PCT WORLD INTELLECTUAL PROPERTY ORGANIZATION
International Bureau

INTERNATIONAL APPLICATION PUBLISHED UNDER THE PATENT COOPERATION TREATY (PCT)

(51) International Patent Classification 6 : G06T 15/00 | A1

(11) International Publication Number: **WO 97/05575**

(43) International Publication Date: 13 February 1997 (13.02.97)

(21) International Application Number: PCT/US96/12301

(22) International Filing Date: 26 July 1996 (26.07.96)

(30) Priority Data:
60/001,529 26 July 1995 (26.07.95) US

(71) Applicant *(for all designated States except US)*: SILICON ENGINES, INC. [US/US]; 950 North California Avenue, Palo Alto, CA 94304 (US).

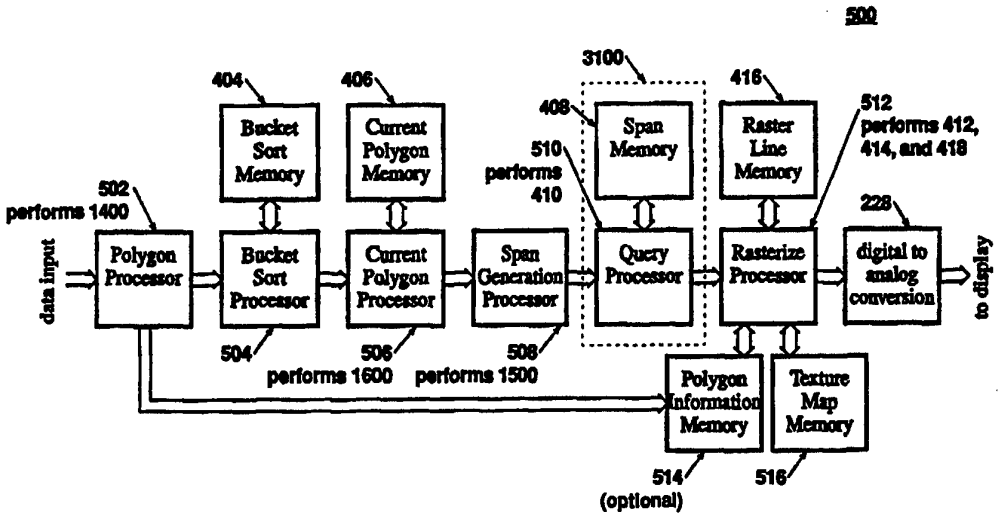
(72) Inventor; and
(75) Inventor/Applicant *(for US only)*: DULUK, Jerome, F., Jr. [US/US]; 950 North California Avenue, Palo Alto, CA 94304 (US).

(74) Agents: SHERIDAN, James, A. et al.; Flehr, Hohbach, Test, Albritton & Herbert, Suite 3400, 4 Embarcadero Center, San Francisco, CA 94111-4187 (US).

(81) Designated States: AL, AM, AT, AU, AZ, BB, BG, BR, BY, CA, CH, CN, CU, CZ, DE, DK, EE, ES, FI, GB, GE, HU, IL, IS, JP, KE, KG, KP, KR, KZ, LK, LR, LS, LT, LU, LV, MD, MG, MK, MN, MW, MX, NO, NZ, PL, PT, RO, RU, SD, SE, SG, SI, SK, TJ, TM, TR, TT, UA, UG, US, UZ, VN, ARIPO patent (KE, LS, MW, SD, SZ, UG), Eurasian patent (AM, AZ, BY, KG, KZ, MD, RU, TJ, TM), European patent (AT, BE, CH, DE, DK, ES, FI, FR, GB, GR, IE, IT, LU, MC, NL, PT, SE), OAPI patent (BF, BJ, CF, CG, CI, CM, GA, GN, ML, MR, NE, SN, TD, TG).

Published
With international search report.
With amended claims.

(54) Title: METHOD AND APPARATUS FOR SPAN SORTING RENDERING SYSTEM



(57) Abstract

The invention (Figure 5) provides for a data shifting capability that permits sorting the data in addition to searching for obtaining real-time performance in color, with high quality imagery through a simple search of a spatial database based on a rectangularly shaped search region or range search (see figure 13). A Sorting Magnitude Comparison Content Addressable Memory (SMCCAM) performs a range search, introducing a conservative approximation of the ideal Occluding Region, and provides an MCCAM wherein the data words stored in the fields is shifted to corresponding fields in an adjacent word, based on the magnitude comparisons (see figure 7). The 3D graphics method stores the parameters of a polygon span in a spatial database (804-814) and a query operation is performed on the database to determine which of those spans, or portions of spans, are visible (816 and 818), and applies a rule for comparing new span portion to an old span portion on a subspan-by-subspan basis, thereby providing additional polygon edge information within a raster line, providing antialiasing.

FOR THE PURPOSES OF INFORMATION ONLY

Codes used to identify States party to the PCT on the front pages of pamphlets publishing international applications under the PCT.

AM	Armenia	**GB**	United Kingdom	**MW**	Malawi
AT	Austria	**GE**	Georgia	**MX**	Mexico
AU	Australia	**GN**	Guinea	**NE**	Niger
BB	Barbados	**GR**	Greece	**NL**	Netherlands
BE	Belgium	**HU**	Hungary	**NO**	Norway
BF	Burkina Faso	**IE**	Ireland	**NZ**	New Zealand
BG	Bulgaria	**IT**	Italy	**PL**	Poland
BJ	Benin	**JP**	Japan	**PT**	Portugal
BR	Brazil	**KE**	Kenya	**RO**	Romania
BY	Belarus	**KG**	Kyrgystan	**RU**	Russian Federation
CA	Canada	**KP**	Democratic People's Republic of Korea	**SD**	Sudan
CF	Central African Republic			**SE**	Sweden
CG	Congo	**KR**	Republic of Korea	**SG**	Singapore
CH	Switzerland	**KZ**	Kazakhstan	**SI**	Slovenia
CI	Côte d'Ivoire	**LI**	Liechtenstein	**SK**	Slovakia
CM	Cameroon	**LK**	Sri Lanka	**SN**	Senegal
CN	China	**LR**	Liberia	**SZ**	Swaziland
CS	Czechoslovakia	**LT**	Lithuania	**TD**	Chad
CZ	Czech Republic	**LU**	Luxembourg	**TG**	Togo
DE	Germany	**LV**	Latvia	**TJ**	Tajikistan
DK	Denmark	**MC**	Monaco	**TT**	Trinidad and Tobago
EE	Estonia	**MD**	Republic of Moldova	**UA**	Ukraine
ES	Spain	**MG**	Madagascar	**UG**	Uganda
FI	Finland	**ML**	Mali	**US**	United States of America
FR	France	**MN**	Mongolia	**UZ**	Uzbekistan
GA	Gabon	**MR**	Mauritania	**VN**	Viet Nam

Method and Apparatus for Span Sorting Rendering System

Field of the Invention

The field of this invention is twofold: 1) three-dimensional computer graphics; and 2) computer memories, and more specifically, Content Addressable Memories (CAM).

Background of the Invention

Three-dimensional Computer Graphics

Computer graphics is the art and science of generating pictures with a computer. Generation of pictures, or images, is commonly called rendering. Generally, in three-dimensional (3D) computer graphics, geometry that represents surfaces (or volumes) of objects in a scene is translated into pixels stored in a frame buffer, and then displayed on a display device. Real-time display devices, such as CRTs used as computer monitors, refresh the display by continuously displaying the image over and over. This refresh usually occurs row-by-row, where each row is called a raster line or scan line. In this document, raster lines are numbered from bottom to top, but are displayed in order from top to bottom.

In a 3D animation, a sequence of images is displayed, giving the illusion of motion in three-dimensional space. Interactive 3D computer graphics allows a user to change his viewpoint or change the geometry in real-time, thereby requiring the rendering system to create new images on-the-fly in real-time.

In 3D computer graphics, each renderable object generally has its own local object coordinate system, and therefore needs to be translated 202 (or transformed) from object coordinates to pixel display coordinates. Conceptually, this is a 4-step process: 1) translation (including scaling for size enlargement or shrink) from object coordinates to world coordinates, which is the coordinate system for the entire scene; 2) translation from world coordinates to eye coordinates, based on the viewing point of the scene; 3) translation from eye coordinates to perspective translated eye coordinates, where perspective scaling (farther objects appear smaller) has been performed; and 4) translation from perspective translated eye coordinates to pixel coordinates, also called screen coordinates. Screen coordinates are points in three-dimensional space, and can be in either screen-precision (i.e., pixels) or object-precision (high precision numbers, usually floating-point), as described later. These translation steps can be compressed into one or two steps by precomputing appropriate translation matrices before any translation occurs. Once the geometry is in screen coordinates, it is broken into a set of pixel color values (that is "rasterized") that are stored into the frame buffer. Many techniques are used for generating pixel color values, including Gouraud shading, Phong shading, and texture mapping.

A summary of the prior art rendering process can be found in: "Fundamentals of Three-dimensional Computer Graphics", by Watt, Chapter 5: The Rendering Process, pages 97 to 113, published by Addison-Wesley Publishing Company, Reading, Massachusetts, 1989, reprinted 1991, ISBN 0-201-15442-0 (hereinafter referred to as the Watt Reference).

Figure 1 shows a three-dimensional object, a tetrahedron 110, with its own coordinate axes (x_{obj}, y_{obj}, z_{obj}). The three-

dimensional object 110 is translated, scaled, and placed in the viewing point's 130 coordinate system based on (x_{eye}, y_{eye}, z_{eye}). The object 120 is projected onto the viewing plane 102, thereby correcting for perspective. At this point, the object appears to have become two-dimensional; however, the object's z-coordinates are preserved so they can be used later by hidden surface removal techniques. The object is finally translated to screen coordinates, based on (x_{screen}, y_{screen}, z_{screen}), where z_{screen} is going perpendicularly into the page. Points on the object now have their x and y coordinates described by pixel location (and fractions thereof) within the display screen 104 and their z coordinates in a scaled version of distance from the viewing point 130.

Because many different portions of geometry can affect the same pixel, the geometry representing the surfaces closest to the scene viewing point 130 must be determined. Thus, for each pixel, the visible surfaces within the volume subtended by the pixel's area determine the pixel color value, while hidden surfaces are prevented from affecting the pixel. Non-opaque surfaces closer to the viewing point than the closest opaque surface (or surfaces, if an edge of geometry crosses the pixel area) affect the pixel color value, while all other non-opaque surfaces are discarded. In this document, the term "occluded" is used to describe geometry which is hidden by other non-opaque geometry.

Many techniques have been developed to perform visible surface determination, and a survey of these techniques are incorporated herein by reference to: "Computer Graphics: Principles and Practice", by Foley, van Dam, Feiner, and Hughes, Chapter 15: Visible-Surface Determination, pages 649 to 720, 2nd edition published by Addison-Wesley Publishing Company, Reading, Massachusetts, 1990, reprinted with corrections 1991, ISBN 0-201-12110-7 (hereinafter referred to as the Foley Reference). In the Foley Reference, on page 650, the terms "image-precision" and "object-precision" are defined: "Image-precision algorithms are typically performed at the resolution of the display device, and determine the visibility at each pixel. Object-precision algorithms are performed at the precision with which each object is defined, and determine the visibility of each object."

As a rendering process proceeds, most prior art renderers must compute the color value of a given screen pixel multiple times because multiple surfaces intersect the volume subtended by the pixel. The average number of times a pixel needs to be rendered, for a particular scene, is called the depth complexity of the scene. Simple scenes have a depth complexity near unity, while complex scenes can have a depth complexity of ten or twenty. As scene models become more and more complicated, renderers will be required to process scenes of ever increasing depth complexity. Thus, for most renders, the depth complexity of a scene is a measure of the wasted processing. For example, for a scene with a depth complexity of ten, 90% of the computation is wasted on hidden pixels. This wasted computation is typical of hardware renderers that use the simple Z-buffer technique (discussed later herein), generally chosen because it is easily built in hardware. Methods more complicated than the Z-buffer technique have heretofore generally been too complex to build in a cost-effective manner. An important feature of the method and apparatus invention presented here is the avoidance of this wasted computation by eliminating hidden portions of geometry before they are rasterized, while still being simple enough to build in cost-effective hardware.

When a point on a surface (frequently a polygon vertex) is translated to screen coordinates, the point has three coordinates: 1) the x-coordinate in pixel units (generally including a fraction); 2) the y-coordinate in pixel units (generally including a fraction); and 3) the z-coordinate of the point in either eye coordinates, distance from the virtual screen, or some other coordinate system which preserves the relative distance of surfaces from the viewing point. In this document, positive z-coordinate values are used for the "look direction" from the viewing point, and smaller values indicate a position closer to the viewing point.

When a surface is approximated by a set of planar polygons, the vertices of each polygon are translated to screen coordinates. For points in or on the polygon (other than the vertices), the screen coordinates are interpolated from the coordinates of vertices, typically by the processes of edge walking 218 and span interpolation 220. Thus, a z-coordinate value is generally included in each pixel value (along with the color value) as geometry is rendered.

Generic 3D Graphics Pipeline

Many hardware renderers have been developed, and an example is incorporated herein by reference: "Leo: A System for Cost Effective 3D Shaded Graphics", by Deering and Nelson, pages 101 to 108 of SIGGRAPH 93 Proceedings, 1-6 August 1993, Computer Graphics Proceedings, Annual Conference Series, published by ACM SIGGRAPH, New York, 1993, Softcover

ISBN 0-201-58889-7 and CD-ROM ISBN 0-201-56997-3 (hereinafter referred to as the Deering Reference). The Deering Reference includes a diagram of a generic 3D graphics pipeline **200** (i.e., a renderer, or a rendering system) that it describes as "truly generic, as at the top level nearly every commercial 3D graphics accelerator fits this abstraction", and this pipeline diagram is reproduced here as Figure 2. Such pipeline diagrams convey the process of rendering, but do not describe any particular hardware. This document presents a new graphics pipeline **400** that shares some of the steps of the generic 3D graphics pipeline **200**. Each of the steps in the generic 3D graphics pipeline **200** will be briefly explained here, and are also shown in the method flow diagram **300** of Figure 3. Processing of polygons is assumed throughout this document, but other methods for describing 3D geometry could be substituted. For simplicity of explanation, triangles are used as the type of polygon in the described methods.

As seen in Figure 2, the fist step within the floating-point intensive functions **250** of the generic 3D graphics pipeline **200** is the transformation step **202**, which was described above. The transformation step **202** is also shown in Figure 3 as the first step in the outer loop of the method flow diagram **300**, and also includes "get next polygon". The second step, the clip test **204**, checks the polygon to see if it is at least partially contained in the view volume **106** (sometimes shaped as a frustum). If the polygon is not in the view volume **106**, it is discarded; otherwise processing continues. The third step is face determination **206**, where polygons facing away from the viewing point are discarded. Generally, face determination **206** is applied only to objects that are closed volumes. The fourth step, lighting computation **208**, generally includes the set up for Gouraud shading and/or texture mapping with multiple light sources of various types, but could also be set up for Phong shading or one of many other choices. The fifth step, clipping **210**, deletes any portion of the polygon that is outside of the view volume **106** because that portion would not project within the rectangular area of the viewing plane **102**. Generally, polygon clipping **210** is done by splitting the polygon into two smaller polygons that both project within the area of the viewing plane **102**. Polygon clipping is computationally expensive, but its need is avoided in the invention presented here, thus providing computational savings. The sixth step, perspective divide **212**, does perspective correction for the projection of objects onto the viewing plane **102**. At this point, the points representing vertices of polygons are converted to pixel-space coordinates by step seven, the screen space conversion **214** step. The eighth step, set up for incremental render **216**, computes the various begin, end, and increment values needed for edge walking **218** and span interpolation **220** (e.g.: x, y, and z-coordinates; RGB color; texture map space u and v-coordinates; etc.).

Within the drawing intensive functions **260**, edge walking **218** incrementally generates horizontal spans for each raster line of the display device by incrementing values from the previously generated span (in the same polygon), thereby "walking" vertically along opposite edges of the polygon. Similarly, span interpolation **220** "walks" horizontally along a span to generate pixel values, including a z-coordinate value indicating the pixel's distance from the viewing point **130**. By comparing this z-coordinate value to the corresponding value stored in the Z-buffer, the z-buffered blend **222** either keeps the new pixel values (if it is closer to the viewing point than previously stored value for that pixel location) by writing it into the frame buffer **224**, or discards the new pixel values (if it is farther). At this step, antialiasing methods (discussed in the next section) can blend the new pixel color with the old pixel color.

The generic 3D graphics pipeline **200** includes a double buffered frame buffer **224**, so a double buffered MUX **226** is also included. An output lookup table **226** is included for translating color map values. Finally, digital to analog conversion **228** makes an analog signal for input to the display device.

A major drawback to the generic 3D graphics pipeline **200** is its drawing intensive functions **260** are not deterministic at the pixel level given a fixed number of polygons. That is, given a fixed number of polygons, more pixel-level computation is required as the average polygon size increases. However, the floating-point intensive functions **250** are proportional to the number of polygons, and independent of the average polygon size. Therefore, it is difficult to balance the amount of computational power between the floating-point intensive functions **250** and the drawing intensive functions **260** because this balance depends on the average polygon size.

An ideal renderer's pixel drawing computational requirement would be proportional to the number of pixels in the dis-

play screen 104, not the total number of pixels in all the polygons in the view volume 106. This ideal is achieved by the invention described here because hidden geometry is removed before most drawing intensive functions are performed. In the invention described here, computational load balancing is not a problem because the amount of floating-point computation is essentially independent of the amount of drawing computation.

Antialiasing

In this document, pixels are defined to be the smallest individually controllable element of the display device. But, because images are quantized into discrete pixels, spatial aliasing occurs. A typical aliasing artifact is a "staircase" effect caused when a straight line or edge cuts diagonally across rows of pixels. An ideal antialiased image eliminates this "staircase" effect by calculating, for each pixel, an average color by taking into account partial coverage by the visible surfaces within the pixel's area.

Some rendering systems reduce aliasing effects by dividing pixels into subpixels, where each sub-pixel can be colored independently. When the image is to be displayed, the colors for all sub-pixels within each pixel are blended together to form an average color for the pixel. A renderer that uses 16 sub-pixels per pixel is described in "RealityEngine Graphics", by Akeley, pages 109 to 116 of SIGGRAPH 93 Proceedings, 1-6 August 1993, Computer Graphics Proceedings, Annual Conference Series, published by ACM SIGGRAPH, New York, 1993, Softcover ISBN 0-201-58889-7 and CD-ROM ISBN 0-201-56997-3 (hereinafter referred to as the Akeley Reference). The drawback with using subpixels is the increase in computation due to computing color values at every subpixel. In the Akeley Reference, the increase in computation is reduced by only dividing a pixel into subpixels when the pixel is crossed by a line or an edge of a polygon. This reduction becomes less significant as the number of polygons increases. In other words, if the image is made up of lots of small overlapping polygons, then most pixels will need to be divided. Utilization of subpixels is an image-precision antialiasing technique.

Another prior art antialiasing method is the A-Buffer used to perform alpha blending (this technique is also included in the Akeley Reference), and is described in "The A-buffer, an Antialiased Hidden Surface Method" by L. Carpenter, SIGGRAPH 1984 Conference Proceedings, pp. 103-108 (hereinafter referred to as the Carpenter Reference). The A-buffer is an image-precision antialiasing technique that reduces aliasing by keeping track of the percent coverage of a pixel by a rendered polygon. The main drawback to this technique is the need to sort polygons front-to-back (or back-to-front) at each pixel in order to get acceptable antialiased polygons.

An ideal antialiasing method would perform object-precision computations to precisely identify the visible portions of geometry. This would require comparing edges of polygons to each other in order to determine the fraction of each pixel covered by each polygon. The invention of this document performs object-precision antialiasing within each scan line, thus achieving this ideal.

Z-buffers

Stated simply, the Z-buffer stores, for every pixel, the z-coordinate of the closest geometry (to the viewing point) that affects the pixel. Hence, as new pixel values are generated, each new pixel's z-coordinate is compared to the corresponding location in the Z-buffer. If the new pixel's z-coordinate is smaller (i.e., closer to the viewing point), this value is stored into the Z-buffer and the new pixel's color value is written into the frame buffer. If the new pixel's z-coordinate is larger (i.e., farther from the viewing point), the frame buffer and Z-buffer values are unchanged and the new pixel is discarded. The Z-buffer is an image-precision visible surface determination technique.

A flow diagram including the prior art Z-buffer method is shown in Figure 3. The main drawback to the Z-buffer hidden surface removal method is the requirement for geometry to be converted to pixel values before hidden surface removal can be done. This is because the keep/discard decision is made on a pixel-by-pixel basis. In contrast, the invention of this document performs hidden surface removal at a higher level by processing spans rather than pixels. For scenes with any significant depth complexity, pixel-by-pixel hidden surface removal introduces much wasted computation by requiring all geometry within the view volume to be converted to pixels, even though most are hidden and, therefore, thrown away. In hardware rendering systems,

pixel color generation (shading, texture mapping, etc.) often happens in parallel with the Z-buffer comparison test, thereby compounding the wasted computation because much of the computation is associated with color generation, and most of the pixels are thrown away. Furthermore, the Z-buffer memory operation is a read-modify-write cycle, generally requiring the Z-buffer memory input/output bus to change directions twice when writing pixels into the frame buffer, thereby causing a bottleneck in the renderer. This bottleneck does not occur in the apparatus and method of the document's invention.

Prior art Z-buffers are based on conventional Random Access Memory (RAM), Video RAM (VRAM), or special purpose DRAMs. One example of a special purpose DRAM is presented in "FBRAM: A new Form of Memory Optimized for 3D Graphics", by Deering, Schlapp, and Lavelle, pages 167 to 174 of SIGGRAPH 94 Proceedings, 24-29 July 1994, Computer Graphics Proceedings, Annual Conference Series, published by ACM SIGGRAPH, New York, 1994, Softcover ISBN 0-201-60795-6.

Geometry Databases

The geometry needed to generate a renderable scene is stored in a database. This geometry database can be a simple display list of graphics primitives or a hierarchically organized data structure. In the hierarchically organized geometry database, the root of the hierarchy is entire database, and the first layer of subnodes in the data structure is generally all the objects in the "world" which can be seen from the viewpoint. Each object, in turn, contains subobjects, which contain sub-subobjects; thus resulting in a hierarchical "tree" of objects. Hereinafter, the term "object" shall refer to any node in the hierarchial tree of objects. Thus, each subobject is an object. The term "root object" shall refer to a node in the first layer of subnodes in the data structure. Hence, the hierarchical database for a scene starts with the scene root node, and the first layer of objects are root objects.

Hierarchical databases of this type are used by the Programmer's Hierarchical Interactive System (PHIGS) and PHIGS PLUS standards An explanation of these standards can be found in the book, "A Practical Introduction to PHIGS and PHIGS PLUS", by T. L. J. Howard, et. al., published by Addison-Wesley Publishing Company, 1991, ISBN 0-201-41641-7 (incorporated herein by reference and hereinafter called the Howard Reference). The Howard Reference describes the hierarchical nature of 3D models and their data structure on pages 5 through 8. Hierarchical models can provide a separate transformation matrix at each layer of the hierarchy, thereby making it possible to move models or parts of a models simply by changing a transformation matrix. This allows non-changing model geometry (in object coordinates) to be used as moving objects in an animation.

Content Addressable Memories

Most Content Addressable Memories (CAM) perform a bit-for-bit equality test between an input vector and each of the data words stored in the CAM. This type of CAM frequently provides masking of bit positions in order to eliminate the corresponding bit in all words from affecting the equality test. It is inefficient to perform magnitude comparisons in a equality-testing CAM because a large number of clock cycles is required to do the task.

CAMs are presently used in translation look-aside buffers within a virtual memory systems in some computers. CAMs are also used to match addresses in high speed computer networks. CAMs are not used in any practical prior art renders.

Magnitude Comparison CAM (MCCAM) is defined here as any CAM where the stored data are treated as numbers, and arithmetic magnitude comparisons (i.e. less-than, greater-than, less-than-or-equal-to, etc.) are performed on the data in parallel. This is in contrast to ordinary CAM which treats stored data strictly as bit vectors, not as numbers. An MCCAM patent, included herein by reference, is U.S. Patent Number 4,996,666, by Jerome F. Duluk Jr., entitled "Content-Addressable Memory System Capable of Fully Parallel Magnitude Comparisons", granted February 26, 1991 (hereinafter referred to as the Duluk Patent). Structures within the Duluk Patent specifically referenced shall include the prefix "Duluk Patent" (for example, "Duluk Patent MCCAM Bit Circuit"). MCCAMs are not used in any prior art renderer.

The basic internal structure of an MCCAM is a set of memory bits organized into words, where each word can perform one or more arithmetic magnitude comparisons between the stored data and input data. In general, for an MCCAM, when a vector of numbers is applied in parallel to an array of words, all arithmetic comparisons in all words occur in parallel. Such a parallel search comparison operation is called a "query" of the stored data.

The invention described here augments the capability of the MCCAM by adding various features, including the ability to perform sorting. This new type of MCCAM is call Sorting Magnitude Comparison CAM (SMCCAM).

Summary of the Invention

Computer graphics is the art and science of generating pictures or images with a computer. This picture generation is commonly referred to as rendering. The appearance of motion, for example in a 3-Dimensional animation is achieved by displaying a sequence of images. Interactive 3-Dimensional (3D) computer graphics allows a user to change his or her viewpoint or to change the geometry in real-time, thereby requiring the rendering system to create new images on-the-fly in real-time. Therefore, real-time performance in color, with high quality imagery is becoming increasingly important.

The inventive apparatus and method provide a data shifting capability that permits sorting the data in addition to searching. It also provides a new way to perform a simple search of a spatial database based on a rectangularly shaped search region or range search. The range search may be performed in a special new Sorting Magnitude Comparison Content Addressable Memory (SMCCAM) apparatus. This SMCCAM provides a magnitude comparison content addressable memory wherein the data stored in the fields in each word of the memory may be shifted to corresponding fields in an adjacent word, where this shifting is conditionally performed based on the results of the magnitude comparisons.

The 3D graphics method stores the parameters of a polygon span in a spatial database, and a query operation is performed on the database to determine which of those spans, or portions of spans, are visible. The spatial database of spans can be stored in an SMCCAM. The SMCCAM apparatus is significant because its operation is fast enough to support real time operation, such as for standard and emerging video technologies (NTSC, PAL, HDTV, and the like), visual flight simulators, virtual reality image generators, and the like, among other applications.

Since the range search is used, it introduces a conservative approximation of the ideal Occluding Region. An Occluding Test is provided which defines an Occluding Test Region as an approximation to the ideal Occluding Region. A rule based method is used when comparing the Current Span Portion with the New Span, where the New Span is found by the Occluding Test and read out of the data structure stored in memory. That memory may advantageously be the SMCCAM, or it may be a conventional memory that implements the spatial data structure with conventional spatial database methods. Based on the results of the comparison, one rule out of several possible simple rues are selected and applied to determine which piece of the Current Span is visible, if any. Several alternative rules are described.

Multiple alternative span representations are provided, including a Segment Span, Trapezoidal Span, and Quadrilateral Span. Each of these Spans is defined by its own set of span parameters, of which there are multiple selections of each. The span parameters are stored in a data structure in memory, which may be implemented by the SMCCAM, and are used during the query search operation. Each of these Span types have their own advantages. For example, a Trapezoidal Span or a Quadrilateral Span provides superior anti-aliasing performance than provided by a Segment Span. This performance advantage is achieved by preserving the left and right edge information of the polygon within a particular raster line. Several ways to handle the query operation for Trapezoidal and Quadrilateral Spans are described.

Spatial searching and sorting apparatus and methods are provided such that spatial searching and sorting can be used to properly render transparent polygon spans in front-to-back or back-to-front order.

The inventive apparatus, system, and method also provide several options and enhancements to the basic system. These include, by way of example: preservation of rendering order, utilization of a single-buffered frame buffer for increased compatibility with conventional video cards, the ability to selectively turn on or off anti-aliasing on specific polygon edges, the ability to store edge-pairs rather than polygons thereby simplifying downstream processing, the ability to eliminate the Current Polygon Memory by replacing it with a list of pointers, the ability to add clipping planes (front, rear, and/or arbitrary), provision of a Geometry Cache for storing geometry as it is input for rendering, flexible options for the number of pages of Span Memory and Page Memory I/O bus architecture, support for geometry primitives other than polygons (such as CSG primitives), and alternatives that support the inventive method in conventional hardware with some performance compromises.

A Brief Description of the Drawings

Figure 1: An object in three-dimensional space, its projection onto the viewing plane, and its image one a display screen.

Figure 2: Generic rendering pipeline from the Deering Reference (prior art).

Figure 3: Method flow diagram for the standard Z-buffer method (prior art).

Figure 4: The Span Sorting Rendering Pipeline. The Span Memory can be implemented with a SMCCAM**800**.

Figure 5: A block diagram of the Span Sorting Renderer architecture.

Figure 6: An example of nine polygons showing how they intersect the display screen and how they are included in a particular raster line.

Figure 7: Three ways to model a polygon span: 1) a line segment; 2) a trapezoid; and 3) a quadrilateral.

Figure 8: Data organization within the Sorting Magnitude Comparison Content Addressable Memory **800** (SMCCAM **800**).

Figure 9: SMCCAM Word **900** block diagram.

Figure 10: Examples of a span's Occluding Region and a span's SOT Region generated by a Span Occluding Test Query.

Figure 11: An example set of segment spans on one raster line, shown in x-z space, with the visible span portions shown as thick lines and hidden span portions shown as narrow lines.

Figure 12: The same example set of spans as shown in Figure 11, except with bounding boxes around each span.

Figure 13: The Span Sorting Rendering Method **1300** flow diagram.

Figure 14: The Process Polygon Method **1400** flow diagram, part of the Span Sorting Rendering Method **1300**.

Figure 15: The Process Current Polygon Memory Method **1500** flow diagram, part of the Span Sorting Rendering Method **1300**.

Figure 16: The Process Bucket Sort Memory Method **1600** flow diagram, part of the Span Sorting Rendering Method **1300**.

Figure 17: The Write Span Parameters Method **1700** flow diagram.

Figure 18: The Simplified Span Rasterization Method **1800** flow diagram, part of the Span Sorting Rendering Method **1300**.

Figure 19: Span interaction nomenclature definitions.

Figure 20: The 49 varieties of Span Interaction Types.

Figure 21: Segment Span Rasterization Method **2100** flow diagram, part of the Span Sorting Rendering Method **1300**.

Figure 22: Rule 1 **2200** method flow diagram, part of Segment Span Rasterization Method **2100**.

Figure 23: Rule 2 **2300** method flow diagram, part of Segment Span Rasterization Method **2100**.

Figure 24: Rule 3 **2400** method flow diagram, part of Segment Span Rasterization Method **2100**.

Figure 25: Rule 4 **2500** method flow diagram, part of Segment Span Rasterization Method **2100**.

Figure 26: Rule 5 **2600** method flow diagram, part of Segment Span Rasterization Method **2100**.

Figure 27: An example raster line with fifteen spans. The bounding box of each span is shown as well as each span's x_{nL} location along the x-axis.

Figure 28: Timing diagram for phase-locked raster line processing and display.

Figure 29: Timing diagram for single buffered frame buffer.

Figure 30: Timing diagram for single buffered frame buffer where span rasterization covers more than one screen display time.

Figure 31: Block diagram of the Query Processor **510** hardware architecture, also including two pages within the Span

Memory **408**.

Figure 32: Block diagram of the Comparison and Offset Computation block **3202** within the Query Processor **510**.

Figure 33: Prior art MCCAM CMOS bit cell.

Figure 34: A "TTL type" circuit for an SMCCAM bit cell.

Figure 35: A CMOS circuit for the SMCCAM bit cell.

Figure 36: An array of SMCCAM bit cells.

Figure 37: Multiple spans vertically within a raster line.

Figure 38: Two trapezoidal spans working together to occlude farther spans that would be visible if the shown trapezoidal spans did not work together.

Figure 39: An example set of trapezoidal spans, shown in x-z space.

Figure 40: SOT Query for Processing Top and Bottom Separately

Figure 41: An example set of trapezoidal spans used to illustrate the Trapezoidal Span Rasterization Method version that does occlusion processing on span tops and span bottoms separately.

Figure 42: SOT Query for processing every visibility transition

Figure 43: SOT Query with a complex shape

Figure 44: SOT Query with a wider search area

Figure 45: A set of segment spans in a raster line where the visible opaque surfaces are shown as thick black lines and the visible translucent spans are shown as thick shaded lines.

Figure 46: Approximating trapezoidal spans using only one z-value per endpoint changes the spans into quadrilateral spans.

Figure 47: An example set of quadrilateral spans, shown in x-z space.

Detailed Description of the Invention

Span Sorting Rendering Pipeline

Figure 4 shows the Span Sorting 3D Graphics Pipeline **400**, where the first six steps are defined as the process polygon **1400** steps. The first five steps (transformation **202**, clip test **204**, face determination **206**, lighting **208**, and perspective divide **212**) are the same as the five of the first six steps in the Generic 3D Graphics Pipeline **200**. The clip **210** step is omitted because the Span Sorting 3D Graphics Pipeline **400** operates in object-precision, and coordinates are not limited to the area of the display screen **104**. Elimination of the clip **210** step allows all polygons in the view volume **106** to be treated the same. However, the clip **210** step can be inserted if desired. The Span Sorting 3D Graphics Pipeline **400** can be built with dedicated hardware, done completely in software, or a combination of the two. Hereinafter, a dedicated hardware implementation is assumed, and a new apparatus is described.

The top-level block diagram of the new apparatus is shown in Figure 5. The process polygon **1400** steps are performed in the Polygon Processor **502**. The last step performed by the Polygon Processor **502** is the computation of the Polygon Parameters **402**. The Polygon Parameters, output by the Polygon Processor **502**, describe a polygons as needed by the rest of the Span Sorting 3D Graphics Pipeline **400**.

For each polygon, Polygon Parameters are written into the Bucket Sort Memory **400**, and include: 1) the location in object-precision screen coordinates of the polygon vertices, V_1, V_2, V_3, etc., defined respectively as: (x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3), etc.; 2) color information, including such things as vertex colors for Gouraud (or Phong) shading and/or texture mapping parameters; 3) the edge derivatives of x and z with respect to y (i.e., $\delta x/\delta y$ and $\delta z/\delta y$) for each edge; 4) starting raster line (or topmost), y_{START}; 5) ending raster line (or bottommost), y_{END}; and 6) the span derivative of z with respect to x (i.e., $\delta x/\delta y$), assigned to the variable d. The last four in this list need to be computed **402**, and this is done in the Polygon Processor **502**.

Assuming the polygons are triangles, the edge derivatives are computed as:

$$\frac{\delta x_{12}}{\delta y_{12}} = \frac{x_1 - x_2}{y_1 - y_2} \qquad \frac{\delta x_{23}}{\delta y_{23}} = \frac{x_2 - x_3}{y_2 - y_3} \qquad \frac{\delta x_{31}}{\delta y_{31}} = \frac{x_3 - x_1}{y_3 - y_1}$$
$$\frac{\delta z_{12}}{\delta y_{12}} = \frac{z_1 - z_2}{y_1 - y_2} \qquad \frac{\delta z_{23}}{\delta y_{23}} = \frac{z_2 - z_3}{y_2 - y_3} \qquad \frac{\delta z_{31}}{\delta y_{31}} = \frac{z_3 - z_1}{y_3 - y_1} \qquad \text{(EQ 1)}$$

When the starting raster line, y_{START}, and the ending raster line, y_{END}, are computed, the edges of the display screen **104** must generally be taken into account. Figure 6 shows a display screen **104** with eight polygons (in this case, triangles), some of which intersect the edges of the display screen **104**. If a polygon does not intersect the right or left edges of the display screen **104**, then y_{START} and y_{END} are computed by:

$$y_{START} = \text{MinOf}\,(y_{MAX}, \text{IntPartOf}\,(\text{MaxOf}\,(y_1, y_2, y_3, \ldots)))$$
$$y_{END} = \text{MaxOf}\,(0, \text{IntPartOf}\,(\text{MinOf}\,(y_1, y_2, y_3, \ldots))) \qquad \text{(EQ 2)}$$

The computation of y_{START} first finds the maximum of the y-coordinates of the vertices of the polygon, thereby finding the "uppermost" y-coordinate in the polygon. Then, the function IntPartOf( ) takes the "integer part of" the uppermost y-coordinate, thereby computing the raster line of the "uppermost" y-coordinate in the polygon. Since the start of the polygon can not be above the topmost raster line in the display screen **104**, the MinOf( ) function substitutes y_{MAX} if the polygon would start in a non-existent raster line. Computation of y_{END} is done similarly. Figure 6 shows a polygon **602** that starts at y_{MAX}, and another **604** that ends at raster line zero.

If a polygon intersects the right or left edges of the display screen **104**, then y_{START} and y_{END} should be the first and last raster lines that is affected by the polygon within the display screen **104**. In Figure 6, an example polygon **606** begins at y_A **608** and ends at y_B **610**. Another example polygon **612** begins at y_C **614** and ends at raster line zero. The effects of the right and left edges of the display screen **104** can be ignored, but then span generation **1500** will create spans that are not within the display screen **104**, and therefore must test for this, and throw away such spans.

Computation of the span derivative, d (i.e., $\delta z/\delta x$), can be computed in one of many conventional ways.

$$d = \frac{\delta z}{\delta x} \qquad \text{(EQ 3)}$$

The span derivative is constant for each planar polygon and can have either a positive or a negative value.

Writing into the Bucket Sort Memory **404** is done according to y_{START} for each polygon. There is one "bucket" per raster line, and a polygon is placed into the one bucket that corresponds to its starting raster line. Within each bucket, polygons do not need to be sorted. Management of the data in the Bucket Sort Memory **404** is done by the Bucket Sort Processor **504**, which can do the sort with one linked list per bucket. In addition, the Bucket Sort Memory **404** can be double buffered so the write operations associated with a scene can be performed concurrently with read operations from the previous scene.

If all the color information described above is stored into the Bucket Sort Memory **404**, it must be carried along, through the Span Sorting Renderer **500** (see Figure 5) all the way to the Rasterize Processor **512**. Alternatively, the color information in the Bucket Sort Memory **404** can be replaced by an information pointer, and then stored into a separate Polygon Information Memory **514**. The information pointer is an address into the Polygon Information Memory **514** where all the color information for the polygon is stored, and can be used by the Rasterize Processor **512** to read the color information. The information pointer is carried along to the Rasterize Processor **512**. This saves expensive hardware memory throughout the Span Sorting Renderer **500** because the information pointer has very few bits when compared to all the color information for a polygon. This savings can be done because: 1) the color information is constant over the entire polygon, and so can be shared by all the spans

of the polygon; and 2) the color information is not needed until visible span portions are colored by the Rasterize Processor 512. The use of the information pointer and the Polygon Information Memory 514 will be assumed in the rest of this document.

Within the Span Sorting Renderer 500, a list is kept in the Current Polygon Memory 406 of all the polygons that affect the raster line that is currently being rendered. Hence, when the rendering of a scene begins, the Current Polygon Memory 406 is empty. During rendering, when a particular raster line is reached, all the polygons that have their y_{START} at that raster line (i.e., that particular bucket) are read from the Bucket Sort Memory 404 and added to the Current Polygon Memory 406. As rendering proceeds, polygons are deleted from the Current Polygon Memory 406 as their y_{END} values are reached. The data in the Current Polygon Memory 406 is managed by the Current Polygon Processor 506, which performs the operation "process data from Bucket Sort Memory 404" 1600, as shown in the pipeline diagram 400. Also, the Current Polygon Processor 506 feeds the polygons that affect the raster line to the Span Generation Processor 508.

Starting at the beginning of the rendering of a raster line, each polygon that has a span in that raster line is input to the Span Generation Processor 508, which performs span generation 1600. The Span Generation Processor 508 uses the geometric properties of each polygon, including its edge derivatives of Equation 1, to compute the location in object-precision screen coordinates of the left and right endpoints of its span within the current raster line. The span can be modeled as a simple line segment, a trapezoid, or a quadrilateral, as shown in Figure 7. The geometric properties of the span are sent to the Query Processor 510, and those properties include: 1) coordinates of the left endpoint of the span; 2) coordinates of the right endpoint of the span; 3) the span derivative, d; and 4) the information pointer. The coordinates of a span endpoint can be: 1) an (x, z) point within the current raster line, used when spans are modeled as line segments as in Figure 7A; 2) two (x, z) points, one for the top edge of the current raster line and one for the lower edge, used when spans are modeled as trapezoids as in Figure 7B; or 3) an (x_U, x_L, z) triplet including one z-coordinate and an x-coordinate for the upper and lower edges of the current raster line, used when spans are modeled as quadrilaterals as in Figure 7C. Of the eight example polygons shown in Figure 6, four of them 616, 618, 620, 622 have spans 626, 628, 630, 632 on the example current raster line 640. Note that one of the example spans 626 starts at a negative x-value.

For a given raster line, as geometric properties of spans are output from the Span Generation Processor 508, they are received by the Query Processor 510 that then writes them all into one bank of the double buffered Span Memory 406. As the spans are written into the Span Memory 406, they are sorted, in order of precedence, by: 1) the left x-coordinate; 2) the left z-coordinate; and 3) the span derivative. Simultaneous to writing span data into one bank, the span data in the other bank of Span Memory 406 is processed to discover visible span portions.

When the Query Processor 510 has completed processing the spans stored in one bank of the Span Memory 406, and all the spans in the next raster line have been received from the Span Generation Processor 508 and written into the other bank, the two banks of the Span Memory 406 are swapped. After the bank swap, the Query Processor 510 and the Span Memory 406 jointly perform arithmetic comparisons on the span data (hereinafter called query operations) and various arithmetic operations to process 1800 or 2100 the spans and determine which spans or portions of spans are visible in the scene. These visible spans (or portions of spans) are sent to the Rasterize Processor 512. A more detailed block diagram of the Query Processor 510 and the Span Memory 406 is shown as Figure 31.

The Rasterize Processor 512 receives only spans (or portions of spans) that are fully visible. To process each span, the Rasterize Processor 512 performs the set up for incremental span render 412 and then performs span interpolation 220. Pixel color values are generated by utilizing the data stored in the Polygon Information Memory 514 and possibly also the Texture Map Memory 516. As pixel color values are generated, they are written into the Raster Line Memory 416.

The Raster Line Memory 416 can store all the color pixel values for several complete raster lines. That is, it 416 stores the color values for the raster line currently being fed to the digital to analog converter 228 (via the Rasterize Processor 512) as well as several subsequent raster lines. Once all the values for a particular raster line have been sent to the digital to analog converter 228, the corresponding part of the Raster Line Memory 416 can be overwritten with another raster line. In this way, the

total amount of memory required is only a few raster lines worth (the minimum is two), rather than the typical doubled buffered full display screen **104** frame buffer **224** required by prior art 3D graphics renders. This saves many megabytes of memory. The Raster Line Memory **416** can store data for several raster lines in order for the rasterization process to "work ahead" of the digital to analog converter **228**, thereby providing some time cushion for raster lines that require more than the average amount of time to process.

Having only enough memory for a few raster lines requires the Span Sorting Renderer **500** architecture to keep up with the average raster line display rate. Therefore, if very large geometry databases (larger than the maximum where the Span Sorting Renderer **500** can keep up) need to be rendered, the Raster Line Memory **416** can be replaced with a doubled buffered full display screen **104** frame buffer **224**.

Sorting Magnitude Comparison Content Addressable Memory 800

A diagram of data storage **800** within a page of Span Memory **406** is shown in Figure 8. This data array can reside in typical random access memory (RAM) as a sorted data structure, or reside in Sorting Magnitude Comparison Content Addressable Memory (SMCCAM). The SMCCAM implementation will be assumed in the rest of this document.

An SMCCAM is a new type of MCCAM, and is comprised of a set of memory registers (or words), each word composed of a multiplicity of fields, where each field can: 1) store a number, either as an integer or as a floating-point number; 2) perform arithmetic comparisons between the stored number and another number broadcast to all words; and 3) shift its contents to the corresponding field in the next word. When used as the Span Memory **406** within the Span Sorting Renderer **500**, the SMCCAM stores a set of spans and performs parallel searching and sorting operations to find the visible span portions.

As shown in Figure 8, the data storage within the SMCCAM **800** is divided into SMCCAM Words **900**, where each word **900** stores and processes (by performing query operations) data corresponding to one span in the current raster line. The figure shows a total of W SMCCAM Words **900**, numbered 0 to $W-1$. Each SMCCAM Word **900** includes seven numeric fields (and the variable name shown here is for the nth word): 1) the Word Number Field **802**, w, that is a fixed (i.e., read-only) unique identifying number for each word **900**, akin to an address; 2) the Left X Field **804**, x_{nL}, that stores the x-coordinate of the left endpoint of a span; 3) the Left Z Field **806**, z_{nL}, that stores the z-coordinate of the left endpoint of a span; 4) the Right X Field **808**, x_{nR}, that stores the x-coordinate of the right endpoint of a span; 5) the Right Z Field **810**, z_{nR}, that stores the z-coordinate of the right endpoint of a span; 6) the Span Derivative Field **812**, d_n, that stores the $\delta z/\delta x$ slope of the span; and 7) the Information Pointer Field **814**, i_n, that stores a pointer into the Polygon Information Memory **514** for color information for the span's polygon.

Each SMCCAM Word **900** also stores: 1) a Valid Flag **816**, F_{nV}, a single bit value indicating whether the SMCCAM Word **900** is storing valid data; and 2) a Query Flag **818**, F_{nQ}, a single bit value indicating whether the SMCCAM Word **900** responded positively to a query operation. Both flag bits **816, 818** each have a corresponding "wired-nor" bus that indicates whether all words **900** have that flag bit turned off. Specifically, for the Valid Flag **816**, F_{nV}, if F_{nV} is false for all n (i.e., all words), then the signal *AllWordsInvalid* **990** is asserted. Similarly, if F_{nQ} is false for all n, then the signal *NullQueryResponse* **992** is asserted. The two wired-nor signals, *AllWordsInvalid* **990** and *NullQueryResponse* **992**, provide the mechanism for query results to be fed back to the external controller (located in the Query Processor **510**), so it can make decisions (i.e. "branches) about how to proceed within the method.

The nomenclature for fields and flags **804** to **818** include the Word Number Field **802** value as the first part of the field subscript (e.g., z_{3L} is in word 3). Later in this document, the number of fields in each word is increased to add various features to the Span Sorting Renderer **500**. Figure 8 shows two variable word indices, n and w, that are used throughout this document as references to SMCCAM Words **900**.

A block diagram of an SMCCAM Word **900** is shown as Figure 9, where each of the seven fields **802** to **814** is shown. The seven fields, x_{wL} **804**, z_{wL} **806**, x_{wR} **808**, z_{wR} **810**, d_w **812**, and i_w **814**, each have a corresponding data bus within the set of Array Busses **910**: *BusW* **912**, *BusXL* **914**, *BusZL* **916**, *BusXR* **918**, *BusZR* **920**, *BusD* **922**, and *BusI* **924**. Six of the

fields 802 to 812 are broken into a memory 902 or 904 and a comparator 906 or 908. The Word Number Field 802 includes: 1) a fixed memory 902; and 2) an equality comparison circuit 906, used for query operations, that compares the fixed memory value 902 to the data on the input bus, *BusW* 912. Five of the fields 804 to 812 include: 1) a read/write memory 904; and 2) an arithmetic comparison circuit 906 (tests for less-than, etc.), used for query operations, that compares the stored memory value 904 to the data on the corresponding input bus. The seventh field, the Information Pointer Field 814, i_w, is simply a read/write memory that does not generally participate in query operations.

Query operations are used for searching, sorting, reading, and writing into the array 800 of SMCCAM Words 900. A query operation is performed by all SMCCAM Words 900 in parallel by supplying query data to all words 900 via the Array Busses 910. The SMCCAM 800 includes the Query Logic Array 850, which is a set of Query Logic 930 circuits, one in each SMCCAM Word 900.

In parallel, within each SMCCAM Word 900, query operation results are computed by the Query Logic 930. The Query Logic 930 receives the results from the comparators 906 and 908 as well at the Valid Flag 816 and Query Flag 818 values, performs a selectable Boolean operation on these values (selected by *QueryCntrl* 932) to generate a query result bit, and then writes the query result bit back into either the Valid Flag 816 or the Query Flag 818.

When a query operation is performed, every word 900 generates a query result, which is stored into either its 900 Valid Flag 816 or the Query Flag 818. The set of all Valid Flags 816 is called the Valid Flag Word 830, and the set of all Query Flags 818 is called the Valid Flag Word 834. When a search is done, the query results designate which words fulfilled the query operation parameters. An example query operation is:

$$F_{nQ} = F_{nV} \wedge (x_{nL} \leq x_{CL}) \qquad \text{(EQ 4)}$$

where the following occurs: 1) x_{CL} is broadcast to all words 900 via *BusZL* 916; 2) the Left X Field 804 in each word 900 performs $(x_{nL} \leq x_{CL})$, by comparing its 804 contents, x_{nL}, to the value on *BusZL* 916; 3) the Query Logic 930 in each word 900 performs the right side of Equation 4, which is the query result for that word 900; 4) in each word 900, the query result is stored into the Query Flag 818 shown as the left side of Equation 4; and 5) the signal, *NullQueryResponse* 992, is asserted if the Query Flag 818 is false in all words 900.

Additional example Boolean operations are shown in Equation 5, where x_{CL}, S_p, x_{CR}, z_{CF}, z_{CL}, and z_{CR} are data broadcast to all SMCCAM Words 900 via the Array Busses 910, and where n is the particular word 900 where the operation is taking place (this happens for all n, which means for all SMCCAM Words 900).

$$\begin{gathered} F_{nQ} = F_{nV} \wedge (n \neq S_p) \wedge (x_{nL} < x_{CR}) \wedge (x_{nR} > x_{CL}) \wedge [(z_{nL} < z_{CF}) \vee (z_{nR} < z_{CF})] \\ F_{nQ} = F_{nV} \wedge F_{nQ} \wedge [(z_{nL} < z_{CL}) \vee (z_{nR} < z_{CL})] \\ F_{nQ} = F_{nV} \wedge (x_{nL} = x_{CR}) \wedge (z_{nL} = z_{CR}) \\ F_{nV} = F_{nV} \wedge (x_{nR} > x_{CR}) \end{gathered} \qquad \text{(EQ 5)}$$

$$F_{nQ} = \overline{F_{nV}} \vee (x_{nL} > x_{AL}) \vee [(x_{nL} = x_{AL}) \wedge (z_{nL} > z_{AL})] \vee [(x_{nL} = x_{AL}) \wedge (z_{nL} = z_{AL}) \wedge (d_n > d_A)]$$

The query operation mechanism inherently performs a search operation on the data stored in the SMCCAM 800, and the search results are stored into the Query Flags 818. When data needs to be written into the SMCCAM 800, the following sequence occurs: 1) the Valid Flag Word 830 (or, for certain applications, the Query Flag Word 834 could be used) is chosen to determine the word 900 to be written; 2) the Valid Flag Word 830 contents are input to its 830 corresponding Priority Resolver 840; 3) the Priority Resolver 840 finds the first occurrence of a logic "zero" within the flag word 830, this is the first invalid word; 4) the SMCCAM Word 900 with the first occurrence of logic "zero" is the selected word 900, and the data on the Array Busses 910 is stored into that word 900; and 5) the selected word 900 has its Valid Flag 816 asserted, indicating that word now contains valid data.

A read operation works similarly, with the following sequence: 1) the Valid Query Flag Word 834 is chosen to deter-

mine the word **900** to be read; 2) the Query Flag Word **834** contents are input to its **834** corresponding Priority Resolver **840**; 3) the Priority Resolver **840** finds the first occurrence of a logic "one" within the flag word **834**; 4) the SMCCAM Word **900** with the first occurrence of logic "one" is the selected word **900**, and its **900** contents are output onto the Array Busses **910**; 5) the selected word **900** has its Query Flag **818** de-asserted, indicating that word has had its contents read, and subsequent reads (without intervening queries) will read other words; and 6) the signal, *NullQueryResponse* **992**, is asserted if the Query Flag **818** is false in all words **900**, indicating that no other words would respond to a read operation. The query, write, and read operations work somewhat similarly to that described in the Duluk Patent.

The portion of the Priority Resolver **840** within an SMCCAM Word **900** is called a Priority Resolver Section **934**, and they **934** communicate across SMCCAM Words **900** via the busses: *ValidPriOut*[*w*−1] **936**, *QueryPriOut*[*w*−1] **937**, *ValidPriOut*[*w*] **938**, and *QueryPriOut*[*w*] **939**. Alternatively, the Priority Resolver **840** could be built with a treelike structure, thereby making the worst case circuit propagation delay proportional to $\log W$ rather than W.

The SMCCAM **800** also performs a sorting operation, which is not provided in the apparatus described in the Duluk Patent. As described above, the Span Sorting Renderer **500** sorts spans as they are written into the Span Memory **406** according to: 1) the left x-coordinate; 2) the left z-coordinate; and 3) the span derivative. In the SMCCAM **800**, sorting is done by locating the place in the span list where a span needs to be inserted, then, from this place, shifting the SMCCAM **800** contents down one word **900**, and then writing the new span into the vacated word **900**.

The SMCCAM **800** can shift data from word-to-word to make room in the sorted span list. The sorting operation is done in the following sequence: 1) the geometric properties (as described above) of the span to be written are input to the SMCCAM **800** for a query operation; 2) the query operation of Equation 6 is performed in every SMCCAM Word **900** with the query results written into the Query Flags **818**, thereby identifying all words that contain either invalid data or contain a span that should be sorted later in the span list that the span being written; 3) the Query Flag Word **834** contents are input to the corresponding Priority Resolver **840**; 4) the Priority Resolver **840** finds the first occurrence of a logic "one" within the Query Flag Word **834**, thereby determining the selected word **900**; 5) for the selected word **900** and all SMCCAM Words **900** after the selected word **900**, stored data is shifted to the next word **900**; and 6) at the selected word **900**, the data on the Array Busses **910** is stored into the word **900**.

$$F_{nQ} = \overline{F_{nV}} \vee (x_{nL} > x_{AL}) \vee [(x_{nL} = x_{AL}) \wedge (z_{nL} > z_{AL})] \vee [(x_{nL} = x_{AL}) \wedge (z_{nL} = z_{AL}) \wedge (d_n > d_A)] \qquad \text{(EQ 6)}$$

Shifting data in the fields **802** to **814** from one SMCCAM Word **900** to the next is done via the Shift In Busses **950** that are connected to the Shift Out Busses **970** of the previous word **900**. The list of Shift In Busses **950** includes: *ShiftOutXL*[*w*−1] **952**; *ShiftOutXR*[*w*−1] **954**; *ShiftOutZL*[*w*−1] **956**; *ShiftOutZR*[*w*−1] **958**; *ShiftOutD*[*w*−1] **960**; and *ShiftOutI*[*w*−1] **962**. The list of Shift Out Busses **970** includes: *ShiftOutXL*[*w*] **972**; *ShiftOutXR*[*w*] **974**; *ShiftOutZL*[*w*] **976**; *ShiftOutZR*[*w*] **978**; *ShiftOutD*[*w*−1] **980**; and *ShiftOutI*[*w*] **982**. The circuit signal nomenclature used in this document includes the use of a bracketed index, such as "[*w*]", indicating it is part of an array of busses (similar to the array nomenclature of the "C" computer language). Here, "[*w*]" indicates the *w*th word **900**, and "[*w*−1]" indicates the word **900** immediately prior in the set of words **800**.

Span Occluding Test

Figure 10 shows three spans **1002, 1004, 1006** represented by line segments (i.e., "segment spans") in the x-z plane, corresponding to one raster line of the display screen **104**. One of the spans **1002** is shown with a corresponding Occluding Region **1008**. If there are no spans that Occluding Region **1008**, then the corresponding span **1002** is not hidden. The shape of the Occluding Region **1008** is trapezoidal, and therefore it is computationally expensive to determine if another span lies within it **1008**, and is particularly expensive if many spans need to tested against the region **1008**. A simplifying approximation for an Occluding Region **1008** is a rectangular Span Occluding Test Region **1010** (hereinafter called an SOT Region **1010**). The SOT Region **1010** is generated from the span (or span portion) being tested for occlusion, hereinafter called the Current Portion,

S_C **1004**. Determining if a span **1006** with endpoints (x_{nL}, z_{nL}) and (x_{nL}, z_{nL}) lies within an SOT Region **1010** for a Current Portion, **1004**, S_C, with endpoints (x_{CL}, z_{CL}) **1012** and (x_{CR}, z_{CR}) **1014** is done by:

$$\text{In SOT Region} = (x_{nL} < x_{CR}) \wedge (x_{nR} > x_{CL}) \wedge [(z_{nL} < \text{MaxOf}(z_{CR}, z_{CL})) \vee (z_{nR} < \text{MaxOf}(z_{CR}, z_{CL}))] \quad \text{(EQ 7)}$$

The arithmetic computation required for Equation 7 is four comparisons and one "choose-the-maximum-of" function (the two needed in the equation are identical). For the SMCCAM **800** to perform the operations of Equation 7, the "choose-the-maximum-of" would be performed outside the SMCCAM **800** to determine:

$$z_{CF} = \text{MaxOf}(z_{CR}, z_{CL}) \quad \text{(EQ 8)}$$

and then the values x_{CL}, x_{CR}, and z_{CF} would be input on the Array Busses **910** and the comparisons would be performed in the appropriate fields **804** to **810**. In this way, all the spans stored in the SMCCAM **800** would be tested in parallel, performing the search for occluding spans in a matter of nanoseconds. In this document, any span that is found by an SOT Query is called an occluding span.

