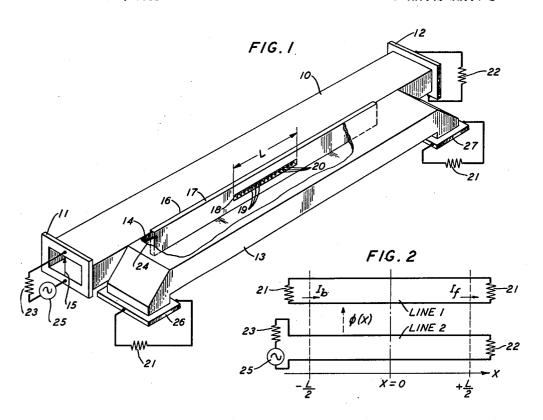
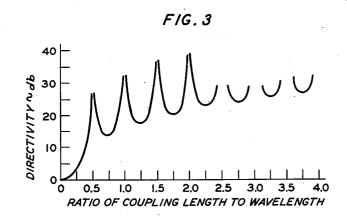
Filed March 17, 1951

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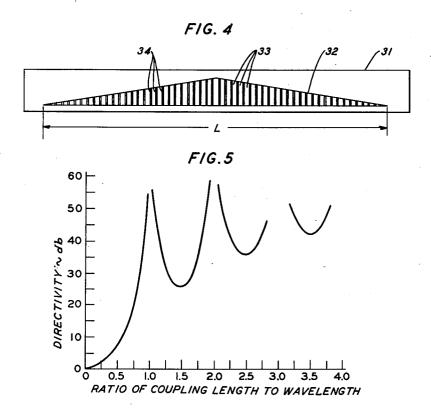


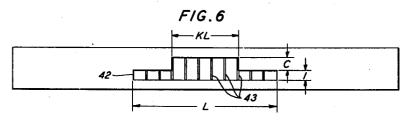


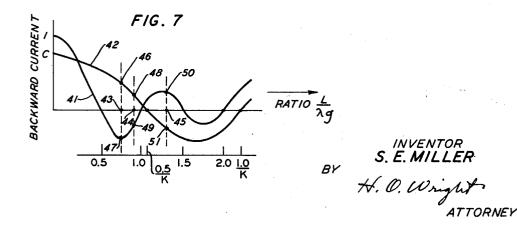
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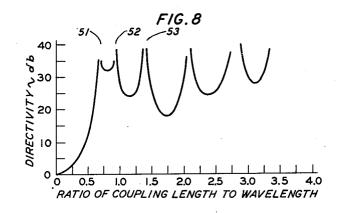


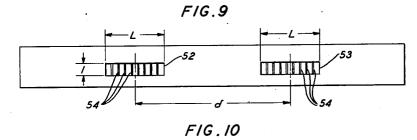


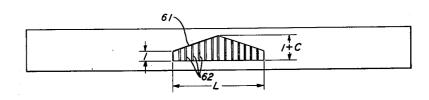


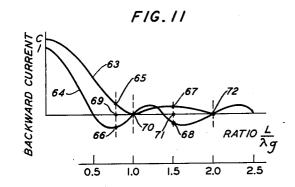
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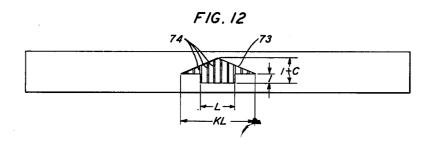
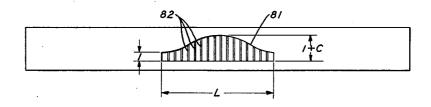


FIG. 13

FIG. 13 K=2 77 79 RATIO  $\frac{L}{\lambda g}$ 

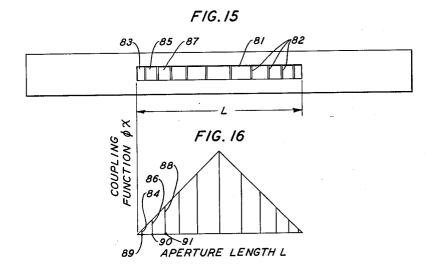
FIG. 14

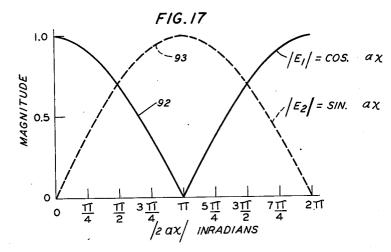


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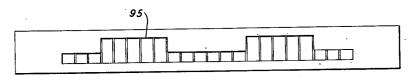
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# United States Patent Office

Patented Feb. 1, 1955

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#### 2,701,340

## HIGH-FREQUENCY DIRECTIONAL COUPLER

Stewart E. Miller, Middletown, N. J., assignor to Bell Telephone Laboratories, Incorporated, New N. Y., a corporation of New York

Application March 17, 1951, Serial No. 216,132 10 Claims. (Cl. 333-10)

This invention relates to electrical wave transmission 15 systems and, more particularly, to improved electromagnetic wave energy couplers providing a directional coupling characteristic between two transmission lines, such as wave guides, coaxial lines or the like.

One of the early practical types of directional couplers 20 was described in an article in the Proceedings of the Institute of Radio Engineers, February 1947, vol. 35, pages 160 to 165 by W. W. Mumford. The couplers there disclosed are now well known in the art, and countless uses and applications thereof have been described 25 in the published art. In general, all presently known directional couplers are formed by a short section of main transmission line coupled to a short section of auxiliary The coupling between the two sections is arranged so that an electromagnetic wave traveling in a single 30 direction along the main line induces a principal secondary wave, known as the forward wave, traveling in a single direction along the auxiliary line. Likewise, this coupling operates so that a wave traveling in the opposite direction in the main transmission line induces a principal secondary wave traveling only in the opposite direction in the auxiliary line.

direction in the auxiliary line.

In most practical directional couplers there is also an induced or secondary wave traveling in the opposite direction from each forward wave, known as the backward 40 wave. The forward and the backward waves are desirably greatly unequal in strength. Their relative strength is called the directivity of the coupler and is usually expressed as the decibel ratio of the forward wave current to the backward wave current. The The 45 strength of the desired induced forward wave in the auxiliary line compared to the inducing wave in the main line is called the "coupling loss" and is also expressed as the decibel ratio of the desired or forward induced wave to the inducing wave in the main line. 50 This "coupling loss" is actually the transfer ratio between the two lines and there is no power dissipated in the structure. The performance of a directional coupler may be described in terms of this directivity and coupling. To operate satisfactorily, the directivity of a coupler must 55 exceed some minimum design value at all frequencies within its operating range. Thus, the plot of directivity versus the operating frequency or wavelength of the coupled energy is known as the directivity characteristic of the coupler and will be so designated beginning. of the coupler and will be so designated herein.

The coupler, as disclosed in one embodiment by Mum-

ford in the above-mentioned publication, consists of a short section of auxiliary wave guide located contiguous to the main wave guide. Coupling is provided between the main wave guide and the auxiliary wave guide by a pair of longitudinally spaced holes in a common side wall. A traveling wave in the main guide will induce a traveling wave in the auxiliary guide traveling in the same direction and, since the path length of the energy coupled through each of the holes is equal, no electrical interference in the forward direction results. The path lengths of the oppositely directed or backward waves induced through the two holes into the auxiliary guide are unequal and, due to the spacing of one-quarter wavelength between the holes, cancellation results and no re- 75 sulting wave will be induced in the auxiliary guide in the backward direction or in a direction opposite to that

of the wave in the main guide.

This results in the directivity of the coupler being frequency sensitive since the desired high directivity occurs only at the frequency at which the coupling holes

are separated by one-quarter wavelength. This frequency is known as the design or center frequency. At frequencies slightly higher and slightly lower than the design frequency the directivity decreases, giving a finite operating range over which the directivity exceeds the

minimum design value.

