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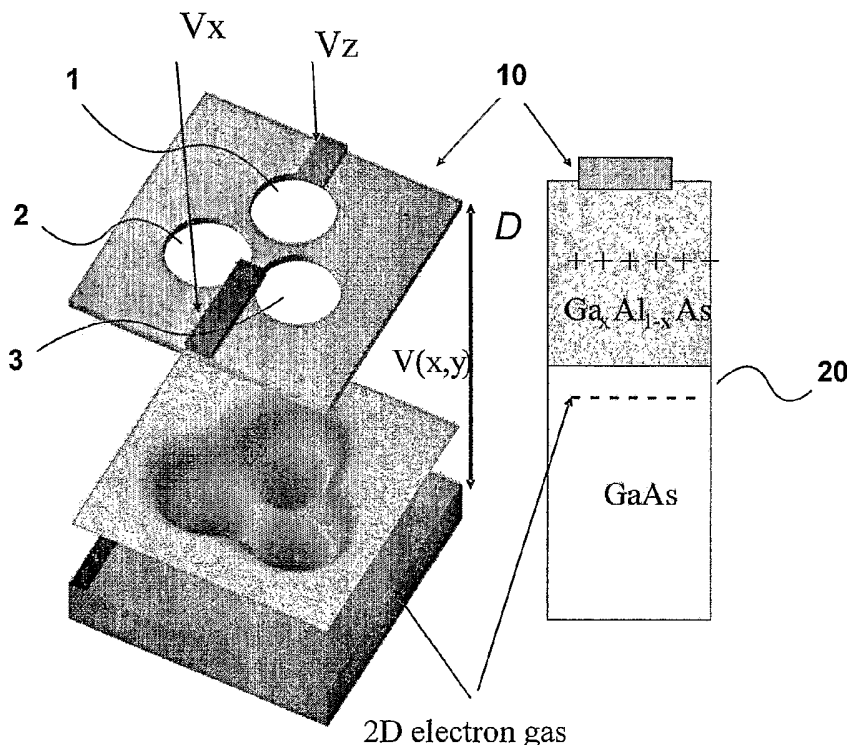
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(54) Title: VOLTAGE-CONTROLLED COMPUTING ELEMENT FOR QUANTUM COMPUTER



(57) Abstract: A computing element for use in a quantum computer has at least three coupled quantum dots, and at least one gate for applying an electric field to manipulate the state of said qubit.



For two-letter codes and other abbreviations, refer to the "Guidance Notes on Codes and Abbreviations" appearing at the beginning of each regular issue of the PCT Gazette.

Voltage-Controlled Computing Element for Quantum Computer

Cross-reference to Related Application

This application claims the benefit under 35 USC 119(e) of prior United States provisional application no. 60/579,234 filed June 15, 2004.

Field of the invention

[0001] This invention relates to the field of quantum computing, and in particular to a novel computing element that provides a coded qubit that can be suitably manipulated.

Background of the Invention

[0002] As is well known in the art, conventional computers work with binary digits or bits, which can exist in either a logical state 1 or 0. In a quantum computer, the fundamental unit of information (called a quantum bit or *qubit*), can exist not only in a state corresponding to the logical state 0 or 1 as in a classical bit, but also in states corresponding to a blend or *superposition* of these classical states. In other words, a qubit can exist as a zero, a one, or simultaneously as both 0 and 1, with a numerical coefficient representing the probability for each state. This qubit property arises as a direct consequence of its adherence to the laws of quantum mechanics which differ radically from the laws of classical physics. A physical realization of a qubit is a spin of an electron.

[0003] Quantum computation requires qubits which simultaneously satisfy contradictory requirements of both maintaining coherence and yet allowing for easy contact to perform quantum operations. Feynman, R. P., Quantum mechanical computers, *Foundations of Physics* **16**, 507 (1986); Bennett, C. H. & DiVincenzo, D. P., Quantum information and computation, *Nature* **404**, 247 (2000); Nielsen, M. A., Knill, E., & Laflamme, R., Complete quantum teleportation using nuclear magnetic resonance, *Nature* **396**, 52

(1998); and Vandersypen, L. M., *et al.*, Experimental realization of Shor's quantum factoring algorithm using nuclear magnetic resonance, *Nature* **414**, 883 (2001).

[0004] It is known that coherence is well preserved in electron spin while operations are best performed on its charge using gates and voltages. Unfortunately, voltage does not couple to electron spin. The quantum operations on a localized electron spin require either nanoscale magnetic fields or complex g-factor engineering combined with spatial manipulation of electron position using voltage controlled surface gates.

[0005] Despite tremendous progress in localizing and controlling single electron spin so far no-one has been able to demonstrate a single-qubit operation for qubits based on electron spin.

[0006] An alternative solution to the problem of qubit operations on a single spin was proposed by DiVincenzo and collaborators, DiVincenzo, D. P., *et al.*, Universal quantum computation with the exchange interaction, *Nature* **408**, 339 (2000), who suggested quantum computation with exchange interaction. DiVincenzo's basic idea is to replace the two level system based on single-electron spin with selected levels of a composite object consisting of several spins. The manipulation of selected quantum levels proceeds not through operations on single spins but through the manipulation of the coupling J between neighbouring spins due to exchange interaction. For example, two qubit states $|0\rangle$ and $|1\rangle$ for a single total spin $S=1/2$ can be identified with two opposite spin directions (S_z) up and down: $|0\rangle = |\downarrow\rangle$ and $|1\rangle = |\uparrow\rangle$.

Summary of the Invention

[0007] In accordance with the principles of the present invention the advantage of long coherence times intrinsic to electron spin is combined with the advantages of voltage control of electron charge. This results in the design of a realistic coded qubit that can be operated by electrical means.

[0008] According to a first aspect of the invention there is provided a computing element for a quantum computer comprising at least three coupled quantum dots that can adopt more than one state, and at least one gate for applying an electric to manipulate the collective state of said coupled quantum dots.

[0009] In one embodiment the invention employs a three-electron complex in three tunable gated quantum dots. The two logical states $|0L\rangle$ and $|1L\rangle$ of a qubit reside in a degenerate subspace of total spin $S=1/2$ states. Applying voltages to selected gates allows for the manipulation of the coded qubit. The technology can be scaled up to produce a working quantum computer.

[0010] In another aspect the invention provides a method of creating a qubit for use in quantum computing comprising creating a two dimensional electron gas; creating at least three coupled quantum dots in said two dimensional electron gas, said couple quantum dots having more than one collective state; and manipulating said qubit by selectively applying an electric field to said quantum dots.

