during the second and third correction periods. The output of multiplier 24a is selectively coupled during the second and third correction periods through gates N7 and U7 to correction adders 42d and 42e respectively.

Referring now to FIGURE 8 we have shown in detail the reference frame accelerations computer 6. The outputs of storage flip-flops 1d, 1e, and 1f provide the respective quantities A_x , A_y , and A_z which are coupled to one input of each of the respective multipliers 7a, 7b, and 7c. The outputs of storage registers 46a, 46b, and 46c provide the respective quantities $\cos (XE)$, $\cos (YE)$, and $\cos (ZE)$ which are respectively coupled to the other input of each of multipliers 7a, 7b, and 7c. The outputs of multipliers 7a and 7b are combined in an adder 56a, the output of which is combined in an adder 53a with the output of multiplier 7c. If the direction cosines computer 3a of FIGURE 6 embodies either the exact correction circuit shown in FIGURE 6 (which incorporates a square root circuit) or the approximate correction circuit shown in FIGURE 7 (which incorporates a halver), then the output of adder 53a will be exactly equal to A_e . For each of the accelerations computers 6 of FIGURES 2 through 5

$$\begin{aligned} (34) \quad A_e &= A_x \cos (XE) + A_y \cos (YE) + A_z \cos (ZE) \\ (35) \quad A_n &= A_x \cos (XN) + A_y \cos (YN) + A_z \cos (ZN) \\ (36) \quad A_u &= A_x \cos (XU) + A_y \cos (YU) + A_z \cos (ZU) \end{aligned}$$

The circuit of FIGURE 8 mechanizes Equation 34. FIGURE 8 also includes a correction circuit alternative to those of FIGURES 6 and 7 in which corrections are applied not to the direction cosines but instead to the resultant resolved accelerations along the E, U, and N axes of the reference frame. The output of storage registers 46a through 46c are coupled through respective gates E1, E2, and E3 to squaring circuits 22a through 22c. The outputs of the three squaring circuits are combined by means of adders 48 and 50. The resultant output of adder 50 is coupled to square root circuit 23. The output of square root circuit 23 is coupled serially through gates 5h and E4 to a storage register flip-flop 54a. The output

of adder 58a, which in the embodiment of FIGURE 8 is now approximately equal to A_e , is coupled to the numerator of dividing circuit 8a. The output of flip-flop storage register 54a is coupled to the denominator of dividing circuit 8a. The output of dividing circuit 8a will now be exactly equal to A_e . In the correction circuit of FIGURE 8, we prefer to derive the exact square root of the output of the adder 50, since an approximation circuit involving a halver, as in FIGURE 7, would not be sufficiently accurate unless the sum of the squares of the three direction cosines were very close to unity.

Referring now to FIGURE 9 we have shown one of the circuits 14b of FIGURES 2, 3, and 4 for calculating $d\theta_1$ from the direction cosines of latitude which are computed by dividing circuits 13c and 13d from distances along the reference frame axes. In FIGURE 9 the input $\cos(NR)$ is obtained from dividing circuits 13c of FIGURES 2, 3 and 4. The input $\cos(QR)$ represents the output of dividing circuits 13d of FIGURES 2 and 3 and also represents the $\cos(UR)$ output of dividing circuit 13d of FIGURE 4. The $\cos(NR)$ input is coupled through a gate 21a to a storage flip-flop 59. The output of register 59 is coupled to one input of an approximate magnitude comparator 72 and through a gate 20a to a second flip-flop storage register 60. The output of flip-flop 60 is coupled to a subtraction circuit 63. The outputs of flip-flop 59 and subtraction circuit 63 are combined in an adder 65 which generates the quantity $d \cos(NR)$. The $\cos(QR)$ input is coupled through a gate 21b to a flip-flop 61. The output of flip-flop 61 is coupled through a gate 20b to a second flip-flop 62. The output of flip-flop 62 is coupled to a subtraction circuit 64. The output of flip-flop 61 and subtraction circuit 64 are combined in an adder 66 which generates the quantity, $d \cos(QR)$. The output of flip-flop 59 is coupled to a subtraction circuit 67. The output of adder 66 is coupled through the first of a pair of simultaneously actuated gates, indicated generally by the reference character 68a, to the numerator of a dividing circuit 15. The output of subtraction circuit 67 is coupled through the second of gates 68a to the denominator of dividing circuit 15. The output of adder 65 is coupled through the first of a pair of gates 68b to the numerator of dividing circuit 15. The output of flip-flop register 61 is coupled through the second of gates 68b to the denominator of dividing circuit 15. We provide a switch 73, which, in the position shown, couples the output of flip-flop 61 to the second input of comparator 72. Alternatively we may actuate switch 73 so that the second input of comparator 72 is provided by circuit 71, which generates a fixed quantity approximately equal to the sine or cosine of 45° and hence approximately equal to a 0.7 value. Comparator 72 provides an output when the magnitude of its first input from flip-flop 59 exceeds that of its second input from the armature of switch 73. The output of comparator 72 is coupled to simultaneous gates 68a. The output of comparator 72 is also coupled to an inhibiting circuit 70, the output of which is coupled to gates 68b.