It is shown in the above-mentioned publication that the frequency range over which the directivity exceeds a minimum design value may be broadened by employing an increased number of coupling elements which are still spaced one-quarter wavelength apart, but which have their coupling effects related to each other in accordance with the coefficients of a binomial expansion. Other coupler designs have been disclosed in which the Other coupler designs have been disclosed in which the frequency range of directivity has been increased by providing an infinite series of couplings or a distributed coupling between the two guides. This has in general been provided either by a plurality of probes or by a long narrow slot. This coupling is located in either case in a side of the guide walls which is perpendicular to the electric vector of the wave energy, that is, a wider wall assuming dominant mode energy, and provides coupling extending over an unusually long distance which must extending over an unusually long distance which must exceed several wavelengths at the operating frequency.

It is an object of the present invention to improve the directivity characteristic of directional couplers by increasing the directivity over a substantially increased op-

erating frequency range.

It is a further object to shorten the wavelength distance over which coupling is required for improved directivity and thereby decrease the physical size of directional

couplers.

In the prior directional couplers, particularly those of the types mentioned above, in which the coupling is ob-tained either by one-quarter wave spaced holes, or by distributed coupling in the wider side wall of rectangular guides, the directivity characteristic and also the cou-pling loss characteristic is inflexibly tied to the physical coupling mechanism which gives the directional effect. In other words, certain types of coupling have been determined to be directive, and to give an inherent directivity characteristic, but it was in no way possible to adjust or change this characteristic if it was not already particularly desirable.

It is therefore a further object of the present invention to produce in directional couplers predetermined direc-

tivity characteristics having arbitrarily large or small band width characteristics.

It has been determined, in accordance with the invention, that when the particular type of coupling, known as divided aperture coupling, to be described with reference to specific embodiments hereinafter, is employed to relate the two transmission lines, the directivity characteristics are small to the two transmission lines, the directivity characteristics. to relate the two transmission lines, the directivity characteristic of the coupler is directly related by a Fourier transform equation, to the shape of the plot of the magnitude of distributed coupling versus the coupling distance along the region of distributed coupling. In other words, the directivity is the transform of the shape of the coupling distribution, and conversely, the coupling distribu-tion shape is the transform of the directivity. While this is true theoretically and is subject to rigorous mathematical proof, the practical difficulty of handling the mathematical relations precludes literal application of the principle in the design of commercial directional couplers. This is so since, with the outstanding exception of certain fundamental characteristics designated hereinafter as the basic geometric shapes, the Fourier transform is an extremely complicated function. The transforms of these basic geometric shapes, however, are not so complicated and are relatively familiar mathematically in the art, having been used extensively, for example, in antenna design problems.

By the unusual and novel combination of the coupling distribution of two or more of these basic geometric distributions, a composite coupling distribution is obtained in accordance with the invention which is, in effect, a superposition of the basic distributions. This composite coupling is easy to handle mathematically, simple to construct in physical directional couplers, and allows substantially any predetermined band width and directivity to be achieved without experimentation.

It is a further object of the invention to regulate in a predetermined manner the amunot of power coupled in the forward direction of transmission in directional complets.

Another object of the invention is to transfer a portion of electromagnetic wave energy from one transmission line into another, which portion may vary as desired from a very small fraction of the power in the one

line to complete power transfer thereof.

In accordance with the last two mentioned objects 10 of the invention, it has been determined that the coupling loss in the forward direction of transmission in directional couplers in which divided aperture coupling is employed is determined by the "size" of the coupling distribution characteristic. In the specific embodiments 15 to be disclosed, this "size" is ideally varied by either the length over which distributed coupling is maintained by an individual divided aperture, or by the number of individual divided apertures of smaller length which is employed.

These and other objects, the nature of the present invention, and its various features and advantages, will appear more fully upon consideration of the various specific illustrative embodiments, shown in the accompanying drawings and in the following detailed description 25

of these embodiments.

In the drawings: Fig. 1 illustrates a specific embodiment of a micro-wave directional coupler in which directional coupling is

provided by a rectangular divided aperture;
Fig. 2, given by way of illustration, is a diagrammatic representation of the coupler of Fig. 1;
Fig. 3, given by way of illustration, is a directivity characteristic of the type to be expected for the coupler

of Fig. 1;
Fig. 4 represents a modification of Fig. 1 whereby directional coupling is provided by a triangular divided aperture in accordance with the invention;
Fig. 5, given by way of illustration, is a directivity characteristic of the type to be expected for the coupler of 40

Fig. 1 when modified in accordance with Fig. 4;
Fig. 6 represents a modification of Fig. 1 whereby directional coupling is provided by a composite divided aperture in accordance with the invention;
Fig. 7 is given by way of explanation of the coupler of 45

Fig. 1 when modified in accordance with Fig. 6;

Fig. 8, given by way of illustration, is the directivity characteristic of the type to be expected for the coupler of Fig. 1 when modified in accordance with Fig. 6;

Fig. 9 represents a modification whereby directional 50 coupling is provided by two identical divided rectangular apertures in accordance with the invention;

Fig. 10 represents a modification of Fig. 1 whereby directional coupling is provided by a second species of a composite aperture in accordance with the invention;

Fig. 11 is given by way of explanation of the coupler

of Fig. 1 when modified in accordance with Fig. 10;

Fig. 12 shows a generalized composite divided aperture of which the aperture of Fig. 10 is a species;

Fig. 13 is given by way of explanation of the coupler 60 of Fig. 1 when modified in accordance with Fig. 12;
Fig. 14 represents a modification of Fig. 1 whereby di-

rectional coupling is provided by a third special of a composite aperture in accordance with the invention;
Fig. 15 represents a modification of Fig. 1 whereby di-

rectional coupling is provided by an unequally divided aperture:

Fig. 16, given by way of explanation, is the current coupling function of the unequally divided aperture of

Fig. 15;
Fig. 17, given by way of explanation, shows the magnitude of forward waves in a divided aperture directional coupler as a function of the length and coupling of the aperture; and

Fig. 18 represents a modification of Fig. 1 whereby the 75 magnitude of power transmitted in a forward direction

may be controlled.

Fig. 1 shows a directional coupler of the type disclosed and claimed in the copending application of A. G. Fox, Serial No. 236,556, filed July 13, 1951, and similar to a 80 related type disclosed and claimed in the copending application of A. E. Bowen and W. W. Mumford, Serial No. 219,426, filed April 5, 1951. This directional coupler comprises a main section 10 of shielded transmission line for guiding wave energy, which may be a rectangular

wave guide, as shown, having terminal connections 11 and 12 at each of its ends. Located adjacent to the main line 10 and having a portion of its length contiguous to a portion of the main line, is an auxiliary shielded transmission line 13 for guiding wave energy, which may be a rectangular wave guide, as shown, and having terminal connections 26 and 27 at its respective ends. A portion of the adjacent walls 14 and 24 of each of the two guides 10 and 13, respectively, which walls are parallel to the electric vector 15 of wave energy, or the narrower wall of each guide 10 and 13 assuming normal dominant mode excitation therein, has been removed providing a slot 16 between the guides when viewed from the direction of the wider wall side. Into slot 16 is placed an insert 17 which forms a common wall between the adjacent wave guides 10 and 13. Insert 17 is provided with a divided rectangular aperture 18 providing coupling means be-tween guides 10 and 13. The exact dimensions of the divided aperture 18 will be given in detail hereinafter, but it may now be stated in general, that the longitudinal dimension L is greater than one-half wavelength, while the smaller dimension thereof is, in the usual case, con-siderably less than one-half wavelength. The aperture is termed divided since extending parallel across the transverse or narrower dimension of aperture 18 and dividing it longitudinally into a plurality of smaller spaces 19, a grid comprising a plurality of dividers or wire 20. Wires 20 may be soldered into recesses placed in insert 17 along the edges of aperture 18 or the resulting structure may be stamped or punched from a blank insert. The preferable number of wires 20, their dimension and the maximum size of spaces 19 will be discussed in detail hereinafter. Each end of main guide 10 and auxiliary guide 13 is terminated in its characteristic impedance as indicated diagrammatically by impedances 21 for guide 13, and 22 and 23 for guide 10. A source 25 of microwave energy is shown diagrammatically connected in se-

ries with impedance 23 to terminal 11 of main guide 10.