Brief Description of the Drawings

[0011] The invention will now be described in more detail, by way of example only, with reference to the accompanying drawings, in which:-

[0012] Figure 1a is a cross sectional view of a coded cubit realized on three coupled gated lateral quantum dots; the right hand side shows the layer structure of the device: the metal gates, the GaAlAs semiconductor barrier layer doped with silicon donors, the GaAs layer containing electrons at the GaAs/GaAlAs interface.

[0013] Figure 1b shows the effect of the voltage applied to metallic gate V_x on the potential $V(x,y)$ seen by the electron at the GaAs/GaAlAs interface.

[0014] Figure 2a shows the energies of the nine lowest single-electron levels of the three-dot system as a function of voltage applied to the gate V_x ;

[0015] Figures 2b-2d show the wave functions corresponding to the three lowest single-electron energy levels of the three-dot system;

[0016] Figure 3 shows the three lowest energies of the three-electron coded qubit as a function of the voltage applied to the control gate V_x measured from the ground state. Black lines show energies of total-spin-1/2 states, the red line shows the energy of the spin-3/2 state. The inset shows the higher energies separated by a gap as a function of the gate voltage;

[0017] Fig. 4 shows the energies of three electrons localized on three Hubbard sites as a function of the tunnelling amplitude t_{12} measured from the ground state. Onsite interaction energy U and $t_{12} = t_{23} = 1$. Black (B) and red (R) lines show energies corresponding to low- (high-) spin states; and

[0018] Figure 5a is a scanning electron microscopy micrograph of an existing lateral quantum dot device capable of localizing a single electron and Figure 5b shows two quantum dots 1, 2 with a single electron (schematically indicated by an arrow) in each.

Detailed Description of the Preferred Embodiments

[0019] In order to understand the invention, it will be first necessary to understand the underlying theory. In a coded qubit consisting of three localised electron spins (a, b, c) it is possible to select a total direction (total S_z) of the three spins, e.g., $|\downarrow\rangle$. A state with this direction can be realised by three different states: $|\downarrow a, \downarrow b, \uparrow c\rangle$, $|\downarrow a, \uparrow b, \downarrow c\rangle$, and $|\uparrow a, \downarrow b, \downarrow c\rangle$, which differ by the position of the spin pointing up. These three states can be grouped into a single state with total spin $S=3/2$ and two orthogonal states with total spin $S=1/2$. Examples of the two orthogonal spin states forming the two logical qubit states $|0L\rangle$ and $|1L\rangle$ are $|0L\rangle = \frac{1}{\sqrt{2}}(|\downarrow a, \downarrow b, \uparrow c\rangle - |\downarrow a, \uparrow b, \downarrow c\rangle)$ and $|1L\rangle = \frac{1}{\sqrt{6}}(|\downarrow a, \downarrow b, \uparrow c\rangle + |\downarrow a, \uparrow b, \downarrow c\rangle - 2|\uparrow a, \downarrow b, \downarrow c\rangle)$.

[0020] The properties of a three-electron complex in a single quantum dot, including the two logical spin states can be analyzed theoretically and experimentally. The three-electron states are a product of the orbital and spin part. The spin part has been discussed in terms of the logical qubit states. But it is precisely the orbital part that can be manipulated using voltages, and hence it plays an important role in a coded qubit.

[0021] Following the methodology which allowed for a quantitative understanding of a single dot with controlled number of electrons to study the energy levels and spin and orbital wave functions of a realistic coded qubit, the inventors have shown that there exist two low-energy states, which map very well onto the two logical qubit states, which can be manipulated by applied voltages.

[0022] One embodiment of a proposed device in accordance with the invention is shown schematically in Fig.1a. It consists of a metallic gate 10 providing a mask defining three coupled lateral quantum dots 1,2,3. The dots are defined by locally depleting the two-dimensional electron gas (2DEG) at a distance D below the surface.

[0023] The quantum dots can be formed from a two dimensional gas in a GaAs/ $\text{Al}_x\text{Ga}_{1-x}\text{As}$ heterostructure generally designated 20 with a metal electrode 10. Control gates V_x and V_z are provided as will be described in more detail. These gates must be insulated from the electrode 10. For example, the electrode 10 can be etched away in the vicinity of the gates. Typically a AlGaAs layer may be grown by MBE (Molecular Beam Epitaxy) on an GaAs sublayer and doped with silicon atoms, as shown in Fig.1a. A two-dimensional electron gas can be formed at the interface by the combination of doping and the application of a suitable electric field. One method of making a lateral quantum dot is to electrostatically define it by four gates.

[0024] Figure 5a shows one such device. In this case, the dot is electrostatically defined by four gates above a GaAs/ $\text{Al}_x\text{Ga}_{1-x}\text{As}$ heterostructure. In Figure 5a, the gates 10a, 10b, 22, 24 can be etched from the applied metal electrode 10 defining the quantum dots, or

they can be separately deposited. The structure in Figure 5a uses a top "T" gate 22 in combination with left and right finger gates 10a, 10b to define the dot geometry. A narrow plunger gate 24, located in the gap between the left and right finger gates is used to vary the number of electrons. The lithographic width and height of the triangular dots was approximately 0.45 μm . The bulk density (mobility) of the $\text{Al}_x\text{Ga}_{1-x}\text{As}/\text{GaAs}$ wafer used were $1.73 \times 10^{11} \text{ cm}^{-2}$ ($2 \times 10^6 \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$).

[0025] Figure 5b shows two quantum dots 1, 2 defined by gates 10a, 22, and 22a; and 10b, 22, 22a. Gates 24a and 24b provide control electrodes. The three gate structure shown in Figure 1 can be made in a similar manner.

[0026] Referring again to the structure shown in Figure 1a, the application of a negative voltage creates a broad region depleted of electrons, with three identical minima. An additional gate, the control gate V_x is positioned in-between the two adjacent dots 1 and 2. The gate V_x lowers or increases the tunnelling barrier between the two dots 1,2 as shown in Figure 1b. The effect of the gate is identical to the effect of the σ_x operation acting on logical qubit states. The σ_x acting on logical qubit $|0L\rangle$ transforms it into qubit $|1L\rangle$ and vice versa. The manifestation of the σ_x operation in the energy spectrum is the energy splitting Δ_x of the two degenerate logical qubit states. Each energy level corresponds to a linear combination of the $|0L\rangle$ and $|1L\rangle$ logical states. In order to have complete logical qubit rotation, a second gate is required. This gate, labelled V_z , plays the role of σ_z spin operation, and tunes the potential minimum of the third dot. Finally, we need to precisely control the number of electrons $N=3$ in our device. This can be accomplished by following the detailed design, which allows the number of electrons in single and coupled quantum dots to be controlled.

