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O. J. ZOBEL

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PHASE SHIFTING NETWORK

Filed March 12, 1927

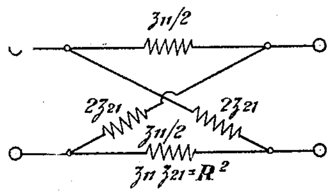

Fig. 1

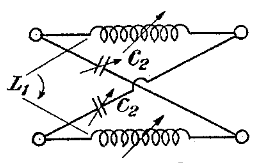

Fig. 2

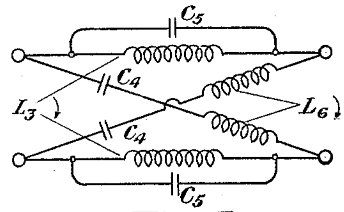

Fig. 3

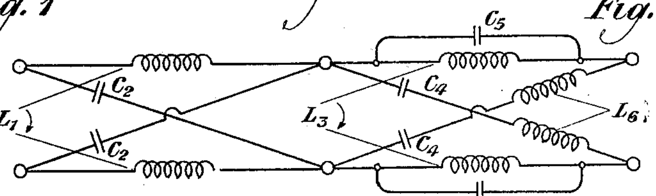

Fig. 4

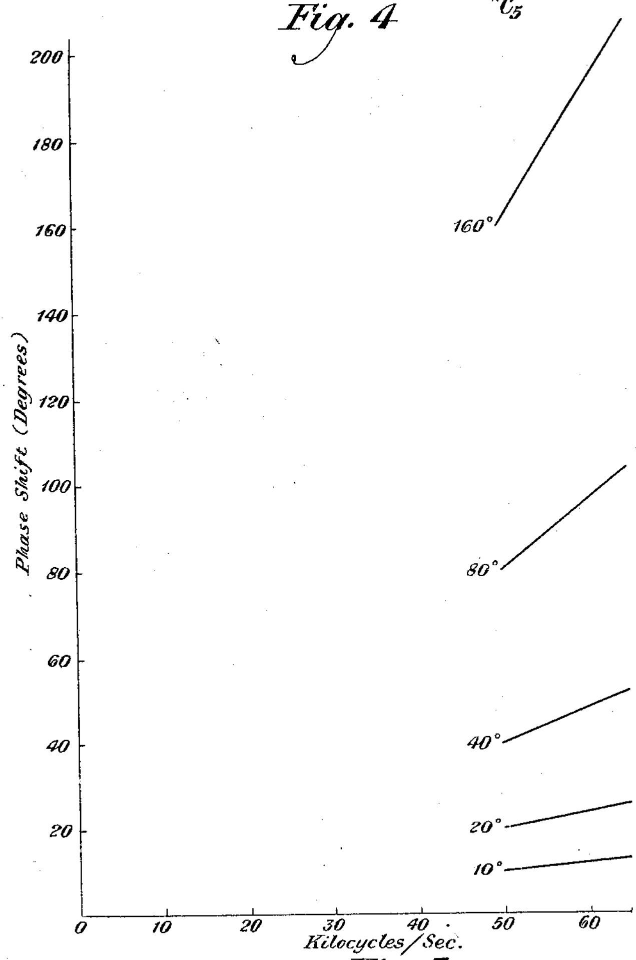

Fig. 5

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UNITED STATES PATENT OFFICE

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PHASE-SHIFTING NETWORK

Application filed March 12, 1927. Serial No. 174,984.

This invention relates to phase shifting networks and particularly to a network of that type having constant characteristic impedance, negligible attenuation, and phase constants which, over a desired frequency range, are proportional to frequency to a very high degree.

In the design and operation of electrical circuits, it is sometimes desirable to change the phase of the current in a part of the system in order to bring it into phase or in opposition to the current in another part of the system. Thus, for example, in a radio receiving system, employing a plurality of antennæ, directional selectivity is produced by varying the phase of the current in one antenna so that it will oppose the current from the same source produced in the other antenna, thus, effectively balancing out an undesired signal or other source of interference.

My invention resides in a phase shifting network having a constant characteristic impedance, negligible attenuation, and phase constants which, over the desired frequency range, are very accurately proportional to frequency.

The invention will be clearly understood from the following description when read in connection with the attached drawing of which Figure 1 shows an impedance network of the lattice type representing an infinite number of elements and having a constant characteristic impedance, the said figure illustrating the description of the underlying principle upon which the invention is based; Fig. 2 represents a network of the one parameter type, which network may employ either fixed units or variable units, in which case the phase shift may be varied through a predetermined range of values; Fig. 3 is a network of two parameter type; Fig 4 is a composite network of the three parameter type and Fig. 5 are characteristic curves representing the characteristics of the networks shown in Figs. 2 to 4, inclusive.

