United States Patent [19]

Unz

[11] **3,780,372**

[45] **Dec. 18, 1973**

[54] **NONUNIFORMLY OPTIMALLY SPACED ANTENNA ARRAY**

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[22] Filed: **Jan. 17, 1972**

[21] Appl. No.: **218,495**

[52] **U.S. Cl.** **343/844**, 343/719

[51] **Int. Cl.** **H01q 21/00**

[58] **Field of Search** 343/719, 844, 853, 343/854

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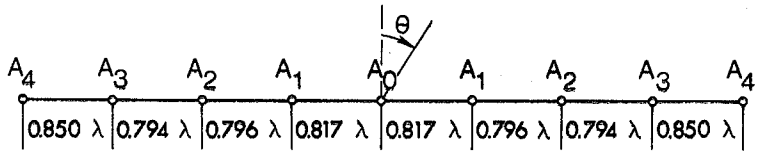
[57] **ABSTRACT**

The object of this invention is to provide a new nonuniformly spaced antenna array, where the most optimum positions of the array elements, and their corresponding amplitudes, are systematically determined by a rigorous synthesis technique of the given radiation pattern, and/or its requirements and specifications, in order to achieve it with the minimum possible number of array elements. This new array will be designated the Nonuniformly, Optimally Spaced Antenna Array, or in short, the NOSA Array.

6 Claims, 2 Drawing Figures

n =	0	1	2	3	4	5	6	7	8
f_n =	1.0000	0.4800	0.4500	0.3700	0.2500	0.1300	0.0540	0.0170	0.0044
a_n =	1.0000	0.4800	0.4500	0.3700	0.2500	0.1300	0.0540	0.0170	0.0044
A_n =	1.3136	0.7508	0.5453	0.2427	0.0561	----	----	----	----

TABLE A



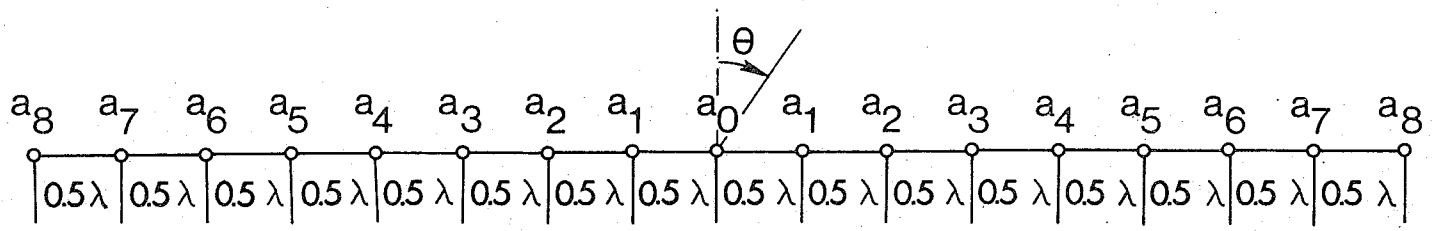


FIGURE 1

n =	0	1	2	3	4	5	6	7	8
f_n =	1.0000	0.4800	0.4500	0.3700	0.2500	0.1300	0.0540	0.0170	0.0044
a_n =	1.0000	0.4800	0.4500	0.3700	0.2500	0.1300	0.0540	0.0170	0.0044
A_n =	1.3136	0.7508	0.5453	0.2427	0.0561	- - - -	- - - -	- - - -	- - - -