[0027] The operation of gates translates into the changes of a potential seen by each of the three electrons in a coded qubit. The electrostatic potential $V_E(\vec{r})$ in the electronic plane due to the gates on the surface is given by an integral over the potential

$V_G(\vec{r})$ applied to the gates as $V_E(\vec{r}) = \int \frac{d\vec{r}'}{2\pi} |D| \frac{V(\vec{r}')}{(D^2 + (\vec{r} - \vec{r}')^2)^{3/2}}$, where D is the distance between the surface and the 2DEG layer, and $V_G(\vec{r})$ is the electrical potential on the surface, corresponding to the appropriate voltage on the gates, and equal to zero in the openings (the holes). All energies and distances are expressed in the units of effective Rydberg, $Ry = m^* e^4 / 2\varepsilon^2 \hbar^2$, and effective Bohr radius, $a_B = \varepsilon \hbar^2 / m^* e^2$, respectively. Here, e and m^* are the electronic charge and effective mass, respectively, ε is the dielectric constant of the semiconductor, and $\hbar$ is the Dirac's constant. For GaAs these units are $1 Ry = 5.93 \text{ meV}$, and $1 a_B = 97.9 \text{ \AA} \sim 10 \text{ nm}$.

[0028] The candidate for a coded qubit investigated here has the length of the side of the rectangular gate of $22.4 a_B$ (for better visibility of the potential minima, in Fig. 1 we only show a central part of the gate, with side length of $14 a_B$). The diameter of the opening (the hole) in the gate is taken to be $4.2 a_B$, the distance between the centers of each pair of holes is $4.85 a_B$, and the distance between the gate and the 2DEG layers is $14 a_B$. The voltage applied to the main gate corresponds to the electronic potential energy $-eV = 10 Ry$ in the plane of the gates. As shown in Fig.1, the pattern of the holes in the surface gate translates into three potential minima on the electronic plane. As illustrated in Fig. 1a, if the voltage of the control gate V_x is set to zero, the three potential minima are identical. But if we apply voltage $-eV = -10 Ry$, i.e., opposite to that of the main gate, the barrier between two of the dots is lowered as shown in Fig.1b and the dots 1 and 2 are expected to be strongly coupled. The two other tunnelling barriers are weakly affected by the control gate. The voltage applied to the gates couples to the charge of the electron and determines the potential V_E acting on each individual electron. The potential determines the single-electron energies E_n and wave functions $\phi_n(x, y)$. The external voltages applied to control gates affect directly only single-electron properties, and through the modification of single-electron energies and wave functions – the states of the three-

electron complex. To calculate the single-electron spectrum we discretize the area under the gates, and define the single-electron states $\phi_n(i, j)$ on a lattice (x_i, y_j) with spacing h .

[0029] Electrons move on a lattice, and their spectrum is described by the tight-binding Hamiltonian:

$$(E_{i,j} + V_E(i, j))\phi_n(i, j) + \sum_{k,l} t_{i,j;k,l} \phi_n(k, l) = E_n \phi_n(i, j), \quad (1)$$

where the site energy and hopping matrix elements are given by $E_{i,j} = 4h^{-2}$ and $t_{i,j;k,l} = -h^{-2} \delta_{k,i \pm 1} \delta_{l,j \pm 1}$, respectively. As a result we obtain a large, but sparse Hamiltonian matrix, which is diagonalized using the conjugate gradient method. In Fig. 2a it will be seen that the calculated energies E_n corresponding to nine lowest single-particle states as a function of the potential V_x of the control gate. At zero bias the spectrum consists of three energy levels, the ground state and two degenerate excited states, separated by a gap from the rest of the spectrum. The corresponding three lowest single-particle wave functions at zero bias are shown in Fig. 2b. The states and the energy spectrum can be understood by considering a linear combination of wave functions $f_m(x, y)$ localized on each m -th dot.

[0030] The corresponding ground state can be written as

$$|0\rangle = \frac{1}{\sqrt{3}}(f_1(x, y) + f_2(x, y) + f_3(x, y)), \text{ and the two degenerate excited states as } |1\rangle = \frac{1}{\sqrt{6}}(f_1(x, y) + f_2(x, y) - 2f_3(x, y)) \text{ and } |2\rangle = \frac{1}{\sqrt{2}}(f_1(x, y) - f_2(x, y)).$$

In all of these states the electron is delocalised and shared between all dots. For negative gate voltages, i.e., when the barrier between the two dots is lowered, the two excited states mix and energy levels split. When the voltage is zero, the three dots are identical, and the two excited states are degenerate. As the barrier between the two dots is increased even further (positive gate voltages), the energies of degenerate states are split again. In this case, however, the splitting is not large. We attribute it to the fact that, in this regime, all interdot barriers are already high and the three dots are almost isolated. Therefore, a

further increase of the control gate voltage increases the total energy of the system, but does not lead to a significant symmetry breaking. Now that we understand the effect of gate voltages on a single electron, we proceed to considering three electrons localized in our three-dot potential. We describe simultaneously the spin and orbital three-electron states in the language of second quantization as $|i\sigma, j\sigma', k\sigma''\rangle = c_{i\sigma}^+ c_{j\sigma'}^+ c_{k\sigma''}^+ |0\rangle$. The operator $c_{i\sigma}^+$ ($c_{i\sigma}$) creates (annihilates) an electron with spin σ on the single-particle state ϕ_i calculated from Eq. 2.

[0031] The electron-electron interactions mix different three-electron configurations for a given set of applied voltages. The mixing is governed by the matrix elements of the Hamiltonian, which takes the form:

$$H = \sum_{i\sigma} E_{i\sigma} c_{i\sigma}^+ c_{i\sigma} + \frac{1}{2} \sum_{ijkl, \sigma\sigma'} \langle i, j | V | k, l \rangle c_{i\sigma}^+ c_{j\sigma'}^+ c_{k\sigma} c_{l\sigma'}, \quad (3) \text{ where}$$

the energies $E_{i\sigma}(V_x, V_z)$ and matrix elements $\langle i, j | V | k, l \rangle(V_x, V_z)$ of the Coulomb potential are implicit functions of the applied voltages V_x, V_z . These matrix elements are independent of spin. They are calculated in real space as

$$\langle i, j | V | k, l \rangle = 2h^3 \sum_{sp, uv} \frac{\phi_i(s, p) \phi_j(u, v) \phi_k(u, v) \phi_l(s, p)}{[(s-u)^2 + (p-v)^2 + d^2]^{1/2}}, \text{ with parameter } d \text{ accounting for}$$

the finite thickness of the electron layer (in the following example we take $d = 0.2 a_B$).