The circuit of FIGURE 9 mechanizes the following equations:

$$(37) \quad d\theta_1 = -\frac{d \cos(QR)}{\cos(NR)}$$

if $|\cos(NR)| > |\cos(QR)|$
or if $|\cos(NR)| > 0.7$

and

$$(38) \quad d\theta_1 = \frac{d \cos(NR)}{\cos(QR)}$$

if $|\cos(NR)| \leq |\cos(QR)|$
or if $|\cos(NR)| \leq 0.7$

It will be seen that there are two equations available for solving $d\theta_1$. Comparator circuit 72 causes that equa-

tion to be selected which results in the larger value of the denominator. This prevents the ambiguity which would result were the value of the denominator to approach zero and the value of the fraction to become undefined. No sign bit inputs are required for comparator 72. Furthermore comparator 72 requires very few digits for satisfactory switching between Equations 37 and 38. Of course, if the number of digital places coupled to comparator 72 is increased then switching will occur closer to the ideal value $\theta_1 = \pm 45^\circ$.

As will be subsequently shown, gates 20a and 20b are first actuated, thereby storing in flip flops 60 and 62 the quantities stored in flip flops 59 and 61 during the previous operational cycle. Then gates 21a and 21b are actuated to update the information in flip flops 59 and 61. Since the previous values of $\cos(NR)$ and $\cos(QR)$ are stored in flip flops 60 and 62, while the present values of these quantities are stored in flip flops 59 and 61, respectively, we may obtain the incremental changes in these quantities by subtracting the previous values from the present values. This is accomplished by subtraction circuits 63 and 64 in conjunction with adders 65 and 66.

Referring now to FIGURE 10 we have shown a circuit for digitally correcting the inertial measurements units of FIGURE 1 by factors obtained during calibration and alignment.

In FIGURE 10a we have shown the desired orthogonal set of X, Y, and Z inertial measurements unit axes. Suppose that the input axes of the gyroscopes are slightly in error and correspond to the skewed axes X', Y', and Z'.

For such skewed gyroscope axes,

$$(39) \quad d\theta_{x'} = d\theta_x \cos(X'X) + d\theta_y \cos(X'Y) + d\theta_z \cos(X'Z)$$

Solving Equation 39 for $d\theta_x$ we obtain

$$(40) \quad d\theta_x = \frac{d\theta_{x'}}{\cos(X'X)} - d\theta_y \frac{\cos(X'Y)}{\cos(X'X)} - d\theta_z \frac{\cos(X'Z)}{\cos(X'X)}$$

By cyclic permutation we obtain

$$(41) \quad d\theta_y = \frac{d\theta_{y'}}{\cos(Y'Y)} - d\theta_x \frac{\cos(Y'X)}{\cos(Y'Y)} - d\theta_z \frac{\cos(Y'Z)}{\cos(Y'Y)}$$

$$(42) \quad d\theta_z = \frac{d\theta_{z'}}{\cos(Z'Z)} - d\theta_x \frac{\cos(Z'X)}{\cos(Z'Z)} - d\theta_y \frac{\cos(Z'Y)}{\cos(Z'Z)}$$