TABLE A

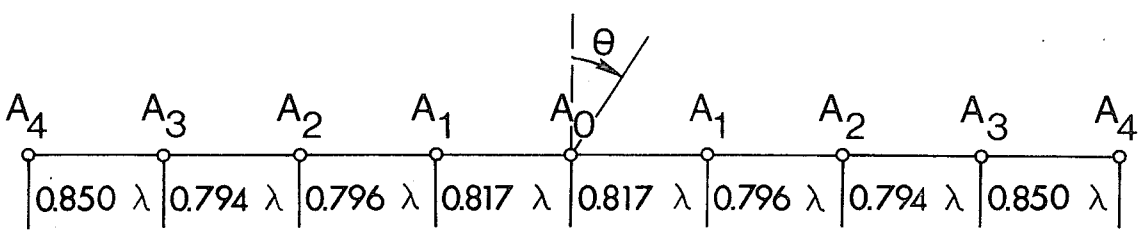


FIGURE 2

NONUNIFORMLY OPTIMALLY SPACED ANTENNA ARRAY

The general idea of the non-uniformly spaced antenna arrays was invented by Unz in 1955, and was publicly proposed and published for the first time in his University of California (Berkeley, 1956) doctoral dissertation, where the first significant work on the subject was reported. From 1960 on many additional contributions have been made on non-uniformly spaced arrays by numerous authors, and references to most of them may be found in recent books on antennas. However, the main synthesis problem of the non-uniformly spaced arrays, namely, finding the most optimum positions of the elements of the array in order to produce a given specified radiation pattern, has not been solved rigorously to-date. This is primarily due to the great difficulties in the solution of this highly non-linear problem. The trial and error computer techniques and the other pseudo-optimum synthesis methods suggested so far are of very limited utility, and are almost impossible to employ with arrays of a large number of elements. Thus, the non-uniformly spaced arrays cannot be technically designed at the present time to their full potential advantage, if at all, and therefore are not generally used.

The object of this invention is to provide a nonuniformly spaced antenna array, where the array elements are distributed in the most optimum positions along the axis of the array, with reference to the given radiation pattern requirements and specifications. The optimal distribution of the elements along the array axis is determined so that the Nonuniformly Optimally Spaced Antenna Array (the NOSA Array) of this invention will require a smaller number of elements, than an equivalent uniformly spaced array, giving the same required approximate radiation pattern, or its specified required characteristics, such as the sidelobe level, and/or beamwidth, and/or gain, etc. Alternatively, the NOSA Array will give a better performance in its radiation pattern and its characteristics than an equivalent uniformly spaced array of an equal number of elements. A rigorous synthesis technique is provided for the systematic determination of the most optimum positions of the elements in the Nonuniformly Optimally Spaced Antenna Array (the NOSA Array), and their corresponding amplitudes, in order to accomplish the radiation pattern given specifications with the minimum possible number of elements in the array. Thus this invention will provide a substantial savings in the number of elements required in an antenna array for a given performance. While the rigorous synthesis technique described later is for the symmetric linear NOSA Array, the invention by no means is limited to this particular case. It could be extended quite easily to the non-symmetric NOSA Array. Furthermore, since the NOSA Array is the building stone for any number of other more sophisticated arrays, this invention covers all other arrays, where they include in whole or in part the NOSA Array. Thus this invention covers, but is not limited to the following arrays, if they include in whole or in part the NOSA Array: symmetric and non-symmetric arrays, linear, two and three dimensional arrays, arrays on curved surfaces, transmitting, receiving and retrodirective arrays, arrays of arrays, correlation arrays, steerable beam, scanning arrays and phased arrays, active and adaptive arrays, time modulated arrays, radio-

astronomy arrays, broadband arrays and frequency independent arrays, superdirective arrays, slot arrays, omnidirectional arrays, broadside arrays, end-fire arrays, resonant and nonresonant arrays and many others.

The invention is illustrated by the accompanying drawing in which:

Table A gives the seventeen (17) complex Fourier series coefficients $f_n = f_{-n}$ specified from the given real and symmetric radiation pattern to be approximated by the symmetric linear antenna array. FIG. 1 and Table A give the seventeen (17) element half-wavelength uniformly spaced antenna array which will give the specified Fourier coefficients f_n listed in Table A. The synthesis has been done in accordance with well established procedures for uniformly spaced arrays. FIG. 2 and Table A give the nine (9) element Nonuniformly Optimally Spaced Antenna Array (the NOSA Array) which will give the same specified Fourier coefficients f_n listed in Table A. The systematic determination in FIG. 2 of the most optimum array element distribution and their corresponding amplitudes for the given Fourier coefficients in Table A, have been accomplished by using the rigorous synthesis technique described below. Thus the NOSA Array in FIG. 2 will give the same required performance as the half-wavelength uniformly spaced antenna array in FIG. 1, but with a substantial saving of almost half of the number of the elements required by the standard uniformly spaced array. A substantial saving of the number of the elements of the array for equivalent performance is of essential importance especially in arrays with a large number of elements.