Divided aperture 18 provides a current coupling between the lines 10 and 13 which is effectively distributed to a substantial degree along the length L of the aperture. The magnitude of the current coupling is uniform along this length and is proportional to the transverse dimension of the aperture. A single undivided rectangular aperture of the same size and shape as the divided aperture 18 does not provide this desired coupling. To the contrary the undivided aperture behaves as a transmission line, more or less independently of either of the adjacent wave guides. This is true no matter how thin the common wall thickness is made since a standing wave is produced in the undivided aperture from end to end thereof as a result of the high coefficient of reflection at the aperture ends. Such an aperture indeed behaves substantially like a resonant single point coupling, and it therefore does not have the particular attributes, including particularly distributed coupling, to be described in detail hereinafter, which attributes are necessary to the practice of the pres-

ent invention.

In Fig. 1 the plurality of dividers or wires 20 are placed across the divided aperture 18 at equal intervals along the length thereof. The standing waves which, in the absence of wires 20, would tend to form in aperture 18 are now localized between the wires 20 and destroyed to an extent dependent upon the spacing between them being chosen in accordance with the following general considerations. Generally speaking, if the space 19 between adjacent wires 20, or the longitudinal dimensions of the spaces 19 into which the length of the aperture 18 is divided, is less than one-half wavelength, no standing waves can be supported. Such a spacing, i. e., merely less than one-half wavelength, would tend to produce discrete couplings located at the center point between the wires 20. However, since the amount of departure from the desired continuous coupling is inversely related to the number of such discrete couplings per wavelength, it is necessary that more than only the two couplings per wavelength obtained with this one-half wavelength spacing, be employed. It has been determined that if the number of spaces 19 is in the range of three or more per wavelength, a useful approximation of continuous coupling is achieved, and if the number of spaces 19 exceeds eight or more per wavelength, the desired continuous coupling assumed as a basis for the mathematical analysis given hereafter is actually realized for all practical purposes.

The transverse dimension of the spaces 19, and therefore of dividing aperture 18, is, of course, limited by the smaller of the transverse dimensions of the narrow walls 14 and 24 of the wave guides 10 and 13, respectively, and will therefore usually be less than one-half wavelength. The exact value of the transverse dimension of the divided rectangular aperture 18 determines the power coupling from the main transmission line 10 to the auxiliary transmission line 13. It may be easily shown that the transfer or power coupling loss through this path varies substantially as the square of the transverse dimension of the aperture 18. This fact is a most important attribute of the divided aperture in the narrow wall, but the full significance of this fact will more readily be appreciated when certain of the embodiments to be described hereinafter are considered. It should be noted, however, that since the power transferred does vary as the square of the transverse dimension, the current coupling through the divided aperture varies directly as the transverse dimension.

The width of the dividers or wires 20, i. e., the diameter thereof if the dividers are wires of circular cross section, does not affect the directivity within reasonable limits. In the preferred embodiment of the invention as shown in Fig. 1, this dimension of the dividers, shown as wires 20, is comparable to and perhaps somewhat less than the common wall thickness between guides 10 and 13, but the operable minimum dimension thereof is principally controlled by physical considerations such as rigidity of the dividers 20. The width of the dividers may be increased dividers 20. The width of the dividers may be increased substantially beyond the thickness of the common wall without causing any substantial departure from the desired continuous coupling discussed above. The power transferred through the divided aperture, however, is affected by the dimensions of the dividers. For example, if the dimension of dividers 20 is increased in the plane of the aperture, i. e., decreasing the area of the spaces 19, or if the dimension of dividers 20 is increased in the dior if the dimension of dividers 20 is increased in the direction of wall thickness which would not change the area of spaces 19, the power transferred is reduced. It is found that the width of spaces 19 changes the current transfer much more rapidly than linearly and as a consequence, for example, a given aperture when divided into ten spaces provides appreciably more power transfer than the identical aperture divided into twenty spaces.

As pointed out above, and as shown in Fig. 1, the divided aperture 18 is located in a common wall of the two rectangular guides 10 and 13 which wall is in the case of both guides parallel to the electric vector of wave en-

of both guides parallel to the electric vector of wave energy in the guide. As stated above, in this position the current coupling between the guides at each point along the aperture 18 is proportional to the transverse dimension of the divided aperture 18. It should be noted, however, that this current coupling relation obtains if the divided aperture 18 is located in a common wall parallel to the electric vector in only one guide, that is, assuming dominant mode excitation, if the common wall includes only the narrower wall of one wave guide. The relation also obtains if the divided aperture is located in a com-

also obtains if the divided aperture is located in a common wall which is perpendicular to the electric vectors in both guides, provided that the aperture is displaced from the center line of the wider wall in both guides.

The manner in which directional coupling operation is obtained from the structure of Fig. 1 will most easily be understood upon a consideration of the diagrammatic representation of this structure in Fig. 2. On Fig. 2 are shown two identical transmission lines 1 and 2 corresponding, respectively, to lines 13 and 10 of Fig. 1. These transmission lines are assumed parallel and the direction of propagation is along the x-axis. The region in which coupling exists, corresponding to the divided aperture coupling in the structure of Fig. 1, is confined to the interval length L and is designated on Fig. 2 by the interval from interval from

$$-\frac{L}{2}$$
 to  $+\frac{L}{2}$ 

The coupling distribution or the variation of coupling between the lines in the interval

$$-\frac{L}{2}$$
 to  $+\frac{L}{2}$ 

is described by the function  $\varphi(x)$ . Assume further that the exciting wave generated by the source 25 is travel-

ing to the right in line 2. When all the forward current elements are summed and referred to the plane of

$$x = \pm \frac{L}{2}$$

the equation

$$I_{f} = kF \int_{-\frac{L}{2}}^{+\frac{L}{2}} \varphi(x) dx \tag{1}$$

is obtained in which the factor F is expressed

$$F = \left[ \frac{2Z}{e^{-i2\pi \frac{L}{\lambda g}}} \right]^{-1}$$

The term Z represents the characteristic impedance of either line and its terminal impedances 21, 22 and 23. The term  $\lambda_g$  represents the guide wavelength of electro-

magnetic energy.

The factor k represents the fraction of the total induced current which travels forward in the auxiliary line 1. The factor k is thus a measure of the directionality of the coupling on a differential length basis. If all the backward current elements are summed and referred to the plane

$$x = -\frac{L}{2}$$

the equation

$$I_{b} = (1-k)F \int_{-\frac{L}{2}}^{+\frac{L}{2}} \varphi(x) e^{-i\frac{4\pi}{\lambda g}x} dx$$

$$(2)$$

is obtained. The ratio of the forward current (Equation 1) to the backward current (Equation 2) is the directivity of the coupler defined above. So long as the phase of the coupling function  $\varphi(x)$  does not change between  $-\frac{L}{2}$  and  $+\frac{L}{2}$ 

$$-\frac{L}{2}$$
 and  $+\frac{L}{2}$ 

the forward current elements all add in phase in line 1. However, the backward current elements add in a form of destructive interference. The backward current expression (Equation 2) is in the form of a Fourier transform. Thus, the directivity characteristic is directly related to the coupling function  $\varphi(x)$  by a Fourier transform of  $\varphi(x)$ . Theoretically, then, it is possible to design a coupling function which would produce any desired directivity characteristic.

