Aug. 20, 1929. O. J. ZOBEL 1,724,987

SELECTIVE CONSTANT RESISTANCE NETWORK

Filed April 13, 1928 2 Sheets-Sheet 1

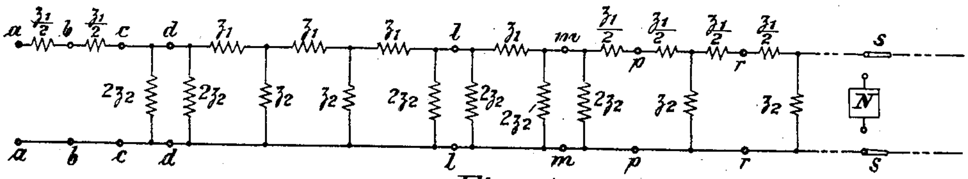

Fig. 1

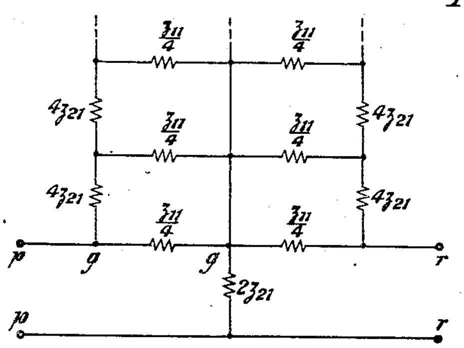

Fig. 2

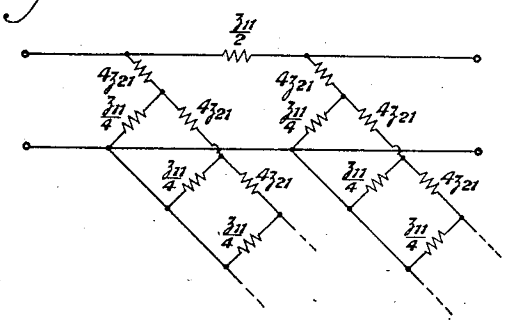

Fig. 3

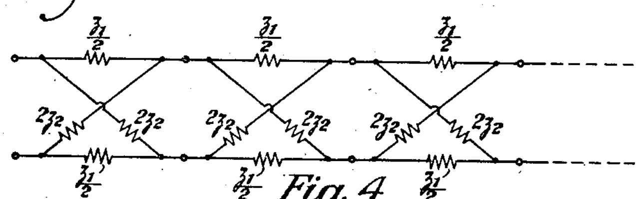

Fig. 4

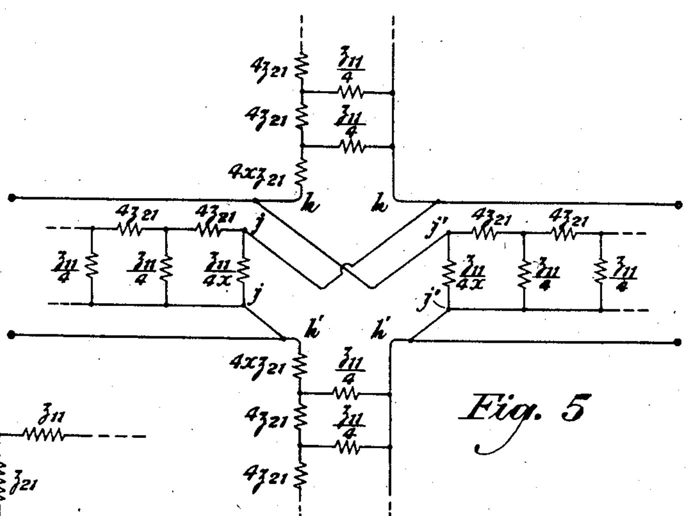

Fig. 5

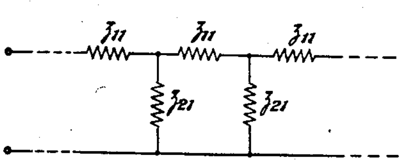

Fig. 6

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SELECTIVE CONSTANT RESISTANCE NETWORK

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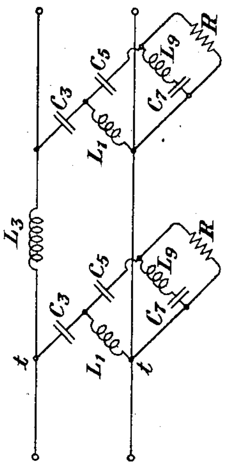