The rigorous synthesis technique for the symmetric linear Non-uniformly Optimally Spaced Antenna Array (NOSA Array) for a given real and symmetric radiation pattern or its complex Fourier series coefficients $f_{-n} = f_n$ = real specified in Table A, in order to determine the most optimum positions of the elements of the NOSA Array, and their corresponding amplitudes, in FIG. 2, is described as follows:

The radiation pattern $F(\theta)$ of a linear array with nonuniform spacings of the elements is given by:

$$F(\theta) = \sum_{p} A_p e^{ikd_p \sin\theta} \qquad (1.a)$$

where $i = \sqrt{-1}$, $k = 2\pi/\lambda$, λ being the wavelength, θ is the angle with the normal to the array axis ($-\pi/2 \leq \theta \leq +\pi/2$), d_p is the distance of the p array element from the origin on the array axis, and A_p is its corresponding complex amplitude, given in magnitude and phase.

Equation (1.a) may be rewritten in the form:

$$F(u) = \sum_{p} A_p e^{ix_p u} \qquad (1.b)$$

where $u = \pi \sin\theta$ ($-\pi \leq u \leq +\pi$) and $x_p = d_p/\lambda/2$ gives the distance of the array element p from the origin in terms of half wavelengths.

For the synthesis problem the given radiation pattern $F(u)$ may be expanded in a complex Fourier series, for n = integer:

$$F(u)=\sum_{n=-\infty}^{+\infty} f_n e^{inu} \tag{2.a}$$

$$f_n=\frac{1}{2\Pi}\int_{-\Pi}^{+\Pi} F(u)\,e^{-inu}du \tag{2.b}$$

For the particular case of half wavelength $d_p = p\ \lambda/2$ uniformly spaced linear array one has $x_p = p$ = integer in (1.b), which then has the same form as (2.a). Thus, each element of the uniformly spaced array corresponds to one complex Fourier series term $A_n = f_n$, and $F(u)$ in general may be approximated to any degree of accuracy by a finite number of array elements, corresponding to the same number of complex Fourier series terms.

In case of a non-uniformly spaced array, x_p ($\neq p$) in (1.b) has to be determined, as well as A_p, for the best approximation of $F(u)$. Substituting (1.*b*) in (2.*b*) one obtains after integrating:

$$f_n=\sum_p A_p \frac{\sin\,(n-x_p)\Pi}{(n-x_p)\Pi} \tag{3.a}$$

Using the trigonometric indentity and taking $\sin n\pi = 0$ and $\cos n\pi = (-1)^n$, one has:

$\sin(n - x_p)\pi = \sin n\pi \cos x_p\pi - \cos n\pi \sin x_p\pi = +(-1)^{n+1}\sin x_p\pi$ and one obtains from (3.a):

$$f_n=\sum_p A_p \frac{(-1)^{n+1}\sin x_p\Pi}{(n-x_p)\Pi} \tag{3.b}$$

From (3.b) one has:

$$(-1)^{n+1}\Pi f_n=\sum_p \frac{B_p}{n-x_p} \tag{4.a}$$

where we define:

$$B_p = A_p \sin x_p\pi \tag{4.b}$$

Taking the array element $p = 0$ of the amplitude A_o to be at the origin $x_o = 0$, one obtains from (4.a) and (4.b):

$$(-1)^{n+1}\Pi f_n=\sum_{p\neq 0} \frac{B_p}{n-x_p}\, n\neq 0 \tag{5.a}$$

$$f_0=A_0+\sum_{p\neq 0} A_p \frac{\sin x_p\Pi}{x_p\Pi}\, n=0 \tag{5.b}$$

Let us assume that the given radiation pattern $F(u)$ is a real and symmetric function $F(-u) = F(u)$. Thus one finds from (2.*b*):