In FIGURE 10a the axis X'' is one of the skewed accelerometer axes. For the set of skewed accelerometer axes, X'', Y'', and Z'',

$$(43) \quad A_x = \frac{A_{x''}}{\cos(X''X)} - A_y \frac{\cos(X''Y)}{\cos(X''X)} - A_z \frac{\cos(X''Z)}{\cos(X''X)}$$

$$(44) \quad A_y = \frac{A_{y''}}{\cos(Y''Y)} - A_x \frac{\cos(Y''X)}{\cos(Y''Y)} - A_z \frac{\cos(Y''Z)}{\cos(Y''Y)}$$

$$(45) \quad A_z = \frac{A_{z''}}{\cos(Z''Z)} - A_x \frac{\cos(Z''X)}{\cos(Z''Z)} - A_y \frac{\cos(Z''Y)}{\cos(Z''Z)}$$

The first terms in each of Equations 40 through 45 relate to the basic sensitivity of the instrument, which is conveniently adjusted by varying the amount of current provided by regulator 84 of FIGURE 1. We calibrate the gyros of the inertial measurements unit, at the known latitude of the calibration site, by selectively aligning the X, Y, and Z axes with the earth's polar axis. We rotate the measurements unit a large number of revolutions about the earth's polar axis in a short period of time to minimize gyro drift rate. The outputs of counter 86 are

integrated, as shown on the copending application of Charles B. Brahm, so that the total rotation is digitally indicated. The digitally indicated rotation is compared with the sum of the total mechanical rotation and the calculated amount of earth's rotation which has occurred during the testing interval. The current regulator 84 is adjusted to bring indicated rotation into correspondence with actual rotation for each gyroscope. Thus, the gains of the gyros are adjusted to satisfy the first term of Equations 40 through 42. The accelerometers are calibrated by selectively aligning the X, Y, and Z axes with the indicated or apparent gravity vector. The output indications of counter 86 are integrated as shown in the aforementioned copending application to provide velocity. Since the precise gravity value at the site of calibration is known, the indicated velocity may be compared with that calculated by multiplying the known value of gravity by the time interval of testing. The current supplied by regulator 84 may be adjusted to bring indicated velocity into correspondence with calculated velocity and thereby satisfy the first terms of Equations 43 through 45. With the gyros and accelerometers now calibrated for inputs associated with their respective axes, the cross-coupling terms may be obtained. For example, if the measurements unit is rotated about its X axis, then the skewed θ_y and θ_z gyros will sense a small component of such motion. By dividing the rotation indicated by the skewed θ_y and θ_z gyros by the total rotation about the X axis, we obtain the cross-coupling coefficients of the third term of Equation 41 and the second term of Equation 42. Thus, in Equation 40, the second term represents cross-coupling into the X axis due to rotation about the Y axis. The third term of Equation 40 represents the cross-coupling into the X axis due to rotation about the Z axis. Cross-coupling coefficients are similarly obtained for the accelerometers in Equations 43 through 45.

