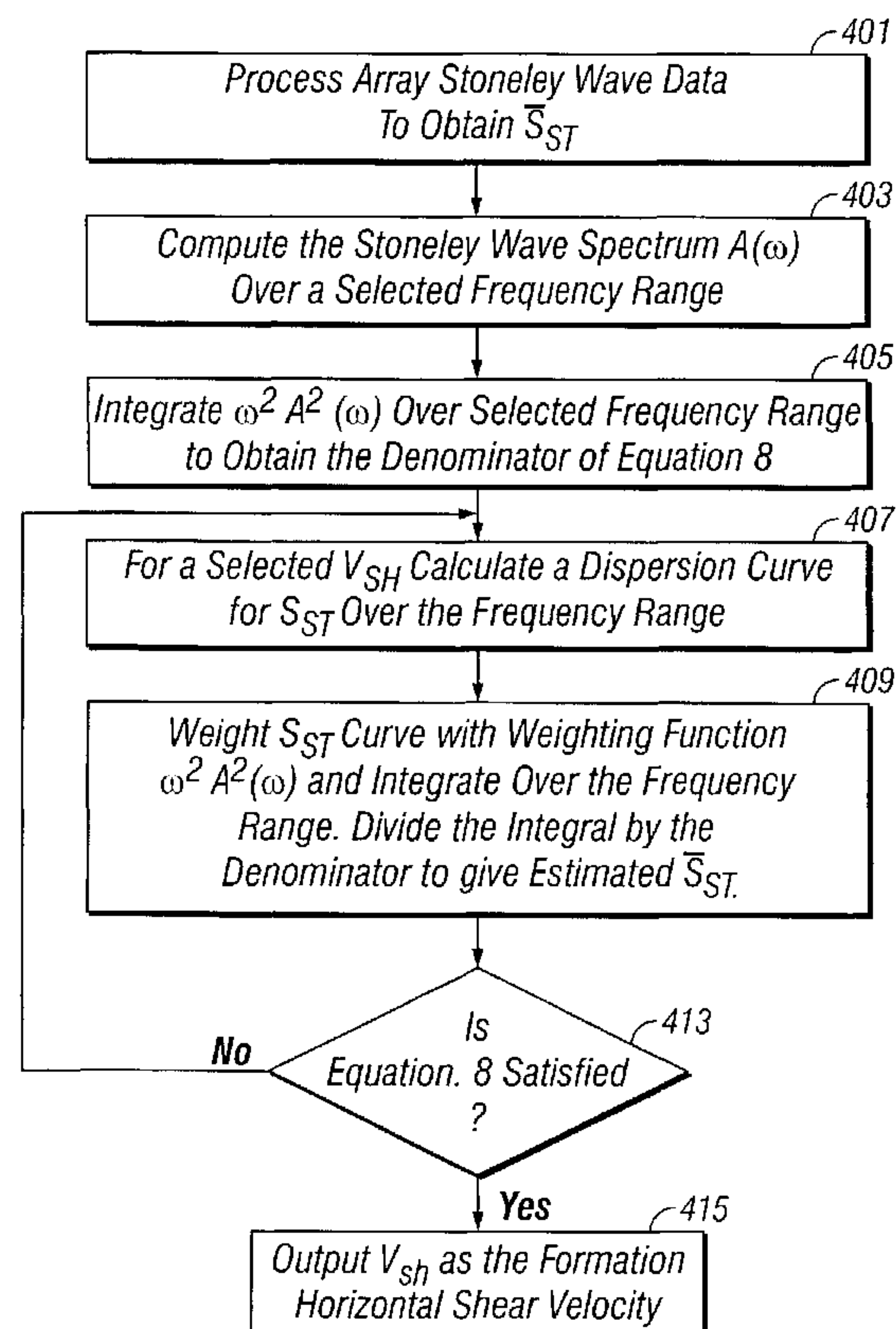




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(54) Titre : PROCEDE ET APPAREIL DE DETERMINATION DE L'ISOTROPIE TRANSVERSALE D'ONDES
EQUIVOLUMIQUES D'UNE FORMATION TERRESTRE A PARTIR DE MESURES D'ONDES DE STONELEY D'UN
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(54) Title: METHOD AND APPARATUS FOR DETERMINING EARTH FORMATION SHEAR-WAVE TRANSVERSE
ISOTROPY FROM BOREHOLE STONELEY-WAVE MEASUREMENTS



(57) Abrégé/Abstract:

The present invention is a method of determining formation horizontal shear wave velocity, formation transverse isotropy and an effective logging tool modulus. The method comprises determining an effective logging tool modulus by modeling the logging tool

(57) **Abrégé(suite)/Abstract(continued):**

as a fluid filled cylindrical shell. Measured Stoneley-wave slowness values are acquired for a formation. A horizontal formation shear wave velocity, V_{sh} , is calculated as a function of the measured Stoneley-wave slowness and an estimated Stoneley-wave slowness wherein the estimated average Stoneley-wave slowness is computed using the effective tool modulus. A difference between the measured Stoneley-wave slowness and the estimated Stoneley-wave slowness is minimized and the horizontal shear-wave velocity value for the minimized difference is output. Transverse isotropy may then be calculated as a function of the determined V_{sh} and vertical shear wave velocity, V_{sh} , determined from standard logging techniques.

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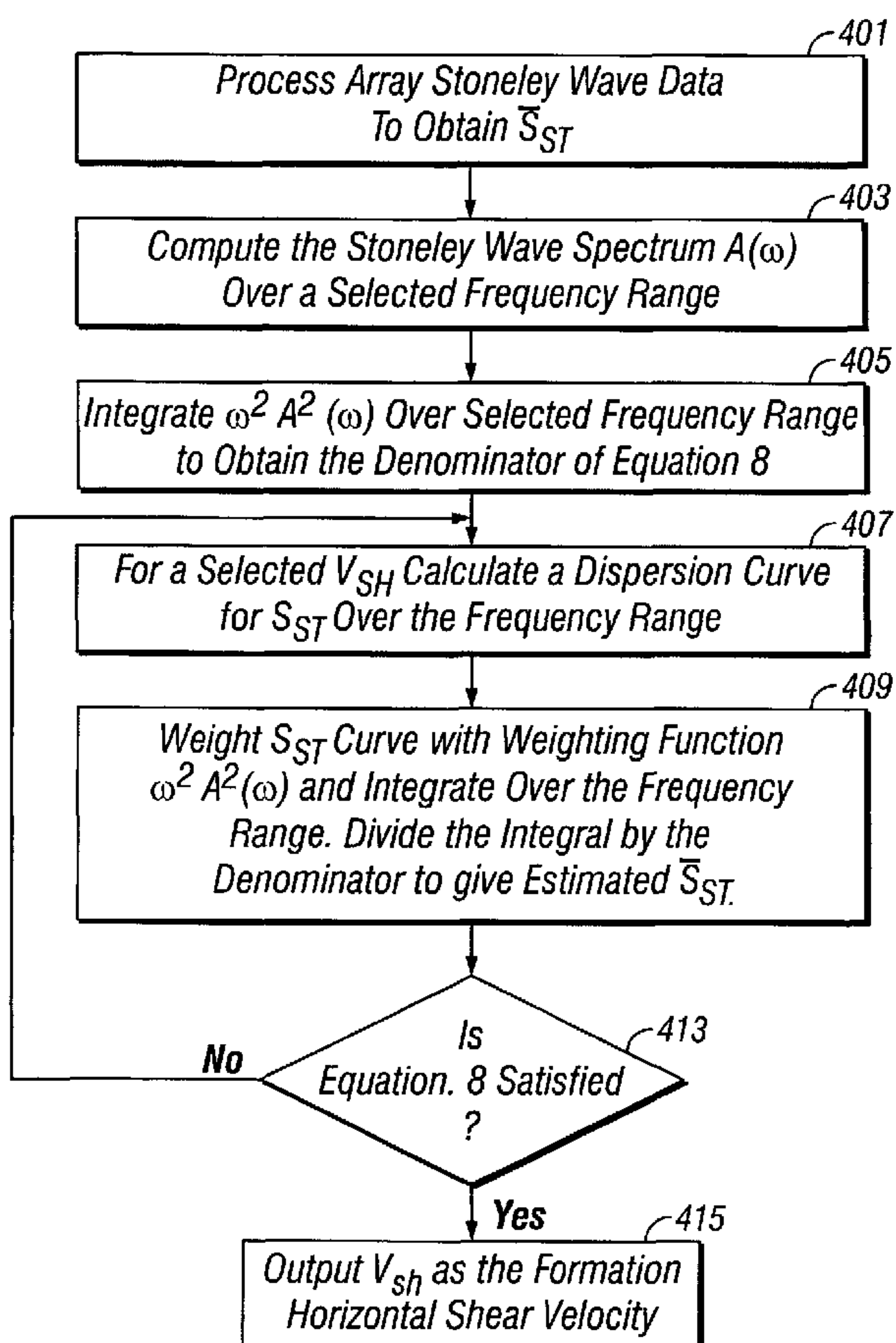
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(57) Abstract: The present invention is a method of determining formation horizontal shear wave velocity, formation transverse isotropy and an effective logging tool modulus. The method comprises determining an effective logging tool modulus by modeling the logging tool as a fluid filled cylindrical shell. Measured Stoneley-wave slowness values are acquired for a formation. A horizontal formation shear wave velocity, V_{sh} , is calculated as a function of the measured Stoneley-wave slowness and an estimated Stoneley-wave slowness wherein the estimated average Stoneley-wave slowness is computed using the effective tool modulus. A difference between the measured Stoneley-wave slowness and the estimated Stoneley-wave slowness is minimized and the horizontal shear-wave velocity value for the minimized difference is output. Transverse isotropy may then be calculated as a function of the determined V_{sh} and vertical shear wave velocity, V_{sh} , determined from standard logging techniques.

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**Method and Apparatus for Determining Earth Formation Shear-Wave
Transverse Isotropy from Borehole Stoneley-Wave Measurements**

FIELD OF THE INVENTION

5 [001] This invention generally relates to a method and apparatus for measuring and processing a characteristic of subsurface earth formations penetrated by a borehole. More specifically this invention relates to a method and apparatus for measuring and processing an acoustic characteristic such as slowness of subsurface sonic waves after these waves traverse earth
10 formations adjoining a borehole or passing through a portion of the subsurface.

BACKGROUND OF THE INVENTION

15 [002] Sonic wave information is used by the oil industry to examine and evaluate the earth's subsurface in the exploration and evaluation of valuable oil and gas, or hydrocarbon, reservoirs. Sonic waves are generated and recorded in oil well logging. This is called sonic or acoustic logging. The sonic wave measurement taken in well boreholes is typically the formation
20 compressional slowness (the reciprocal of velocity). However, many different acoustic wave types may measured, for example shear waves and Stoneley waves.

[003] Acoustic logging is performed in order to resolve the slowness or velocity structure of subsurface earth formations. One aspect of the velocity structure is that subsurface earth formations may be directional in nature such that velocities vary depending on the direction of acoustic wave propagation and the polarization of the wave. This variance in velocity structure due to direction of wave propagation is called seismic anisotropy. The subsurface earth formation information accuracy or resolution of velocity structure that is possible is directly related to both the acquisition and the processing of the acquired data for the sake of accurately locating the subsurface oil and gas reservoirs.

[004] Seismic wave anisotropy plays an increasingly important role in the exploration and production of hydrocarbon reservoirs. In seismic exploration, information about the earth formation anisotropy is needed to accurately delineate the reservoir from seismic wave data. In formation evaluation using borehole measurements, the anisotropy provides an important indication of pay zones, laminations and fractures. Determining seismic wave anisotropy from borehole acoustic wave measurements is thus an important task.

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SUMMARY

[005] The present invention is method of using an acoustic logging tool conveyed in a borehole in a formation for determining a characteristic of the formation, the logging tool having at least one source and a plurality of receivers. An effective tool modulus value for the logging tool is determined.

An Stoneley-wave slowness from array Stoneley wave data is measured. A horizontal formation shear velocity, V_{sh} , is calculated as a function of the measured Stoneley-wave slowness and an estimated Stoneley-wave slowness. The estimated Stoneley-wave slowness is computed using the effective tool
5 modulus value. A shear-wave transverse isotropy for the formation may be calculated using the calculated horizontal shear-wave value, V_{sh} , and a vertical shear wave value, V_{sv} , from standard logging techniques.

BRIEF DESCRIPTION OF THE DRAWINGS

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[006] The present invention and its advantages will be better understood by referring to the following detailed description and the attached drawings in which:

FIGURE 1 Illustrates Stoneley wave sensitivity for a fluid-filled borehole
15 with a TI formation.

FIGURE 2a Illustrates modeling the effect of the phase velocity of Stoneley waves in a formation, and the sensitivity of Stoneley wave modes due to a logging tool in a formation.

FIGURE 2b Illustrates Stoneley wave sensitivities in a TI formation with a
20 logging tool.

FIGURE 3a Illustrates synthetic array Stoneley wave data.

FIGURE 3b Illustrates a wave spectrum and dispersion curve for Stoneley wave data.

FIGURE 3c Illustrates a comparison between the results of wave-form averaging and the results of spectral averaging to determine slowness values for Stoneley wave data.

FIGURE 4 Illustrates a flow chart for determining a formation horizontal shear velocity.

FIGURE 5 Illustrates a flow chart for determining a logging tool compliance value, the tool effective modulus.

FIGURE 6a Illustrates a Stoneley wave waveform.

FIGURE 6b Illustrates a Stoneley wave amplitude spectrum.

FIGURE 7 Illustrates a well log of Stoneley wave TI anisotropy results in an earth formation.

FIGURE 8 Illustrates a well log of Stoneley wave TI anisotropy results in an earth formation.

[007] While the invention will be described in connection with its preferred embodiments, it will be understood that the invention is not limited thereto. On the contrary, it is intended to cover all alternatives, modifications, and equivalents which may be included within the spirit and scope of the invention, as defined by the appended claims.