$$f_n = \text{real};\ f_{-n} = f_{+n} \tag{6.a}$$

Let us further assume that the non-uniformly spaced array has an odd number of elements and is symmetric with respect to the element A_o at the origin $x_o = 0$. Using (4.*b*) one obtains:

$$x_{-p} = -x_p;\ A_{-p} = A_p;\ B_{-p} = -B_p \tag{6.b}$$

One obtains from (5.*a*) and (5.*b*) for $s \geq 1$:

$$(-1)^{n+1}\Pi f_n=\sum_{p=1}^{s}\left[\frac{B_{-p}}{n-x_{-p}}+\frac{B_p}{n-x}\right] n\neq 0 \tag{7.a}$$

$$f_0=A_0+\sum_{p=1}^{s}\left[A_{-p}\frac{\sin x_{-p}\Pi}{x_{-p}\Pi}+A_p\frac{\sin x_p\Pi}{x_p\Pi}\right] n=0 \tag{7.b}$$

Substituting (6.*b*) in (7.*a*) and (7.*b*) one obtains after rearranging:

$$(-1)^{n+1}\Pi f_n=\sum_{p=1}^{s} B_p\frac{2x_p}{n^2-x_p^2}\, n\neq 0 \tag{8.a}$$

$$f_0=A_0+2\sum_{p=1}^{s} A_p \frac{\sin x_p\Pi}{x_p\Pi}\, n=0 \tag{8.b}$$

From (8.*a*) one finds that $f_{-n} = f_n$ as in (6.*a*). Taking in (8.*a*) $D_p = B_p x_p$, $N = n^2$, $y_p = x_p^2$ and using (4.*b*) one obtains:

$$(-1)^{n+1}\frac{\Pi}{2} f_n=\sum_{p=1}^{s}\frac{D_p}{N-y_p}\, n\neq 0 \tag{9.a}$$

$$D_p=x_pB_p=A_px_p\sin x_p\Pi;\ N=n^2;\ y_p=x_p^2 \tag{9.b}$$

Thus equation (9.*a*) for the symmetric nonuniformly spaced array has almost an identical form as equation (4.*a*) for the general non-uniformly spaced array, and both of them could be solved in a very similar manner for the synthesis problem.

For the limiting case of uniformly spaced array $x_p \rightarrow p$ = integer one may find from (5.*a*) or (9.*a*) and from (5.*b*) or (8.*b*) after the suitable substitution and the correct limiting process:

$$f_n = A_n;\ f_o = A_o \text{ for } x_p = p \tag{10}$$

We will discuss here only the solution of (9.*a*) for the symmetric non-uniformly spaced array.

In equation (9.*a*) we have two sets of unknowns, y_p and D_p, which have to be solved, and thus give us x_p and A_p from (9.*b*). Writing (9.*a*) explicitly for s terms and multiplying by the common denominator on both sides, one obtains for $N = n^2 \geq 1$:

$(-1)^{n+s}\pi/2\, f_n(N-y_1)\,(N-y_2)\,(N-y_3)\,\ldots\,(N-y_s)$ =

$$= D_1 (N - y_2) (N - y_3) \ldots\ldots (N - y_s) +$$
$$+ D_2 (N - y_1) (N - y_3) \ldots\ldots (N - y_s) +$$
$$+ D_3 (N - y_1) (N - y_2) \ldots\ldots (N - y_s) + \ldots\ldots$$
$$+ D_s (N - y_1) (N - y_2) \ldots\ldots (N - y_{s-1}) \quad (11)$$

where (11) for given f_n is a linear equation for D_p and a non-linear equation for y_p. Opening the brackets on both sides and rearranging in descending powers of N, one obtains:

$$(-1)^{n+1}\, \pi/2\, f_n[N^s - \alpha_1 N^{s-1} + \alpha_2 N^{s-2} - \alpha_3 N^{s-3} + \ldots . + (-1)^s \alpha_s] =$$
$$= \beta_1 N^{s-1} - \beta_2 N^{s-2} + \beta_3 N^{s-3} - \ldots\ldots\ldots\ldots + (-1)^{s-1} \beta_s \quad (12.a)$$