[0032] To capture spin effects we generate all possible configurations of three electrons on N_S single-particle states and classify them by total spin. This allows us to construct the Hamiltonian matrix separately in the spin 3/2 and spin 1/2 basis and diagonalize these matrices numerically.

[0033] Figure 3 shows the low-energy segment of the three-electron energy spectrum as a function of the voltage on the control gate V_x (the energies are measured from the ground-state energy). The low-energy spectrum consists of two low-energy states corresponding to the two states of the $S=1/2$ Hilbert space (black lines), while the energy of the higher

state corresponds to the high-spin $S=3/2$ state (red line). Hence the central result of this paper rests on the identification of the two lowest $S=1/2$ states as the two states of the coded qubit. Fig.3 shows that they can be manipulated by applying gate voltage V_x . The voltage V_x acts analogously to the σ_x operation, while the σ_z operation can be implemented by applying voltage V_z . Hence we have presented a design of a lateral quantum dot device acting as a voltage-tunable coded qubit.

[0034] The principle of operation of this device, and the analogy to the two-spin logical qubit states can be understood by considering a simplifying model of the coded qubit. The model starts with the single-particle spectrum of Fig.2, described in terms of linear combinations of orbitals localised on each dot “1, 2, 3”. With the energies of one localised orbital per dot denoted by E_n ($n=1, 2, 3$), hopping matrix element $t_{n,m}$ from dot n to dot m , and on-site Coulomb repulsion U , we can analyze the coded qubit in the framework of a three-site Hubbard model. We start with the completely polarized system, i.e., one with total spin $S=3/2$. We can distribute our electrons on the three sites in only one way: one electron on each site, which gives a spin-polarized state

$|a_{3/2}\rangle = c_{3\downarrow}^+ c_{2\downarrow}^+ c_{1\downarrow}^+ |0\rangle$. As the basis of our Hilbert space consists of one configuration only, $|a_{3/2}\rangle$ is the eigenstate of our system, and its energy $E_{3/2} = E_1 + E_2 + E_3$. Let us

now flip the spin of one of the electrons. This electron can be placed on any orbital, and as a result we can generate nine different configurations. Three of those configurations involve single occupancy of the orbitals. They can be written as $|A\rangle = c_{3\downarrow}^+ c_{2\downarrow}^+ c_{1\uparrow}^+ |0\rangle$,

$|B\rangle = c_{1\downarrow}^+ c_{3\downarrow}^+ c_{2\uparrow}^+ |0\rangle$, and $|C\rangle = c_{2\downarrow}^+ c_{1\downarrow}^+ c_{3\uparrow}^+ |0\rangle$. Out of these three configurations we

construct the three eigenstates of the total spin operator. One of those eigenstates is $|a_{3/2}\rangle = \frac{1}{\sqrt{3}}(|A\rangle + |B\rangle + |C\rangle)$, and it corresponds to the total spin $S=3/2$. The two

other eigenstates, $|a_{1/2}\rangle = \frac{1}{\sqrt{2}}(|A\rangle - |B\rangle)$ and $|b_{1/2}\rangle = \frac{1}{\sqrt{6}}(|A\rangle + |B\rangle - 2|C\rangle)$,

correspond to the total spin $S=1/2$. The remaining six configurations involve doubly-occupied orbitals. They are $|c_{1/2}\rangle = c_{2\downarrow}^+ c_{1\downarrow}^+ c_{1\uparrow}^+ |0\rangle$, $|d_{1/2}\rangle = c_{3\downarrow}^+ c_{1\downarrow}^+ c_{1\uparrow}^+ |0\rangle$

$|e_{1/2}\rangle = c_{2\downarrow}^+ c_{1\downarrow}^+ c_{2\uparrow}^+ |0\rangle$, $|f_{1/2}\rangle = c_{3\downarrow}^+ c_{2\downarrow}^+ c_{2\uparrow}^+ |0\rangle$, $|g_{1/2}\rangle = c_{3\downarrow}^+ c_{1\downarrow}^+ c_{3\uparrow}^+ |0\rangle$, and

$|h_{1/2}\rangle = c_{3\downarrow}^+ c_{2\downarrow}^+ c_{3\uparrow}^+ |0\rangle$. All these configurations are eigenstates of the total spin with $S = 1/2$. Thus, among our nine spin-unpolarized states we have one high-spin, and eight low-spin states. In this basis the Hamiltonian matrix is block-diagonal, with the high-spin state completely decoupled. The energy corresponding to this state is equal to that of the fully polarized system discussed above, and is $E_{3/2}$. We construct the Hamiltonian matrix in the basis of the nine $S=1/2$ -spin configurations by dividing them into three groups, each containing one of the singly-occupied configurations $|A\rangle = c_{3\downarrow}^+ c_{2\downarrow}^+ c_{1\uparrow}^+ |0\rangle$, $|B\rangle = c_{1\downarrow}^+ c_{3\downarrow}^+ c_{2\uparrow}^+ |0\rangle$, and $|C\rangle = c_{2\downarrow}^+ c_{1\downarrow}^+ c_{3\uparrow}^+ |0\rangle$ which are needed to construct a coded qubit. By labelling each group with an index of spin-up electron, the Hamiltonian takes the form of a 3×3 matrix:

$$\hat{H}_{1/2} = \begin{bmatrix} \hat{H}_1 & \hat{T}_{12} & \hat{T}_{13} \\ \hat{T}_{12} & \hat{H}_2 & \hat{T}_{23} \\ \hat{T}_{13} & \hat{T}_{23} & \hat{H}_3 \end{bmatrix}. \quad (4)$$