The phase shifting networks in which the invention is embodied are of the lattice type, being either single sections or composite sections. The general form of network of single section type having a constant impedance R is illustrated in Fig. 1 and the propagation constant of that network is given by

$$e^{\Gamma}=\frac{1+z_{11}/2R}{1-z_{11}/2R} \quad (1)$$

where $z_{11}z_{21}=R^2$.

In Fig. 1, z_{11} represents the total series impedance of the network and z_{21} represents the total lattice impedance. The series impedance is represented as being equally divided between the two sides of the circuit, each half having a value $z_{11}/2$. In order that the total lattice impedance shall equal z_{21}, each shunt unit has the value $2z_{21}$. It will be remembered that if we terminate a plurality of networks in series, each of the types shown in Fig. 1, by an impedance R, the impedance looking into the opposite end of the said plurality of networks will equal R.

The derivation of the formulæ for the different sections will now be shown, the number of parameters corresponding to the number of different series elements involved.

One parameter lattice.

In this network, which is shown in Fig. 2, the series elements is an inductance L_1 which is preferably uniformly distributed between the two sides of the circuit in order to maintain a balanced condition. The lattice elements are the condensers C_2.

$$\frac{z_{11}}{2R}=ifg, \quad (2)$$

where $i=\sqrt{-1}$
$f=$frequency
$g=\pi L_{11}/R$, and $L_{11}=gR/\pi=L_1$

Substituting in equation (1), the value for

$$\frac{z_{11}}{2R}$$

given by (2) equation (1) becomes

$$e^{\Gamma}=\frac{1+ifg}{1-ifg}$$

Substituting for Γ its equivalent $A+iB$

$$e^{A}\cdot e^{iB}=\frac{1+ifg}{1-ifg},$$

which when multiplied by

$$\frac{1+ifg}{1+ifg}=\frac{(1+ifg)^2}{1+f^2g^2}$$

Since

$$e^A=1$$
$$A=0$$

then the above becomes

$$e^{iB}=\frac{(1+ifg)^2}{1+f^2g^2}$$

$$e^{\frac{iB}{2}}=\frac{1+ifg}{\sqrt{1+f^2g^2}}=\frac{1+i(fg)}{\sqrt{1+f^2g^2}}$$

$$\therefore \tan\frac{B}{2}=fg \text{ or } G=P\cdot f \quad (3)$$

where B is the phase constant, G is introduced for convenience, and $P=g$.

If at $f=f_1$, $B=B_1$, then $P=G_1/f_1$. (4)

In the notation of Fig. 2,

$$L_1=L_{11}=gR/\pi,$$
$$C_2=C_{21}/2=L_{11}/2R=g/2\pi R. \quad (5)$$

Two parameter lattice

In this network, which is shown in Fig. 3, the total series element is an anti-resonant inductance L_3 and capacity $C_5/2$. The lattice elements are resonant paths each made up of an inductance $L_6/2$ in series with a capacity C_4. As will be seen

$$\frac{z_{11}}{2R}=\frac{ifh}{1-f^2m} \quad (6)$$

where

$$h=\pi L_{12}/R, \qquad L_{12}=hR/\pi=L_3,$$
$$m=4\pi^2L_{12}C_{13}, \quad C_{13}=m/4\pi hR=C_5/2.$$

$$e^{\Gamma}=\frac{1+\dfrac{ifh}{1-f^2m}}{1-\dfrac{ifh}{1-f^2m}}$$

$$\therefore \tan\frac{B}{2}=G=\frac{P\cdot f}{1-Q\cdot f^2} \quad (7)$$

where

$P=h$,
$Q=m$. Both are positive.

From (7) the linear relation is

$$P+fG\cdot Q=G/f. \quad (8)$$

The coefficients P and Q are determined by fixing the phase at two frequencies.

Finally in Fig. 3,

$$L_3=L_{12}=hR/\pi,$$
$$C_4=C_{22}/2=L_{12}/2R^2=h/2\pi R, \quad (9)$$
$$C_5=2C_{13}=m/2\pi hR,$$
$$L_6=4L_{23}=4R^2C_{13}=mR/\pi h.$$

Three parameter composite lattice

This network, shown in Fig. 4, is made up of one each of the two preceding sections. For this composite section the phase constant is given from (1), (2), and (6) by

$$e^{iB}=\frac{1+ifg}{1-ifg}\cdot\frac{1+ifh-f^2m}{1-ifh-f^2m}, \quad (10)$$

with the same meanings attached to g, h, and m as above.