The complete equation for the Span Occluding Test Query (hereinafter called the SOT Query) is shown as Equation 9.

$$F_{nQ} = F_{nV} \wedge (n \neq S_P) \wedge (x_{nL} < x_{CR}) \wedge (x_{nR} > x_{CL}) \wedge [(z_{nL} < z_{CF}) \vee (z_{nR} < z_{CF})] \quad \text{(EQ 9)}$$

The complete SOT Query equation adds two more terms to the conjunction: 1) the Valid Flag **816**, needed to keep words with invalid data from mistakenly identifying themselves as storing an occluding span; and 2) a test, $n \neq S_P$, where S_P is the Word Number Field **802** corresponding to the Current Portion, done to keep the span from being identified as occluding itself. The complete SOT Query equation stores the query result into the Query Flag **818** of each SMCCAM Word **900**. The result of the SOT Query is available at the signal, *NullQueryResponse* **992**, where an asserted value indicates that no occluding spans were found.

By using the SOT Region as an approximation of the Occluding Region **1008**, a conservative error is introduced. That is, the SOT Query may find occluding spans that are within the SOT Region **1010**, but are actually behind the Current Portion. However, the SOT Query will never fail to find spans that actually occlude the Current Portion.

Span Sorting Rendering Method 1300

In order for the SOT Query to inadvertently find an occluding span that does not actually occlude the Current Portion, the bounding box (described in the next section) of the Current Portion must overlap the bounding box of the occluding span. Therefore, the simplest form of the Span Sorting Rendering Method **1300** assumes that bounding boxes of spans do not overlap, thereby eliminating the need to handle occluding spans that do not actually occlude the Current Portion. While this assumption is not practical for most rendering applications, it does simplify part of the description of the method **1300**. Therefore, this simplified (though not generally practical) version will be described first. In a later section, the method **1300** will be expanded to process spans with overlapping bounding boxes.

Figure 11 shows a set of fourteen spans in the same raster line (numbered S_0 **1100** to S_{13} **1113**), where each span is represented by a line segment in the x-z plane of the raster line (the y-coordinate is fixed). The visible parts of the spans are shown as thick lines, and the hidden parts are shown as thin lines. For example, S_4 **1104** and S_6 **1106** are hidden, and S_0 **1100** has two visible portions. The spans have been sorted according to the x-coordinate location of their left endpoint. Hence, the spans are numbered from left to right according to their Word Number Field **802** assigned when they are all stored in the SMCCAM **800**. For example, span S_4 **1104** is stored in SMCCAM Word **900** number four, and has its left endpoint located at the point (x_{4L}, z_{4L}) and its right endpoint located at (x_{4R}, z_{4R}).

Figure 12 shows the same fourteen spans, except that each span is enclosed by a bounding box. The set of spans in Figure 11 and Figure 12 were chosen so that the span bounding boxes do not overlap. The spans of Figure 12 will be used to describe the simplest form of the Span Sorting Rendering Method **1300**, the version that includes the Simplified Span Rasteriza-

tion Method 1800 which assumes span bounding boxes do not overlap.

The Span Sorting Rendering Method 1300 is described in the context of the apparatus described herein, but can also be performed by software on a general purpose computer. Figure 13 is the top-level method diagram of the Span Sorting Rendering Method 1300. This method 1300 is performed by the Span Sorting Rendering Pipeline 400 of Figure 4, in the same way the method flow diagram 300 of Figure 3 is performed by the generic 3D graphics pipeline 200.

In the Span Sorting Rendering Method 1300, scenes are generated 1302 (object locations in world coordinates, etc.), and within each scene, polygons are obtained 1304 for input to the Span Sorting Rendering Pipeline 400 and processed 1400.

Figure 14 shows the process polygon step 1400. Included are the previously described transformation 202, clip test 204, face determination 206, lighting 208, perspective divide 212 and compute polygon parameter 402 steps. After these steps, polygons are written into the Bucket Sort Memory 404, each according to its y_{START} parameter.

Getting back to Figure 13, once all 1306 the polygons for a scene have been processed 1400 and therefore written into the Bucket Sort Memory 404, each raster line is processed. The variable, R, is used to keep track of the current raster line number, and is initialized 1308 to zero. Before the first raster line can be processed, the Current Polygon Memory 406 must be cleared 1310, thereby indicating that no polygons are on the current raster line. Also, the Span Memory 408 must be cleared 1312, thereby indicating that no spans are on the current raster line. These operations 1310, 1312 can be done by marking the entire contents of the memories 406, 408 as invalid. Since, at the first raster line, there are no polygons in the Current Polygon Memory 406, the next step 1600 is to transfer polygons that start in the current raster line form the Bucket Sort Memory 404 to the Current Polygon Memory 406 and to make spans for these polygons. Beginning on the second raster line (i.e., $R = 1$), there is probably some polygons in the Current Polygon Memory 406, and spans need to be made 1500 for those polygons.

Figure 15 is the Process Current Polygon Memory 406 step 1500 within the Span Sorting Rendering Method 1300. Each 1502 polygon in the Current Polygon Memory 406, is read 1504, and if 1506 that polygon is not included in the current raster line, it is deleted 1508 from the Current Polygon Memory 406 so subsequent raster lines do not need to consider it. If 1506 the polygon is included in the present raster line, then a span within the current raster line and its Span Parameters (as described above) are generated 1510 for that polygon. Then, the Span Parameters are written 1700 into the Span Memory 408.

Figure 16 is the Process Bucket Sort Memory 404 step 1600 within the Span Sorting Rendering Method 1300. If 1602 there are more polygons to be read from the Bucket Sort Memory 404 for the current raster line, then for each such polygon: 1) the polygon is read 1604 from the Bucket Sort Memory 404; 2) the polygon is written 1606 into the Current Polygon Memory 406; 3) the polygon has a span in the current raster line, and Span Parameters (as described above) are generated 1510 for the span; and 4) the Span Parameters are written 1700 into the Span Memory 408.

When writing 1700 Span Parameters into the Span Memory 408, a sort operation is done. The sort operation is performed by the SMCCAM 800 as a query operation (Equation 6) and a special write operation, as described above. The apparatus of the SMCCAM 800 performs this step 1700 in parallel, but it is described in the flow diagram 1700 of Figure 17 as if it is a sequential search process. A counter, n, used as an index into the set of SMCCAM Words 900, is initialized 1702 to zero. The flow diagram shows the query operation of Equation 6 broken down into six conditionals 1704 that test to see if the new span should be inserted at the nth word 900. Words 900 are stepped through 1706 by incrementing n until such an insertion point is found. When the insertion point is found, the counter n stops incrementing, and the contents of all the words from $W - 1$ to n (using the counter, m) are transferred 1708 to the next word and the Span Parameters are written 1710 into word n. A test 1712 for reaching the last word 900 (i.e., word $W - 1$) is included to detect an exception 1714 condition if there are more than a total of W spans in the raster line, thus exceeding the total number of words 900. Strategies to work within a fixed number of words 900 are presented later in this document.

Once again returning to Figure 13, once all the spans within the display screen at raster line R have been written into the Span Memory 408, visible span portions are identified 1800 or 2100. One version of this portion of the method is the Simpli-

fied Span Rasterization Method 1800, which assumes span bounding boxes do not overlap. Other versions that do not make this assumption are the Segment Span Rasterization Method 2100 and the Trapezoidal Span Rasterization Method, described in later sections.

Simplified Span Rasterization Method 1800

The Simplified Span Rasterization Method 1800, shown in Figure 18, maintains three sets of variables: 1) the current left x-coordinate position, x_{CL}, used to indicate how much of the current raster line has been rendered so far; 2) the Present Span, S_P, used to keep track of the frontmost span at x_{CL}, including its Span Parameters x_{PL}, z_{PL}, x_{PR}, z_{PR}, d_P, i_P, and word number; and 3) the New Span, S_N, used to keep track of the span most recently read form the Span Memory 408 (generally found by a query), including its Span Parameters x_{NL}, z_{NL}, x_{NR}, z_{NR}, d_N, i_N, and word number. In the Simplified Span Rasterization Method 1800, the Current Portion, S_C, is always equal to the Present Span, S_P.

The rasterization method 1800 will be explained by using the set of spans in Figure 12 as an example. In the rest of this section, the method 1800 will be followed step-by-step as the example spans are processed.

The method 1800 starts by setting 1802 x_{CL} to zero (thereby starting at the left edge of the display screen) and declaring the variables S_P and S_N to be invalid. Because S_P is invalid 1804 and there are valid 1806 spans still stored in the Span Memory 408, a search 1808 must be done to find either the frontmost span at x_{CL} (i.e., zero) or, if there is no span at x_{CL}, then find the leftmost valid span in the Span Memory 408.

The first step in this search 1808 is to do a query 1810 to find all valid spans in the Span Memory 408 that include x_{CL} (i.e., zero). Looking at Figure 12, it is seen that there are no 1812 spans at x-coordinate zero (the query 1810 finds nothing). Because there are no spans 1812 at x-coordinate zero, the leftmost span in the Span Memory 408 (i.e., the first valid span because the spans are sorted) is read 1818 thereby making S_P to be equal to S_0 1200. There were no valid spans at x_{CL} (i.e., zero), and the leftmost span does not start until x_{0L} (left endpoint of span S_0 1200), so the range from zero to x_{0L} does not have any spans at all, and therefore the background is rendered 1820 from x_{CL} (i.e., zero) to x_{0L}. Some rendering has been done, so the value for x_{CL} is updated 1822 to become x_{PL} (i.e., x_{0L}), indicating rendering has been done up to this point. An SOT Query is done 1824 for the Present Span (i.e., S_0 1200) to find any occluding spans, and this finds all the other spans 1201 to 1213 since they are all in the SOT Region of S_0 1200. The first of these found spans are read 1824, making the value of the New Span, S_N, equal to S_1 1201.

The method 1800 has not reached 1826 the right edge of the display screen (i.e., $x_{CL} < x_{MAX}$), both S_P and S_N are valid 1804 and 1828, and the next step 1830 is to render the S_P (i.e., S_0 1200) from x_{CL} (i.e., x_{0L}) to x_{NL} (i.e., x_{1L}). This rendering can be done because the SOT Query 1824 found the leftmost occluding span in front of S_0 1200, meaning any part of S_0 1200 to the left of this occluding span must be visible. This step 1830 also updates the value for x_{CL} to be x_{NL} (i.e., x_{1L}), and also makes S_N (i.e., S_1 1201) into the new value of S_P. An SOT Query is done 1832 for the Present Span (i.e., S_1 1201) to find any occluding spans, and this finds only one span, S_2 1202, since it 1202 is the only span in the SOT Region of S_1 1201. This span is read 1832, making the value of the New Span, S_N, equal to S_2 1202.

The span S_1 1201 is processed in a way similar to S_0 1200. The method 1800 has not reached 1826 the right edge of the display screen (i.e., $x_{CL} < x_{MAX}$), both S_P and S_N are valid 1804 and 1828, and S_P (i.e., S_1 1201) is rendered 1830 from x_{CL} (i.e., x_{1L}) to x_{NL} (i.e., x_{2L}). The value for x_{CL} is updated 1830 to be x_{NL} (i.e., x_{2L}), and S_N (i.e., S_2 1202) is made into the new value of S_P. An SOT Query is done 1832 for the Present Span (i.e., S_2 1202), but the query does not find any spans because nothing occludes S_2 1202, and the value of the New Span, S_N, is set to invalid.

The right edge of the display screen has not been reached 1826, S_P is valid 1804, S_N is invalid 1828, and the next portion 1834 of the method 1800 includes a search for an abutting span for S_P. An abutting span is a span whose left endpoint is located at the same point as the right endpoint of the Current Portion (or, for the Simplified Span Rasterization Method 1800, the Present Span). In Figure 12, three example pairs of abutting spans are: 1) S_2 1202 and S_5 1205; 2) S_5 1205 and S_7 1207; and 3) S_6 1206 and S_9 1209. Abutting spans occur frequently because 3D objects are generally described by a set of polygons that

share edges. Therefore, for highly tessellated objects, most spans will be part of at least one abutting span pair.

The portion 1834 of the method 1800 starts with rendering 1836 S_P (i.e., S_2 1202) from x_{CL} (i.e., x_{2L}) to x_{PR} (i.e., x_{2R}). This can be done because the SOT Query did not find any occluding spans, thus proving S_P (i.e., S_2 1202) is not hidden at all, and should be rendered. Also, x_{CL} is set 1836 to x_{PR} (i.e., x_{2R}). Next, a query is done 1838 to find an abutting span for S_P (i.e., S_2 1202), which finds span S_5 1205. Because 1840 an abutting span was found, the abutting span, S_5 1205, is read 1846 and assigned to S_P. An SOT Query is done 1848 for the Present Span (i.e., S_5 1205), but the query does not find any spans because nothing occludes S_5 1205, and the value of the New Span, S_N, is set to invalid.

The Present Span, S_5 1205, is processed similarly to S_2 1202. The right edge of the display screen has not been reached 1826, S_P is valid 1804, S_N is invalid 1828, S_P (i.e., S_5 1205) is rendered 1836 from x_{CL} (i.e., x_{2R}) to x_{PR} (i.e., x_{5R}), and x_{CL} is set 1836 to x_{PR} (i.e., x_{5R}). A query is done 1838 for an abutting span, which finds span S_7 1207. The abutting span, S_7 1207, is read 1846 and assigned to S_P. An SOT Query is done 1848 for the Present Span (i.e., S_7 1207), finding only S_8 1208 which is read 1832, making the value of the S_N equal to S_8 1208.

The Present Span, S_7 1207 is processed similarly to S_1 1201. The right edge of the display screen has not been reached 1826, both S_P and S_N are valid 1804 and 1828, and S_P (i.e., S_7 1207) is rendered 1830 from x_{CL} (i.e., x_{5R}) to x_{NL} (i.e., x_{8L}). The value for x_{CL} is updated 1830 to be x_{NL} (i.e., x_{8L}), and S_P is set to S_N (i.e., S_2 1202). An SOT Query is done 1832 for the Present Span (i.e., S_8 1208), but the query does not find any spans, and the value of the New Span, S_N, is set to invalid.

The Present Span, S_8 1208 is processed similarly to S_2 1202 and S_5 1205. The right edge of the display screen has not been reached 1826, S_P is valid 1804, S_N is invalid 1828, S_P (i.e., S_8 1208) is rendered 1836 from x_{CL} (i.e., x_{8L}) to x_{PR} (i.e., x_{8R}), and x_{CL} is set 1836 to x_{PR} (i.e., x_{8R}). A query is done 1838 for an abutting span, which finds span S_{10} 1210. The abutting span, S_{10} 1210, is read 1846 and assigned to S_P. An SOT Query is done 1848 for the Present Span (i.e., S_{10} 1210), but the query does not find any spans, and the value of the New Span, S_N, is set to invalid.

As the Present Span, S_{10} 1210 is processed, the search for an abutting span does not find such a span. The right edge of the display screen has not been reached 1826, S_P is valid 1804, S_N is invalid 1828, S_P (i.e., S_{10} 1210) is rendered 1836 from x_{CL} (i.e., x_{8R}) to x_{PR} (i.e., x_{10R}), and x_{CL} is set 1836 to x_{PR} (i.e., x_{10R}). A query is done 1838 for an abutting span, but no abutting span is found. Since 1840 no abutting span was found, a search must be done to find the frontmost span at x_{CL} (i.e., x_{10R}). However, before the search is done, a query is done to invalidate 1842 all spans that are completely to the left of x_{CL} (i.e., x_{10R}). This invalidation 1842 can be done because the current raster line has been rendered up to x_{CL} (i.e., x_{10R}), and spans to the left of x_{CL} (i.e., x_{10R}) were either rendered or hidden (or some of each). The invalidation 1842 turns off the Valid Flags 816 for S_1 1201 through S_{10} 1210, thereby leaving only S_0 1200 and S_{11} 1211 through S_{13} 1213 as validly stored spans. The value of the Present Span, S_N, and the value of the New Span, S_N, are both set 1844 to invalid.

The right edge of the display screen has not been reached 1826, S_P is invalid 1804, there are still 1806 valid spans, and the next part 1808 of the method includes a search for the frontmost span at x_{CL} (i.e., x_{10R}). First, a query 1810 is done to find all spans that include x_{CL} (i.e., x_{10R}), which finds two spans: S_0 1200 and S_{11} 1211. Since 1812 spans were found, these two spans 1200 and 1211 are read 1814, the z-coordinates of the spans are computed 1814 at x_{CL} (i.e., x_{10R}), and are compared to determine 1814 that S_{11} 1211 is the frontmost span. The z-coordinate computation is simplified by having the span derivative (from the Span Derivative Field 812) available for use in extrapolating from the left endpoint of the span. The frontmost span, S_{11} 1211, is made 1814 the Present Span, S_P. An SOT Query is done 1824 for the Present Span (i.e., S_{11} 1211), but the query does not find any spans, and the value of the New Span, S_N, is set to invalid.

The Present Span, S_{11} 1211, is processed similarly to S_2 1202, S_5 1205, and S_8 1208. The right edge of the display screen has not been reached 1826, S_P is valid 1804, S_N is invalid 1828, S_P (i.e., S_{11} 1211) is rendered 1836 from x_{CL} (i.e., x_{10R}) to x_{PR} (i.e., x_{11R}), and x_{CL} is set 1836 to x_{PR} (i.e., x_{11R}). A query is done 1838 for an abutting span, which finds span S_{12} 1212. The abutting span, S_{12} 1212, is read 1846 and assigned to S_P. An SOT Query is done 1848 for the Present Span (i.e., S_{12} 1212),

but the query does not find any spans, and the value of the New Span, S_N, is set to invalid.

The Present Span, S_{12} **1212** is processed similarly to S_{10} **1210**. The right edge of the display screen has not been reached **1826**, S_P is valid **1804**, S_N is invalid **1828**, S_P (i.e., S_{12} **1212**) is rendered **1836** from x_{CL} (i.e., x_{11R}) to x_{PR} (i.e., x_{12R}), and x_{CL} is set **1836** to x_{PR} (i.e., x_{12R}). A query is done **1838** for an abutting span, but no abutting span is found. Since **1840** no abutting span was found, a query is done to invalidate **1842** all spans that are completely to the left of x_{CL} (i.e., x_{12R}), thereby turning off the Valid Flags **816** for S_{11} **1211** and S_{12} **1212**, thereby leaving only S_0 **1200** and S_{13} **1213** as validly stored spans. The value of the Present Span, S_N, and the value of the New Span, S_N, are both set **1844** to invalid.

The right edge of the display screen has not been reached **1826**, S_P is invalid **1804**, there are still **1806** valid spans, and the next part **1808** of the method includes a search for the frontmost span at x_{CL} (i.e., x_{12R}). First, a query **1810** is done to find all spans that include x_{CL} (i.e., x_{12R}), which finds only one span, S_0 **1200**. Since **1812** a span was found, it is made **1814** the Present Span, S_P. An SOT Query is done **1848** for the Present Span (i.e., S_0 **1200**), finding only S_{13} **1213** which is read **1832**, making the value of the S_N equal to S_{13} **1213**.

The right edge of the display screen has not been reached **1826**, both S_P and S_N are valid **1804** and **1828**, and S_P (i.e., S_0 **1200**) is rendered **1830** from x_{CL} (i.e., x_{12R}) to x_{NL} (i.e., x_{13L}). The value for x_{CL} is updated **1830** to be x_{NL} (i.e., x_{13L}), and S_P is set to S_N (i.e., S_{13} **1213**). An SOT Query is done **1832** for the Present Span (i.e., S_{13} **1213**), but the query does not find any spans, and the value of the New Span, S_N, is set to invalid.

The right edge of the display screen has not been reached **1826**, S_P is valid **1804**, S_N is invalid **1828**, S_P (i.e., S_{13} **1213**) is rendered **1836** from x_{CL} (i.e., x_{13L}) to x_{PR} (i.e., x_{13R}), and x_{CL} is set **1836** to x_{PR} (i.e., x_{13R}). A query is done **1838** for an abutting span, but no abutting span is found. Since **1840** no abutting span was found, a query is done to invalidate **1842** all spans that are completely to the left of x_{CL} (i.e., x_{13R}), thereby turning off the Valid Flags **816** for S_0 **1200** and S_{13} **1213**, thereby eliminating the last valid spans. The value of the Present Span, S_N, and the value of the New Span, S_N, are both set **1844** to invalid.

The right edge of the display screen has not been reached **1826**, S_P is invalid **1804**, and there are **1806** no valid spans. Therefore, the background is rendered from x_{CL} (i.e., x_{13R}) to x_{MAX} (i.e.,the right edge of the raster line). The entire current raster line has now been rendered, and the next raster line can be processed, as shown in the method flow diagram of Figure 13.

It is important to note that the method **1800** rendered the visible portions of the spans, and completely ignored the spans that are totally hidden (i.e., S_3 **1203**, S_4 **1204**, S_6 **1206**, and S_9 **1209**). For scenes with greater depth complexity, a larger fraction of the spans will be ignored in a similar way, thereby providing a greater computational savings. A unique feature of this method **1800** is the ability of abutting spans to work together to occlude spans behind them.

Another important feature of the method **1800** is the pixel coloration processing within a raster line is roughly proportional to the number of pixels in the raster line, rather than proportional to the number of pixels in all the spans in the raster line. This is because hidden portions within spans are never sent to the pixel coloration process. Pixel coloration processing is "roughly" proportional to the number of pixels within the raster line because a pixel can be affected by two spans because, for example, the span to the right may end within the pixel's boundary. In this case, color from two spans are blended together to form the final color for the pixel.

Span Interaction Types and Span Interaction Parameters

As described above, the SOT Query is a set of comparison operations simple enough to be performed in hardware by an SMCCAM **800**. However, in the general case where span bounding boxes overlap, the SOT Query will sometimes find spans that do not occlude the Current Portion. Furthermore, the SOT Query will also sometimes find spans that intersect the Current Portion, thus making both spans partially visible. The Segment Span Rasterization Method **2100** solves this shortcoming by categorizing the interaction between the Current Portion and a span in the SOT Region into one of many Span Interaction Types (hereinafter abbreviated SIT), and then applying rendering rules based on the SIT. The SIT is determined from the Span Interaction Parameters (hereinafter SIP) of the two spans.

As the Segment Span Rasterization Method **2100** proceeds, it **2100** maintains four spans and their associated parame-

ters: 1) the Present Span, S_P the frontmost span at x_{CL}, as described above; 2) the New Span, S_N, a span in the SOT Region of S_P as described above; 3) the Current Portion, S_C, a subsection of the Present Span, S_P; and 4) the Saved Span, S_S, used as a single depth stack for temporarily saving S_N so that more spans in the SOT Region of S_P can be read.

The nomenclature for spans, span endpoints, SIPs, and SITs is illustrated by the examples shown in Figure 19. The Present Span, S_P **1902**, has the endpoints (x_{PL}, z_{PL}) **1904, 1906** and (x_{PR}, z_{PR}) **1908, 1910**, is enclosed by a dashed oval, and is shown as the union of a solid black line and a finely dotted line. The solid black line subsection of S_P **1902** is the Current Portion, S_C **1912**, and has the endpoints (x_{CL}, z_{CL}) **1914, 1916** and (x_{CR}, z_{CR}) **1918, 1920**. The SOT Region **1922** corresponding to S_C **1912** is enclosed by a dashed line. The New Span, S_N **1932**, has the endpoints (x_{NL}, z_{NL}) **1934, 1936** and (x_{NR}, z_{NR}) **1938, 1940**, and is enclosed by its bounding box **1942**.

The Span Interaction Parameters (i.e., SIPs) are arithmetic comparison results between the z-coordinates of S_C **1912** and S_N **1932** at four particular x-coordinate values, specifically, the endpoints of the two spans: x_{NL} **1934**, x_{NR} **1938**, x_{CL} **1914**, and x_{CR} **1918**. In detailed terms, the four Span Interaction Parameters (i.e., SIPs) are: 1) the comparison of the z-coordinate of S_C **1912** at x_{NL} **1934** to z_{NL} **1936**, shown in the example as "Nonexistent" because S_C **1912** does not have a point at x_{NL} **1934**; 2) the comparison of the z-coordinate of S_C **1912** at x_{NR} **1938** to z_{NR} **1940**, shown in the example as "Farther" because the point on S_C **1912** is father (i.e., has a greater z-coordinate) than z_{NR} **1940**; 3) the comparison of the z-coordinate of S_N **1932** at x_{CL} **1914** to z_{CL} **1916**, shown in the example as "Farther" because the point on S_N **1932** is father (i.e., has a greater z-coordinate) than z_{CL} **1916**; and 4) the comparison of the z-coordinate of S_N **1932** at x_{CR} **1918** to z_{CR} **1920**, shown in the example as "Nonexistent" because S_N **1932** does not have a point at x_{CR} **1918**. Each of the four SIPs can take on one of the values: 1) "Nonexistent" (abbreviated "Non"); 2) "Farther" (abbreviated "Far"); 3) "Nearer" (abbreviated "Near"); or 4) "Equal". In the example of Figure 19, the four SIPs are Non, Far, Far, Non, as shown in the figure. This particular set of four is SIT 23, causing invocation of Rule 4, as will be described later in this document.

The SIPs were chosen so as to be easy to compute, avoiding expensive computations such as division. The computation of z-coordinates used in the comparison is done by:

$$\begin{aligned} \text{z-coordinate of } S_C \text{ at } x_{NL} &= z_{CL} + (x_{NL} - x_{CL})\, d_C \\ \text{z-coordinate of } S_C \text{ at } x_{NR} &= z_{CL} + (x_{NR} - x_{CL})\, d_C \\ \text{z-coordinate of } S_N \text{ at } x_{CL} &= z_{NL} + (x_{CL} - x_{NL})\, d_N \\ \text{z-coordinate of } S_N \text{ at } x_{CR} &= z_{NL} + (x_{CR} - x_{NL})\, d_N \end{aligned} \qquad \text{(EQ 10)}$$

Figure 20 is a listing of all 49 possible SITs. For example, Figure 20C illustrates SIT 3, showing three examples of a S_C paired with an S_N, where the SIPs are Near, Equal, Non, Non. Each possible combination of SIPs correspond to one SIT. When a particular SIT is encountered during the Segment Span Rasterization Method **2100**, one of several rules is invoked, and each SIT in Figure 20 identifies the rule it invokes. Some SITs, such as SIT 19 and SIT 22, cannot ever be encountered by the method **2100**, therefore, no rule is invoked, and the tag "impossible" is shown in Figure 20.

***Segment Span Rasterization Method* 2100**

The Segment Span Rasterization Method **2100**, shown in Figure 21, processes spans that are represented by line segments (as in Figure 7A). This method can be utilized within Span Sorting Rendering Method **1300**, shown in Figure 13. In this section of the document, the method flow diagrams of Figure 21 through Figure 26 will be described in very general terms. In the next section, an complex set of example spans is used to describe the step-by-step functioning of the method **2100**.

The Segment Span Rasterization Method **2100** starts by initializing **2102** and **2104** variables. The method **2100** calls sub-methods, called Rule 1 **2200** through Rule 5 **2600**. Every rule considers: 1) what part of S_C should be rendered, if any; 2) the updating of S_P and/or S_C; 3) updating of S_S; 4) invalidation of spans stored in the Span Memory **408**; 5) doing an SOT Query; and/or 6) updating of S_N.

Rule 1 **2200** (shown in Figure 22) is invoked if S_P is invalid. It **2200** searches for the frontmost span at x_{CL}, makes the frontmost span S_P, performs an SOT Query, and updates S_N.

Rule 2 **2300** (shown in Figure 23) is invoked if S_C has no Potentially Occluding Spans remaining. Therefore, it **2300** renders the Present Span, S_P, from x_{CL} to x_{CR}. Then, S_C is set to, in order of precedence: 1) the Saved Span, S_S; 2) an Abutting Span; or 3) the first valid span after invalidation of all spans completely to the left of x_{CR}. In any case, x_{CL} is set to x_{CR}, and S_S is invalidated.

Rule 3 **2400** (shown in Figure 24) is invoked if the SIT is such that the Current Portion, S_C, is occluded at x_{NL}, but is visible from x_{CL} to x_{NL}. Therefore, render the Present Span, S_P, from x_{CL} to x_{NL}. Then, make the New Span, S_N, the Present Span and begin processing it. Since S_C is not being rendered to x_{CR}, invalidate the Saved Span, S_S.

Rule 4 **2500** (shown in Figure 25) is invoked if the SIT is such that S_C intersects S_N and the point of intersection is visible. The intersection point, at (x_I, z_I), between the Current Portion, S_C, and the New Span, S_N is computed. Since S_C is occluded to the right of x_I, S_C is modified by changing x_{CR} to x_I. The New Span, S_N, is stored as the Saved Span, S_S, in case it is needed later under Rule 1 **2200**. Then, the next Potentially Occluding Span is read and made S_N.

Rule 5 **2600** (shown in Figure 26) is invoked if the Current Portion, S_C, occludes the New Span, S_N, such that none of S_N is visible from x_{CL} to x_{CR}. Therefore, S_N is discarded, and the next Potentially Occluding Span is read and made S_N. Since S_C is not being rendered when this rule is applied, leave the status of the Saved Span, S_S, unchanged.

***Detailed example of the Segment Span Rasterization Method* 2100**

In order to fully describe the Segment Span Rasterization Method **2100** in a step-by-step fashion, an example set of fifteen spans, shown in Figure 27, is used to illustrate how the method **2100** works. The spans in this example were chosen so as to exercise all parts of the Segment Span Rasterization Method **2100**. The example, when compared to typical raster lines in typical scenes, is abnormally complex due to the large fraction of overlapping bounding boxes and intersecting spans. As each step in the Segment Span Rasterization Method **2100** is described, the corresponding reference designators in the method flow diagrams are listed at the beginning of the step description. Within a step description, equations for query operations are first shown as they appear in the method flow diagrams, then, on a second line, the same equation is shown with all its variables substituted by values from the example of Figure 27. Also within a step description, assignment statements will be cascaded (with multiple "=" signs) showing how to arrive at the fully substituted variables.

For the example of Figure 27, when the Segment Span Rasterization Method **2100** begins, there are fifteen valid spans in the SMCCAM **800**, designated S_0 through S_{14}. These spans are stored in SMCCAM Words **900** 0 through 14, and therefore each have $(F_{nV} = 1)$, indicating those SMCCAM Words **900** have valid contents. The rest of the SMCCAM Words **900**, namely words 15 through W - 1, each have $(F_{nV} = 0)$, indicating those SMCCAM Words **900** have invalid contents. Hence, at the start of the Segment Span Rasterization Method **2100**, the Valid Flag Word **840**, F_V, designates the set of valid spans, and this set is not empty.

For the example set of fifteen spans shown in Figure 27, the step-by-step processing using the Segment Span Rasterization Method **2100** is:

1) **2102** The left x value of the Current Portion, x_{CL}, indicates how far the rendering has proceeded along the present raster line. Hence, the start of a raster line causes setting $(x_{CL} = 0)$. The left z value of the Current Portion, z_{CL}, is set to infinity to allow any span to be in front of the background. The depth location of the background is considered to be at infinity. The background is not composed of polygons, and for may applications, is set to the color black. The right x and y values of the Current Portion, x_{CR} and x_{CR}, are set to "invalid" because there is no valid Present Span, S_P. With assignment statements, this is described as:

$$x_{CL} = 0; \qquad x_{CR} = \text{invalid};$$
$$z_{CL} = \text{infinity}; \qquad z_{CR} = \text{invalid}; \qquad \text{(EQ 11)}$$
$$z_{CF} = \text{infinity};$$

2) **2104** The data within registers for storing span information external to the SMCCAM **800** (i.e., S_P, S_N, and S_S) are also designated as invalid.

$$S_P = \text{invalid};$$
$$S_N = \text{invalid}; \qquad \text{(EQ 12)}$$
$$S_S = \text{invalid};$$

3) **2106, 2108, and 2200** Because the Present Span, S_P, is not valid and F_V is not empty (indicating that at least some spans are valid), Rule 1 is invoked. Here, Rule 1 is used to search for the first span to make the Present Span, S_P.

3A) **2202** Do a query to find all valid spans that could affect the current raster line at x_{CL} (i.e., at x coordinate zero). The query performs, in each SMCCAM Word **900**:

$$F_{nQ} = F_{nV} \wedge (x_{nL} \leq x_{CL})$$
$$F_{nQ} = F_{nV} \wedge (x_{nL} \leq 0) \qquad \text{(EQ 13)}$$

This query does not find any spans, and therefore, the set F_Q is empty.

3B) **2204, 2206, and 2208** Because F_Q is empty and S_P is not valid, there must not be a valid span that could affect the current raster line at x_{CL}. Therefore, the first span in F_V is read and made the Present Span, S_P. This read operation sets the following values:

$$S_P = \text{read}(n) = 0;$$
$$x_{PL} = \text{read}(x_L) = x_{0L}; \qquad x_{PR} = \text{read}(x_R) = x_{0R};$$
$$z_{PL} = \text{read}(z_L) = z_{0L}; \qquad z_{PR} = \text{read}(z_R) = z_{0R}; \qquad \text{(EQ 14)}$$
$$d_P = \text{read}(d) = d_0; \qquad i_P = \text{read}(i) = i_0;$$

Reading the first span from F_V yields the leftmost valid span, S_0, because the spans are stored in the SMCCAM **800** in a left to right ordering.

3C) **2210** Because there is not a valid span that could affect the current raster line at x_{CL} (i.e., at x coordinate zero), and the leftmost valid span does not start until x_{PL} (i.e., x_{0L}), the background must be rendered from x_{CL} to x_{PL} (i.e., from 0 to x_{0L}).

As possible alternate methods, rendering of the background can be avoided if: 1) it is know that polygons will always cover 100% of the viewing plane; or 2) the raster buffer is initialized with the background colors prior to processing the first span in each raster line.

3D) **2212** There is now a Present Span, S_P (i.e., S_0), and the Current Portion, S_C, is set equal to S_P by setting:

$$x_{CL} = x_{PL} = x_{0L}; \qquad x_{CR} = x_{PR} = x_{0R};$$
$$z_{CL} = z_{PL} = z_{0L}; \qquad z_{CR} = z_{PR} = z_{0R}; \qquad \text{(EQ 15)}$$
$$_{CF} = \max(z_{PL}, z_{PR}) = \max(z_{0L}, z_{0R}) = z_{0L};$$

The Current Portion, S_C, is equal to S_0. The Far Z Value for the Current Portion, z_{CF}, is set to z_{0L} by the "find the maximum" function.

3E) **2214** There is now a defined Present Span, S_P (i.e., S_0), and a defined Current Portion, S_C (i.e., S_0), so a search is done for spans that could possibly occlude the Current Portion, S_C, (i.e., S_0). The Span

Occluding Test query performs, in each SMCCAM Word **900**:

$$F_{nQ} = F_{nV} \wedge (n \neq S_P) \wedge (x_{nL} < x_{CR}) \wedge (x_{nR} > x_{CL}) \wedge [(z_{nL} < z_{CF}) \vee (z_{nR} < z_{CF})]$$
$$F_{nQ} = F_{nV} \wedge (n \neq 0) \wedge (x_{nL} < x_{0R}) \wedge (x_{nR} > x_{0L}) \wedge [(z_{nL} < z_{0L}) \vee (z_{nR} < z_{0L})] \quad \text{(EQ 16)}$$

This query creates the set of all Potentially Occluding Spans for the Current Portion, S_C (i.e., S_0). The query finds S_1 through S_{14}. In general, queries used for search operations include F_{nV} in the conjunction in order to prevent invalid data stored in a SMCCAM Word **900** from causing a bit in F_Q from being mistakenly asserted.

3F) **2216 and 2218** Because F_Q is not empty, there must spans that possibly occlude the Current Portion, S_C (i.e., S_0). Therefore, the first span in F_Q is read and made the New Span, S_N. This read operation sets the following values:

$$S_N = \text{read}(n) = 1;$$
$$x_{NL} = \text{read}(x_L) = x_{1L}; \qquad x_{NR} = \text{read}(x_R) = x_{1R};$$
$$z_{NL} = \text{read}(z_L) = z_{1L}; \qquad z_{NR} = \text{read}(z_R) = z_{1R};$$
$$d_N = \text{read}(d) = d_1; \qquad i_N = \text{read}(i) = i_1; \quad \text{(EQ 17)}$$

Reading the first span in F_Q yields S_1, the first span in the set. The read process also deletes S_1 from F_Q by setting $(F_{1Q} = 0)$.

4) **2109, 2106, 2110, and 2112** The right edge of the display screen has not been reached (i.e., $x_{CL} < x_{max}$) and both the Present Span, S_P (i.e., S_0), and the New Span, S_N (i.e., S_1), are valid. Therefore, the Span Interaction Parameters for these two spans are computed, and are found to be: (Far, Non, Non, Far).

5) **2114** The Span Interaction Type is determined from the Span Interaction Parameters, and turns out to be Type 14.

6) **2116 and 2400** Because the Span Interaction Type is Type 14, Rule 3 invoked to process S_0:

6A) **2402** The Current Portion, S_C (i.e., S_0), is hidden starting at x_{NL} (i.e., x_{1L}), but is visible to the left of x_{NL} (i.e., x_{1L}). Therefore, render the Present Span, S_P (i.e., S_0), from x_{CL} to x_{NL} (i.e., from x_{0L} to x_{1L}).

6B) **2404** The processing of Present Span is completed (at this point), so make S_N (i.e., S_1) into the Present Span, S_P, by setting:

$$S_P = S_N = 1;$$
$$x_{PL} = x_{NL} = x_{1L}; \qquad x_{PR} = x_{NR} = x_{1R};$$
$$z_{PL} = z_{NL} = z_{1L}; \qquad z_{PR} = z_{NR} = z_{1R};$$
$$d_P = d_N = d_1; \qquad i_P = i_N = i_1; \quad \text{(EQ 18)}$$

6C) **2406** The Current Portion, S_C, is set equal to S_N by setting:

$$x_{CL} = x_{NL} = x_{1L}; \qquad x_{CR} = x_{NR} = x_{1R};$$
$$z_{CL} = z_{NL} = z_{1L}; \qquad z_{CR} = z_{NR} = z_{1R};$$
$$_{CF} = \max(z_{NL}, z_{NR}) = \max(z_{1L}, z_{1R}) = z_{1L}; \quad \text{(EQ 19)}$$

Alternatively, the Current Portion, S_C, could be set to the Present Span, S_P, (rather than S_N) because it has the same values. However, this alternate way is not desirable for hardware implementation because this step and the previous step could not be done in parallel.

6D) **2408** The Saved Span, S_S, is invalidated, but at this point in this example, it was already invalid.

6E) **2410** There is a new Current Portion, S_C (i.e., S_1), so a search is done for spans that could possibly occlude it. The Span Occluding Test query performs, in each SMCCAM Word **900**:

$$F_{nQ} = F_{nV} \wedge (n \neq S_P) \wedge (x_{nL} < x_{CR}) \wedge (x_{nR} > x_{CL}) \wedge [(z_{nL} < z_{CF}) \vee (z_{nR} < z_{CF})]$$
$$F_{nQ} = F_{nV} \wedge (n \neq 1) \wedge (x_{nL} < x_{1R}) \wedge (x_{nR} > x_{1L}) \wedge [(z_{nL} < z_{1L}) \vee (z_{nR} < z_{1L})] \quad \text{(EQ 20)}$$

This query creates the set of all Potentially Occluding Spans for the Current Portion, S_C (i.e., S_1). The query finds S_0 and S_2 through S_{14}.

6F) **2412 and 2414** Because F_Q is not empty, there must be spans that possibly occlude the Current Portion, S_C (i.e., S_1). Therefore, the first span in F_Q is read and made the New Span, S_N. This read operation sets the following values:

$$S_N = \text{read}(n) = 0;$$
$$x_{NL} = \text{read}(x_L) = x_{0L}; \qquad x_{NR} = \text{read}(x_R) = x_{0R};$$
$$z_{NL} = \text{read}(z_L) = z_{0L}; \qquad z_{NR} = \text{read}(z_R) = z_{0R}; \quad \text{(EQ 21)}$$
$$d_N = \text{read}(d) = d_0; \qquad i_N = \text{read}(i) = i_0;$$

Reading the first span in F_Q yields S_0, the first span in the set. The read process also deletes S_0 from F_Q by setting $(F_{0Q} = 0)$.

7) **2109, 2106, 2110, and 2112** The right edge of the display screen has not been reached (i.e., $x_{CL} < x_{max}$) and both the Present Span, S_P (i.e., S_1), and the New Span, S_N (i.e., S_0), are valid. Therefore, the Span Interaction Parameters for these two spans are computed, and are found to be: (Far, Non, Far, Non).

8) **2114** The Span Interaction Type is determined from the Span Interaction Parameters, and turns out to be Type 23.

9) **2116, 2118, and 2500** Because the Span Interaction Type is Type 23, Rule 4 is invoked to process S_1:

9A) **2502** The Current Portion, S_C (i.e., S_1), and the New Span, S_N (i.e., S_0), interact to cause S_C to transition from being visible to being hidden somewhere along its length: at point of intersection between S_C (i.e., S_1) and S_N (i.e., S_0). The intersection is computed, and found to be at (x_I, z_I) **2702**, as shown in Figure 27.

9B) **2504** The Current Portion, S_C (i.e., S_1), is shortened by changing its right end point to the intersection point. The right endpoint of S_C is modified by setting

$$x_{CR} = x_I;$$
$$z_{CR} = z_I; \quad \text{(EQ 22)}$$
$$_{CF} = \max(z_{CL}, z_I) = z_{CL} = z_{1L};$$

The Present Span, S_P, is still equal to all of S_1.

9C) **2506** The New Span, S_N (i.e., S_0), is saved as the Saved Span, S_S. This is done because S_S will be needed later if S_P (i.e., S_1) is rendered to x_I.

$$S_S = S_N = 0;$$
$$x_{SL} = x_{NL} = x_{0L}; \qquad x_{SR} = x_{NR} = x_{0R};$$
$$z_{SL} = z_{NL} = z_{0L}; \qquad z_{SR} = z_{NR} = z_{0R}; \quad \text{(EQ 23)}$$
$$d_S = d_N = d_0; \qquad i_S = i_N = i_0;$$

The portion of the Saved Span, S_S, to the right x_I abuts S_C at the x coordinate x_I. If, later in the method, S_C is rendered to x_I, then S_S will become the Present Span, S_P.

9D) **2508** The Current Portion, S_C (i.e., S_1 from x_{1L} to x_I) has been changed, so a search is done for spans that could possibly occlude it. The Span Occluding Test query performs, in each SMCCAM Word **900**:

$$F_{nQ} = F_{nV} \wedge F_{nQ} \wedge (n \neq S_P) \wedge (x_{nL} < x_{CR}) \wedge (x_{nR} > x_{CL}) \wedge [(z_{nL} < z_{CF}) \vee (z_{nR} < z_{CF})]$$
$$F_{nQ} = F_{nV} \wedge F_{nQ} \wedge (n \neq 1) \wedge (x_{nL} < x_I) \wedge (x_{nR} > x_{1L}) \wedge [(z_{nL} < z_{1L}) \vee (z_{nR} < z_{1L})] \quad \text{(EQ 24)}$$

This query creates the set of all Potentially Occluding Spans for the Current Portion, S_C (i.e., S_1 between x_{1L} and x_I). The query finds only S_2. The conjunction performed by the query includes F_Q to prevent any previously read (and discarded) Potentially Occluding Span of S_1 from being re-included into F_Q and needlessly reprocessed.

A variation of this method could skip this step and continue reading from the existing F_Q rather than doing a query to eliminate spans from F_Q. This variation saves execution time by eliminating the query, but possibly increases execution time by failing to reduce the number of spans in F_Q.

9E) **2510 and 2512** Because F_Q is not empty, there must be spans that possibly occlude the Current Portion, S_C (i.e., S_1 from x_{1L} to x_I). Therefore, the first span in F_Q is read and made the New Span, S_N. This read operation sets the following values:

$$S_N = \text{read}(n) = 2;$$
$$x_{NL} = \text{read}(x_L) = x_{2L}; \quad x_{NR} = \text{read}(x_R) = x_{2R};$$
$$z_{NL} = \text{read}(z_L) = z_{2L}; \quad z_{NR} = \text{read}(z_R) = z_{2R};$$
$$d_N = \text{read}(d) = d_2; \quad i_N = \text{read}(i) = i_2; \quad \text{(EQ 25)}$$

Reading the first span in F_Q yields S_2, the first (and only) span in the set. The read process also deletes S_2 from F_Q by setting $(F_{2Q} = 0)$, making F_Q empty.

10) **2109, 2106, 2110, and 2112** The right edge of the display screen has not been reached (i.e., $x_{CL} < x_{max}$) and both the Present Span, S_P (i.e., S_1), and the New Span, S_N (i.e., S_2), are valid. Therefore, the Span Interaction Parameters for these two spans are computed, and are found to be: (Far, Non, Non, Near).

11) **2114** The Span Interaction Type is determined from the Span Interaction Parameters, and turns out to be Type 13.

12) **2116 and 2400** Because the Span Interaction Type is Type 13, Rule 3 is invoked to continue processing S_1:

12A) **2402** The Current Portion, S_C (i.e., S_1 between x_{1L} and x_I), is hidden starting at x_{NL} (i.e., x_{2L}), but is visible to the left of x_{NL} (i.e., x_{2L}). Therefore, render the Present Span, S_P (i.e., S_1), from x_{CL} to x_{NL} (i.e., from x_{1L} to x_{2L}).

12B) **2404** The processing of Present Span is completed (at this point), so make S_N (i.e., S_2) into the Present Span, S_P, by setting:

$$S_P = S_N = 2;$$
$$x_{PL} = x_{NL} = x_{2L}; \quad x_{PR} = x_{NR} = x_{2R};$$
$$z_{PL} = z_{NL} = z_{2L}; \quad z_{PR} = z_{NR} = z_{2R};$$
$$d_P = d_N = d_2; \quad i_P = i_N = i_2; \quad \text{(EQ 26)}$$

12C) **2406** The Current Portion, S_C, is set equal to S_N by setting:

$$x_{CL} = x_{NL} = x_{2L}; \quad x_{CR} = x_{NR} = x_{2R};$$
$$z_{CL} = z_{NL} = z_{2L}; \quad z_{CR} = z_{NR} = z_{2R};$$
$$CF = \max(z_{NL}, z_{NR}) = \max(z_{2L}, z_{2R}) = z_{2L}; \quad \text{(EQ 27)}$$

12D) **2408** The Saved Span, S_S, is invalidated. In Step 9C, the values for S_S were set to those of S_1, but these are now labelled invalid.

$$S_S = \text{invalid};$$ (EQ 28)

12E) **2410** There is a new Current Portion, S_C (i.e., S_2), so a search is done for spans that could possibly occlude it. The Span Occluding Test query performs, in each SMCCAM Word **900**:

$$F_{nQ} = F_{nV} \wedge (n \neq S_P) \wedge (x_{nL} < x_{CR}) \wedge (x_{nR} > x_{CL}) \wedge [(z_{nL} < z_{CF}) \vee (z_{nR} < z_{CF})]$$
$$F_{nQ} = F_{nV} \wedge (n \neq 2) \wedge (x_{nL} < x_{2R}) \wedge (x_{nR} > x_{2L}) \wedge [(z_{nL} < z_{2L}) \vee (z_{nR} < z_{2L})]$$ (EQ 29)

This query creates the set of all Potentially Occluding Spans for the Current Portion, S_C (i.e., S_2). The query finds S_0, S_3, and S_4.

12F) **2412 and 2414** Because F_Q is not empty, there must be spans that possibly occlude the Current Portion, S_C (i.e., S_2). Therefore, the first span in F_Q is read and made the New Span, S_N. This read operation sets the following values:

$$S_N = \text{read}(n) = 0;$$
$$x_{NL} = \text{read}(x_L) = x_{0L}; \quad x_{NR} = \text{read}(x_R) = x_{0R};$$
$$z_{NL} = \text{read}(z_L) = z_{0L}; \quad z_{NR} = \text{read}(z_R) = z_{0R};$$
$$d_N = \text{read}(d) = d_0; \quad i_N = \text{read}(i) = i_0;$$ (EQ 30)

Reading the first span in F_Q yields S_0, the first span in the set. The read process also deletes S_0 from F_Q by setting $(F_{0Q} = 0)$.

13) **2109, 2106, 2110, and 2112** The right edge of the display screen has not been reached (i.e., $x_{CL} < x_{max}$) and both the Present Span, S_P (i.e., S_2), and the New Span, S_N (i.e., S_0), are valid. Therefore, the Span Interaction Parameters for these two spans are computed, and are found to be: (Non, Non, Far, Far).

14) **2114** The Span Interaction Type is determined from the Span Interaction Parameters, and turns out to be Type 32.

15) **2116, 2118, and 2600** Because the Span Interaction Type is Type 32, Rule 5 is invoked to process S_2.

15A) **2602 and 2604** The New Span, S_N (i.e., S_0), is hidden within the x axis projection of S_C (i.e., S_2). Therefore, S_N (i.e., S_0) is discarded. Because F_Q is not empty, there must be other spans that possibly occlude the Current Portion, S_C (i.e., S_2). Therefore, the first span in F_Q is read and made the New Span, S_N. This read operation sets the following values:

$$S_N = \text{read}(n) = 3;$$
$$x_{NL} = \text{read}(x_L) = x_{3L}; \quad x_{NR} = \text{read}(x_R) = x_{3R};$$
$$z_{NL} = \text{read}(z_L) = z_{3L}; \quad z_{NR} = \text{read}(z_R) = z_{3R};$$
$$d_N = \text{read}(d) = d_3; \quad i_N = \text{read}(i) = i_3;$$ (EQ 31)

Reading the first span in F_Q yields S_3, the first span in the set. The read process also deletes S_3 from F_Q by setting $(F_{3Q} = 0)$. S_2 is still the Present Span, S_P. The status of the Saved Span, S_S, is left unchanged, thereby remaining invalid.

16) **2109, 2106, 2110, and 2112** The right edge of the display screen has not been reached (i.e., $x_{CL} < x_{max}$) and both the Present Span, S_P (i.e., S_2), and the New Span, S_N (i.e., S_3), are valid. Therefore, the Span Interaction Parameters for these two spans are computed, and are found to be: (Far, Non, Non, Near).

17) **2114** The Span Interaction Type is determined from the Span Interaction Parameters, and turns out to be Type 13.

18) **2116 and 2400** Because the Span Interaction Type is Type 13, Rule 3 is invoked to continue processing S_2.

18A) **2402** The Current Portion, S_C (i.e., S_2), is hidden starting at x_{NL} (i.e., x_{3L}), but is visible to the left of x_{NL} (i.e., x_{3L}). Therefore, render the Present Span, S_P (i.e., S_2), from x_{CL} to x_{NL} (i.e., from x_{2L} to x_{3L}).

18B) **2404** The processing of Present Span is completed, so make the New Span, S_N (i.e., S_3), into the Present Span, S_P, by setting:

$$\begin{aligned} S_P &= S_N = 3; \\ x_{PL} = x_{NL} = x_{3L}; &\qquad x_{PR} = x_{NR} = x_{3R}; \\ z_{PL} = z_{NL} = z_{3L}; &\qquad z_{PR} = z_{NR} = z_{3R}; \\ d_P = d_N = d_3; &\qquad i_P = i_N = i_3; \end{aligned} \tag{EQ 32}$$

18C) **2406** The Current Portion, S_C, is set equal to S_N (i.e., S_3) by setting:

$$\begin{aligned} x_{CL} = x_{NL} = x_{3L}; &\qquad x_{CR} = x_{NR} = x_{3R}; \\ z_{CL} = z_{NL} = z_{3L}; &\qquad z_{CR} = z_{NR} = z_{3R}; \\ {}_{CF} = \max(z_{NL}, z_{NR}) &= \max(z_{3L}, z_{3R}) = z_{3L}; \end{aligned} \tag{EQ 33}$$

18D) **2408** The Saved Span, S_S, is invalidated, but it was already labelled invalid in a previous step.

$$S_S = \text{invalid}; \tag{EQ 34}$$

18E) **2410** There is a new Current Portion, S_C (i.e., S_3), so a search is done for spans that could possibly occlude it. The Span Occluding Test query performs, in each SMCCAM Word **900**:

$$\begin{aligned} F_{nQ} &= F_{nV} \wedge (n \neq S_P) \wedge (x_{nL} < x_{CR}) \wedge (x_{nR} > x_{CL}) \wedge [(z_{nL} < z_{CF}) \vee (z_{nR} < z_{CF})] \\ F_{nQ} &= F_{nV} \wedge (n \neq 3) \wedge (x_{nL} < x_{3R}) \wedge (x_{nR} > x_{3L}) \wedge [(z_{nL} < z_{3L}) \vee (z_{nR} < z_{3L})] \end{aligned} \tag{EQ 35}$$

This query creates the set of all Potentially Occluding Spans for the Current Portion, S_C (i.e., S_3). The query finds only S_0.

18F) **2412 and 2414** Because F_Q is not empty, there must be spans that possibly occlude the Current Portion, S_C (i.e., S_3). Therefore, the first span in F_Q is read and made the New Span, S_N. This read operation sets the following values:

$$\begin{aligned} S_N &= \text{read}(n) = 0; \\ x_{NL} = \text{read}(x_L) = x_{0L}; &\qquad x_{NR} = \text{read}(x_R) = x_{0R}; \\ z_{NL} = \text{read}(z_L) = z_{0L}; &\qquad z_{NR} = \text{read}(z_R) = z_{0R}; \\ d_N = \text{read}(d) = d_0; &\qquad i_N = \text{read}(i) = i_0; \end{aligned} \tag{EQ 36}$$

Reading the first span in F_Q yields S_0, the first (and only) span in the set. The read process also deletes S_0 from F_Q by setting $(F_{0Q} = 0)$. The read process leaves set F_Q empty.

19) **2109, 2106, 2110, and 2112** The right edge of the display screen has not been reached (i.e., $x_{CL} < x_{max}$) and both the Present Span, S_P (i.e., S_3), and the New Span, S_N (i.e., S_0), are valid. Therefore, the Span Interaction Parameters for these two spans are computed, and are found to be: (Non, Non, Far, Far).

20) **2114** The Span Interaction Type is determined from the Span Interaction Parameters, and turns out to be Type 32.

21) **2116, 2118, and 2600** Because the Span Interaction Type is Type 32, Rule 5 is invoked to process S_3.

21A) **2602 and 2606** The New Span, S_N (i.e., S_0), is hidden within the x axis projection of S_C (i.e., S_3). Therefore, S_N (i.e., S_0) is discarded. Because F_Q is empty, there must be no other spans that possibly occlude the Current Portion, S_C (i.e., S_3). Therefore, the New Span, S_N, is labelled as invalid.

$$S_N = \text{invalid}; \tag{EQ 37}$$

A read operation is not done. S_3 is still the Present Span, S_P. The status of the Saved Span, S_S, is left

unchanged, thereby remaining invalid.

22) **2109, 2106, 2110, and 2300** Because the right edge of the display screen has not been reached (i.e., $x_{CL} < x_{max}$) and also because S_P is valid and S_N is not valid, Rule 2 is invoked to continue processing S_3.

22A) **2302** The Current Portion, S_C (i.e., S_3), must be completely visible. Therefore, render the Present Span, S_P (i.e., S_3), from x_{CL} to x_{CR} (i.e., from x_{3L} to x_{3R}).

22B) **2304 and 2306** Because the Saved Span, S_S, is not valid, there was not an intersecting span that reduced the size of S_C. Therefore, a query is done to look for an Abutting Span. The query performs, in each SMCCAM Word **900**:

$$\begin{aligned} F_{nQ} &= F_{nV} \wedge (x_{nL} = x_{CR}) \wedge (z_{nL} = z_{CR}) \\ F_{nQ} &= F_{nV} \wedge (x_{nL} = x_{3R}) \wedge (z_{nL} = z_{3R}) \end{aligned} \qquad \text{(EQ 38)}$$

This query creates the set of all Abutting Spans for the Current Portion, S_C (i.e., S_3) and stores them in F_Q. The query finds only S_7.