For specific example, the coupling characteristic of the

For specific example, the coupling characteristic of the divided rectangular aperture 18 of Fig. 1 is a rectangular wave, i. e., the magnitude of the current coupling at each point along the length of the aperture is uniform over the coupling interval L, the length of aperture 18, and the magnitude is zero outside this interval. The exact manner in which this characteristic is obtained by a divided rectangular aperture such as 18 has been explained in detail hereinbefore. In terms of the notations used above, the function  $\varphi(x)$  for this coupling characteristic is equal to unity as x varies from

$$-\frac{L}{2}$$
 to  $+\frac{L}{2}$ 

Equations 1 and 2 above may therefore be easily evalu-Equations 1 and 2 above may ineretore be easily evaluated for this coupling function. In each equation the factor k becomes ½ since, as demonstrated by Mumford in the above publication, one-half of the current coupled by an aperture in the side wall of a wave guide will travel in each direction. The forward current expression of Equation 1 becomes

$$I_f = \frac{FL}{2} \tag{3}$$

The backward current expression of Equation 2 when evaluated for the rectangular coupling distribution of Fig. 1 is the known Fourier transform of a rectangular wave; that is, the backward current is a constant times the Fourier transform of the coupling distribution and

$$I_b = \frac{FL}{2} \frac{\sin u}{u} \tag{4}$$

in which

$$u = \frac{2\pi L}{\lambda_z} \tag{5}$$

and where  $\lambda_g$  is the guide wavelength of the electromagnetic energy in both guides. Thus, the directivity is given by the ratio of the forward current to the backward current or the ratio

$$Dir. = \frac{u}{\sin u} \tag{6}$$

This function is plotted on Fig. 3, in which the ordinate represents the directivity in decibels and the abscissa represents the ratio of the coupling length L to the wavelength  $\lambda_B$ . It will be noted that perfect directivity, i. e., infinite decibel discrimination in a backward direction, is found in all regions in which the coupling length L is an integral number of half wavelengths of the guide wavelength  $\lambda_B$ . It will be noted the minima of directivity length  $\lambda_g$ . It will fall off as the ratio

$$\frac{L}{\lambda a}$$

and a coupling interval of approximately three wavelengths is desirable in order to obtain broad band directivity of the order of 25 decibels.

The nature of the coupling in accordance with the invention as thus far described, and the particular characteristics of this coupling may well be summarized at this point, to provide a firm foundation from which to proceed point, to provide a firm foundation from which to proceed to the description of the more refined embodiments of the invention hereinafter. Thus, the coupling is provided by what has been termed, and will continue to be termed hereinafter and in the appended claims, a "divided aperture." A "divided aperture" may be considered as an original opening, the perimeter of which defines a given geometric shape, but which original opening has been broken down into many smaller openings or spaces. If the number of smaller spaces is large, their exact individ-ual size and shape need not be considered, but rather attention should be directed to the size and shape defined by the perimeter of the original opening or the "divided aperture." This shape will in general be designated as having a "basic geometric shape." The magnitude of current coupled at any point through the divided aperture, located as described, is directly proportional to the transverse dimension of the divided aperture so that the distribution along the aperture of the coupled current is identical to the physical shape of the divided aperture. This current characteristic will be designated as a "basic geometric distribution." The Fourier transform of the basic geometric distribution, and therefore, the transform of the basic geometric shape is the characteristic of the backward current in the auxiliary transmission line which characteristic is directly related to the directivity of the coupler. The total current coupled, which determines the coupling loss in the forward direction, depends upon the coupling loss in the forward direction, depends upon the magnitude of the transverse dimension of the divided aperture. So long as this magnitude is varied without altering the shape of the divided aperture, as may be done with the basic geometric shapes considered herein, the coupling loss may be independently chosen by the transverse dimension without affecting the directivity characteristics.

Fig. 4 shows an insert 31 which may replace the insert 65 17 of Fig. 1 and form the common wall between adjacent wave guides 10 and 13. Insert 31 is provided with a diwave guides 10 and 13. Insert 31 is provided with a divided aperture 32 having a basic geometric shape of an isosceles triangle having a base of length L. As with the rectangular aperture 18 of Fig. 1, the triangular aperture 32 of Fig. 4 is broken into smaller spaces 33 by a plurality of grids or wires 34. The same considerations with regard to the spacing and number of wires 34 treated above in connection with Fig. 1 obtain in the case of Fig. 4. As pointed out in the paragraph just preceding, the magnitude of the current coupling at any point along the length L is directly proportional to the transverse dimension of the divided aperture at that point. Therefore, the coupling distribution characteristic of aperture 32 of Fig. 4 takes a form identical to the physical shape of the triangular aperture. In terms of the notations already used, the function  $\varphi(x)$  over the interval from

 $-\frac{L}{2}$  to 0

may be expressed as

$$\varphi(x) = \frac{2}{L} \left( \frac{L}{2} + x \right) \tag{7}$$

and over the interval

0 to 
$$\pm \frac{L}{2}$$

may be expressed as

$$\varphi(x) = \frac{2}{L} \left( \frac{L}{2} - x \right) \tag{8}$$

As in the case of a rectangular coupling distribution characteristic, this triangular coupling distribution characteristic is one of the basic geometric shapes for which the Fourier transform is well known. Thus, Equation 2 evaluated for this coupling gives a backward current which is expressed as

$$I_{b} = \frac{FL}{4} \frac{\sin^{2}\left(\frac{u}{2}\right)}{\left(\frac{u}{2}\right)^{2}} \tag{9}$$

The forward current will be

$$\frac{FL}{4}$$

Thus the directivity is given by

$$Dir. = \frac{\left(\frac{u}{2}\right)^2}{\sin^2\left(\frac{u}{2}\right)} \tag{10}$$

This function is plotted on Fig. 5 which shows that perfect directivity is found in the regions in which the coupling length L is an integral number of guide wavelengths. The locus of minimum directivity falls off as

$$\left(\frac{L}{\lambda_{\ell}}\right)^2$$

and a coupling interval of the order of one wavelength produces broad band directivity in excess of 25 decibels. By going to a length slightly greater than two wavelengths, 35 decibels directivity can be maintained over a very broad band. A comparison with the characteristic of Fig. 3 will, therefore, show that the aperture shape of Fig. 4 has provided a substantial improvement in the directivity characteristic with a much shorter required coupling interval.

It has thus been shown with reference to two basic geometric coupling aperture shapes, i. e., rectangular and triangular, that the backward current and thus directivity characteristic of of the resulting coupler employing that aperture in divided form is related by the Fourier transform to the shape of the distributed current coupling along the length of the divided aperture which is in turn directly related to the transverse dimension of the aperture. Other basic geometric shapes includes a one-half period of a cosine wave, a whole period of a cosine wave measured from one minimum point to another and which is mathematically the same as one-half period of a sin2 wave, and positive and negative exponential waves. For each of these shapes the Fourier transform is well known in the art and is expressible in its simplest form as a function of a single angular variable, for example, the variable u in Equation 4 for the basic rectangular geometrical shape and the variable

$$\frac{u}{2}$$

in Equation 9 for the basic triangular geometrical shape. Use of the transform will give the evaluation of Equation 2 for the backward current when a divided aperture, physically of the shape considered, provides the coupling between the main guide and the auxiliary guide.

Merely for the convenience of those who may practice the invention, the following relations for certain of these basic geometric shapes are given herein. By no means is this listing to be considered as exclusive of other basic shapes for which Fourier transforms are functions of a single angular variable and can readily be derived by those familiar with the principles of mathematical 85 analysis.

Thus, for a one-half period cosine wave for which the coupling function (and the aperture shape) is

$$\varphi(x) = \cos\left(\frac{\pi x}{L}\right) \tag{11}$$

over the interval

$$-\frac{L}{2}$$
 to  $\frac{L}{2}$ 

and the backward current (transform of the wave form of  $\varphi(x)$ ) is

$$I_b = \frac{FL}{\pi} \frac{\cos u}{\left[1 - \left(\frac{2u}{\pi}\right)^2\right]} \tag{12}$$

and the directivity characteristic

Dir. = 
$$\frac{\left[1 - \left(\frac{2u}{\pi}\right)^2\right]}{\cos u} \tag{13}$$

For a whole period cosine for which the coupling function is

$$\varphi(x) = 1 + \cos\left(\frac{2\pi x}{L}\right) \tag{14}$$

over the interval

$$-\frac{L}{2}$$
 to  $\frac{L}{2}$ 

and the backward current is

$$I_{b} = \frac{FL}{2} \frac{\sin u}{u \left[1 - \left(\frac{u}{\pi}\right)^{2}\right]} \tag{15}$$

and the directivity characteris

Dir. = 
$$\frac{u\left[1-\left(\frac{u}{\pi}\right)^2\right]}{\sin u} \tag{16}$$

The same may be shown for a positive exponential wave and for a negative exponential wave. In each of the above, the functions u and F are the same as defined