Whence,

$$\tan(B/2)=G=\frac{P\cdot f-S\cdot f^3}{1-Q\cdot f^2}, \quad (11)$$

where

$P=g+h$,
$Q=gh+m$,
$S=gm$, all positive.

From (11) follows the linear relation

$$P+fG\cdot Q-f^2S=G/f. \quad (12)$$

Fixing the phase at three frequencies determines these coefficients P, Q, and S. The relations between g, h, m and the former are the following:

$$g^3-Pg^2+Qg-S=0,$$

from which cubic

$$g=P/3+\left\{-c+\sqrt{b^3+c^2}\right\}^{1/3}+\left\{-c-\sqrt{b^3+c^2}\right\}^{1/3},$$

where

$$b=-(P/3)^2+Q/3,$$
$$c=-(P/3)^3+PQ/6-S/2;$$
$$h=P-g, \quad (13)$$
$$m=Q-gh.$$

The final network elements, as in (5) and (9) and Fig. 4, are

$$L_1=gR/\pi,$$
$$C_2=g/2\pi R,$$
$$L_3=hR/\pi,$$
$$C_4=h/2\pi R, \quad (14)$$
$$C_5=m/2\pi hR,$$
$$L_6=mR/\pi h.$$

The general equations set forth hereinbefore, for the one, two, and three parameter networks, will now be applied to the design of phase shifting networks having specific values, the purpose of which application is to more fully describe the invention.

Let it be assumed that the requirements to be met by the networks are as follows: (*a*) a constant characteristic impedance of R equal to 600 ohms; (*b*) a continuously varied phase shift which is proportional to frequency through the frequency range from 50 to 65 kilocycles per second, the total phase shift range being from zero to 250 degrees at 50 kilocycles per second; (*c*) over any frequency band of 5 kilocycles per second in the range set forth in (*b*), the variations should be less than 1/10 degree for the phase and less than .025 TU or the attenuation; and

(d) the structure of the network shall be balanced.

0 to 15 degree continuously variable section

This section has one parameter which at any frequency is determined from (3) and (4), and the elements from (5). The inductances and capacities, L_1 and C_2 must be adjustable over a continuous range and have the magnitude-phase characteristics contained in the following relations between magniture and phase at 50,000 cycles/sec.

f_1	B_1 (degree)	$P=g$	L_1 (mh.)	C_2 (mf.)
50,000	0	0	0	0
50,000	1	.0174 times 10^{-5}	.0332	.0461 times 10^{-3}
50,000	2	.0350 times 10^{-5}	.0669	.0928 times 10^{-3}
50,000	3	.0524 times 10^{-5}	.1001	.1390 times 10^{-3}
50,000	4	.0698 times 10^{-5}	.1333	.1851 times 10^{-3}
50,000	5	.0874 times 10^{-5}	.1669	.2318 times 10^{-3}
50,000	6	.1048 times 10^{-5}	.2002	.2779 times 10^{-3}
50,000	7	.1224 times 10^{-5}	.2338	.3246 times 10^{-3}
50,000	8	.1398 times 10^{-5}	.2670	.3707 times 10^{-3}
50,000	9	.1574 times 10^{-5}	.3006	.4174 times 10^{-3}
50,000	10	.1750 times 10^{-5}	.3342	.4641 times 10^{-3}
50,000	11	.1926 times 10^{-5}	.3679	.5108 times 10^{-3}
50,000	12	.2102 times 10^{-5}	.4015	.5574 times 10^{-3}
50,000	13	.2278 times 10^{-5}	.4351	.6041 times 10^{-3}
50,000	14	.2456 times 10^{-5}	.4691	.6513 times 10^{-3}
50,000	15	.2634 times 10^{-5}	.5031	.6985 times 10^{-3}

(15)

10-degree section

This one parameter section has the constants corresponding at $f_1=50{,}000$ to $B_1=10$ degrees, as above.

$$L_1=.3342mh.,\quad C_2=.4641\cdot10^{-3}mf. \qquad (16)$$

20-degree section

$f_1=50{,}000$ $B_1=20.00$ degrees
$f_2=65{,}000$ $B_2=26.00$ degrees
$P=h=.34907\cdot10^{-5}$ $Q=m=.040707\cdot10^{-10}$
$L_3=.6667mh.$ $C_5=.3093\cdot10^{-3}mf.$
$C^4=.9259\cdot10^{-3}mf.$ $L_6=.22272mh.$