22C) **2308 and 2310** Because F_Q is not empty, there must be an Abutting Span. Therefore, the first span in F_Q is read and made the Present Span, S_P. This read operation sets the following values:

$$\begin{aligned} S_P &= \text{read}(n) = 7; \\ x_{PL} = \text{read}(x_L) &= x_{7L}; \quad x_{PR} = \text{read}(x_R) = x_{7R}; \\ z_{PL} = \text{read}(z_L) &= z_{7L}; \quad z_{PR} = \text{read}(z_R) = z_{7R}; \\ d_P = \text{read}(d) &= d_7; \quad i_P = \text{read}(i) = i_7; \end{aligned} \qquad \text{(EQ 39)}$$

Reading the first span in F_Q yields S_7, the first (and only) span in the set. The read process also deletes S_7 from F_Q by setting $(F_{7Q} = 0)$. The read process leaves set F_Q empty.

22D) **2312** The Current Portion, S_C, is set equal to S_P (i.e., S_7) by setting:

$$\begin{aligned} x_{CL} = x_{PL} &= x_{7L}; \quad x_{CR} = x_{PR} = x_{7R}; \\ z_{CL} = z_{PL} &= z_{7L}; \quad z_{CR} = z_{PR} = z_{7R}; \\ CF &= \max(z_{PL}, z_{PR}) = \max(z_{7L}, z_{7R}) = z_{7R}; \end{aligned} \qquad \text{(EQ 40)}$$

This step can be done concurrently with Step 22C.

22E) **2314** There is a new Current Portion, S_C (i.e., S_3), so a search is done for spans that could possibly occlude it. The Span Occluding Test query performs, in each SMCCAM Word **900**:

$$\begin{aligned} F_{nQ} &= F_{nV} \wedge (n \neq S_P) \wedge (x_{nL} < x_{CR}) \wedge (x_{nR} > x_{CL}) \wedge [(z_{nL} < z_{CF}) \vee (z_{nR} < z_{CF})] \\ F_{nQ} &= F_{nV} \wedge (n \neq 7) \wedge (x_{nL} < x_{7R}) \wedge (x_{nR} > x_{7L}) \wedge [(z_{nL} < z_{7R}) \vee (z_{nR} < z_{7R})] \end{aligned} \qquad \text{(EQ 41)}$$

This query creates the set of all Potentially Occluding Spans for the Current Portion, S_C (i.e., S_7). The query does not find any spans, so F_Q is empty.

22F) **2316 and 2318** Because F_Q is empty, there must be no spans that possibly occlude the Current Portion, S_C (i.e., S_7). Therefore, the New Span, S_N, is labelled as invalid.

$$S_N = \text{invalid}; \qquad \text{(EQ 42)}$$

A read operation is not done. S_7 is still the Present Span, S_P.

23) **2109, 2106, 2110, and 2300** Because the right edge of the display screen has not been reached (i.e., $x_{CL} < x_{max}$) and also because S_P is valid and S_N is not valid, Rule 2 is invoked to process S_7.

23A) **2302** The Current Portion, S_C (i.e., S_7), must be completely visible. Therefore, render the Present Span,

S_P (i.e., S_7), from x_{CL} to x_{CR} (i.e., from x_{7L} to x_{7R}).

23B) **2304 and 2306** Because the Saved Span, S_S, is not valid, there was not an intersecting span that reduced the size of S_C. Therefore, a query is done to look for an Abutting Span. The query performs, in each SMCCAM Word **900**:

$$F_{nQ} = F_{nV} \wedge (x_{nL} = x_{CR}) \wedge (z_{nL} = z_{CR})$$
$$F_{nQ} = F_{nV} \wedge (x_{nL} = x_{7R}) \wedge (z_{nL} = z_{7R}) \quad \text{(EQ 43)}$$

This query creates the set of all Abutting Spans for the Current Portion, S_C (i.e., S_7) and stores them in F_Q. The query does not find any spans, so F_Q is empty.

23C) **2308 and 2320** Because F_Q is empty, there is not an Abutting Span. All spans completely to the left of x_{CR} (i.e., x_{7R}) must be hidden, so they are declared invalid. This is done by a query that performs, in each SMCCAM Word **900**:

$$F_{nV} = F_{nV} \wedge (x_{nR} > x_{CR})$$
$$F_{nV}' = F_{nV} \wedge (x_{nR} > x_{7R}) \quad \text{(EQ 44)}$$

This query reduces the number of valid spans by reducing the spans in set F_V from 15 to 10. At this point, F_V consists of S_0, S_1, S_6, and S_8 through S_{14}.

23D) **2322** There is neither a valid Present Span, S_P, nor a valid New Span, S_N. Hence, both are labelled as invalid.

$$S_P = \text{invalid};$$
$$S_N = \text{invalid}; \quad \text{(EQ 45)}$$

23E) **2324** The Current Portion, S_C, is set equal to:

$$x_{CL} = x_{CR} = x_{7R}; \qquad x_{CR} = \text{invalid};$$
$$z_{CL} = \text{infinity}; \qquad z_{CR} = \text{invalid}; \quad \text{(EQ 46)}$$
$$z_{CF} = \text{infinity};$$

24) **2109, 2106, 2108, and 2200** Because the right edge of the display screen has not been reached (i.e., $x_{CL} < x_{max}$) and also because the Present Span, S_P, is not valid and F_V is not empty (indicating that at least some spans are valid), Rule 1 is invoked. Here, Rule 1 is used to search for the frontmost span at x_{7R}. When found, this frontmost span is processed as the Present Span, S_P.

24A) **2202** Do a query to find all valid spans that could affect the current raster line at x_{CL} (i.e., x_{7R}). The query performs, in each SMCCAM Word **900**:

$$F_{nQ} = F_{nV} \wedge (x_{nL} \leq x_{CL})$$
$$F_{nQ} = F_{nV} \wedge (x_{nL} \leq x_{7R}) \quad \text{(EQ 47)}$$

This query finds S_0, S_1, S_6, and S_8. The query operation could include z coordinate comparisons that check for values less than z_{CF}, which has been set to infinity.

24B) **2204 and 2222** Because F_Q is not empty, there must be at least one span that could affect the current raster line at x_{CL} (i.e., x_{7R}). Therefore, the first span in F_Q is read and made the New Span, S_N. This read operation sets the following values:

$$
\begin{aligned}
S_N &= \text{read}(n) = 0; \\
x_{NL} = \text{read}(x_L) &= x_{0L}; \qquad x_{NR} = \text{read}(x_R) = x_{0R}; \\
z_{NL} = \text{read}(z_L) &= z_{0L}; \qquad z_{NR} = \text{read}(z_R) = z_{0R}; \\
d_N = \text{read}(d) &= d_0; \qquad i_N = \text{read}(i) = i_0;
\end{aligned}
\tag{EQ 48}
$$

Reading the first span yields the leftmost span in F_Q, which is S_0. The read process also deletes S_0 from F_Q by setting $(F_{0Q} = 0)$.

24C) **2224** The z value for the New Span, S_N (i.e., S_0), at the x coordinate x_{CL} (i.e., x_{7R}) is computed as z_{temp}.

$$z_{temp} = z_{NL} + (x_{CL} - x_{NL})\, d_N = z_{0L} + (x_{7R} - x_{0L})\, d_0 \tag{EQ 49}$$

The position (x_{7R}, z_{temp}) **2704** is shown in Figure 27.

24D) **2226, 2228, and 2232** The computed value, z_{temp}, is less than z_{CL} (i.e., infinity), so make the New Span, S_N (i.e., S_0), into the Present Span, S_P, by setting:

$$
\begin{aligned}
S_P &= S_N = 0; \\
x_{PL} = x_{NL} &= x_{0L}; \qquad x_{PR} = x_{NR} = x_{0R}; \\
z_{PL} = z_{NL} &= z_{0L}; \qquad z_{PR} = z_{NR} = z_{0R}; \\
d_P = d_N &= d_0; \qquad i_P = i_N = i_0;
\end{aligned}
\tag{EQ 50}
$$

24E) **2236** The Current Portion, S_C, is set to the portion of the Present Span, S_P (i.e., S_0) to the right of x_{CL} (i.e., x_{7R}) by setting:

$$
\begin{aligned}
x_{CR} &= x_{NR} = x_{0R}; \\
z_{CL} = z_{temp}; &\qquad z_{CR} = z_{NR} = z_{0R}; \\
z_{CF} = \max(z_{temp}, z_{NR}) &= \max(z_{temp}, z_{0R}) = z_{temp};
\end{aligned}
\tag{EQ 51}
$$

24F) **2237 and 2238** As stated above, the objective of this part of the method is to find the frontmost span at x_{CL} (i.e., x_{7R}). At x_{CL} (i.e., x_{7R}), the z coordinate of the Present Span, S_P (i.e., S_0), is equal to z_{temp}. Therefore, any span in front of S_P (i.e., S_0) at x_{CL} (i.e., x_{7R}) must be at least partially located closer to the observer than z_{temp}. The set F_Q is not empty and includes all the candidates for frontmost span at x_{CL} (i.e., x_{7R}), and some of these candidates maybe eliminated with the query:

$$
\begin{aligned}
F_{nQ} &= F_{nV} \wedge F_{nQ} \wedge [(z_{nL} < z_{CL}) \vee (z_{nR} < z_{CL})] \\
F_{nQ} &= F_{nV} \wedge F_{nQ} \wedge [(z_{nL} < z_{temp}) \vee (z_{nR} < z_{temp})]
\end{aligned}
\tag{EQ 52}
$$

This query operation keeps candidate spans that have either endpoint z coordinate less than z_{temp}. After the query, F_Q contains only S_8.

A variation of this method could skip this query to save time, but does not delete spans hidden by S_P (i.e., S_0) at x_{CL} (i.e., x_{7R}), and so must read all the spans that could affect the current raster line at x_{CL} (i.e., x_{7R}). If the depth complexity is small, then this variation would yield a net savings. However, for large depth complexity, the time saved by eliminating candidate spans would outweigh the time spent doing the query operations. Hence, the method choice depends on the expected depth complexity of the scene.

To avoid making an a priori assumption about scene complexity, the SMCCAM**800** could include a mechanism for determining the depth complexity at x_{CL} by counting the number of spans in the set F_Q. If the number of spans in F_Q is small, the query operation of this step is skipped, and each span in F_Q is

read. If the number of spans in F_Q is sufficiently large, the query operation is not skipped, and some spans in F_Q are deleted, and a net saving is achieved.

24G) **2204 and 2222** Because F_Q is not empty, there must be at least one more candidate span that could be frontmost at x_{CL} (i.e., x_{7R}). Therefore, the first span in F_Q is read and made the New Span, S_N. This read operation sets the following values:

$$\begin{aligned} S_N &= \text{read}(n) = 8; \\ x_{NL} &= \text{read}(x_L) = x_{8L}; \quad x_{NR} = \text{read}(x_R) = x_{8R}; \\ z_{NL} &= \text{read}(z_L) = z_{8L}; \quad z_{NR} = \text{read}(z_R) = z_{8R}; \\ d_N &= \text{read}(d) = d_8; \quad i_N = \text{read}(i) = i_8; \end{aligned} \tag{EQ 53}$$

Reading the first span yields the only span in F_Q, S_8. The read process also deletes S_8 from F_Q by setting $(F_{8Q} = 0)$. The read process leaves set F_Q empty.

24H) **2224** The z value for the New Span, S_N (i.e., S_8), at the x coordinate x_{CL} (i.e., x_{7R}) is computed as z_{temp}.

$$z_{temp} = z_{NL} + (x_{CL} - x_{NL})\, d_N = z_{8L} + (x_{7R} - x_{8L})\, d_8 \tag{EQ 54}$$

The position (x_{7R}, z_{temp}) **2704** is shown in Figure 27.

24I) **2226, 2228, and 2232** The computed value, z_{temp}, is less than z_{CL} (i.e., z coordinate of S_0 at x_{7R}, computed as z_{temp} in Step 24C), so make the New Span, S_N (i.e., S_8), into the Present Span, S_P, by setting:

$$\begin{aligned} S_P &= S_N = 8; \\ x_{PL} &= x_{NL} = x_{8L}; \quad x_{PR} = x_{NR} = x_{8R}; \\ z_{PL} &= z_{NL} = z_{8L}; \quad z_{PR} = z_{NR} = z_{8R}; \\ d_P &= d_N = d_8; \quad i_P = i_N = i_8; \end{aligned} \tag{EQ 55}$$

24J) **2236** The Current Portion, S_C, is set to the portion of the Present Span, S_P (i.e., S_8) to the right of x_{CL} (i.e., x_{7R}) by setting:

$$\begin{aligned} x_{CR} &= x_{NR} = x_{8R}; \\ z_{CL} &= z_{temp}; \quad z_{CR} = z_{NR} = z_{8R}; \\ z_{CF} &= \max(z_{temp}, z_{NR}) = \max(z_{temp}, z_{8R}) = z_{8R}; \end{aligned} \tag{EQ 56}$$

24K) **2237, 2204, 2206, and 2214** Because F_Q is empty and S_P (i.e., S_8) is valid, a search is done for spans that could possibly occlude the Current Portion, S_C, (i.e., S_8 to the right of x_{7R}). The Span Occluding Test query performs, in each SMCCAM Word **900**:

$$\begin{aligned} F_{nQ} &= F_{nV} \wedge (n \neq S_P) \wedge (x_{nL} < x_{CR}) \wedge (x_{nR} > x_{CL}) \wedge [(z_{nL} < z_{CF}) \vee (z_{nR} < z_{CF})] \\ F_{nQ} &= F_{nV} \wedge (n \neq 8) \wedge (x_{nL} < x_{8R}) \wedge (x_{nR} > x_{7R}) \wedge [(z_{nL} < z_{8R}) \vee (z_{nR} < z_{8R})] \end{aligned} \tag{EQ 57}$$

This query creates the set of all Potentially Occluding Spans for the Current Portion, S_C (i.e., S_8 to the right of x_{7R}). The query finds S_0 and S_9.

24L) **2216 and 2218** Because F_Q is not empty, there must spans that possibly occlude the Current Portion, S_C (i.e., S_8 to the right of x_{7R}). Therefore, the first span in F_Q is read and made the New Span, S_N. This read operation sets the following values:

$$S_N = \text{read}(n) = 0;$$
$$x_{NL} = \text{read}(x_L) = x_{0L}; \quad x_{NR} = \text{read}(x_R) = x_{0R};$$
$$z_{NL} = \text{read}(z_L) = z_{0L}; \quad z_{NR} = \text{read}(z_R) = z_{0R};$$
$$d_N = \text{read}(d) = d_0; \quad i_N = \text{read}(i) = i_0; \qquad \text{(EQ 58)}$$

Reading the first span in F_Q yields S_0, the first span in the set. The read process also deletes S_0 from F_Q by setting $(F_{0Q} = 0)$.

25) **2109, 2106, 2110, and 2112** The right edge of the display screen has not been reached (i.e., $x_{CL} < x_{max}$) and both the Present Span, S_P (i.e., S_8), and the New Span, S_N (i.e., S_0), are valid. Therefore, the Span Interaction Parameters for these two spans are computed, and are found to be: (Non, Non, Far, Near).

26) **2114** The Span Interaction Type is determined from the Span Interaction Parameters, and turns out to be Type 31.

27) **2116, 2118, and 2500** Because the Span Interaction Type is Type 31, Rule 4 invoked to process S_8:

27A) **2502** The Current Portion, S_C (i.e., S_8 to the right of x_{7R}), and the New Span, S_N (i.e., S_0), interact to cause S_C to transition from being visible to being hidden somewhere along its length: at point of intersection between S_C (i.e., S_8 to the right of x_{7R}) and S_N (i.e., S_0). The intersection is computed, and found to be at (x_I, z_I) **2708**, as shown in Figure 27.

27B) **2504** The Current Portion, S_C (i.e., S_8 to the right of x_{7R}), is shortened by changing its right end point to the intersection point. The right endpoint of S_C is modified by setting

$$x_{CR} = x_I;$$
$$z_{CR} = z_I; \qquad \text{(EQ 59)}$$
$$z_{CF} = \max(z_{CL}, z_I) = z_I;$$

The Current Portion, S_C is now the portion of S_8 between x_{7R} and x_I. The Present Span, S_P, is still equal to all of S_8.

27C) **2506** The New Span, S_N (i.e., S_0), is saved as the Saved Span, S_S. This is done because S_S will be needed later if S_P (i.e., S_8) is rendered to x_I.

$$S_S = S_N = 0;$$
$$x_{SL} = x_{NL} = x_{0L}; \quad x_{SR} = x_{NR} = x_{0R};$$
$$z_{SL} = z_{NL} = z_{0L}; \quad z_{SR} = z_{NR} = z_{0R}; \qquad \text{(EQ 60)}$$
$$d_S = d_N = d_0; \quad i_S = i_N = i_0;$$

The portion of the Saved Span, S_S, to the right x_I abuts S_C at the x coordinate x_I. If, later in the method, S_C is rendered to x_I, then S_S will become the Present Span, S_P.

27D) **2508** The Current Portion, S_C (i.e., S_8 from x_{7R} to x_I) has been changed, so a search is done for spans that could possibly occlude it. The Span Occluding Test query performs, in each SMCCAM Word **900**:

$$F_{nQ} = F_{nV} \wedge F_{nQ} \wedge (n \neq S_P) \wedge (x_{nL} < x_{CR}) \wedge (x_{nR} > x_{CL}) \wedge [(z_{nL} < z_{CF}) \vee (z_{nR} < z_{CF})]$$
$$F_{nQ} = F_{nV} \wedge F_{nQ} \wedge (n \neq 8) \wedge (x_{nL} < x_I) \wedge (x_{nR} > x_{7R}) \wedge [(z_{nL} < z_I) \vee (z_{nR} < z_I)] \qquad \text{(EQ 61)}$$

This query creates the set of all Potentially Occluding Spans for the Current Portion, S_C (i.e., S_8 from x_{7R} to x_I). The query does not find any spans; therefore, F_Q is empty.

27E) **2510 and 2514** Because F_Q is empty, there must not be any spans that possibly occlude the Current Portion, S_C (i.e., S_8 from x_{7R} to x_I). Therefore, the New Span, S_N, is declared invalid.

$$S_N = \text{invalid};$$ (EQ 62)

28) **2109, 2106, 2110, and 2300** Because the right edge of the display screen has not been reached (i.e., $x_{CL} < x_{max}$) and also because S_P is valid and S_N is not valid, Rule 2 is invoked to process S_8.

28A) **2302** The Current Portion, S_C (i.e., S_8 from x_{7R} to x_I), must be completely visible. Therefore, render the Present Span, S_P (i.e., S_8), from x_{CL} to x_{CR} (i.e., from x_{7R} to x_I).

28B) **2304 and 2326** Because the Saved Span, S_S, is valid, there was an intersecting span that reduced the size of S_C. Therefore, the Saved Span, S_S (i.e., S_0), is made into the Present Span, S_P, by setting:

$$\begin{aligned} S_P = S_S = 0; \\ x_{PL} = x_{SL} = x_{0L}; \quad x_{PR} = x_{SR} = x_{0R}; \\ z_{PL} = z_{SL} = z_{0L}; \quad z_{PR} = z_{SR} = z_{0R}; \\ d_P = d_S = d_0; \quad i_P = i_S = i_0; \end{aligned}$$ (EQ 63)

28C) **2328** The Current Portion, S_C, is set to the portion of S_P (i.e., S_0) that is to the right of x_I. That way, the Saved Span, S_S (i.e., S_0), acts the same as an Abutting Span for the portion of S_8 to the left of x_I, where the abutting takes place at x_I. This is done by setting:

$$\begin{aligned} x_{CL} = x_{CR} = x_I; \quad x_{CR} = x_{SR} = x_{0R}; \\ z_{CL} = z_{CR} = z_I; \quad z_{CR} = z_{SR} = z_{0R}; \\ z_{CF} = \max(z_{CR}, z_{SR}) = \max(z_I, z_{0R}) = z_I; \end{aligned}$$ (EQ 64)

This step can be done concurrently with Step 28B.

28D) **2330** Because the values for the Saved Span, S_S, have been transferred to the Present Span, S_P, S_S is invalidated.

$$S_S = \text{invalid};$$ (EQ 65)

28E) **2314** There is a new Current Portion, S_C (i.e., S_0 to the right of x_I), so a search is done for spans that could possibly occlude it. The Span Occluding Test query performs, in each SMCCAM Word900:

$$\begin{aligned} F_{nQ} = F_{nV} \wedge (n \neq S_P) \wedge (x_{nL} < x_{CR}) \wedge (x_{nR} > x_{CL}) \wedge [(z_{nL} < z_{CF}) \vee (z_{nR} < z_{CF})] \\ F_{nQ} = F_{nV} \wedge (n \neq 0) \wedge (x_{nL} < x_{0R}) \wedge (x_{nR} > x_I) \wedge [(z_{nL} < z_I) \vee (z_{nR} < z_I)] \end{aligned}$$ (EQ 66)

This query creates the set of all Potentially Occluding Spans for the Current Portion, S_C (i.e., S_0 to the right of x_I). The query finds S_8 through S_{13}.

28F) **2316 and 2332** Because F_Q is not empty, there must spans that possibly occlude the Current Portion, S_C (i.e., S_0 to the right of x_I). Therefore, the first span in F_Q is read and made the New Span, S_N. This read operation sets the following values:

$$\begin{aligned} S_N = \text{read}(n) = 8; \\ x_{NL} = \text{read}(x_L) = x_{8L}; \quad x_{NR} = \text{read}(x_R) = x_{8R}; \\ z_{NL} = \text{read}(z_L) = z_{8L}; \quad z_{NR} = \text{read}(z_R) = z_{8R}; \\ d_N = \text{read}(d) = d_8; \quad i_N = \text{read}(i) = i_8; \end{aligned}$$ (EQ 67)

Reading the first span in F_Q yields S_8, the first span in the set. The read process also deletes S_8 from F_Q by setting $(F_{8Q} = 0)$.

An alternate method could include a way to prevent the left side partner of an intersecting span pair from

being considered as a Potentially Occluding Span of the right side partner. This could be done with an additional query of an SMCCAM **800** that can do two simultaneous inequality tests on n in each SMCCAM Word **900**.

29) **2109, 2106, 2110, and 2112** The right edge of the display screen has not been reached (i.e., $x_{CL} < x_{max}$) and both the Present Span, S_P (i.e., S_0), and the New Span, S_N (i.e., S_8), are valid. Therefore, the Span Interaction Parameters for these two spans are computed, and are found to be: (Non, Near, Equal, Non).

30) **2114** The Span Interaction Type is determined from the Span Interaction Parameters, and turns out to be Type 21.

31) **2116, 2118, and 2600** Because the Span Interaction Type is Type 21, Rule 5 is invoked to process S_0.

31A) **2602 and 2604** The New Span, S_N (i.e., S_8), is hidden within the x axis projection of S_C (i.e., S_0 to the right of x_I). Therefore, S_N (i.e., S_8) is discarded. Because F_Q is not empty, there must be other spans that possibly occlude the Current Portion, S_C (i.e., S_0 to the right of x_I). Therefore, the first span in F_Q is read and made the New Span, S_N. This read operation sets the following values:

$$\begin{aligned} S_N &= \text{read}(n) = 9; \\ x_{NL} &= \text{read}(x_L) = x_{9L}; \quad & x_{NR} &= \text{read}(x_R) = x_{9R}; \\ z_{NL} &= \text{read}(z_L) = z_{9L}; \quad & z_{NR} &= \text{read}(z_R) = z_{9R}; \\ d_N &= \text{read}(d) = d_9; \quad & i_N &= \text{read}(i) = i_9; \end{aligned} \tag{EQ 68}$$

Reading the first span in F_Q yields S_9, the first span in the set. The read process also deletes S_9 from F_Q by setting $(F_{9Q} = 0)$. S_0 is still the Present Span, S_P. The status of the Saved Span, S_S, is left unchanged, thereby remaining invalid.

32) **2109, 2106, 2110, and 2112** The right edge of the display screen has not been reached (i.e., $x_{CL} < x_{max}$) and both the Present Span, S_P (i.e., S_0), and the New Span, S_N (i.e., S_9), are valid. Therefore, the Span Interaction Parameters for these two spans are computed, and are found to be: (Far, Far, Non, Non).

33) **2114** The Span Interaction Type is determined from the Span Interaction Parameters, and turns out to be Type 5.

34) **2116 and 2400** Because the Span Interaction Type is Type 5, Rule 3 is invoked to continue processing S_0.

34A) **2402** The Current Portion, S_C (i.e., S_0 to the right of x_I), is hidden starting at x_{NL} (i.e., x_{9L}), but is visible to the left of x_{NL} (i.e., x_{9L}). Therefore, render the Present Span, S_P (i.e., S_0), from x_{CL} to x_{NL} (i.e., from x_I to x_{9L}).

34B) **2404** The processing of Present Span is completed (for now), so make the New Span, S_N (i.e., S_3), into the Present Span, S_P, by setting:

$$\begin{aligned} S_P &= S_N = 9; \\ x_{PL} &= x_{NL} = x_{9L}; \quad & x_{PR} &= x_{NR} = x_{9R}; \\ z_{PL} &= z_{NL} = z_{9L}; \quad & z_{PR} &= z_{NR} = z_{9R}; \\ d_P &= d_N = d_9; \quad & i_P &= i_N = i_9; \end{aligned} \tag{EQ 69}$$

34C) **2406** The Current Portion, S_C, is set equal to S_N (i.e., S_9) by setting:

$$\begin{aligned} x_{CL} &= x_{NL} = x_{9L}; \quad & x_{CR} &= x_{NR} = x_{9R}; \\ z_{CL} &= z_{NL} = z_{9L}; \quad & z_{CR} &= z_{NR} = z_{9R}; \\ \text{CF} &= \max(z_{NL}, z_{NR}) = \max(z_{3L}, z_{3R}) = z_{9L}; \end{aligned} \tag{EQ 70}$$

34D) **2408** The Saved Span, S_S, is invalidated, but it was already labelled invalid.

$$S_S = \text{invalid}; \tag{EQ 71}$$

34E) **2410** There is a new Current Portion, S_C (i.e., S_9), so a search is done for spans that could possibly occlude it. The Span Occluding Test query performs, in each SMCCAM Word **900**:

$$F_{nQ} = F_{nV} \wedge (n \neq S_P) \wedge (x_{nL} < x_{CR}) \wedge (x_{nR} > x_{CL}) \wedge [(z_{nL} < z_{CF}) \vee (z_{nR} < z_{CF})]$$
$$F_{nQ} = F_{nV} \wedge (n \neq 9) \wedge (x_{nL} < x_{9R}) \wedge (x_{nR} > x_{9L}) \wedge [(z_{nL} < z_{9L}) \vee (z_{nR} < z_{9L})] \quad \text{(EQ 72)}$$

This query creates the set of all Potentially Occluding Spans for the Current Portion, S_C (i.e., S_{11}). The query does not find any spans, so F_Q is empty.

34F) **2412 and 2416** Because F_Q is empty, there must be no other spans that possibly occlude the Current Portion, S_C (i.e., S_9). Therefore, the New Span, S_N, is labelled as invalid.

$$S_N = \text{invalid}; \quad \text{(EQ 73)}$$

A read operation is not done. S_9 is still the Present Span, S_P. The status of the Saved Span, S_S, is left unchanged, thereby remaining invalid.

35) **2109, 2106, 2110, and 2300** Because the right edge of the display screen has not been reached (i.e., $x_{CL} < x_{max}$) and also because S_P is valid and S_N is not valid, Rule 2 is invoked to process S_9.

35A) **2302** The Current Portion, S_C (i.e., S_9), must be completely visible. Therefore, render the Present Span, S_P (i.e., S_9), from x_{CL} to x_{CR} (i.e., from x_{9L} to x_{9R}).

35B) **2304 and 2306** Because the Saved Span, S_S, is not valid, there was not an intersecting span that reduced the size of S_C. Therefore, a query is done to look for an Abutting Span. The query performs, in each SMCCAM Word **900**:

$$F_{nQ} = F_{nV} \wedge (x_{nL} = x_{CR}) \wedge (z_{nL} = z_{CR})$$
$$F_{nQ} = F_{nV} \wedge (x_{nL} = x_{9R}) \wedge (z_{nL} = z_{9R}) \quad \text{(EQ 74)}$$

This query creates the set of all Abutting Spans for the Current Portion, S_C (i.e., S_3) and stores them in F_Q. The query finds only S_{11}.

35C) **2308 and 2310** Because F_Q is not empty, there must be an Abutting Span. Therefore, the first span in F_Q is read and made the Present Span, S_P. This read operation sets the following values:

$$S_P = \text{read}(n) = 11;$$
$$x_{PL} = \text{read}(x_L) = x_{11L}; \qquad x_{PR} = \text{read}(x_R) = x_{11R};$$
$$z_{PL} = \text{read}(z_L) = z_{11L}; \qquad z_{PR} = \text{read}(z_R) = z_{11R}; \quad \text{(EQ 75)}$$
$$d_P = \text{read}(d) = d_{11}; \qquad i_P = \text{read}(i) = i_{11};$$

Reading the first span in F_Q yields S_{11}, the first (and only) span in the set. The read process also deletes S_{11} from F_Q by setting $(F_{11Q} = 0)$. The read process leaves set F_Q empty.

35D) **2312** The Current Portion, S_C, is set equal to S_P (i.e., S_{11}) by setting:

$$x_{CL} = x_{PL} = x_{11L}; \qquad x_{CR} = x_{PR} = x_{11R};$$
$$z_{CL} = z_{PL} = z_{11L}; \qquad z_{CR} = z_{PR} = z_{11R}; \quad \text{(EQ 76)}$$
$$_{CF} = \max(z_{PL}, z_{11R}) = \max(z_{11L}, z_{11R}) = z_{11R};$$

35E) **2314** There is a new Current Portion, S_C (i.e., S_{11}), so a search is done for spans that could possibly occlude it. The Span Occluding Test query performs, in each SMCCAM Word **900**:

$$F_{nQ} = F_{nV} \wedge (n \neq S_P) \wedge (x_{nL} < x_{CR}) \wedge (x_{nR} > x_{CL}) \wedge [(z_{nL} < z_{CF}) \vee (z_{nR} < z_{CF})]$$
$$F_{nQ} = F_{nV} \wedge (n \neq 11) \wedge (x_{nL} < x_{11R}) \wedge (x_{nR} > x_{11L}) \wedge [(z_{nL} < z_{11R}) \vee (z_{nR} < z_{11R})] \quad \text{(EQ 77)}$$

This query creates the set of all Potentially Occluding Spans for the Current Portion, S_C (i.e., S_7). The query finds only S_0.

35F) **2316 and 2432** Because F_Q is not empty, there must be spans that possibly occlude the Current Portion, S_C (i.e., S_{11}). Therefore, the first span in F_Q is read and made the New Span, S_N. This read operation sets the following values:

$$S_N = \text{read}(n) = 0;$$
$$x_{NL} = \text{read}(x_L) = x_{0L}; \qquad x_{NR} = \text{read}(x_R) = x_{0R};$$
$$z_{NL} = \text{read}(z_L) = z_{0L}; \qquad z_{NR} = \text{read}(z_R) = z_{0R};$$
$$d_N = \text{read}(d) = d_0; \qquad i_N = \text{read}(i) = i_0; \quad \text{(EQ 78)}$$

Reading the first span in F_Q yields S_0, the first (and only) span in the set. The read process also deletes S_0 from F_Q by setting $(F_{0Q} = 0)$. The read process leaves set F_Q empty.

36) **2109, 2106, 2110, and 2112** The right edge of the display screen has not been reached (i.e., $x_{CL} < x_{max}$) and both the Present Span, S_P (i.e., S_{11}), and the New Span, S_N (i.e., S_0), are valid. Therefore, the Span Interaction Parameters for these two spans are computed, and are found to be: (Non, Non, Far, Far).

37) **2114** The Span Interaction Type is determined from the Span Interaction Parameters, and turns out to be Type 32.

38) **2116, 2118, and 2600** Because the Span Interaction Type is Type 32, Rule 5 is invoked to process S_{11}.

38A) **2602 and 2606** The New Span, S_N (i.e., S_0), is hidden within the x axis projection of S_C (i.e., S_{11}). Therefore, S_N (i.e., S_0) is discarded. Because F_Q is empty, there must be no other spans that possibly occlude the Current Portion, S_C (i.e., S_{11}). Therefore, the New Span, S_N, is labelled as invalid.

$$S_N = \text{invalid}; \quad \text{(EQ 79)}$$

A read operation is not done. S_3 is still the Present Span, S_P. The status of the Saved Span, S_S, is left unchanged, thereby remaining invalid.

39) **2109, 2106, 2110, and 2300** Because the right edge of the display screen has not been reached (i.e., $x_{CL} < x_{max}$) and also because S_P is valid and S_N is not valid, Rule 2 is invoked to continue processing S_{11}.

39A) **2302** The Current Portion, S_C (i.e., S_{11}), must be completely visible. Therefore, render the Present Span, S_P (i.e., S_{11}), from x_{CL} to x_{CR} (i.e., from x_{11L} to x_{11R}).

39B) **2304 and 2306** Because the Saved Span, S_S, is not valid, there was not an intersecting span that reduced the size of S_C. Therefore, a query is done to look for an Abutting Span. The query performs, in each SMCCAM Word **900**:

$$F_{nQ} = F_{nV} \wedge (x_{nL} = x_{CR}) \wedge (z_{nL} = z_{CR})$$
$$F_{nQ} = F_{nV} \wedge (x_{nL} = x_{11R}) \wedge (z_{nL} = z_{11R}) \quad \text{(EQ 80)}$$

This query creates the set of all Abutting Spans for the Current Portion, S_C (i.e., S_{11}) and stores them in F_Q. The query does not find any spans, so F_Q is empty.

39C) **2308 and 2320** Because F_Q is empty, there is not an Abutting Span. All spans completely to the left of x_{CR} (i.e., x_{11R}) must be hidden, so they are declared invalid. This is done by a query that performs, in each SMCCAM Word **900**:

$$F_{nV} = F_{nV} \wedge (x_{nR} > x_{CR})$$
$$F_{nV} = F_{nV} \wedge (x_{nR} > x_{11R}) \quad \text{(EQ 81)}$$

This query reduces the number of valid spans by reducing the spans in set F_V from 10 to 5. At this point, F_V consists of S_0, S_1, and S_{12} through S_{14}.

39D) **2322** There is neither a valid Present Span, S_P, nor a valid New Span, S_N. Hence, both are labelled as invalid.

$$S_P = \text{invalid};$$
$$S_N = \text{invalid}; \quad \text{(EQ 82)}$$

39E) **2324** The Current Portion, S_C, is set equal to:

$$x_{CL} = x_{CR} = x_{11R}; \qquad x_{CR} = \text{invalid};$$
$$z_{CL} = \text{infinity}; \qquad z_{CR} = \text{invalid}; \quad \text{(EQ 83)}$$
$$z_{CF} = \text{infinity};$$

40) **2109, 2106, 2108, and 2200** Because the right edge of the display screen has not been reached (i.e., $x_{CL} < x_{max}$) and also because the Present Span, S_P, is not valid and F_V is not empty (indicating that at least some spans are valid), Rule 1 is invoked. Here, Rule 1 is used to search for the frontmost span at x_{11R}. When found, this frontmost span is processed as the Present Span, S_P.

40A) **2202** Do a query to find all valid spans that could affect the current raster line at x_{CL} (i.e., x_{11R}). The query performs, in each SMCCAM Word **900**:

$$F_{nQ} = F_{nV} \wedge (x_{nL} \le x_{CL})$$
$$F_{nQ} = F_{nV} \wedge (x_{nL} \le x_{11R}) \quad \text{(EQ 84)}$$

This query finds S_0, S_1, and S_{12}. The query operation could include z coordinate comparisons that check for values less than z_{CF}, which has been set to infinity.

40B) **2204 and 2222** Because F_Q is not empty, there must be at least one span that could affect the current raster line at x_{CL} (i.e., x_{11R}). Therefore, the first span in F_Q is read and made the New Span, S_N. This read operation sets the following values:

$$S_N = \text{read}(n) = 0;$$
$$x_{NL} = \text{read}(x_L) = x_{0L}; \qquad x_{NR} = \text{read}(x_R) = x_{0R};$$
$$z_{NL} = \text{read}(z_L) = z_{0L}; \qquad z_{NR} = \text{read}(z_R) = z_{0R}; \quad \text{(EQ 85)}$$
$$d_N = \text{read}(d) = d_0; \qquad i_N = \text{read}(i) = i_0;$$

Reading the first span yields the leftmost span in F_Q, which is S_0. The read process also deletes S_0 from F_Q by setting $(F_{0Q} = 0)$.

40C) **2224** The z value for the New Span, S_N (i.e., S_0), at the x coordinate x_{CL} (i.e., x_{11R}) is computed as z_{temp}.

$$z_{temp} = z_{NL} + (x_{CL} - x_{NL})\, d_N = z_{0L} + (x_{11R} - x_{0L})\, d_0 \quad \text{(EQ 86)}$$

The position (x_{11R}, z_{temp}) **2710** is shown in Figure 27.

40D) **2226, 2228, and 2232** The computed value, z_{temp}, is less than z_{CL} (i.e., infinity), so make the New Span, S_N (i.e., S_0), into the Present Span, S_P, by setting:

$$S_P = S_N = 0;$$
$$x_{PL} = x_{NL} = x_{0L}; \quad x_{PR} = x_{NR} = x_{0R};$$
$$z_{PL} = z_{NL} = z_{0L}; \quad z_{PR} = z_{NR} = z_{0R};$$
$$d_P = d_N = d_0; \quad i_P = i_N = i_0; \qquad \text{(EQ 87)}$$

40E) **2236** The Current Portion, S_C, is set to the portion of the Present Span, S_P (i.e., S_0) to the right of x_{CL} (i.e., x_{11R}) by setting:

$$x_{CR} = x_{NR} = x_{0R};$$
$$z_{CL} = z_{temp}; \quad z_{CR} = z_{NR} = z_{0R};$$
$$_{CF} = \max(z_{temp}, z_{NR}) = \max(z_{temp}, z_{0R}) = z_{temp}; \qquad \text{(EQ 88)}$$

40F) **2237 and 2238** As stated above, the objective of this part of the method is to find the frontmost span at x_{CL} (i.e., x_{11R}). At x_{CL} (i.e., x_{11R}), the z coordinate of the Present Span, S_P (i.e., S_0), is equal to z_{temp}. Therefore, any span in front of S_P (i.e., S_0) at x_{CL} (i.e., x_{11R}) must be at least partially located closer to the observer than z_{temp}. The set F_Q is not empty and includes all the candidates for frontmost span at x_{CL} (i.e., x_{11R}), and some of these candidates maybe eliminated with the query:

$$F_{nQ} = F_{nV} \wedge F_{nQ} \wedge [(z_{nL} < z_{CL}) \vee (z_{nR} < z_{CL})]$$
$$F_{nQ} = F_{nV} \wedge F_{nQ} \wedge [(z_{nL} < z_{temp}) \vee (z_{nR} < z_{temp})] \qquad \text{(EQ 89)}$$

This query operation keeps candidate spans that has either endpoint z coordinate less than z_{temp}. After the query, F_Q is empty.

40G) **2204, 2206, and 2214** Because F_Q is empty, there can not be any other candidate span that could frontmost at x_{CL} (i.e., x_{11R}). Also, because S_P (i.e., S_0) is valid, perform, in each SMCCAM Word **900**, the Span Occluding Test query:

$$F_{nQ} = F_{nV} \wedge (n \neq S_P) \wedge (x_{nL} < x_{CR}) \wedge (x_{nR} > x_{CL}) \wedge [(z_{nL} < z_{CF}) \vee (z_{nR} < z_{CF})]$$
$$F_{nQ} = F_{nV} \wedge (n \neq 0) \wedge (x_{nL} < x_{0R}) \wedge (x_{nR} > x_{11R}) \wedge [(z_{nL} < z_{temp}) \vee (z_{nR} < z_{temp})] \qquad \text{(EQ 90)}$$

This query creates the set of all Potentially Occluding Spans for the Current Portion, S_C (i.e., S_0 to the right of x_{11R}). The query does not find any spans, and F_Q is empty.

40H) **2216 and 2220** Because F_Q is empty, there must be no other spans that possibly occlude the Current Portion, S_C (i.e., S_0 to the right of x_{11R}). Therefore, the New Span, S_N, is labelled as invalid.

$$S_N = \text{invalid}; \qquad \text{(EQ 91)}$$

A read operation is not done. S_0 is still the Present Span, S_P. The status of the Saved Span, S_S, is left unchanged, thereby remaining invalid.

41) **2109, 2106, 2110, and 2300** Because the right edge of the display screen has not been reached (i.e., $x_{CL} < x_{max}$) and also because S_P is valid and S_N is not valid, Rule 2 is invoked to process S_0.

41A) **2302** The Current Portion, S_C (i.e., S_0 to the right of x_{11R}), must be completely visible. Therefore, render the Present Span, S_P (i.e., S_0), from x_{CL} to x_{CR} (i.e., from x_{11R} to x_{0R}).

41B) **2304 and 2306** Because the Saved Span, S_S, is not valid, there was not an intersecting span that reduced the size of S_C. Therefore, a query is done to look for an Abutting Span. The query performs, in each SMCCAM Word **900**:

$$F_{nQ} = F_{nV} \wedge (x_{nL} = x_{CR}) \wedge (z_{nL} = z_{CR})$$
$$F_{nQ} = F_{nV} \wedge (x_{nL} = x_{0R}) \wedge (z_{nL} = z_{0R}) \quad \text{(EQ 92)}$$

This query creates the set of all Abutting Spans for the Current Portion, S_C (i.e., S_0) and stores them in F_Q. The query does not find any spans, so F_Q is empty.

41C) **2308 and 2320** Because F_Q is empty, there is not an Abutting Span. All spans completely to the left of x_{CR} (i.e., x_{0R}) must be hidden, so they are declared invalid. This is done by a query that performs, in each SMCCAM Word **900**:

$$F_{nV} = F_{nV} \wedge (x_{nR} > x_{CR})$$
$$F_{nV} = F_{nV} \wedge (x_{nR} > x_{0R}) \quad \text{(EQ 93)}$$

This query reduces the number of valid spans by reducing the spans in set F_V from 10 to 2. At this point, F_V consists of S_1 and S_{14}.

41D) **2322** There is neither a valid Present Span, S_P, nor a valid New Span, S_N. Hence, both are labelled as invalid.

$$S_P = \text{invalid};$$
$$S_N = \text{invalid}; \quad \text{(EQ 94)}$$

41E) **2324** The Current Portion, S_C, is set equal to:

$$x_{CL} = x_{CR} = x_{0R}; \qquad x_{CR} = \text{invalid};$$
$$z_{CL} = \text{infinity}; \qquad z_{CR} = \text{invalid}; \quad \text{(EQ 95)}$$
$$z_{CF} = \text{infinity};$$

42) **2109, 2106, 2108, and 2200** Because the right edge of the display screen has not been reached (i.e., $x_{CL} < x_{max}$) and also because the Present Span, S_P, is not valid and F_V is not empty (indicating that at least some spans are valid), Rule 1 is invoked. Here, Rule 1 is used to search for the frontmost span at x_{0R}. When found, this frontmost span is processed as the Present Span, S_P.

42A) **2202** Do a query to find all valid spans that could affect the current raster line at x_{CL} (i.e., x_{0R}). The query performs, in each SMCCAM Word **900**:

$$F_{nQ} = F_{nV} \wedge (x_{nL} \leq x_{CL})$$
$$F_{nQ} = F_{nV} \wedge (x_{nL} \leq x_{0R}) \quad \text{(EQ 96)}$$

This query finds S_1 and S_{14}. The query operation could include z coordinate comparisons that check for values less than z_{CF}, which has been set to infinity.

42B) **2204 and 2222** Because F_Q is not empty, there must be at least one span that could affect the current raster line at x_{CL} (i.e., x_{0R}). Therefore, the first span in F_Q is read and made the New Span, S_N. This read operation sets the following values:

$$S_N = \text{read}(n) = 1;$$
$$x_{NL} = \text{read}(x_L) = x_{1L}; \qquad x_{NR} = \text{read}(x_R) = x_{1R};$$
$$z_{NL} = \text{read}(z_L) = z_{1L}; \qquad z_{NR} = \text{read}(z_R) = z_{1R}; \quad \text{(EQ 97)}$$
$$d_N = \text{read}(d) = d_1; \qquad i_N = \text{read}(i) = i_1;$$

Reading the first span yields the leftmost span in F_Q, which is S_1. The read process also deletes S_1 from

F_Q by setting $(F_{1Q} = 0)$.

42C) **2224** The z value for the New Span, S_N (i.e., S_1), at the x coordinate x_{CL} (i.e., x_{0R}) is computed as z_{temp}.

$$z_{temp} = z_{NL} + (x_{CL} - x_{NL})\, d_N = z_{1L} + (x_{0R} - x_{1L})\, d_1 \qquad \text{(EQ 98)}$$

The position (x_{0R}, z_{temp}) **2712** is shown in Figure 27.

42D) **2226, 2228, and 2232** The computed value, z_{temp}, is less than z_{CL} (i.e., infinity), so make the New Span, S_N (i.e., S_1), into the Present Span, S_P, by setting:

$$\begin{aligned} S_P &= S_N = 1; \\ x_{PL} &= x_{NL} = x_{1L}; \quad x_{PR} = x_{NR} = x_{1R}; \\ z_{PL} &= z_{NL} = z_{1L}; \quad z_{PR} = z_{NR} = z_{1R}; \\ d_P &= d_N = d_1; \quad i_P = i_N = i_1; \end{aligned} \qquad \text{(EQ 99)}$$

42E) **2236** The Current Portion, S_C, is set to the portion of the Present Span, S_P (i.e., S_1) to the right of x_{CL} (i.e., x_{0R}) by setting:

$$\begin{aligned} x_{CR} &= x_{NR} = x_{1R}; \\ z_{CL} &= z_{temp}; \quad z_{CR} = z_{NR} = z_{1R}; \\ CF &= \max(z_{temp}, z_{NR}) = \max(z_{temp}, z_{1R}) = z_{temp}; \end{aligned} \qquad \text{(EQ 100)}$$

42F) **2237 and 2238** As stated above, the objective of this part of the method is to find the frontmost span at x_{CL} (i.e., x_{0R}). At x_{CL} (i.e., x_{0R}), the z coordinate of the Present Span, S_P (i.e., S_1), is equal to z_{temp}. Therefore, any span in front of S_P (i.e., S_1) at x_{CL} (i.e., x_{0R}) must be at least partially located closer to the observer than z_{temp}. The set F_Q is not empty and includes all the candidates for frontmost span at x_{CL} (i.e., x_{0R}), and some of these candidates maybe eliminated with the query:

$$\begin{aligned} F_{nQ} &= F_{nV} \wedge F_{nQ} \wedge [(z_{nL} < z_{CL}) \vee (z_{nR} < z_{CL})] \\ F_{nQ} &= F_{nV} \wedge F_{nQ} \wedge [(z_{nL} < z_{temp}) \vee (z_{nR} < z_{temp})] \end{aligned} \qquad \text{(EQ 101)}$$

This query operation keeps candidate spans that has either endpoint z coordinate less than z_{temp}. After the query, F_Q contains only S_{14}, and did not get changed by the query.

42G) **2204 and 2222** Because F_Q is not empty, there must be at least one more candidate span that could be frontmost at x_{CL} (i.e., x_{0R}). Therefore, the first span in F_Q is read and made the New Span, S_N. This read operation sets the following values:

$$\begin{aligned} S_N &= \text{read}(n) = 14; \\ x_{NL} &= \text{read}(x_L) = x_{14L}; \quad x_{NR} = \text{read}(x_R) = x_{14R}; \\ z_{NL} &= \text{read}(z_L) = z_{14L}; \quad z_{NR} = \text{read}(z_R) = z_{14R}; \\ d_N &= \text{read}(d) = d_{14}; \quad i_N = \text{read}(i) = i_{14}; \end{aligned} \qquad \text{(EQ 102)}$$

Reading the first span yields the only span in F_Q, that is, S_{14}. The read process also deletes S_{14} from F_Q by setting $(F_{14Q} = 0)$. The read process leaves set F_Q empty.

42H) **2224** The z value for the New Span, S_N (i.e., S_{14}), at the x coordinate x_{CL} (i.e., x_{0R}) is computed as z_{temp}.

$$z_{temp} = z_{NL} + (x_{CL} - x_{NL})\, d_N = z_{14L} + (x_{0R} - x_{14L})\, d_{14} \qquad \text{(EQ 103)}$$

The position (x_{0R}, z_{temp}) **2714** is shown in Figure 27.

42I) **2226, 2228, and 2232** The computed value, z_{temp}, is less than z_{CL} (i.e., z coordinate of S_0 at x_{7R}, computed as z_{temp} in Step 42C), so make the New Span, S_N (i.e., S_{14}), into the Present Span, S_P, by setting:

$$\begin{aligned} S_P &= S_N = 14; \\ x_{PL} &= x_{NL} = x_{14L}; \quad x_{PR} = x_{NR} = x_{14R}; \\ z_{PL} &= z_{NL} = z_{14L}; \quad z_{PR} = z_{NR} = z_{14R}; \\ d_P &= d_N = d_{14}; \quad i_P = i_N = i_{14}; \end{aligned} \tag{EQ 104}$$

42J) **2236** The Current Portion, S_C, is set to the portion of the Present Span, S_P (i.e., S_{14}) to the right of x_{CL} (i.e., x_{0R}) by setting:

$$\begin{aligned} x_{CR} &= x_{NR} = x_{14R}; \\ z_{CL} &= z_{temp}; \quad z_{CR} = z_{NR} = z_{14R}; \\ CF &= \max(z_{temp}, z_{NR}) = \max(z_{temp}, z_{14R}) = z_{14R}; \end{aligned} \tag{EQ 105}$$

Because, as shown Figure 27, z_{temp} is equal to z_{CL}, the max( ) function could choose either z_{temp} or z_{CL}.

42K) **2237, 2204, 2206, and 2214** Because F_Q is empty and S_P (i.e., S_{14}) is valid, a search is done for spans that could possibly occlude the Current Portion, S_C, (i.e., S_{14} to the right of x_{0R}). The Span Occluding Test query performs, in each SMCCAM Word **900**:

$$\begin{aligned} F_{nQ} &= F_{nV} \wedge (n \neq S_P) \wedge (x_{nL} < x_{CR}) \wedge (x_{nR} > x_{CL}) \wedge [(z_{nL} < z_{CF}) \vee (z_{nR} < z_{CF})] \\ F_{nQ} &= F_{nV} \wedge (n \neq 8) \wedge (x_{nL} < x_{14R}) \wedge (x_{nR} > z_{temp}) \wedge [(z_{nL} < z_{14R}) \vee (z_{nR} < z_{14R})] \end{aligned} \tag{EQ 106}$$

This query creates the set of all Potentially Occluding Spans for the Current Portion, S_C (i.e., S_{14} to the right of x_{0R}). The query does not find any spans, so F_Q is empty.

42L) **2216 and 2220** Because F_Q is empty, there must be no other spans that possibly occlude the Current Portion, S_C (i.e., S_{14} to the right of x_{0R}). Therefore, the New Span, S_N, is labelled as invalid.

$$S_N = \text{invalid}; \tag{EQ 107}$$

A read operation is not done. S_{14} is still the Present Span, S_P. The status of the Saved Span, S_S, is left unchanged, thereby remaining invalid.

43) **2109, 2106, 2110, and 2300** Because the right edge of the display screen has not been reached (i.e., $x_{CL} < x_{max}$) and also because S_P is valid and S_N is not valid, Rule 2 is invoked to process S_{14}.

43A) **2302** The Current Portion, S_C (i.e., S_{14} to the right of x_{0R}), must be completely visible. Therefore, render the Present Span, S_P (i.e., S_{14}), from x_{CL} to x_{CR} (i.e., from x_{0R} to x_{14R}).

43B) **2304 and 2306** Because the Saved Span, S_S, is not valid, there was not an intersecting span that reduced the size of S_C. Therefore, a query is done to look for an Abutting Span. The query performs, in each SMCCAM Word **900**:

$$\begin{aligned} F_{nQ} &= F_{nV} \wedge (x_{nL} = x_{CR}) \wedge (z_{nL} = z_{CR}) \\ F_{nQ} &= F_{nV} \wedge (x_{nL} = x_{14R}) \wedge (z_{nL} = z_{14R}) \end{aligned} \tag{EQ 108}$$

This query creates the set of all Abutting Spans for the Current Portion, S_C (i.e., S_{14} to the right of x_{0R}) and stores them in F_Q. The query does not find any spans, so F_Q is empty.

43C) **2308 and 2320** Because F_Q is empty, there is not an Abutting Span. All spans completely to the left of x_{CR} (i.e., x_{14R}) must be hidden, so they are declared invalid. This is done by a query that performs, in

each SMCCAM Word **900**:

$$F_{nV} = F_{nV} \wedge (x_{nR} > x_{CR})$$
$$F_{nV} = F_{nV} \wedge (x_{nR} > x_{14R}) \quad \text{(EQ 109)}$$

This query reduces the number of valid spans by reducing the spans in set F_V from 2 to 1. At this point, F_V consists only of S_1.

43D) **2322** There is neither a valid Present Span, S_P, nor a valid New Span, S_N. Hence, both are labelled as invalid.

$$S_P = \text{invalid};$$
$$S_N = \text{invalid}; \quad \text{(EQ 110)}$$

43E) **2324** The Current Portion, S_C, is set equal to:

$$x_{CL} = x_{CR} = x_{14R}; \qquad x_{CR} = \text{invalid};$$
$$z_{CL} = \text{infinity}; \qquad z_{CR} = \text{invalid}; \quad \text{(EQ 111)}$$
$$z_{CF} = \text{infinity};$$

44) **2109, 2106, 2108, and 2200** Because the right edge of the display screen has not been reached (i.e., $x_{CL} < x_{max}$) and also because the Present Span, S_P, is not valid and F_V is not empty (indicating that at least some spans are valid), Rule 1 is invoked. Here, Rule 1 is used to search for the frontmost span at x_{14R}. When found, this frontmost span is processed as the Present Span, S_P.

44A) **2202** Do a query to find all valid spans that could affect the current raster line at x_{CL} (i.e., x_{14R}). The query performs, in each SMCCAM Word **900**:

$$F_{nQ} = F_{nV} \wedge (x_{nL} \le x_{CL})$$
$$F_{nQ} = F_{nV} \wedge (x_{nL} \le x_{14R}) \quad \text{(EQ 112)}$$

This query finds only S_1. The query operation could include z coordinate comparisons that check for values less than z_{CF}, which has been set to infinity.

44B) **2204 and 2222** Because F_Q is not empty, there must be at least one span that could affect the current raster line at x_{CL} (i.e., x_{14R}). Therefore, the first span in F_Q is read and made the New Span, S_N. This read operation sets the following values:

$$S_N = \text{read}(n) = 1;$$
$$x_{NL} = \text{read}(x_L) = x_{1L}; \qquad x_{NR} = \text{read}(x_R) = x_{1R};$$
$$z_{NL} = \text{read}(z_L) = z_{1L}; \qquad z_{NR} = \text{read}(z_R) = z_{1R}; \quad \text{(EQ 113)}$$
$$d_N = \text{read}(d) = d_1; \qquad i_N = \text{read}(i) = i_1;$$

Reading the first span yields the leftmost span in F_Q, which is S_1. The read process also deletes S_1 from F_Q by setting $(F_{1Q} = 0)$.

44C) **2224** The z value for the New Span, S_N (i.e., S_1), at the x coordinate x_{CL} (i.e., x_{14R}) is computed as z_{temp}.

$$z_{temp} = z_{NL} + (x_{CL} - x_{NL})\, d_N = z_{1L} + (x_{14R} - x_{1L})\, d_1 \quad \text{(EQ 114)}$$

The position (x_{14R}, z_{temp}) **2716** is shown in Figure 27.