FIGURE 10 mechanized Equation 40. Counter 86a, which is of the same type as counter 86 of FIGURE 1, provides outputs which are coupled through a sampling gate 87a to a flip flop register 1a, which stores the quantity $d\theta_x$. The reset input of counter 86a is derived from the output of the first of gates 118 of FIGURE 1. The plus and minus inputs of counter 86a are derived from a pair of AND circuits as shown in FIGURE 1. Because of the adjustment of the feedback current to the forcing winding, the inputs to counter 86a from these AND circuits represents $d\theta_x/\cos(X'X)$. Storage registers 1b and 1c, providing the quantities $d\theta_y$ and $d\theta_z$, are coupled to respective multipliers 88a and 88b. The coefficient $\cos(X'Y)/\cos(X'X)$ obtained during alignment is provided by circuit 90a and is coupled to multiplier 88a. The coefficient $\cos(X'Z)/\cos(X'X)$ likewise obtained during alignment is provided by circuit 90b and coupled to multiplier 88b. The outputs of multipliers 88a and 88b are coupled to respective adders 89a and 89b. The "1" output of adder 89a is coupled to one input of each of AND circuits 95a and 95b; and the "1" output of adder 89b is connected to one input of each of AND circuits 95c and 95d. The sign and fractional bit outputs of adder 89a are coupled through a gate 91a and a delay network 93a to a storage flip flop 94a. The sign and fractional bit outputs of adder 89b are coupled through a gate 92a and a delay network 93b to a storage flip flop 94b. The output of each of flip flops 94a and 94b is coupled to the other input of respective adders 89a and 89b. The sign output of adder 89a is connected to the other input of AND circuit 95b and to an inhibiting input of AND circuit 95a. The sign output of adder 89b is coupled to the other input of AND circuit 95d and to an inhibiting input of AND circuit 95c. The outputs of AND circuits 95a and 95b are coupled through a pair of simultaneous gates 91b to the plus and minus inputs, respectively, of counter 86a. The outputs of AND circuits 95c and 95d are coupled through simultaneous gates

92b to the plus and minus inputs, respectively, of counter 86a.

The error in the gyro and accelerometer axes is readily held to five milliradians, which corresponds to 0.3° and represents an error of approximately one part in two hundred. Since the maximum output of counter 86 or 86a is one hundred, counter corrections by the circuit of FIGURE 10 will not occur more frequently than once during two integration periods. The outputs of multipliers 88a and 88b represent a fraction of a whole count of counter 86a. These fractions are cumulatively summed by the adders in conjunction with the gates and storage flip flops. Suppose that the magnitude of the output of flip flop 94a is only slightly less than unity and that the increment from multiplier 88a causes the output of adder 89a to exceed unity. The "1" output of adder 89a enables flip flops 95a and 95b. The sign output of adder 89a causes one of circuits 95a and 95b to produce an output in accordance with the polarity of the accumulated error. Upon the actuation of gates 91b, an additional pulse is supplied to counter 86a. Gate 91a is actuated simultaneously with gates 91b, thus storing in flip flop 94a the excess by which the magnitude of the output of adder 89a exceeds unity. As will be pointed out hereinafter, gates 92b are actuated subsequently to gates 91b. Furthermore, the normal counting pulses from AND circuits 82 and 83 of FIGURE 1 are not coincident with the actuation of either of gates 91b or 92b. Thus correction pulses may be applied to counter 86a without interference from the normal counting pulses or from other correction pulses. It will be appreciated that during one integration period, counter 86a may receive a correction pulse not only from gates 91b but also from gates 92b.

Referring now to FIGURE 5b, we have shown a circuit for determining heading H or track T in the reference frames of FIGURES 4 and 5. We have assumed that the X measurements axis defines the heading of the craft. For the present position reference frame of FIGURE 5, switch 73a should be in the position shown, where it couples either $\cos(XN)_5$ or $(dN)_5$ to a squaring circuit 11e. Either $\cos(XE)$ or dE is coupled to a squaring circuit 11d. The output of squaring circuits 11d and 11e are combined in an adder 28a, the output of which is coupled to a square root circuit 12c. The output of square root circuit 12c is coupled to the denominator of each of dividing circuits 13e and 13f. The input of squaring circuit 11d is coupled to the numerator of dividing circuit 13e; and the input of squaring circuit 11e is coupled to the numerator of dividing circuit 13f. The output of dividing circuit 13e represents either $\cos(HE)$ or $\cos(TE)$; that is, the direction cosine of either heading or track relative to the E reference frame axis. The output of dividing circuit 13f represents either $\cos(HN)$ or $\cos(TN)$; that is, the direction cosine of either heading or track relative to the N present position reference frame axis of FIGURE 5. The outputs of circuits 13e and 13f are coupled to a circuit 14c, similar to the incremental latitude circuit 14b of FIGURE 9, which computes the incremental change in heading $d\theta_h$ or the incremental change in track $d\theta_t$. The output of circuit 14c is coupled to integrator 19c, the output of which represents the heading θ_h or the track θ_t .