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DETAILED DESCRIPTION OF THE PREFERRED EMBODIMENTS

[008] The present invention derives earth formation horizontal shear-wave slowness or velocity profiles from the acoustic data measured by an array acoustic tool. The derived horizontal shear-wave slowness is then used to

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determine earth formation shear wave anisotropy, by the virtue of determining transverse isotropic (TI) effects. The present invention takes advantage of the characteristic of Stoneley waves as these waves are the only borehole wave mode that has a significant sensitivity to transverse isotropic effects, especially when the formation is acoustically slow compared to borehole fluid.

[009] Many earth formation rocks exhibit anisotropic wave characteristics. A very common scenario is the anisotropy existing in many sedimentary rocks, such as shales. In many, if not most, cases anisotropy is modeled by its simplest form: Transverse Isotropy (TI). The TI anisotropy has a symmetry axis such that along any direction transverse to this axis one will see the same material property for a selected polarization (i.e., seismic wave velocity/slowness). Between the symmetry axis and any direction perpendicular to it, one will see a material property difference. A TI formation is described by five elastic constants:

$$[c_{11}, c_{13}, c_{33}, c_{44}, c_{66}] \quad (1)$$

[0010] The present invention is directed to resolving the last two parameters, c_{44} and c_{66} , which are related to the shear-wave propagation in a TI medium. Acoustic logging in a borehole surrounded by a TI formation (axis of symmetry coinciding with borehole) has been studied, among others, by White and Tongtaow (1981) and Tongtaow (1982) for the monopole case, and by Ellefsen (1990) for the multipole (monopole, dipole, and quadrupole) cases. Schmitt (1989) extended the work to study multipole-wave propagation

in poro-elastic formations where both the elastic parameters and permeability have the TI characteristics.

[0011] For almost all types of borehole acoustic waves, acoustic logging can only determine the vertical wave propagation velocity. The commonly used acoustic waves are: P- and S-waves in monopole logging, the flexural wave from dipole logging, and the screw wave from quadrupole logging, etc. The vertical propagation wave velocity is $V_p = \sqrt{c_{33}/\rho}$ for the P wave, and is $V_{sv} = \sqrt{c_{44}/\rho}$ for the S wave (including flexural and screw waves). This is because most borehole waves (monopole and dipole, etc.) involve wave motion or vibration transverse to the borehole. The only exception is the Stoneley wave. At low frequencies, the Stoneley wave, or tube wave, involves radial displacements that distort the circumference of the borehole. This circumferential distortion involves the shear modulus c_{66} . In fact, White and Tongtaow (1981) showed that the low-frequency limit of the tube-wave velocity is $V_{ST} = V_f / \sqrt{1 + \rho_f V_f^2 / c_{66}}$, (V_f and ρ_f are borehole fluid velocity and density, respectively), which is controlled by c_{66} , and is therefore related to horizontally rather than vertically propagating shear waves.

c_{11}	c_{33}	c_{13}	c_{44}	c_{66}	K_f	R
23.87	15.33	9.79	2.77	4.27	2.25	0.1

Table I. Formation TI constants, borehole fluid modulus (in GPa), and hole radius (in m) used in Figures 1 and 2.

[0012] To further demonstrate the sensitivity of the Stoneley wave to c_{66} over the frequency range of interest (0-5 kHz), **Figure 1** plots the wave's phase velocity sensitivity in a slow TI formation (see Table I) to three controlling parameters: the borehole fluid velocity V_f , formation horizontal shear velocity $V_{sh} = \sqrt{c_{66}/\rho}$, and vertical shear velocity $V_{sv} = \sqrt{c_{44}/\rho}$. The sensitivity is defined as the normalized velocity partial derivatives $[V_\gamma/V_{ST}(\omega)][\partial V_{ST}(\omega)/\partial V_\gamma]$, where the subscript γ denotes the subscripts f , sv , and sh , respectively. The Stoneley wave phase velocity $V_{ST}(\omega)$ is calculated using the dispersion equation (given in Equation 5 below) in the absence of a logging tool. As **Figure 1** shows, Stoneley wave sensitivity to the TI formation is mostly controlled by V_{sh} or c_{66} at low frequencies. Only at higher frequencies, does V_{sv} or c_{44} become an important parameter to affect the wave propagation. The borehole fluid velocity V_f is a dominating parameter at low frequencies. It can be seen that the sum of the three sensitivity curves is close to 1, especially at low frequencies (below 1.5 kHz). According to the theory of partition coefficients (equivalent to the normalized velocity partial derivatives, see Cheng et al., 1982), this indicates that the Stoneley wave sensitivity to other TI parameters, c_{11} , c_{13} , and c_{33} , is low for this slow TI formation.

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[0013] Because of the sensitivity of the Stoneley wave to c_{66} of a TI formation, it is desirable to estimate the TI parameter from borehole acoustic logging data, as this will yield a continuous log profile for assessing the

formation anisotropic characteristics (Ellefsen, 1990). Although the effects of TI on the Stoneley wave is well understood from the theoretical analyses, application of the theory to real acoustic logging data has been hindered by two major factors. The first is the lack of understanding for the effect of a compliant logging tool on the Stoneley wave propagation and an effective model to handle the effect. The second is the lack of an efficient inversion method to estimate the TI effects from the Stoneley wave data. The method of the present invention addresses these two problems.

[0014] The acoustic logging tool is modeled as a cylindrically-shaped structure, for example a cylindrical rod, with an effective modulus, which can be determined using a calibration method. This cylindrically-shaped structure model gives a good approximation when compared with exact solution of a cylindrical shell model. With this approach, the effects of the tool can be accounted for regardless of the actual structure and make-up of the tool. In the inversion processing of the Stoneley wave data, the Stoneley wave slowness is a weighted average of the Stoneley wave dispersion curve over the wave's frequency band. This provides a fast and effective method for the inversion. With this method and the known tool modulus and other formation and borehole parameters, V_{sh} or c_{66} can be efficiently and reliably estimated.

[0015] Application of the method of this invention to Stoneley wave logging data yields a continuous profile for the shear-wave TI parameter, commonly known as the Thomsen parameter γ (Thomsen, 1986). Many shale formations

exhibit a substantial TI characteristic, with γ reaching the value of 40%. A specific example for the Lewis shale formation in Braggs, Wyoming is presented. The obtained profile provides valuable information for seismic data processing and interpretation, as this indicates where and when anisotropy
5 needs to be considered when migrating the seismic data to image subsurface reservoirs.

[0016] Stoneley wave propagation in a TI formation with a logging tool may be modeled. Acoustic logging in a vertical well with a transversely isotropic
10 formation has been theoretically analyzed [see White and Tongtaow (1981); Tongtaow (1982); Ellefsen (1990)]. Although the analyses are very complicated as compared with the isotropic case, analytical solutions can still be derived to describe the wave propagation. The main focus of this section is to further develop the analyses to incorporate the presence of an acoustic-
15 logging tool in the borehole. A common practice was to model the tool as a rigid rod of radius a (Tang and Cheng, 1993), which is appropriate when the tool housing is made of steel that is almost incompressible compared to borehole fluid (Norris, 1990). Modern multi-pole (monopole and dipole) array acoustic tools, however, are quite compliant because of the requirement to
20 isolate wave propagation along the tool during dipole logging (e.g., Cowles et al., 1994). The tool compliance will have a substantial effect on the Stoneley wave propagation during monopole logging. The tool's compliance effect needs to be accounted for in the analysis of Stoneley wave data acquired by a logging tool.

[0017] The model of a logging tool should provide a simple and sufficiently accurate description of the tool compliance effects. The actual tool make-up is quite complicated. It may be slotted or cut to allow coupling of receivers to borehole fluid. The tool may also contain sound isolation materials and electronics, etc. However, if the wavelength is long compared to the diameter of the tool, the tool can be effectively modeled as an elastic rod for the Stoneley wave propagation. Further, at low frequencies, the tool's elasticity can be adequately modeled by an effective modulus, as will be described in the following analysis.

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[0018] Consider an acoustic wave propagating along a borehole containing a logging tool of radius a . The borehole wave will induce acoustic motion in the fluid annulus between the tool and formation (modeled as a TI medium). For the Stoneley wave problem, the general solution for fluid radial displacement u and pressure p is given by (Ellefsen, 1990)

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$$\begin{Bmatrix} u \\ p \end{Bmatrix} = \begin{Bmatrix} AfI_1(fr) - BfK_1(fr) \\ \rho_f \omega^2 [AI_0(fr) + BK_0(fr)] \end{Bmatrix}, \quad (a < r < R) \quad (2)$$

where $f = (k^2 - \omega^2 / V_f^2)^{1/2}$ is the radial acoustic wavenumber in fluid and r is the radial distance in the fluid annulus between tool radius a and borehole radius R ; The symbols k , ω , V_f , and ρ_f denote axial wavenumber, angular frequency, fluid acoustic velocity, and fluid density, respectively; I_n and K_n ($n = 0, 1$) denote the first and second kind modified Bessel functions of the n th order, respectively; The wave amplitude coefficients A and B are to

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be determined by the boundary conditions at the tool ($r=a$) and borehole ($r=R$) interfaces.

[0019] Using a quasi-static analysis, Norris (1990) showed that borehole fluid
 5 conductance (defined as u/p) of an elastic rod concentric with the borehole is
 given by

$$\left(\frac{u}{p}\right)_{r=a} = -\frac{a}{M_T}, \quad (3)$$

where M_T is a tool modulus, which, in this elastic rod case, is approximately
 given by the rod's Young's modulus E and Poisson's ratio ν , as
 10 $M_T \approx E/(1-\nu)$. Equation (3) is extended to the frequency regime assuming
 that (1), the quasi-static result still holds for low-frequency Stoneley waves
 and (2), the compliance of a cylindrical tool can be modeled by an effective
 modulus M_T regardless of the tool's internal structure. Equations (2) and (3)
 give a relation between the unknown coefficients, as

$$\frac{B}{A} = \frac{(M_T/a)fI_1(fa) + \rho_f\omega^2 I_0(fa)}{(M_T/a)fK_1(fa) - \rho_f\omega^2 K_0(fa)}. \quad (4)$$

Thus only one unknown coefficient (A or B) needs to be determined from the
 boundary condition at the interface between the borehole and the TI
 formation. This leads to a dispersion equation for computing the Stoneley
 wave propagation in a fluid-filled borehole centered by a logging tool and
 20 surrounded by a TI formation.

$$D(k, \omega, c_{66}, c_{44}, M_T, R, a, \rho, V_f, \rho_f, c_{11}, c_{13}, c_{33}) = 0. \quad (5)$$

The parameters in the above dispersion equation are ordered by their importance and relevance to the present problem. The functional form of the dispersion equation is given in the Appendix.

- 5 [0020] By solving the above dispersion equation in the frequency range of interest to find the root values of k for the Stoneley wave, as denoted by k_{ST} , the Stoneley-wave phase velocity (or slowness) dispersion curve is obtained:

$$V_{ST}(\omega) = \omega / k_{ST}, \text{ or } S_{ST}(\omega) = 1 / V_{ST}(\omega) = k_{ST} / \omega. \quad (6)$$

- 10 [0021] The validity of the simplified tool model is verified by comparing the modeling result with that of a cylindrically-shaped structure such as a cylindrical elastic shell model. The inner and outer radii of the shell are 0.035 m and 0.045 m, respectively. The shell's elastic parameters are $E = 48.8$ GPa and $\nu = 0.25$. The interior of the shell is filled with fluid (water). For this
15 cylindrical shell tool model, the effective tool modulus in Equation (3) can be derived using a method given by Norris (1990) in the low-frequency limit.

$$M_T = \frac{2a^2 / R^2}{\left(1 - \frac{a^2}{R^2}\right) \left(\frac{1}{\rho_f V_{ST}^2(0)} - \frac{1}{\rho_f V_f^2} \right) - \frac{1}{c_{66}}}, \quad (7)$$

- where $V_{ST}(0)$ is the zero-frequency limit of velocity for the Stoneley wave in the fluid annulus between borehole and the cylindrical shell. (Formation and
20 borehole parameters are in Table I.) This gives a value of 6.73 GPa for M_T . With this M_T value, the performance and accuracy of the approximate tool

model can now be tested. The tool is now modeled by its effective modulus M_T regardless of its internal structure. Considering the parameters involved even in this relatively simple tool model (i.e., the shell and fluid elastic constants and dimensions), the effective modulus model is truly a drastic simplification. Using the M_T value in the dispersion relation given in Equation (5), one obtains the Stoneley wave phase velocity curve as a function of frequency. **Figure 2a** shows the result of the approximate cylindrically-shaped structure model (dashed curve **201**) compared with the exact result (solid curve **203**) for the fluid-filled cylindrical shell model. The dispersion curve in the absence of the tool is also shown **205 Without Tool**. There is an excellent agreement between the two results at low frequencies as may be seen when the **201** curve and **203** curve merge **With Tool** (especially below 2.5 kHz), with only small differences as frequency increases. These differences are only of academic importance because in practice they are well within the errors of field Stoneley wave measurements. The approximate model therefore provides simple but sufficiently accurate modeling for the effect of a logging tool on Stoneley wave propagation.

[0022] The need to incorporate the logging tool effect in the Stoneley wave propagation and inversion problem can now be demonstrated. As shown in **Figure 2a**, the presence of the tool can substantially reduce the Stoneley wave velocity and changes the character of the wave dispersion curve. The presence of a tool can also significantly modify the wave's sensitivity to borehole fluid and formation elastic parameters. Using Equation (5), the Stoneley wave sensitivities of **Figure 1** for the TI formation are recalculated

for the presence of the logging tool. (The tool is modeled by $a = 0.045$ m and $M_T = 6.73$ GPa, as derived from the fluid-filled cylindrical shell model.)

Figure 2b shows the sensitivity analysis result. Compared with its respective counterpart in **Figure 1**, the Stoneley wave sensitivity to the fluid and formation parameters is reduced by the presence of the tool. The sum of the three sensitivities is appreciably less than one, as can be easily seen by adding their zero-frequency values. The reduction of the sensitivity to borehole fluid is quite significant. This is understandable because the presence of the tool replaces a portion of the borehole fluid and the wave's sensitivity is now partitioned to the tool's compliance. Fortunately, the Stoneley wave sensitivity to V_{sh} or c_{66} is still quite significant in the low-frequency range (below 2 kHz in this case), which ensures the estimation of this parameter from Stoneley wave measurements. The analysis results shown in **Figure 2a** and **Figure 2b** demonstrates the necessity to incorporate the effects of logging tool in the analysis of field Stoneley wave data.