44D) **2226, 2228, and 2232** The computed value, z_{temp}, is less than z_{CL} (i.e., infinity), so make the New

Span, S_N (i.e., S_1), into the Present Span, S_P, by setting:

$$\begin{aligned} S_P &= S_N = 1; \\ x_{PL} &= x_{NL} = x_{1L}; \quad x_{PR} = x_{NR} = x_{1R}; \\ z_{PL} &= z_{NL} = z_{1L}; \quad z_{PR} = z_{NR} = z_{1R}; \\ d_P &= d_N = d_1; \quad i_P = i_N = i_1; \end{aligned} \tag{EQ 115}$$

44E) **2236** The Current Portion, S_C, is set to the portion of the Present Span, S_P (i.e., S_1) to the right of x_{CL} (i.e., x_{14R}) by setting:

$$\begin{aligned} x_{CR} &= x_{NR} = x_{1R}; \\ z_{CL} &= z_{temp}; \quad z_{CR} = z_{NR} = z_{1R}; \\ CF &= \max(z_{temp}, z_{NR}) = \max(z_{temp}, z_{1R}) = z_{temp}; \end{aligned} \tag{EQ 116}$$

44F) **2237, 2204, 2206, and 2214** Because F_Q is empty and S_P (i.e., S_1) is valid, a search is done for spans that could possibly occlude the Current Portion, S_C, (i.e., S_1 to the right of x_{14R}). The Span Occluding Test query performs, in each SMCCAM Word **900**:

$$\begin{aligned} F_{nQ} &= F_{nV} \wedge (n \neq S_P) \wedge (x_{nL} < x_{CR}) \wedge (x_{nR} > x_{CL}) \wedge [(z_{nL} < z_{CF}) \vee (z_{nR} < z_{CF})] \\ F_{nQ} &= F_{nV} \wedge (n \neq 8) \wedge (x_{nL} < x_{1R}) \wedge (x_{nR} > x_{14R}) \wedge [(z_{nL} < z_{temp}) \vee (z_{nR} < z_{temp})] \end{aligned} \tag{EQ 117}$$

This query creates the set of all Potentially Occluding Spans for the Current Portion, S_C (i.e., S_1 to the right of x_{14R}). The query does not find any spans, so F_Q is empty.

44G) **2216 and 2220** Because F_Q is empty, there must not be any spans that possibly occlude the Current Portion, S_C (i.e., S_1 to the right of x_{14R}). Therefore, the New Span, S_N, is labelled as invalid.

$$S_N = \text{invalid}; \tag{EQ 118}$$

A read operation is not done. S_1 is still the Present Span, S_P. The status of the Saved Span, S_S, is left unchanged, thereby remaining invalid.

45) **2109, 2106, 2110, and 2300** Because the right edge of the display screen has not been reached (i.e., $x_{CL} < x_{max}$) and also because S_P is valid and S_N is not valid, Rule 2 is invoked to process S_1.

45A) **2302** The Current Portion, S_C (i.e., S_1 to the right of x_{14R}), must be completely visible. Therefore, render the Present Span, S_P (i.e., S_1), from x_{CL} to x_{CR} (i.e., from x_{14R} to x_{1R}).

45B) **2304 and 2306** Because the Saved Span, S_S, is not valid, there was not an intersecting span that reduced the size of S_C. Therefore, a query is done to look for an Abutting Span. The query performs, in each SMCCAM Word **900**:

$$\begin{aligned} F_{nQ} &= F_{nV} \wedge (x_{nL} = x_{CR}) \wedge (z_{nL} = z_{CR}) \\ F_{nQ} &= F_{nV} \wedge (x_{nL} = x_{1R}) \wedge (z_{nL} = z_{1R}) \end{aligned} \tag{EQ 119}$$

This query creates the set of all Abutting Spans for the Current Portion, S_C (i.e., S_1 to the right of x_{14R}) and stores them in F_Q. The query does not find any spans, so F_Q is empty.

45C) **2308 and 2320** Because F_Q is empty, there is not an Abutting Span. All spans completely to the left of x_{CR} (i.e., x_{14R}) must be hidden, so they are declared invalid. This is done by a query that performs, in each SMCCAM Word **900**:

$$F_{nV} = F_{nV} \wedge (x_{nR} > x_{CR})$$
$$F_{nV} = F_{nV} \wedge (x_{nR} > x_{1R}) \quad \text{(EQ 120)}$$

This query reduces the number of valid spans by reducing the spans in set F_V from one to zero. At this point, F_V is empty.

45D) **2322** There is neither a valid Present Span, S_P, nor a valid New Span, S_N. Hence, both are labelled as invalid.

$$S_P = \text{invalid};$$
$$S_N = \text{invalid}; \quad \text{(EQ 121)}$$

45E) **2324** The Current Portion, S_C, is set equal to:

$$x_{CL} = x_{CR} = x_{1R}; \qquad x_{CR} = \text{invalid};$$
$$z_{CL} = \text{infinity}; \qquad z_{CR} = \text{invalid}; \quad \text{(EQ 122)}$$
$$z_{CF} = \text{infinity};$$

46) **2106, 2108, and 2120** Because the Present Span, S_P, is not valid and F_V is empty, all the spans have been processed. Since x_{CL} (i.e., x_{1R}) must be less than x_{max}, there is still a fraction of the current raster line to the right of x_{CL} (i.e., x_{1R}) that has not been covered by spans. Therefore, the background is rendered from x_{CL} (i.e., x_{1R}) to x_{max}. The Segment Span Rasterization Method **2100** is complete for the current raster line.

Timing of Processing within the Span Sorting Pipeline 400

The Generic 3D Graphics Pipeline **200** generally utilizes a double buffered frame buffer **224** with two pages of memory. This allows an image in one page to be displayed on the display device, while the rendering process writes an image into the other page. Image data is written to random locations within the display screen **104** because the geometry is fed into the pipeline without any spatial sorting. When scene rendering is complete, the pages are swapped and the next scene is rendered.

Using only a few raster lines of display memory

The Span Sorting Rendering Pipeline **400** generates the complete pixel coloring for a raster line before proceeding to the next raster line. If raster line rendering is done at a rate to keep up with the display, then only a few raster lines of Raster Line Memory **416** are needed.

Figure 28 shows how the timing of processing within the Span Sorting Rendering Pipeline **400** can be accomplished with only two raster lines of Raster Line Memory **416**. In this figure, the horizontal axis is time, where each tick mark represents the time to display one raster line on the physical display device (CRT, etc.). Polygon processing **1400** for scene 1 **2802** writes all the polygons in the scene into the Bucket Sort Memory **404**. Once this **2802** is completed, raster line 0 is rendered **2804** for scene 1. When rendering this first raster line **2804** is completed, the displaying of scene 1 **2806** on the display device can begin, provided that subsequent raster lines are rendered before they are needed by the display device. In this way, the rendering of raster lines **2808** is in lock step with displaying of raster lines **2806**, and only two raster lines of Raster Line Memory **416** are needed, one for writing while a raster line is processed, and one for reading to send data to the display device. Once polygon processing **1400** for scene 1 **2802** is complete, polygon processing **1400** for scene 2 **2810** can begin, and this **2810** is done in parallel (in a pipelined fashion) with processing raster lines for scene 1 **2808** and displaying raster lines for scene 1 **2806**.

The diagram of Figure 28 is an ideal case because processing a raster line **2808** is assumed to be very closely matched to the time it takes to display a raster line. In practical systems, there is some variation in the time it takes to process a raster line. Therefore, several raster lines should be processed before the corresponding scene is displayed, thereby providing some "slack" in the timing requirements. The diagram of Figure 28 also neglects display timing issues such as horizontal and vertical retrace times, but these can be used to introduce some of the "slop" in the timing.

Having less memory than a full frame buffer is possible because pixel coloration processing is proportional to the number of pixels in the display screen rather than to the number of pixels in all the polygon spans in the raster line. When rendering timing is done this way, the latency through the Span Sorting Rendering Pipeline **400** is mostly in the process polygon **1400** portion. If the polygon processing **1400** can keep up with the display screen rate, then a new scene can be displayed every time the screen of the display device is refreshed.

The diagram of Figure 28 assumes, when a new scene is displayed, raster line 0 of the scene must be displayed first. This assumption requires polygon processing **2802** and raster line processing **2808** to "sync up" with the display device **2806**. An alternate approach is to let the polygon processing complete at any point with in the cycle **2806** of the display device, and to begin raster line rendering at the raster line that is about to be displayed (taking into account the latency of processing a raster line). This alternate approach requires the Bucket Sort Memory **404** to be able to start its read operation at any raster line.

Some display screens, such as standard televisions, use an interlaced display of the raster lines. That is the even numbered raster lines are displayed first (in the even "field") and the odd numbered raster lines are displayed next (in the odd "field"). For this type of display, the Span Sorting Rendering Pipeline **400** generates raster lines in the order needed by the display. The two fields of the display can be treated as separate scenes, or as a single interlaced scene.

Using a single buffered display memory

Figure 28 assumes only a few raster lines are stored in the Raster Line Memory **416**. An alternate approach is to have memory for all the raster lines in the Raster Line Memory **416**. This increases the required memory by quite a lot (maybe by a factor of 100), but most computer video display systems have memory for every pixel in the screen, and 3D rendering maybe considered an add-on to this type of computer display. Hence, the required memory may already present in the video memory of the host computer system.

This section of the document assumes the Raster Line Memory **416** stores all pixel color information for every raster line. Figure 29 shows the timing for a display device with ten raster lines (practical displays generally have on the order of 1000 raster lines) being continuously refreshed, where three screens worth of raster lines are displayed **2902**. As in the previous figure, the horizontal axis is time, where each tick mark represents the time to display one raster line on the physical display device. At at some point within the display of a scene **2904**, polygon processing **1400** will be completed **2906** for the next scene, and this point **2906** is not assumed to be synchronized with the scene display in any particular way. Completion **2906** of polygon processing **1400** means that raster line processing **2907** can begin. Processing raster line 0 **2908** is done first, and this raster line is displayed the next time **2910** raster line 0 is sent to the display device. Similarly, the last raster line is processed **2912** and subsequently displayed **2914**, thereby displaying **2916** the entire scene. Figure 29 shows how, upon completion **2906** of a scene's polygon processing **1400**, the scene can be displayed beginning at the very next refresh cycle **2916** of the display, even though the raster line processing of the last raster line **2912** occurs after display of the first raster line **2910**. If both the polygon processing and the raster line processing for entire scenes can keep up with the display device's screen refresh rate, then a new scene can be displayed at every screen refresh. If, once in a while, either the polygon processing or the raster line processing does not keep up, then the display device may show the same scene on two consecutive display refresh cycles.

The concept behind Figure 29 can be extended to raster line processing that does not, on the average, keep up with the display refresh cycles. That is, as long as the raster line processing of the last raster line **2912** is completed before it is displayed **2914**, then the entire scene will be displayed correctly. Furthermore, raster line processing can start on the raster line that has most recently been displayed. Figure 30 shows polygon processing **1400** completing **3002** just before raster line 7 is completely displayed. Raster line processing **3004** can begin with raster line 7 **3006**, and this raster line is displayed **3008** the next time the display device shows it. Display of the scene is not intermixed with display of the previous scene (that is, there is a clean transition) provided the last raster line processed **3010** is completed before it is displayed **3012**. In this way, the maximum time to process all rasters **3014** is almost as long as two screen refresh cycles. Hence, the processing of each raster line can be about twice as long as the time to display it without requiring the need for double buffering, only a single buffer is needed.

The concept illustrated in Figure 40 of having raster line processing **3004** cover more that one screen refresh cycles, can be modified to have raster line processing **3004** wait until raster line 0 is displayed, thereby synchronizing with the refresh cycles. Polygon processing **1400** for the next scene does not need to wait because the Bucket Sort Memory **404** is double buffered.

Using a double buffered display memory

The Span Sorting Rendering Pipeline **400** can function with a fully double buffered Raster Line Memory **416**, where every pixel in the display screen **104** corresponds to two words of memory (one in each buffer page). In this configuration, polygon processing **1400** for a scene is done until completed, then raster line processing is done until all raster lines in the display screen **104** have been processed, then the scene is shown on the display device. This approach has the advantage of being able to take as long as necessary without being constrained to display device timing. This approach has the disadvantage of introducing an additional stage of latency.

Query Processor Architecture

Within the Span Sorting Rendering Architecture **500** of Figure 5, for each raster line, the Query Processor **510** receives spans from the Span Generation Processor **508**, determines the visible span portions (as described above), and sends the visible span portions to the Rasterize Processor **510**. A block diagram of the Query Processor **510** is shown in Figure 31, along with the two pages within the Span Memory **408**.

While one Span Memory Page **800** is receiving Span Parameters from the Span Generation Processor **508**, the other page **800** is being queried to determine the visible span portions. The two pages **800** are swapped via the Query Processor Crossbar **3102**, which is a set of multiplexors. The Query Processor **510** includes a register file **3104** that is subdivided into five sets of registers for storing Span Parameters: 1) registers **3106** for the Current Portion, S_C; 2) registers **3108** for the Present Span, S_P; 3) registers **3110** for the New Span, S_N; 4) registers **3112** for the Saved Span, S_S; and 5) temporary registers **3114** used for things such as span intersection locations.

The Query Processor Control **3116** determines the sequence of operations in the Query Processor **510**, including the sequence of query operations. As raster line processing proceeds, branches within the method **1800** or **2100** depend on query outcomes, span status, and Span Interaction Types (SITs). For each pair of S_C and S_N, the SIT and SIP Logic **3118** generates the Span Interaction Type. Computing a SIT requires the operations of Equation 10 to be performed, as well as various comparison operations. These operations are performed in the Compare and Offset Computation **3200** block on data received from the register file **3104**. Whenever Rule 4 **2500** is invoked, the intersection point between S_C and S_N is computed in the Intersection Computation block.

For maximum throughput, Compare and Offset Computation **3200** can be implemented in parallel hardware. Figure 32 shows four identical blocks **3202**, where each block **3202** includes two arithmetic comparators **3204** and **3206**, a subtracter **3208**, and a multiplier **3210**. The subtracter **3208** and multiplier **3210** compute the offsets as shown in Equation 10. The comparators generate signals that can be used to generate the Span Interaction Parameters (SIPs). Inputs to the blocks **3202** come from the register file **3104**, and outputs go to the SIT and SIP Logic **3118**.

Sorting Magnitude Comparison CAM Circuits

Figure 33 shows a prior art MCCAM bit cell **3300**, taken from the Duluk Patent, constructed out of CMOS transistors and gates. A field within an MCCAM Word can be built with a multiplicity of these cells **3300**. In this cell **3300**, one bit is stored in a Six Transistor RAM Subcircuit **3302**, where reading and writing is controlled by the signal *CamWord* **3304** and data is written and read via the differential bit lines, *BitH* **3306** and *BitL* **3308** (the "H" and "L" at the end of signal names indicates active high and active low, respectively). During a query operation, the XOR Subcircuit **3310** performs an exclusive-or function to produce the signal *NotEqual* **3312**, which indicates the bit value stored in the RAM **3302** is not equal to the bit value on the bit lines **3306** and **3308**. During a query operation, the signal *EnableIn* **3314** being asserted indicates higher order bits in the field are all equal, and this bit position is enabled for comparison. If *NotEqual* **3312** is at logic "0" when *EnableIn* **3314** is

asserted, the Precharged Enable Propagation Subcircuit **3316** enables the next lower bit position by asserting *EnableOut* **3318**. If *NotEqual* **3312** is at logic "1" when *EnableIn* **3314** is asserted, the Two Transistor Query Resolution Subcircuit **3320** conditionally pulls down the *QueryTrue* **3322** signal, depending on the value of *ComparisonOp* **3324** (as described in the Duluk Patent), and *EnableOut* **3318** remains unasserted.

In contrast to an MCCAM, an SMCCAM **800** (recall, the "S" stands for "sorting") can shift data from one word **900** to the next during a sorting operation. Figure 34 is a "TTL type" circuit for one bit in an SMCCAM Word **900**. This cell **3400** includes a master-slave flip-flop **3402** for storing the bit value because it must function as a bit in a shift register. The cell is shown with separate I/O lines, *InputBit* **3404** and *OutputBit* **3406**, but a single bidirectional I/O line or a bidirectional differential pair could be used. A read operation is done by asserting the *Read* **3408** control line, which enables the tri-state buffer **3410**. A write operation is done on the rising edge of *Clock* **3412** when *WriteEn* **3414** is asserted and *ShiftEn* **3416** is not asserted. The shift within a sort operation is done on the rising edge of *Clock* **3412** when *WriteEn* **3414** is asserted and *ShiftEn* **3416** is also asserted. The signal *ShiftEn* **3416** controls a multiplexor **3418** that chooses whether data on *InputBit* **3404** or data on *ShiftIn* **3420** is stored into the flip-flop **3402**. Data on *ShiftIn* **3420** come from *ShiftOut* **3422** in the corresponding cell **3400** in the previous SMCCAM Word **900**. Query operations function similarly to the MCCAM of the Duluk Patent. That is, an exclusive-or **3424** generates the signal *NotEqual* **3312**, which is used to generate *EnableOut* **3318** and the word's query result, *StoredGreaterL* **3426**. The key differences in the query operation function are: 1) the absence of *ComparisonOp* **3324** in the cell; and 2) an open collector circuit **3428** to generate the query result.

Figure 35 is a 27-transistor CMOS circuit for one bit in an SMCCAM Word **900**. This cell **3500** includes a Six Transistor RAM Subcircuit **3302** as the master part of the flip-flop, and a Slave Memory Subcircuit **3502** as the slave part of the flip-flop. The Six Transistor RAM Subcircuit **3302** is used in the conventional way for read and write operations, and the Slave Memory Subcircuit **3502** is only used during shifting. To allow a bit to be shifted into the Six Transistor RAM Subcircuit **3302**, a Shift Write Subcircuit is included in the cell **3504**. The following sequence occurs (in this example cell **3500**) when shifting is done: 1) the differential clock signal *ShiftToSlaveH* **3506** and *ShiftToSlaveL* **3508** is asserted then de-asserted in order to latch the data bit into the Slave Memory Subcircuit **3502**; 2) the data bit is output from the cell to the corresponding cell **3500** in the next word **900** via the differential signal *ShiftDataOutH* **3510** and *ShiftDataOutL* **3512**; and 3) the data bit is received on the differential signal *ShiftDataInH* **3514** and *ShiftDataInL* **3516** in the next word **900** and is written into that cell **3500** by asserting *ShiftEnable* **3518**.

Certainly, there are many bit cell circuits that could be designed to do an equivalent function. To illustrate some of the other choices, the cell in Figure 35 uses a Non-Precharged Enable Propagation Subcircuit **3520** and a Three Transistor Query Resolution Subcircuit **3522**. Note that the Slave Memory Subcircuit **3502** is a dynamic circuit, and it could be replaced with many other types of latches, including a static latch.

Figure 36 shows an array of CMOS SMCCAM bit cells forming a set of fields. The signals, *ShiftDataInH* **3514** and *ShiftDataInL* **3516**, are grounded in the first word **900** because the first word **900** cannot have data shifted into it **900**.

When signed numbers need to be stored into the SMCCAM, it is simplest to invert the sign bit (i.e., "1" implies positive) during both the reading and writing processes. That way, in the hardware, positive numbers are "bigger" than negative numbers. An alternate approach is to change the logic in the sign bit positions in every field where a signed number needs to be stored and queried.

Multiple Spans Vertically within a Raster Line

In the Segment Span Rasterization Method **2100** as described above, each polygon can have only one span in a given raster line, and that span is represented by a line segment (hence, the name **2100**) parallel to the *x*-*y* plane in screen coordinates. The location of the endpoints of spans are in object-precision, and therefore can be located anywhere horizontally within a pixel's boundaries. For example, if the end a span covers 20% of the width of a pixel, then that span accounts for 20% of the color value for that pixel, and is blended with color value for the other 80% of the pixel width. This provides for horizontal anti-

aliasing, but not vertical antialiasing. That is, blending occurs horizontally, but not vertically. If the edge of a polygon is closer to horizontal than vertical, then the edge has a good chance of crossing through multiple pixel areas, meaning that the polygon color should be blended into multiple pixels. However, when a single line segment is used to represent the span, the polygon's color is blended into one pixel (the one that includes the segment span endpoint), and pixels on either side are assumed to be covered 100%, and are not blended (assuming visible span portions are wider than one pixel). Lack of vertical antialiasing is acceptable for some applications, but high quality images should be both antialiased in both directions.

One method of adding vertical antialiasing is to divide each raster line vertically into several "sub-raster" lines. Then, the Segment Span Rasterization Method **2100** can be used for each sub-raster, and pixel color values can be generated by blending vertically as well as horizontally. Figure 37A shows a raster line **3702** and its pixels **3704**, two edges of a polygon **3706 and 3708**, and two sub-raster spans **3710 and 3712** for that polygon. Similarly, Figure 37B shows four sub-raster spans **3714, 3716, 3728, and 3720** for the polygon. As a possible variation, if a pixel is covered by only two such sub-raster spans from the same polygon, it would be possible to compute one color value at the center of the pixel.

Using sub-raster lines reduces vertical aliasing, but does not eliminate it. For many applications, this is an acceptable approximation.

Trapezoidal Span Rasterization Method

Spans can be represented by two-dimensional trapezoids rather than one-dimensional line segments. Both line segments and planar trapezoids are in three-dimensional object-precision eye coordinates. The advantage of a trapezoidal representation is the preservation of edge information within a raster line, thereby providing correct antialiasing in both the horizontal and vertical dimensions.

Figure 38 shows a triangle mesh with two triangles **3802, and 3804**, and one raster line **3806** within the display screen **104**. The area within the raster line **3806** for each triangle **3802, and 3804** is a trapezoidally shaped span, labelled S_{11} **3808** and S_{12} **3810**. For trapezoidal spans, the span endpoint is an edge of the trapezoid, and is described by two points in three-dimensional screen coordinates: one at the top edge of the raster line, and one at the bottom edge of the raster line. Hence, a trapezoidal span is described by four points in screen coordinates. For example, the trapezoidal span on the left **3808** in the figure has endpoints described in x-z space by: 1) the top of the left endpoint located at (x_{11LT}, z_{11LT}); 2) the bottom of the left endpoint located at (x_{11LB}, z_{11LB}); 3) the top of the right endpoint located at (x_{11RT}, z_{11RT}); and 4) the bottom of the right endpoint located at (x_{11RB}, z_{11RB}). These four points are actually in x-y-z space, but the y-coordinate is dictated by the location of either the top or the bottom of the raster line **3806**. Note that the subscripts in the coordinate variables indicate three things: 1) the span number (e.g., "11"); 2) the left or right endpoint (i.e., "L" or "R"); and 3) the top or bottom of the raster line (i.e., "T" or "B").

An opaque trapezoidal span occludes more distant geometry over most of its length, but at endpoints **3812, and 3814** that are not abutted to another trapezoidal span, more distant geometry is partially visible. However, for abutting trapezoidal spans, as shown in Figure 38, the common endpoint **3816** does occlude more distant geometry. This presents a problem when utilizing trapezoidal spans. That is, non-abutting trapezoidal span endpoints do not occlude more distant geometry, but abutting endpoints work together to occlude **3818** more distant geometry. The invention of this document successfully solves this problem by searching for abutting spans.

Figure 39 shows a group of trapezoidal spans in the same raster line, and these spans are trapezoidal versions of the segment spans shown in Figure 11. In Figure 39, the visible portions of the spans are shaded. Notice that for spans in a polygon mesh (i.e. abutting spans), span tops lay end-to-end with each other, and span bottoms do likewise. Span endpoints cast an edge "shadow" onto spans that are farther from the viewing point. For example, the left endpoint **3902** of S_2 casts such a shadow **3904** onto S_1.

An additional concern when using trapezoidal spans occurs when a polygon crosses either the top or the bottom of the raster line, but not both. This would generally happen at: 1) the corner of a polygon 2) at a horizontal edge of a polygon; or 3) when occluding spans leave only a sliver of a span visible. One example of this is shown in Figure 39, where the far right

end **3906** of S_0 is visible. In this type of situation, the trapezoidal span is actually shaped like a triangle, and therefore could be considered a "degenerate trapezoid" where the top or bottom is of length zero.

This document has explained in great detail the Segment Span Rasterization Method **2100**, and the Trapezoidal Span Rasterization Method is similar in that it follows the frontmost spans from left to right and sending the visible portions of spans to the Rasterization Processor **512**. The key difference is that non-abutting endpoints of trapezoidal spans do not fully occlude more distant geometry. The Trapezoidal Span Rasterization Method must correctly handle these types of spans.

There is a variety of ways to handle trapezoidally shaped spans. Rather than having a method flow diagram for each of the four choices for the Trapezoidal Span Rasterization Method presented here, the features of each choice will be explained, along with the corresponding choices for SOT Queries. These four particular choices were selected to illustrate how the rasterization method presented here can be expanded to process trapezoidally shaped spans.

Version 1: Occlusion processing done separately for span tops and span bottoms

A trapezoidal span can be described as the collection of: 1) a span top, the line segment on the boundary of the trapezoidal span along the top of the raster line; 2) a span bottom, the line segment along the bottom of the raster line; and 3) the area between the span top and the span bottom. The version of the Trapezoidal Span Rasterization Method described in the section of the document does occluding processing on span tops separately from span bottoms. This can be done because, for the same raster line, span tops and span bottoms never occlude each other. Occlusion processing for span tops can be done in the same way occlusion processing is done for segment spans in the Segment Span Rasterization Method **2100**. Occlusion processing for span bottoms can be done separately in the same way, and should be done in parallel with occlusion processing of span tops in order for pairs of tops and bottoms to be correctly paired for pixel color generation.

This version of the Trapezoidal Span Rasterization Method keeps track of the following "span-like" things (including appropriate endpoints): 1) the Present Span, S_P, the trapezoidal span that is currently being tested to see what part of it should be rendered; 2) the Current Span Top, S_{CT}, the top edge of the S_P that is being tested for visibility; 3) the Current Span Bottom, S_{CB}, the bottom edge of the S_P that is being tested for visibility; 4) the New Span, S_N, the trapezoidal span recently read from the Span Memory **408**; 5) the New Span Top, S_{NT}, the top edge of S_N; 6) the New Span Bottom, S_{NB}, the bottom edge of S_N; 7) the Top Saved Span, S_{ST}, a trapezoidal span that is temporarily saved because S_{CT} intersects S_{NT}, in a situation analogous to Rule 4 described above; and 8) the Bottom Saved Span, S_{SB}, a trapezoidal span that is temporarily saved because S_{CB} intersects S_{NB}. Two different saved spans are needed because span tops are processed separately from span bottoms, and they could have intersections with different spans. Keeping track of more information during rasterization requires more registers to be added to the Span Parameter Registers **3104** within the Query Processor **510**.

Figure 40 shows an enlarged version of the example spans S_1 **4001** and S_2 **4002** from Figure 39, and, S_1 **4001** is assumed to be the Present Span, S_P. The span top of S_1 **4001**, called S_{1T} **4003**, has its corresponding SOT Region **4004** that finds S_{2T} **4006** occluding it **4003**. This SOT Region **4004** does not find S_{2B} **4008** because it **4004** is only searching for span tops. Similarly, the span bottom of S_1, S_{1B} **4010**, has its SOT Region **4012** that finds S_2 **4008** occluding it **4010**.

Figure 41 shows two views of a partial set of spans within a raster line **4100**, labelled S_{31} **4101** through S_{39} **4109**. The upper diagram is the *x*-*z* view (the look direction is parallel to the *y*-axis) of the spans, and the lower diagram is *x*-*y* view (the look direction is parallel to the *z*-axis) of the same spans. In the lower view, hidden portions of the spans are not shown, and therefore represent how an infinite precision rendering of the spans would be drawn for that raster line **4100**. But, since the display device does not have infinite precision, span colors must be blended together in correct proportions for each pixel. In the figure, visible portions of span tops and bottoms are shown as thicker lines. Visible portions of span tops and bottoms are passed to the Rasterization Processor **512** were they are used along with polygon edge derivatives to determine pixel colors. The spans in Figure 41 were chosen to illustrate handling of various cases, including spans where only the span top or span bottom is visible and also spans that do not cover the entire height of the raster line **4100**.

Just as in the Segment Span Rasterization Method **2100**, the spans are processed in a left-to-right manner. The exam-

ple of Figure 41 starts in the middle (i.e., horizontally) of the raster line where S_{31} **4101** is S_P, S_{31T} **4111** is S_{CT}, and S_{31B} **4121** is S_{CB}. The SOT Queries for S_{CT} and S_{CB} find nothing, S_N is set to invalid, and by a rendering rule similar to Rule 2 **2300** of the Segment Span Rasterization Method **2100**, S_{31} **4101** is rendered to its right endpoint. The rendering of S_{31} **4101** covers 100% of the pixels up to the one that includes x_{31RT} **4151**. From that point, the edge derivative for the corresponding polygon edge (see Equation 1) is used to determine the percent coverage of each pixel, and this percentage weights the pixel color. The span **4101** can be rendered to its right endpoint because the entire length of the endpoint (the endpoint of a trapezoidal span is a line segment) is guaranteed to be visible. Also by the Rule 2 **2300** analog, queries for abutting span tops and span bottoms find S_{32T} **4112** and S_{32B} **4122**. At this point in the method, S_{32} **4102** is S_P, S_{32T} **4112** is S_{CT}, and S_{32B} **4122** is S_{CB}. The SOT Queries for S_{CT} and S_{CB} find nothing, and abutting spans are also not found. Hence, S_N is set to invalid.

By a rule similar to Rule 2 **2300**, S_{32} **4102** is rendered in its entirety (its right endpoint is guaranteed visible), and edge derivatives are again used to compute pixel coverage. To sight an example, for the pixels at least partly contained between x_{31RT} **4151** and x_{31RB} **4152**, the sum of percent coverage due to S_{31} **4101** and S_{32} **4102** totals 100%. Queries to find abutting span tops or bottoms fail, so all spans completely to the left of x_{32RB} **4152** are invalidated, and both S_P and S_N are set to invalid.

By a rule similar to Rule 1 **2200**, a search is done to find the frontmost span tops and bottoms at x_{32RT} **4153** and x_{32RB} **4154**, which finds S_{33} **4103** and assigns it to S_P, sets S_{CT} to the part of S_{33T} **4113** to the left of x_{32RT} **4154**, and sets S_{CB} to the part of S_{33B} **4123** to the left of x_{32RB} **4153**. The SOT Queries for S_{CT} and S_{CB} find S_{34T} **4114** and S_{35B} **4125** (note these are from different spans) which are assigned to S_{NT} and S_{NB}, respectively. S_N is set to S_{34} **4104**, rather than S_{35} **4105** because the visible portion of S_{34} **4104** is farther to the left, and the method proceeds from left to right.

By a rule similar to Rule 3 **2400**, S_{33} **4103** is rendered, but only partially. At this point, the Rasterization Processor **512** can only render it **4103** to x_{34LT} **4155** because its **4103** area to the right of x_{34LT} **4155** still needs to have its visibility determined (generally the case when S_{NT} and S_{NB} are not from the same span). Because x_{34RT} **4155** almost certainly does not align with a pixel edge, the pixel that includes x_{34RT} **4155** is partially colored by the portion of S_{33} **4103** that is know to cover it (and the rest added later). Alternatively, the entire color contribution to that pixel by S_{33} **4103** can be postponed until the entire amount of pixel coverage is known. At this point in the method, S_{34} **4104** is S_P, S_{34T} **4114** is S_{CT}, and S_{35B} **4125** is S_{CB}. The SOT Query for S_{CT} (i.e., S_{34T} **4114**) finds S_{35T} **4115** and makes it S_{NT}. Because this S_{NT} matches S_{NB} of the previous step (i.e., they are from the same span, S_{35} **4105**), the SOT Query for S_{CB} (i.e., S_{35B} **4125**) is postponed to the next step in the method. S_N is set to S_{35} **4105**.

During the method, when S_{NT} and S_{NB} are not from the same span, previous spans can only be rendered to the leftmost of either the current x-value position along the top of the raster line and the current x-value position along the bottom of the raster line. At the leftmost of the two x-values, all span to the left has been considered, and so rendering can be done.

By a rule similar to Rule 3 **2400**, S_{34} **4104** is rendered by the Rasterization Processor **512**. This time, S_{NT} and S_{NB} are from the same span, so S_{34} **4104** is rendered up to the left endpoint of S_{35} **4105**. Also, rendering of S_{33} **4103** can be completed because its visibility has now been determined. At this point in the method, S_{35} **4102** is made S_P, S_{35T} **4115** is made S_{CT}, and S_{35B} **4125** is made S_{CB}. The SOT Query for S_{CT} (i.e., S_{35T} **4115**) finds S_{37T} **4117** making it S_{NT}, while the SOT Query for S_{CB} (i.e., S_{35B} **4125**) finds nothing, and makes S_{NB} invalid. S_N is set to S_{37} **4107**.

By a rule similar to Rule 3 **2400**, S_{35} **4105** is rendered from its left endpoint to x_{35RB} **4156** because x_{35RB} **4156** is the leftmost choice between x_{35RB} **4156** and x_{37LT} **4157**. A search for an abutting span bottom to S_{CB} (i.e., S_{35B} **4125**) finds S_{36B} **4126**. At this point in the method, S_{36} **4106** is S_P, S_{37T} **4117** is S_{CT}, S_{36B} **4126** is S_{CB}, S_{37T} **4117** is S_{NT}, and an SOT Query for S_{CB} (i.e., S_{36B} **4126**) finds S_{39B} **4129** and makes it S_{NB}.

By a rule similar to Rule 3 **2400**, S_{36} **4106** is rendered up to x_{37LT} **4157** because x_{37LT} **4157** is the leftmost choice between x_{37LT} **4157** and x_{39LB} **4158**. Also, rendering of S_{35} **4105** can be continued up to x_{37LT} **4157**. Because S_{37} **4107** is not hidden and is known to have only a span top (no span bottom), it **4107** is made S_P. Also at this point, S_{37T} **4117** is still S_{CT}, S_{39B} **4129** is S_{CB}, and query to find an abutting span to S_{37} **4107** finds noting, so S_{NT} is invalid, and S_{39B} **4129** is still S_{NB}.

By a rule similar to Rule 2 **2300**, S_{37} **4107** is rendered in its entirety. Both S_{35} **4105** and S_{36} **4106** can now be rendered up to x_{37RT} **4159** (completing S_{35} **4105**) because it is the leftmost choice between x_{37RT} **4159** and x_{39LB} **4158**. At this point, S_{38} **4108** is S_P, S_{CT} is invalid, S_{39B} **4129** is S_{CB}, S_{NT} is invalid, and an SOT Query for S_{CB} (i.e., S_{39B} **4129**) finds nothing, so S_{NB} is invalid.

By a rule similar to Rule 1 **2200**, a search is done to find the frontmost span top at x_{37RT} **4159**. This search finds S_{38T} **4118**, and S_{CT} is set to the portion of S_{38T} **4118** to the right of x_{37RT} **4159**. An SOT Query for S_{CT} finds S_{39T} **4119** which is assigned to S_{NT}. At this point, S_{38} **4108** is still S_P, S_{39B} **4129** is still S_{CB}, and S_{NB} is still invalid.

By a rule similar to Rule 3 **2400**, S_{38} **4108** is rendered up to the left endpoint of S_{39B} **4129**. Because S_{39} **4109** is going to be made into the next S_P, the other remaining uncompleted span, S_{36} **4106**, is completed by rendering it up to the left endpoint of S_{39B} **4129**. Care must be taken to correctly account for all the polygons edge derivatives correctly.

The raster line **4100** processing continues with S_{39} **4109** as S_P, but the right edge of our example has been reached. If there had been some completely hidden spans within the left and right boundaries of this example, they would have been completely avoided by either never being found by an SOT Query or by being discarded by a rendering rule similar to Rule 5 **2600**. Similarly, if there had been an intersection between two span tops or two span bottoms, a rule similar to Rule 4 **2500** would have been applied, and Saved Spans would have been used.

A special case may be added for handling spans that either do not have either a span top or a span bottom (they are less than one pixel in height) or have both their span top and span bottom hidden, but are visible within the raster line (just peeking out from between two nearly horizontal polygon edges). In most practical applications, such marginally visible items can be ignored.

It is possible that color information from several different spans are blended together for a particular pixel. Under such circumstances (and even for blending only two span's colors), only a reasonable approximation of the percent coverage is needed to make a high quality image.

When implemented in hardware, this version of the Trapezoidal Span Rasterization Method uses one SMCCAM Word **900** for each span top and another one for each span bottom. Both the span tops and span bottoms could be stored into the same page within the Span Memory **408**. This would mean time-sharing the Span Memory **408** between two essentially independent occlusion processes, thereby reducing performance when compared to storing span tops and span bottoms into different pages. Using one page for occlusion implies the Span Memory **408** needs a total of two pages, as shown in Figure 38, to double buffer the spans.

If two pages within the Span Memory **408** are used, the two occlusion processes can be done in parallel. However, some fields, such as the Span Derivative Field **812** and the Information Pointer Field **812**, need to be duplicated because they are needed by both occluding processes. Also, using two pages for occlusion processing tends to imply a total of four pages are needed in the Span Memory **408**. But, if a non-interlaced display device is used, then the set of span bottoms for the current raster line is the same as the set of span tops for the next raster line. Hence, three total pages are needed in the Span Memory **408** because one page is used for writing, and two pages are used for reading, and one of those two pages is reused for the next raster line. If an interlaced display device is used, then the same technique can be used, but raster line rendering order will not match the display device.

Another alternative is saving the list of the visible bottom span portions (i.e., after occlusion processing) in another memory and then using them as the span tops in the next raster line. Such a memory could be included in the Rasterization Processor **512**. While this is certainly possible, it is probably more economical to recompute the visible top spans.

Version 2: Perform an SOT Query at every transition in the raster line

In this version of the Trapezoidal Span Rasterization Method, spans are processed from left to right, but occlusion processing treats each span as a unit, not as a separate span top and a span bottom. To do this, SOT Regions are bounded on the right and left by a visibility transition on either span tops or span bottoms.

Figure 42 shows an enlarged version of the example spans S_1 **4201** and S_2 **4202** from Figure 39. At the start of this example, S_1 **4201** is the Present Span, S_P, and x_{1LB} **4204** is the current x-coordinate, x_{CL}. Hence, the raster line has been rendered up to x_{1LB} **4204**, including the part of S_1 **4201** between x_{1LT} **4203** and x_{1LB} **4204**. The Current Span, S_C, is the part of S_P (i.e., S_1 **4201**) to the right of x_{CL} (i.e., x_{1LB} **4204**) that has the same visibility characteristic, namely the part between x_{1LB} **4204** and x_{1RT} **4206** (making x_{1RT} **4206** the value for x_{CR}). The visibility characteristic for S_1 **4201** changes at x_{1RT} **4206** because to the right, the span **4201** does not occlude more distant geometry, while to the left of that point **4206**, it does. The SOT Region **4208** for this S_C is shown in Figure 42 and described by Equation 123.

(EQ 123)

$$F_{nQ} = F_{nV} \wedge F_{nQ} \wedge (n \neq S_C) \wedge [(x_{nLT} < x_{CR}) \vee (x_{nLB} < x_{CR})] \wedge [(x_{nRT} < x_{CL}) \vee (x_{nRB} < x_{CL})]$$
$$\wedge [[(z_{nLT} < z_{CFT}) \vee (z_{nLB} < z_{CFB})] \vee [(z_{nRT} < z_{CFT}) \vee (z_{nRB} < z_{CFB})]]$$
$$F_{nQ} = F_{nV} \wedge F_{nQ} \wedge (n \neq 1) \wedge [(x_{nLT} < x_{1RT}) \vee (x_{nLB} < x_{1RT})] \wedge [(x_{nRT} < x_{1LB}) \vee (x_{nRB} < x_{1LB})]$$
$$\wedge [[(z_{nLT} < z_{temp1}) \vee (z_{nLB} < z_{1LB})] \vee [(z_{nRT} < z_{temp1}) \vee (z_{nRB} < z_{1LB})]]$$

There are two different z-values used in this query, z_{CFT} and z_{TFB}, because the span portion being tested for occlusion has different depth values for its top and bottom. The value for z_{CFT} is set to the maximum z-value of the span top within the x-coordinate interval of S_C, which happens to be x-coordinate with the value x_{1LB} **4204**, labelled z_{temp1} **4209**. The value z_{temp1} **4209** is computed using the span derivative for S_1 **4201**. Similarly, the value for z_{CFB} is set to the maximum z-value of the span bottom within the x-coordinate interval of S_C, and this value is z_{1LB} **4210**. The SOT Query finds S_2 **4202**, and it **4202** is made S_N.

By a rule similar to Rule 3 **2400** of the Segment Span Rasterization Method **2100**, S_P (i.e., S_1 **4201**) is rendered from x_{CL} (i.e., x_{1LB} **4204**) to the leftmost point of S_N (i.e., S_2 **4202**), which is x_{2LB} **4204**. Over this interval, the visibility characteristic does not change for the raster line. That is, S_1 **4201** is the only geometry that is visible within the interval.

The value of x_{CL} is changed to x_{2LB} **4211**, and S_C is the top edge of S_1 **4201** from x_{2LB} **4211** to x_{2LT} **4212** combined with the bottom edge of S_2 **4202** over the same interval. The valid for S_N is set to invalid. For this S_C, the visibility characteristic is consistent in that it includes S_1 **4201** gradually being covered by S_2 **4202** over its entire width. The corresponding Query Region **4213** is over the interval of S_C in the x-coordinate dimension. But, in the z-coordinate dimension, span top coordinates are tested to see if they are in front of z_{temp2} **4214** and span bottom coordinates are tested to see if they are in front of z_{2LT} **4216**. The coordinate z_{temp2} **4214** is the z-coordinate of the span top of S_1 **4201** at x-coordinate x_{2LB} **4204**. This Query Region **4213** needs to include a search for span tops that are between z_{temp2} **4214** and z_{2LT} **4216** because a span with its left top corner in this range would be visible. The query operation is shown as Equation 124.

(EQ 124)

In this example, the Query Region **4213** finds nothing, so S_N remains invalid.

By a rule similar to Rule 2 **2300**, S_C is not hidden, so it is rendered, resulting in a blending of S_1 **4201** and S_2 **4202** over the entire width of S_C. The rule then "checks for" the equivalent of an abutting span, and this is known to exist because S_2 **4202** continues to the right from x_{2LT} **4212**. Therefore, S_C is set to the part of S_2 **4202** between x_{2LT} **4212** and x_{2RT} **4218**, and x_{CL} is set to x_{2LT} **4212**. Once again (as always for this version of the method), S_C has a consistent visibility characteristic over its entire width. The SOT Region **4220** for this S_C is shown in Figure 42 and described by Equation 125. The value z_{temp3} **4222** is the z-value of the span bottom of S_2 **4202** at the x-coordinate x_{2LT} **4212**.

(EQ 125)

The rasterization of the spans in this raster line would continue from this point. This version of the method also has

$$F_{nQ} = F_{nV} \wedge F_{nQ} \wedge (n \neq S_C) \wedge [(x_{nLT} < x_{CR}) \vee (x_{nLB} < x_{CR})] \wedge [(x_{nRT} < x_{CL}) \vee (x_{nRB} < x_{CL})]$$
$$\wedge [[(z_{nLT} < z_{CFT}) \vee (z_{nLB} < z_{CFB})] \vee [(z_{nRT} < z_{CFT}) \vee (z_{nRB} < z_{CFB})]]$$
$$F_{nQ} = F_{nV} \wedge F_{nQ} \wedge (n \neq 1) \wedge (n \neq 2)$$
$$\wedge [(x_{nLT} < x_{2LT}) \vee (x_{nLB} < x_{2LT})] \wedge [(x_{nRT} < x_{2LB}) \vee (x_{nRB} < x_{2LB})]$$
$$\wedge [[(z_{nLT} < z_{temp2}) \vee (z_{nLB} < z_{2LB})] \vee [(z_{nRT} < z_{temp2}) \vee (z_{nRB} < z_{2LB})]]$$

$$F_{nQ} = F_{nV} \wedge F_{nQ} \wedge (n \neq S_C) \wedge [(x_{nLT} < x_{CR}) \vee (x_{nLB} < x_{CR})] \wedge [(x_{nRT} < x_{CL}) \vee (x_{nRB} < x_{CL})]$$
$$\wedge [[(z_{nLT} < z_{CFT}) \vee (z_{nLB} < z_{CFB})] \vee [(z_{nRT} < z_{CFT}) \vee (z_{nRB} < z_{CFB})]]$$
$$F_{nQ} = F_{nV} \wedge F_{nQ} \wedge (n \neq 2) \wedge [(x_{nLT} < x_{2RT}) \vee (x_{nLB} < x_{2RT})] \wedge [(x_{nRT} < x_{2LT}) \vee (x_{nRB} < x_{2LT})]$$
$$\wedge [[(z_{nLT} < z_{2LT}) \vee (z_{nLB} < z_{temp3})] \vee [(z_{nRT} < z_{2LT}) \vee (z_{nRB} < z_{temp3})]]$$

rules analogous to Rule 1 **2200**, Rule 4 **2500**, and Rule 5 **2600**.

In general, this version of the method takes spans, and subdivides them horizontally in order to create sections that have a consistent visibility characteristic. This means that the endpoint regions of a span (the part that does not occlude distant geometry, such as the endpoints **3812 and 3814** in Figure 38) are treated separately from the central portion of the span. This causes more query operations to be needed, but can be done in a single occlusion process.

This version of the Trapezoidal Span Rasterization Method requires more fields per SMCCAM Word **900** because there are four fields per endpoint rather than just two. Additional comparison logic is also needed in the Word Number Field **802** because it **802** must be able to check inequality with a Current Span that is the combination of two trapezoidal spans. In the above example, the value of S_C corresponding to the Query Region **4213** covering x_{2LB} **4210** to x_{2LT} **4212** is the combination of S_1 **4201** and S_2 **4202**. Hence, the SOT Query must check for both ($n \neq 1$) and ($n \neq 2$), thereby requiring two inequality comparisons during that query operation. This version of the method has the advantage of needing only one word **900** to store a span.

Version 3: More comparisons per word to allow more complex SOT Regions

In this version of the Trapezoidal Span Rasterization Method, occlusion processing treats each span as a unit, just as in Version 2. However, this version performs more comparison operations in each SMCCAM Word **900**, thereby providing the ability to make more complicated SOT Regions. Rather than splitting spans at every visibility transition, the Current Span, SC, can contain one visibility transition. In essence, an SOT Region for this version is the union of two SOT Regions as defined in Version 2. Figure 43 shows the same two spans **4301 and 4302** as shown in Figure 42, and an SOT Region **4304** that is the union of two SOT Regions **4213 and 4220** from Version 2 of the method. To perform this SOT Query, the query operation is the disjunction (i.e., logical "or") of the right sides of Equation 124 and Equation 125.

To perform the more complicated query function to generate the SOT Region **4304**, the fields in each SMCCAM Word **900** each need to perform two comparison operations in parallel (if the two operations are done sequentially, then it is the same as Version 2). This makes the hardware more complicated, but will reduce the time to perform occlusion processing by about a factor of two.

Version 4: Use a looser approximation of the Occluding Region

Another choice to reduce the occlusion processing time is to use a looser approximation of the Occluding Region. For example, the "L-shaped" SOT Region **4304** in Figure 43 (which is a good approximation) can be made into a simple rectangular region by increasing its area. Figure 44 shows the same two spans **4401 and 4402** as shown in Figure 43, and an SOT Region **4404** that is larger and simpler than the SOT Region **4304** from Version 3 of the method. The SOT Query that generates this SOT Region **4404** is the combination of Equation 124 and Equation 125 such that the looser constraint is used for each field. The query operation is:

(EQ 126)

$$F_{nQ} = F_{nV} \wedge F_{nQ} \wedge (n \neq S_C) \wedge [(x_{nLT} < x_{CR}) \vee (x_{nLB} < x_{CR})] \wedge [(x_{nRT} < x_{CL}) \vee (x_{nRB} < x_{CL})]$$
$$\wedge [[(z_{nLT} < z_{CFT}) \vee (z_{nLB} < z_{CFB})] \vee [(z_{nRT} < z_{CFT}) \vee (z_{nRB} < z_{CFB})]]$$
$$F_{nQ} = F_{nV} \wedge F_{nQ} \wedge (n \neq 1) \wedge (n \neq 2) \wedge$$
$$\wedge [(x_{nLT} < x_{2RT}) \vee (x_{nLB} < x_{2RT})] \wedge [(x_{nRT} < x_{2LB}) \vee (x_{nRB} < x_{2LB})]$$
$$\wedge [[(z_{nLT} < z_{temp2}) \vee (z_{nLB} < z_{2LB})] \vee [(z_{nRT} < z_{temp2}) \vee (z_{nRB} < z_{2LB})]]$$

When this looser approximation is used for the SOT Region **4404**, more Potentially Occluding Spans (on the average) that are actually hidden will be found, thereby reducing performance. However, this may yield a net gain because the number of query operations is reduced.

Transparency

The Span Sorting Rendering Architecture **500** can render transparent geometry (colored geometry that can be seen through) by generating its spans in the same way as those for opaque geometry. Translucency can be handled in a similar way, but this document focuses on transparency. To aid in the rasterization process, the transparency of each span is marked in the Span Memory **408** by one of three methods: 1) another flag bit, F_{nT}, is added to each word **900**; 2) a bit in the Information Pointer Field **814**; or 3) another field, a Translucency Field, is added to each word **900**. The opacity coefficient (i.e., the amount of light transmitted by the transparent geometry) is stored in the Polygon Information Memory **514** along with the other color information.

There are various ways to handle transparent spans within the rasterization method presented in this document. As an aid in the following explanations, Figure 45 (a modified version of Figure 11) shows a set of segment spans **4500 to 4513** in a raster line where the visible opaque surfaces are shown as thick black lines and the visible portions of the transparent spans **4502, 4505, and 4507** are shown as thick shaded lines. This section of the document uses an example with segment spans, but the transparency methods can be applied to trapezoidal spans and quadrilateral spans.

Version 1: Keeping visibility characteristic consistent over the Current Span

This version of the method keeps the visibility characteristic of the Current Span, S_C, consistent by making S_C smaller, if necessary, when a transparent span occludes it. The method follows the frontmost opaque spans, and adds in the effect of transparent spans if they are found to occlude S_C.

For example, in Figure 45, when the SOT Query for S_1 **4501** finds the transparent span S_2 **4502**, S_C is changed to the part of S_1 **4501** between x_{1L} **4524** and x_{2L} **4526**, and S_1 **4501** is also kept as the Saved Span, S_S, so that its part to the right of x_{2L} **4524** will be the next S_C. When this part becomes S_C (i.e, the part of S_1 **4501** between x_{2L} **4526** and x_{3L} **4528**), its SOT Query also finds S_2 **4502**, and the combination of S_1 **4501** and S_2 **4502** is rendered. The combination of S_3 **4503** and S_2 **4502** is processed in a similar way.

When S_4 4504 is reached, it is first processed as its part located between x_{4L} 4530 and x_{5L} 4532 and includes the effect of S_2 4502. Then, when S_C is the portion of S_4 4504 to the right of x_{5L} 4532, the SOT Query finds both S_5 4505 and S_6 4506. At this point, the method and apparatus can be designed to either: 1) read the leftmost opaque occluding span; 2) read the leftmost transparent occluding span; or 3) read the leftmost span or either type. The first of these choices is assumed here. Hence, by a rendering rule similar to Rule 3 2400, S_C is reduced to the portion of S_4 4504 between x_{5L} 4532 and x_{6L} 4534. But, before S_C is rendered, transparent spans in the SOT Region are read, thus obtaining S_5 4505. Because S_5 4505 covers all of S_C (i.e., S_4 4504 between x_{5L} 4532 and x_{6L} 4534), S_C is not further divided, and is rendered as the combination of S_4 4504 and S_5 4505 over the interval from x_{5L} 4532 to x_{6L} 4534. Within the same rendering rule, S_6 4506 is made S_P, and all of it 4506 is designated as S_C.

The span S_6 4506 is processed similarly to S_4 4504 in that its first portion, from x_{6L} 4534 to x_{7L} 4536, is combined with the transparent span S_5 4505. Its second portion, from x_{7L} 4535 to x_{8L} 4536, is combined with the transparent span S_7 4507. And, its third portion, to the right of x_{8L} 4536, is hidden, and then S_7 4507 becomes S_P. From x_{8L} 4536 to the right, the spans are processed be described in previous section because there are no more transparent spans.

Version 2: Dividing the raster line into pieces where depth visibility changes

This version of the method follows, in left-to-right order, the frontmost spans whether they are opaque or transparent. When opaque spans are rendered from left to right up to the x-coordinate where a visible transparent span starts, rendering of the opaque spans is temporarily suspended (in essence, that rendering process is pushed onto a stack), and rendering shifts to the transparent span. Then frontmost transparent spans are rendered from left to right until either: 1) the transparent spans end at an x-coordinate where either only opaque spans or no spans exist; or 2) the frontmost transparent span is occluded by an opaque span. At that point, rendering returns to the rendering of opaque spans that were temporarily suspended (the rendering process stack is popped).

As transparent spans are rendered, they are deleted from the Span Memory 408 in order to keep them from being re-rendered later in the method. Because a transparent span can transition from visible to hidden and then become visible again as the method renders farther to the right, the visible part that is already rendered needs to be deleted to keep it from being re-rendered. This can be done by overwriting the transparent span in the Span Memory 408 and changing its left endpoint. Thus, when a transparent span is rendered, if it is rendered to its right endpoint, it is deleted completely. When a transparent span is partially rendered, only the rendered part is deleted.

In portions of the raster line that have one visible transparent span, the stack-oriented approach renders the transparent spans first, then the opaque spans. In portions of the raster line that have two visible transparent spans (one in front of the other), the stack-oriented approach renders the frontmost transparent spans first, then the second from the front transparent spans, and finally the opaque spans. As long as the rendering process stack does not overflow, this method can render any depth of visible transparent spans.

The raster line can be thought of as being divided into pieces at x-coordinate locations where the visibility depth changes (e.g., from one to two, from two to three, from two to one, etc.). Within these pieces, visible spans are rendered in a front-to-back order. Having the rendering done in front-to-back order allows pixel color values to be correctly blended. This is in contrast to prior art methods (such as the Z-buffer technique) that renders geometry in random order.

Using the spans shown in Figure 45 as an example, this version of the method renders the visible portions of S_0 4500 and S_1 4501 up to x_{2L} 4526. At that point in the method, S_C is the portion of S_1 4501 between x_{2L} 4526 and x_{3L} 4528, and it is pushed onto the rendering process stack. Next, the transparent spans S_2 4502, S_5 4505, and S_7 4507 are rendered from x_{2L} 4526 to x_{8L} 4536. As these spans are rendered, S_2 4502 is deleted, S_5 4505 is deleted, and part of S_7 4507 is deleted by changing its left endpoint to (x_{8L}, z_{temp1}) 4540. Next, the rendering process stack is popped, and S_C is restored to the portion of S_1 4501 between x_{2L} 4526 and x_{3L} 4528. There, rendering of opaque spans resumes, and the visible portions of spans S_1 4501, S_3 4503, S_4 4504, and S_6 4506 are rendered from x_{2L} 4526 to x_{8L} 4536. The next S_C is set to all of S_8 4508, and the balance of

the opaque spans are rendered. The remaining piece of S_6 **4506** is hidden, so the method avoids reading it **4506** from the Span Memory **408**.

Version 3: Perform multiple passes

In this version of the method, a multiple pass strategy is used. In the first pass, all the opaque geometry is rendered with the previously described method of this document. At the end of rendering the opaque spans, all the Valid Flags **816** for the opaque spans have been turned off. Then, in the second pass that renders the transparent spans, the Valid Flags **816** for all the opaque spans are turned on, and each transparent span is then sequentially tested to determine if it is occluded by an opaque span, and then its visible portions are rendered.

As a possible option, as an opaque span is rendered, a query is done to find all transparent spans that are completely behind the opaque span and de-assert the Valid Flags **816** of such transparent spans. This reduces the number of transparent spans that are processed and are hidden, but increases the processing required by each opaque span by adding another query. To reduce this overhead, the query that invalidates transparent spans can be done once after every set of abutting opaque spans. Hence, when the rendering of a set of abutting spans ends, the maximum z-value of any of the visible portions of spans in the set is used as the minimum z-value in the query to find hidden transparent spans. This option is assumed in the example described in the next paragraph. This option may consider a single span as a one-span set for use as occluding transparent spans.

Using the spans shown in Figure 45 as an example, in the first pass of this version of the method renders the visible portions of S_0 **4500**, S_1 **4501**, S_3 **4503**, and S_4 **4504**. At this point in the method, rendering of a set of abutting spans has ended, and transparent spans hidden by the set are invalidated by a query that finds transparent spans completely within the x-coordinate range of x_{1L} **4524** to x_{6L} **4534** and having the z-value of both its endpoints greater than z_{temp2} **4542**. Next, the visible part of S_6 **4506** is rendered. Then, the abutting span set of S_8 **4508** and S_{10} **4510** is rendered, and a query is done to invalidate the transparent spans behind the set. Likewise, when S_{11} **4511** and S_{12} **4512** are rendered, the same sort of query is done. The other opaque spans in the raster line are processed normally. At the beginning of the second pass, all the Valid Flags **816** of the opaque spans are asserted, and then the transparent spans are sequentially processed in a left-to-right order. In the example, S_2 **4502** is read from the Span Memory **408**, an SOT Query is done for it **4502** which finds no occluding spans, and it is therefore rendered. The span S_6 **4506** is processed similarly. For S_7 **4507**, its SOT Query finds S_8 **4508**, so S_7 **4507** is only rendered up to x_{8L} **4536**. Since no other transparent spans are left, this raster line has been completed.