Fig. 9

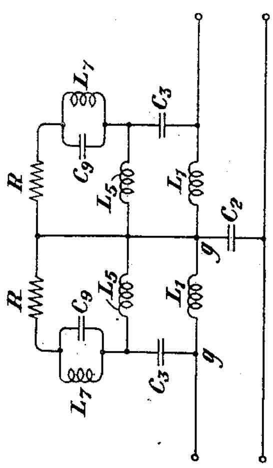

Fig. 8

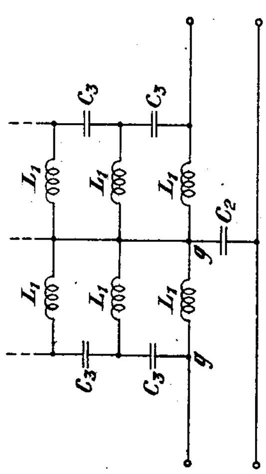

Fig. 7

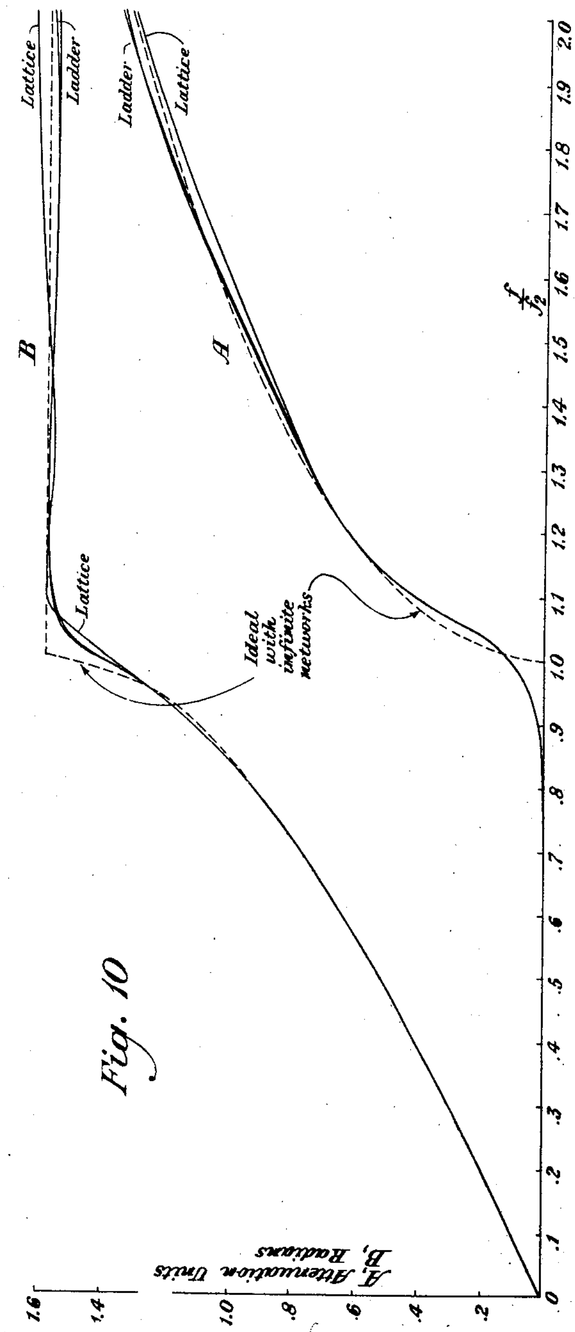

Fig. 10

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Patented Aug. 20, 1929. **1,724,987**

UNITED STATES PATENT OFFICE.

OTTO J. ZOBEL, OF NEW YORK, N. Y., ASSIGNOR TO AMERICAN TELEPHONE AND TELEGRAPH COMPANY, A CORPORATION OF NEW YORK.

SELECTIVE-CONSTANT-RESISTANCE NETWORK.

Application filed April 13, 1928. Serial No. 269,845.