[0023] This section describes a simple and effective formulation to estimate the shear-wave TI parameter from borehole Stoneley waves using the dispersion Equation (5). The Stoneley waves acquired by an array acoustic tool are usually processed using array coherence stacking (e.g., semblance, Kimball and Marzetta, 1986) or waveform inversion (Tang, 1997) methods, yielding a Stoneley wave slowness profile over the logging depths. These methods are non-dispersive techniques that do not consider the dispersion effect in the waveform data. However, the Stoneley waves, as shown in **Figure 2a**, can exhibit a quite significant dispersion effect in the low

frequency range of interest. (In the following text, Stoneley wave slowness is used, instead of velocity, to conform to the convention used in acoustic logging.)

5 [0024] Fortunately, there exists a theoretical relationship between the wave's dispersion characteristics and the wave slowness derived from a non-dispersive processing technique. This relationship is given by

$$\bar{S}_{ST} = \frac{\int_{-\infty}^{+\infty} S_{ST}(\omega, V_{sh}) \omega^2 A^2(\omega) d\omega}{\int_{-\infty}^{+\infty} \omega^2 A^2(\omega) d\omega}, \quad (8)$$

where $\bar{S}_{ST}$ is the Stoneley-wave slowness, as obtained from the non-dispersive array processing; $S_{ST}(\omega, V_{sh})$ is the Stoneley-wave phase slowness dispersion curve, as computed from Equations (5) and (6). This dispersion curve is parameterized by the horizontal shear velocity V_{sh} or c_{66} , assuming all other formation, borehole, and tool parameters, as needed to calculate Equation (5), are known; $A(\omega)$ is the Stoneley-wave amplitude spectrum. The integration in Equation (8) is over the entire wave spectrum covered by $A(\omega)$. The theoretical result given by Equation (8) can be called the Weighted Spectral Average Slowness theorem. The theorem states that the slowness resulting from a non-dispersive time-domain array processing method is a weighted spectral average of the wave's slowness dispersion curve over the frequency range of the wave spectrum. The weighting function is given by $\omega^2 A^2(\omega)$. Equation (8) is used to obtain an estimated Stoneley-wave slowness.

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[0025] An example of the weighted spectral average slowness theorem uses a numerical example to demonstrate the validity of the important result in Equation (8). Although this result was obtained from studying the dispersion effect of dipole-flexural waves, we use a synthetic Stoneley wave data set to demonstrate the general applicability of this result to dispersive waves. **Figure 3a** shows an array of synthetic Stoneley waves **301** for the slow TI formation listed in **Table I**. The waveforms also contain a high-frequency P-wave **303** whose frequency content is outside the frequency range of interest. **Figure 3b** shows the corresponding wave amplitude spectrum, **Filtered spectrum $A(\omega)$** , and the wave slowness dispersion curve, **slowness**. Both the waveform and dispersion curve show that the wave is quite dispersive toward the low frequency range, which is needed to test the theoretical result in Equation (8). As illustrated in **Figure 3b**, a band-pass filter of 1.5 kHz width, **Band-pass filter moving window**, is used to pass the wave frequency components within the frequency window specified by the filter. The filtered wave data are then processed using the predictive waveform inversion method (Tang 1997) to yield a wave slowness value for $\bar{S}_{ST}$. This slowness is then assigned to the frequency at the center of the frequency window for the band-pass filter. In the frequency domain, the amplitude spectrum of the filter is multiplied with that of the Stoneley wave, as shown in the same figure, to form the filtered wave amplitude spectrum $A(\omega)$. The wave spectrum is then used in Equation (8) to weight the wave dispersion curve and produce a weighted slowness value. These calculations are repeated for a range of the filter center frequency from 0.75 to 6 kHz, at an increment of 0.2 kHz. **Figure 3c** shows the

comparison of the waveform-processed slowness (open circles **Waveform Processing**) and the weighted spectral average slowness (solid curve **Spectral Averaging**) values, together with the wave's dispersion curve (dashed curve **Dispersion Curve**). As predicted by the theory represented by Equation (8),
 5 the waveform-processed slowness (**Waveform Processing**) does not fall onto the wave's phase dispersion curve (**Dispersion Curve**). Instead, it agrees excellently with the weighted spectral average slowness value (**Spectral Averaging**) given by Equation (8). This numerical example gives a proof of the theoretical relationship given in Equation (8) for the Stoneley wave case.

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[0026] Equation (8) can be used to provide a simple and effective estimation for the horizontal shear velocity V_{sh} or c_{66} from the Stoneley-wave logging data. As the equation shows, the Stoneley-wave slowness, as derived by a non-dispersive array processing method (semblance or waveform inversion),
 15 is related to the wave's phase dispersion curve over the frequency range occupied by the wave's spectrum. Equation (8) contains only one unknown parameter, V_{sh} or c_{66} . All other parameters, as needed to calculate the dispersion Equation (5), are assumed known or available from log data. For example, c_{44} , c_{33} , ρ , and R are available from conventional dipole shear-wave
 20 slowness, monopole P-wave slowness, density, and caliper logs. The borehole fluid parameters ρ_f and V_f can be estimated from the type of borehole fluid used. The TI parameters c_{11} and c_{13} can be treated as follows. The difference between c_{11} and c_{33} defines the P-wave TI parameter ε (Thomsen, 1986), which is generally correlated with the shear-wave TI parameter γ . Assuming

that ε and γ are about the same order gives $c_{11} = c_{33}(c_{66}/c_{44})$. We also assume $c_{13} = c_{33} - 2c_{44}$. These treatments, although approximate, will not significantly alter the estimation for V_{sh} or c_{66} , because the Stoneley wave is relatively insensitive to the parameters c_{11} , c_{13} , and c_{33} . The tool radius a is always available and the tool's effective modulus can be obtained from a calibration procedure to be described later.

[0027] By specifying all necessary parameters in the dispersion Equation (5), the shear-wave TI parameter, V_{sh} or c_{66} , can be estimated using Equation (8) by the following procedure:

Step 1: Process array Stoneley wave data to obtain $\bar{S}_{ST}$, the measured Stoneley-wave slowness.

Step 2: Compute the Stoneley wave spectrum $A(\omega)$. Integrate $\omega^2 A^2(\omega)$ over the frequency range of $A(\omega)$ to obtain the denominator of Equation (8).

Step 3: For a selected horizontal shear velocity V_{sh} , calculate the dispersion curve $S_{ST}(\omega, V_{sh})$ using Equations (5) and (6) including an effective tool modulus M_T .

Step 4: Weight $S_{ST}(\omega, V_{sh})$ with the weighting function $\omega^2 A^2(\omega)$ and integrate the weighted slowness curve over the frequency range of $A(\omega)$. Divide the integral value with the denominator value from step 2 to obtain an estimated Stoneley-wave slowness, then equate the

result to the measured Stoneley-wave slowness $\bar{S}_{ST}$ determined from the acquired data of Step 1.

Step 5: Repeat steps 3 and 4 for selected values of V_{sh} until Equation (8) is satisfied; that is, until an acceptable difference between the measured and estimated values for $\bar{S}_{ST}$ are achieved. Output V_{sh} as the formation horizontal shear-wave velocity when the difference is acceptable.

[0028] Repeating the above procedure for the logging depth range of interest yields a continuous formation horizontal shear-wave velocity (or slowness) profile. The vertical shear velocity (or slowness) profile may be obtained from standard logging techniques. With the vertical and horizontal shear velocity (slowness) profiles, a continuous profile of the shear-wave TI parameter, commonly known as the Thomsen parameter γ (Thomsen, 1986), can be calculated, as

$$\gamma = \frac{c_{66} - c_{44}}{2c_{44}} \approx 2 \frac{V_{sh} - V_{sv}}{V_{sv} + V_{sh}}. \quad (9)$$

The anisotropy profile can then be used to assess the degree of formation anisotropy for seismic and/or formation evaluation applications.

[0029] Deriving a formation horizontal shear-wave velocity profile is further illustrated by referring to the flow chart of **Figure 4**. Array Stoneley wave data are processed 401 to obtain $\bar{S}_{ST}$. For example, standard non-dispersive

predictive waveform inversion techniques may be used. The Stoneley wave spectrum $A(\omega)$ is computed 403 over a selected frequency range. For example, the frequencies in the range of from .75 Hz to 6 Hz has been found to be acceptable. For use as a denominator in the calculation of Equation (8),

5 $\omega^2 A^2(\omega)$ is integrated 405 over the selected frequency range. For a selected horizontal shear wave velocity, V_{sh} , a dispersion curve for $S_{ST}(\omega, V_{sh})$ over the frequency range is determined 407, using, for example, the bandpass method commonly known in signal processing, and processing with the predictive waveform inversion method. A range for values for selecting the

10 horizontal shear wave velocity, V_{sh} , is well known by practitioners in the art and may be preselected. The dispersion curve $S_{ST}(\omega, V_{sh})$ is then weighted with the weighting function $\omega^2 A^2(\omega)$ and integrated 409 over the frequency range of $A(\omega)$. This integration is then divided by the previously determined denominator to produce an estimated $\bar{S}_{ST}$, an estimate which is based on the

15 calculation of Equation (8). This estimated $\bar{S}_{ST}$ is then compared 413 to the originally acquired array Stoneley-wave slowness data $\bar{S}_{ST}$ of 401. When the difference between the estimated Stoneley wave slowness data $\bar{S}_{ST}$ of 409 and the acquired array Stoneley wave slowness data $\bar{S}_{ST}$ of 401 is at or within a selected minimum difference, the selected V_{sh} value may be output 415 as the

20 formation horizontal shear velocity from the selected range of values of V_{sh} . Many standard techniques are available for determining whether the differences determined in step 413 are acceptable.

[0030] As shown in the previous modeling example (Figure 2a and Figure 2b), the compliance of the tool can have an important influence on the Stoneley-wave propagation characteristics. A calibration procedure is used to determine the tool compliance, which is specified by the effective modulus M_T . For a formation interval of known anisotropy (e.g., zero anisotropy), the same form of equation as Equation (8) is used to determine the tool modulus M_T . For example, for a formation without anisotropy, the formation parameters (V_p , V_s , and ρ) needed to calculate the dispersion Equation (5) are available from log data, and the fluid parameters (V_f and ρ_f) can be estimated from the borehole fluid used. The only unknown parameter is the tool modulus M_T . The calibration procedure is essentially adjusting the value of M_T in the dispersion equation, in a manner analogous to the method for horizontal shear determination of equation (8), until the following equation is satisfied (e.g., the difference between the measured $\bar{S}_{ST}$ and estimated $\bar{S}_{ST}$ reaches an acceptable minimum.

$$\bar{S}_{ST} = \frac{\int_{-\infty}^{+\infty} S_{ST}(\omega, M_T) \omega^2 A^2(\omega) d\omega}{\int_{-\infty}^{+\infty} \omega^2 A^2(\omega) d\omega}. \quad (10)$$

Once the tool modulus is specified from calibration, it is used in the dispersion Equation (5) to calculate the Stoneley wave dispersion for the inversion processing to determine formation horizontal shear-wave velocity or slowness.

[0031] The method for calibrating the compliance of the logging tool, the tool modulus M_T , is illustrated in the flow chart of **Figure 5**. Array Stoneley wave data are processed **501** to obtain measured Stoneley-wave slowness, $\bar{S}_{ST}$, at a depth where horizontal shear-wave velocity, V_{sh} , is known and/or $V_{sh} = V_{sv}$. For example, standard non-dispersive predictive waveform inversion techniques may be used. The Stoneley wave spectrum $A(\omega)$ is computed **503** over a selected frequency range. For example, the frequency range of from . 0.25 to 3 kHz has been found to be acceptable. For use as a denominator in the calculation of Equation (10), $\omega^2 A^2(\omega)$ is integrated **505** over the selected frequency range. A dispersion curve for $S_{ST}(\omega, M_T)$ over the frequency range $A(\omega)$ is determined **507**, using, for example, the bandpass method previously disclosed, and processing with the predictive waveform inversion method. The dispersion curve $S_{ST}(\omega, M_T)$ is then weighted with the weighting function $\omega^2 A^2(\omega)$ and integrated **509** over the frequency range of $A(\omega)$. This integration is then divided by the previously determined denominator to produce an estimated $\bar{S}_{ST}$, an estimate which is based on the calculation of Equation (10). This estimated $\bar{S}_{ST}$ is then compared **513** to the originally acquired array Stoneley wave data $\bar{S}_{ST}$ of **501**. When the difference between the estimated slowness $\bar{S}_{ST}$ of **509** and the measured slowness, $\bar{S}_{ST}$, from the acquired array Stoneley wave data of **501** is at or within a selected minimum difference, the value of M_T used may be output **515** as the tool modulus M_T . Many standard techniques, well known

to practitioners of the art, are available for determining whether the differences determined in step 513 are acceptable.

[0032] The method of this invention has been applied to an acoustic logging data set from a well near Braggs, Wyoming. The well was drilled through the Lewis shale formation. The goal was to characterize the TI property of the Lewis shale formation in this area. Acoustic dipole and monopole waveform logging data were acquired throughout the formation. The low-frequency end of the monopole data was set to 0.2 kHz to allow for the acquisition of Stoneley waves in the waveform data. **Figure 6a** shows an example of the Stoneley waveform **601** at a depth in the Lewis shale formation (only receiver 1 waveform from an eight-receiver array is shown). The Stoneley wave frequency band as shown in **Figure 6b** is 0.2-4 kHz, with a peak frequency shown in the amplitude spectrum **603** at about 1 kHz. As suggested by the theoretical analyses of **Figure 1** and **Figure 2b**, the frequency range below 2 kHz is ideal for the shear-wave TI estimation, because the Stoneley-wave sensitivity to the TI parameter is most significant in the low-frequency range.