In each of the above-described coupling distributions, it will be noted that the single angular variable in each case depends upon the length of the coupling interval as this length appears in the function u. However, in accordance with an important feature of the present inventional control of the coupling interval as this length appears in the function u. tion, two or more of these basic geometric shapes are combined to form a composite shape. One such composite shape is shown in Fig. 6, wherein an insert, suitable for disposition in slot 16 of Fig. 1, is provided with a divided aperture 42 in the shape of two pyramided recorded aperture 42 in the shape of two pyramided recorded aperture 45 in the shape of two pyramided recorded aperture 45 in the shape of two pyramided recorded aperture 45 in the shape of two pyramided recorded aperture 45 in the shape of two pyramided recorded aperture 45 in the shape of two pyramided recorded aperture 45 in the shape of two pyramided recorded aperture 45 in the shape of two pyramided recorded aperture 45 in the shape of two pyramided recorded aperture 45 in the shape of two pyramided recorded aperture 45 in the shape of two pyramided recorded aperture 45 in the shape of two pyramided recorded aperture 45 in the shape of two pyramided recorded aperture 45 in the shape of two pyramided recorded aperture 45 in the shape of two pyramided recorded aperture 45 in the shape of two pyramided recorded aperture 45 in the shape of two pyramided recorded aperture 45 in the shape of two pyramided pyramided recorded aperture 45 in the shape of two pyramided recorded aperture 45 in the shape of two pyramided recorded aperture 45 in the shape of two pyramided pyramid tangles. In other words, a lower rectangular aperture of longitudinal dimension L and transverse dimension of unity, is located immediately adjacent to and below an upper rectangular aperture of longitudinal dimension kL and transverse dimension c, the quantity c being an arbitrary constant and the quantity k designating a fraction of the length L. The two rectangular apertures are positioned so that a side kL is located contiguous to and contrared upon a side of the length L. Thus, the two centered upon a side of the length L. Thus, the two separate rectangular apertures merge into a single composite divided aperture, that is, divided by grids 43, which will provide a coupling  $\varphi(x)$  equal to unity over the intervals from

$$-\frac{L}{2}$$
 to  $-\frac{kL}{2}$ 

and from

$$rac{kL}{2}$$
 to  $rac{L}{2}$ 

Over the interval from

$$-\frac{kL}{2}$$
 to  $\frac{kL}{2}$ 

the coupling function  $\varphi(x)$  is equal to 1+c.

It may be shown that the Fourier transform for such a pyramid-shaped area is the sum of the transforms of each of the related rectangular areas. In other words, the transform of the composite coupling distribution is the

sum of the transforms of each of the basic geometric dissum of the transforms of each of the basic geometric distributions and is a function of both of the separate single variables of the basic transforms. The composite coupling, therefore, represents a distribution the Fourier transform of which expressed in its simplest form is a function of a plurality of angular variables. Therefore, the total backward current resulting from the composite of the contraction. or a plurality of angular variables. Therefore, the total backward current resulting from the composite coupling distribution of Fig. 6 may be obtained by independently substituting the proper transforms in Equation 2 for the two component constant amplitude couplings and adding the two backward currents thus obtained arithmetically. The total backward current expression for a directional coupling the properties the properties of the insert Fig. coupler employing the aperture shown in the insert of Fig. 6 then becomes

$$I_b = \frac{FL}{2} \left[ \frac{\sin u}{u} + \frac{ck \sin (ku)}{(ku)} \right]$$
 (17)

The first term of Equation 17 is a function of a first angular variable u and represents the component backward current resulting from constant amplitude coupling of the lower basic distribution of length L and height unity. The second term of Equation 17 is a function of a second angular variable ku and represents the component backward current resulting from the upper baic distribution of height c and of length kL.

On Fig. 7 the magnitude of these two component backward currents are plotted versus the ratio of the coupling interval L to the wavelength  $\lambda_g$ . Curve 41 is the current component expressed by the first term of Equation 17, and curve 42 is the current component expressed by the second term of Equation 17. The sum of curves 41 and 42, therefore, represents the total backward current, expressed by Equation 17, of the composite distribution of Fig. 6. Whenever the backward current contributions resulting from each of the two basic geometric distributions, i. e., the currents represented by curves 41 and 42, are equal in magnitude and opposite in phase, the total backward current becomes zero and the directivity of the coupler employing the aperture shown in the insert of Fig. 6 becomes very large.

In accordance with an object of the invention, high directivity is obtained in broad frequency regions by obtaining substantially equal backward current magnitudes from the two component basic geometric coupling distributions and, at the same time, arranging these currents to be of opposite phase or sign. This is easily done with the component distribution of Fig. 6. It will be noted that the factor c in Equation 17, representing the relative height of the basic geometric shape, determines the relative amplitudes of the backward currents. Likewise, the factor k, representing the relative lengths of the two basic geometric shapes, determines the relative phase of the backward currents. For example, Fig. 7 shows that the points at which curve 41 represents zero backward current are located at those ratios of the two component basic geometric coupling distributions

$$\frac{L}{\lambda_{g}}$$

which are equal to integral multiples of 0.5. The points at which curve 42 represents zero backward current are those ratios of

$$\frac{L}{\lambda_s}$$

which are equal to integral multiples of 0.5 divided by the factor k. Thus, a variation in the factor k results in an extension or a contraction of curve 42 along the abscissa of Fig. 7 and a variation in the factor c results in an extension or a contraction of the amplitude of curve 42 with respect to curve 41.

As shown on Fig. 7, point 46 on curve 42 represents a positive current equal in magnitude to a negative current represented by point 47 of curve 41. Likewise, points 48 and 51 of curve 42 represent currents equal in magnitude and opposite in phase to the currents represented by points 49 and 50 of curve 41, respectively. Thus, for the ratio values represented by points 43, 44 and 45 of Fig. 7, zero total backward current is obtained. In the entire region between ratio values 43 and 44, substantial cancellation of between ratio values 43 and 44, substantial cancellation of backward current is obtained, and exceedingly high directivity results over this entire broad band region. A like broad band region will be centered around the ratio represented by point 45.

As has been previously shown, the directivity of the

coupler is given by the ratio of the forward current to the total backward current. In Fig. 8 is shown the directivity of a coupler employing the insert of Fig. 6, the backward currents of which have been considered in Fig. 7. More particularly, Fig. 8 shows the directivity of the coupler of Fig. 6 when c is equal to unity and when k is equal to 0.454. A comparison of Fig. 8 with the backward current characteristic of Fig. 7 will show that the points of infinite directivity on Fig. 8, such as points 51, 52 and 53, correspond to those ratios of coupling length to wavelength on Fig. 7 at which the two backward current components were equal in magnitude and opposite in phase. For example, point 51 of Fig. 8 corresponds to the ratio indicated by point 43 on Fig. 7. Likewise, points 52 and 53 of Fig. 8 correspond, respectively, to ratios 44 and 45 of Fig. 7. It should, therefore, be apparent that many and varied directional band-pass characteristics may be obtained by adjusting the values of the constants c and k in accordance with the principles which have just been described.

Fig. 9 shows an insert, suitable for location in slot 16 of Fig. 1, which is provided with two identical divided rectangular apertures 52 and 53. Both divided apertures 52 and 53 are identical to the aperture 18 described hereinbefore in connection with Fig. 1. With regard to details of the considerations involved in the spacing and location of the grids or dividers 54, reference may be had to the discussion of Fig. 1. Each of the apertures 52 and 53 has a longitudinal dimension L and a transverse dimension of unity. The transverse center lines of the apertures are separated by a longitudinal distance d. It should be noted that for all values of the center line spacing d of less than L, the apertures 52 and 53 will overlap and, therefore, theoretically produce a distribution identical to that shown for the aperture of Fig. 6. Therefore, the following analysis of the characteristics of the aperture of Fig. 9 applies with equal validity to the aperture of Fig. 6 and may be considered an alternative of the analysis of the aperture of Fig. 6 given hereinbefore.