The principal object of my invention is to provide apparatus for convenient and effective use to filter components according to frequency from a sequence of composite electric waves. Another object of my invention is to provide apparatus for performing this operation with advantageous impedance values at all frequencies. Another object is to provide an electric wave filter or selective network having substantially constant input impedance at practically all frequencies. These objects, and various other objects of my invention, will become apparent on consideration of a limited number of specific examples of practice according to the invention, which I have chosen for presentation in the following specification. It will be understood that this disclosure relates principally to these particular examples of the invention, and that the scope of the invention will be indicated in the appended claims.

Referring to the drawings, Figure 1 is a diagram of a ladder type filter, with generalized elements indicated symbolically and showing various terminations at the input end; Fig. 2 is a diagram of a mid-series section to be substituted in the filter of Fig. 1. Fig. 3 is a diagram of a mid-shunt section to be substituted in the filter of Fig. 1; Fig. 4 is a diagram of a lattice type filter, of general form; Fig. 5 is a diagram of a section for a more special lattice type filter; Fig. 6 is a diagram of an elementary filter; Fig. 7 is a diagram of a mid-series low pass filter section with infinite networks for certain elements; Fig. 8 is a corresponding diagram with finite terminations for the networks; Fig. 9 is a diagram of a mid-shunt low pass filter section with finite endings for the networks comprised therein; and Fig. 10 is an assembly of diagrams for attenuation and phase shift in various filters considered herein.

In Fig. 1, a ladder type filter is shown, with recurrent alternately disposed series impedances z_1 and shunt impedances z_2. This network has a pair of input terminals at the left. The theory of such networks is ordinarily developed on the initial assumption that the recurrent structure extends to infinity from the input terminals; this is indicated by the dotted lines at the right. Of course, any series impedance z_1 may be separated into two equal parts in series, each of value $\frac{1}{2}z_1$. Also, any shunt element z_2 can be replaced with equivalent effect by two shunt elements in parallel, each of impedance value $2z_2$. It will be readily apparent that if the input terminals are taken at *aa*, the ending may be called full series; if at *bb*, it is mid-series; if at *cc*, it is full shunt; and if at *dd*, it is mid-shunt.

The corresponding input impedances are:

$$K_{fr}=\sqrt{z_1z_2+z_1^2/4}+z_1/2 \quad (1)$$

$$K_{mr}=\sqrt{z_1z_2+z_1^2/4} \quad (2)$$

$$K_{fh}=\sqrt{z_1z_2+z_1^2/4}-z_1/2 \quad (3)$$

$$K_{mh}=z_1z_2/\sqrt{z_1z_2+z_1^2/4} \quad (4)$$

The foregoing results may be readily established by reference to pages 34 and 35 of my paper on "Theory and design of uniform and composite electric wave filters" in the Bell System Technical Journal for January, 1923.

A section of the ladder type filter as from *ll* to *mm* is called a mid-shunt section; from *pp* to *rr* is a mid-series section.

The propagation constant Γ for the filter of Fig. 1 is given by the following formula

$$e^{\Gamma}=1+z_1/2z_2+\sqrt{z_1/z_2+(z_1/2z_2)^2} \quad (5)$$

This formula may be deduced from formula 7 of page 34 of the paper above mentioned.

After starting the development of the theory of the ladder type electric wave filter, as above, with an infinite number of sections, it is usual to show that the practical operation is almost the same with a moderate finite number of sections and a suitable terminal network N; this may be connected in Fig. 1 by throwing the switches *s*. Generally the network N should have an impedance value as nearly as practicable the same as the impedance of the infinitely extending recurrent part of the network which it replaces.

Let the z's of Fig. 1 be subject to the condition that

$$z_1z_2=k^2=R^2 \quad (6)$$

where $k=R$ is a constant real number. The filter, subject to this condition, is called a constant k filter. It may be noted that for a smooth line of series impedance z_1 per unit length, and shunt impedance z_2 per unit length, the characteristic impedance would be a constant pure resistance k, but the characteristic impedance of a constant k filter is, of course, not constant in general, nor a pure resistance at all frequencies, as a glance at Equations (1) to (4) will make readily apparent. However, in a constant k filter, it may readily be seen from Equations (2), (4) and (6) that

$$K_{mr}K_{mh}=k^2=R^2 \quad (7)$$

By substituting in Equation (5) from Equation (6), the formula for the propagation constant of the constant R filter is shown to be

$$e^{\Gamma}=1+z_1^2/2R^2+z_1\sqrt{(4R^2+z_1^2)}/R^2 \quad (8)$$

In the general network of Fig. 1, with mid-series ending at bb, let each mid-series section such as from pp to rr be as shown in Fig. 2, where each half series impedance, $\frac{1}{2}z_1$, is represented by the infinite network between and extending up from the points gg, and where each shunt element, z_2 of Fig. 1, is represented by $2z_{21}$.