The determination of heading and track requires measurements in the plane of the local horizon. For the present position reference frame of FIGURE 5, the plane of the local horizon contains the N_5 and E_5 axes. However, for the present longitude reference frame of FIGURE 4, only the E_4 axis is in the plane of the local horizon. Accordingly it is necessary to transform the components along the N_4 and U_4 axes of FIGURE 4 into components along the N_5 axis of FIGURE 5.

For this transformation the following equations apply:

$$(46) \quad \cos(XN)_5 = \cos(XN)_4 \cos(UR) - \cos(XU)_4 \cos(NR)$$

$$(47) \quad (dN)_5 = (dN)_4 \cos(UR) - (dU)_4 \cos(NR)$$

Equations 46 and 47 are mechanized by multipliers 96a and 96b in conjunction with subtraction circuit 97a and adder 97b. The output of adder 97b represents either $\cos(XN)_5$ or $(dN)_5$. Heading and track calculations in the present position reference frame of FIGURE 5 are accomplished with switch 73a in the position shown. These calculations for the present longitude reference frame of FIGURE 4 are accomplished by actuating switch 73a, so that the output of adder 97b is coupled to squaring circuit 11e. For track computation in response to the velocity components dE and $(dN)_5$, the output of square root circuit 12c represents the resultant velocity V of the craft in the plane of the local horizon.

Referring again to FIGURE 1, for each of the ten outputs of pulse divider 116, fifty pulses appear from pulse generator 110. Since the first and tenth outputs of pulse divider 114 are coupled to the inhibiting input of AND circuit 128, gates 120 are actuated by AND circuit 128 only forty times for each output from pulse divider 116. The outputs of gates 120 supply the clock pulses for the various multipliers, dividers, halvers, squaring, and square root circuits. We have found that between twenty-five and thirty-five binary bits are required for adequate computational accuracy. Thus the provision of forty clock pulses for the output of each of gates 120 contemplates the shifting of as many as forty binary bits through the various computational circuits to insure high accuracy.

During each computational cycle there are 500 pulses from generator 110. As has been previously described, pulse 1 from the first of gates 122 actuates sampling gates 87 (and 87a of FIGURE 10) and resets staircase generator 78; and pulse 3 from the first of gates 118 resets counter 86 (and 86a of FIGURE 10). Pulses 6 through 46 operate all multipliers 4 and 88, and more particularly, 4a through 4d of FIGURE 6, 4e and 4f of FIGURE 7, and 88a and 88b of FIGURE 10. Pulse 51 from the second of gates 122 actuates all gates 5, and more particularly, 5a through 5e of FIGURE 6, 5f and 5g of FIGURE 7, and 5h of FIGURE 8. Pulses 56 through 96 from the second of gates 120 operate all multipliers 7 and 96, and more particularly, 7a through 7c of FIGURE 8 and 96a and 96b of FIGURE 5b. Pulses 106 through 146 operate all dividing circuits 8, and more particularly, 8a of FIGURE 8. Pulse 151 from the third of gates 122 actuates all integrators 9, and more particularly, 9a through 9c of FIGURES 2 through 5. Pulse 151 further indexes ring counter 117. Pulse 153 from the second of gates 118 actuates all integrators 10, and more particularly, 10a through 10c of FIGURES 2 through 5. Pulse 153 further actuates all gates 91, as 91a and 91b of FIGURE 10. Pulses 156 through 196 from the fourth of gates 120 operate all squaring circuits 11 and 22, and more particularly, 11a through 11c of FIGURES 2 through 4, 11d and 11e of FIGURE 5b, and 22a through 22c of FIGURES 6 through 8. Pulses 206 through 246 from the fifth of gates 120 operate all square root circuits 12 and 23, and more particularly, 12a and 12b of FIGURES 2 through 4, 12c of FIGURE 5b, and 23 of FIGURES 6 and 8. Pulses 206 through 246 further operate halver 23 of FIGURE 7 which functions as an approximate square root circuit. Pulses 256 through 296 from the sixth of gates 120 operate all dividing circuits 13, and more particularly, 13a through 13d of FIGURES 2 through 4 and 13e and 13f of FIGURE 5b. Pulse 301 from the fourth of gates 122 actuates all gates 20, and more particularly, 20a and 20b of FIGURE 9. Pulse 303 from the third of gates 118 actuates all gates 21 and 92, and more particularly, 21a and 21b of FIGURE 9 and 92a and 92b of FIGURE 10. Pulses 306 through 346 from the seventh