An optional change to this version of the method is to store the transparent spans in a memory separate from the Span Memory **408**. Then, in the second pass, each transparent span is read from this separate memory, has an SOT Query done for it, and has its visible parts rendered.

Additional considerations for transparencies

Polygon coloration is frequently described by texture maps. Some texture maps are defined to have some pixels be 100% transparent (perfectly clear). A typical example use of such a texture map is an image of a tree mapped onto a rectangular "billboard-type" polygon (billboard polygons automatically rotate to face the viewing point), where the corners of the rectangle are clear, and holes within the tree's image can occur. Polygons with such texture maps need to have their spans processed as transparent spans. This is because they do not completely occlude all the geometry behind them. Hence, when a polygon uses a texture map with transparent pixels, spans in that polygon are assumed to be transparent, and the texture map dictates the opacity coefficient for each of its pixels.

When trapezoidal spans (or quadrilateral spans) are used, spans that include corners and edges of polygons that have either a span top or span bottom but not both, can be processed as transparent spans. This can be done because spans of this type do not completely occlude more distant spans, just as transparent spans do. Hence, this section of the document on transparency can be used to process spans that only partially cover the vertical height of the raster line.

Spans Represented by Quadrilaterals

Representing spans with trapezoids has the disadvantage of requiring the location of four points in x-z plane (not pre-

cisely a plane, it has the thickness of one raster line), which, in turn, requires lots of bits of memory. This is especially true if the z-values are 32-bits each.

In order to reduce the memory requirements, the endpoint location of a span can be approximated by two x-values and one z-value. In essence, the two points that describe the endpoint have the same z-coordinate, but have different x-coordinates. Therefore, the number of fields for storing coordinates is reduced from eight to six. This introduces a small amount of error in the antialiasing methods described above, but the error is only significant for polygons that are nearly parallel to the x-z plane. For these polygons, the look direction is very close to being a tangent to the plane of the polygon. However, for most applications, this errors is acceptable because the projection of these polygons onto the viewing plane **102** has not changed.

Figure 46 shows the trapezoidal spans S_1 **4601**, S_3 **4603**, and S_4 **4604** as dashed lines, and shows the quadrilateral approximations of these spans, S'_1 **4611**, S'_3 **4613**, and S'_4 **4614** as solid lines. In this figure, endpoints for the quadrilateral spans are always a horizontal line segment. Figure 47 is similar to Figure 39, except that all the trapezoidal spans have been replaced by their trapezoidal approximation. Note that some of the trapezoidal spans appear to fold over themselves. Trapezoidal spans can be processed by similar methods to those described in this document for trapezoidal spans.

Preservation of Rendering Order

For some applications, the order of rendering must be preserved. A typical example is the "painter's algorithm" (see Foley Reference, page 674) as used in computer windowing systems. To render an inner rectangle with a boarder, first the boarder is rendered as a filled rectangle, then the inner rectangle is rendered, over-writing most of the pixels in the boarder rectangle. If the rendering order of these two rectangles is reversed, then the entire area will have the color of the boarder rectangle because the inner rectangle will be completely over-written. If these two rectangles have a distinct placement due to different z-coordinates, then they will be rendered correctly. However, if no z-coordinate is specified (the default would be the depth of the viewing plane **102**), then the order must be preserved. This can be done by having the Bucket Sort Processor **504** and the Current Polygon Processor **506** maintain the order in which they are received from the Polygon Processor **502**. The Query Processor would maintain the rendering order by placing spans for later input polygons into later positions in the span list. Then, as rendering proceeds, rendering order is preserved because, in the case of ties in z-coordinate, spans earlier in the list will be rendered first and later geometry with the same z-coordinate will be assumed to be hidden.

Method and Apparatus Options and Possible Enhancements

Add to single buffered VGA graphics card

The video output for computers (for example, VGA and SVGA interfaces on IBM compatibles) generally includes a frame buffer memory that stores the pixel information sent to the display device (i.e., the CRT). The 3D graphics rendering methods and/or apparatus described here can be added to such a display system. The CPU (or DMA controller) writes geometry into the renderer, and it gets processed and stored into the Bucket Sort Memory (which may need to be double buffered). When the geometry is done being written, the rest of the pipeline synchronizes to the computer display, and generates spans before they are needed, overwriting the previous frames's data in the computer's frame buffer. Hence, a low cost, but high performance, 3D graphics accelerator can be added to a computer without adding more memory to the frame buffer.

Turn off antialiasing on specific polygon edges

The antialiasing method described above assumes that every polygon has independent color values; and when abutting spans are found, the pixels that include both of the spans need to have their colors be a blend from the two polygons. However, polygons that share an edge often have exactly the same color information along the edge. For example, this occurs for Gouraud shaded polygons that have the same color and surface normal information at both ends of the same polygon edge. For these types of edges, the pixel blending can be ignored because two identical colors will be blended together. Such polygon edges can be tagged as "skip antialiasing" when they are supplied to the renderer. Then, if such edges are found to be part of an abutting span pair, the pixel color value for one of them (say, the one on the right) is skipped, thereby saving computation. Edges that are tagged but are not part of an abutting span pair are processed by the antialiasing methods described above. Prior art renderers can

not take advantage of this saving because they do not search for abutting spans or polygons. That is, prior art renderers generally assume every polygon is independent from all other polygons. If tagging is not done (due possibly to constraints imposed by the interfacing software), the Span Sorting Rendering Pipeline **400** can automatically generate such tags for edges within polygon meshes and strips where an edge shared between two polygons has the same color information for both polygons.

***Store edge pairs rather than polygons in Bucket Sort Memory* 404**

Rather than store polygons in the Bucket Sort Memory **404**, polygon edge pairs can be stored. This means polygons are horizontally sliced so that each horizontal strip of a polygon does not contain any polygon corners. Hence, polygons are horizontally sliced at the y-coordinate (or nearby raster line boundary) of corners that are at neither y_{START} or y_{END}. This polygon slicing makes downstream processing easier by eliminating the need to test for when to change to a new edge at the corner of a polygon.

***Eliminate Current Polygons Memory* 406 *and read from Bucket Sort Memory* 404**

By adding more "smarts" to the Bucket Sort Processor **504**, the Current Polygon Memory **406** and the Current Polygon Processor **506** can be eliminated. This means polygons within the current raster line are fed, at high speed, directly from the Bucket Sort Processor **504** to the Span Generation Processor **508**.

This has the disadvantage of accessing the Bucket Sort Memory **404** at a much higher rate, thus probably requiring the Bucket Sort Memory **404** to be fast memory, rather than slow memory. Another disadvantage is the need for a more complex data structure within the Bucket Sort Memory **404**, rather than a simple linked list. One possible way to do this is to sort polygons within each "bucket" according to its y_{END} parameter in reverse order. Then, at each raster line, every "bucket" is read until a polygon is reached that has its y_{END} parameter greater than the y-coordinate of the current raster line, thereby indicating that this polygon and those after it in the same bucket are not in the current raster line.

A decision to use this alternative must be based on an engineering decision between a large fast memory versus a large slow memory coupled with a small fast memory. Also, the processing time required to doubly sort (i.e., into buckets and then also within buckets) the Bucket Sort Memory **404** must be considered.

Front and rear clipping planes

By adding clipping planes, the view volume (shaped as an infinite pyramid) is changed into a frustum. Right before spans are written into the Span Memory **408**, the z-value of their endpoints are compared to the z-location of the front and rear clipping planes. If both endpoints are nearer then the front clipping plane or farther than the rear clipping plane, then the span is discarded. If one endpoint is farther than the rear clipping plane, then that end of the span is shortened so that its endpoint is on the rear clipping plane. If one endpoint is nearer than the front clipping plane, then that end of the span is shortened so that its endpoint is on the front clipping plane. This requires a set of comparators to be added to the either the Span Generation Processor **508** or the Query Processor **510**.

Multiple arbitrary clipping planes

An arbitrary clipping plane could be done the same way as simple front and rear clipping planes, except, right before a span in written into Span Memory **408**, Span Interaction Parameters between the span and the span of clipping plane are generated, and these are used, as described above, to determine the Span Interaction Type (SIT). The span is then kept, clipped, or discarded depending on the SIT.

This could also be done by adding dummy spans into the Span Memory **408** that represent the location of an arbitrary clipping plane. Then the rasterization methods described above could be altered to make sure all rendered spans are behind these dummy spans.

Geometry Cache

A memory at the input to the pipeline could be added to store geometry as it is input for rendering. Then, when subsequent scenes are rendered, this stored (or "cached") geometry can be re-rendered (with possibly changed translation matrices), thereby saving cycles on the system bus. This could solve a bottleneck caused by having a renderer that can process more geom-

etry per second than the host computer bus can support. It can only be applied to geometry that has not changed shape since its previous rendering. "Squishy" objects would generally need to be supplied to the renderer for every scene.

Use only one page of Span Memory 408

It is possible to use only one page of Span Memory **408**. This is done by shifting valid spans toward upper words, leaving lower words invalid. Then, as new spans are generated, store them into the lower words. This strategy can be thought of as a single bank of memory that is separated into two pages by a moving boundary. This can be done because, as a raster line is processed, spans are deleted, while the simultaneous process of preparing for the next raster line generates spans.

As an alternate approach, the fixed Word Number Field **802** can be changed to a variable Tag Field. Unique (and random will do) tags for each span are written into the Tag Field, and designate if: 1) a span is in the current raster line or the next raster line; and 2) be a unique identifier for the span. Furthermore, the Tags can be included in the query process such that spans from the current raster line can be intermixed with those in the next raster line. As long as spans in the same raster line are kept in order with respect to each other, then spans from an unrelated raster line can be intermixed as long as they are ignored. This could be done by maintaining two sets of Valid Flags **816**, one for each raster line. Spans from more raster lines could be simultaneously maintained by adding more sets of Valid Flags **816**.

Separate Read and Write Busses in the Span Memory Page

The circuit for the SMCCAM Cell **3400** or **3500** can be modified to have both an input bus and an output bus. This allows simultaneous read and write operations. The is specially useful if there is only one page of Span Memory **408** as described in the previous section.

Computing SIP and SIT only when bounding boxes overlap

Rather than computing the SIT for every New Span, S_N, compute it sequentially with less hardware only when bounding boxes overlap. This sacrifices time in order to save circuitry.

Split raster line into smaller pieces to save on SMCCAM memory

The total number of words in the Span Memory Page **800** imposes a limit on the number of spans that can be processed for a given raster line. This limitation can be overcome by conditionally splitting the raster line in half (or more pieces if necessary), and passing through the span data twice. This decision can be done on the fly as a scene is rendered or by an a priori decision before rendering starts.

Have an overflow RAM to save spans that are shifted out during sort operations

Rather than splitting the raster line in to pieces, the Span Memory Page **800** can be augmented with an "overflow RAM" that stores spans as they are shifted out of the last SMCCAM Word **900**. Shifted out spans would start at far right of raster line, so the left side of the raster line could be processed normally until x_{CL} gets to left of leftmost shifted out span. Then, shifted out spans could be re-written into the Span Memory Page **800** because may spans have been deleted (i.e., invalidated) by the rasterizing process.

Inclusion rendering primitives other than polygons

Geometry primitives other than polygons (such as CSG primitives) can be processed by the Span Sorting Rendering Pipeline **400**. This can be done as long as the y_{START} and y_{END} parameters can be computed (so the primitive can be stored into the Bucket Sort Memory) and spans with bounding boxes can be generated. The corners of the bounding box are used as the Span Parameters, and SOT Queries can be used to find the visible portions of the bounding boxes. Another set of Span Interaction Types would be needed for the specific set of geometry primitives.

Low cost version can use CPU for front end

The host CPU, or a dedicated general-purpose processor, could perform the tasks of the Polygon Processor **502**, thus feeding geometry in screen coordinates (in object-precision) to the rest of the pipeline. Similarly, such a CPU could perform more of the front end of the pipeline **400**. For example, a CPU and its general purpose memory could perform the tasks of the Polygon Processor **502**, the Bucket Sort Processor **504**, and the Bucket Sort Memory **404**, and feeding polygons that start on

the present raster line to the rest of the pipeline.

Alternate sorting mechanism could use up-counting tag

The Word Number Field **802** could be replaced with a conditional up-counter, in order for shifting to be eliminated from the SMCCAM. When a new span is being written to the SMCCAM, a query determines which spans should be later in the sorted list than the new span. Such "later" spans have their up-counter incremented, and no shifting is performed. The up-counter for the new span would be assigned the lowest counter number in the list of "later" spans before the up-count takes place.

This approach means the "get the first span in the sorted list" becomes "find the smallest counter value" in the Span Memory **408**. Hence, the sorting (i.e., shifting) is replaced with a new search type. For this to be fast, the hardware would need to have a "find the smallest" function associated with the set of up-counters. Also, this "find the smallest" could be performed in bit-serial method.

Using a generic CAM

Generic, prior art CAMs could be used, but magnitude comparisons would be bit sequential. This would greatly slow the processing, but may afford some cost savings in the hardware. This alternative would probably use the up-counting tag, as described in the previous section.

Using multiple registers per arithmetic comparator

The above description of the SMCCAM assumes a one-to-one match between memory register and arithmetic comparators. An alternate approach is to have all the memory registers in the same word share a single arithmetic comparator. This saves on circuitry, but slows the processing. However, since not all fields participate in every query operation, the performance loss is not proportional to the total number of fields.

Other alternate approaches could share comparators between words, or between words and fields. These approaches further reduce the amount of hardware, but further reduce the performance. If any of these shared comparator approaches are used, then a standard six-transistor RAM cell should be used because it provides the greatest density. Hence, the searching and sorting hardware could be made from a set of multiple-word memories, where each such memory has one or more comparators associated with it.

Using a RAM-based sorting processor

An alternative to using a CAM of any type can be done by having a fast linked list sorting processor. By utilizing fast memory, spans could be sorted into a linear list in the same order as described above (using an SMCCAM). But, since spans are added and deleted frequently, this approach might be very slow. The advantage is the ability to use fast off-the-shelf RAMs that are readily available.

To aid in SOT Query operations, a two-dimensional data structure could be used that sorts the spans in both the x and z dimensions at the same time. Such a searching and sorting structure is described on pages 24 to 55 of "Data Structures and Algorithms 3: Multi-dimensional Searching and Computational Geometry", by Kurt Mehlhorn, a volume in the EATCS series on Monographs on Theoretical Computer Science, edited by Brauer, W., et. al.,ISBN 0-387-13642-8, published by Springer-Verlag, New York, 1985.

Shadow processing

Because shadow computations are essentially the same as hidden surface removal, the Span Sorting Renderer **500** can be used for shadow computation. This can be done by utilizing multiple Span Sorting Renderers **500** in parallel, or it can be done by time-sharing one Span Sorting Renderer **500**.

What is Claimed is:

1. A sorting magnitude comparison content addressable memory (SMCCAM) comprising:

a plurality of words of memory storage, each said word having a plurality of fields for storing a polygon's span parameters;

query means for performing a query operation between at least two different polygon spans based on comparisons of said stored span parameters between said at least two different polygon spans to determine which of said spans is visible and which of said spans is occluded.

2. The SMCCAM in Claim 1, wherein said query means further comprises Span Occluding Test (SOT) means for performing a Span Occluding Test to determine which of said spans is visible and which of said spans is occluded.

3. The SMCCAM in Claim 2, wherein said Span Occluding Test (SOT) query means comprises:

means for generating a span occulting test (SOT) region based on span parameters for a Current Span Portion (SC) having current span endpoints (x_{CL}, z_{CL}) and (x_{CR}, z_{CR}) being tested to determine if it is occluded;

means for identifying a selected span as an occluding span if said selected span having endpoints (x_{nR}, z_{nL}) and (x_{nR}, z_{nL}) lies within said Span Occulting Test Region according to the relationship: Selected Span is within SOT Region if $\{ (x_{nL} < x_{CR}) \wedge (x_{nR} > x_{CL}) \wedge [(z_{nL} < z_{CF}) \vee (z_{nR} < z_C))]$, where z_{CF} = Maximum Of (z_{CR}, z_{CL}).

4. The SMCCAM in Claim 3, wherein said operation of computing z_{CF} = Maximum Of (z_{CR}, z_{CL}) is performed external to said SMCCAM and said SMCCAM further comprises:

input bus means for receiving values of x_{CL}, x_{CR}, and z_{CF}, into said SMCCAM so that said x_{CL}, x_{CR}, and z_{CF} values are compared to said selected span endpoints to identify said selected span as an occulting span;

said SMCCAM testing all of said polygon spans simultaneously and in parallel;

said SOT query result being stored in a field of said SMCCAM word.

5. The SMCCAM in Claim 4, further comprising:

means for preventing words storing invalid data from being identified as occluding spans; and

means for preventing a current span from being identified as an occulting span for said current span.

6. The SMCCAM in Claim 4, wherein each said span is modeled by geometric properties of said span including: coordinates of the left endpoint of the span, and coordinates of the right endpoint of the span.

7. The SMCCAM in Claim 6, wherein said spans are represented by trapazoids, and wherein said endpoints of each span are two (x,z) points, one for the top edge of the current raster line and one for the lower edge of said current raster line.

8. The SMCCAM in Claim 6, wherein said spans are represented by quadrilaterals, and wherein said endpoints of each span are an (x_U,x_L,z) triplet including one z-coordinate and an x-coordinate for the upper and lower edges of the current raster line.

9. The SMCCAM in Claim 6, wherein said spans are represented by line-segments, and wherein said endpoints of each span are an (x,z) point within the current raster line.

10. A sorting magnitude comparison content addressable memory (SMCCAM) comprising:

a plurality of memory registers, each word comprised of a multiplicity of fields, each said field including:

means for storing a number;

comparison means for performing an arithmetic comparison between said stored number and another number broadcast to and received by all of said memory registers (words); and

means for shifting its stored number contents to the corresponding field in the next word, where said shifting is conditionally based upon the results of said arithmetic comparisons;

said memory registers storing a set of spans and said comparison means performing parallel searching and sorting operations to find the visible span portions.

11. The SMCCAM in Claim 10, wherein each said n^{th} SMCCAM word includes:

seven numeric fields including:

a Word Number Field (w) that stores a unique identifying number address for each word;

a Left X Field (x_{nL}) that stores the x-coordinate of the left endpoint of a span;

a Left Z Field (z_{nL}) that stores the z-coordinate of the left endpoint of a span;

a Right X Field (x_{nR}) that stores the x-coordinate of the right endpoint of a span;

a Right Z Field (z_{nR}) that stores the z-coordinate of the right endpoint of a span;

a Span Derivative Field (d_n) that stores the dz/dx slope of the span; and

an Information Pointer Field (i_n) that stores a pointer into a Polygon Information Memory for color information for the span's polygon; and

a plurality of flag fields including:

a Valid Flag (F_{nV}), a single bit value indicating whether the SMCCAM Word is storing valid data; and

a Query Flag (F_{nQ}), a single bit value indicating whether the SMCCAM Word responded positively to a query operation;

each said flag bit having a corresponding wired-nor bus that indicates whether all words have said flag bit turned off, such that for the Valid Flag (F_{nV}), if FnV is false for all words (n), then a signal AllWordsInvalid is asserted, and if FnQ is false for all words (n), then a signal NullQueryResponse is asserted;

said two wired-nor signals, AllWordsInvalid and NullQueryResponse, providing means for feeding back query results to an external controller in a Query Processor, so that said query Processor can make decisions, including branch decisions.

12. The SMCCAM in Claim 11, wherein each of said numeric field w, x_{nL}, z_{nL}, x_{nR}, z_{nR}, d_n, and i_n have a corresponding data bus BusW, BusXL, BusZL, BusXR, BusZR, BusD, and BusI for communicating a value to each of said numeric fields;

said numeric field w, x_{nL}, z_{nL}, x_{nR}, z_{nR}, and d_n, each comprising a memory portion and a comparator portion;

said w field memory portion providing fixed readable memory and said w comparator portion providing an equality comparison circuit used for query operations, that compares the fixed memory value to data received over said input bus, BusW;

said numeric field x_{nL}, z_{nL}, x_{nR}, z_{nR}, d_n, and i_n field memory portions each providing readable and writable memory and said comparator portions providing an arithmetic magnitude comparison including greater-than, less-than, equal-to, operations and combinations thereof, used for query operations, that compares the stored memory value in said memory portion to the data on the corresponding input bus;

said Information Pointer Field, iw, having a readable writable memory portion.

said array busses supplying query data to all said words via said Array Busses;

means for performing a query operation by all said SMCCAM Words simultaneously and in parallel in response to receipt of said query data;

a Query Logic Array comprising a Query Logic circuits associated with each said SMCCAM Word for computing

query operation results;

said Query Logic receiving the query operation results from said equality and arithmetic comparators and said Valid Flag and said Query Flag values;

a query controller generating a QueryCntrl signal for selecting a selected Boolean operation;

means for performing said selectable Boolean operation on said query operation results, said Valid Flag values, and said Query Flag values, for every word, to generate a query result bit for every word;

means for writing said query result bit back into either said Valid Flag or said Query Flag depending upon a control signal;

the set of all Valid Flags for all said SMCCAM words comprising a Valid Flag Word, and the set of all Query Flags for all SMCCAM words comprising a Valid Flag Word;

said query results identifying which of said words satisfied the query operation parameters.

13. A query processor apparatus comprising:

means receiving each raster line of data;

means for receiving spans from the Span Generation Processor;

means for determining the visible span portions;

means for sending visible span portions of said received spans to an external Rasterize Processor;

first Span Memory Page Means for receiving Span Parameters from an external Span Generation Processor;

second Span Memory Page Means for query interrogation to determine the visible span portions;

multiplexer means (Query Processor crossbar) for swapping data between said span memory page means and said second span memory page means;

a register file subdivided into a plurality of registers for storing Span Parameters including:

a plurality of registers for storing the Current Portion of a span, SC;

a plurality of registers for storing the Present Span, SP;

a plurality of registers for storing the New Span, SN;

a plurality of registers for storing the Saved Span, SS; and

a plurality of temporary registers for storing for storing data including span intersection location data;

a Query Processor Controller for determines the sequence of operations in said Query Processor, including the sequence of query operations;

SIT Generation Logic means coupled to receive said Current Portion Span Parameter (SC) and said New Span Parameter (SN) and generating a Span Interaction Type (SIT) Result;

compare and offset computation means coupled to said span parameter registers and receiving parameters from said registers for computing said SIT Result; and

selectably operable Intersection Computation Block means for computing an intersection point between said Current Span (SC) and said New Span (SN), said operation being selected by predetermined rules.

14. A span sorting rendering apparatus comprising:

a polygon processor;

a bucket sort processor;

a bucket sort memory coupled to said bucket sort processor for storing data;

a current polygon processor;

a current polygon memory coupled to said current polygon;

a span generation processor;

a query processor;

a span memory coupled to said processor;

a rasterize processor; and

a raster line memory coupled to said rasterizer.

15. In a memory apparatus having a plurality of storage registers for storing data, and a data structure defined within said memory apparatus, a method for testing for span occultation of a current span by a selected different span, said method comprising the steps of:

receiving a polygons' span parameters and storing said parameters in predetermined storage locations within said data structure;

performing a span occluding test query operation between at least two different polygon spans based on comparisons of said stored span parameters between said at least two different polygon spans to determine which of said spans is visible and which of said spans is occluded; and

storing said span occluding test query results in said data structure.

16. The method in Claim 15, wherein said Span Occluding Test (SOT) query comprises the steps of:

generating a span occulting test (SOT) region based on span parameters for a Current Span Portion (SC) having current span endpoints (x_{CL}, z_{CL}) and (x_{CR}, z_{CR}) being tested to determine if it is occluded;

identifying a selected span as an occluding span if said selected span having endpoints (x_{nR}, z_{nL}) and (x_{nR}, z_{nL}) lies within said Span Occulting Test Region according to the relationship: Selected Span is within SOT Region if $\{ (x_{nL} < x_{CR}) \wedge (x_{nR} > x_{CL}) \wedge [(z_{nL} < z_{CF}) \vee (z_{nR} < z_C)]\}$, where z_{CF} = Maximum Of (z_{CR}, z_{CL}).

17. The method in Claim 16, wherein said method further comprises:

receiving values of x_{CL}, x_{CR}, and z_{CF}, into said memory so that said x_{CL}, x_{CR}, and z_{CF} values are compared to said selected span endpoints to identify said selected span as an occulting span;

testing all of said polygon spans simultaneously and in parallel; and

storing said query result in a field of said memory.

18. The method in Claim 17, wherein each said span is modeled by geometric properties of said span including: coordinates of the left endpoint of the span, and coordinates of the right endpoint of the span.

19. The method in Claim 17, wherein said spans are represented by trapazoids, and wherein said endpoints of each span are two (x,z) points, one for the top edge of the current raster line and one for the lower edge of said current raster line.

20. The method in Claim 17, wherein said spans are represented by quadrilaterals, and wherein said endpoints of each span are an (x_U,x_L,z) triplet including one z-coordinate and an x-coordinate for the upper and lower edges of the current raster line.

AMENDED CLAIMS
[received by the International Bureau on 03 January 1997 (03.01.97); original claims 1-20 replaced by amended claims 1-37 (8 pages)]

1. A sorting magnitude comparison content addressable memory (SMCCAM) apparatus comprising:

Means for storing a plurality of words, each of said words comprising a plurality of data fields, each of said data fields being divided into a plurality of data bits;

Means for providing an input comprising a plurality of input fields matching some of said data fields, each of said input fields divided into input bits so as to have a one-to-one bit correspondence to said data bits in said data fields in said words;

Query means for simultaneously comparing said plurality of input fields to all said words, with simultaneous field comparisons such that each said data field is compared to its corresponding input field, and for generating a one-bit query result for each said word which query result is true when all said data fields within said word which are compared to one of said input fields compare favorably to each corresponding input field;

Flag memory storage means for storing a flag bit equal to said query result for each of said words; and

Shifting means for conditionally shifting data stored in said data fields of each said word to corresponding fields of a different adjacent word said flag bits stored in said words.

2 A sorting magnitude comparison content addressable memory (SMCCAM) apparatus as in Claim 1 wherein said flag memory means include means for storing a plurality of said flag bits associated with each of said words.

3. A sorting magnitude comparison content addressable memory (SMCCAM) apparatus as in Claim 2 wherein said flag memory means includes means for outputting a multiplicity of output flag bits for each said word into query logic means which performs a selectable Boolean operation for each said word on both said output flag

bits and said query result to form a flag result for each said word and stores said flag result into said flag memory.

4. A sorting magnitude comparison content addressable memory (SMCCAM) apparatus memory as in Claim 3 including control means for causing said fields comparisons to be one of a plurality of different query comparison operations including either equality, less-than, less-than-or-equal-to, greater-than, greater-than-or-equal-to, inequality, of don't care.

5. A sorting magnitude comparison content addressable memory (SMCCAM) apparatus as in Claim 4 including at least one priority resolver means for selecting the highest priority said output flag bit from the multiplicity of said output flag bits for purposes of selecting one said word for:

reading or writing; or

determining the set of set words perform said conditional shifting.

6. In a graphical processing system for processing 3-dimensional object geometry data and rendering at least some of said object geometry data on a 2-dimensional display screen, a method for rendering a scan line comprising the steps of:

(a) Transforming an object geometry data into a polygonal representation, said polygonal representation comprising at least one polygon, each or said polygon defined by a set of polygon parameters including polygon vertices having display screen relative coordinates;

(b) Sorting said polygons using a bucket sorting routine wherein a separate memory bucket is allocated for each display raster line and a polygon is placed into the one particular bucket that corresponds to its starting raster line; and

(c) For each display raster scan line:

(i) Maintaining a list of all current polygons that intersect the raster scan line currently being rendered;

(ii) Generating a span for each polygon that intersects the current raster scan line based on geometric properties of said polygon including said polygon parameters;

(iii) Storing geometric properties of each said generated span into a span memory; and

(iv) Performing span occluding tests to determine which spans or portions of spans are visible in the rendered scene; where said span occluding test comprises the steps:

(1) selecting a current span portion which is part of a span that is potentially visible;

(2) determining the leftmost, rightmost, and farthest spatial coordinates in the said current span portion;

(3) performing a query operation on the said stored geometric properties in said span memory to find all said spans whose stored geometric properties include: a spatial coordinate located between said leftmost and said rightmost spatial coordinates of the said current span portion; and a spatial coordinate closer than the said farthest spatial coordinate of the said current span portion.

7. The method in Claim 6, wherein said span memory comprises a data array in a random access memory (RAM).

8. The method of Claim 6, further comprising the steps of: rasterizing said spans or said portions of spans that are visible.

9. The method in Claim 6, wherein said step of performing span query operations further comprises the steps of:

(a) comparing said span data being stored into said span memory to span data previously stored into said span memory to thereby compare first stored data directly with second stored data; and

(b) inserting said span data being stored into said span memory so as to maintain a sorted list of said span data within said span memory.

10. The method in Claim 9, wherein said step of storing geometric properties of each said generated span further comprises sorting said spans in order of precedence by 1) left x-coordinate, 2) left z-coordinate, and 3) span derivative.

11. The method in Claim 6, wherein said rendering is performed by identifying visible polygon spans than can be rendered without performing pixel-by-pixel z-buffer comparisons.

12. The method in Claim 6, wherein said step of maintaining a list of all current polygons includes the steps of: discarding the sorted list of polygon spans from processing a previous raster line and building a new sorted list for each raster line as it is processed.

13. The method in Claim 6, wherein said display screen relative coordinates include the location in object - precision screen coordinates of polygon vertices for said object geometry.

14. The method in Claim 6, wherein said coordinates are not limited to coordinates within the bounding area of said display screen.

15. The method in Claim 6, wherein said step of sorting said polygons using a bucket sorting routine includes the steps of generating one linked list per bucket, and using said linked list to perform said sorting.

16. The method in Claim 6, wherein said polygon parameters include: the location of said polygon vertices in object-precision screen coordinates, color information, edge derivatives for each polygon edge, starting raster line, ending raster line, and span derivative.

17. The method in Claim 6, wherein each said span can be modeled as a member of the set consisting of a line segment, a trapezoid, a quadrilateral, or combinations thereof.

18. The method in Claim 6, wherein said span is defined by span parameters including:

(a) coordinates of a first span endpoint;

(b) coordinates of a second span endpoint;

(c) a span derivative dz/dx; and

(d) an information pointer comprising an address into a polygon information memory store where color information for said polygon is stored.

19. The method in Claim 6, wherein said span memory comprises a double buffered span memory having first and second banks, wherein said span geometric properties are written to said first bank; and wherein simultaneous to writing span geometric properties into said first bank, span geometric properties in said second bank are processed to identify visible span portions.

20. The method in Claim 6, wherein said rendering is performed with sufficient speed to provide real time image generation.

21. The method in Claim 6, wherein said span memory comprises a said sorting magnitude comparison content addressable memory (SMCCAM), and said sorting magnitude comparison content addressable memory (SMCCAM) performs said query operations.

22. The method of Claim 21, further comprising the steps of: rasterizing said spans or said portions of spans that are visible.

23. The method in Claim 22, wherein said step of performing span query operations further comprises the steps of:

(a) comparing said span data being stored into said span memory to span data previously stored into said span memory to thereby compare first stored data directly with second stored data; and

(b) inserting said span data being stored into said span memory so as to maintain a sorted list of said span data within said span memory.

24. The method in Claim 21, wherein said step of storing geometric properties of each said generated span further comprises sorting said spans in order of precedence by 1) left x-coordinate, 2) left z-coordinate, and 3) span derivative.

25. The method in Claim 21, wherein said rendering is performed by identifying visible polygon spans than can be rendered without performing pixel-by-pixel z-buffer comparisons.

26. The method in Claim 21, wherein said step of maintaining a list of all current polygons includes the steps of: discarding the sorted list of polygon spans from processing a previous raster line and building a new sorted list for each raster line as it is processed.

27. The method in Claim 21, wherein said display screen relative coordinates include the location in object - precision screen coordinates of polygon vertices for said object geometry.

28. The method in Claim 21, wherein said coordinates are not limited to coordinates within the bounding area of said display screen.

29. The method in Claim 21, wherein said step of sorting said polygons using a bucket sorting routine includes the steps of generating one linked list per bucket, and using said linked list to perform said sorting.

30. The method in Claim 21, wherein said polygon parameters include: the location of said polygon vertices in object-precision screen coordinates, color information, edge

derivatives for each polygon edge, starting raster line, ending raster line, and span derivative.

31. The method in Claim 21, wherein each said span can be modeled as a member of the set consisting of a line segment, a trapezoid, a quadrilateral, or combinations thereof.

32. The method in Claim 21, wherein said span is defined by span parameters including:

(a) coordinates of a first span endpoint;

(b) coordinates of a second span endpoint;

(c) a span derivative dz/dx; and

(d) an information pointer comprising an address into a polygon information memory store where color information for said polygon is stored.

33. The method in Claim 21, wherein said span memory comprises a double buffered span memory having first and second banks, wherein said span geometric properties are written to said first bank; and wherein simultaneous to writing span geometric properties into said first bank, span geometric properties in said second bank are processed to identify visible span portions.

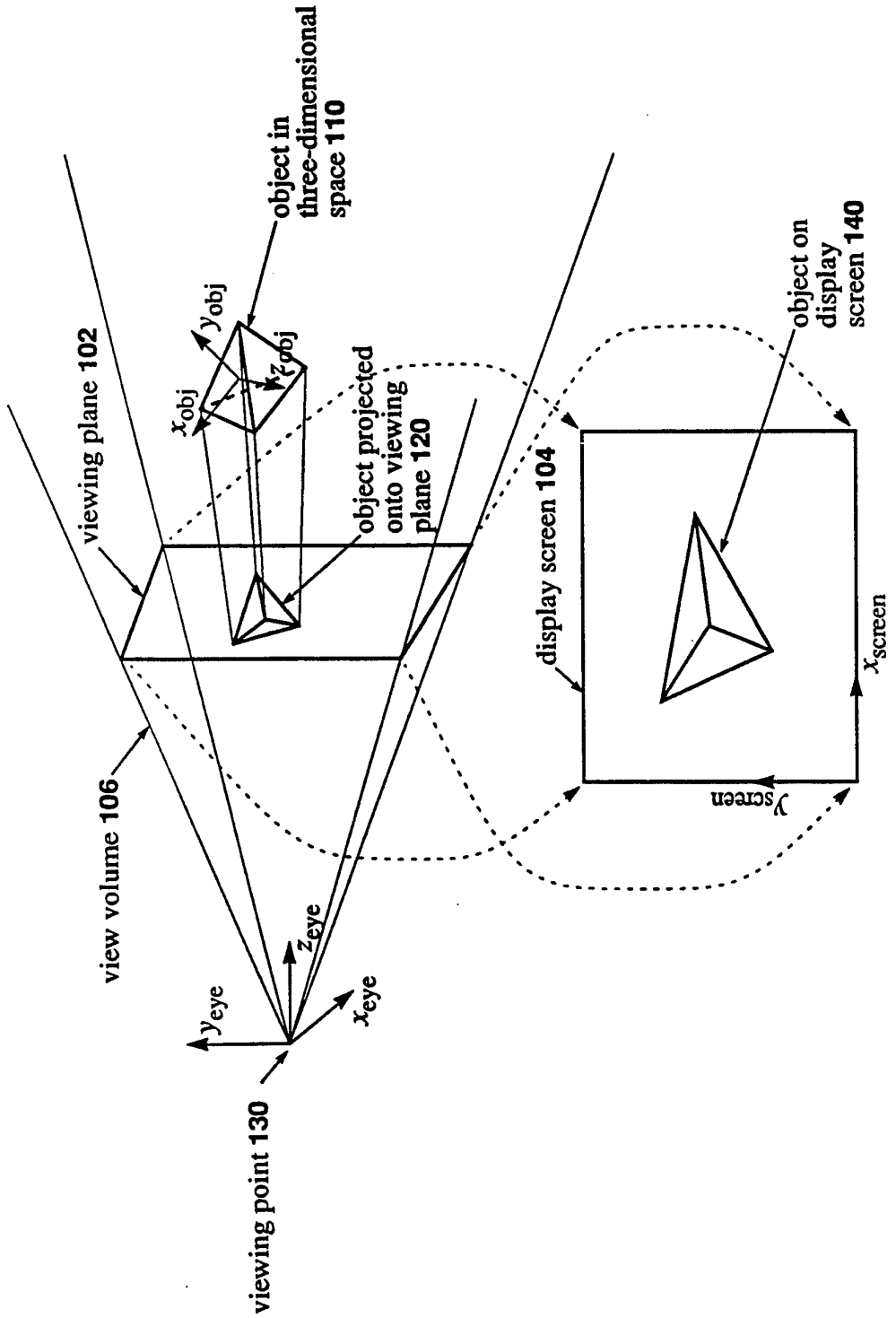
34. The method in Claim 21, wherein said rendering is performed with sufficient speed to provide real time image generation.

35. The method in Claim 21, wherein said new sorted list is generated within about two clock cycles per stored data value, whereas during first said clock cycle, a query operation determines which said words perform said conditional shifting; and during second said clock cycle, said geometric properties of said span are written into said span memory.

36. The method in Claim 21, wherein hidden surface removal is performed by processing spans rather than pixels, thereby reducing computation for scenes with any significant depth complexity.

37. The method in Claim 21, wherein said polygon spans are stored in a SMCCAM, and wherein said step of performing a query further comprises directly querying the entire set of polygon spans stored in said SMCCAM to identify visible spans so that the number of comparison operations is proportional to the number of visible polygon spans which is a number smaller than the total number of spans.

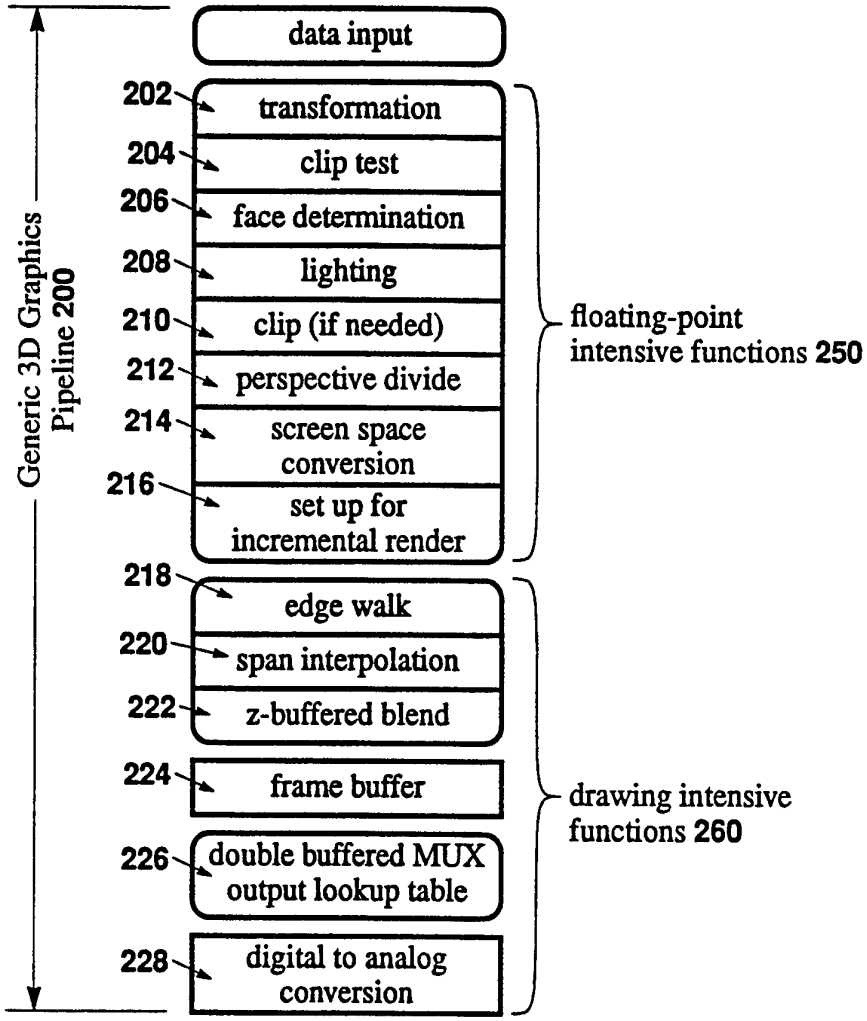
Figure 1


object in three-dimensional space 110
y_{obj}
z_{obj}
x_{obj}
viewing plane 102
object projected onto viewing plane 120
object on display screen 140
display screen 104
x_{screen}
y_{screen}
view volume 106
z_{eye}
y_{eye}
x_{eye}
viewing point 130

2 / 56

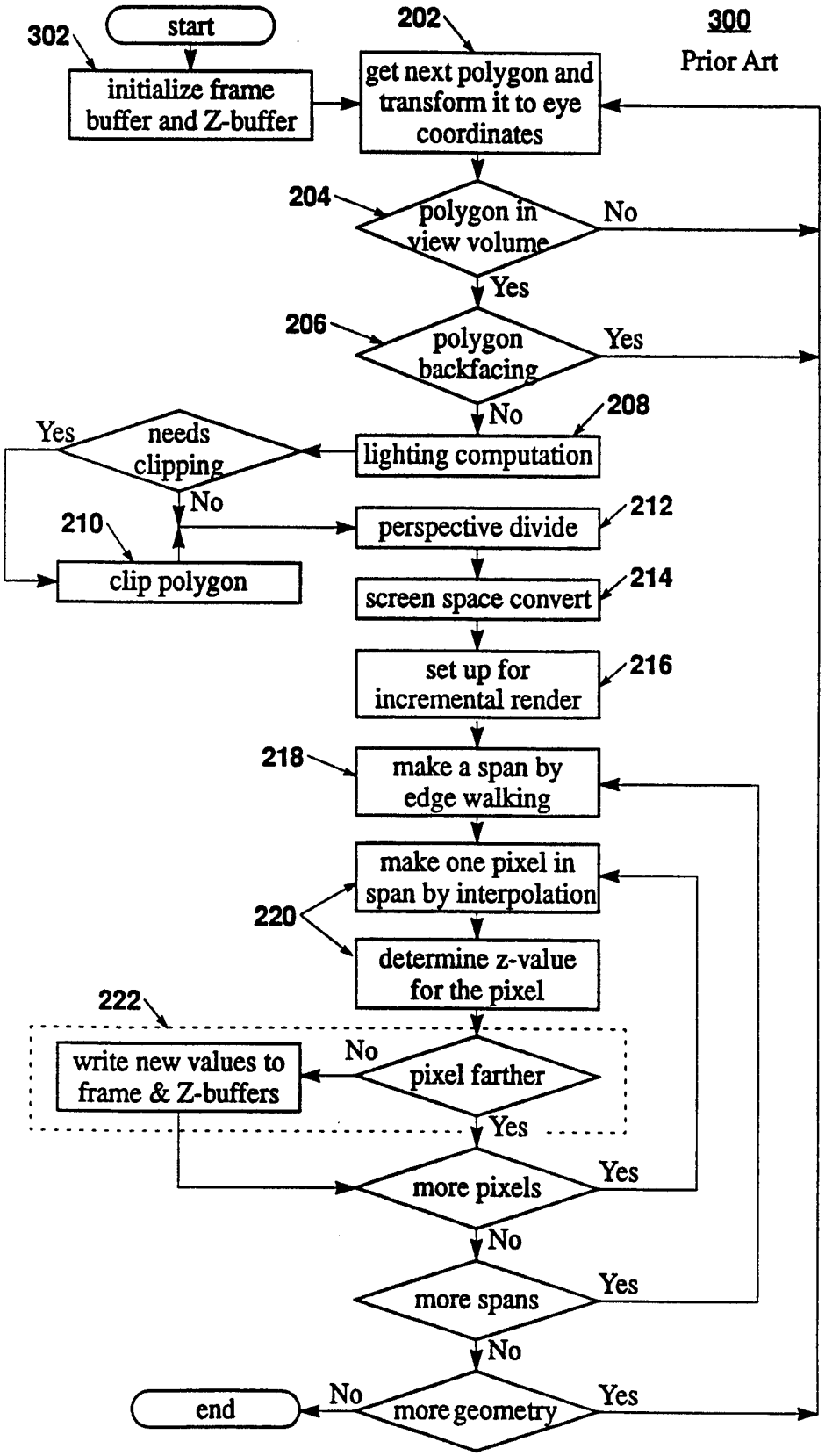
Figure 2

200
Prior Art



3 / 56

Figure 3


start
302
initialize frame buffer and Z-buffer
202
get next polygon and transform it to eye coordinates
300
Prior Art
204
polygon in view volume
No
Yes
206
polygon backfacing
Yes
No
208
lighting computation
Yes
needs clipping
No
210
clip polygon
perspective divide
212
screen space convert
214
set up for incremental render
216
218
make a span by edge walking
make one pixel in span by interpolation
220
determine z-value for the pixel
222
write new values to frame & Z-buffers
No
pixel farther
Yes
more pixels
Yes
No
more spans
Yes
No
end
No
more geometry
Yes

4 / 56

Figure 4

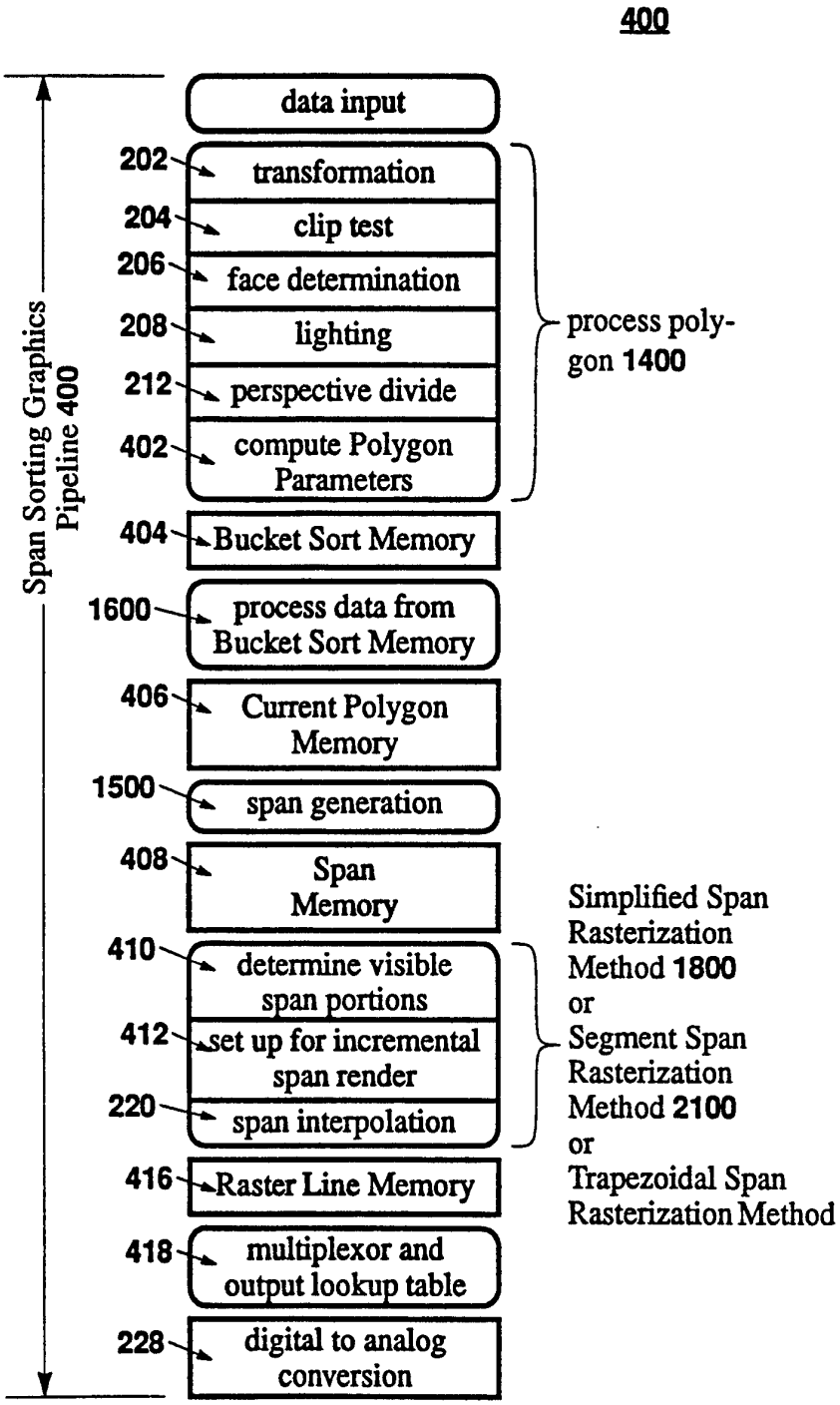

400
Span Sorting Graphics Pipeline 400
data input
202
transformation
204
clip test
206
face determination
208
lighting
212
perspective divide
402
compute Polygon Parameters
process polygon 1400
404
Bucket Sort Memory
1600
process data from Bucket Sort Memory
406
Current Polygon Memory
1500
span generation
408
Span Memory
410
determine visible span portions
412
set up for incremental span render
220
span interpolation
Simplified Span Rasterization Method 1800
or
Segment Span Rasterization Method 2100
or
Trapezoidal Span Rasterization Method
416
Raster Line Memory
418
multiplexor and output lookup table
228
digital to analog conversion

Figure 5

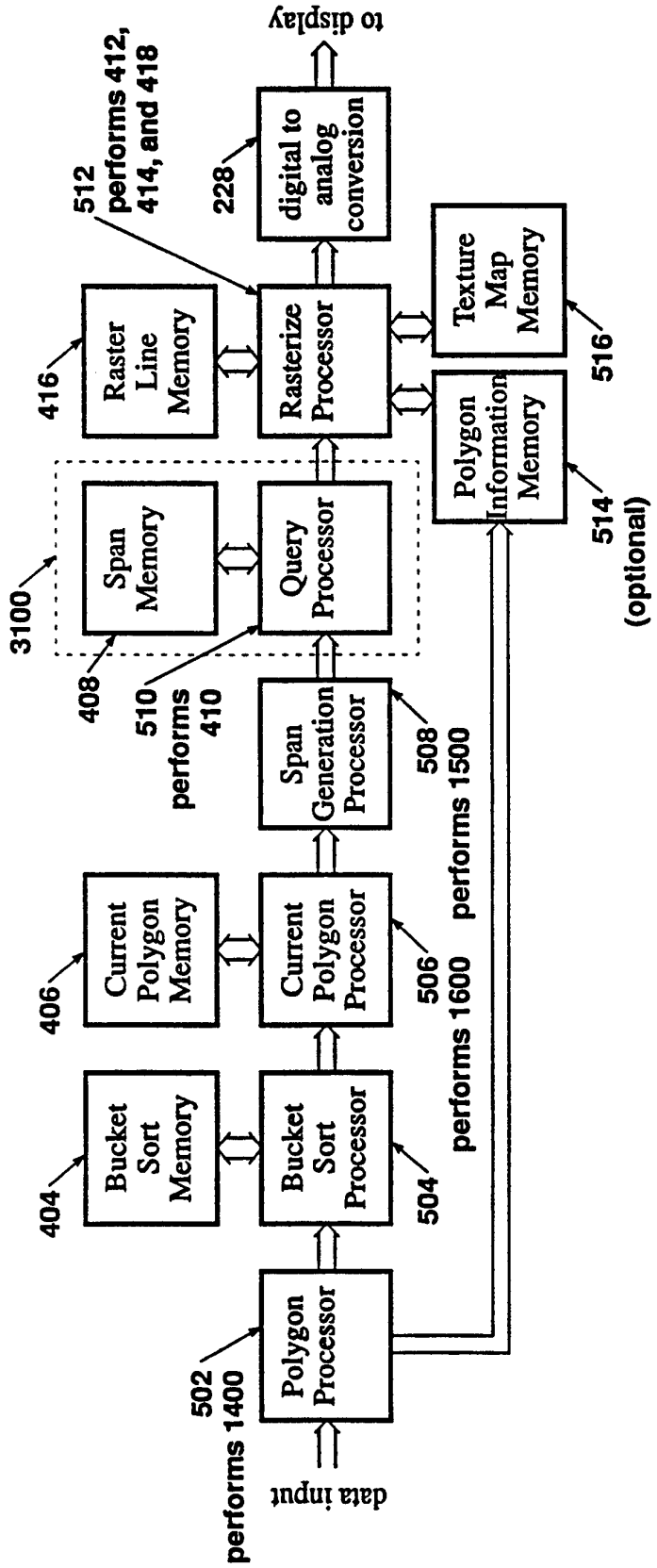

500
data input
performs 1400
502
Polygon Processor
404
Bucket Sort Memory
Bucket Sort Processor
504
406
Current Polygon Memory
Current Polygon Processor
506
performs 1600
408
Span Memory
3100
Query Processor
510
performs 410
Span Generation Processor
508
performs 1500
416
Raster Line Memory
512
performs 412, 414, and 418
Rasterize Processor
228
digital to analog conversion
to display
Texture Map Memory
516
Polygon Information Memory
514
(optional)

Figure 6

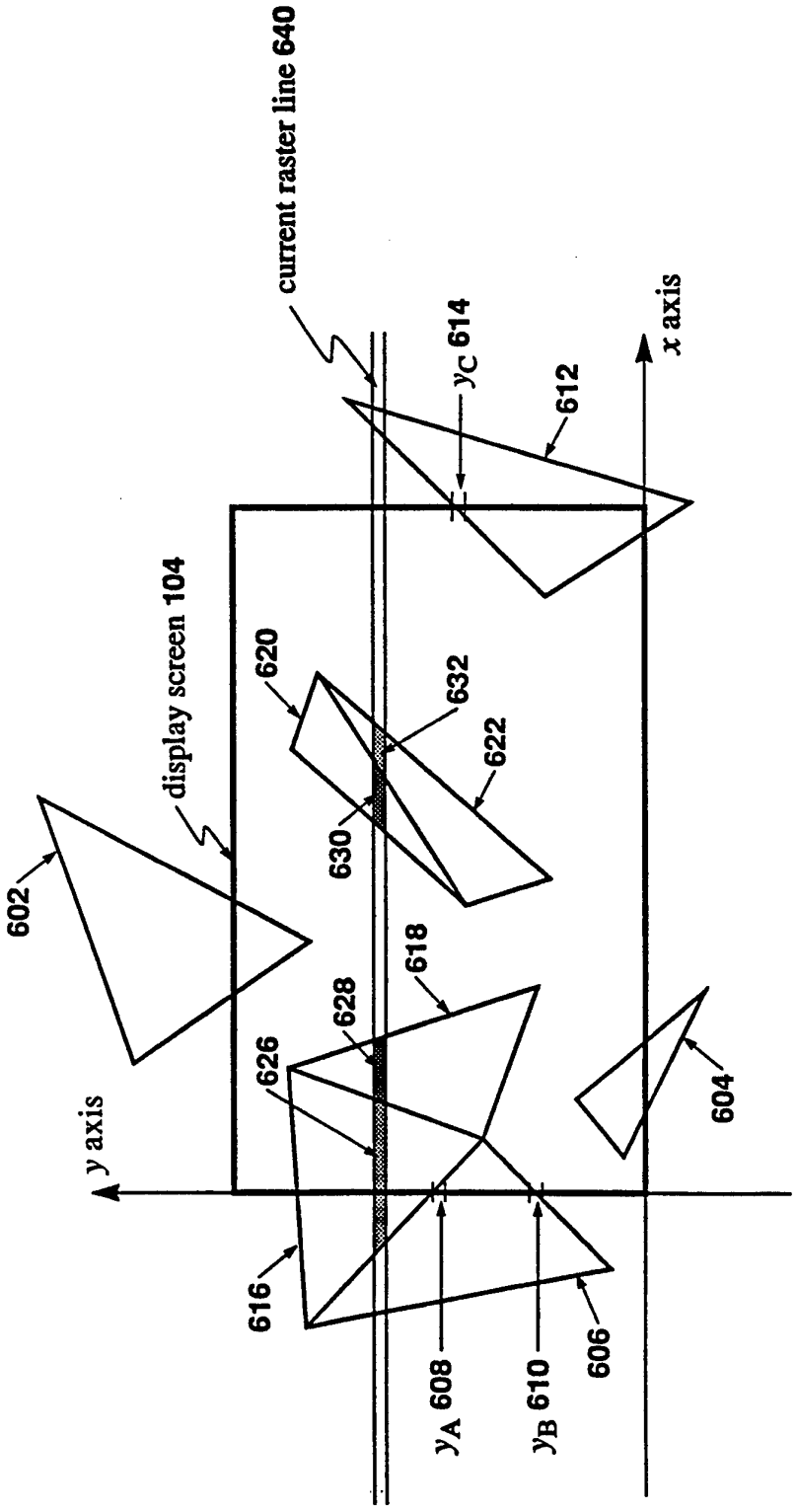

current raster line 640
y_C 614
612
x axis
display screen 104
620
632
622
630
602
618
628
626
y axis
604
616
606
y_A 608
y_B 610