The backward current expression for either divided aperture 52 or divided aperture 53 is expressed by Equations 4 and 5 given hereinbefore. Combining these equations the backward current from either divided aperture 52 or 53 may be expressed

$$I_{b} = \frac{FL}{2} \frac{\sin \frac{2\pi L}{\lambda_{s}}}{\frac{2\pi L}{\lambda_{s}}}$$
(18)

It may be easily shown that the total backward current resulting from both divided apertures 52 and 53 may be expressed as the function of a plurality of angular variables

$$I_{bT} = [2I_b] \left[ \cos \frac{2\pi d}{\lambda_s} \right] \tag{19}$$

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The total backward current will be zero, giving infinite directivity, for all values at which either the first term 2I<sub>b</sub>, dependent upon the variable

$$\frac{2\pi L}{\lambda_{g}}$$

or the second term cos

$$\frac{2\pi d}{\lambda_z}$$

dependent upon the variable

$$\frac{2\pi d}{\lambda}$$

is zero. Thus, a first set of frequencies of infinite directivity may be independently chosen by selecting the proper length L as it appears in the first term, and a second set by choosing the proper separation d as it appears in the second term. By properly locating these points of infinite directivity a large number of useful directivity characteristics may be obtained of which the characteristic already shown in Fig. 8 is one example.

ready shown in Fig. 8 is one example.

The divided apertures 52 and 53 may be replaced by indentical divided apertures of any of the basic geometric shapes described hereinbefore, including the triangular shape described with reference to Fig. 4. Furthermore,

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divided apertures 52 and 53 may be replaced by the composite apertures, such as the one disclosed in Fig. 6, if desired, with the resulting advantage of an increased num-

ber of points of infinite directivity.

Fig. 10 shows an insert, suitable for location in slot 16 of Fig. 1, which is provided with a composite divided aperture 61. The shape of composite divided aperture 61 is a combination in accordance with the invention of a basic rectangular shape, such as aperture 18 of Fig. 1, located adjacent to a basic triangular shape, such as divided aperture 32 of Fig. 4. As with the composite aperture 42 of Fig. 6, the two basic shapes and their coupling distributions are merged to form a single divided aperture. As shown, the longitudinal dimension of the rectangle and the base dimension of the triangle are adjacent and are equal to L. The maximum transverse dimension of aperture  $\bf 61$  is equal to 1+c inasmuch as the transverse dimension of the rectangle is unity and the altitude of the triangle is c. posite area is divided by grids 62 into smaller spaces in accordance with the considerations detailed hereinbe-fore. As has been demonstrated, the total backward current produced by the coupling of such a composite coupling distribution is the arithmetic sum of the backward currents produced by each of the component basic distributions. These component backward currents are plotted in Fig. 11. Thus, the backward current component contributed by the triangular distribution, which is given by Equation 9 hereinbefore, is represented by curve 63 of Fig. 11 which has the same coordinates as Fig. 7 hereinbefore. The backward current component contributed by the rectangular distribution which is contributed by the rectangular distribution, which is given by Equation 4 hereinbefore, is represented by curve 64 of Fig. 11. The initial amplitude of curve 63 is equal to c so that by properly choosing the value of c, the magnitude of the positive current represented by point 65 on curve 63, may be made equal to the magnitude of the negative current represented by point 66 on curve 64. The same will, therefore, be true for points 67 and 68 on curves 63 and 64 respectively. Thus in the re-68 on curves 63 and 64, respectively. Thus, in the region between the ratio values indicated by points 69 and 70 and in the region between points 71 and 72, substantial cancellation of the total backward current is

In Fig. 12 the same two basic geometric shapes employed in Fig. 10 have been used to form the composite aperture 73 except that the base dimension of the component basic triangular aperture has been increased by a factor k. As in the previous figures, the total aperture area has been divided by grids 74. It will be readily appreciated that the composite aperture of Fig. 10 is one species of the aperture of Fig. 12 in which the factor k is equal to 1. Similarly, the factor k may be a quantity less than unity, in which case the base of the triangular distribution will be shorter than the longitudinal dimension of the rectangular distribution.

A general expression for the total backward current of a composite divided aperture resulting from the combination of the basic geometric shapes of a rectangle and a triangle, as shown in Fig. 12, may be expressed

$$I_{bT} = \frac{FL}{2} \left[ \frac{\sin u}{u} + \frac{ck}{2} \frac{\sin^2 \left(\frac{ku}{2}\right)}{\left(\frac{ku}{2}\right)^2} \right]$$
(20)

This equation will be seen to be a combination of Equation 4, expressing the component backward current of the rectangular distribution, and Equation 9, expressing the component backward current of the triangular distribution and taking into account the factors c and k.

In Fig. 13 the component backward current expressed by the first term of Equation 20 is represented by curve 77, and the component backward current expressed by the second term of Equation 20, with the factor k equal to the specific value of 2, is represented by curve 76. It will be observed that in the entire interval between the ratio values of 0.5 and 1.0, substantial cancellation of the total backward current will result. The same is also true for the interval between the ratio values of 1.5 and 2. Thus, specific bands, each having a width of one-half wavelength, have been obtained over which the directivity is very large for a directional coupler having divided aperture coupling provided by the insert shown in Fig. 12.

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One final composite distribution should serve to illustrate the principles of the invention. Thus, Fig. 14 shows an insert, suitable for location in slot 16 of Fig. 1, which is provided with a composite divided aperture 81. The shape of composite divided aperture 81. The shape of composite divided aperture 81 is a combination in accordance with the invention of a basic rectangular shape, such as aperture 18 of Fig. 1, located adjacent to the two minima points of a basic whole period of a cosine wave, as defined by Equations 14, 15 and 16 hereinbefore. As with the other composite apertures herein, the two basic shapes and their coupling distributions are merged to form a single divided aperture. As shown, the longitudinal dimension of the rectangle and the whole period dimension of the cosine wave are adjacent and are equal to L. The maximum transverse dimension of aperture 81 is equal to 1+c inasmuch as the transverse dimension of the rectangle is unity and the total amplitude of the cosine wave is c. The composite arrangement is divided by grids 82 into smaller spaces in accordance with the considerations detailed hereinbefore. It will be noted that this distribution is similar to the distribution of Fig. 10, except that the linear taper of the triangular distribution of the insert of Fig. 10 has been replaced by a cosinusoidal taper of Fig. 14.

10 has been replaced by a cosinusoidal taper of Fig. 14.

The total backward current of the composite divided 25 aperture resulting from the combination of the basic geometric shapes of a cosine wave and a rectangle, as shown in Fig. 14, may be expressed

$$I_{bT} = \frac{FL}{2} \left[ \frac{\sin u}{u} + \frac{c \sin u}{u \left[ 1 - \left( \frac{u}{\pi} \right)^2 \right]} \right] \tag{21}$$

This equation will be seen to be a combination of Equation 4, expressing the component backward current of the rectangular distribution, and Equation 15, expressing the component backward current of the cosine distribution, taking into account the factor c. If the factor c is selected by empirical and graphical methods of the type illustrated hereinbefore by Figs. 7 and 8 for the embodiment of Fig. 6, to be equal to the specific value of 22.4, it may be shown that a directivity characteristic of more than 50 decibels may be obtained at all frequencies in which the coupling length L is at least 1.5 times the wavelength of the coupled energy.

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It should be apparent that further combinations of basic geometric shapes for which the Fourier transform

It should be apparent that further combinations of basic geometric shapes for which the Fourier transform is known, may readily be made by those skilled in the art in accordance with the principles of the invention which have been disclosed. The particular directivity characteristic desired will be obtained in each case by proportioning the relative dimensions of the component geometric shapes in the manner demonstrated with ref-

erence to several specific embodiments herein.

The particular coupling distributions contemplated by the invention have been obtained in each of the preceding embodiments by varying the shape of the coupling aperture in a predetermined manner. In Fig. 15 an insert is shown, suitable for location in slot 16 of Fig. 1, which provides a predetermined coupling distribution by varying the spacing of the grids or dividers 82 placed across aperture 81 of constant transverse dimension. For example, assume that the triangular coupling function of Fig. 16 is desired along the aperture length L. Thus, the spacing of dividers 82 will be chosen to render the areas 83, 85 and 87, etc., proportional to substantially the square root amplitudes 84, 86 and 88 of the coupling function at points such as 89, 90 and 91, etc. along the aperture length which correspond to the centers of the spaces 83, 85 and 87, etc. A composite distribution may be obtained in this manner by varying the spacing of the divider elements in accordance with a first predetermined function and also varying the transverse dimension of the aperture in accordance with a second predetermined function.