of gates 120 operate all dividing circuits 15, and more particularly, dividing circuit 15 of FIGURE 9. Pulses 356 through 396 from the eighth of gates 120 operate all multipliers 16 and 24, and more particularly, multiplier 16 of FIGURE 5 and 24a through 24c of FIGURES 6 and 7. Pulses 406 through 446 from the ninth of gates 120 operate all dividing circuits 17, and more particularly, 17a and 17b of FIGURES 4 and 5. Pulse 451 from the fifth of gates 122 actuates all integrators 19, and more particularly, 19a and 19b of FIGURES 2 through 5. Pulses 456 through 496 operate all multipliers 18, and more particularly, 18a and 18b of FIGURE 5.

The correction circuits associated with FIGURES 6, 7 and 8 are time-shared as has been previously described. During the first computational cycle, the direction cosines associated with the E axis are corrected as in FIGURE 6 or alternatively the A_e reference frame acceleration is corrected as in FIGURE 8. The second operational cycle corrects either the N direction cosines or A_n ; and the third operational cycle corrects the U direction cosines or A_u . The fourth operational cycle corrects the auxiliary direction cosines of latitude θ_1 as in FIGURE 7. The time-sharing sequence is controlled by the E, N, U, and θ outputs of pulse divider 117. It is necessary that ring-counter 117 be indexed subsequent to the actuation of gates 5 by pulse 51 from the second of gates 122 and prior to the operation of squaring circuits 22 by pulses 156 through 196 from the fourth of gates 122. Accordingly, we may employ pulse 151 from the third of gates 122 to index ring counter 117. The E output from ring counter 117 actuates all E gates, and more particularly, E1 through E9 of FIGURE 6, or alternatively, E1 through E4 of FIGURE 8. The N output from ring counter 117 actuates all N gates, and more particularly, N1, N4, and N7 of FIGURE 6. The U output from ring counter 117 actuates all U gates, and more particularly, U1, U4, and U7 of FIGURE 6. The θ output from ring counter 117 actuates all θ gates, and more particularly, $\theta 1$ through $\theta 6$ of FIGURE 7.

It will be seen that we have accomplished the objects of our invention. In our random orientation inertial system, the measurements unit need not be mechanically stabilized. We employ single-degree-of-freedom gyroscopes which are semi-proportionally pulse-torqued by the circuit of FIGURE 1. The purely arithmetic incremental computers of FIGURES 6 and 7 generate the direction cosines defining the orientation of the measurements unit relative to a given reference frame. The correction circuits of FIGURES 6, 7 and 8 prevent the accumulation of errors in the computation of the reference frame accelerations. We compute latitude and longitude without employing inverse trigonometric memories by the incremental computer of FIGURE 9. We further compute heading and track by the incremental circuit of FIGURE 5b. In our inertial system, the measurements unit is computationally calibrated by the circuit of FIGURE 10. Our system may have either an inertial reference frame as in FIGURE 2, an earth-fixed reference frame as in FIGURE 3, a present longitude reference frame as in FIGURE 4, or a present position reference frame as in FIGURE 5.

It will be understood that certain features and subcombinations are of utility and may be employed without reference to other features and subcombinations. This is contemplated by and is within the scope of our claim. It is further obvious that various changes may be made in details within the scope of our claims without departing from the spirit of our invention. It is therefore to be understood that our invention is not to be limited to the specific details shown and described.