Let the z's of Fig. 2 be subject to the condition

$$z_{11}z_{21}=R^2 \quad (9)$$

The impedance of each network representing $\frac{1}{2}z_1$, that is, the impedance across the points gg, may be obtained by the aid of Equation (3); then, with the result so obtained, the mid-series characteristic impedance of Fig. 1, with its elements replaced as in Fig. 2, may be worked out. In this case, the result will be R.

That is, if the general impedances of the filter of Fig. 1 are replaced according to the more special values shown in Fig. 2, subject to Equation (9), then the doubly infinite structure obtained in this way, with mid-series input termination, has a constant input impedance R at all frequencies.

Again, suppose that the general filter of Fig. 1 is given the more special form in which each mid-shunt section as from ll to mm is replaced by the structure of Fig. 3. By a procedure similar to that for Fig. 2, it can be shown that the filter obtained according to Fig. 3 will have the mid-shunt characteristic impedances of constant value R.

In both cases, that is, for Fig. 1 made definite according to Fig. 2 or Fig. 3, the propagation constant will be given by the following formula derived from Equation (5), above:

$$e^{\Gamma}=z_{11}/2R+\sqrt{1+(z_{11}/2R)^2} \quad (10)$$

Instead of the ladder type filters considered heretofore, attention is now directed to the lattice type filter shown in general form in Fig. 4. The characteristic impedance is given by

$$K_l=\sqrt{z_1z_2} \quad (11)$$

and the propagation constant by

$$e^{\Gamma}=\frac{2\sqrt{z_1z_2}+z_1}{2\sqrt{z_1z_2}-z_1} \quad (12)$$

Equations (11) and (12) follow from Equation (23) on page 19 of my paper mentioned above.

In the filter shown in Fig. 4, let the generally expressed impedances, $\frac{z_1}{2}$ and $2z_2$, be replaced by the corresponding infinite networks shown in Fig. 5. In other words, Fig. 5 represents a single section of the filter of Fig. 4, with the general impedances of Fig. 4 made more specific, as shown in Fig. 5. Let the z's of Fig. 5 be subject to the condition

$$z_{11}z_{21}=R^2 \quad (13)$$

The coefficient x appearing in the expressed values of the terminal impedance elements of Fig. 5 may have any convenient value from zero to +1. The impedance across the points hh in Fig. 5 can be obtained by the aid of Equation (3), and likewise across the points $h'h'$. The impedance across the points jj can be found by the aid of Equation (1), and likewise across the points $j'j'$. With these impedance values so obtained, the impedance of the infinite network of Fig. 4, with sections as in Fig. 5, may be obtained by the aid of Equation (11), and, subject to the condition expressed in Equation (13), it works out to the value R.

Also, if the impedance values across hh, $h'h'$, jj and $j'j'$ are substituted in Equation (12), the result will be obtained that the propagation constant for the network of Fig. 4, with its elements replaced as in Fig. 5, is given by

$$e^{\Gamma}=\frac{z_{11}/2R+(\sqrt{1+(z_{11}/2R)^2}+2x-1)}{z_{11}/2R-(\sqrt{1+(z_{11}/2R)^2}+2x-1)} \quad (14)$$

In Figs. 2, 3 and 5, I have indicated constructions for wave filters of constant resistance characteristic impedance; Fig. 2 has a mid-series ending, Fig. 3, a mid-shunt ending; both these are of ladder type, and Fig. 5 is of lattice type. These Figs. 2, 3 and 5 are specific when compared with the more general Figs. 1 and 4, but, on the other hand, they are very general for the class of constant resistance characteristic impedance filters; so general that they may represent low pass or high pass, or band pass, or low-and-high pass filters.