The preceding discussion of the invention in connection with the figures already considered in detail, has been directed essentially to the directional effects obtained by the disclosed coupling means, i. e., directed to the character of the backward current in the auxiliary transmission line. It has been demonstrated that the directivity of a particular coupling depends upon the "shape" of the coupling distribution characteristic. It has been noted in passing, however, that conduction in the forward direction in the auxiliary transmission line de-

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pends upon factors which do not affect this "shape" of the coupling distribution characteristic. Further useful advantages may be obtained by increasing the magnitude of the coupling, either by increasing the length of the coupling interval or by increasing the number of individual coupling distributions, and thereby affecting the forward current in the manner to be described. This is done without varying the shape of the distribution and, therefore, without changing the directivity characteristic.

therefore, without changing the directivity characteristic.

Consider again, therefore, Fig. 1 and particularly the current transmitted in the forward direction in the auxiliary line 13 and within a length interval so small that negligible power is transmitted between the lines 10 and 13. The envelope of the traveling wave in the main transmission line 10 may be expressed

$$\frac{dE_1}{dx} = -\alpha E_1 + \alpha E_2 \tag{22}$$

and the traveling wave in the auxiliary transmission line 13 may be expressed as

$$\frac{dE_2}{dx} = -\alpha E_2 + \alpha E_1 \tag{23}$$

wherein  $\alpha$  represents the continuous coupling in radians per unit length between the lines. Assuming an input wave of magnitude unity impressed on the main transmission line and no wave impressed on the auxiliary transmission line, Equation 22 becomes

$$E_1 = \frac{1}{2} (1 + \epsilon^{-2\alpha x}) \tag{24}$$

and Equation 23 becomes

$$E_2 = \frac{1}{2} (1 - \epsilon^{-2a\pi}) \tag{25}$$

wherein x represents the distance over which the coupling is maintained.

The coupling factor  $\alpha$  in Equations 22 through 25 is a complex mathematical quantity. In a practical structure, however, the requirement of conservation of energy leads to the restriction that  $|E_1|^2 + |E_2|^2$  equal a constant for all values of x. Therefore, the quantity  $\alpha$  must be a pure imaginary. This results in a 90 degree phase difference between energy in lines 10 and 11 at all times. In Fig. 17 the magnitudes of waves on the two lines expressed by Equations 24 and 25 are plotted versus the product of coupling  $\alpha$  in radians per unit length times the distance x over which the coupling is maintained.

In Fig. 17 the magnitudes of waves on the two lines expressed by Equations 24 and 25 are plotted versus the product of coupling  $\alpha$  in radians per unit length times the distance x over which the coupling is maintained. The wave magnitude in the main transmission line 10 represented by curve 92 of Fig. 17 is seen to decline cosinusoidally and the wave magnitude in the auxiliary transmission line 13 represented by curve 93 of Fig. 17 is seen to increase sinusoidally as the coupling length x is increased. Complete power transfer between the lines takes place when the product  $|2\alpha x|$  is equal to  $m\pi$  radians, where m is any odd integer, and repeats cyclically as long as coupling is maintained. The coupling may be broken at any point where the waves in the two lines have a relation which it is desired to preserve. In other words, the power  $P_1$  in the main transmission line 10 and the power  $P_2$  in the auxiliary transmission line 13 will be divided in accordance with the ratio

$$\frac{P_1}{P_2} = \frac{\cos^2 |\alpha x|}{\sin^2 |\alpha x|} \tag{26}$$

The quantity  $\alpha x$  is chosen to provide hybrids with any desired transfer ratio.

An alternative method for increasing the coupling and controlling the amount of power transmitted in the forward direction is demonstrated in Fig. 18 by increasing the number of individual coupling units. Fig. 18 shows an insert, suitable for location in slot 16 of Fig. 1, which is provided with a composite divided aperture 95. The shape of aperture 95 is a combination of a plurality of cascaded apertures of the type shown and described hereinbefore with reference to Fig. 6. Only two such cascaded apertures are shown in Fig. 18, which two will be designated the left half of aperture 95 and the right half of aperture 95, respectively, but the number of cascaded apertures may be increased to n as will be shown. Thus, the left half of aperture 95 and the right half of aperture 95 may be considered as separate discrete couplings each having a good directivity and a determinable coupling loss. Energy transferred from

the main transmission line 10 to the auxiliary transthe main transmission line 10 to the auxiliary transmission line 13 through the left-hand coupling experiences a 90 degree phase delay. This energy travels along the auxiliary transmission line 13 to the right-hand coupling and part of this energy returns to the main transmission line 10 with the further phase delay of 90 degrees. Thus, energy which goes from the main transmission line 10 to the auxiliary transmission line 13 and back to the main transmission line 10 at a later 13 and back to the main transmission line 10 at a later coupling point arrives in the main transmission line 10 10 out of phase with the energy which travels straight through in the main transmission line 10. A summation of such components eventually results in cancellation of the wave in the main transmission line 10.

Designating the magnitude of the coupling through 15 either the left half or the right half of aperture 95 as C, the voltage V<sub>1</sub> in the auxiliary transmission line 13 after the first coupling unit, i. e., the left half of aperture 95, may be expressed as

$$V_1 = C \tag{27} 20$$

and the voltage E1 in the main transmission line 10 as

$$E_1 = \sqrt{1 - C^2}$$
 (28)

Upon passing the second coupling unit, i. e., the right 25 half of aperture 95, these voltages become

$$V_2 = \sqrt{1 - C^2} V_1 + CE_1 = 2C\sqrt{1 - C^2}$$
 (29)

$$E_2 = \sqrt{1 - C^2} E_1 - CV_1 = 1 - 2C^2 \tag{30}$$

Employing the transformation

$$C = \sin \theta$$
 (31)

Equations 29 and 30 may be expressed after n coupling units as

$$E_n = \cos n\theta$$
 (32)

$$V_n = \sin n\theta$$
 (33)

Equations 32 and 33 may be rewritten in the form

$$E_n = \cos \left[ n \sin^{-1}C \right] \tag{34}$$

$$V_n = \sin \left[ n \sin^{-1}C \right] \tag{35}$$

Thus, the required number n of identical coupling units having a coupling factor C may be determined for any required ratio in the voltages  $E_n$  and  $V_n$ . The desired ratio of power division would, therefore, be the square of the voltage expression. On the other hand, the desired relation between voltage or power may be obtained by a fixed number of coupling units having the required coupling factor C. In either case complete power transfer will take place between the lines when the quantity  $n \sin^{-1}C$  is equal to

$$\frac{m\pi}{2}$$

where m is any odd integer.

It is desirable in certain applications to cascade a plurality of directive apertures having different coupling losses. The desired power division may then be deter-The desired power division may then be deter- 60 mined as

$$E = \cos (n_1\theta_1 + n_2\theta_2 + n_3\theta_3 + \dots n_k\theta_k)$$
 (36)

$$V = \sin \left( n_1 \theta_1 + n_2 \theta_2 + \dots n_k \theta_k \right) \tag{37}$$

where E and V are the voltages in the main and auxiliary transmission lines, respectively, at the end of the series of couplings, and there are employed:

n<sub>1</sub> apertures of individual coupling C<sub>1</sub>  $n_2$  apertures of individual coupling  $C_2$   $n_3$  apertures of individual coupling  $C_3$ nk apertures of individual coupling Ck

and the transformation

$$C_1 = \sin \theta_1$$
 $C_2 = \sin \theta_2$ 
 $C_k = \sin \theta_k$ 

has been employed in writing the above expression for E and V. Equations 36 and 37 may be rewritten in the

$$E = \cos (n_1 \sin^{-1}C_1 + n_2 \sin^{-1}C_2 + n_3 \sin^{-1}C_3 + \dots n_k \sin^{-1}C_k$$
(38)  

$$V = \sin (n_1 \sin^{-1}C_1 + n_2 \sin^{-1}C_2 + n_3 \sin^{-1}C_2 + n_3 \sin^{-1}C_3 + \dots n_k \sin^{-1}C_k)$$
(39)

Again complete power transfer between the lines will take place when the expression in the parentheses of Equation 39 is equal to

> $m\pi$ 2

in which m may be any odd integer. It will be apparent that the coupling factors  $\alpha$  and Cas employed in the equations for power transfer in the forward direction of transmission hereinbefore are general expressions of distributed and discrete coupling, respectively. For this reason, the relations given obtain in the case of each type of electrical wave transmission system known in the art. The coupling therebetween may be obtained by any of the particular coupling means employed in each of these transmission systems.