Figure 7

Figure 7A

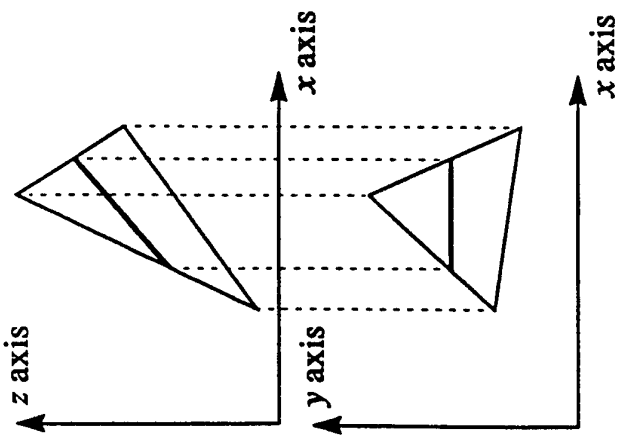


Figure 7B

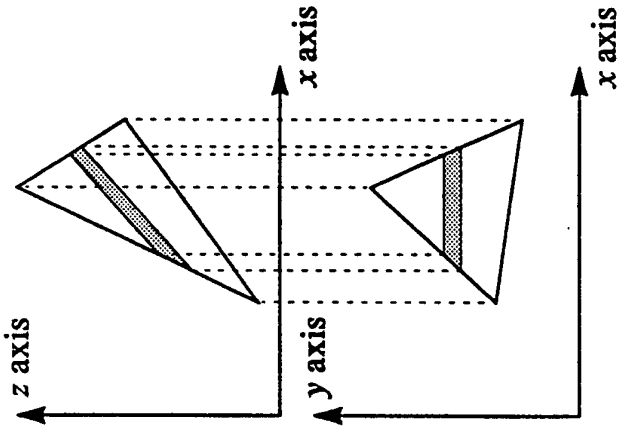


Figure 7C

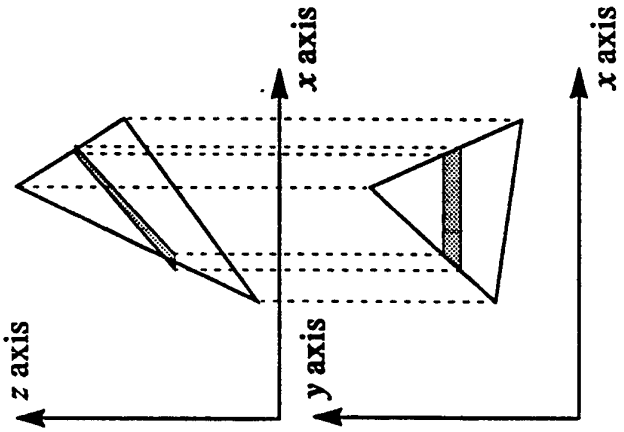


Figure 8

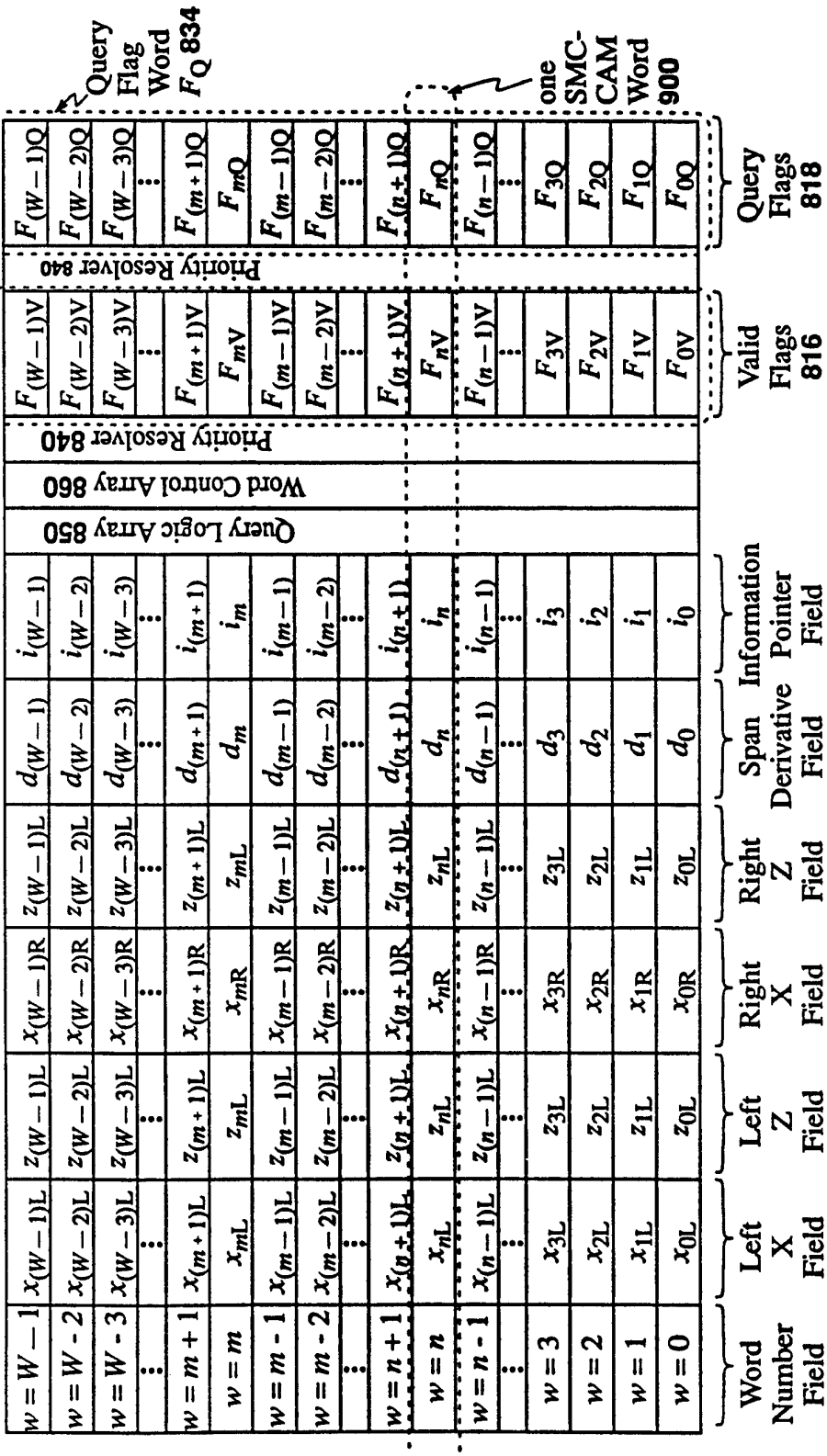

800
Valid Flag Word F_V 830
Query Logic Array 850
Word Control Array 860
Priority Resolver 840
Priority Resolver 840
Query Flag Word F_Q 834
one SMC-CAM Word 900
Word Number Field 802
Left X Field 804
Left Z Field 806
Right X Field 808
Right Z Field 810
Span Derivative Field 812
Information Pointer Field 814
Valid Flags 816
Query Flags 818
word W - 1
word W - 2
word W - 3
word m + 1
word m
word m - 1
word m - 2
word n + 1
word n
word n - 1
word 3
word 2
word 1
word 0
w = W − 1
w = W - 2
w = W - 3
w = m + 1
w = m
w = m - 1
w = m - 2
w = n + 1
w = n
w = n - 1
w = 3
w = 2
w = 1
w = 0

Figure 9

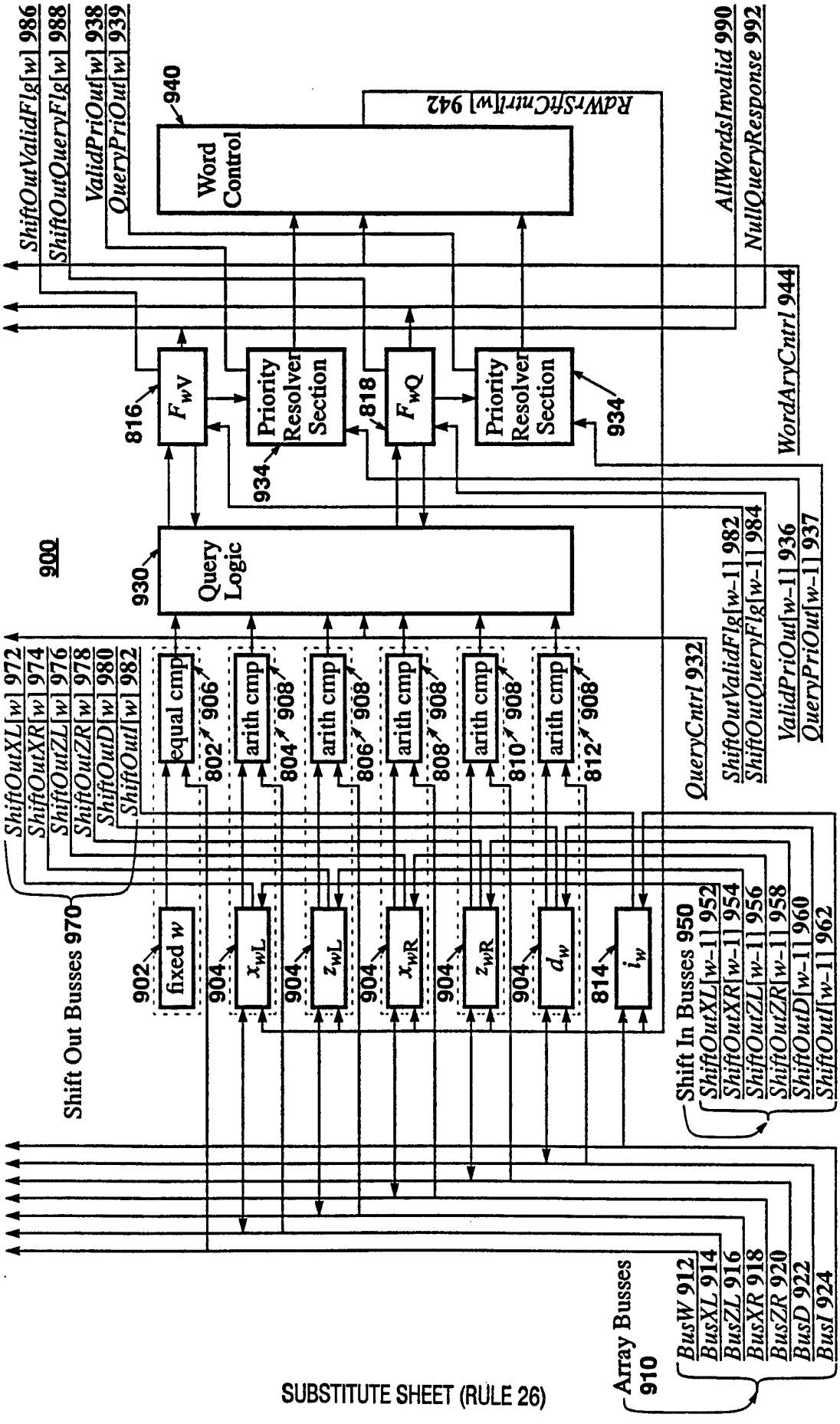

900
Word Control
940
RdWrSftCntrl[w] 942
ShiftOutValidFlg[w] 986
ShiftOutQueryFlg[w] 988
ValidPriOut[w] 938
QueryPriOut[w] 939
AllWordsInvalid 990
NullQueryResponse 992
WordAryCntrl 944
Priority Resolver Section
934
816
818
Query Logic
930
QueryCntrl 932
ShiftOutValidFlg[w-1] 982
ShiftOutQueryFlg[w-1] 984
ValidPriOut[w-1] 936
QueryPriOut[w-1] 937
ShiftOutXL[w] 972
ShiftOutXR[w] 974
ShiftOutZL[w] 976
ShiftOutZR[w] 978
ShiftOutD[w] 980
ShiftOutI[w] 982
equal cmp
arith cmp
802
804
806
808
810
812
906
908
902
fixed w
904
814
Shift Out Busses 970
Shift In Busses 950
ShiftOutXL[w-1] 952
ShiftOutXR[w-1] 954
ShiftOutZL[w-1] 956
ShiftOutZR[w-1] 958
ShiftOutD[w-1] 960
ShiftOutI[w-1] 962
Array Busses 910
BusW 912
BusXL 914
BusZL 916
BusXR 918
BusZR 920
BusD 922
BusI 924

Figure 10

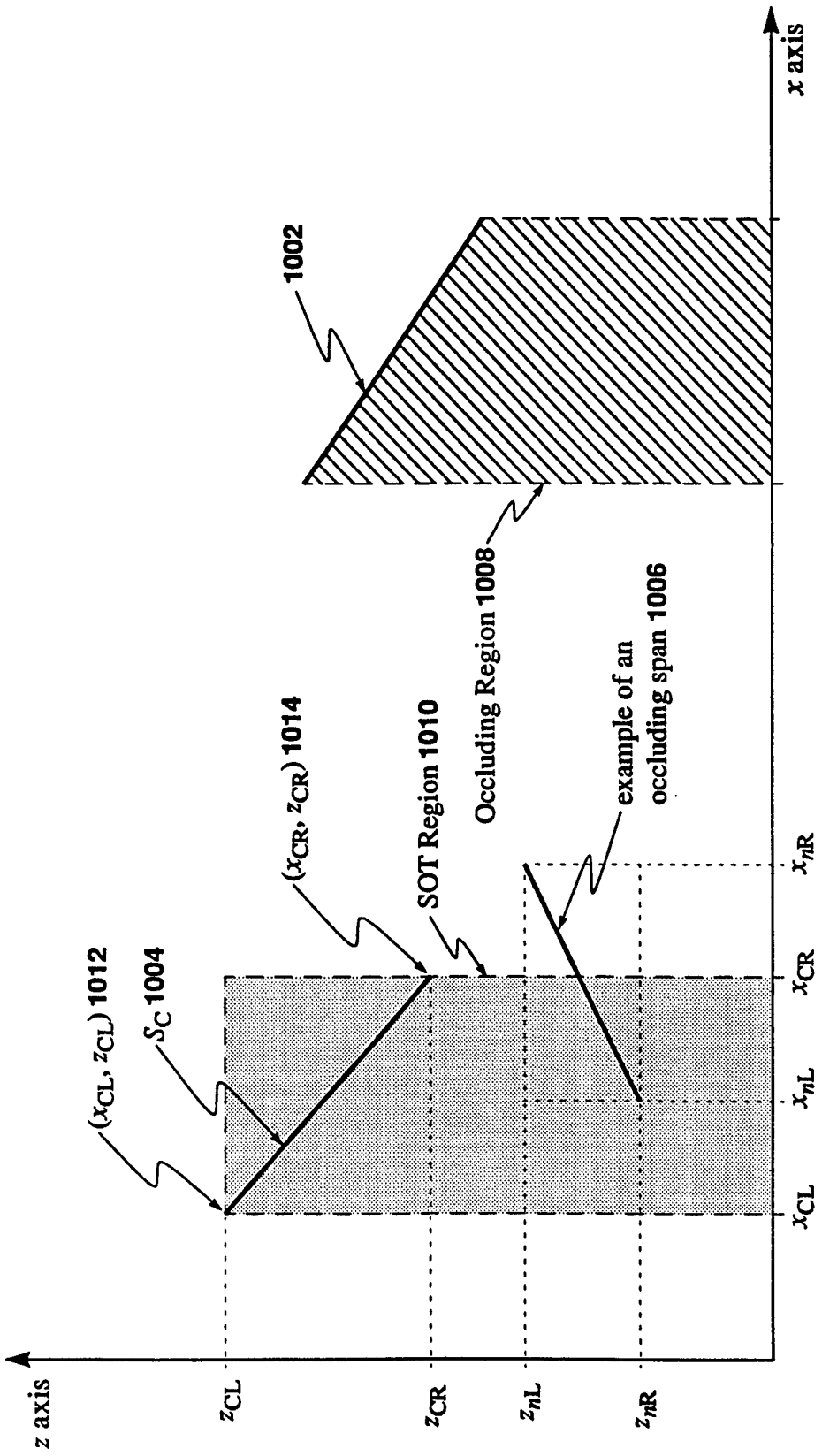

x axis
1002
Occluding Region 1008
example of an occluding span 1006
(x_{CR}, z_{CR}) 1014
SOT Region 1010
(x_{CL}, z_{CL}) 1012
S_C 1004
x_{nR}
x_{CR}
x_{nL}
x_{CL}
z axis
z_{CL}
z_{CR}
z_{nL}
z_{nR}

Figure 11

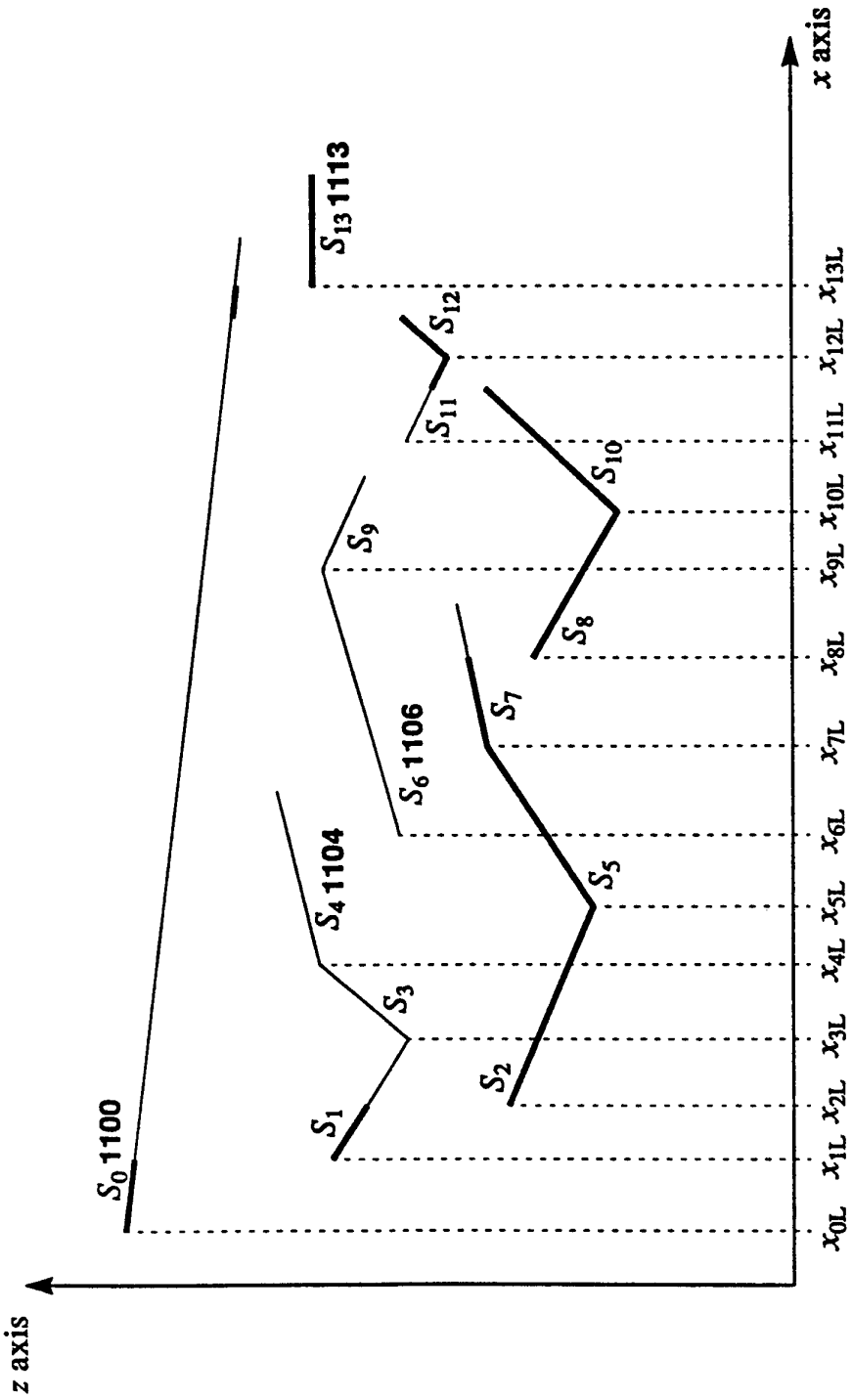
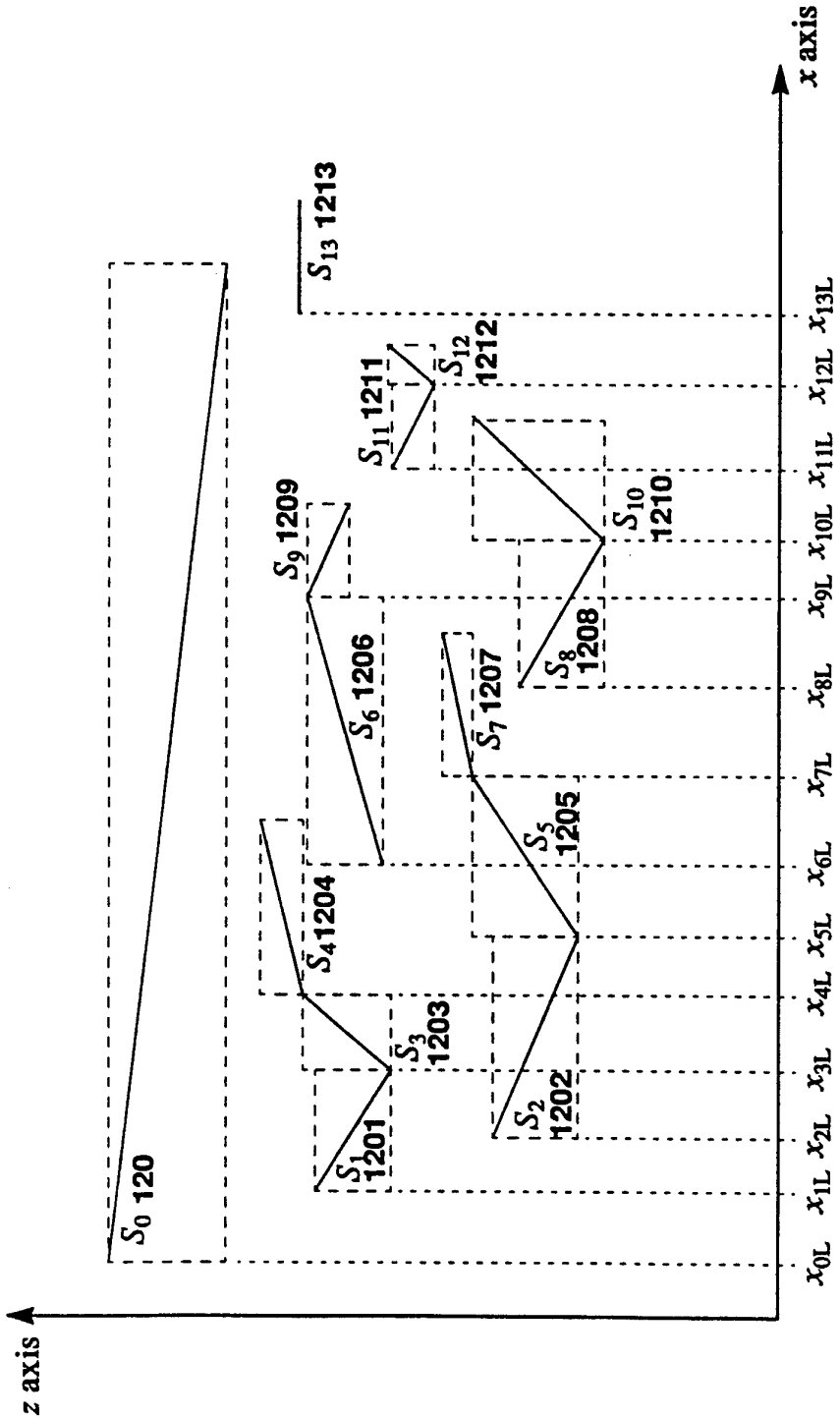

x axis
z axis
S_0 1100
S_1
S_2
S_3
S_4 1104
S_5
S_6 1106
S_7
S_8
S_9
S_{10}
S_{11}
S_{12}
S_{13} 1113
x_{0L} x_{1L} x_{2L} x_{3L} x_{4L} x_{5L} x_{6L} x_{7L} x_{8L} x_{9L} x_{10L} x_{11L} x_{12L} x_{13L}

Figure 12


x axis
z axis
S_0 120
S_1 1201
S_2 1202
S_3 1203
S_4 1204
S_5 1205
S_6 1206
S_7 1207
S_8 1208
S_9 1209
S_{10} 1210
S_{11} 1211
S_{12} 1212
S_{13} 1213
x_{0L} x_{1L} x_{2L} x_{3L} x_{4L} x_{5L} x_{6L} x_{7L} x_{8L} x_{9L} x_{10L} x_{11L} x_{12L} x_{13L}

13 / 56

Figure 13

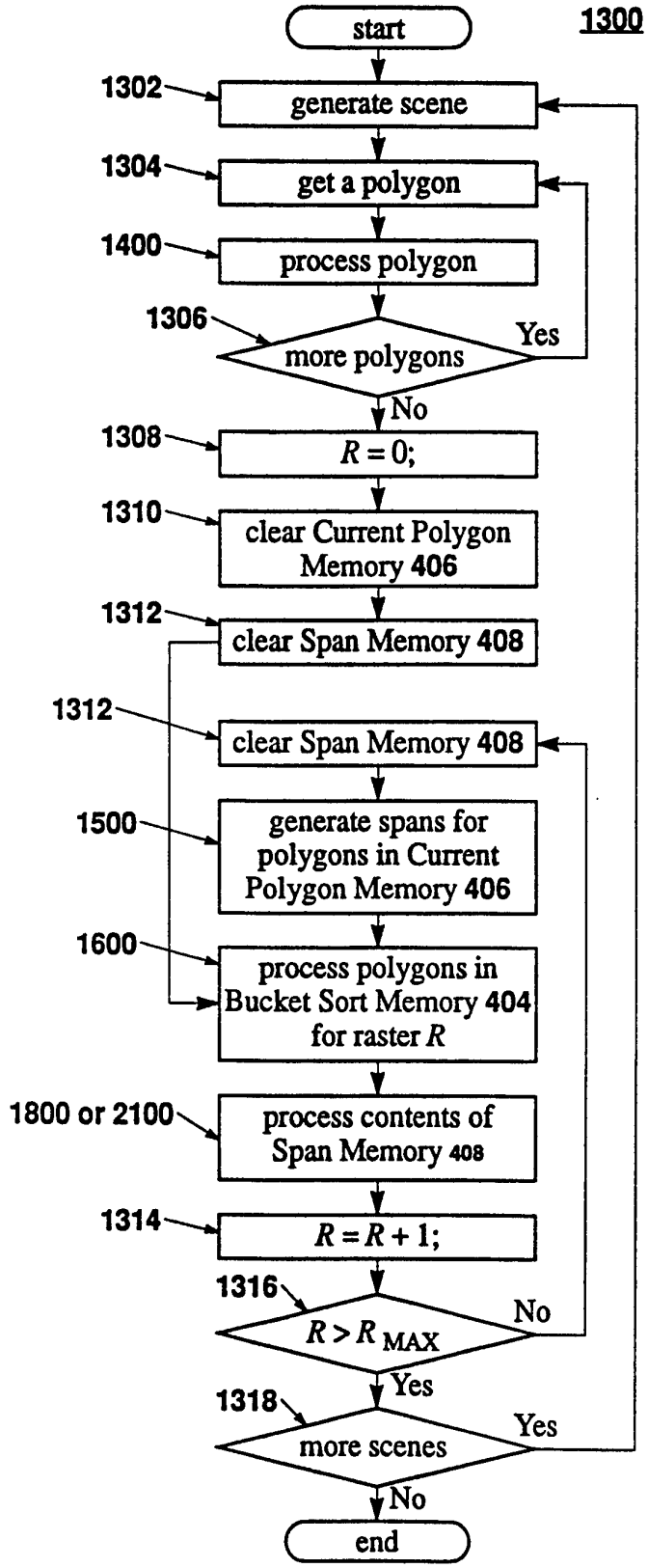


Figure 14

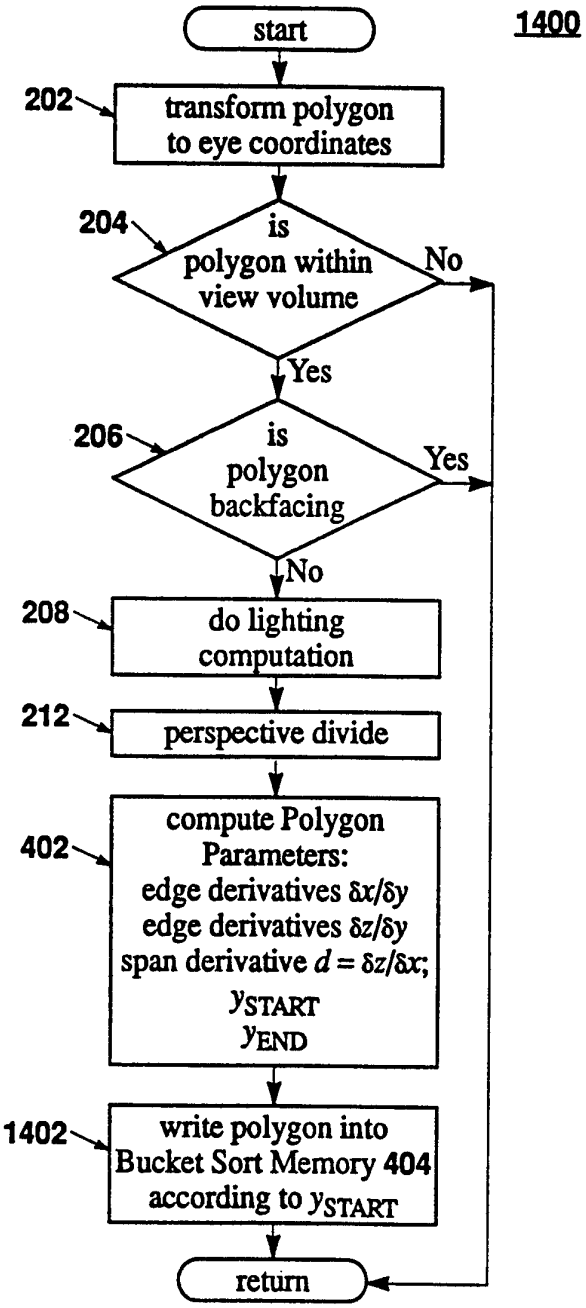
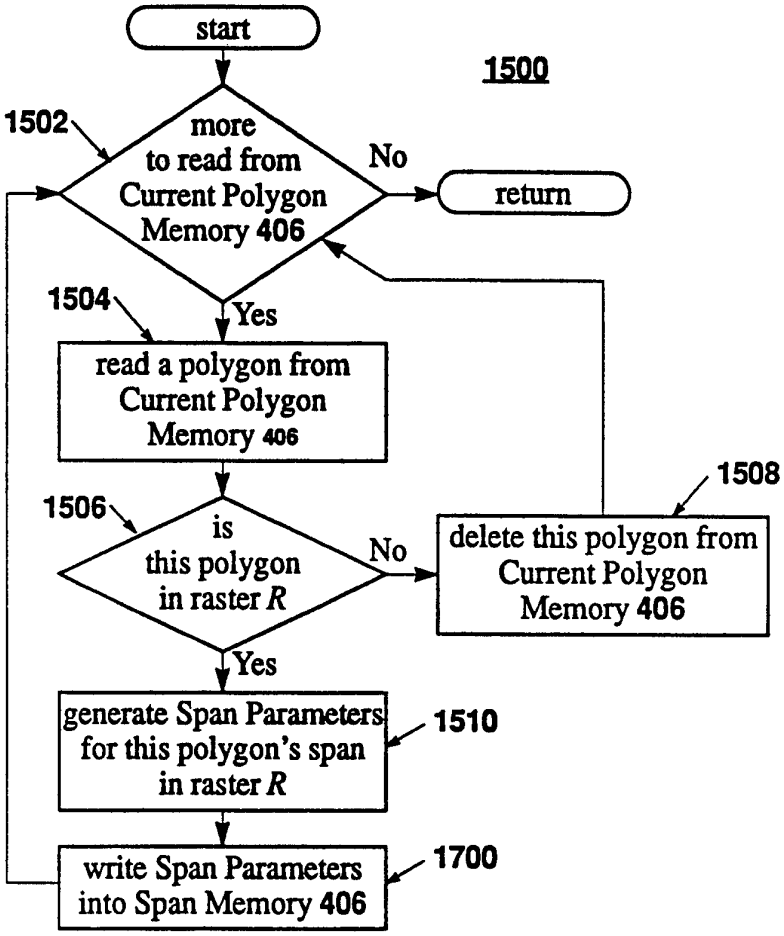

start
1400
202
transform polygon to eye coordinates
204
is polygon within view volume
No
Yes
206
is polygon backfacing
Yes
No
208
do lighting computation
212
perspective divide
402
compute Polygon Parameters:
edge derivatives δx/δy
edge derivatives δz/δy
span derivative d = δz/δx;
y_{START}
y_{END}
1402
write polygon into Bucket Sort Memory 404 according to y_{START}
return

Figure 15


start
1500
1502
more to read from Current Polygon Memory 406
No
return
Yes
1504
read a polygon from Current Polygon Memory 406
1506
is this polygon in raster R
No
1508
delete this polygon from Current Polygon Memory 406
Yes
generate Span Parameters for this polygon's span in raster R
1510
write Span Parameters into Span Memory 406
1700

16 / 56

Figure 16

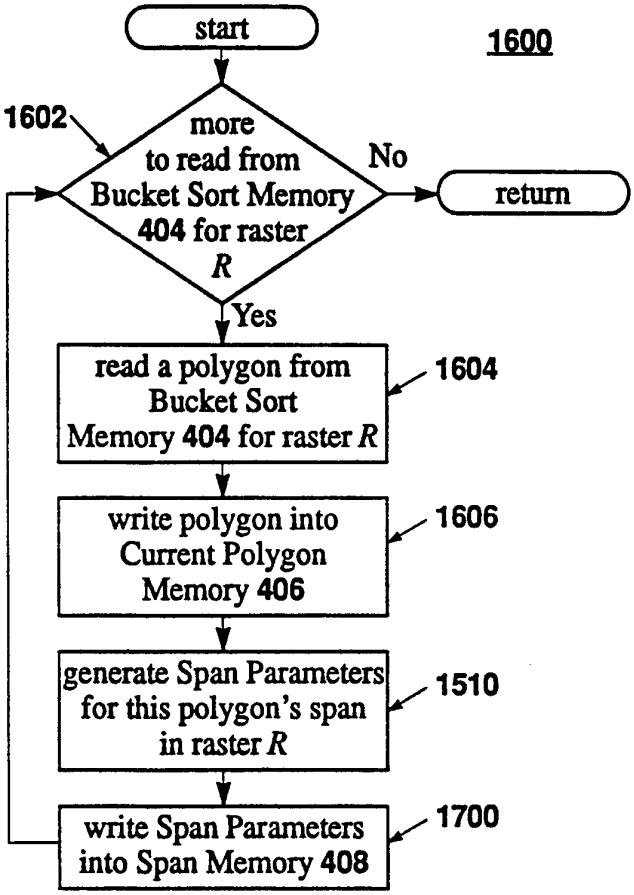


Figure 17

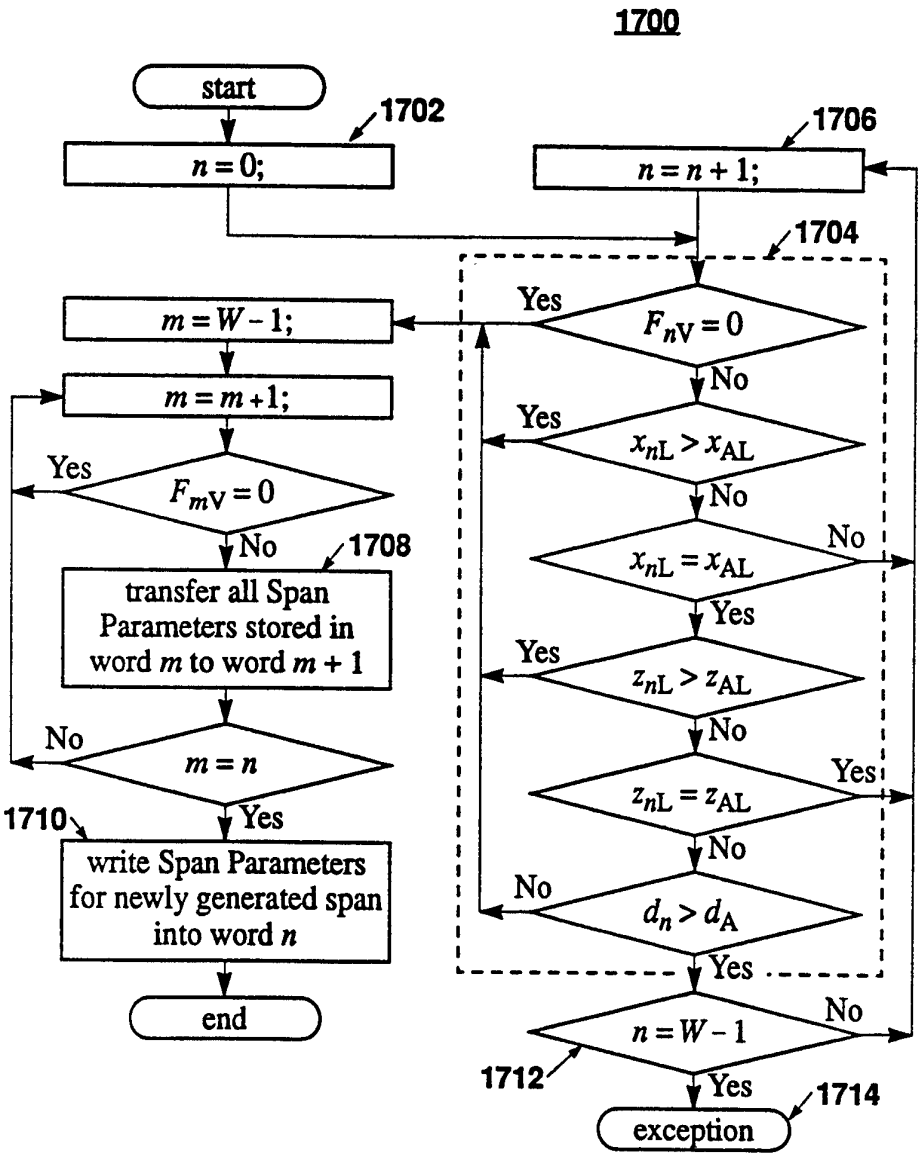

1700
start
1702
n = 0;
1706
n = n + 1;
1704
m = W - 1;
Yes
F_nV = 0
No
m = m + 1;
Yes
x_nL > x_AL
No
Yes
F_mV = 0
No
x_nL = x_AL
No
1708
transfer all Span Parameters stored in word m to word m + 1
Yes
Yes
z_nL > z_AL
No
No
m = n
Yes
z_nL = z_AL
Yes
1710
write Span Parameters for newly generated span into word n
No
No
d_n > d_A
Yes
end
No
n = W - 1
1712
Yes
1714
exception

Figure 18

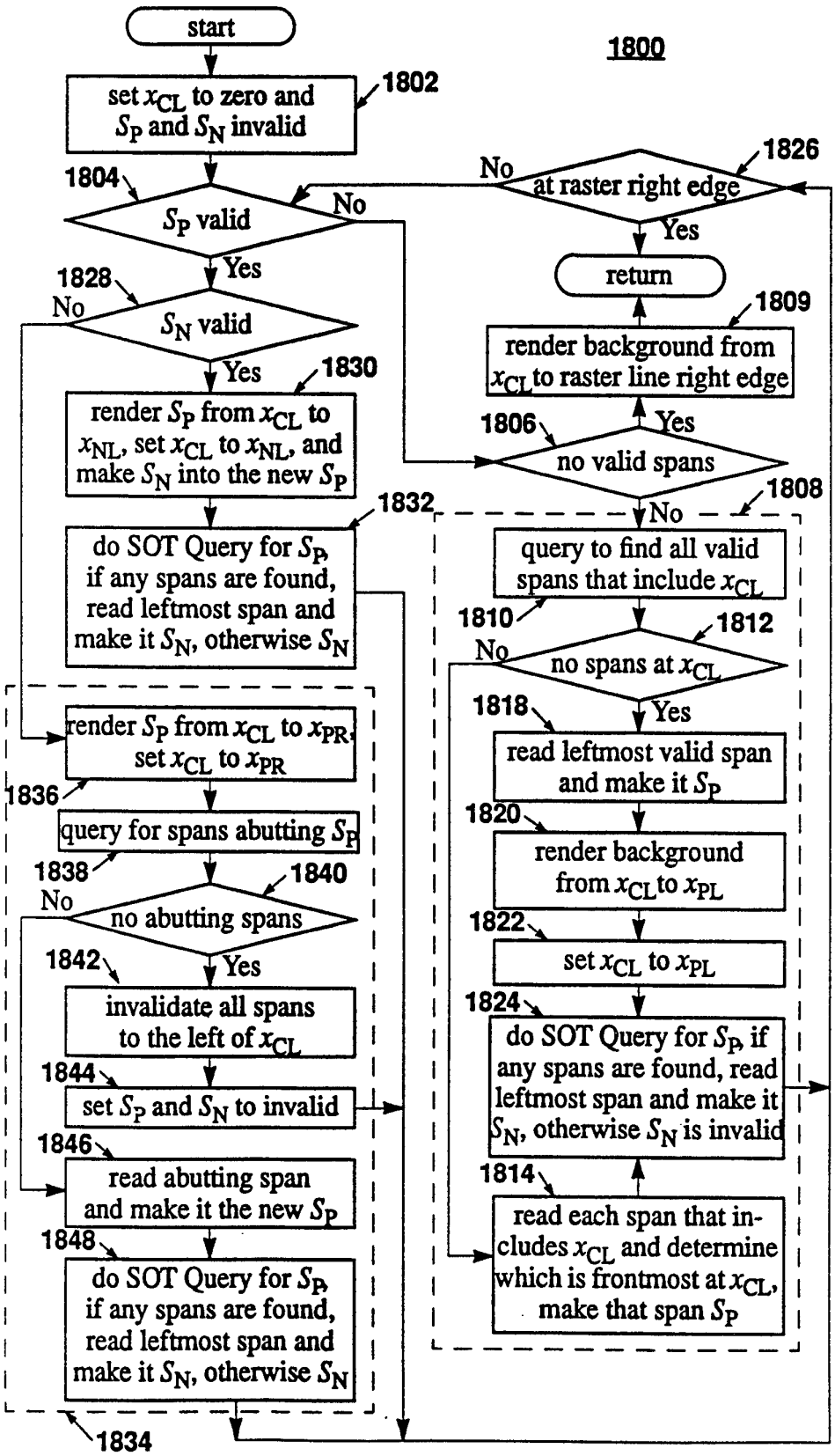

start
1800
set x_{CL} to zero and S_P and S_N invalid
1802
1804
S_P valid
No
Yes
1826
No
at raster right edge
Yes
return
1828
No
S_N valid
Yes
1809
render background from x_{CL} to raster line right edge
1830
render S_P from x_{CL} to x_{NL}, set x_{CL} to x_{NL}, and make S_N into the new S_P
1806
Yes
no valid spans
1808
No
1832
do SOT Query for S_P, if any spans are found, read leftmost span and make it S_N, otherwise S_N
query to find all valid spans that include x_{CL}
1810
1812
No
no spans at x_{CL}
Yes
render S_P from x_{CL} to x_{PR}, set x_{CL} to x_{PR}
1818
read leftmost valid span and make it S_P
1836
query for spans abutting S_P
1820
render background from x_{CL} to x_{PL}
1838
1840
No
no abutting spans
Yes
1822
set x_{CL} to x_{PL}
1842
invalidate all spans to the left of x_{CL}
1824
do SOT Query for S_P, if any spans are found, read leftmost span and make it S_N, otherwise S_N is invalid
1844
set S_P and S_N to invalid
1846
read abutting span and make it the new S_P
1814
read each span that includes x_{CL} and determine which is frontmost at x_{CL}, make that span S_P
1848
do SOT Query for S_P, if any spans are found, read leftmost span and make it S_N, otherwise S_N
1834

Figure 19

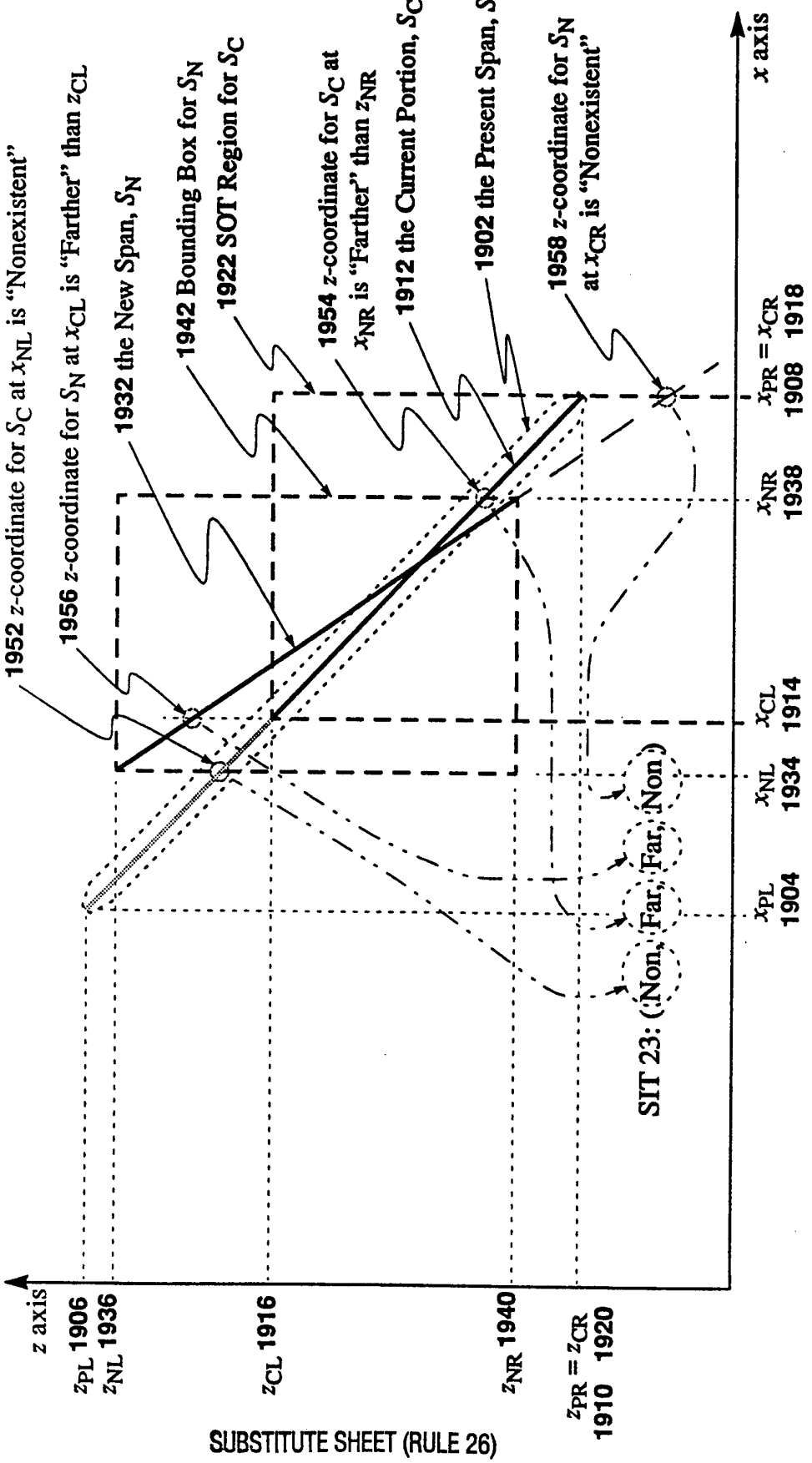

1952 z-coordinate for S_C at x_{NL} is "Nonexistent"
1956 z-coordinate for S_N at x_{CL} is "Farther" than z_{CL}
1932 the New Span, S_N
1942 Bounding Box for S_N
1922 SOT Region for S_C
1954 z-coordinate for S_C at x_{NR} is "Farther" than z_{NR}
1912 the Current Portion, S_C
1902 the Present Span, S_P
1958 z-coordinate for S_N at x_{CR} is "Nonexistent"
x axis
z axis
z_{PL} 1906
z_{NL} 1936
z_{CL} 1916
z_{NR} 1940
$z_{PR} = z_{CR}$
1910 1920
$x_{PR} = x_{CR}$
1908 1918
x_{NR}
1938
x_{CL}
1914
x_{NL}
1934
x_{PL}
1904
SIT 23: (Non, Far, Far, Non)

Figure 20

Figure 20A: Interaction Type 1 = (Near, Near, Non, Non) ⇒ Rule 5

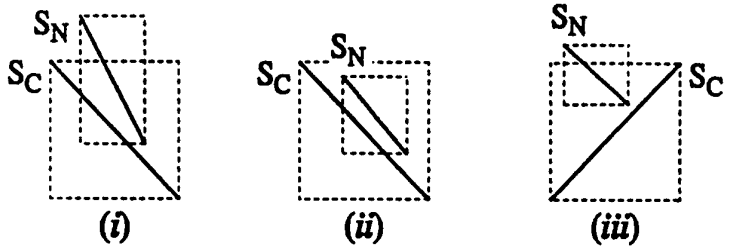


Figure 20B: Interaction Type 2 = (Near, Far, Non, Non) ⇒ Rule 4

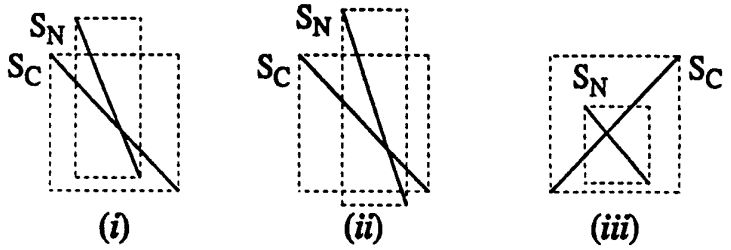


Figure 20C: Interaction Type 3 = (Near, Equal, Non, Non) ⇒ Rule 5

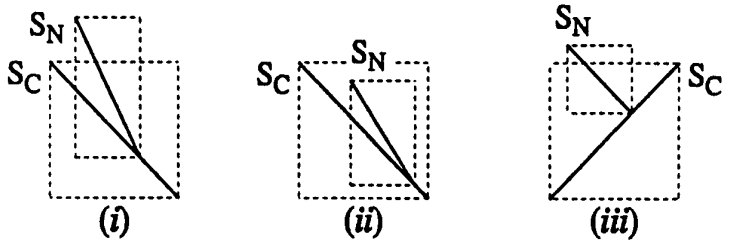


Figure 20D: Interaction Type 4 = (Far, Near, Non, Non) ⇒ Rule 3

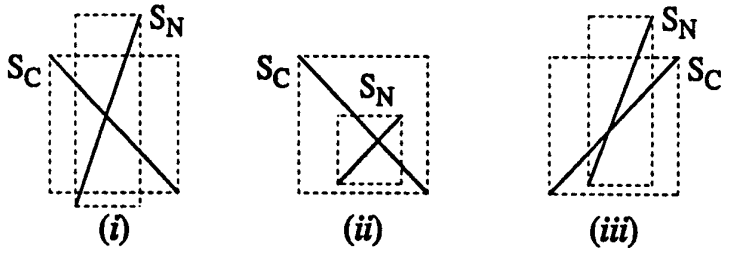


Figure 20E: Interaction Type 5 = (Far, Far, Non, Non) ⇒ Rule 3

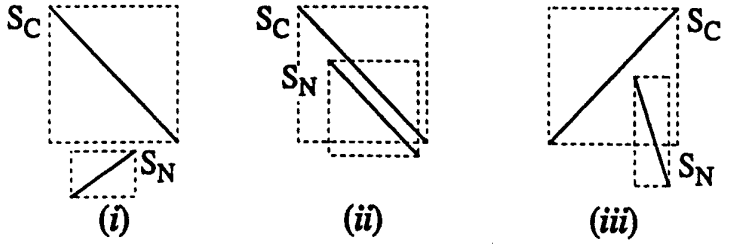


Figure 20 (continued)

Figure 20F: Interaction Type 6:(Far, Equal, Non, Non) ⇒ Rule 3

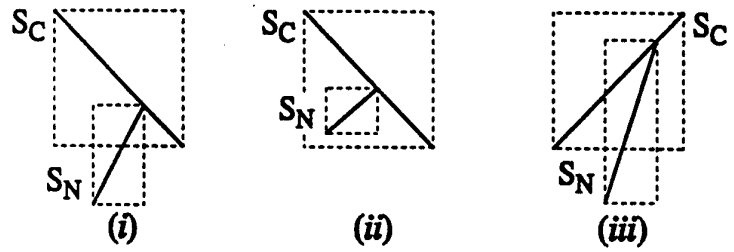


Figure 20G: Interaction Type 7 = (Equal, Near, Non, Non) ⇒ Rule 5

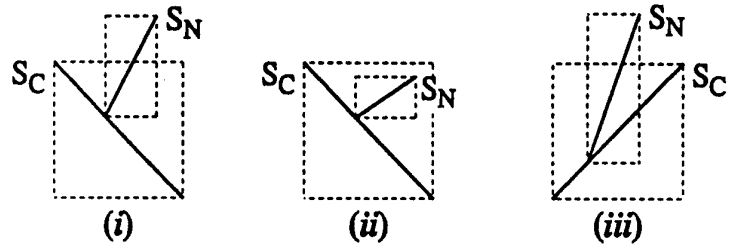


Figure 20H: Interaction Type 8 = (Equal, Far, Non, Non) ⇒ Rule 3

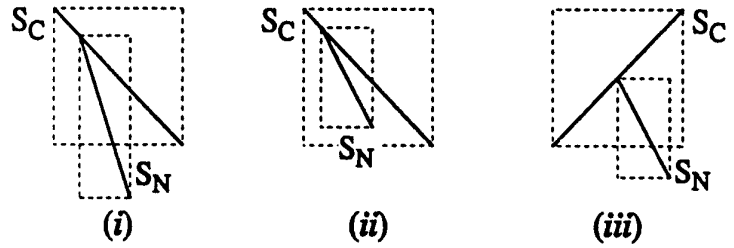


Figure 20I: Interaction Type 9 = (Equal, Equal, Non, Non) ⇒ Rule 5

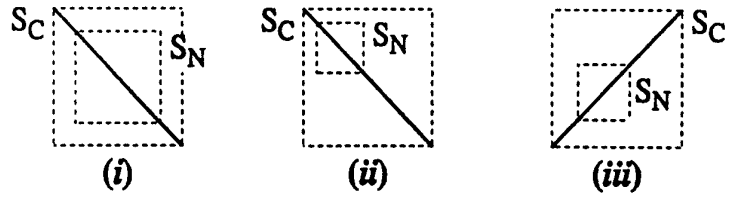


Figure 20J: Interaction Type 10 = (Near, Non, Non, Near) ⇒ Rule 4

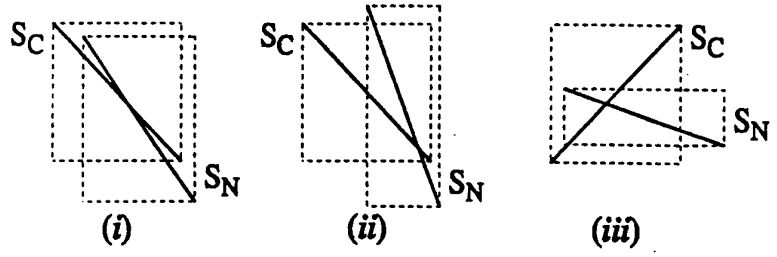


Figure 20 (continued)