In my paper in the Bell System Technical Journal, referred to above, I have shown on page 6 and context how to design the series and shunt impedances (respectively z_{1k} and z_{2k}) of a constant k wave filter, so that it will have any pre-assigned transmitting and attenuating bands, subject to the condition that the product of the said series and shunt impedances shall be k^2. The principles of this paper may be applied to the filters of Figs. 2, 3 and 5, and those figures can be made more specific than they are shown, to get any desired attenuating and transmitting ranges; this I will now demonstrate.

Consider the ladder type filter of Fig. 6, whose series impedances are each z_{11} of Figs. 2 and 3 and whose shunt impedances are each z_{21} of those figures. Fig. 6 represents a "constant k" wave filter, according to Equations (6) and (9), where, in the notation of my paper just referred to, $z_{11}=z_{1k}$ and $z_{21}=z_{2k}$. Its propagation constant Γ_k is given by replacing z_1 by z_{11} and z_2 by z_{21} in Equation (5). But the same value will be obtained by squaring Equation (10). Hence

$$\Gamma_k=2\Gamma \quad (15)$$

This shows that at each frequency the propagation constant of the filter of Figs. 2 or 3 is half that of Fig. 6. In other words, two sections of Fig. 2 or Fig. 3 have just the same propagation constant as one section of Fig. 6. Now by the principles of my paper above referred to I can assign values to z_{11} and z_{21} of Fig. 6 to get any desired transmitting and attenuating bands; and the values of z_{11} and z_{21}, so determined, are to be introduced in Fig. 2 or Fig. 3.

As to the filter of Fig. 5, if $x=0$, it can be shown that two sections of this lattice type filter become equivalent to a single section of the ladder type filter of Fig. 6. Hence, the procedure to design Fig. 5 for a special purpose is to begin with Fig. 6, as outlined above for the ladder type.

Specifically to illustrate the further procedure, let us assume that the desired filter is to be a low pass filter cutting off at a definite given frequency f_2. Such a filter may be of ladder type as in Fig. 6, with series inductances L_{1k} and shunt capacities C_{2k}, so that

$$\left.\begin{aligned} z_{11}&=i2\pi fL_{1k} \\ \text{and}\quad z_{21}&=-i/2\pi C_{2k} \end{aligned}\right\}(16)$$

By Equation (6),

$$R^2=L_{1k}/C_{2k} \quad (17)$$

By page 39 of my paper heretofore mentioned, the critical or cut-off frequency of this filter is given by

$$f_2^{\,2}=1/\pi^2L_{1k}C_{2k} \quad (18)$$

Hence the design equations for this filter are

$$L_{1k}=R/\pi f_2 \text{ and } C_{2k}=1/\pi f_2R \quad (19)$$

and accordingly the filter of Fig. 2 takes the form shown in Fig. 7 with

$$\left.\begin{aligned} C_2&=1/2\pi f_2R \\ C_3&=1/4\pi f_2R \\ L_1&=R/4\pi f_2 \end{aligned}\right\} \quad (20)$$

We have now progressed to the point for passing from the infinitely upward extending networks to the finite networks with proper terminations; these are shown in Fig. 8. The design and the values for the terminal elements are determined according to the principles set forth in my paper hertofore mentioned. Extending upwardly from the points gg, in succession, are

One and one fourth mid-shunt sections,

One mid-half section, M-type, where $m=3/5$.

Resistance termination R.

These are embodied as follows: L_1, C_2 and C_3 have the values stated heretofore;

$$L_5=\frac{5}{4}L_1,\ L_7=\frac{15}{8}L_1,\ C_9=\frac{10}{3}C_3.$$

The recurrent network extending upwardly from the points gg is in itself a high pass filter, and it is terminated in such manner that within its pass range its input impedance is a pure resistance and approximately constant.

Again, dealing with Fig. 3 instead of Fig. 2, the desired low pass filter may be shown to take the form presented in Fig. 9, in which L_1 and C_3 have the values stated heretofore, and

$$L_3=2L_1, C_5=\frac{5}{4}C_3,\ C_7=\frac{15}{8}C_3 \text{ and } L_9=\frac{10}{3}L_1.$$

If the desired low pass filter is to be of lattice type as in Fig. 5, then in each section one pair of non-adjacent impedances will be the same as looking upwardly across the points gg in Fig. 8; and the other such pair will be the same as looking into the shunt network across the points tt in Fig. 9. The factor x of Fig. 5 is taken at zero value.