[0033] **Figure 7** shows the detailed analysis results for the transition zone at the top of the Lewis shale formation. Adjacent to the Lewis shale, the formation is characterized by shaly sandstone and sand/shale sequences. **Track 1** of the figure shows the gamma ray curve. The high gamma ray value (> 120 gAPI, arrow at **701** in **Track 1**) marks the beginning of the massive Lewis shale formation. **Track 2** and **Track 3** show, respectively, the

Stoneley- and dipole-wave data (plotted using a variable density display, only receiver 1 of an eight-receiver array is shown). Processing the Stoneley and dipole wave data yields the Stoneley wave slowness in **Track 4** (solid curve labeled '*DTST*') and the shear slowness in **Track 6** (dashed curve labeled '*DTSV*', standing for slowness of vertically propagating shear wave). As a quality-control check for the slowness curves, the curves are integrated over the transmitter to receiver 1 distance to give the respective travel time curves for the Stoneley wave (**Track 2**) and dipole-shear wave (**Track 3**). The travel time curves track the respective waveform quite well, indicating the validity of the slowness results.

[0034] An interesting feature is that the travel-time curve (or waveform) for the dipole data shows more character/variation than that of the Stoneley data. Relative to the Stoneley wave, the dipole-shear wave is much delayed in the shale formation compared to the formation above it. Because the Stoneley- and dipole-waves in a TI formation are controlled, respectively, by $V_{sh}(DTSH)$ and $V_{sv}(DTSV)$ (Ellefsen, 1990), this waveform/travel-time character/delay difference between the two type of waves reveals the difference between the two shear-wave velocities, providing an direct indication of the TI effect for the formation of interest. The dipole shear slowness (*DTSV* in **Track 6**) and the Stoneley wave slowness (*DTST* in **Track 4**) are utilized in Equation (8) to determine the horizontal shear slowness *DTSH*.

[0035] The borehole fluid is a water-based mud with $V_f = 1524$ m/s (5000 ft/s) and $\rho_f = 1.2$ g/cm³. The shaly sandstone interval around 1100' is chosen as the calibration depth to determine the tool modulus using Equation (10). The estimated horizontal shear slowness profile is shown in **Track 6** (solid curve labeled '*DTSH*'). This shear slowness profile shows a substantial difference (shaded area between *DTSH* and *DTSV*) compared with the vertical shear slowness *DTSV*. The two slowness curves are then used in Equation (9) to calculate the shear-wave TI anisotropy parameter γ . The γ profile is shown as the shaded curve in **Track 5**, scaled from 0 to 50%. The anisotropy shows a massive, continuous feature below the top of the Lewis shale formation, but tends to vanish in the shaly-sand interval and sandstone streaks above the formation.

[0036] The anisotropy effect can also be analyzed by comparing the measured Stoneley slowness having the TI effects with a computed slowness without the TI effects. Assuming isotropy for the formation, the vertical shear slowness curve (*DTSV* in **Track 6**) can be used to compute an isotropic Stoneley-wave dispersion curve, which, after the weighted-averaging process using Equation (8), gives the (isotropic) Stoneley slowness in **Track 4** (dashed curve labeled '*DTST (from DTSV)*'). The difference between the measured and computed (isotropic) Stoneley-wave slowness curves (shaded area between the two curves) also indicates the presence of anisotropy. In fact, the Stoneley wave slowness curves in **Track 4** can be regarded as the mapping/transformation, using Equations (5) and (8), of the shear wave slowness curves in **Track 6** for the isotropic and anisotropic (TI) cases.

[0037] **Figure 8** is the continuation of **Figure 7** into the main body of the Lewis shale formation. The data display and scales are the same as **Figure 7**. Notice that a substantial (TI) anisotropy is observed throughout this formation except some thin streaks. The anisotropy is quite significant, generally on the order of 20-30%. A prominent feature can be seen in a depth zone below 1300', where the dipole wave **Track 3** shows a significant delay while the corresponding response on the Stoneley wave **Track 2** is minimal. This feature corresponds to the significant increase in the difference between the two slowness curves in **Track 4** and **Track 6**, and the increase of anisotropy in **Track 5**. The anisotropy exceeds 30% between 1300' and 1350'. The anisotropy estimation from the Stoneley log data delineates the shear-wave (TI) anisotropy magnitude and variation of the Lewis shale formation.

[0038] In the absence of formation anisotropy (TI), the above procedure can also be used to estimate the formation shear-wave velocity (slowness) from Stoneley-wave logging data. In this case, the formation has only two elastic parameters: P- and shear-wave velocities. The former is available from logging and the latter can be estimated from the above procedure. Deriving shear-wave slowness from Stoneley waves logged in slow formations was a common practice before the inception of dipole acoustic logging. However, most calculations either ignored the presence of a tool, or used inappropriate tool models (rigid or having a low impedance). With the effective elastic modulus model and the effective estimation method as described in this study,

the accuracy and reliability of the shear-wave velocity estimation can be enhanced.

[0039] As in the shear slowness analysis using Stoneley waves, the Stoneley wave method is applicable mostly in slow formations where the formation shear rigidity is comparable or below the borehole fluid modulus. In this case the Stoneley wave is quite sensitive to formation shear wave properties, isotropic or anisotropic. However, in fast formations, the sensitivity diminishes (Ellefsen, 1990) and the estimation will suffer large errors or become invalid. Additionally, formation permeability is known to significantly affect the Stoneley wave propagation velocity, especially at low frequencies (Tang et al., 1991). Thus the TI parameter estimation method using low-frequency Stoneley waves may be less effective in highly permeable formations unless the permeability effect can be accounted for.

[0040] The Stoneley-wave estimation method of this invention obtains only the shear-wave anisotropy information. However, in seismic migration/imaging using P- and/or converted waves, the P-wave anisotropy parameter ε (Thomsen, 1986) is desired. Fortunately, in many rocks, such as shales, P-wave anisotropy and shear-wave anisotropy are strongly correlated (Thomsen, 1986). In this case, the shear-wave anisotropy can be correlated with the P-wave anisotropy data to delineate the magnitude and variation of the P-wave anisotropy. In this regard, the obtained shear-wave anisotropy provides important information for seismic migration/imaging.

[0041] In summary, Stoneley waves acquired in a borehole penetrating a transversely isotropic (TI) formation can be utilized to quantify the shear-wave TI property, yielding a continuous TI-parameter profile of the formation.

5 In the TI-parameter estimation, the effect of a logging tool needs to be considered, as the presence of the tool can significantly modify the Stoneley-wave sensitivity to the formation shear-wave property. For low-frequency Stoneley waves, the effect of the tool can be satisfactorily and accurately modeled using an effective tool modulus, which can be determined using a

10 calibration procedure. The Stoneley wave slowness, as obtained from a non-dispersive array processing techniques, is the weighted average of the Stoneley-wave dispersion curve over the frequency range of the wave spectrum. By relating the Stoneley slowness to the Stoneley dispersion curve computed using the formation TI parameter, a fast inversion method is

15 developed to estimate the TI parameter. Application of the method to the Stoneley wave data from the Lewis shale formation in Braggs, Wyoming, yields a continuous shear-wave (TI) anisotropy profile of the formation. This shale formation has high anisotropy values on the order of 20-30%.

20 **Appendix: Stoneley-wave Dispersion Equation for a Fluid-filled Borehole
Centered by a Logging Tool and Surrounded by a TI formation**

[0042] This Appendix extends the work of Tongtaow (1982) and Ellefsen (1990) for a monopole-wave propagation in a fluid-filled borehole with a TI formation to incorporate the presence of a compliant logging tool. In a TI

solid, elastic waves can no longer be generally resolved into independent
 compressional (P) and shear waves, as in the isotropic case. Instead, the waves
 are designated as “quasi-P” (“quasi-S”) to describe the small coupling of the P
 (shear) wave vibration to the shear (P) wave. There is also another shear wave
 5 involving transverse motion confined to the horizontal plane, the SH shear
 wave. In the acoustic logging problem, the elastic waves in the TI formation
 are coupled with the acoustic wave motion in the borehole fluid. The solutions
 for the borehole fluid displacement u and pressure p have already been given
 in Equations (2). At the borehole interface, the borehole fluid conductance
 10 u/p must match that of the (TI) formation, as

$$\left(\frac{u}{p} \right)_{r=R} = \left(\frac{u_r}{-\sigma_{rr}} \right)_{r=R} \quad (\text{A-1})$$

where u_r and σ_{rr} are the radial displacement and stress of the TI solid,
 respectively. The shear stress σ_{rz} of the TI solid must vanish at the borehole
 15 interface.

$$\sigma_{rz} = 0, \text{ (at } r = R \text{)}. \quad (\text{A-2})$$

[0043] The solutions for u_r , σ_{rr} , and σ_{rz} have been given by Tongtaow
 (1982) and Ellefsen (1990). Using these solutions and Equations (3), (A-1),
 and (A-2), a monopole-wave dispersion equation is derived, as given by:

$$D(k, \omega) = \det \begin{pmatrix} TI_{11} & TI_{12} & TI_{13} \\ TI_{21} & TI_{22} & TI_{23} \\ TI_{31} & TI_{32} & TI_{33} \end{pmatrix} = 0 \quad (\text{A-3})$$

where $\det$ denotes taking the determinant of the matrix (TI_{ij}) for the TI formation, whose elements are given by:

$$\begin{cases} TI_{11} = f [I_1(fR) - E_{tool} K_1(fR)] \\ TI_{12} = -q_p (1 + ika') K_1(q_p R) \\ TI_{13} = -q_s (b' + ik) K_1(q_s R) \\ TI_{21} = -\rho_f \omega^2 [I_0(fR) + E_{tool} K_0(fR)] \\ TI_{22} = [c_{11} q_p^2 - c_{13} k^2 + (c_{11} - c_{13}) ika' q_p^2] K_0(q_p R) + \frac{2q_p c_{66}}{R} (1 + ika') K_1(q_p R) \\ TI_{23} = [(c_{11} q_s^2 - c_{13} k^2) b' + (c_{11} - c_{13}) ik q_s^2] K_0(q_s R) + \frac{2q_s c_{66}}{R} (b' + ik) K_1(q_s R) \\ TI_{31} = 0 \\ TI_{32} = c_{44} [a' (k^2 + q_p^2) - 2ik] q_p K_1(q_p R) \\ TI_{33} = c_{44} [(k^2 + q_s^2) - 2ikb'] q_s K_1(q_s R) \end{cases}$$

5 (A-4)

where I_n and K_n ($n=0,1$) denote the first and second kind modified Bessel functions of the n th order, respectively. The radial wavenumbers for the borehole fluid, formation quasi-P and quasi-S waves are given by:

$$10 \quad f = \left(k^2 - \frac{\omega^2}{V_f^2} \right)^{1/2} ; q_p = \omega \sqrt{\frac{-B + \sqrt{B^2 - 4AC}}{2A}} ; q_s = \omega \sqrt{\frac{-B - \sqrt{B^2 - 4AC}}{2A}}$$

(A-5)

with the symbols A , B , and C given by

$$\begin{cases} A = c_{11}c_{44} \\ B = \rho(c_{11} + c_{44}) - (c_{11}c_{33} - c_{13}^2 - 2c_{13}c_{44})(k^2 / \omega^2) \\ C = c_{33}c_{44}(\rho / c_{44} - k^2 / \omega^2)(\rho / c_{33} - k^2 / \omega^2) \end{cases} \quad (\text{A-6})$$

[0044] The shear coupling to the quasi-P wave, a' , and the compressional coupling to the quasi-S wave, b' , coefficients, as appear in Equations (A-4),
5 are given by

$$\begin{aligned} a' &= -\frac{1}{ik} \frac{(c_{13} + 2c_{44})k^2 - c_{11}q_p^2 - \rho\omega^2}{c_{44}k^2 - (c_{11} - c_{13} - c_{44})q_p^2 - \rho\omega^2}, \\ b' &= -ik \frac{c_{44}k^2 - (c_{11} - c_{13} - c_{44})q_s^2 - \rho\omega^2}{(c_{13} + 2c_{44})k^2 - c_{11}q_s^2 - \rho\omega^2}. \end{aligned} \quad (\text{A-7})$$

Of particular importance for this study is the coefficient, E_{tool} , as appears in TI_{11} and TI_{21} of Equations (A-4), which models the effect of a compliant acoustic logging tool on the wave propagation. This coefficient is given in
10 Equation (4), and is rewritten here for discussions.

$$E_{tool} = \frac{(M_T / a)f I_1(fa) + \rho_f \omega^2 I_0(fa)}{(M_T / a)f K_1(fa) - \rho_f \omega^2 K_0(fa)}. \quad (\text{A-8})$$

Two special cases can be derived from the above expression. The first is the fluid-filled borehole without tool. This is case is derived by setting the tool radius a to zero, leading to $E_{tool} = 0$. Equation (A-4) then reduces the result of
15 Ellefsen (1990). The second case is the rigid tool case studied by Tang and Cheng (1993). This case is modeled by letting $M_T \rightarrow \infty$, resulting in $E_{tool} = I_1(fa) / K_1(fa)$, in agreement with Tang and Cheng (1993).

[0045] By setting the determinant in Equation (A-3) to zero, the resulting equation is called the dispersion equation. The wavenumbers, k , satisfying the dispersion equation are found numerically for each given angular frequency ω .