where we define:

$$\alpha_1 = (y_1 + y_2 + y_3 + \ldots\ldots + y_s)$$
$$\alpha_2 = y_1 (y_2 + y_3 + \ldots + y_s) + y_2 (y_3 + y_4 + \ldots + y_s) + \ldots + y_{s-1}\, y_s$$
$$\alpha_3 = y_1\, y_2 (y_3 + y_4 + \ldots + y_s) + \ldots\ldots + y_{s-2}\, y_{s-1}\, y_s$$
$$\alpha_s = (y_1\, y_2\, y_3 \ldots y_s) \quad (12.b)$$
$$\beta_1 = (D_1 + D_2 + D_3 + \ldots + D_s)$$
$$\beta_2 = D_1 (y_2 + y_3 + \ldots + y_s) + D_2 (y_1 + y_3 + \ldots + y_s) + \ldots$$
$$\ldots + D_s (y_1 + y_2 + \ldots y_{s-1})$$
$$\beta_3 = D_1 (y_2\, y_3 + y_2\, y_4 + \ldots + y_{s-1}\, y_s) + \ldots$$
$$\ldots + D_s (y_1\, y_2 + \ldots + y_{s-2}\, y_{s-1})$$
$$\beta_s = D_1 (y_2\, y_3 \ldots y_s) + D_2 (y_1\, y_3 \ldots y_s) + \ldots$$
$$\ldots + D_s (y_1\, y_2\, y_3 \ldots y_{s-1}) \quad (12.c)$$

Since y_p and D_p are the unknowns to be found, and thus α_p and β_p defined above are also the unknowns to be found. Thus (12.a) could be rearranged and rewritten as follows for $N = n^2 \geq 1$:

$$\left[(-1)^{n+1} \frac{\Pi}{2} f_n\right] \sum_{q=1}^{s} (-1)^{q+1} N^{s-q} \alpha_q + \sum_{q=1}^{s} (-1)^{q+1} N^{s-q} \beta_q = \left[(-1)^{n+1} \frac{\Pi}{2} f_n\right] N^s \quad (13.a)$$

or alternatively, dividing (13.a) by N^s one obtains:

$$\left[(-1)^{n+1} \frac{\Pi}{2} f_n\right] \sum_{q=1}^{s} \frac{(-1)^{q+1} \alpha_q}{N^q} + \sum_{q=1}^{s} \frac{(-1)^{q+1} \beta_q}{N^q} = \left[(-1)^{n+1} \frac{\Pi}{2} f_n\right] \quad (13.b)$$

alternatively (13.b) may be rewritten in the following form:

$$\sum_{q=1}^{s} \frac{(-1)^{q+1} \alpha_q}{N^q} + \frac{1}{\left[(-1)^{n+1} \frac{\Pi}{2} f_n\right]} \sum_{q=1}^{s} \frac{(-1)^{q+1} \beta_q}{N^q} = 1 \quad (13.c)$$

Equation (13.a) or (13.b) or (13.c) represents a linear equation with $2s$ unknowns, namely, s unknown terms of α_q, and s unknown terms of β_q. One may rewrite one of the equations (13), i.e. (13.c), $2s$ times for the given $2s$ Fourier coefficients f_n of the given radiation pattern F(u), namely for $(f_1, f_2, f_3 \ldots f_{2s})$ where $N = n^2 \geq 1$. Thus one obtains $2s$ linear equations with $2s$ unknowns α_q and β_q. The method of solving $2s$ linear equations with $2s$ unknowns by the use of determinants and by well established and modified computer techniques is very well known.

Thus if $(f_1, f_2, \ldots, f_{2s})$ are given, one may solve (13.c) for $\alpha_q(\alpha_1, \alpha_2 \ldots, \alpha_s)$, by determinants with the aid of a computer. As it will be shown later, there is no need to solve for β_q.