The diagonal matrix $\hat{H}_i = \begin{bmatrix} 2E_1 + E_2 + U_1 & & \\ & 2E_1 + E_3 + U_1 & \\ & & E_1 + E_2 + E_3 \end{bmatrix}$ describes the interaction

of three configurations which contain spin up electron on site “1”, i.e., two doubly-occupied configurations $|c_{1/2}\rangle = c_{2\downarrow}^+ c_{1\downarrow}^+ c_{1\uparrow}^+ |0\rangle$ and $|d_{1/2}\rangle = c_{3\downarrow}^+ c_{1\downarrow}^+ c_{1\uparrow}^+ |0\rangle$, and a singly-occupied configuration $|A\rangle = c_{3\downarrow}^+ c_{2\downarrow}^+ c_{1\uparrow}^+ |0\rangle$ (in this order). The configurations with double occupancy acquire the diagonal interaction term U . The three configurations involve a pair of spin-polarized electrons (spin triplet) moving on a triangular plaquette in the presence of a “spectator” spin-up electron. Because of the triplet character of the two electrons, the phase of the hopping matrix element $-t_{13}$ from site “1” to site “3” is different from the phase of the hopping matrix element $+t_{23}$ from site “2” to site “3”. The negative phase in $-t_{13}$ distinguishes singlet and triplet electron pairs, lowers the energy of the spin-polarized pair in the absence of interactions, and is a manifestation of the “molecular Hund’s rule”. The orbital Hund’s rules are responsible for dressing up of the desired singly occupied configuration by the doubly occupied configuration, and by the

interaction of the coded qubit states. It is clear that the simple model discussed here contains a rich and nontrivial behaviour. The remaining matrices corresponding to spin-up electrons localized on sites “2” and “3” can be constructed in a similar fashion:

$$\hat{H}_2 = \begin{bmatrix} E_1 + 2E_2 + U_2 & t_{23} & -t_{13} \\ t_{23} & E_1 + E_2 + E_3 & t_{12} \\ -t_{13} & t_{12} & 2E_2 + E_3 + U_2 \end{bmatrix}, \quad \hat{H}_3 = \begin{bmatrix} E_1 + E_2 + E_3 & t_{23} & -t_{13} \\ t_{23} & E_1 + 2E_3 + U_3 & t_{12} \\ -t_{13} & t_{12} & E_2 + 2E_3 + U_3 \end{bmatrix}.$$

The interaction between them is given in terms of effective hopping matrix

$$\hat{T}_{ij} = \begin{bmatrix} 0 & t_{ij} & 0 \\ t_{ij} & 0 & t_{ij} \\ 0 & t_{ij} & 0 \end{bmatrix}. \text{ There is no direct interaction between the configurations with single}$$

occupancy, since such scattering process would have to involve two electrons, one with spin up and one with spin down. This cannot be accomplished by the single-particle tunnelling. These states are coupled only indirectly, involving the configurations with double occupancy. The parameters of the Hubbard Hamiltonian can in principle be fitted to the numerical calculations for realistic gates. At this stage we are interested in understanding of the operation of the coded qubit. Assuming model parameters $t = -1$ and $U = 3$ we diagonalize the Hamiltonian matrix as a function of the amplitude t_{31} and show the resulting eigenenergies in Fig. 4.

As predicted, the two $S=1/2$ spin states cross at $t_{31} = -1$. The logical qubit states are given by $|0_L\rangle = \alpha_0 \frac{1}{\sqrt{2}}(|A\rangle - |B\rangle) + \beta_0 |\Delta_0\rangle$ and $|1_L\rangle = \alpha_1 \frac{1}{\sqrt{6}}(|A\rangle + |B\rangle - 2|C\rangle) + \beta_1 |\Delta_1\rangle$ where $|\Delta_0\rangle, |\Delta_1\rangle$ are contributions of doubly-occupied configurations. While these configurations render the form of the qubit not easily expressible in terms of simple singly-occupied configurations, they are responsible for the tunability of the qubit. The Hubbard model presented here both supports the numerical simulations of the voltage-controlled coded qubit as well as facilitates our understanding of its operations.

[0035] The inventors have shown that a lateral gated quantum dot device produce a voltage-tunable qubit based on electron spin.

All references listed below are incorporated herein by reference.

We claim

1. A computing element for a quantum computer comprising at least three coupled quantum dots that can adopt more than one collective state, and at least a first gate means for applying an electric field to manipulate the collective state of said coupled quantum dots.
2. The computing element of claim 1, wherein said coupled quantum dots are laterally adjacent each other.
3. A computing element as claimed in claim 1, wherein said quantum dots are formed in a material providing a two-dimensional electron gas, and dot-forming gate means is arranged on said material to apply a voltage to locally deplete said electron gas and thereby define said quantum dots.
4. A computing element as claimed in claim 3, wherein said material is deposited as layer on a substrate.
5. A computing element as claimed in claim 3, wherein said quantum dots form a tunneling barrier between them, and further comprising a second gate means for applying an electric field voltage to change said tunneling barrier.
6. A computing element as claimed in claim 5, wherein said second gate means is positioned between two of said quantum dots.
7. A computing element as claimed in claim 6, comprising a third said gate means for tuning the potential of a third said dot.
8. A computing element as claimed in claim 7, wherein said third gate means tunes the potential minimum of said third dot.
9. A computing element as claimed in claim 8, comprising a complex of three quantum dots.
10. A computing element as claimed in claim 9, wherein each quantum dot comprises a single electron.

11. A computing element as claimed in claim 4, wherein said first gate means comprises one or more pairs of fingers defining locally depleted regions to form said quantum dots.
12. A computing element as claimed in claim 11, wherein said first gate means further comprises additional electrodes extending between said pairs of fingers.
13. A computing element as claimed in claim 1, wherein said quantum dots are arranged in a triangular formation.
14. A computing element as claimed in claim 3, wherein said two dimensional gas is formed in a heterostructure.
15. A computing element as claimed in claim 14, wherein said two-dimensional gas is formed at the interface of two components of said heterostructure.
16. A computing element as claimed in claim 15, wherein said heterostructure is a GaAs/AlGaAs heterostructure.
17. A computing element as claimed in any one of claims 1 to 8, wherein each quantum dot comprises a single electron.
18. A computing element as claimed in claim 17, wherein the state of said computing element is represented by the collective spin states of said single electrons.
19. A quantum computer comprising multiple computing elements as claimed in any of claims 1 to 18.
20. A method of creating a computing element for use in quantum computing comprising:
 - creating a two dimensional electron gas;
 - creating at least three coupled quantum dots in said two dimensional electron gas, said couple quantum dots having more than one collective state; and
 - manipulating the state of said computing element by selectively applying an electric field to said quantum dots.
21. A method as claimed in claim 20, wherein said quantum dots are created in said two-

dimensional gas by applying an electric field.