SUBSTITUTE CLAIMS

1. A method of using an acoustic logging tool conveyed in a borehole in a formation for determining a characteristic of the formation, said logging tool having at least one source and a plurality of receivers, the method comprising:
 - (a) obtaining an effective tool modulus of said logging tool;
 - (b) measuring a Stoneley-wave slowness from array Stoneley-wave data; and,
 - (c) calculating a horizontal formation shear wave velocity, V_{sh} , using said effective tool modulus of said logging tool, said measured Stoneley wave slowness and a weighted average of a Stoneley wave dispersion curve.
2. The method according to claim 1 further comprising calculating a transverse isotropy for said formation as a function of said calculated horizontal shear wave velocity V_{sh} and a vertical shear wave velocity V_{sv} .
3. The method according to claim 2 wherein a formation shear-wave transverse isotropy parameter, γ , is calculated as

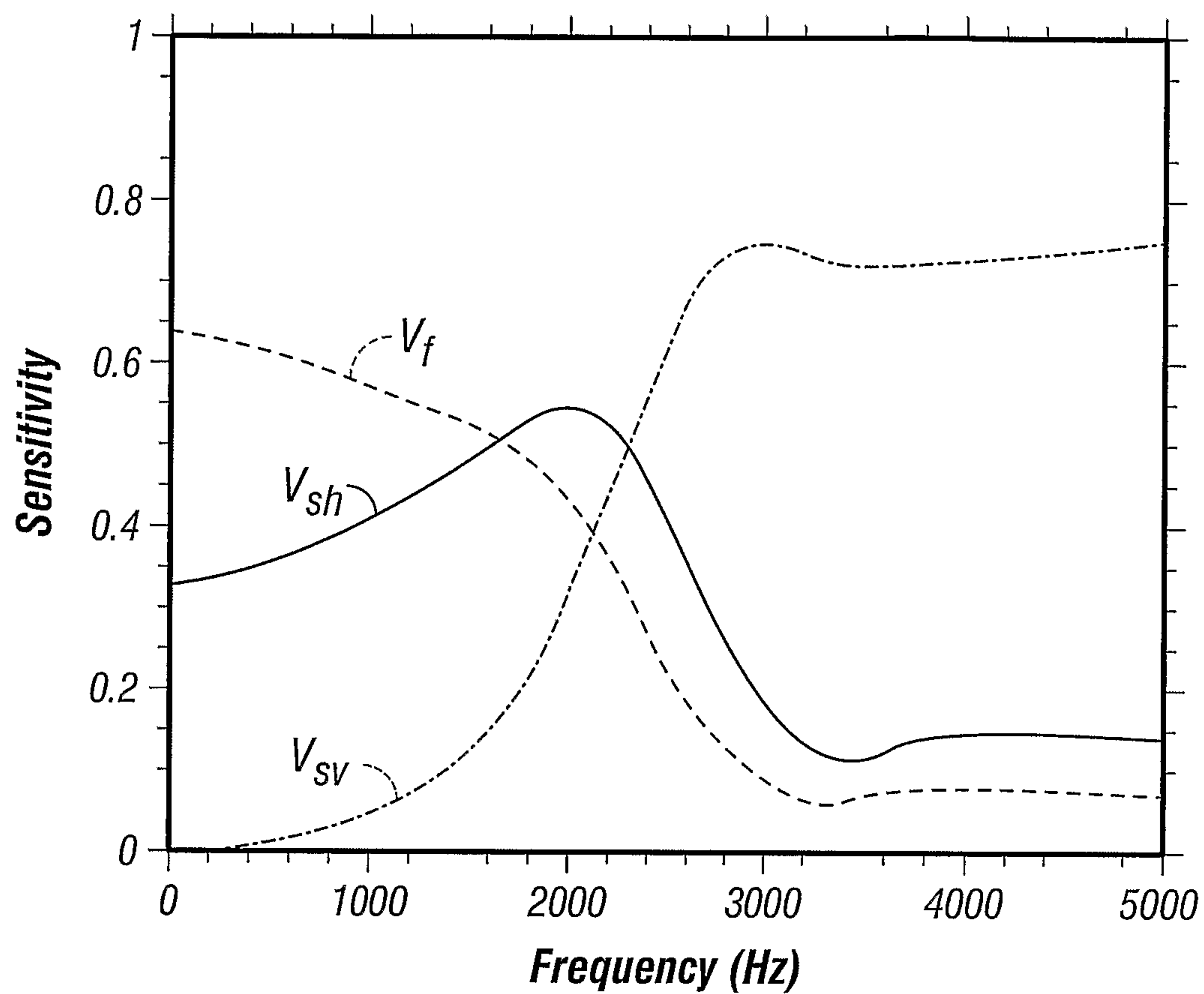
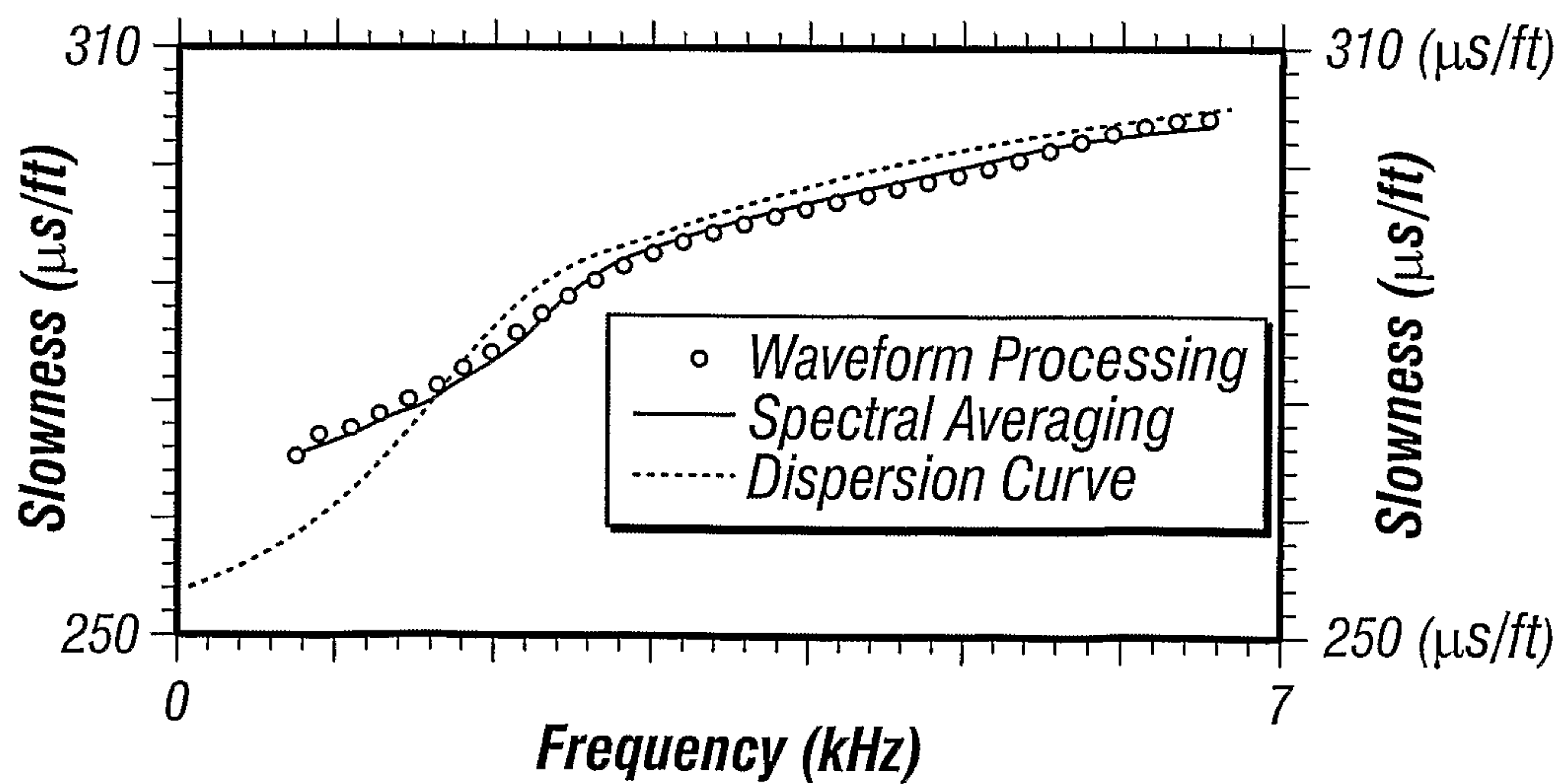
$$\gamma = \frac{c_{66} - c_{44}}{2c_{44}} \approx 2 \frac{V_{sh} - V_{sv}}{V_{sv} + V_{sh}}.$$
4. The method according to claim 1 wherein calculating said horizontal formation shear velocity, V_{sh} , further comprises minimizing a

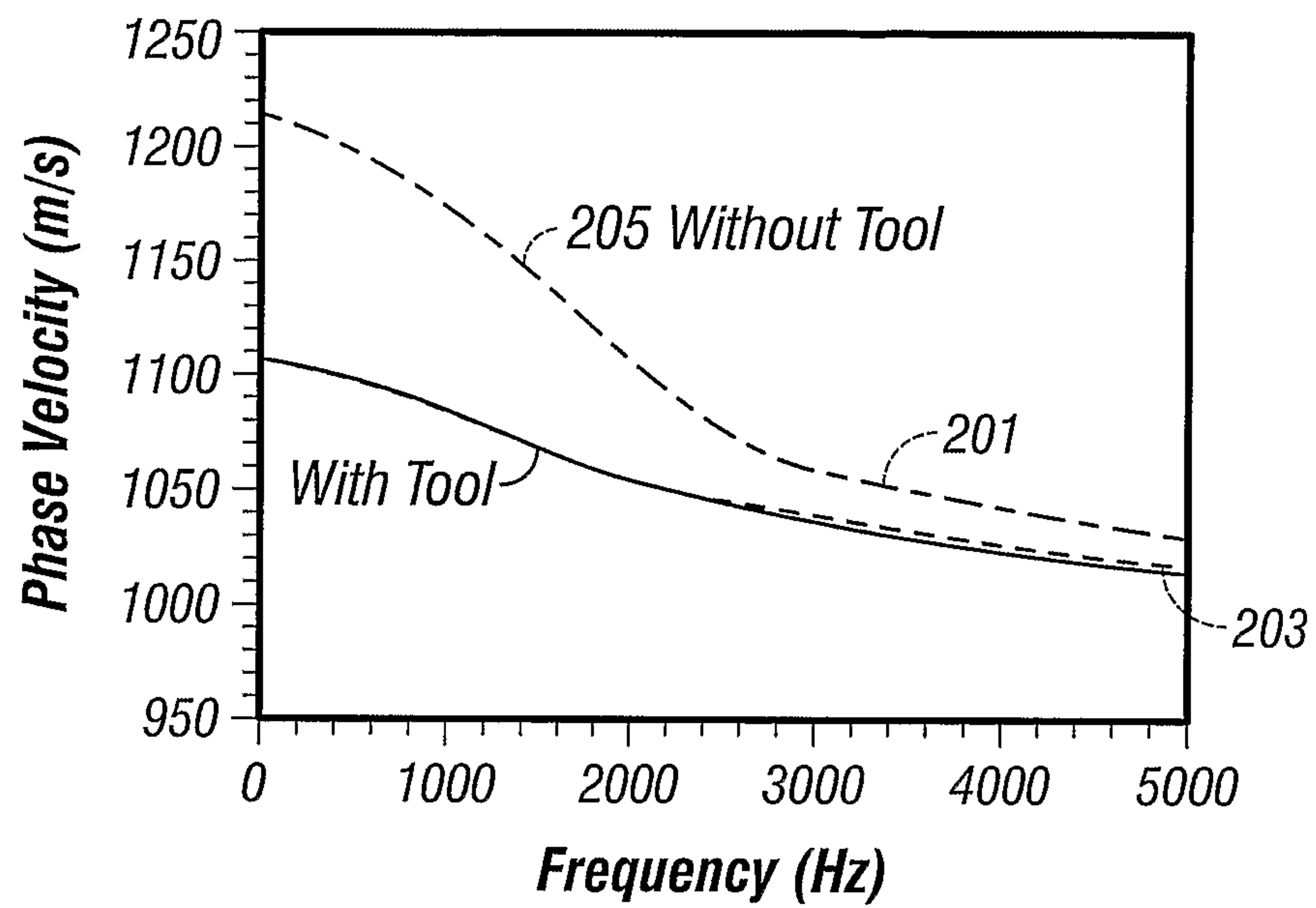
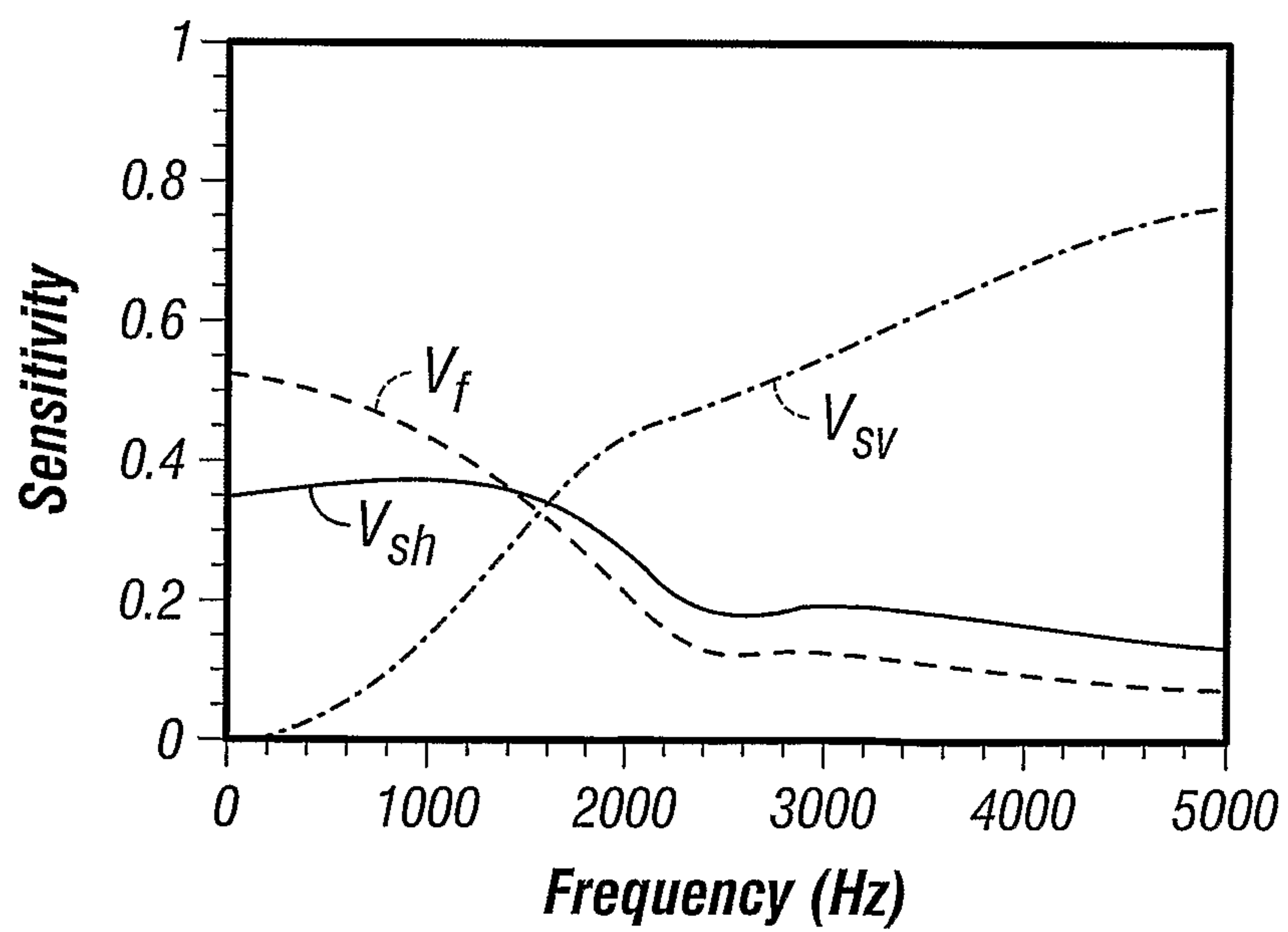
difference between said Stoneley-wave slowness and an estimated Stoneley-wave slowness computed from a calculation of the form