Once $(\alpha_1, \alpha_2 \ldots, \alpha_s)$ are thus found, one should solve the s non-linear equations (12.b) for $(y_1, y_2, \ldots, y_s)$. It is well known that a polynomial equation with the solutions $y = y_1$, $y = y_2, \ldots, y = y_s$ could be written in the form:

$$(y - y_1) (y - y_2) (y - y_3) \ldots . (y - y_s) = 0 \quad (14.a)$$

From the left hand side of (11) and (12.a) and from (12.b) one finds by inspection and by multiplying and opening the brackets of (14.a) one obtains:

$$y^s - \alpha_1 y^{s-1} + \alpha_2 y^{s-2} - \alpha_3 y^{s-3} + \ldots + (-1)^s \alpha_s = 0 \quad (14.b)$$

But the coefficients $(\alpha_1, \alpha_2, \ldots, \alpha_s)$ have been solved above. Thus the positions of the elements x_p along the axis of the non-uniformly spaced array could be found as the roots of the polynomial equation (14.b) $y_p = x_p^2$. Finding the roots of the polynomial equation (14.b) is a well established process with the aid of a computer. The positions x_p thus found from the roots of the algebraic equation (14.b) represent the optimum positions of the symmetric NOSA Array elements.

Once the s roots y_p have been found from (14.b), one may substitute them in (9.a) and rewrite it s times for $(f_1, f_2 \ldots, f_s)$. Thus one obtains s linear equations with s unknown D_p, which could be solved by using determinants and well known computer techniques. With all y_p and D_p thus found, one may use (9.b) to obtain:

$$x_p = + \sqrt{y_p};\ A_p = D_p/(x_p \sin x_p \pi) \quad (15.a)$$

and thus the position of each element x_p and its amplitude A_p are found for all the side elements of the symmetric non-uniformly spaced array. Once this is known, one may use (8.b) and (9.b) to find the amplitude of the center element A_o at $x_o = 0$:

$$A_o = f_o - 2 \sum_{p=1}^{s} A_p \frac{\sin x_p \Pi}{x_p \Pi} = f_o - \frac{2}{\Pi} \sum_{p=1}^{s} \frac{D_p}{y_p} \quad (15.b)$$

and the synthesis problem for a given $F(u)$ is solved for the non-uniformly optimally spaced array.

The details of the procedure of the rigorous synthesis technique for the determination of the most optimum positions of the elements of the NOSA Array and their corresponding amplitudes for the symmetric case in FIG. 2 are given as follows:

Assuming that $2s$ Fourier series coefficients of f_n have to be used, one rewrites the linear equation (13.c) of 2s unknowns α_q and β_q for a total of $2s$ times, using the given f_n for $1 \leq n \leq 2s$ and taking $N = n^2$;

The $2s$ linear equations with $2s$ unknowns are solved for all α_q $(\alpha_1, \alpha_2 \ldots\ldots, \alpha_s)$ by the use of well known computer techniques.

Using the α_q coefficients found previously, one obtains the algebraic equation (14.b) of order s. The positive real roots of the algebraic equation (14.b) are found by using well known computer techniques, and they represent the optimum positions of the symmetric NOSA Array elements, where $x_p = \sqrt{y_p}$.

Substituting the value of y_p found above in (9.a), and rewriting it s times for different f_n for $1 \leq n \leq s$, taking $N = n^2$, one obtains s linear equations for s unknowns D_p, which can be solved by the use of known computer techniques. Once D_p and y_p have been solved

for the symmetric linear NOSA Array, obe obtains the position of the p element x_p, and its amplitude A_p, by using (9.*b*). The amplitude A_0 of the center element at $x_o = O$ is found by using (15.*b*). Using the above rigorous synthesis technique, the symmetric linear NOSA Array in FIG. 2 was technically designed from the given complex Fourier co-efficients in Table A. The nine (9) element NOSA Array in FIG. 2 is equivalent with its required performance to the seventeen (17) element half wavelength uniformly spaced array given in FIG. 1. Both the uniformly spaced array in FIG. 1 and the non-uniformly optimally spaced array in FIG. 2 give almost the same identical radiation pattern which has been prescribed and defined by the f_n complex Fourier series coefficients given in Table A.