In all cases, it is understood that the above-described arrangements are simply illustrative of a small number of the many possible specific embodiments which can represent applications of the principles of the invention. Numerous and varied other arrangements can readily be devised in accordance with said principles by those skilled in the art without departing from the spirit and scope of the invention.

What is claimed is:

1. Directional coupling apparatus for high frequency electrical energy comprising a main length of electrical transmission line, an auxiliary length of electrical trans-(30) 30 mission line coupled substantially distributively to said main line over a section of the length of said lines, the length of said section being greater than one-half wavelength of said energy, a first portion of said section being symmetrically disposed about the center of said section and having a length less than said section, the magnitude distribution of said coupling over said first portion being the sum of the component distribution of one of two discontinuous substantially linear tapers and a constant disposed symmetrically about said center, the magnitude of said coupling over the remaining portions of said section being a continuation of one of the component distributions over said fact requires with the other desired. tributions over said first portion with the other of said component distributions over said first portion with the other of said component distributions equal to zero, said first portion and said remaining portions proportioned so that the maximum amplitudes of the sum of the Fourier transforms of each are less than a value determined from the minimum amplitude of a given directivity characteristic for said apparatus said apparatus.

2. Directional coupling apparatus for electromagnetic wave energy comprising a main shielded transmission line for guiding said wave energy, an auxiliary shielded transmission line having a portion of its length contiguous to a portion of said main transmission line, and means for to a portion of said main transmission line, and means for coupling said transmission lines with a variation in coupling strength along said contiguous portion, said means comprising a common shield in said contiguous portion having a composite aperture therein of length greater than one-half wavelength of said energy, the shape of said composite aperture being the geometrical combination of at least two basic geometrical shapes that are symmetrical about a transverse axis of said lines that passes in the plane of said energy through its center, said agent. in the plane of said aperture through its center, said aperture being asymmetrical about every longitudinal axis of said aperture that passes in the plane of said aperture parallel to the axis of said lines so that the separate current components coupled from said main line into said auxiliary line by each of the composite shapes of said aperture and which each vary in amplitude and phase as functions of the dimensions of one of said shapes are in phase and amplitude to cancel with each other over a substantial band of frequency variations of said wave energy

3. Directional coupling apparatus for electromagnetic wave energy comprising a main shielded transmission line 75 for guiding said wave energy, an auxiliary shielded transmission line having a portion of its length contiguous to a portion of said main transmission line, and means coupling said transmission lines with a variation in coupling strength along said contiguous portion, said means com-prising a common shield in said contiguous portion having an aperture therein, the shape of said aperture being the geometrical combination of at least two rectangles which are each symmetrical about a transverse axis of said lines that passes in the plane of said aperture through (39) 85 its center, said rectangles having different longitudinal

dimensions each greater than one-half wavelength so that the separate current components coupled from said main line into said auxiliary line by each rectangle and which components each vary in amplitude and phase as functions of the dimensions of one of said rectangles are in phase and amplitude to cancel with each other over a substantial band of frequency variations of said wave energy.

4. Directional coupling apparatus for electromagnetic wave energy comprising a main shielded transmission line for guiding said wave energy, an auxiliary shielded transmission line having a portion of its length contiguous to a portion of said main transmission line, and means coupling said transmission lines with a variation in coupling strength along said contiguous portion, said means comprising a common shield in said contiguous portion 15 having an aperture therein, the shape of said aperture being the geometrical combination of a rectangle and a triangle located coextensive with each other along at least a portion of the length of said contiguous portion so that the separate current components coupled from said 20 main line into said auxiliary line by said rectangular portion and by said triangular portion which components each vary in amplitude and phase as functions of the dimensions of one portion are in phase and amplitude to cancel with each other over a substantial band of fre-

quency variations of said wave energy.

5. Directional coupling apparatus for electromagnetic wave energy comprising a main shielded transmission line for guiding said wave energy, an auxiliary shielded transmission line having a portion of its length contiguous to a portion of said main transmission line, and means coupling said transmission lines with a variation in coupling strength along at least one-half wavelength of said contiguous portion, said means comprising a common shield in said contiguous portion having an aperture therein, the shape of said aperture being the geometrical combination of a rectangle and a whole period of a cosine wave which are each symmetrical about a transverse axis of said lines that passes in the plane of said aperture through its center so that the separate current components coupled from said main line into said auxiliary line by said rectangular portion and by said cosine portion which components each

vary in amplitude and phase as functions of the dimensions

of one portion are in phase and amplitude to cancel with each other over a substantial band of frequency variations of said wave energy.

6. Directional coupling apparatus for high frequency electrical energy comprising a main electrical transmission line, an auxiliary electrical transmission line coupled distributively to said main line over a given longitudinal section of said lines, the length of said section being at least greater than one-half wavelength of said energy, the plot of the characteristic of the magnitude versus distance along said section of said distributed coupling and the distance coordinate of said plot forming an enclosed area, said area requiring more than two parameters to define its two characteristics of size and shape and being divisible by lines parallel to said distance coordinate into at least two parts each having their two characteristics of size and shape definable by less than three parameters, at least one of said two characteristics of each part being different from the corresponding characteristic of every other part with any of the parts having the same shape having different longitudinal lengths so that the separate current components coupled from said main line into said auxiliary line by each of said parts and which components each vary in amplitude and phase as functions of the size and shape of said parts are in phase and amplitude to cancel with each other over a substantial band of frequency variations of said wave energy.

7. Directional coupling apparatus for electromagnetic wave energy comprising a main shielded transmission line for guiding said wave energy, an auxiliary shielded transmission line having a portion of its length contiguous to a portion of said main transmission line, means coupling said transmission lines comprising a common shield in

said contiguous portion having an elongated aperture therein, the longitudinal dimension of said aperture being greater than one-half wavelength of said wave energy, the shape of said aperture being the geometrical combination of a rectangle and a triangle located coextensive with each other along at least a portion of said longitudinal dimension so that the separate current components coupled from said main line into said auxiliary line by said rectangular portion and by said triangular portion which components each vary in amplitude and phase as functions of the dimensions of one portion are in phase and amplitude to cancel with each other over a substantial band of frequency variations of said wave energy, and a grid comprising a plurality of wires extending across said aperture and dividing said aperture into a plurality of spaces each having dimensions parallel to said longitudinal dimension of less than one-half wavelength of said wave

8. Directional coupling apparatus for high frequency electrical energy comprising a main electrical transmission line, and an auxiliary electrical transmission line, means for coupling said lines distributively over a given longitudinal section of said lines with the coupling strength of said means varying with distance along the length of said section such that the amplitude of the energy coupled by said means increases by degrees from a first value of zero to a finite value and subsequently increases to at least one different finite value with at least one discontinuity in the degree of said increase between zero and said different finite value and subsequently returns to zero in stages that render the coupling strength characteristic symmetrical about the center of said longitudinal section, the location of said discontinuity along the length of said section which location determines the relative phases of current components coupled from said main line into said auxiliary line being selected relative to said finite values which values determine the relative absolute maximum amplitudes of said components to produce components in said auxiliary line that are in phase and amplitude to cancel with each other over a substantial band of frequency variations of said wave energy.

9. Directional coupling apparatus in accordance with

claim 8 in which at least one of said increases is an abrupt

change between two of said values.

10. Directional coupling apparatus in accordance with claim 8 in which at least one of said increases is an abrupt change between two of said values and at least another of said increases is a tapered increase between two of said values.

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