Having thus described our invention, what we claim is:

1. A random orientation inertial system including in combination an inertial measurements unit comprising a plurality of gyroscopes and a plurality of accelerometers,

means responsive to the gyroscopes for computing direction cosines defining the orientation of the measurements unit relative to an earth-fixed reference frame, means responsive to the direction cosines and to the accelerometers for determining accelerations in the reference frame, means providing the rate of rotation of the earth, and means coupling the earth rate means to said direction cosines computing means.

2. A random orientation inertial system including in combination an inertial measurements unit comprising a plurality of gyroscopes and a plurality of accelerometers, means responsive to the gyroscopes for determining direction cosines defining the orientation of the measurements unit relative to an inertial reference frame, means responsive to the direction cosines and to the accelerometers for computing accelerations in the reference frame means responsive to the computing means for determining the rate of change of sidereal longitude, means providing the earth's rate of rotation, and algebraic combining means responsive to the sidereal longitude rate means and to the earth rate means for providing the rate of change of earth longitude.

3. A random orientation inertial system including in combination an inertial measurements unit comprising a plurality of gyroscopes and a plurality of accelerometers, means responsive to the gyroscopes for determining direction cosines defining the orientation of the measurements unit relative to a present earth longitude reference frame having an axis which intersects the earth's polar axis, means responsive to the direction cosines and to the accelerometers for computing accelerations in the reference frame, means responsive to the computing means for determining the rate of change of earth longitude, means for providing the rate of rotation of the earth about its polar axis, algebraic combining means responsive to the earth longitude rate means and to the earth rotation rate means for providing the rate of change of sidereal longitude, and means coupling the sidereal longitude rate means to the direction cosines determining means.

4. A random orientation inertial system including in combination an inertial measurements unit comprising a plurality of gyroscopes and a plurality of accelerometers, means responsive to the gyroscopes for determining direction cosines defining the orientation of the measurements unit relative to a present position reference frame having an axis which intersects the center of the earth, means responsive to the direction cosines and to the accelerometers for computing accelerations in the reference frame, means responsive to the computing means for determining the rate of change of latitude, means responsive to the computing means for determining the rate of change of earth longitude, means coupling the latitude rate means to the direction cosines computing means, means providing the rate of rotation of the earth, algebraic combining means responsive to the earth longitude rate means and to the earth rotation rate means for providing the rate of change of sidereal longitude, and means coupling the sidereal longitude rate means to the direction cosines determining means.

5. A random orientation inertial system including in combination means for sensing the incremental angular rotations, $d\theta_y$ and $d\theta_z$, about the Y and Z axes of a first set of orthogonal axes, X and Y and Z; means for computing the incremental angular rotation, $d\theta_n$, about the N axis of a second set of orthogonal axes, E and N and U; means for generating the cosines, $\cos(YE)$ and $\cos(ZE)$ and $\cos(XU)$, of the angles between the Y and E and the Z and E and the X and U axes; means responsive to the sensing and the computing and the generating means for solving an equation of the form

$$d \cos \overline{XE} = \pm (d\theta_z \cos \overline{YE} - d\theta_y \cos \overline{ZE}) \pm d\theta_n \cos \overline{XU}$$

to provide an incremental output in accord with the

change, $d \cos(XE)$, in the cosine of the angle between the X and E axes; and means for integrating the incremental output.

6. A random orientation inertial system including in combination means for sending the incremental angular rotations, $d\theta_y$ and $d\theta_z$, about the Y and Z axes of a first set of orthogonal axes, X and Y and Z; means for computing the incremental angular rotations, $d\theta_n$ and $d\theta_u$, about the N and U axes of a second set of orthogonal axes, E and N and U; means for generating the cosines, $\cos(YE)$ and $\cos(ZE)$ and $\cos(XU)$ and $\cos(XN)$, of the angles between the Y and E and the Z and E and the X and U and the X and N axes; means responsive to the sensing and the computing and the generating means for solving an equation of the form

$$d \cos \overline{XE} = \pm (d\theta_z \cos \overline{YE} - d\theta_y \cos \overline{ZE}) \pm (d\theta_n \cos \overline{XU} - d\theta_u \cos \overline{XN})$$

to provide an incremental output in accord with the change, $d \cos(XE)$, in the cosine of the angle between the X and E axes; and means for integrating the incremental output.