Figure 20K: Interaction Type 11 = (Near, Non, Non, Far) ⇒ Rule 5

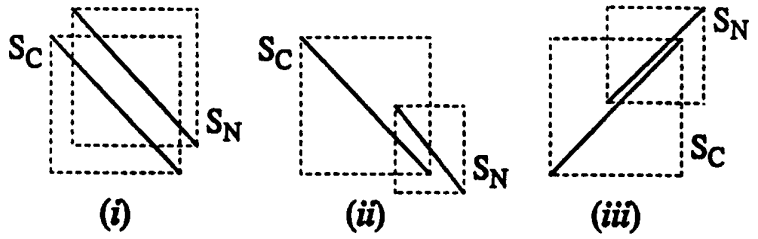


Figure 20L: Interaction Type 12 = (Near, Non, Non, Equal) ⇒ Rule 5

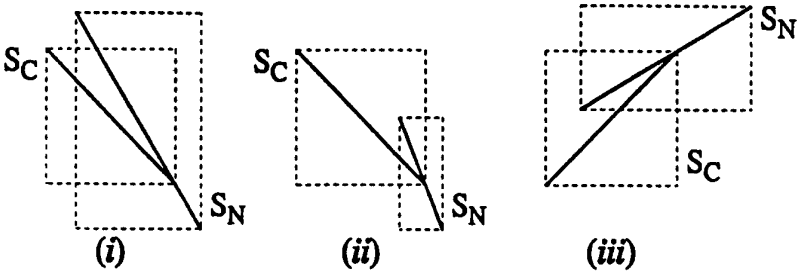


Figure 20M: Interaction Type 13 = (Far, Non, Non, Near) ⇒ Rule 3

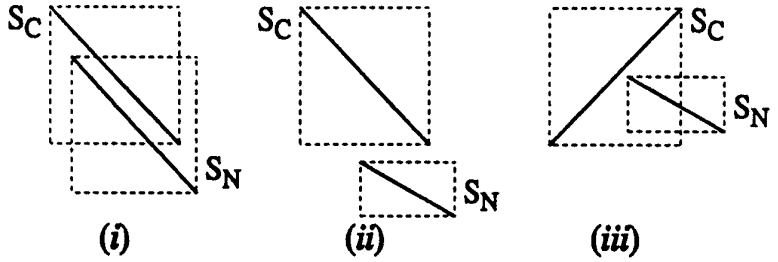


Figure 20N: Interaction Type 14 = (Far, Non, Non, Far) ⇒ Rule 3

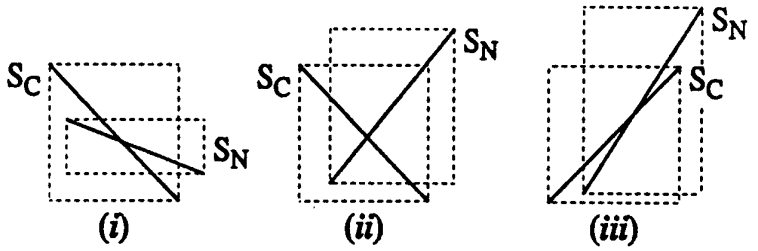


Figure 20O: Interaction Type 15 = (Far, Non, Non, Equal) ⇒ Rule 3

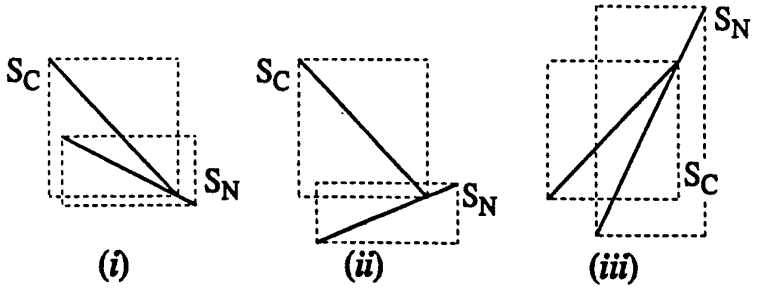


Figure 20 (continued)

Figure 20P: Interaction Type 16 = (Equal, Non, Non, Near) ⇒ Rule 3

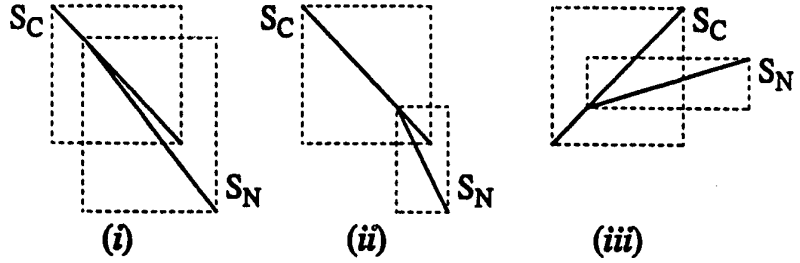


Figure 20Q: Interaction Type 17 = (Equal, Non, Non, Far) ⇒ Rule 5

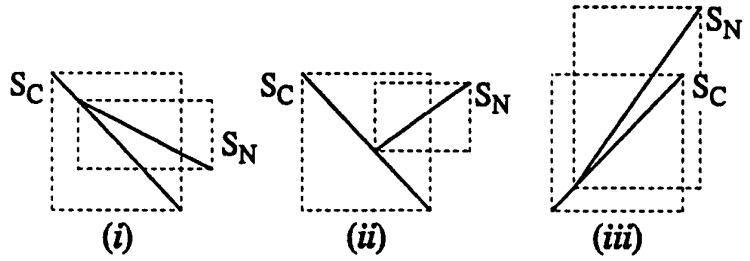


Figure 20R: Interaction Type 18 = (Equal, Non, Non, Equal) ⇒ Rule 5

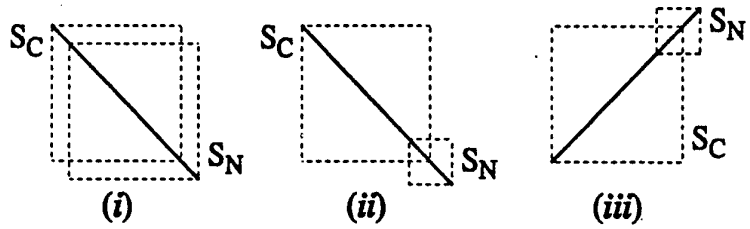


Figure 20S: Interaction Type 19 = (Non, Near, Near, Non) ⇒ impossible

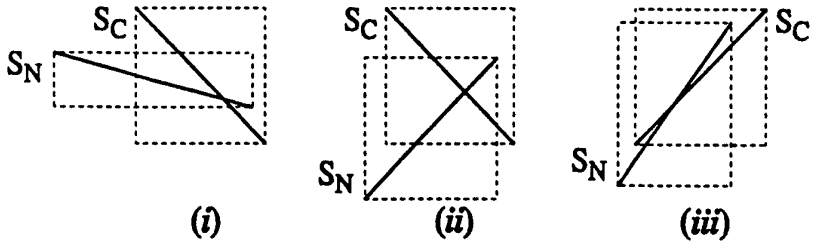


Figure 20T: Interaction Type 20 = (Non, Near, Far, Non) ⇒ Rule 5

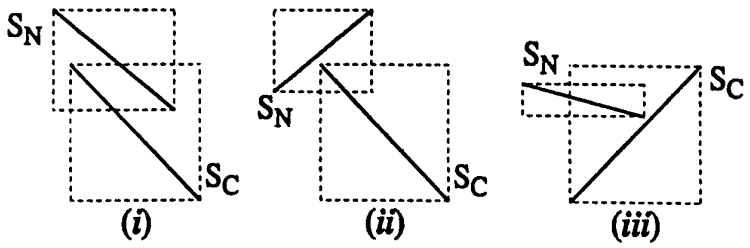


Figure 20 (continued)

Figure 20U: Interaction Type 21 = (Non, Near, Equal, Non) ⇒ Rule 5

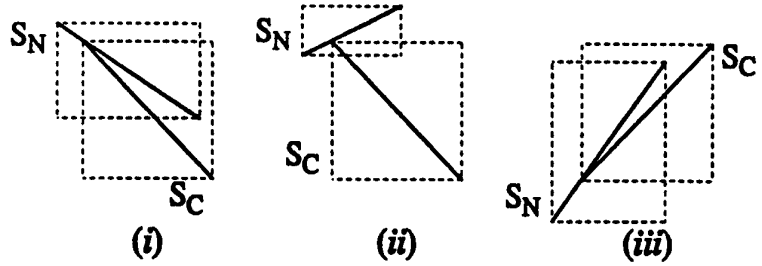


Figure 20V: Interaction Type 22 = (Non, Far, Near, Non) ⇒ impossible

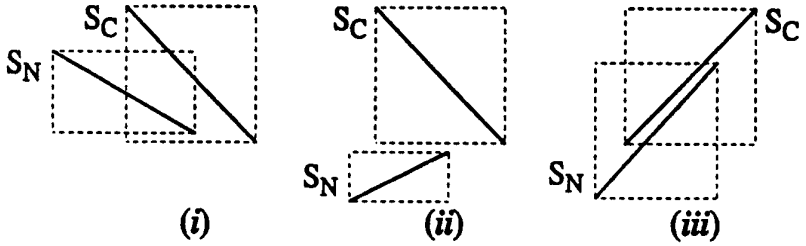


Figure 20W: Interaction Type 23 = (Non, Far, Far, Non) ⇒ Rule 4

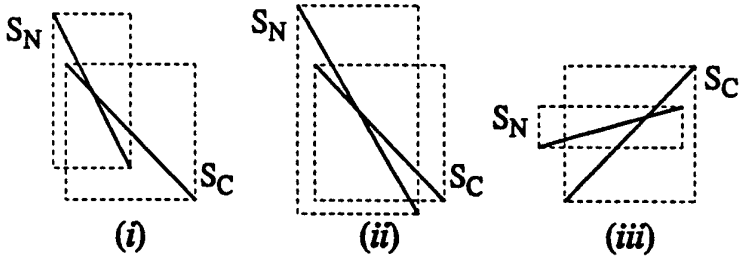


Figure 20X: Interaction Type 24 = (Non, Far, Equal, Non) ⇒ impossible

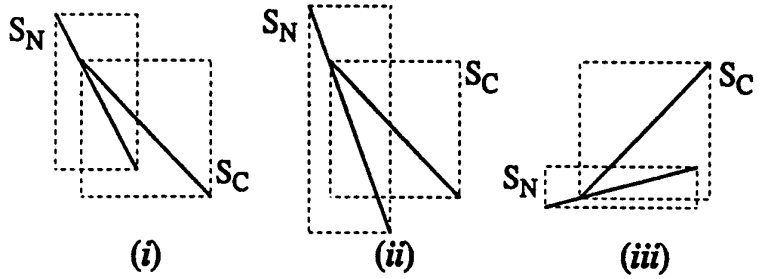


Figure 20Y: Interaction Type 25 = (Non, Equal, Near, Non) ⇒ impossible

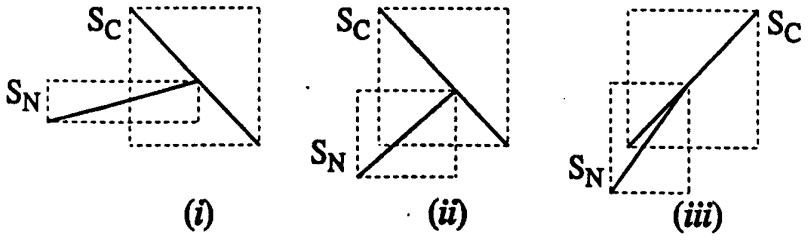


Figure 20 (continued)

Figure 20Z: Interaction Type 26 = (Non, Equal, Far, Non) ⇒ Rule 5

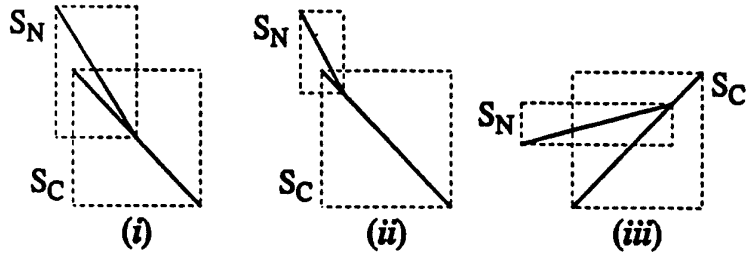


Figure 20AA: Interaction Type 27 = (Non, Equal, Equal, Non) ⇒ Rule 5

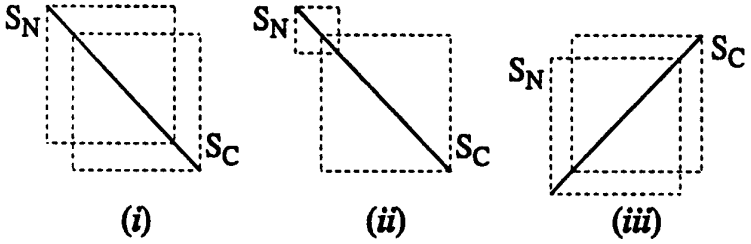


Figure 20BB: Interaction Type 28 = (Non, Non, Near, Near) ⇒ impossible

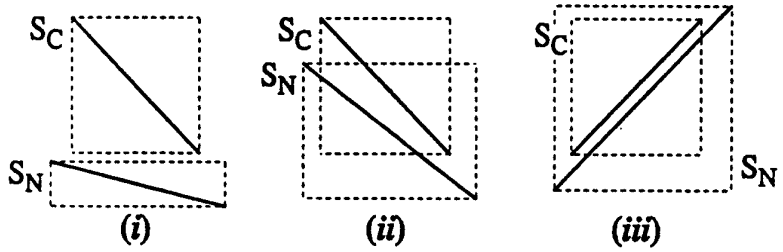


Figure 20CC: Interaction Type 29 = (Non, Non, Near, Far) ⇒ impossible

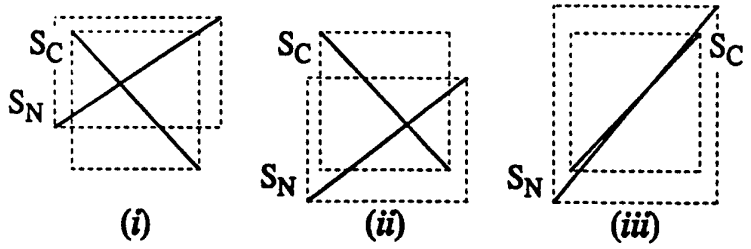


Figure 20DD: Interaction Type 30 = (Non, Non, Near, Equal) ⇒ impossible

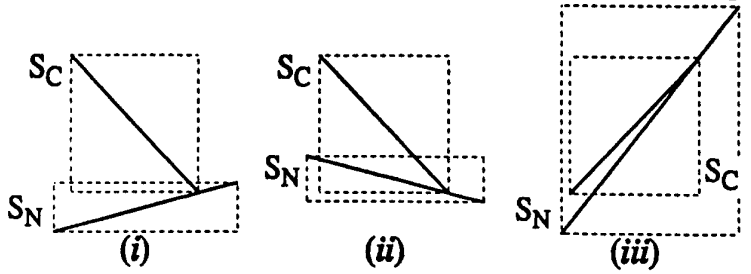


Figure 20 (continued)

Figure 20EE: Interaction Type 31 = (Non, Non, Far, Near) ⇒ Rule 4

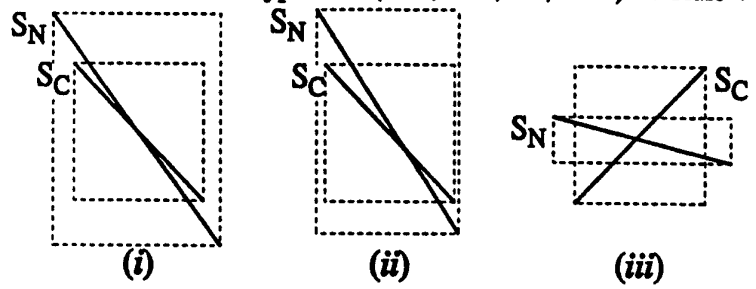


Figure 20FF: Interaction Type 32 = (Non, Non, Far, Far) ⇒ Rule 5

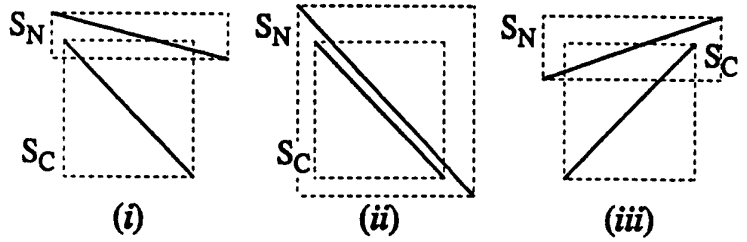


Figure 20GG: Interaction Type 33 = (Non, Non, Far, Equal) ⇒ Rule 5

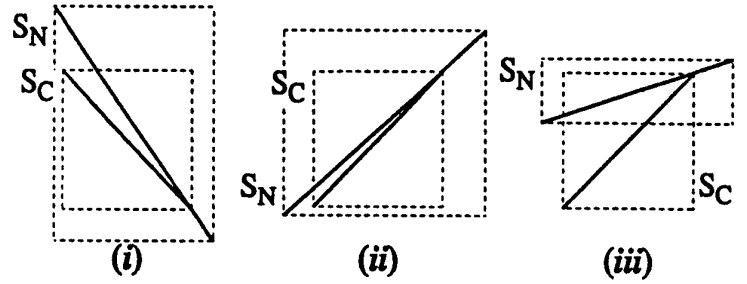


Figure 20HH: Interaction Type 34 = (Non, Non, Equal, Near) ⇒ impossible

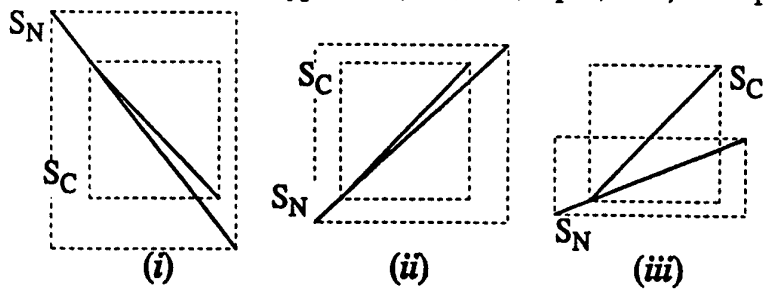


Figure 20II: Interaction Type 35 = (Non, Non, Equal, Far) ⇒ Rule 5

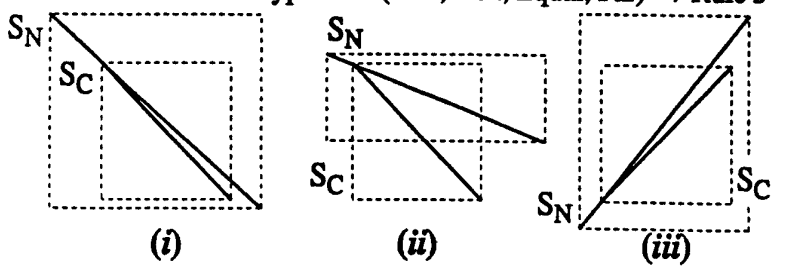


Figure 20 (continued)

Figure 20JJ: Interaction Type 36 = (Non, Non, Equal, Equal) ⇒ Rule 5

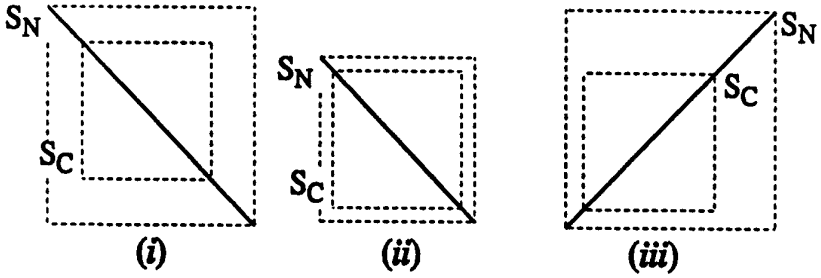


Figure 20KK: Interaction Type 37 = (Equal, Near, Equal, Non) ⇒ Rule 5

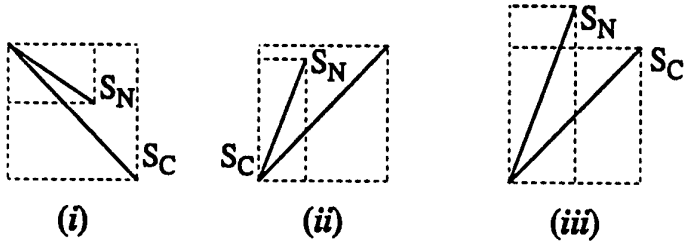


Figure 20LL: Interaction Type 38 = (Equal, Far, Equal, Non) ⇒ impossible

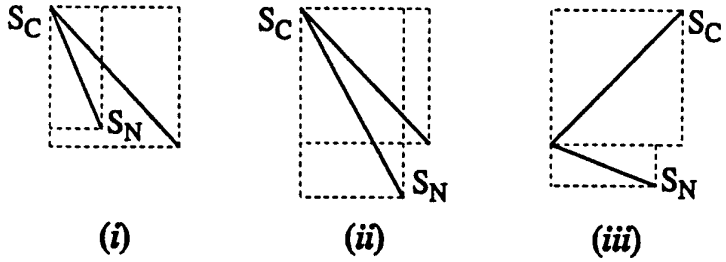


Figure 20MM: Interaction Type 39 = (Equal, Equal, Equal, Non) ⇒ Rule 5

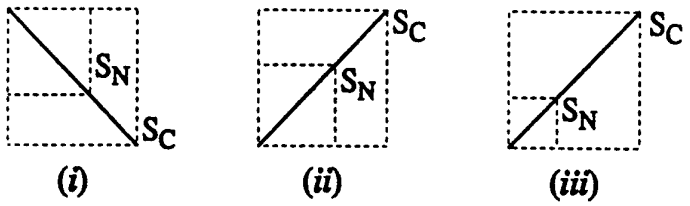


Figure 20NN: Interaction Type 40 = (Equal, Non, Equal, Near) ⇒ impossible

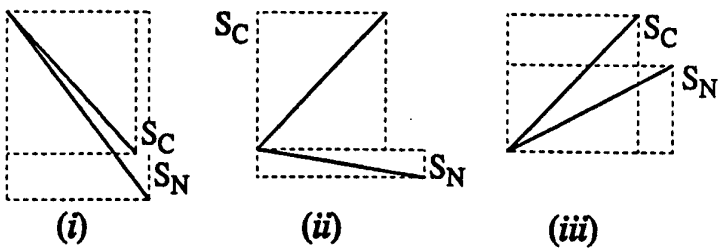


Figure 20 (continued)

Figure 20OO: Interaction Type 41 = (Equal, Non, Equal, Far) → Rule 5

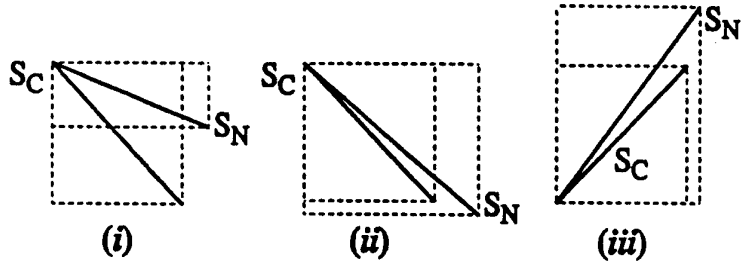


Figure 20PP: Interaction Type 42 = (Equal, Non, Equal, Equal) ⇒ Rule 5

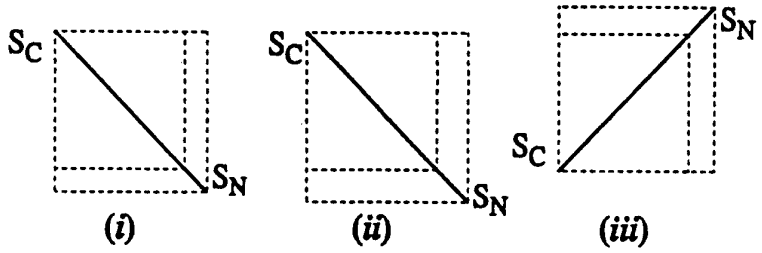


Figure 20QQ: Interaction Type 43 = (Near, Equal, Non, Equal) ⇒ Rule 5

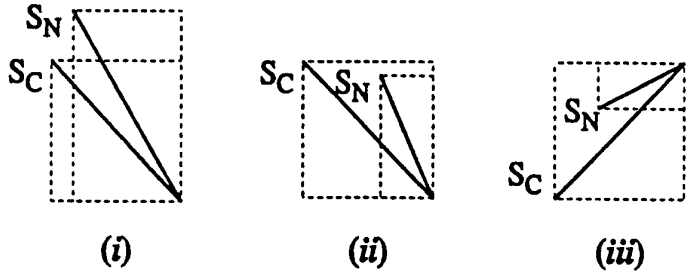


Figure 20RR: Interaction Type 44 = (Far, Equal, Non, Equal) ⇒ Rule 3

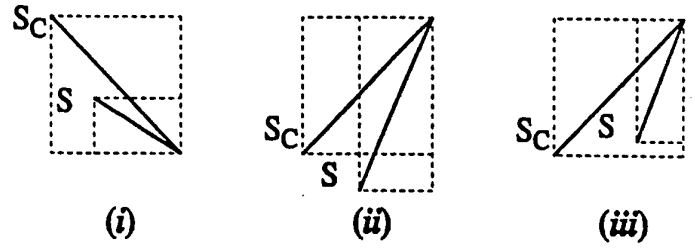


Figure 20SS: Interaction Type 45 = (Equal, Equal, Non, Equal) ⇒ Rule 5

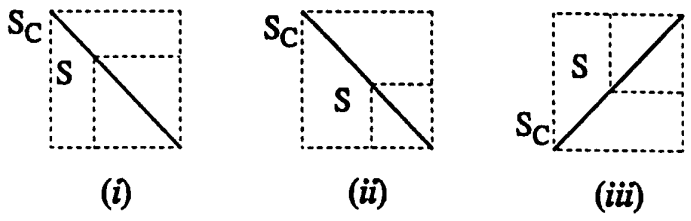


Figure 20 (continued)

Figure 20TT: Interaction Type 46 = (Non, Equal, Near, Equal) ⇒ impossible

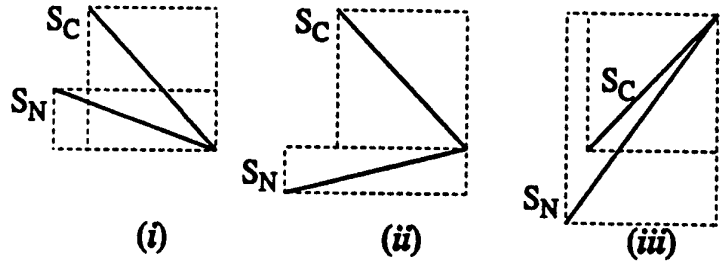


Figure 20UU: Interaction Type 47 = (Non, Equal, Far, Equal) ⇒ Rule 5

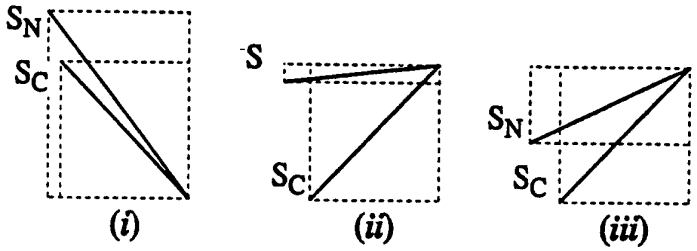


Figure 20VV: Interaction Type 48 = (Non, Equal, Equal, Equal) ⇒ Rule 5

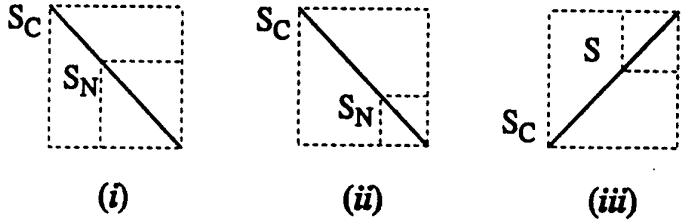


Figure 20WW: Interaction Type 49 = (Equal, Equal, Equal, Equal) ⇒ Rule 5

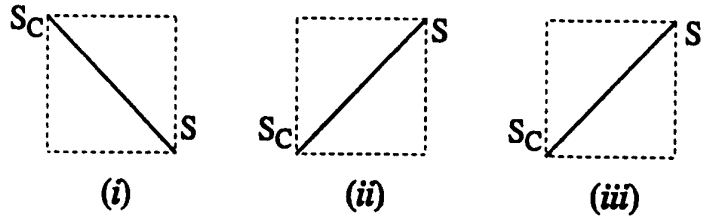


Figure 21

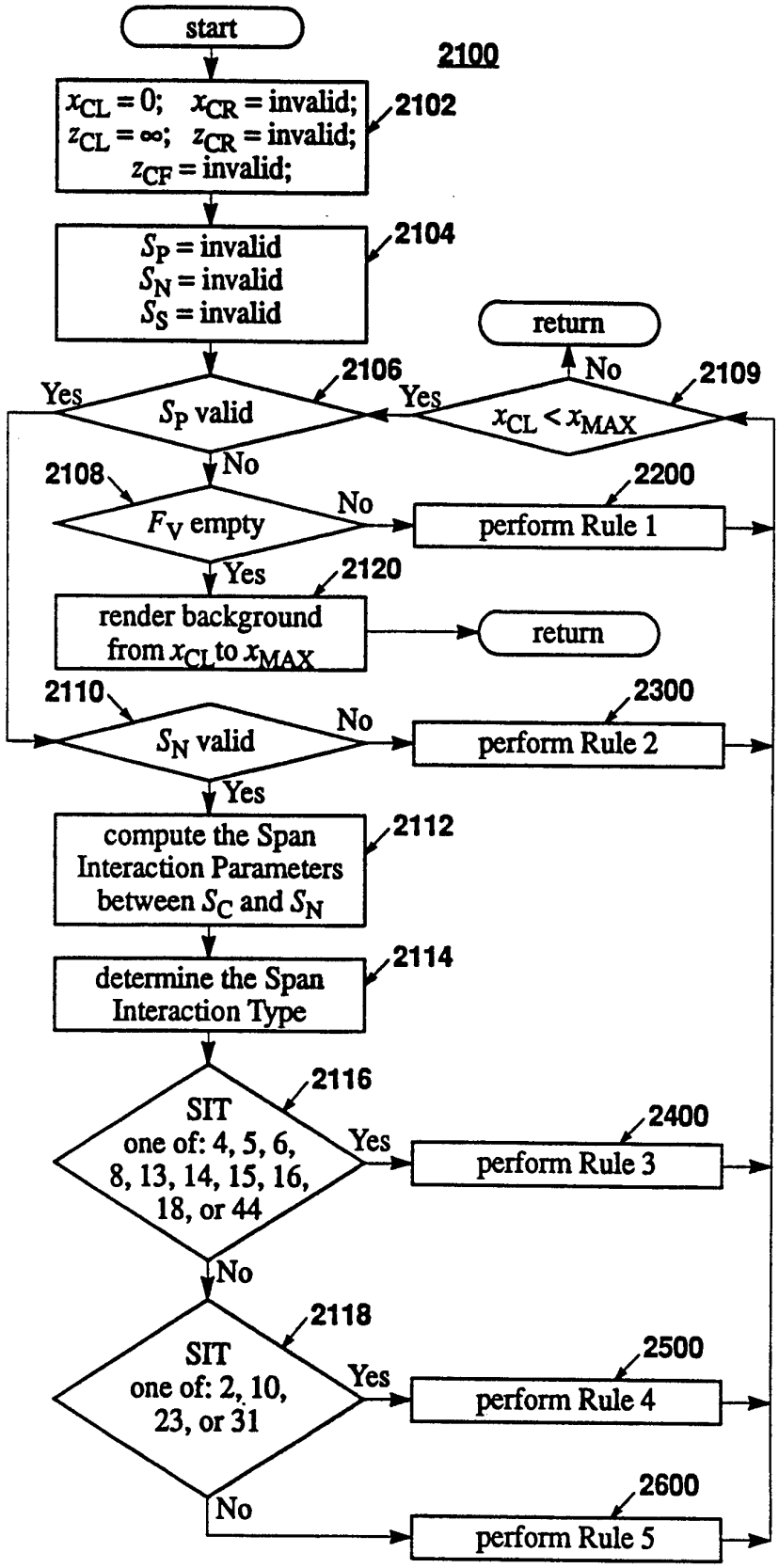

start
2100
$x_{CL} = 0$; x_{CR} = invalid;
$z_{CL} = \infty$; z_{CR} = invalid;
z_{CF} = invalid;
2102
S_P = invalid
S_N = invalid
S_S = invalid
2104
return
No
2109
Yes
$x_{CL} < x_{MAX}$
2106
Yes
S_P valid
No
2108
F_V empty
No
2200
perform Rule 1
Yes
2120
render background from x_{CL} to x_{MAX}
return
2110
S_N valid
No
2300
perform Rule 2
Yes
compute the Span Interaction Parameters between S_C and S_N
2112
determine the Span Interaction Type
2114
SIT one of: 4, 5, 6, 8, 13, 14, 15, 16, 18, or 44
2116
Yes
2400
perform Rule 3
No
SIT one of: 2, 10, 23, or 31
2118
Yes
2500
perform Rule 4
No
2600
perform Rule 5

Figure 22

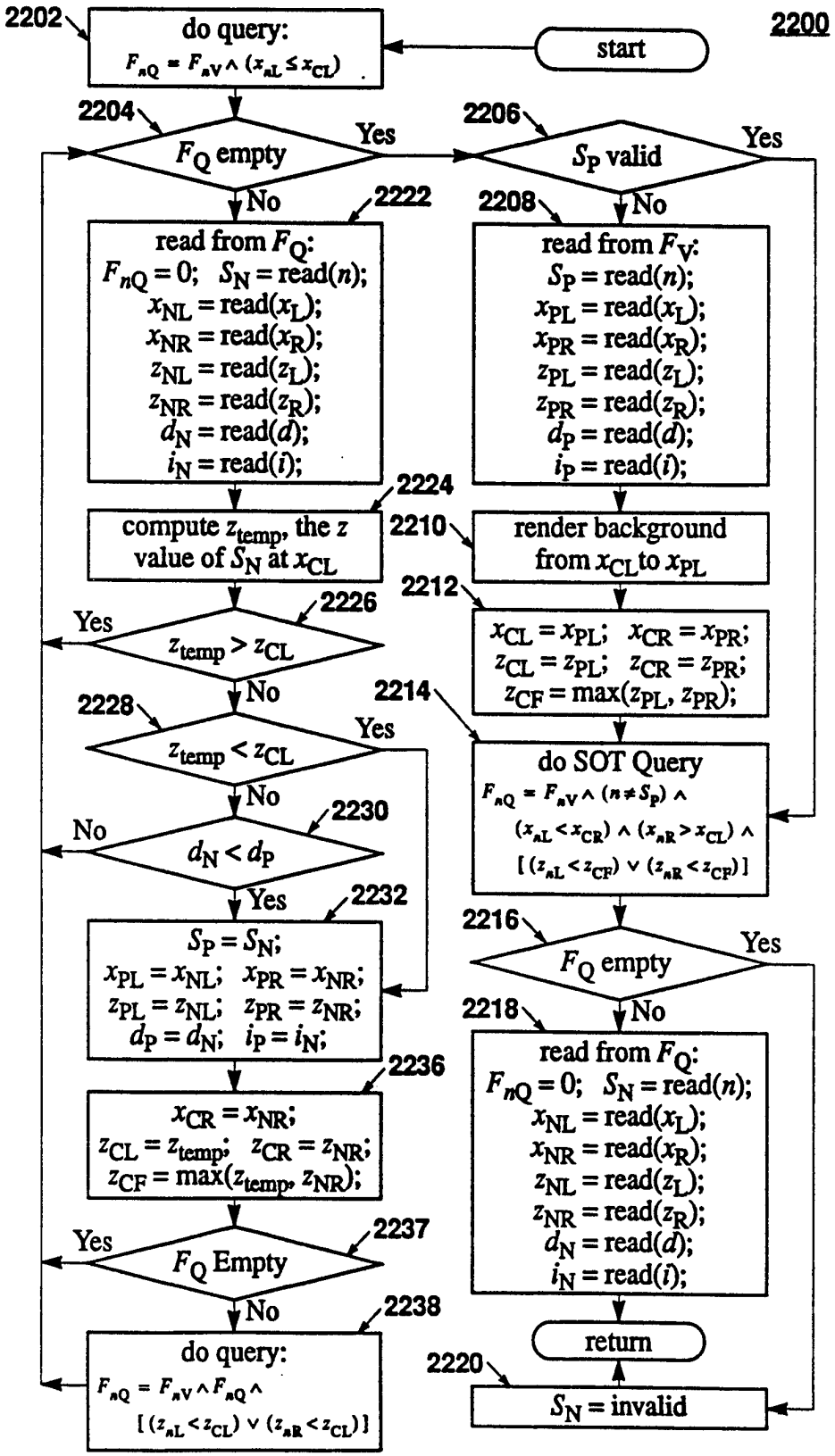

2200
start
2202
do query:
$F_{nQ} = F_{nV} \wedge (x_{nL} \le x_{CL})$
2204
F_Q empty
Yes
No
2206
S_P valid
Yes
No
2208
read from F_V:
$S_P = \text{read}(n)$;
$x_{PL} = \text{read}(x_L)$;
$x_{PR} = \text{read}(x_R)$;
$z_{PL} = \text{read}(z_L)$;
$z_{PR} = \text{read}(z_R)$;
$d_P = \text{read}(d)$;
$i_P = \text{read}(i)$;
2210
render background from x_{CL} to x_{PL}
2212
$x_{CL} = x_{PL}$; $x_{CR} = x_{PR}$;
$z_{CL} = z_{PL}$; $z_{CR} = z_{PR}$;
$z_{CF} = \max(z_{PL}, z_{PR})$;
2214
do SOT Query
$F_{nQ} = F_{nV} \wedge (n \neq S_P) \wedge$
$(x_{nL} < x_{CR}) \wedge (x_{nR} > x_{CL}) \wedge$
$[(z_{nL} < z_{CF}) \vee (z_{nR} < z_{CF})]$
2216
F_Q empty
Yes
No
2218
read from F_Q:
$F_{nQ} = 0$; $S_N = \text{read}(n)$;
$x_{NL} = \text{read}(x_L)$;
$x_{NR} = \text{read}(x_R)$;
$z_{NL} = \text{read}(z_L)$;
$z_{NR} = \text{read}(z_R)$;
$d_N = \text{read}(d)$;
$i_N = \text{read}(i)$;
return
2220
S_N = invalid
2222
read from F_Q:
$F_{nQ} = 0$; $S_N = \text{read}(n)$;
$x_{NL} = \text{read}(x_L)$;
$x_{NR} = \text{read}(x_R)$;
$z_{NL} = \text{read}(z_L)$;
$z_{NR} = \text{read}(z_R)$;
$d_N = \text{read}(d)$;
$i_N = \text{read}(i)$;
2224
compute z_{temp}, the z value of S_N at x_{CL}
2226
$z_{temp} > z_{CL}$
Yes
No
2228
$z_{temp} < z_{CL}$
Yes
No
2230
$d_N < d_P$
No
Yes
2232
$S_P = S_N$;
$x_{PL} = x_{NL}$; $x_{PR} = x_{NR}$;
$z_{PL} = z_{NL}$; $z_{PR} = z_{NR}$;
$d_P = d_N$; $i_P = i_N$;
2236
$x_{CR} = x_{NR}$;
$z_{CL} = z_{temp}$; $z_{CR} = z_{NR}$;
$z_{CF} = \max(z_{temp}, z_{NR})$;
2237
F_Q Empty
Yes
No
2238
do query:
$F_{nQ} = F_{nV} \wedge F_{nQ} \wedge$
$[(z_{nL} < z_{CL}) \vee (z_{nR} < z_{CL})]$

Figure 23

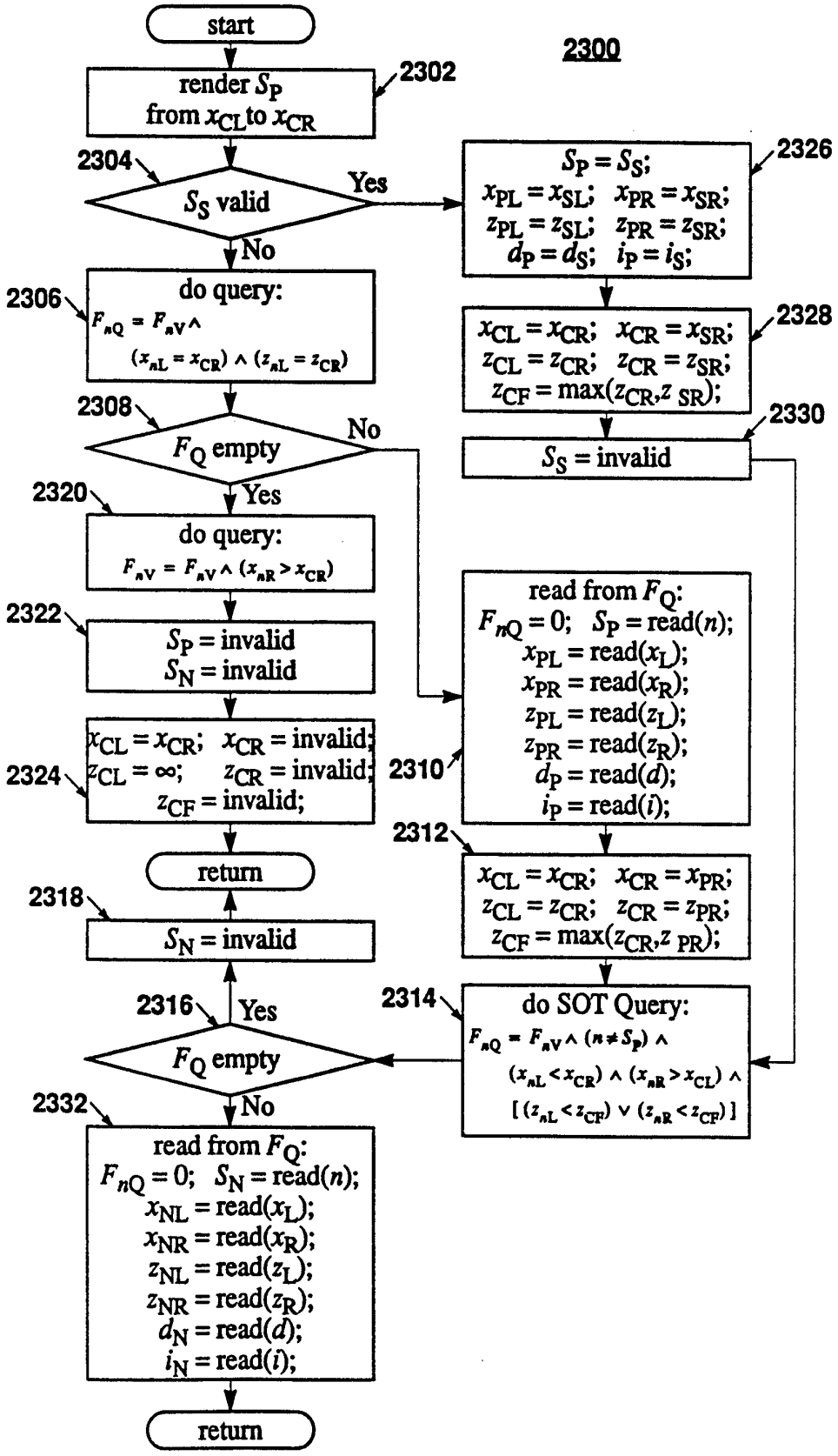

start
2300
render S_P from x_{CL} to x_{CR}
2302
2304
S_S valid
Yes
No
$S_P = S_S$; $x_{PL} = x_{SL}$; $x_{PR} = x_{SR}$; $z_{PL} = z_{SL}$; $z_{PR} = z_{SR}$; $d_P = d_S$; $i_P = i_S$;
2326
2306
do query: $F_{nQ} = F_{nV} \wedge (x_{nL} = x_{CR}) \wedge (z_{nL} = z_{CR})$
$x_{CL} = x_{CR}$; $x_{CR} = x_{SR}$; $z_{CL} = z_{CR}$; $z_{CR} = z_{SR}$; $z_{CF} = \max(z_{CR}, z_{SR})$;
2328
2308
F_Q empty
No
Yes
2330
S_S = invalid
2320
do query: $F_{nV} = F_{nV} \wedge (x_{nR} > x_{CR})$
2322
S_P = invalid
S_N = invalid
read from F_Q: $F_{nQ} = 0$; $S_P = \text{read}(n)$; $x_{PL} = \text{read}(x_L)$; $x_{PR} = \text{read}(x_R)$; $z_{PL} = \text{read}(z_L)$; $z_{PR} = \text{read}(z_R)$; $d_P = \text{read}(d)$; $i_P = \text{read}(i)$;
2310
$x_{CL} = x_{CR}$; x_{CR} = invalid; $z_{CL} = \infty$; z_{CR} = invalid; z_{CF} = invalid;
2324
2312
$x_{CL} = x_{CR}$; $x_{CR} = x_{PR}$; $z_{CL} = z_{CR}$; $z_{CR} = z_{PR}$; $z_{CF} = \max(z_{CR}, z_{PR})$;
return
2318
S_N = invalid
2314
do SOT Query: $F_{nQ} = F_{nV} \wedge (n \neq S_P) \wedge (x_{nL} < x_{CR}) \wedge (x_{nR} > x_{CL}) \wedge [(z_{nL} < z_{CF}) \vee (z_{nR} < z_{CF})]$
2316
Yes
F_Q empty
No
2332
read from F_Q: $F_{nQ} = 0$; $S_N = \text{read}(n)$; $x_{NL} = \text{read}(x_L)$; $x_{NR} = \text{read}(x_R)$; $z_{NL} = \text{read}(z_L)$; $z_{NR} = \text{read}(z_R)$; $d_N = \text{read}(d)$; $i_N = \text{read}(i)$;
return

Figure 24

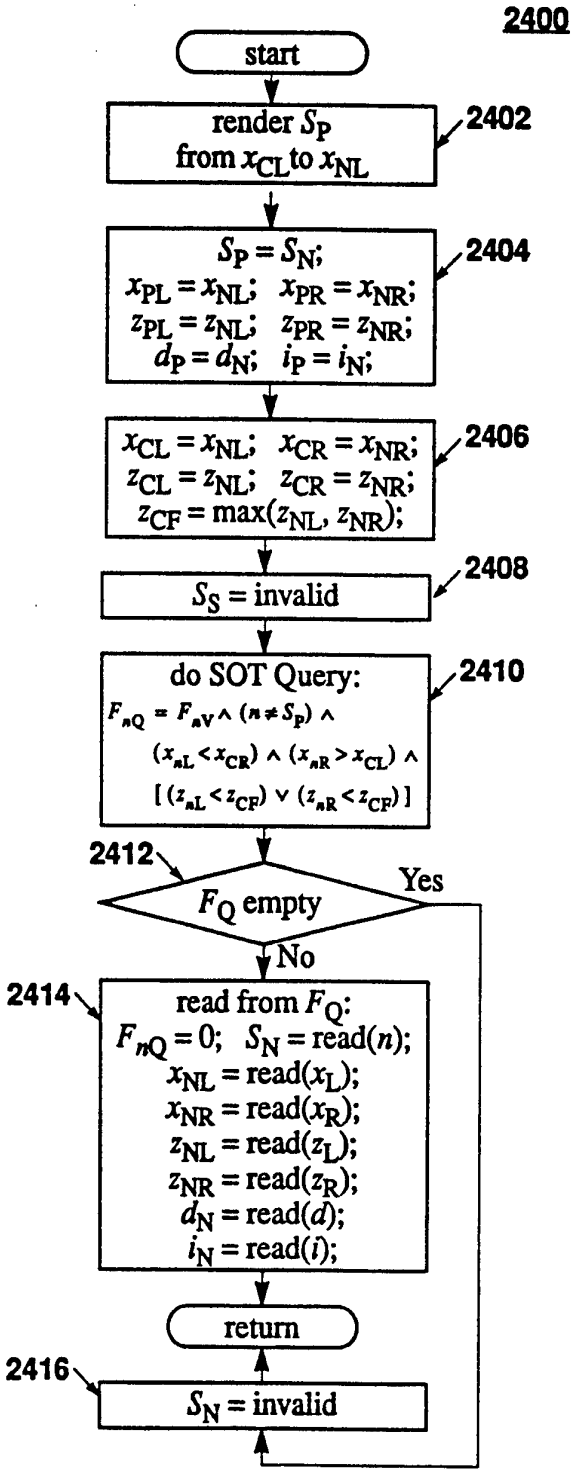

2400
start
render S_P from x_{CL} to x_{NL}
2402
$S_P = S_N$;
$x_{PL} = x_{NL}$; $x_{PR} = x_{NR}$;
$z_{PL} = z_{NL}$; $z_{PR} = z_{NR}$;
$d_P = d_N$; $i_P = i_N$;
2404
$x_{CL} = x_{NL}$; $x_{CR} = x_{NR}$;
$z_{CL} = z_{NL}$; $z_{CR} = z_{NR}$;
$z_{CF} = \max(z_{NL}, z_{NR})$;
2406
S_S = invalid
2408
do SOT Query:
$F_{nQ} = F_{nV} \wedge (n \neq S_P) \wedge$
$(x_{nL} < x_{CR}) \wedge (x_{nR} > x_{CL}) \wedge$
$[(z_{nL} < z_{CF}) \vee (z_{nR} < z_{CF})]$
2410
2412
F_Q empty
Yes
No
2414
read from F_Q:
$F_{nQ} = 0$; S_N = read(n);
x_{NL} = read(x_L);
x_{NR} = read(x_R);
z_{NL} = read(z_L);
z_{NR} = read(z_R);
d_N = read(d);
i_N = read(i);
return
2416
S_N = invalid

Figure 25

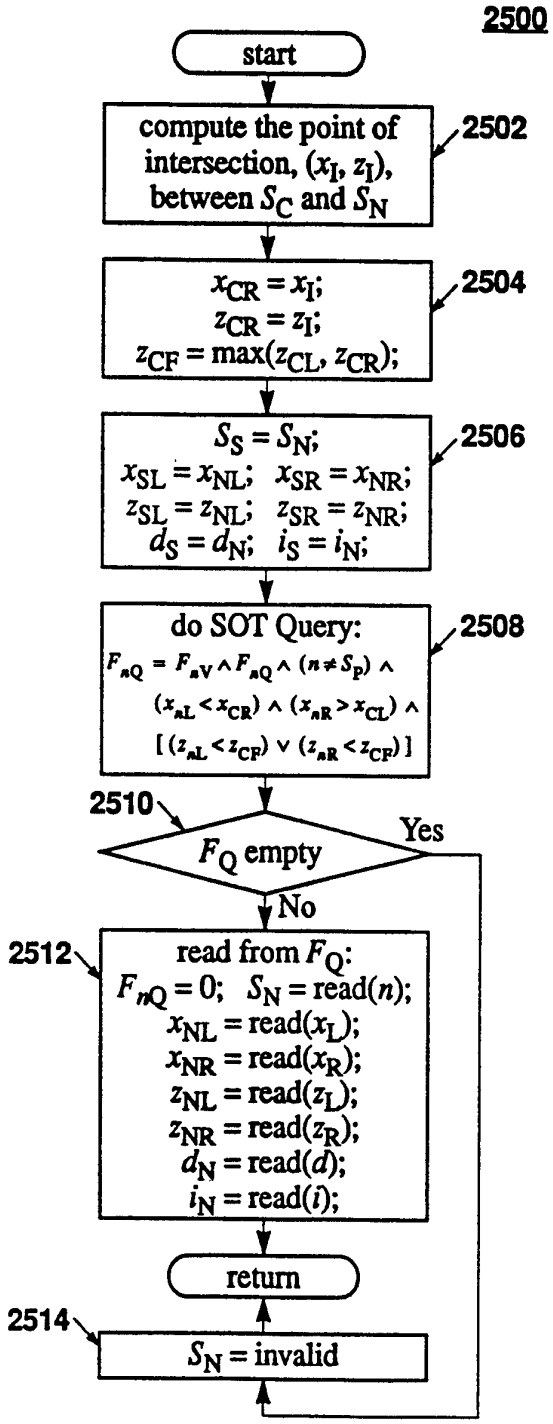

2500
start
compute the point of intersection, (x_I, z_I), between S_C and S_N
2502
$x_{CR} = x_I$;
$z_{CR} = z_I$;
$z_{CF} = \max(z_{CL}, z_{CR})$;
2504
$S_S = S_N$;
$x_{SL} = x_{NL}$; $x_{SR} = x_{NR}$;
$z_{SL} = z_{NL}$; $z_{SR} = z_{NR}$;
$d_S = d_N$; $i_S = i_N$;
2506
do SOT Query:
$F_{nQ} = F_{nV} \wedge F_{nQ} \wedge (n \neq S_P) \wedge$
$(x_{nL} < x_{CR}) \wedge (x_{nR} > x_{CL}) \wedge$
$[(z_{nL} < z_{CF}) \vee (z_{nR} < z_{CF})]$
2508
2510
F_Q empty
Yes
No
2512
read from F_Q:
$F_{nQ} = 0$; $S_N = \text{read}(n)$;
$x_{NL} = \text{read}(x_L)$;
$x_{NR} = \text{read}(x_R)$;
$z_{NL} = \text{read}(z_L)$;
$z_{NR} = \text{read}(z_R)$;
$d_N = \text{read}(d)$;
$i_N = \text{read}(i)$;
return
2514
S_N = invalid

Figure 26

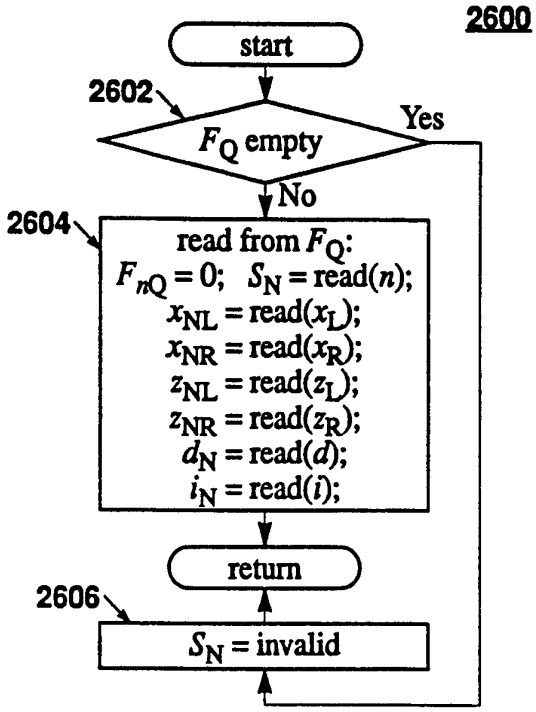

2600
start
2602
F_Q empty
Yes
No
2604
read from F_Q:
$F_{nQ} = 0$; S_N = read(n);
x_{NL} = read(x_L);
x_{NR} = read(x_R);
z_{NL} = read(z_L);
z_{NR} = read(z_R);
d_N = read(d);
i_N = read(i);
return
2606
S_N = invalid

Figure 27

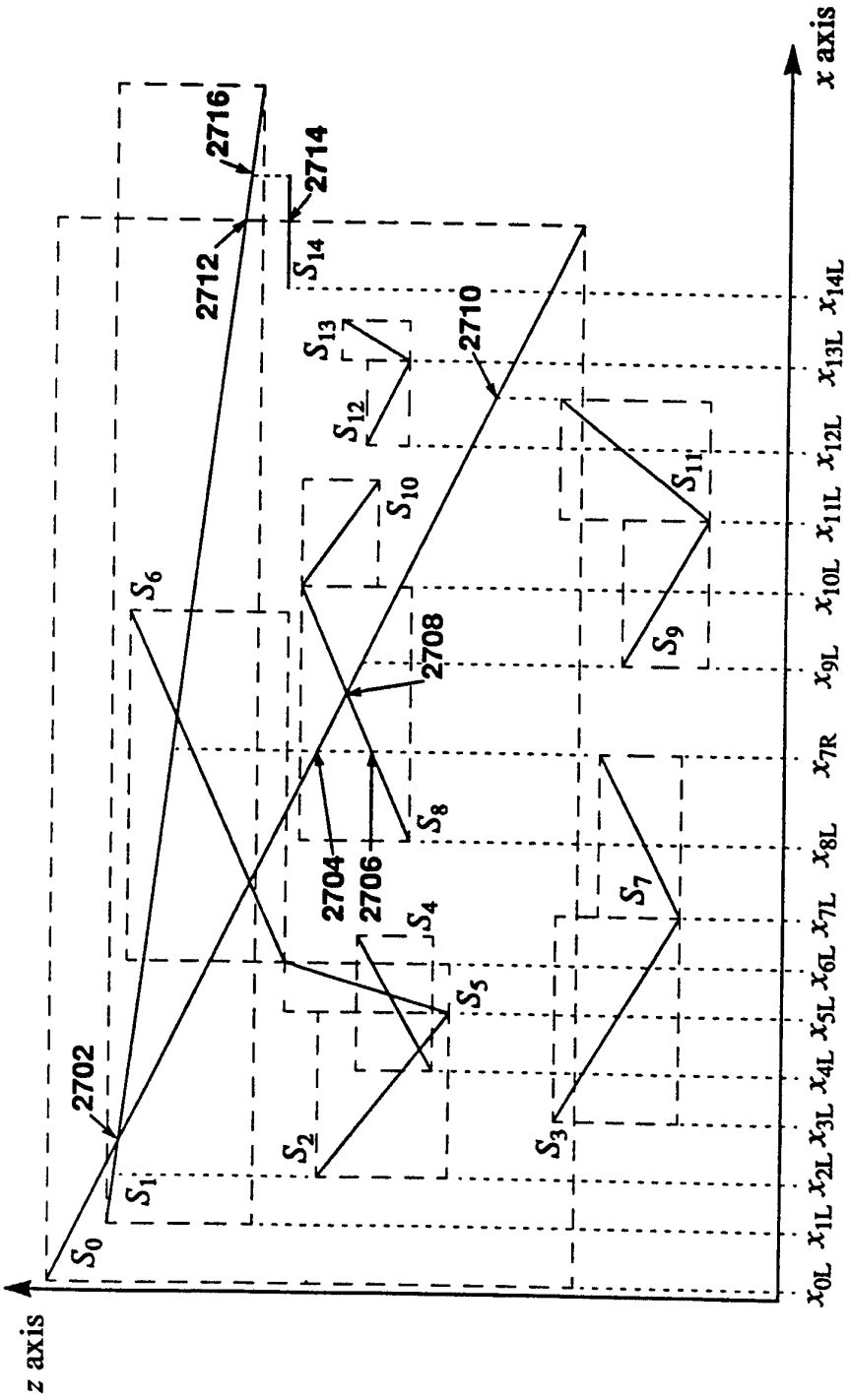

x axis
z axis
2702
2704
2706
2708
2710
2712
2714
2716
S_0
S_1
S_2
S_3
S_4
S_5
S_6
S_7
S_8
S_9
S_{10}
S_{11}
S_{12}
S_{13}
S_{14}
x_{0L} x_{1L} x_{2L} x_{3L} x_{4L} x_{5L} x_{6L} x_{7L} x_{8L} x_{7R} x_{9L} x_{10L} x_{11L} x_{12L} x_{13L} x_{14L}