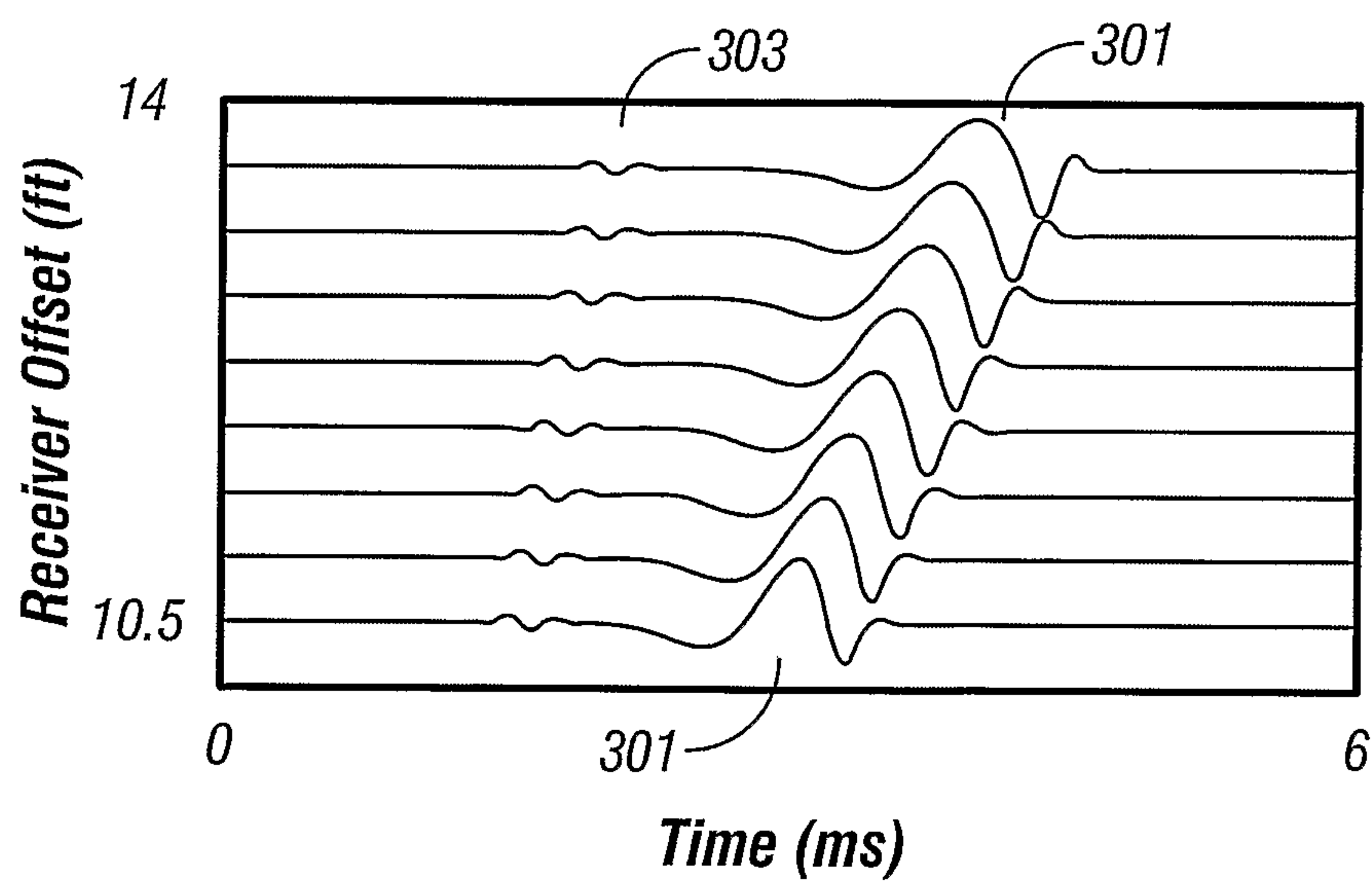
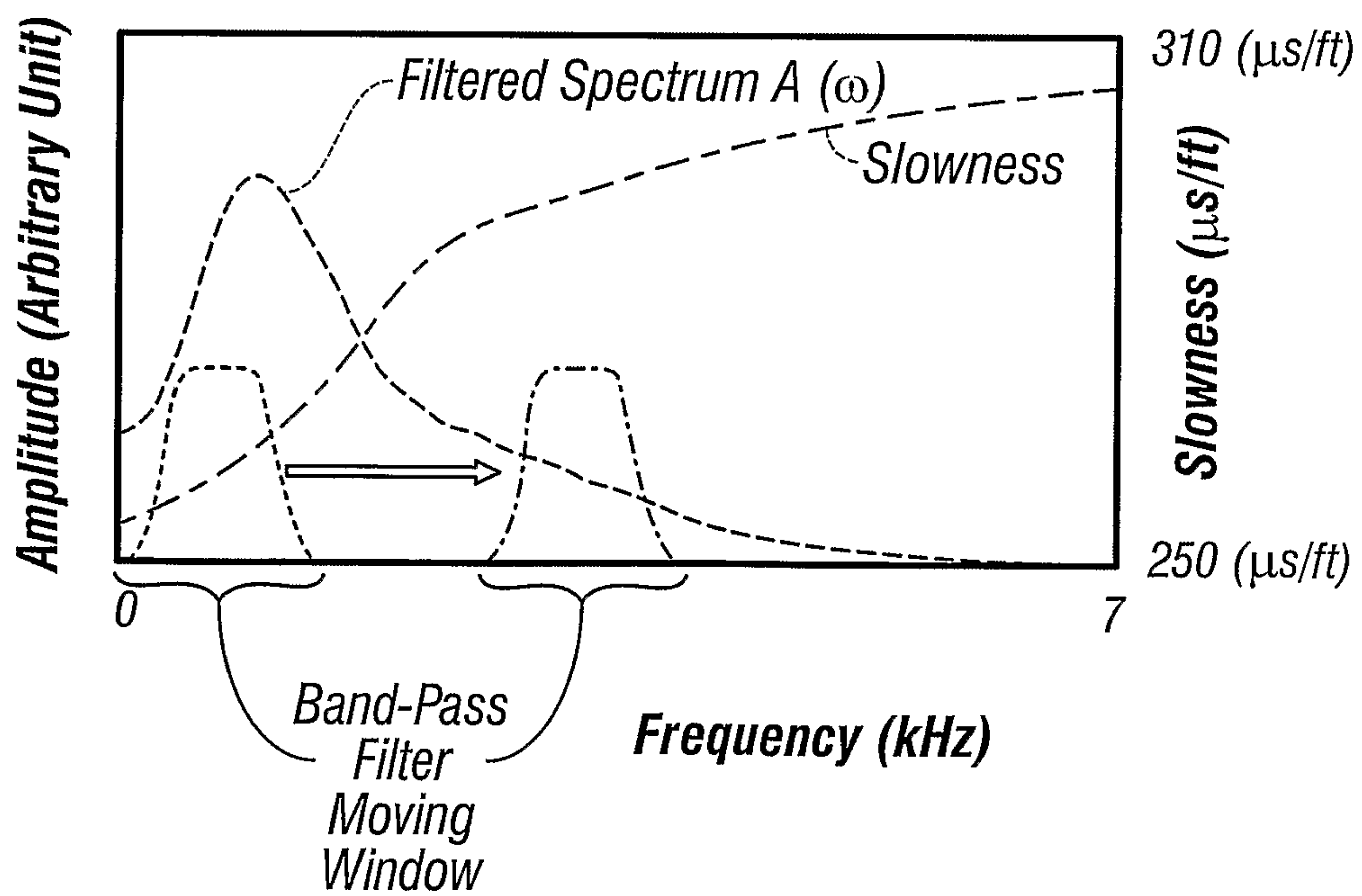
$$\bar{S}_{ST} = \frac{\int_{-\infty}^{+\infty} S_{ST}(\omega, V_{sh}) \omega^2 A^2(\omega) d\omega}{\int_{-\infty}^{+\infty} \omega^2 A^2(\omega) d\omega} \text{ for selected values of } V_{sh}.$$

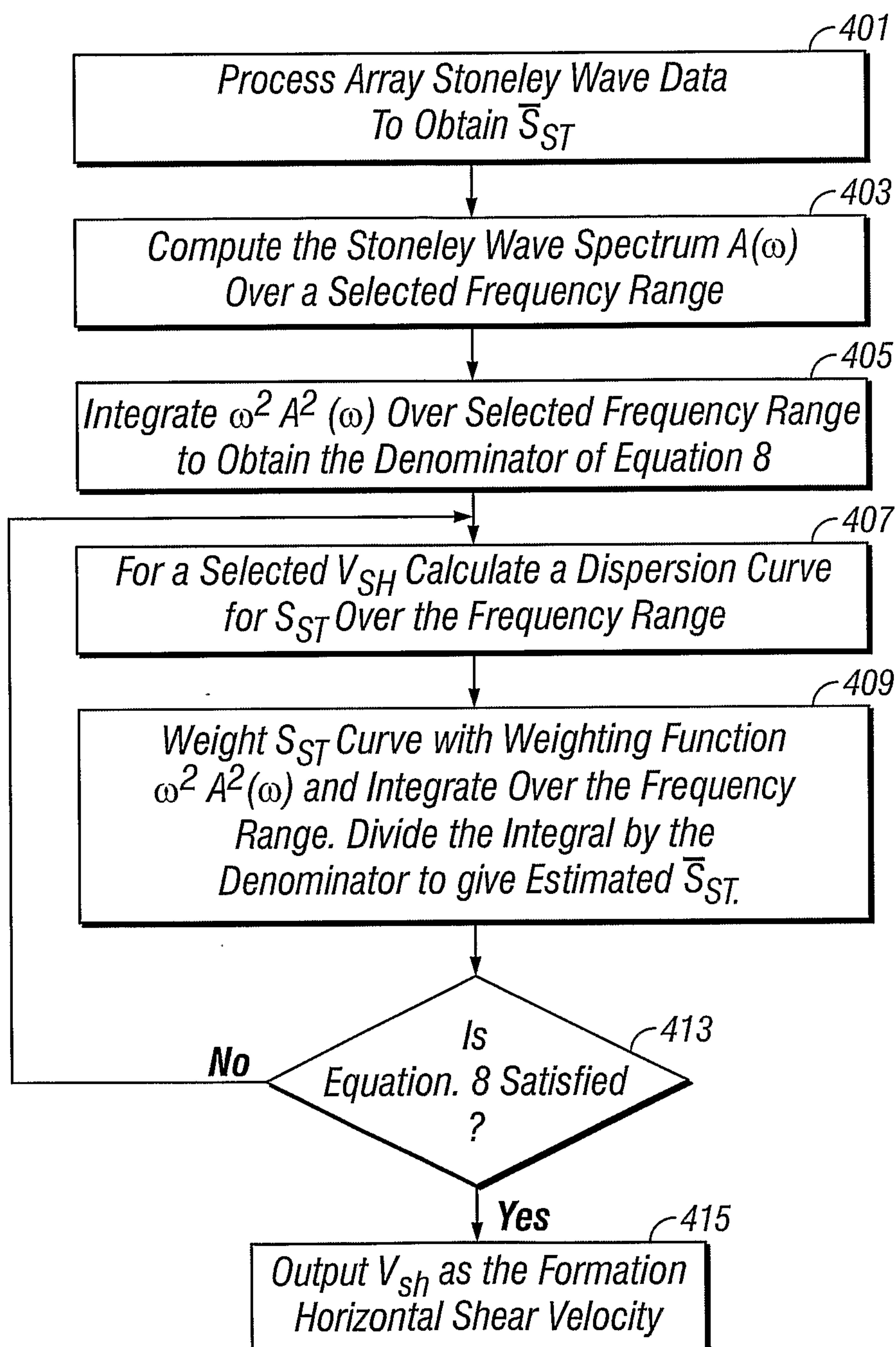
5. The method according to claim 1 wherein obtaining said effective tool modulus of said logging tool further comprises modeling said logging tool as a cylindrically-shaped structure.
6. The method according to claim 1 wherein obtaining said effective tool modulus of said logging tool further comprises:
 - i) measuring a Stoneley-wave slowness from array Stoneley-wave data in a formation where a horizontal shear wave velocity is known; and
 - ii) calculating an effective logging tool modulus for said logging tool wherein said logging tool modulus is calculated as a function of said measured Stoneley-wave slowness and an estimated Stoneley-wave slowness, said estimated Stoneley-wave slowness computed using said known horizontal shear wave velocity.
7. The method according to claim 6 wherein calculating said effective tool modulus further comprises minimizing a difference between said Stoneley-wave slowness and said estimated Stoneley-wave slowness computed using said known horizontal shear wave velocity from