7. A random orientation inertial system including in combination means for generating the cosines, $\cos(NP)$ and $\cos(UP)$, of the angles between a planar pair of orthogonal axes, N and U, and a co-planar axis, P; means for squaring each of the cosines; means for adding the squares of the cosines to provide a quantity; means for determining the error by which the square root of the quantity differs from unity; means for multiplying the error by $\cos(NP)$ to provide a first correction output; means for multiplying the error by $\cos(UP)$ to provide a second correction output; means for changing $\cos(NP)$ by an increment equal to the first correction output; and means for changing $\cos(UP)$ by an increment equal to the second correction output.

8. A random orientation inertial system including in combination means for generating the cosines, $\cos(XE)$ and $\cos(YE)$ and $\cos(ZE)$, of the angles between an axis, E, and a set of orthogonal axes, X and Y and Z; means for squaring each of the cosines; means for adding the squares of the cosines to provide a quantity; means for determining, at least approximately, the error by which the square root of the quantity differs from unity; means for multiplying the error by each of the cosines to provide correction outputs; and means for changing each cosine by an increment equal to its associated correction output.

9. A random orientation inertial system including in combination means for generating the cosines, $\cos(XE)$ and $\cos(YE)$ and $\cos(ZE)$, of the angles between an axis, E, and a set of orthogonal axes, X and Y and Z; means for sensing the accelerations, A_x and A_y and A_z , along the X and Y and Z axes; means responsive to the generating and the sensing means for solving an equation of the form

$$A_e = \pm A_x \cos(XE) \pm A_y \cos(YE) \pm A_z \cos(ZE)$$

to provide the acceleration, A_e , along the E axis; means for squaring each of the cosines; means for adding the squares of the cosines to provide a quantity; means for obtaining the square root of the quantity; and means for dividing A_e by said square root.

10. A random orientation inertial system including in combination means for generating the cosines, $\cos(NR)$ and $\cos(QR)$, of the angles between a planar pair of orthogonal axes, N and Q, and a co-planar axis, R; means for computing the incremental changes, $d \cos(NR)$ and $d \cos(QR)$, in the cosines which have occurred during a time interval; means responsive to the generating and the computing means for selectively solving one of the equations

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$$d\theta = \pm \frac{d \cos (QR)}{\cos (NR)}$$

$$d\theta = \pm \frac{d \cos (NR)}{\cos (QR)}$$

to provide the angular increment, $d\theta$; means responsive to the generating means for selecting that equation having approximately the larger absolute value of denominator; and means for integrating the angular increment.

11. A random orientation inertial system including in combination means for indicating the incremental angular rotation, $d\theta_y$, about the Y axis of a planar pair of orthogonal axes, X and Y; means for digitally sensing the incremental angular rotation, $d\theta_{x'}$, about an axis, X', having a co-planar component which is slightly skewed relative to the X axis; integrating digital means responsive to $d\theta_y$ and providing an output; means providing a reference quantity; means for sensing the amount by which the output of the integrating means exceeds the reference quantity and means responsive to the presence of such excess for changing $d\theta_{x'}$ by one digital increment and for causing the output of the integrating means to accord with such excess amount.

12. A random orientation inertial system including in combination means for indicating the acceleration, A_y , along the Y axis of a planar pair of orthogonal axes, X and Y; means for digitally sensing the acceleration, $A_{x'}$, along an axis, X', having a co-planar component which is slightly skewed relative to the X axis; means for providing the fixed quantity

$$\cos (X'Y)/\cos (X'X)$$

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defining the coefficient of skew; means for multiplying the skew coefficient by A_y to provide a product output; means for digitally integrating the product output; the integrating means having a units output and fractional unit outputs; means responsive to the units output for changing $A_{x'}$ by one digital increment and for causing the units output to revert to zero.

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