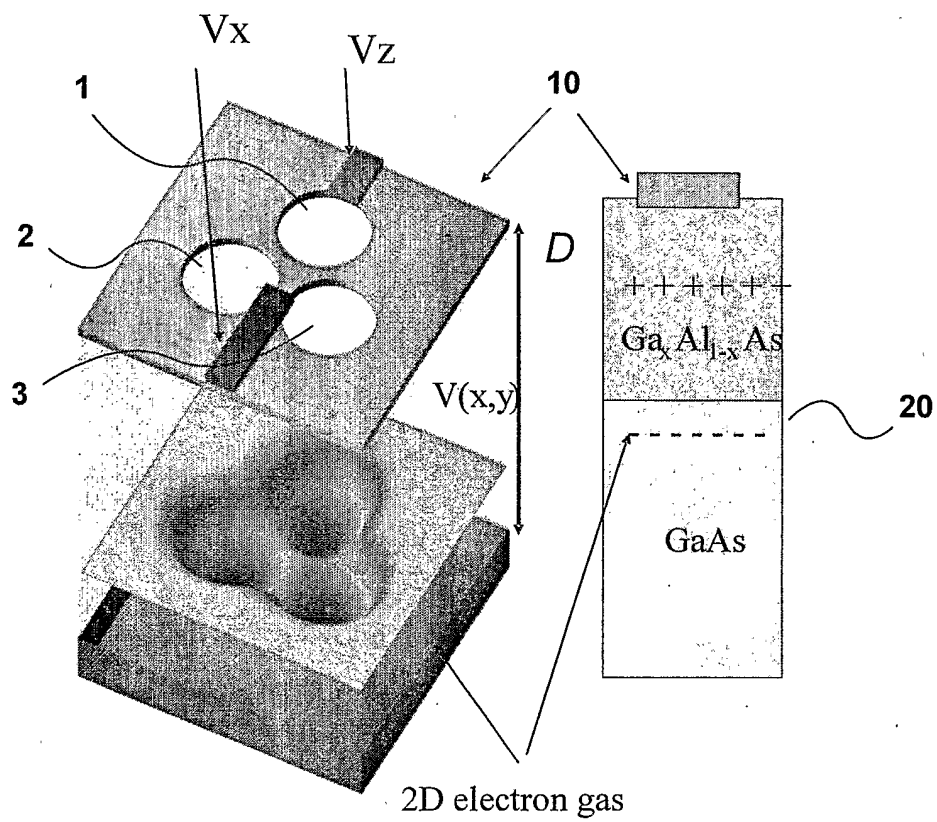
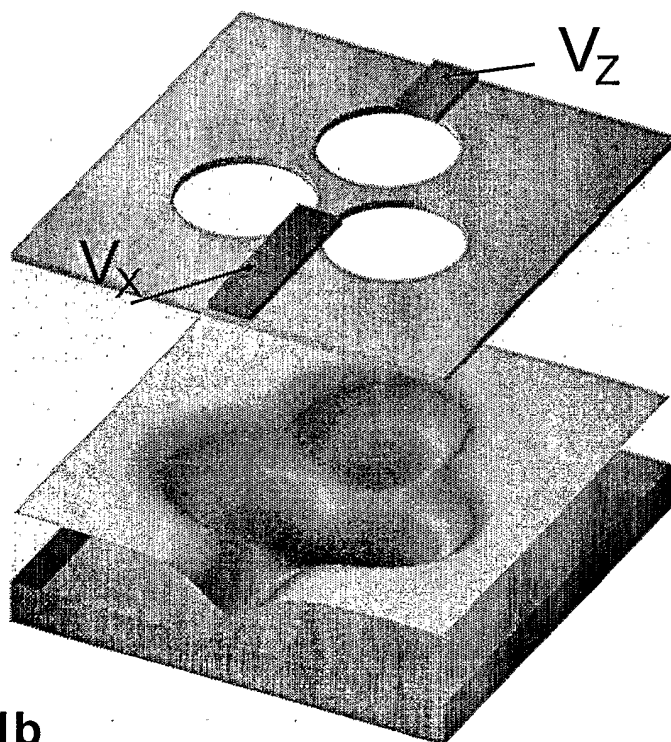
22. A method as claimed in claim 20, wherein said two dimensional gas is created at the interface of a heterostructure.

23. A method as claimed in claim 22, wherein said heterostructure is a GaAs/AlGaAs heterostructure.

24. A method as claimed in claim 23, wherein metal gates are deposited on a surface of said heterostructure to form said quantum dots within said heterostructure.

25. A method as claimed in claim 20, wherein each quantum dot comprises a single electron.

26. A method as claimed in claim 20, wherein the state of said computing element is represented by the collective spin states of said electrons.