Let us assume that in prder to approximate a given symmetric and real function radiation pattern $F(u)$ to a certain given degree of average mean square approximation, one needs to have a symmetric linear array which will give exactly the values of the complex Fourier coefficients f_n for $0 \leq n \leq 2M$. If we chose a symmetric uniformly spaced array of half wavelength spacings, we determine in advance the element positions and will need, in accordance with equation (10), a total of$(4M + 1)$ antenna elements, for the symmetric array with the center element. However, if we choose a non-uniformly spaced symmetric linear array, where both the positions of the elements and their corresponding amplitudes arc to be determined for the best approximation, we have twice as many degrees of freedom, and we could approximate 2M Fourier coefficients by using only M radiation elements in accordance with (13). Thus the nonuniformly spaced symmetric linear array with the center element would require a total of only$(2M + 1)$non-uniformly spaced antenna elements in order to achieve approximately the same radiation pattern; therefore one has for a symmetric linear array, where f_n for $0 \leq n \leq 2M$ is given:

$$\frac{\text{No. of elements uniform}}{\text{No. of elements nonuniform}} = \frac{4M+1}{2M+1} = 2 - \frac{1}{2M+1} \approx 2 \tag{16}$$

One needs therefore half the number of elements in the Non-uniformly Optimally Spaced Antenna Array (the NOSA Array) as compared to a uniformly spaced array giving the same approximate radiation pattern with all its side lobes. In this statement we assume that all the roots in the polynomial equation (14.*b*) are positive and real: thus the result depends on all the coefficients α_q, and on the given Fourier Coefficients f_n through the linear equation (13.*c*). The results of such a numerical example for a given prescribed set of Fourier coefficients f_n are given above.

Furthermore, one should check an additional number of Fourier array coefficients f_n for $n > 2s$, which have not been approximated, but which also may be found from a given radiation pattern $F(u)$, as well as the final resulting radiation pattern $F(u)$ given by the NOSA Array and its elements A_p at positions x_p in accordance with equation (1b). In case the approximation requirements of the NOSA Array radiation pattern with respect to the given prescribed radiation pattern are not acceptable, one may repeat the process by adding one more antenna element on each side of the NOSA Array and repeat the process again, and check again for the approximation acceptability. The number of the elements in the NOSA Array will be still smaller than an equivalent uniform array, but by less than the factor of two given in (16). The final result depends on each indivudal or class of given radiation patterns.

The rigorous analytical method described above that theoretically solves the highly non-linear synthesis problem of finding the optimum positions of the elements in the non-uniformly optimally spaced linear array, with non-uniformly optimally excited amplitudes, for a given prescribed radiation pattern, has been applied to other cases as well. In several different examples of prescribed, broadside, directive radiation patterns with sidelobes, including uniform array patterns and Dolph-Chebyshev patterns, a non-uniformly optimally spaced linear array has been obtained, which required 35 percent – 50 percent fewer elements than a half wavelength uniformly spaced linear array giving the same prescribed radiation pattern with all its sidelobes.

Since the non-uniformly optimally spaced array described in the foregoing specifications could be used as the basis for building more sophisticated arrays with specific or minimax performance indices requirements, this invention covers, but is not limited to, arrays with specific requirements on the radiation patterns, impedance, bandwidth, mutual coupling, gain and directivity, polarization, noise temperature, signal to noise or interference ratio and any other specific indices, if the array includes in whole or in part certain definitive aspects of this invention as a part of its analysis or synethsis. This invention also covers this array, when the elements used in the array are of different types, including but not limited to, dipoles, slots, horns, apertures, parabolic reflectors, dishes and many others. Many of the design techniques in prior art developed for electromagnetic arrays can and has been applied with modification to acoustic arrays, seismic arrays and arrays in other fields. Thus this invention of non-uniformly optimally spaced array described in the foregoing specifications covers, but is not limited to, all such arrays or combination of them in other fields for any purpose. This invention covers non-uniformly optimally spaced acoustic arrays and sonar arrays, used with acoustic elements of any type, when used under water or above water , for whatever purpose. This invention covers non-uniformly optimally spaced seismic arrays, used with seismic elements, geophones, seismometers seismographs, or other elements, when used underground for whatever purpose. Thus the foregoing invention of non-uniformly optimally spaced array covers any array of any shape and size, and used with any kind of elements for any purpose, provided that the invention described here will be used there in whole or in part, or will be used there as a part of its analysis or synthesis during the design procedure.