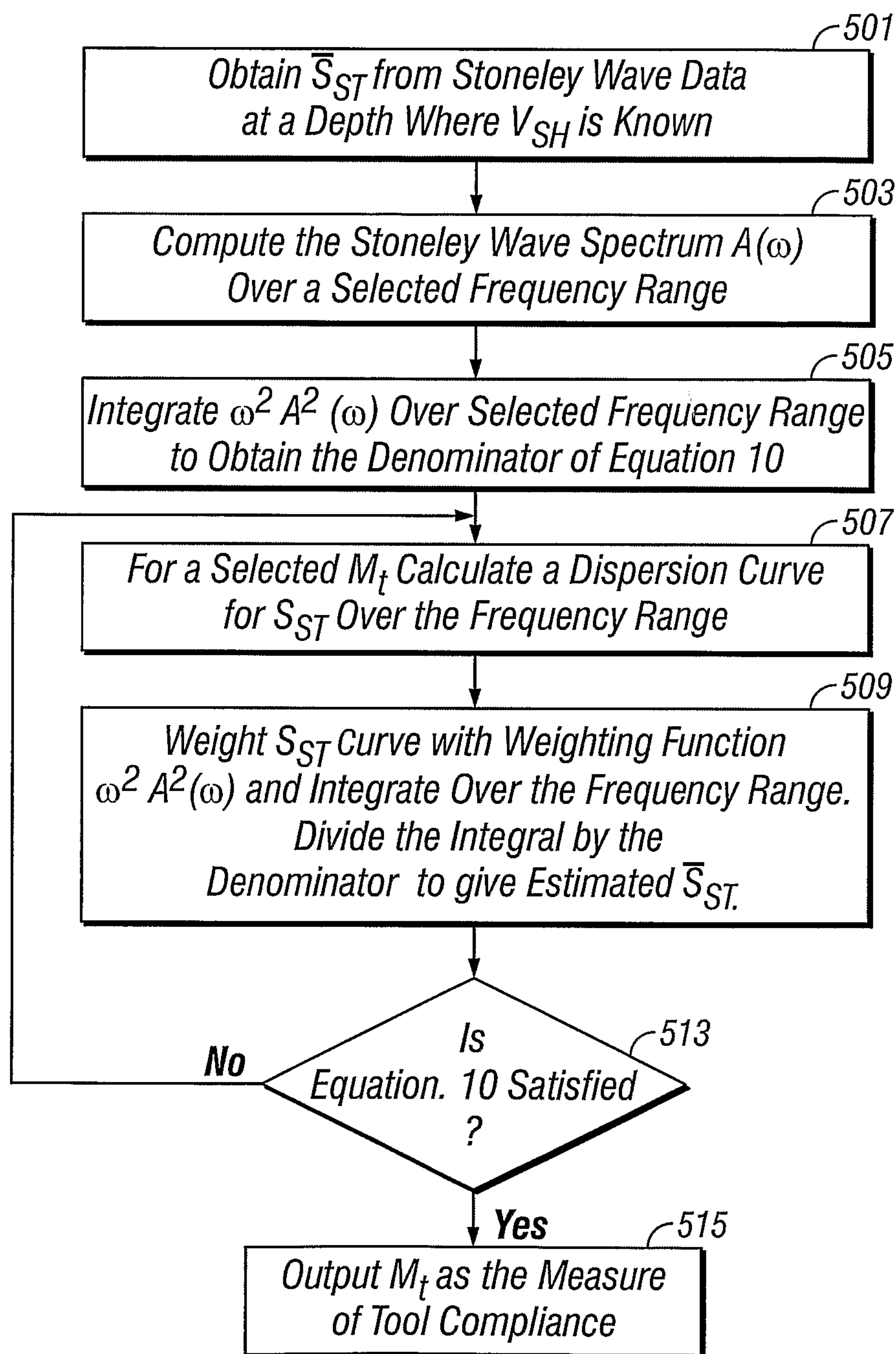
$$\bar{S}_{ST} = \frac{\int_{-\infty}^{\infty} S_{ST}(\omega, M_T) \omega^2 A^2(\omega) d\omega}{\int_{-\infty}^{\infty} \omega^2 A^2(\omega) d\omega} \text{ for selected values of } M_T.$$

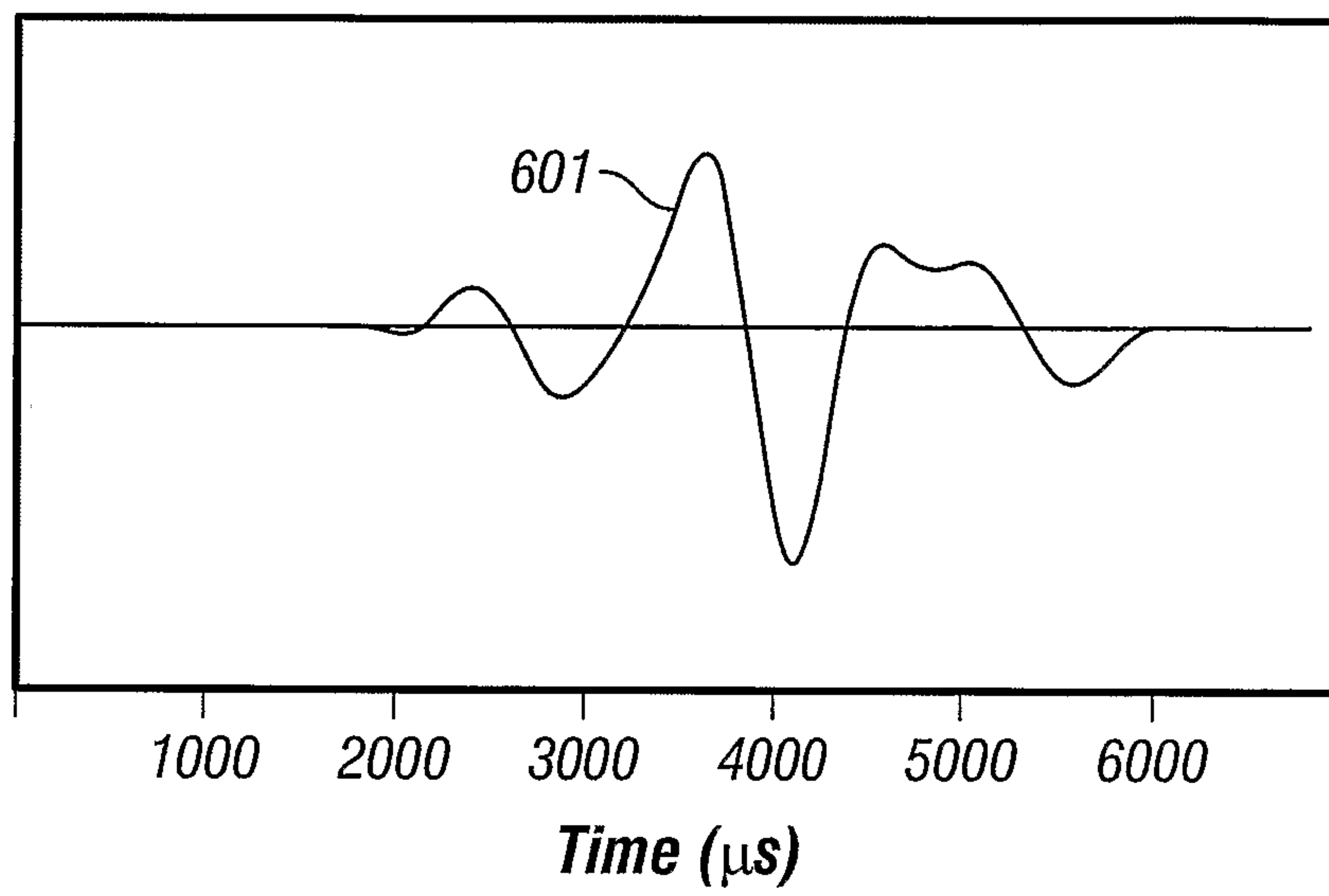
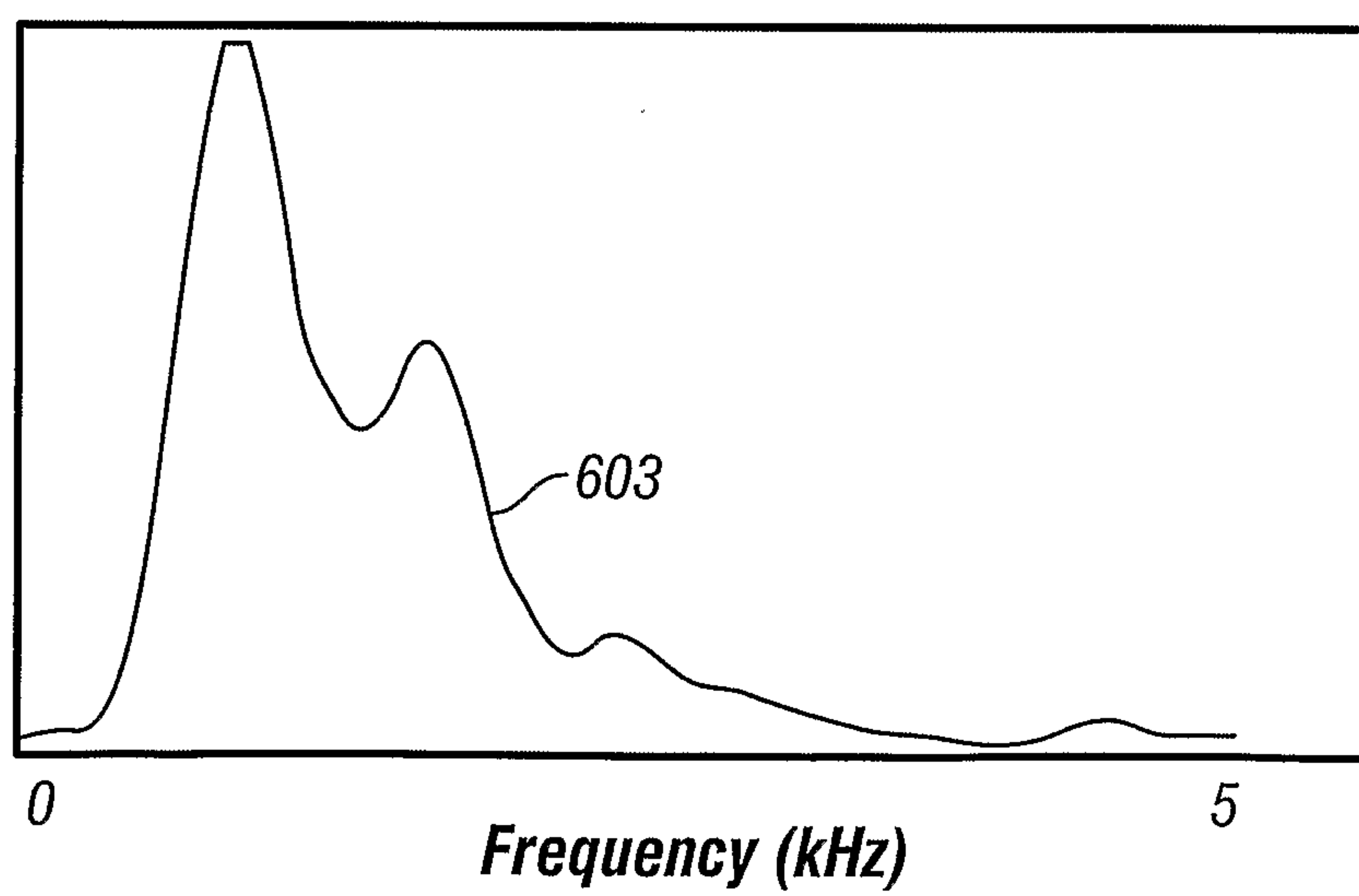
1/10**FIG. 1****FIG. 3C**