**Fig. 1a****Fig. 1b**

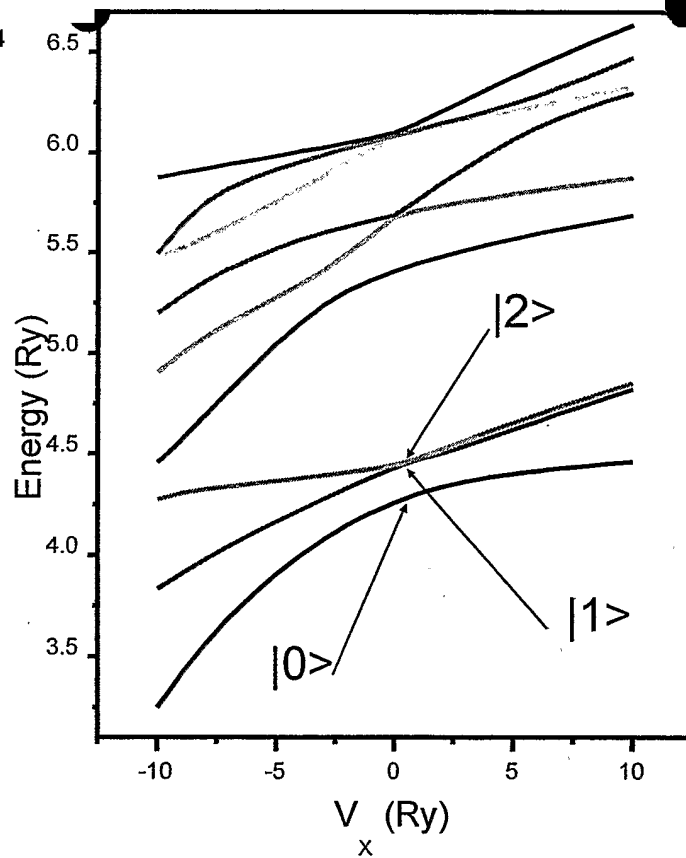
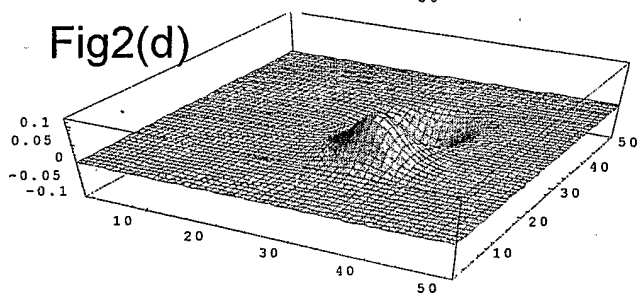
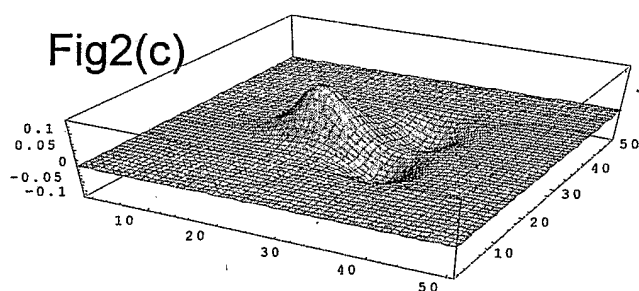
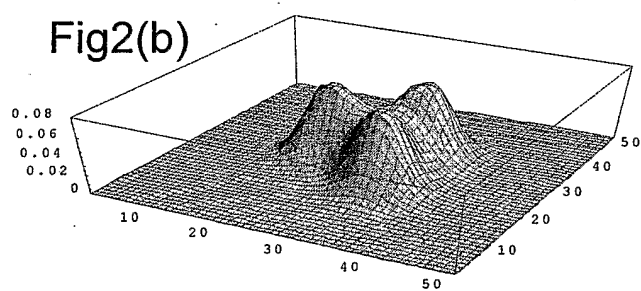
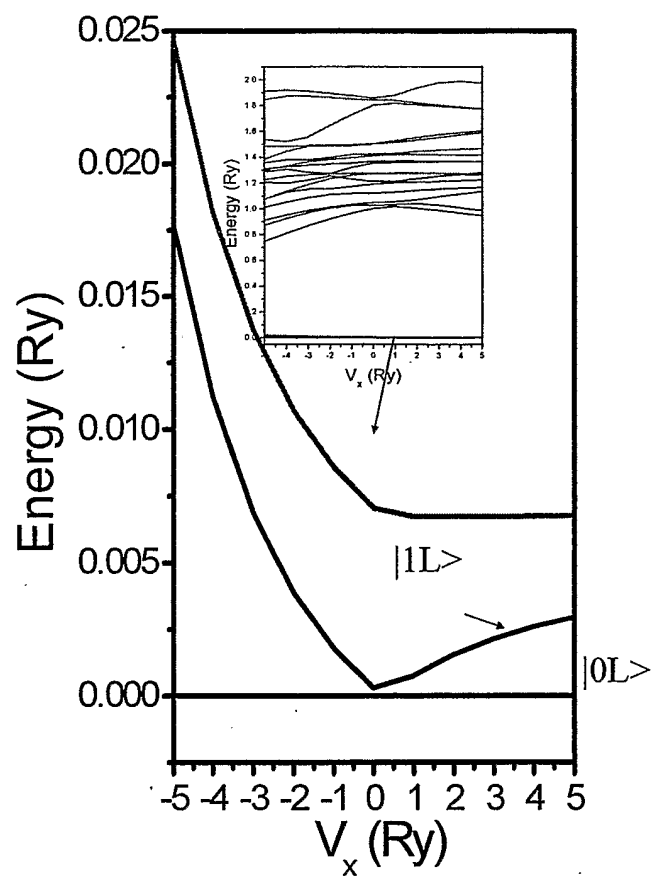
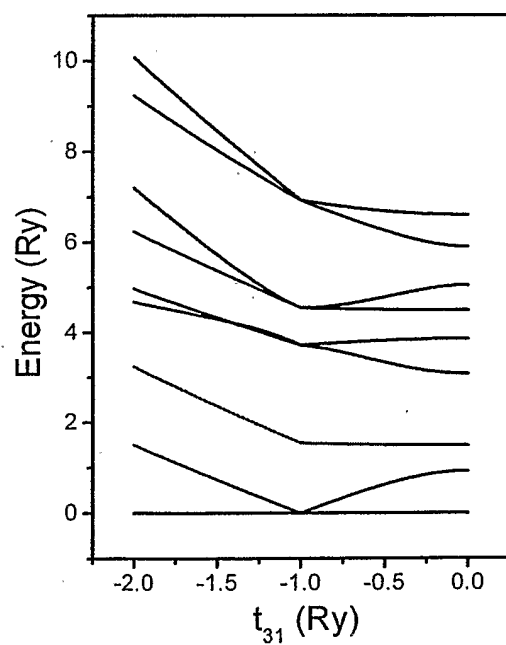