While in the foregoing specification, I have set forth certain details of the Nonuniformly Optimally Spaced Antenna Array (the NOSA Array) and its rigorous synthesis technique for a specified radiation pattern, for the purpose of illustrating one mode of the invention, it will be understood that such details may be varied widely by those skilled in the art without departing from the spirit of my invention, and it is therefore aimed to cover all such changes and modifications in all areas of endeavor where arrays are used, as fall within the true spirit and scope of this invention.

What I claim is :

1. A method for optimizing the spacing and amplitude of the individual elements in a non-uniformly spaced array, as compared to a conventional uniformly spaced array, in order to accomplish a given radiation pattern, and/or its requirements and specifications, with the minimum number of elements, said method based on the following relationship for a symmetric linear array:

$$(-1)^{n+1}\frac{\Pi}{2}f_n=\sum_{p=1}^{s}\frac{D_p}{N-y_p} \text{ for } n\neq 0 \qquad (A)$$

where f_n is the n-th complex Fourier series coefficient of the given radiation pattern, $N=n^2$, $y_p=x_p^2$, where x_p is the distance of the p-th element of the array from the origin in terms of half wavelengths, and $D_p=A_px_p \sin x_p\pi$, where A_p is the amplitude of the p-th element of the array, the said method comprising the selecting of the optimum positions of the individual elements from the s roots $y=y_p=x_p^2$ of the following algebraic equation:

$$y^s-\alpha_1 y^{s-1}+\alpha_2 y^{s-2}-\alpha_3 y^{s-3}+\ldots.+(-1)^s\alpha_s=O \qquad (B)$$

where the coefficients α_q are determined by the solution of the following $2s$ linear equations for the given f_n coefficients:

$$\sum_{q=1}^{s}\frac{(-1)^{q+1}\alpha_q}{N^q}+\frac{1}{\left[(-1)^{n+1}\frac{\pi}{2}f_n\right]}\sum_{q=1}^{s}\frac{(-1)^{q+1}\beta_q}{N^q}=1 \qquad (C)$$

once the optimum positions x_p are determined, the optimum amplitude of each element is found from (A) by solving s linear equations for the unknowns D_p the amplitude of the center element A_o at the origin $x_o=0$ being given by:

$$A_0=f_0-\frac{2}{\pi}\sum_{p=1}^{s}\frac{D_p}{y_p} \qquad (D)$$

2. A method for optimizing the spacing and the amplitude of the individual elements in a non-uniformly spaced non-symmetric array, as compared to a conventional uniformly spaced array, in order to accomplish a given radiation pattern with the minimum number of elements, said method accomplished essentially as in claim 1, except it is based on the following relationship for a non-symmetric array:

$$(-1)^{n+1}\pi f_n=\sum_{p\neq 0}\frac{B_p}{n-x_p} \text{ for } n\neq 0 \qquad (E)$$

where $B_p=A_p \sin x_p\pi$, and (E) is almost identical in form to (A) in claim 1.

3. An arrangement of a set of elements in a non-uniformly optimally spaced array, as specified in claim 1, in which each element is an electromagnetic antenna radiator or receiver, like dipole, slot, horn, aperture, dish, parabolic reflector, etc. or a combination of them.

4. An arrangement of a set of elements in a non-uniformly optimally spaced array, as specified in claim 1, in which each element is an acoustic or sonar radiator or receiver, or a combination of them, used under water or above water.

5. An arrangement of a set of elements in a non-uniformly optimally spaced array, as specified in claim 1, in which each element is a seismic radiator or receiver, like geophone, seismometer, seismograph, etc. or a combination of them, used under ground or above ground.

6. An array comprising a grouping of subarrays each designed in accordance with the method set out in claim 1.

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