Let Γ be separated into its real and imaginary parts, thus $\Gamma=A+iB$, where A is the attenuation constant and B is the phase constant. I have computed A and B over a wide range of frequencies for each of the single filter sections of Figs. 7, 8, 9 and 5 with the special elements for Fig 5 mentioned above. The results are plotted in Fig. 10. The curves for the two ladder type filters of Figs. 8 and 9 are so close together as to be almost indistinguishable and I have not attempted to distinguish them in the labelling of the curves. In this computation I assumed that the inductances for the practical approximate filters of Figs. 8, 9 and 5 (with special elements) were subject to dissipation losses with factor $d=0.01$, i. e., in these coils the resistance is 0.01 times the reactance. This is a fair practical ratio, but it should be noticed that no such assumption is made for the curve for Fig. 7. Thus it is apparent that the departure of the other curves from the curve for Fig. 7 is in part due to the necessary dissipation losses in the coils of a practical filter, and only in part due to the finite endings for the impedance networks within each filter section as compared with the infinite extent of those networks shown in Fig. 7. The curves of Fig. 10 are in each case for a single filter section; for any number of sections the ordinates should be multiplied by that number, but obviously the percentage departures of the practical filters from the ideal filter will remain the same.

The impedance of these three practical networks have also been worked out on the same basis and compared with the impedance R for the corresponding ideal networks. For all three networks and at all frequencies from zero to $2f_2$ the modulus of the impedance is within 2% of the value R and the angle of the impedance is within 2°. For most frequencies the departures are much less than 2% and 2°.

The attenuation constants, phase constants and impedances for the three practical filters mentioned were all worked out on the basis of an infinite number of sections, each section like Fig. 8 or 9 or 5 (with special elements) respectively. On this basis the characteristic impedance is found to be nearly of constant value R. Therefore, a small integral number of sections may be chosen and the terminal network N of Fig. 1 may be employed, this being simply a resistance R. For such a finite filter the input impedance will be very nearly R and the attenuation and phase constant will be very nearly as shown in Fig. 10.

A filter made according to my invention may be connected to receive input from a line or other transducer of resistance R, and the filter output may be connected to receiving apparatus of resistance R. Several filter sections with the same characteristic impedance R but different attenuation values A and different phase shifts B may be connected directly together as desired; in this way specially desirable characteristics for A and B may be built up.

I claim:

1. A selective constant resistance network of the type having like recurrent sections, with means forming a part of each section to make the characteristic impedance substantially a constant pure resistance for a wide range of frequencies both within and without the pass range of the network.

2. A selective constant resistance network of the type having like recurrent sections, an element of substantially constant pure resistance connected with the network output terminals, and means in each section to make the characteristic impedance the same as said resistance for a wide range of frequencies both within and without the pass range of the network.

3. An electric wave filter, a selective constant resistance network consisting of a network of like recurrent sections, each section comprising a plurality of impedances, at least one in series and at least one in shunt, at least one of these impedances in each section being another network of like recurrent sections, the characteristic impedance of the filter being substantially a constant resistance for a wide range of frequencies both within and without the pass range of the filter.

4. An electric wave filter, a selective constant resistance network consisting of a network of like recurrent sections, each section comprising a plurality of impedances, at least one in series and at least one in shunt, at least one of these impedances in each section being another network of like recurrent sections, and a terminal network for each such last mentioned network to give it approximately a constant resistance in its transmitting range, the characteristic impedance of the filter being a constant resistance for a wide range of frequencies both within and without the pass range of the filter.

5. A selective constant resistance network of sectional ladder type, each section having at least one series impedance and at least one shunt impedance and each section also having a network with series and shunt elements associated with one of said impedances to make the characteristic impedance of the main network substantially a constant pure resistance for a wide range of frequencies both within and without the pass range of the network.

6. An electric wave filter, a selective constant resistance network of sectional ladder type with mid-series or mid-shunt ending at both ends, an element of substantially constant pure resistance connected with the filter output terminals, and means in each section to give the filter an input impedance at substantially all frequencies equal to said constant resistance.

In testimony whereof, I have signed my name to this specification this 12th day of April 1928.

OTTO J. ZOBEL.