Fig. 2a



**Fig. 3****Fig. 4**

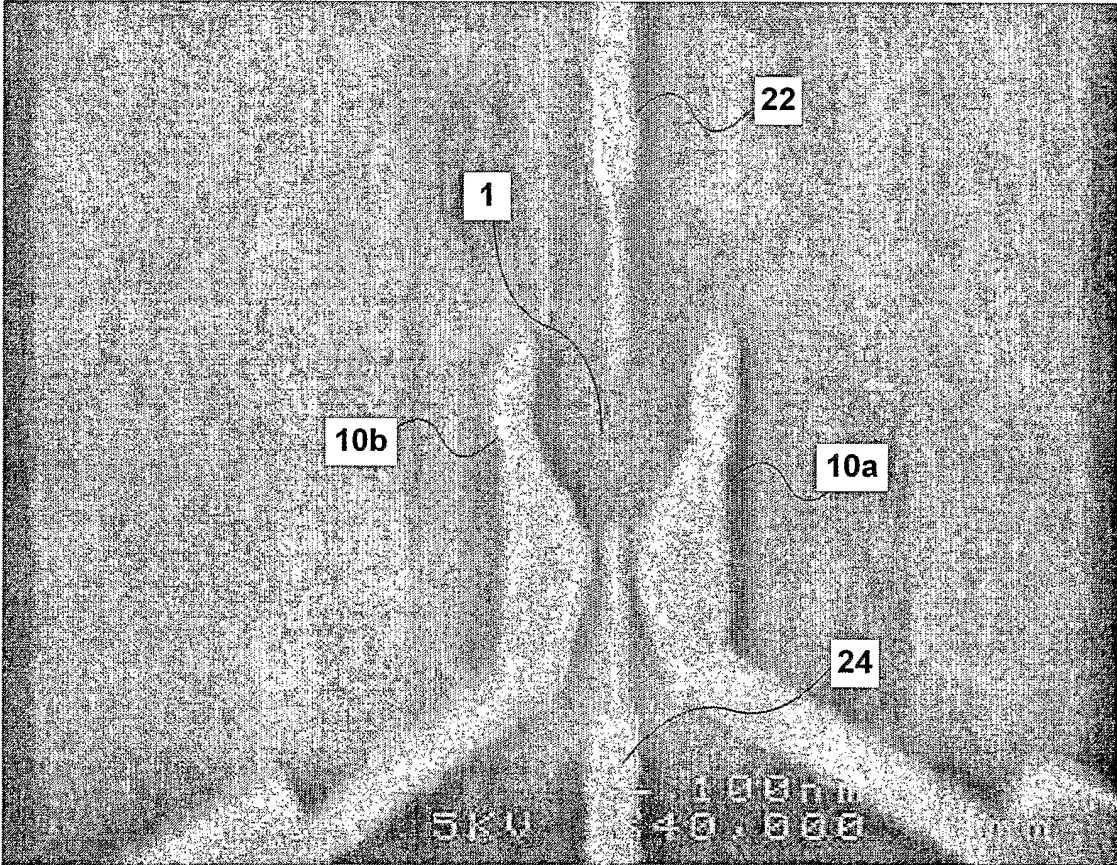


Fig. 5a

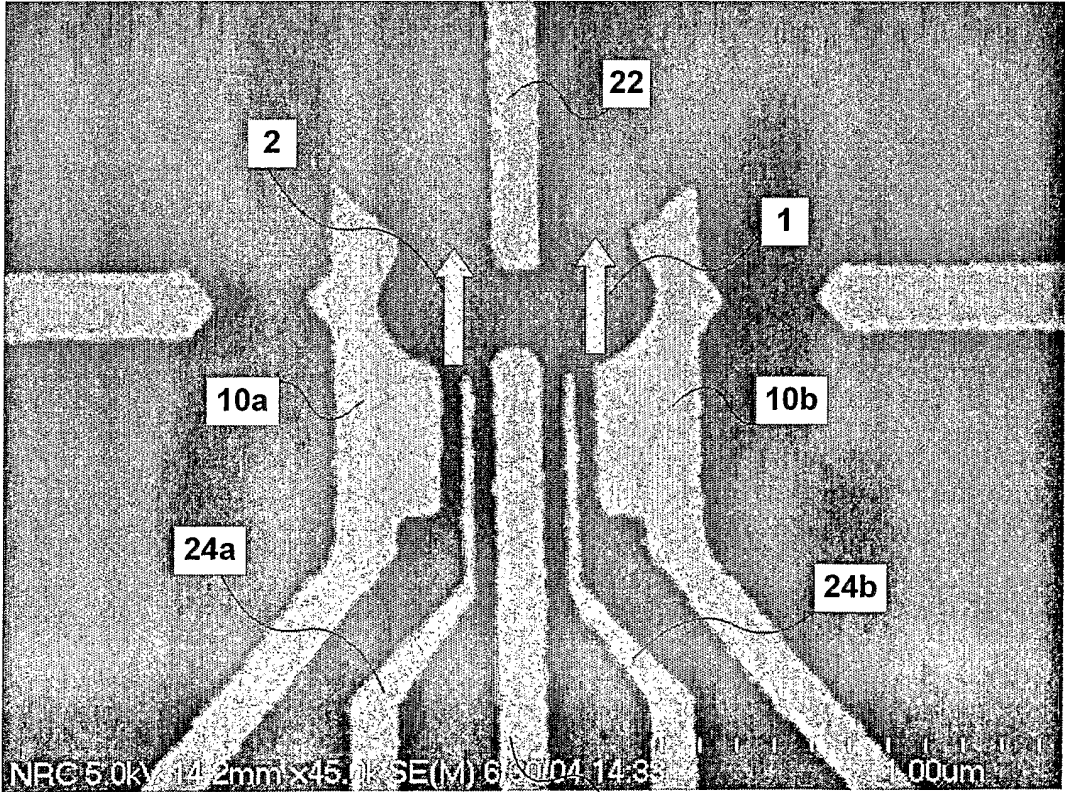


Fig. 5b

INTERNATIONAL SEARCH REPORT

International application No.
PCT/CA2005/000920

A. CLASSIFICATION OF SUBJECT MATTER IPC(7): G06N 1/00, H03K 19/08 According to International Patent Classification (IPC) or to both national classification and IPC		
B. FIELDS SEARCHED Minimum documentation searched (classification system followed by classification symbols) IPC(7): G06N 1/00, H03K 19/08 Documentation searched other than minimum documentation to the extent that such documents are included in the fields searched Electronic database(s) consulted during the international search (name of database(s) and, where practicable, search terms used) Databases searched: Pluspat, CPD (Canadian Patent Database), Google, Delphion, IEEE Xplore Keywords searched: quantum dot/operation, gate, computing element, coupling, collective states, voltage control, electron spin, single qubit		
C. DOCUMENTS CONSIDERED TO BE RELEVANT		
Category*	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
X	US 5 530 263 25. Jun., 1996(25-06-1996) Di Vincenzo **see Abstract, col. 1. line 25 to line 46, col. 1 line 64 to col. 2 line 50, col. 3 line 63 to col. 4 line 4 and see claims**	1
A	US 2002/0190249 19.Dec. 2002 (19-12-2002) Williams et al. ** whole document**	1-26
A	EP 1 310 910(A1) 14 May 2003 (14-05-2003) Suter **whole document**	1-26
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A	US 2002/0179897 5.Dec.2002 (05-12-2002) Eriksson et al. **whole document**	1-26
☐ Further documents are listed in the continuation of Box C. ☒ See patent family annex.		
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Date of the actual completion of the international search 20 September 2005 (20-09-2005)	Date of mailing of the international search report 13 October 2005 (13-10-2005)	
Name and mailing address of the ISA/CA Canadian Intellectual Property Office Place du Portage I, C114 - 1st Floor, Box PCT 50 Victoria Street Gatineau, Quebec K1A 0C9 Facsimile No.: 001(819)953-2476	Authorized officer Karen Oprea (819) 934-2668	

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