2/10**FIG. 2A****FIG. 2B**

3/10**FIG. 3A****FIG. 3B**

4/10**FIG. 4**

5/10**FIG. 5**

6/10**FIG. 6A****FIG. 6B**

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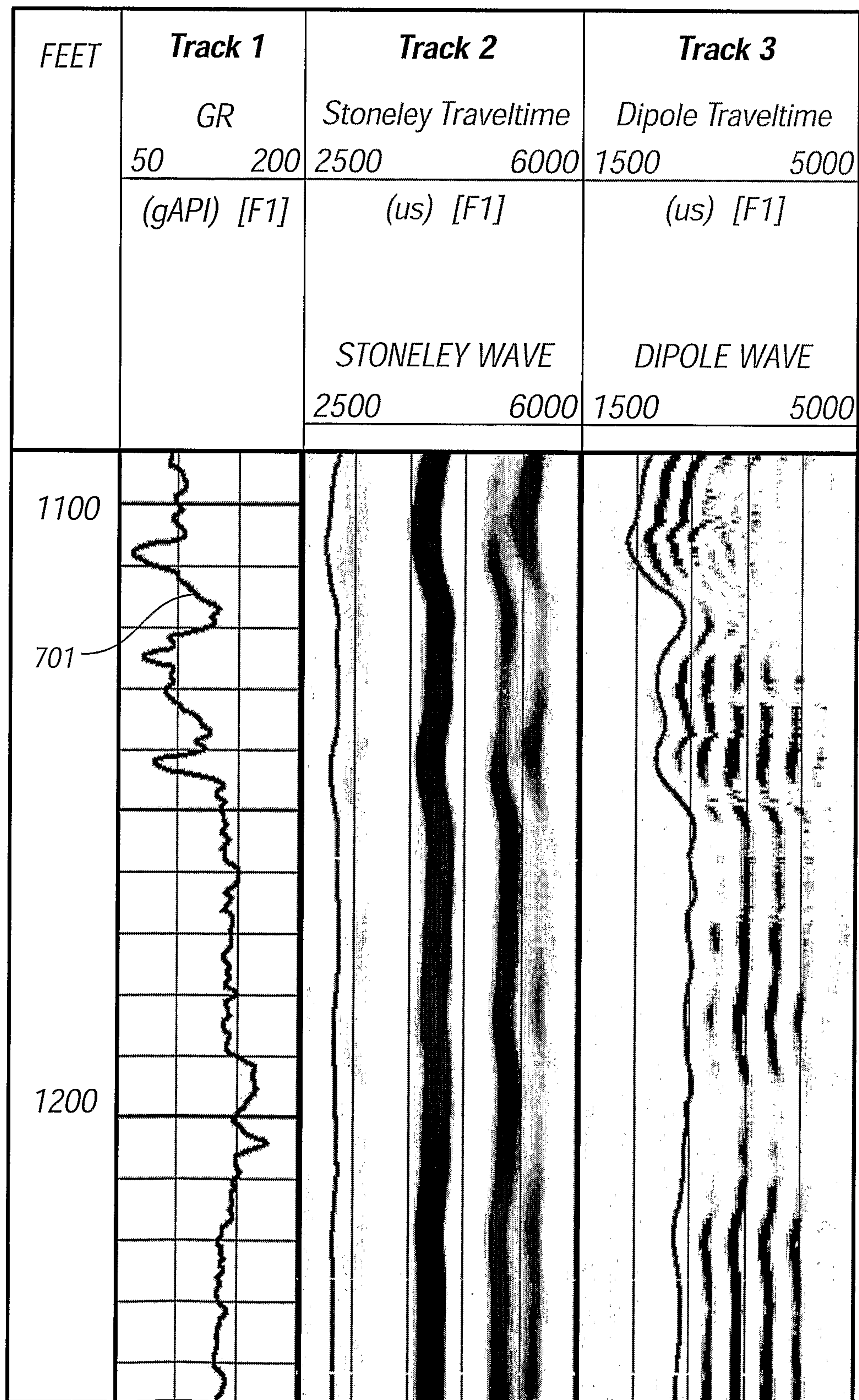


FIG. 7A

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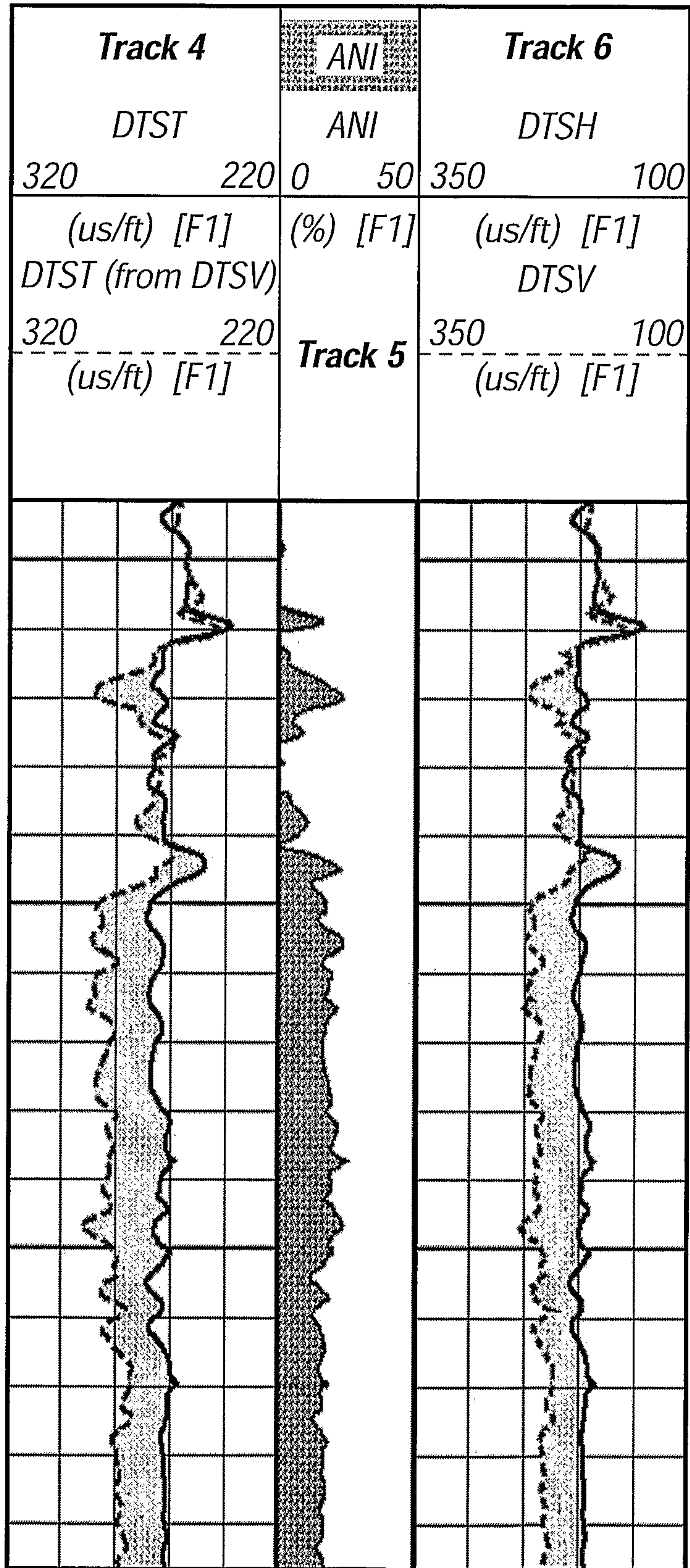


FIG.7 B

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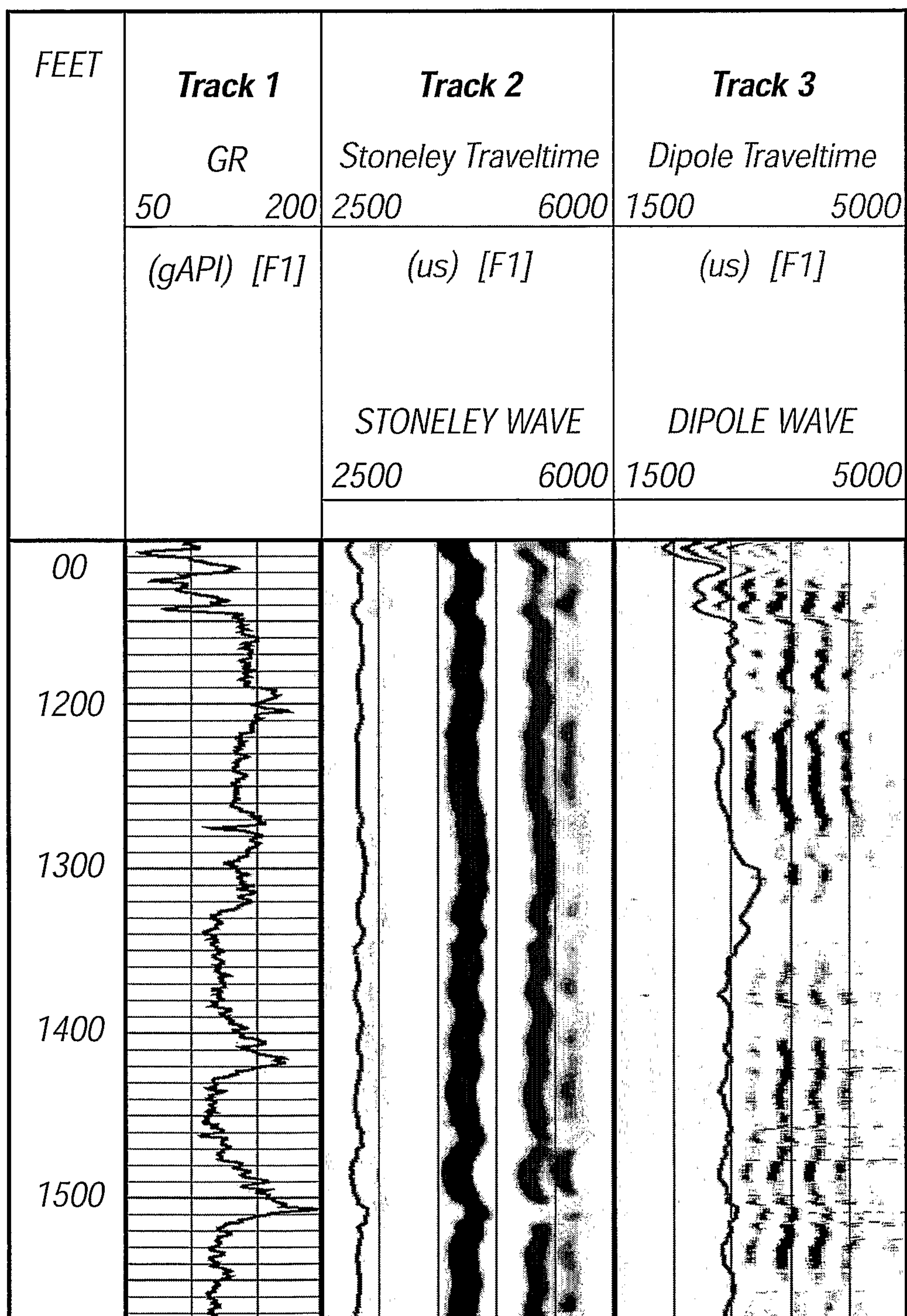


FIG. 8A

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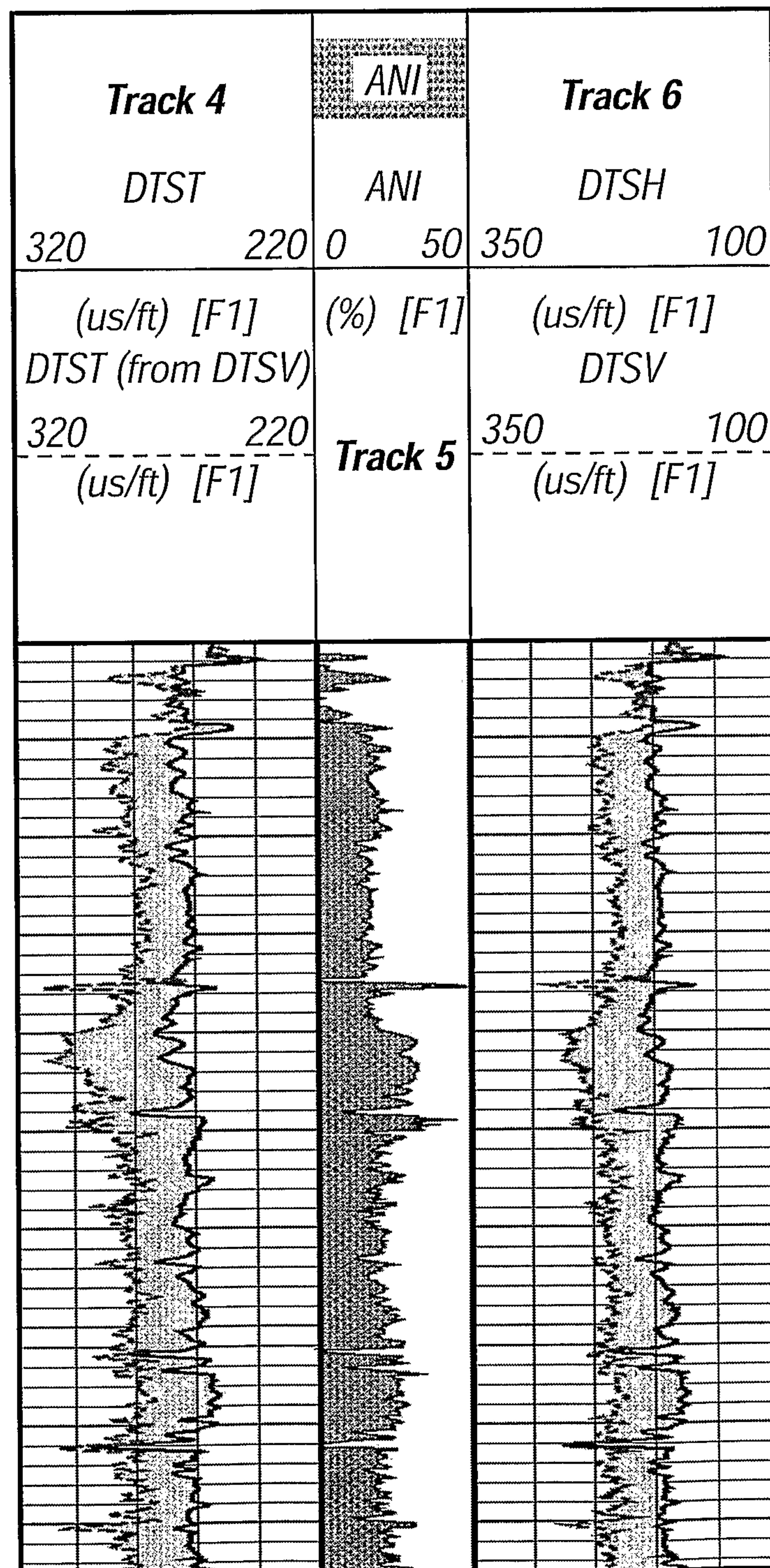


FIG.8 B

401
Process Array Stoneley Wave Data
To Obtain $\bar{S}_{ST}$

403
Compute the Stoneley Wave Spectrum $A(\omega)$
Over a Selected Frequency Range

405
Integrate $\omega^2 A^2(\omega)$ Over Selected Frequency Range
to Obtain the Denominator of Equation 8

407
For a Selected V_{SH} Calculate a Dispersion Curve
for S_{ST} Over the Frequency Range

409
Weight S_{ST} Curve with Weighting Function
 $\omega^2 A^2(\omega)$ and Integrate Over the Frequency
Range. Divide the Integral by the
Denominator to give Estimated $\bar{S}_{ST}$.

413
Is
Equation. 8 Satisfied
?

No

Yes

415
Output V_{sh} as the Formation
Horizontal Shear Velocity