40-degree section

$f_1=50{,}000$ $B_1=40.00$ degrees
$f_2=65{,}000$ $B_2=52.00$ degrees
$P=h=.69774\cdot10^{-5}$ $Q=m=.16595\cdot10^{-10}$
$L_3=1.333mh.$ $C_5=.6309\cdot10^{-3}mf.$
$C_4=1.851\cdot10^{-3}mf.$ $L_6=.4544mh.$

80-degree section

$f_1=50{,}000$ $B_1=80.00$ degrees
$f_2=57{,}500$ $B_2=92.00$ degrees
$f_3=65{,}000$ $B_3=104.00$ degrees
$P=1.3965\cdot10^{-5}$
$Q=.78549\cdot10^{-10}$ $S=.19148\cdot10^{-15}$
$g=.6340\cdot10^{-5}$
$h=.7625\cdot10^{-5}$ $m=.3021\cdot10^{-10}$
$L_1=1.211mh.$ $C_4=2.023\cdot10^{-3}mf.$
$C_2=1.682\cdot10^{-3}mf$ $C_5=1.051\cdot10^{-3}mf.$
$L_3=1.456mh.$ $L_6=.7564mh.$

160-degree section

$f_1=50{,}000$ $B_1=160.00$ degrees
$f_2=57{,}500$ $B_2=184.00$ degrees
$f_3=65{,}000$ $B_3=208.00$ degrees
$P=2.8308\cdot10^{-5}$ $S=1.8046\cdot10^{-15}$
$Q=3.1603\cdot10^{-10}$
$g=1.540\cdot10^{-5}$ $m=1.1724\cdot10^{-10}$
$h=1.291\cdot10^{-5}$
$L_1=2.941mh.$ $C_4=3.425\cdot10^{-3}mf.$
$C_2=4.084\cdot10^{-3}mf.$ $C_5=2.408\cdot10^{-3}mf.$
$L_3=2.466mh.$ $L_6=1.7344mh.$

While the variable section is normally required to give a maximum of but 10 degrees, an extension of this range to 15 degrees has been made to insure phase overlapping at any transition point where a section having a fixed value is put into or taken out of the circuit. By combining these sections, a continuous range of from zero to 325 degrees is obtainable. The attenuation requirements are met by choosing coils having a small dissipation constant d.

An idea of the accuracy of the phase shifting networks, described hereinbefore, through the range from 50 to 65 kilocycles may be derived from the following table which sets forth the departure in degrees of phase for the various frequencies between 50 and 65 kilocycles.

Phase departures (degrees) from ideal proportionality to frequency

f	Section (degrees at 50,000 cycles/sec.)				
Cycles/sec.	10	20	40	80	160
50,000	.000	.000	.000	.000	.000
54,000	−.003	.000	.001	−.001	−.017
57,500	−.008	.001	.002	.000	.000
61,000	−.014	.000	.001	−.001	.013
65,000	−.022	.000	.000	.000	.000

The proportionality of the phase shift to frequency is shown clearly by Fig. 5. All of the curves shown thereon pass through the origin. Furthermore, it will be apparent from an inspection of the curves of Fig. 5 that the factor of proportionality for each section remains constant through a predetermined frequency range. The expression "factor of proportionality" means the ratio

$$\frac{B}{f},$$

that is to say, the slope of the characteristic.

With regard to the feature of resonance, which is presented by Figs. 3 and 4, it is desirable to point out that the resonant frequency of these networks will be well outside the range of frequencies with respect to which the networks are intended to be used.

While the invention has been disclosed as embodied in sections having particular forms and also as embodied in combinations of those sections, it will be apparent that the invention may be embodied in sections having other forms and other combinations of sections without departing from the spirit and scope of the appended claims.

What is claimed is:

1. A phase shifting network comprising two subsidiary networks serially connected, one of said networks comprising series inductance and lattice connected capacity and having a constant characteristic impedance, and the other of said networks comprising series elements each consisting of inductance and capacity that are anti-resonant, and lattice connected elements each consisting of resonant inductions and capacity, the product of the series and lattice branches of said other network being constant with respect to frequency and the said network having a constant characteristic impedance.

2. A phase shifting network comprising series elements each consisting of inductance and capacity which are anti-resonant, and lattice connected elements each consisting of resonant inductance and capacity, the product of the series and the lattice branches being constant with respect to frequency, and the said network having a constant characteristic impedance.

In testimony whereof, I have signed my name to this specification this 11th day of March, 1927.

OTTO J. ZOBEL.