Figure 28

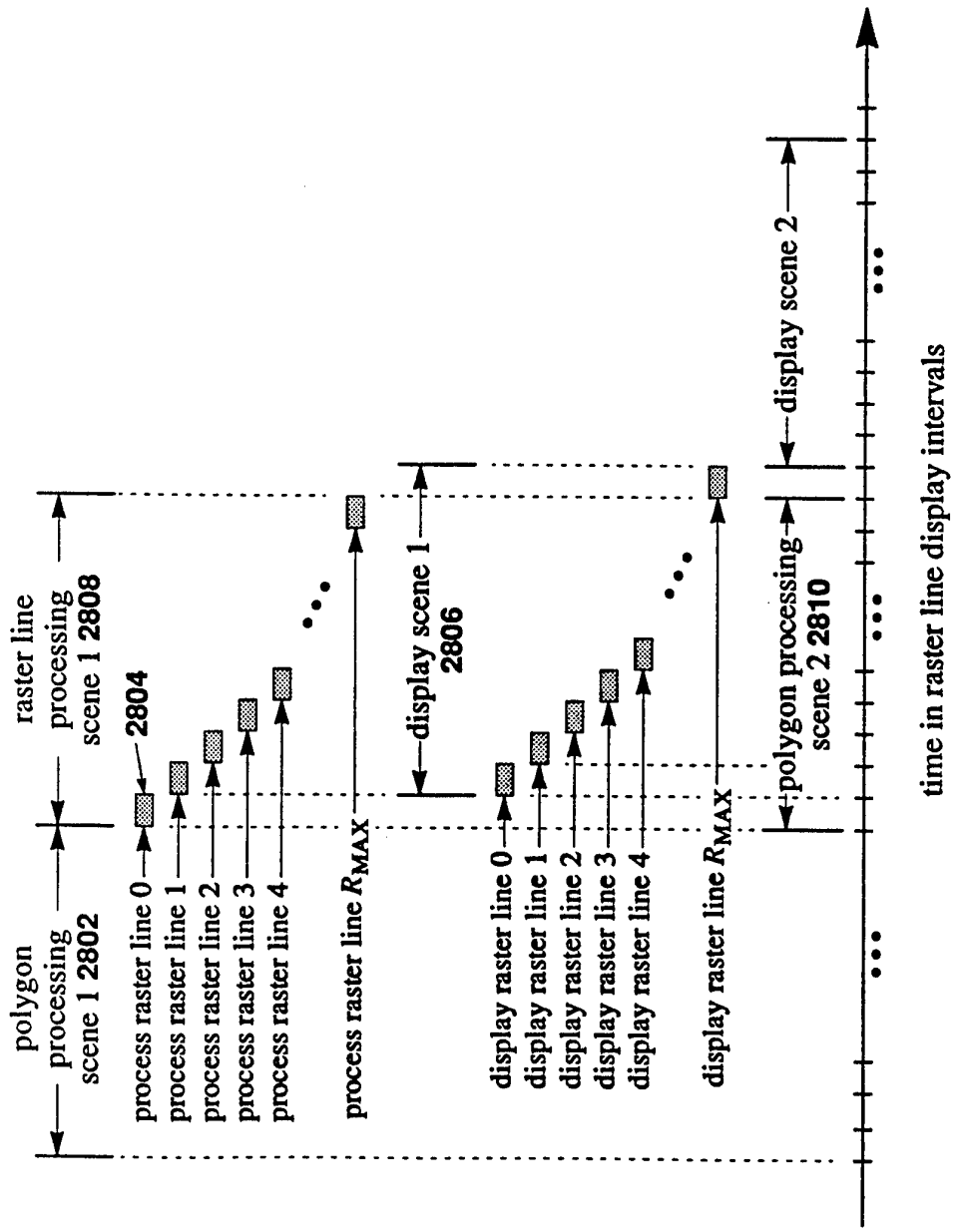

polygon processing scene 1 2802
raster line processing scene 1 2808
2804
process raster line 0
process raster line 1
process raster line 2
process raster line 3
process raster line 4
process raster line R_{MAX}
display scene 1 2806
display raster line 0
display raster line 1
display raster line 2
display raster line 3
display raster line 4
display raster line R_{MAX}
polygon processing scene 2 2810
display scene 2
time in raster line display intervals

Figure 29

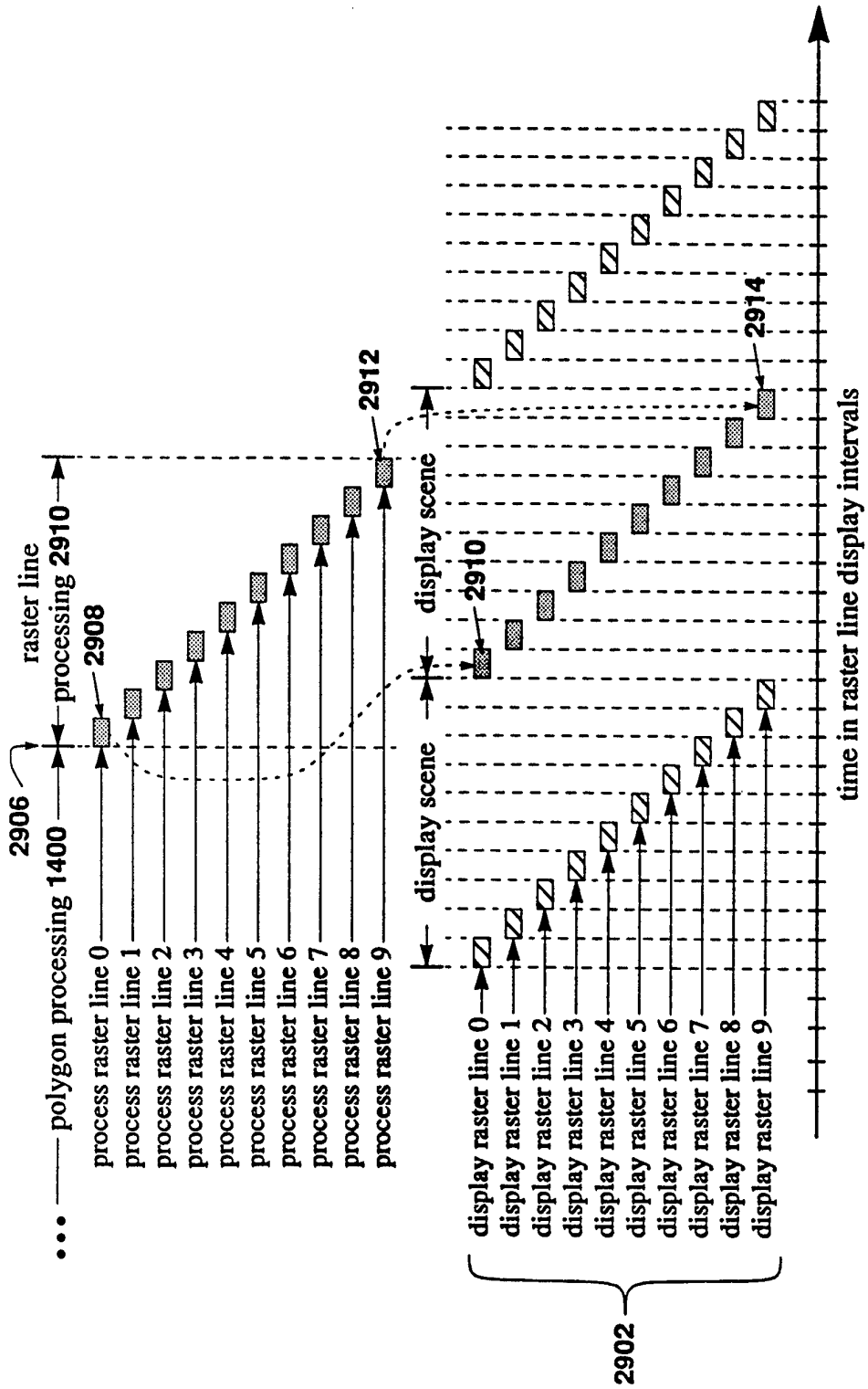

2906
1400
polygon processing
raster line
processing
2910
2908
process raster line 0
process raster line 1
process raster line 2
process raster line 3
process raster line 4
process raster line 5
process raster line 6
process raster line 7
process raster line 8
process raster line 9
2912
display scene
display scene
2910
2914
display raster line 0
display raster line 1
display raster line 2
display raster line 3
display raster line 4
display raster line 5
display raster line 6
display raster line 7
display raster line 8
display raster line 9
2902
time in raster line display intervals

Figure 30

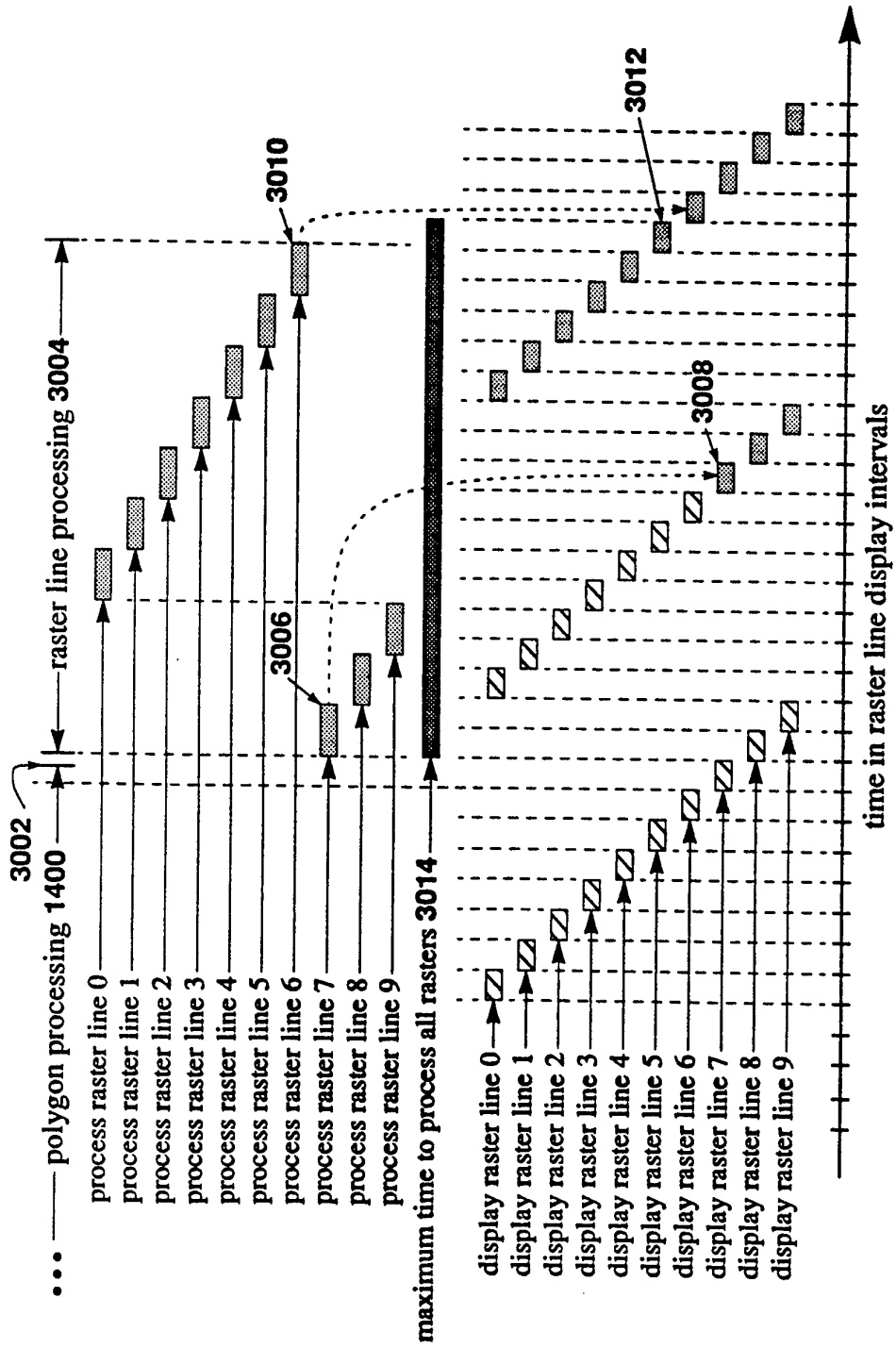

3002
polygon processing 1400
raster line processing 3004
process raster line 0
process raster line 1
process raster line 2
process raster line 3
process raster line 4
process raster line 5
process raster line 6
process raster line 7
process raster line 8
process raster line 9
3010
3006
maximum time to process all rasters 3014
3012
3008
display raster line 0
display raster line 1
display raster line 2
display raster line 3
display raster line 4
display raster line 5
display raster line 6
display raster line 7
display raster line 8
display raster line 9
time in raster line display intervals

Figure 31

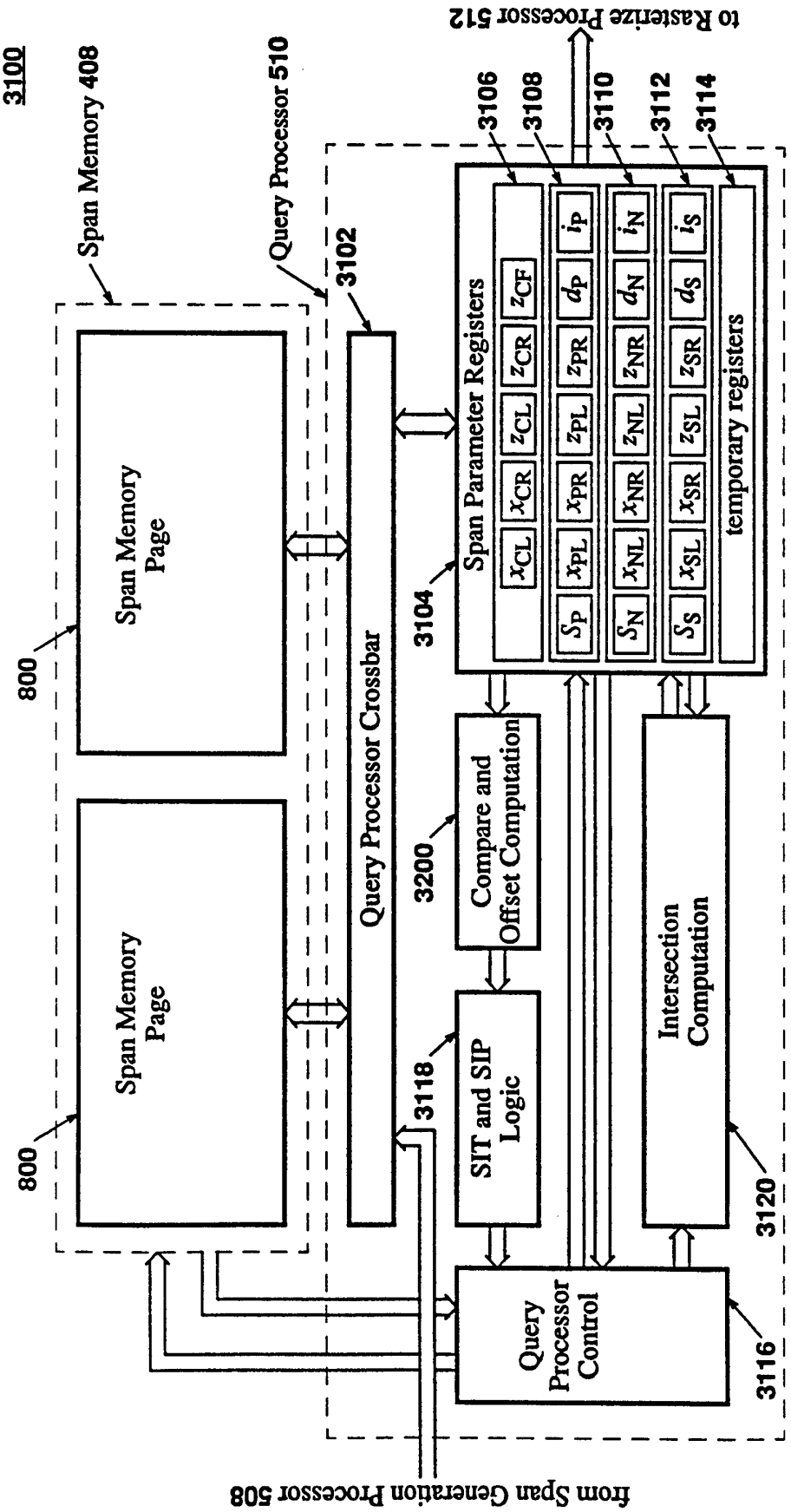

3100
Span Memory 408
Query Processor 510
3102
3106
3108
3110
3112
3114
to Rasterize Processor 512
Span Memory Page
800
Span Memory Page
800
Query Processor Crossbar
3104
Span Parameter Registers
x_{CL} x_{CR} z_{CL} z_{CR} z_{CF}
s_P x_{PL} x_{PR} z_{PL} z_{PR} d_P i_P
s_N x_{NL} x_{NR} z_{NL} z_{NR} d_N i_N
s_S x_{SL} x_{SR} z_{SL} z_{SR} d_S i_S
temporary registers
3200
Compare and Offset Computation
3118
SIT and SIP Logic
Intersection Computation
3120
Query Processor Control
3116
from Span Generation Processor 508

Figure 32

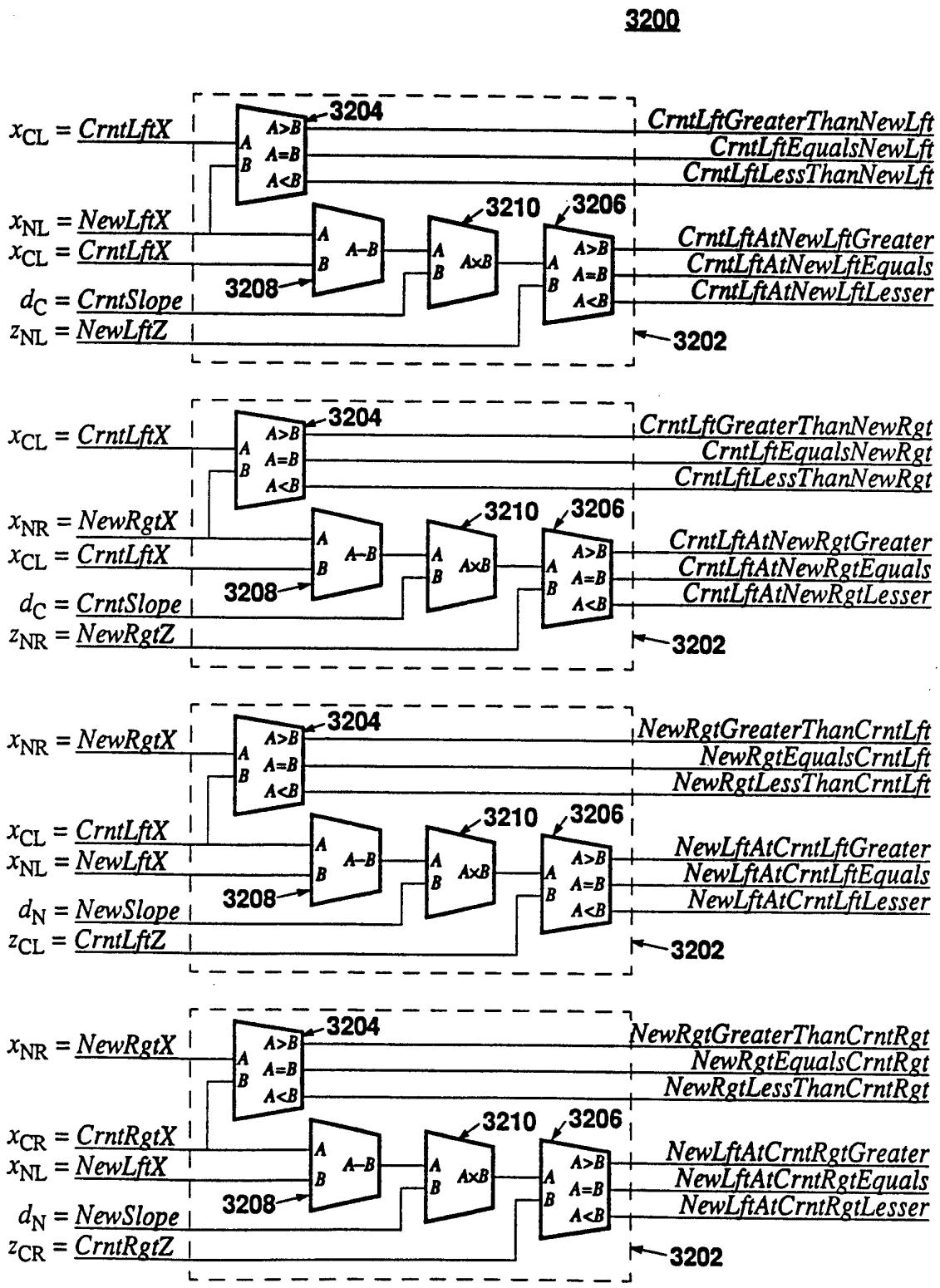

3200
x_{CL} = CrntLftX
x_{NL} = NewLftX
x_{CL} = CrntLftX
d_C = CrntSlope
z_{NL} = NewLftZ
3204
3210
3206
3208
3202
CrntLftGreaterThanNewLft
CrntLftEqualsNewLft
CrntLftLessThanNewLft
CrntLftAtNewLftGreater
CrntLftAtNewLftEquals
CrntLftAtNewLftLesser
x_{CL} = CrntLftX
x_{NR} = NewRgtX
x_{CL} = CrntLftX
d_C = CrntSlope
z_{NR} = NewRgtZ
CrntLftGreaterThanNewRgt
CrntLftEqualsNewRgt
CrntLftLessThanNewRgt
CrntLftAtNewRgtGreater
CrntLftAtNewRgtEquals
CrntLftAtNewRgtLesser
x_{NR} = NewRgtX
x_{CL} = CrntLftX
x_{NL} = NewLftX
d_N = NewSlope
z_{CL} = CrntLftZ
NewRgtGreaterThanCrntLft
NewRgtEqualsCrntLft
NewRgtLessThanCrntLft
NewLftAtCrntLftGreater
NewLftAtCrntLftEquals
NewLftAtCrntLftLesser
x_{NR} = NewRgtX
x_{CR} = CrntRgtX
x_{NL} = NewLftX
d_N = NewSlope
z_{CR} = CrntRgtZ
NewRgtGreaterThanCrntRgt
NewRgtEqualsCrntRgt
NewRgtLessThanCrntRgt
NewLftAtCrntRgtGreater
NewLftAtCrntRgtEquals
NewLftAtCrntRgtLesser

Figure 33

3300

prior art

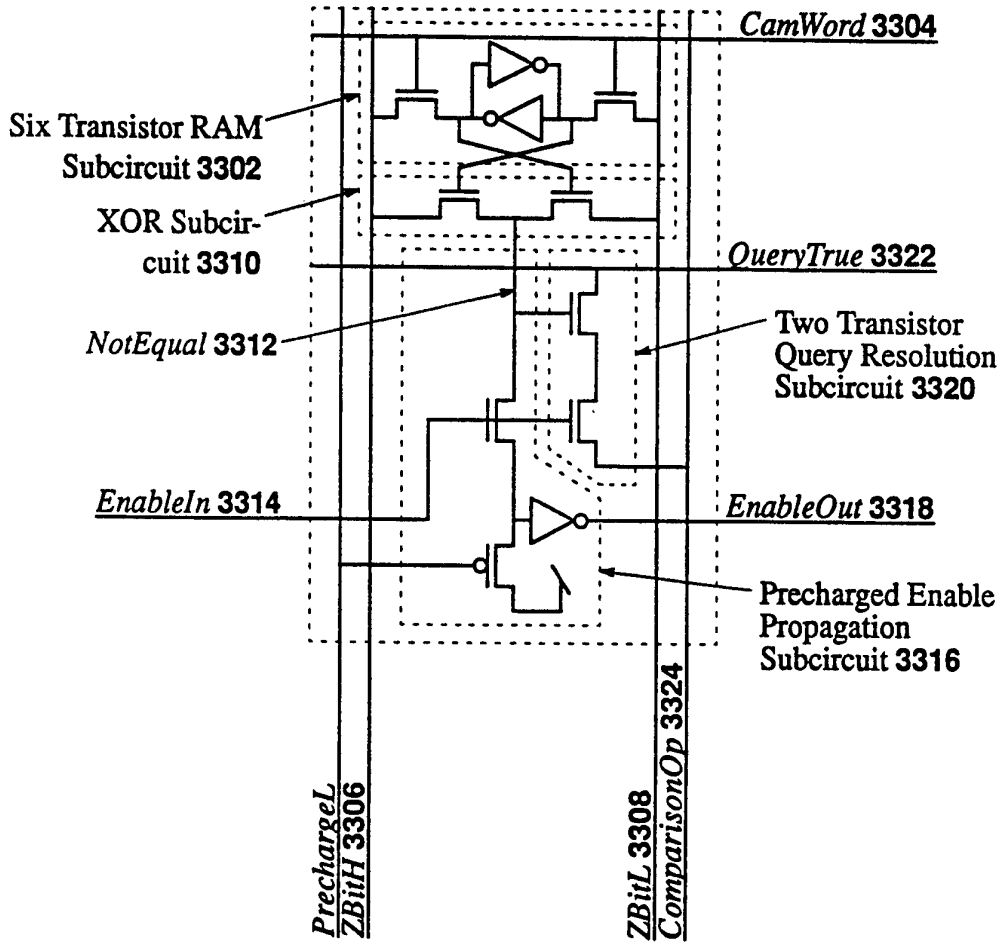


Figure 34

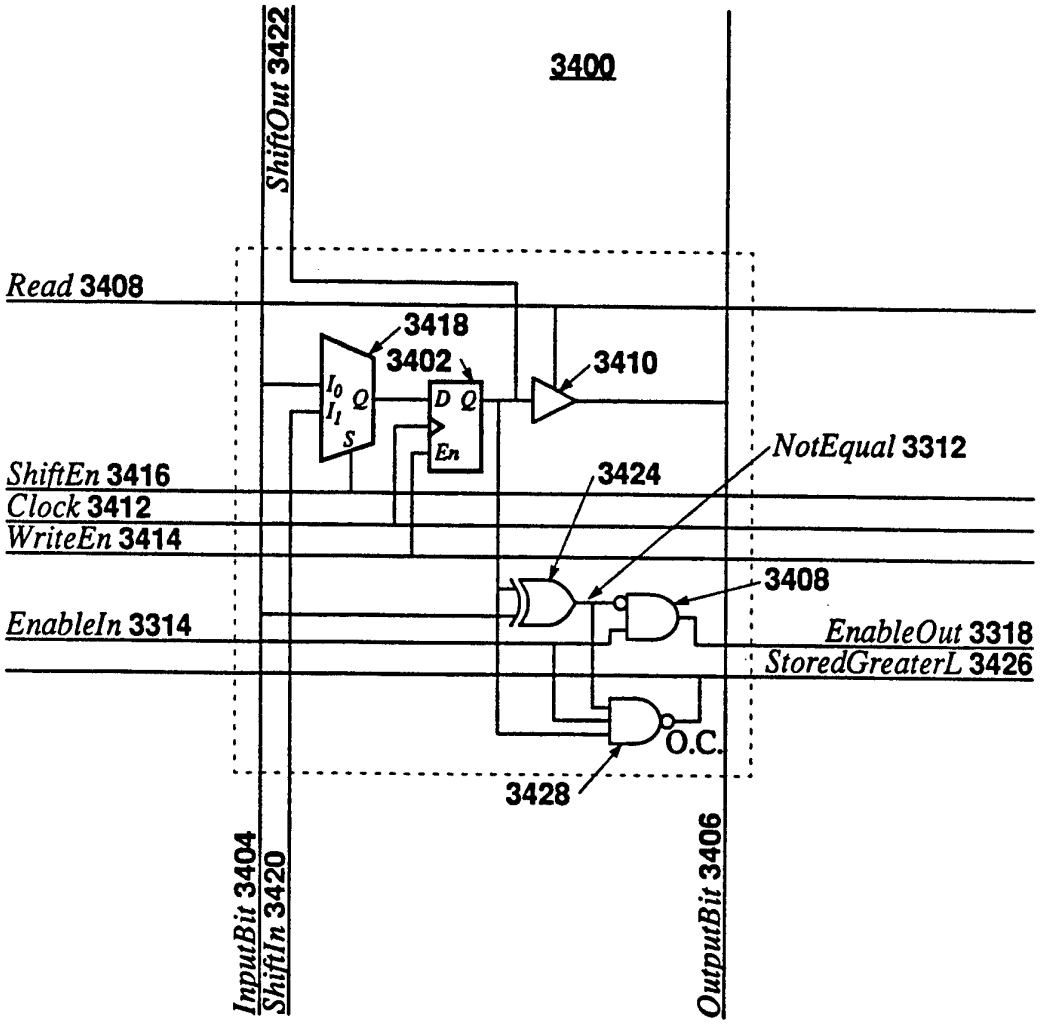

3400
ShiftOut 3422
Read 3408
3418
3402
3410
I_0 I_1 Q S
D Q
En
NotEqual 3312
ShiftEn 3416
3424
Clock 3412
WriteEn 3414
3408
EnableIn 3314
EnableOut 3318
StoredGreaterL 3426
O.C.
3428
InputBit 3404
ShiftIn 3420
OutputBit 3406

Figure 35

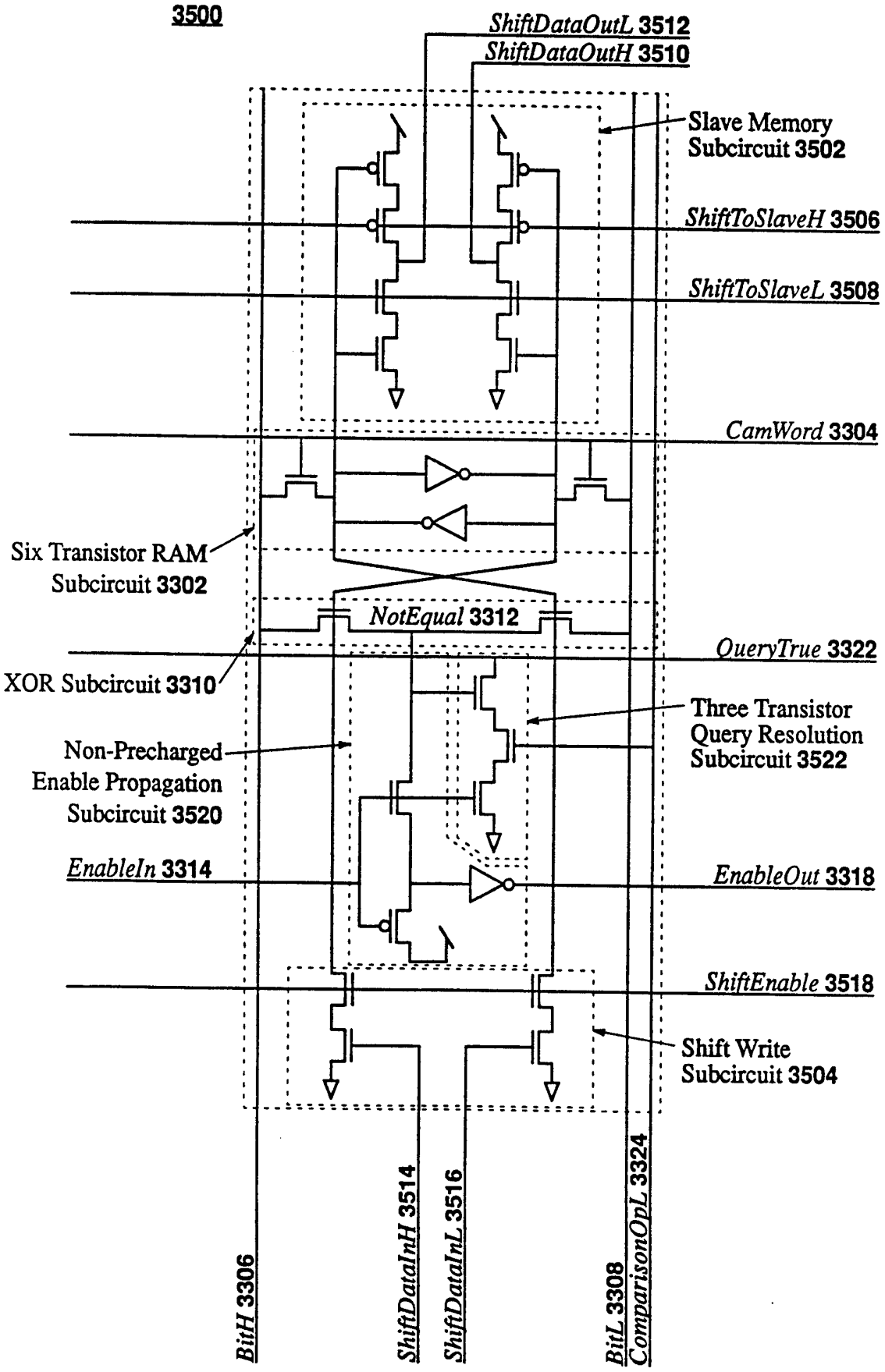

3500
ShiftDataOutL 3512
ShiftDataOutH 3510
Slave Memory Subcircuit 3502
ShiftToSlaveH 3506
ShiftToSlaveL 3508
CamWord 3304
Six Transistor RAM Subcircuit 3302
NotEqual 3312
QueryTrue 3322
XOR Subcircuit 3310
Three Transistor Query Resolution Subcircuit 3522
Non-Precharged Enable Propagation Subcircuit 3520
EnableIn 3314
EnableOut 3318
ShiftEnable 3518
Shift Write Subcircuit 3504
BitH 3306
ShiftDataInH 3514
ShiftDataInL 3516
BitL 3308
ComparisonOpL 3324

Figure 36

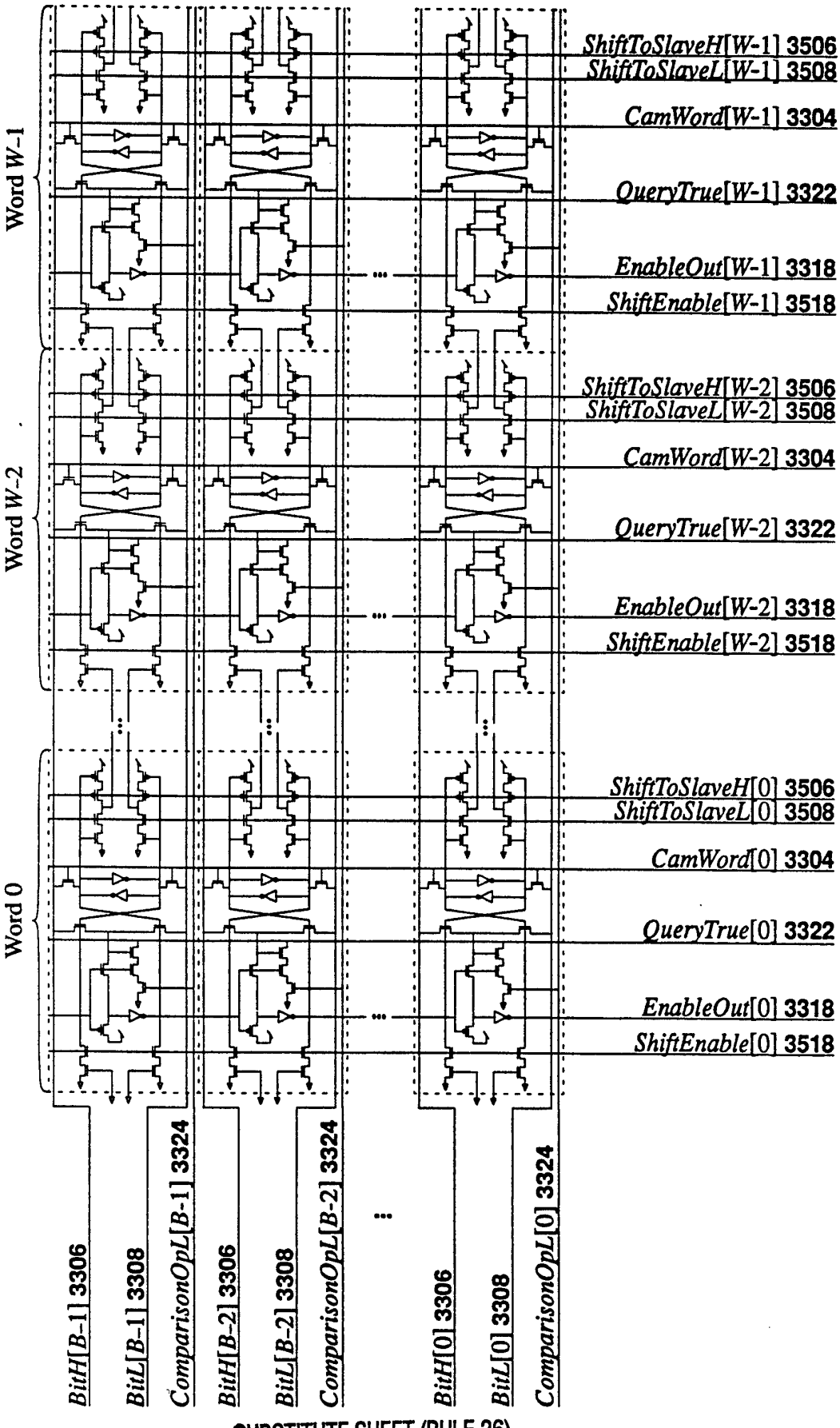

Word W-1
Word W-2
Word 0
ShiftToSlaveH[W-1] 3506
ShiftToSlaveL[W-1] 3508
CamWord[W-1] 3304
QueryTrue[W-1] 3322
EnableOut[W-1] 3318
ShiftEnable[W-1] 3518
ShiftToSlaveH[W-2] 3506
ShiftToSlaveL[W-2] 3508
CamWord[W-2] 3304
QueryTrue[W-2] 3322
EnableOut[W-2] 3318
ShiftEnable[W-2] 3518
ShiftToSlaveH[0] 3506
ShiftToSlaveL[0] 3508
CamWord[0] 3304
QueryTrue[0] 3322
EnableOut[0] 3318
ShiftEnable[0] 3518
BitH[B-1] 3306
BitL[B-1] 3308
ComparisonOpL[B-1] 3324
BitH[B-2] 3306
BitL[B-2] 3308
ComparisonOpL[B-2] 3324
BitH[0] 3306
BitL[0] 3308
ComparisonOpL[0] 3324

Figure 37

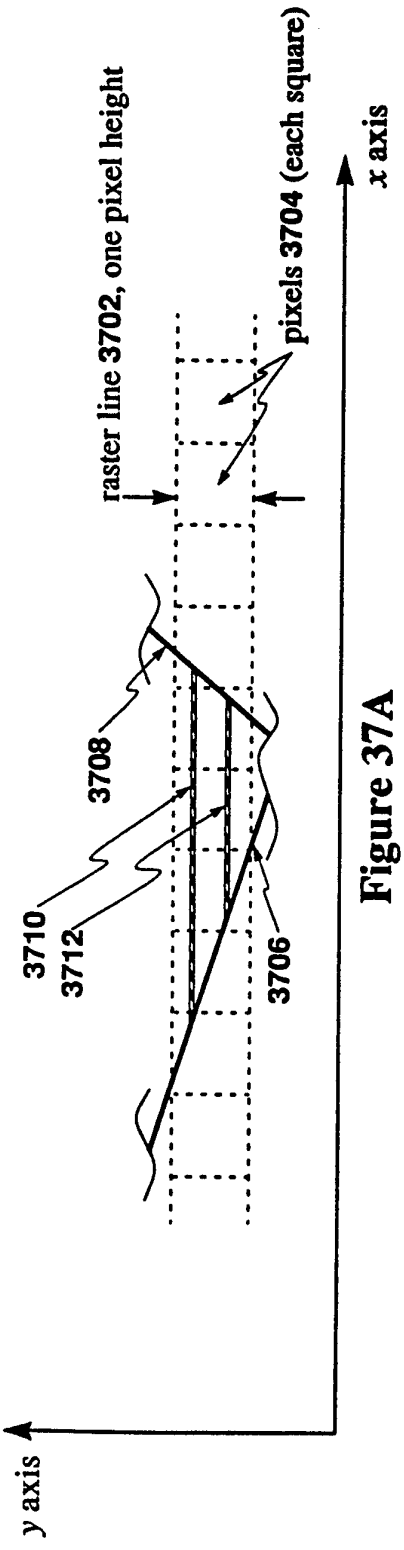


Figure 37A

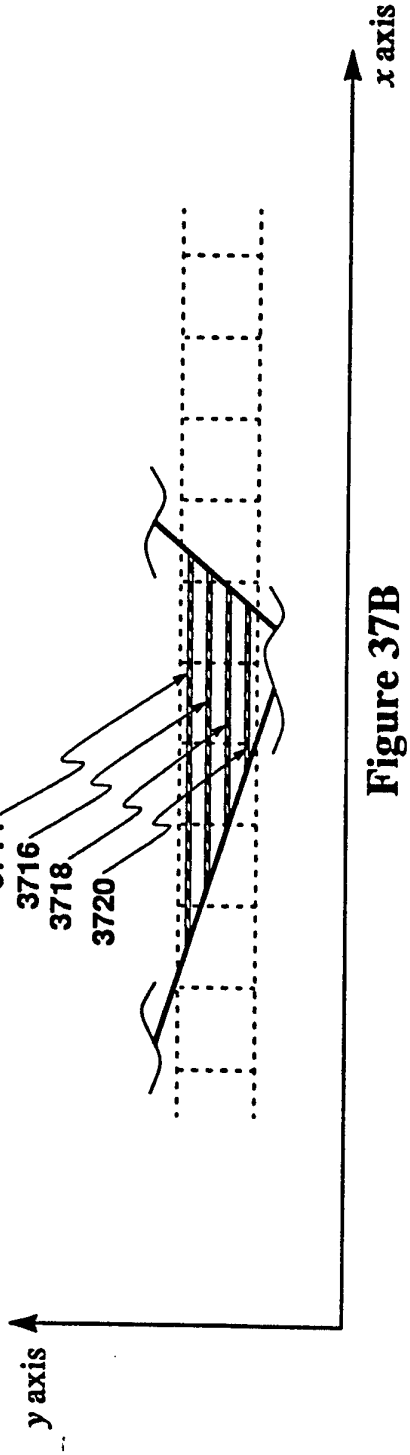


Figure 37B

Figure 38

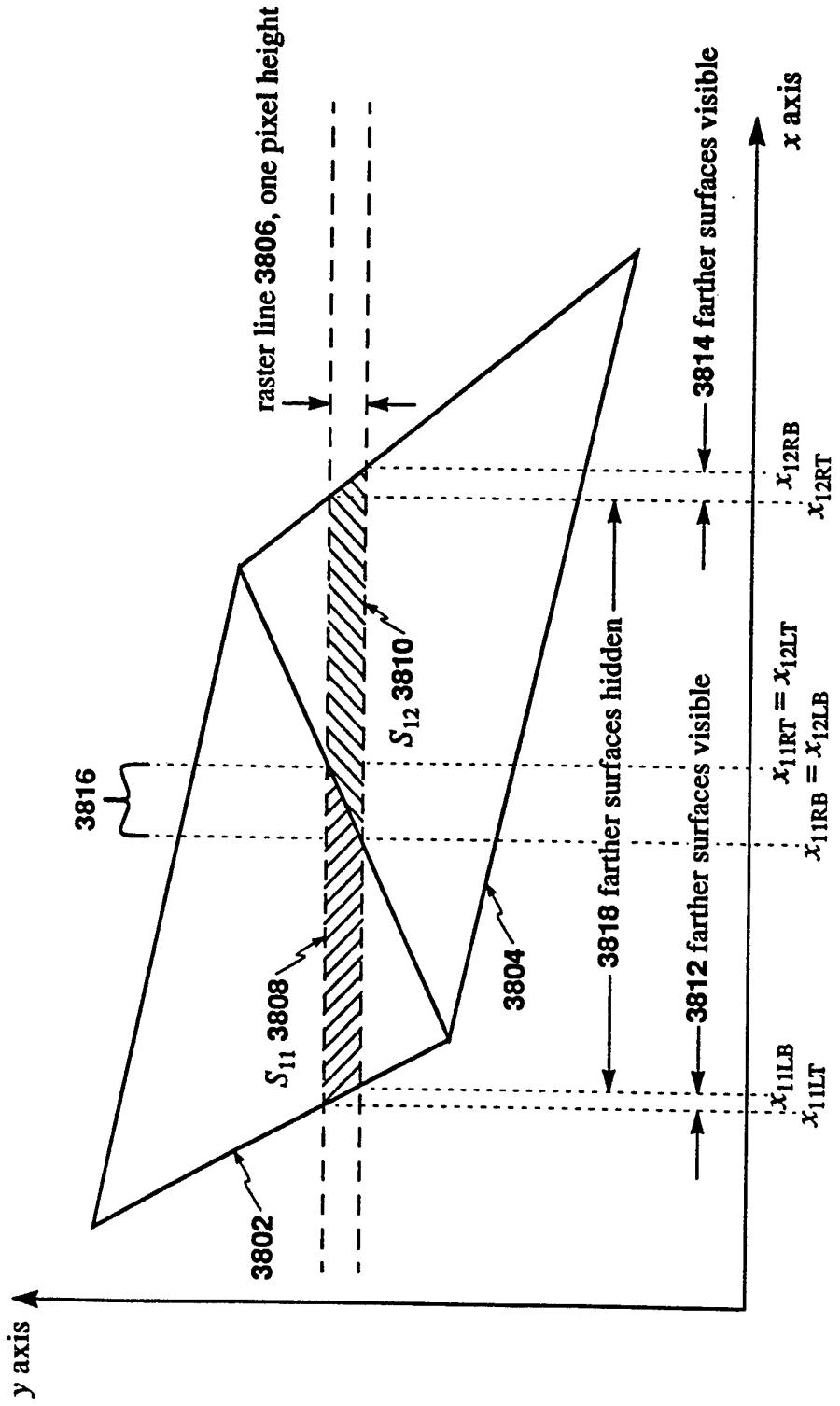

raster line 3806, one pixel height
3814 farther surfaces visible
x axis
x_{12RB}
x_{12RT}
S_{12} 3810
3816
3818 farther surfaces hidden
$x_{11RT} = x_{12LT}$
$x_{11RB} = x_{12LB}$
3812 farther surfaces visible
S_{11} 3808
3804
x_{11LB}
x_{11LT}
3802
y axis

Figure 39

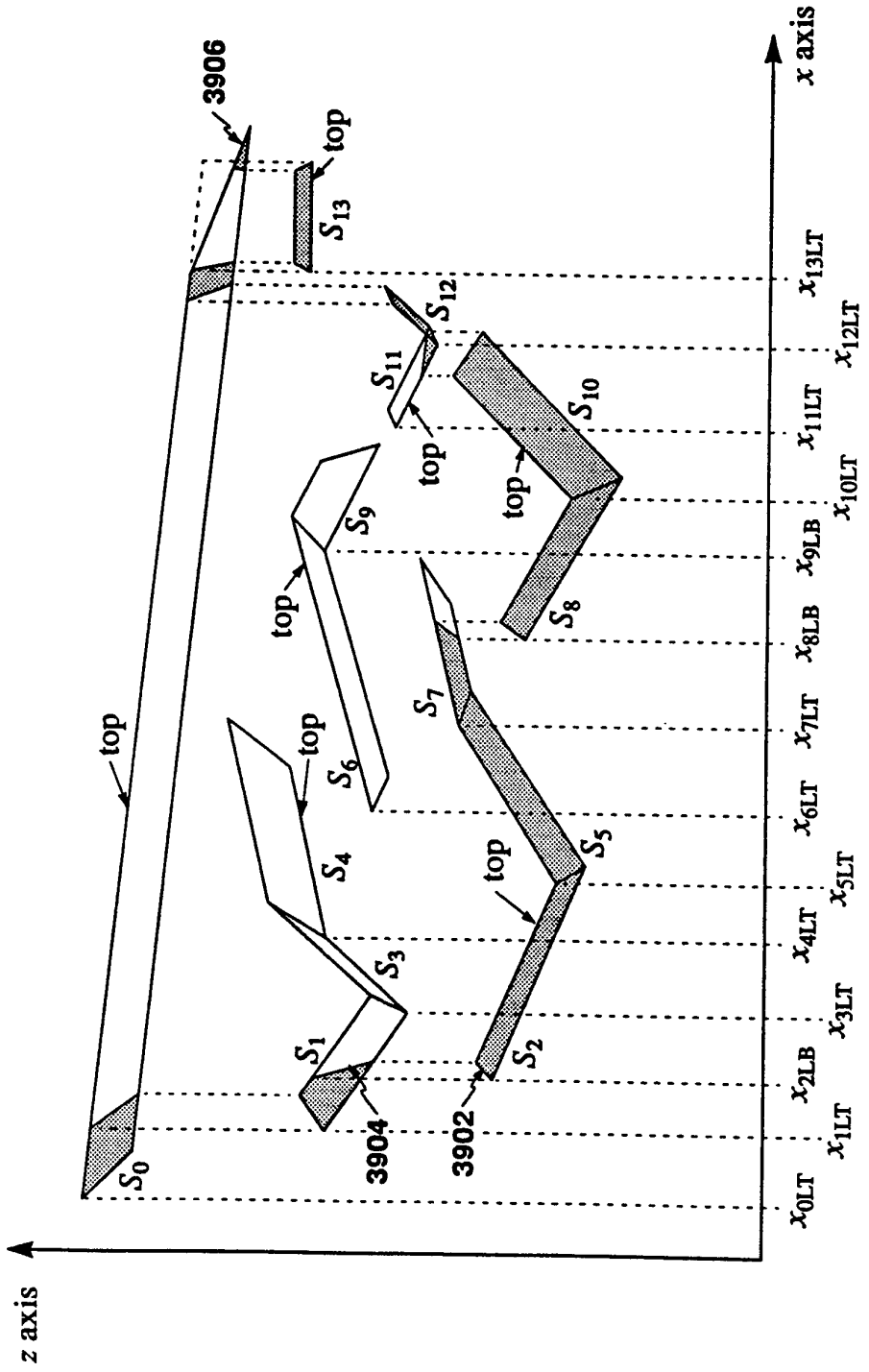

x axis
z axis
3906
3904
3902
top
S_0
S_1
S_2
S_3
S_4
S_5
S_6
S_7
S_8
S_9
S_{10}
S_{11}
S_{12}
S_{13}
x_{0LT}
x_{1LT}
x_{2LB}
x_{3LT}
x_{4LT}
x_{5LT}
x_{6LT}
x_{7LT}
x_{8LB}
x_{9LB}
x_{10LT}
x_{11LT}
x_{12LT}
x_{13LT}

Figure 40

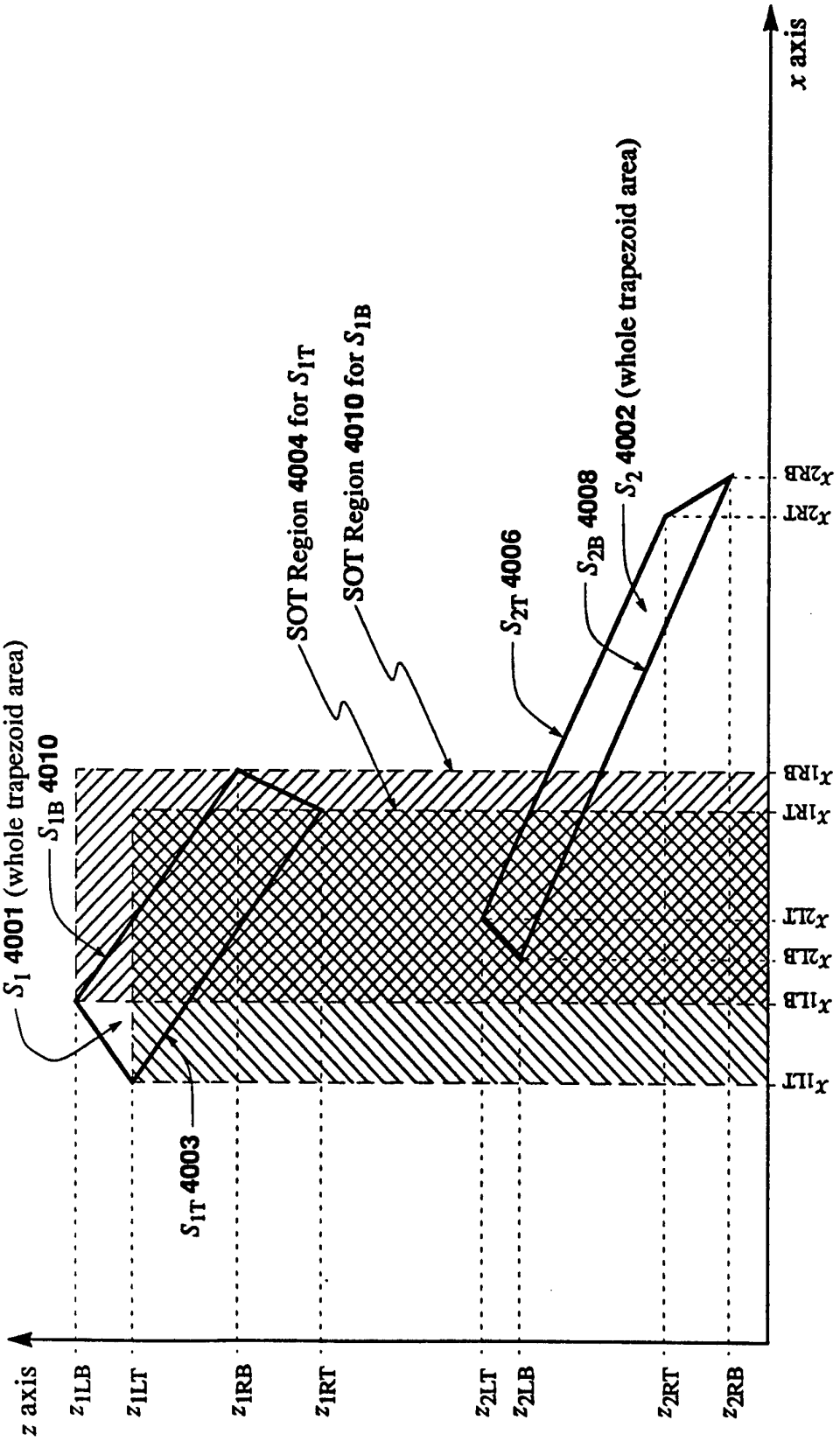

x axis
z axis
S_1 4001 (whole trapezoid area)
S_{1B} 4010
SOT Region 4004 for S_{1T}
SOT Region 4010 for S_{1B}
S_{2T} 4006
S_{2B} 4008
S_2 4002 (whole trapezoid area)
S_{1T} 4003
x_{2RB}
x_{2RT}
x_{1RB}
x_{1RT}
x_{2LT}
x_{2LB}
x_{1LB}
x_{1LT}
z_{1LB}
z_{1LT}
z_{1RB}
z_{1RT}
z_{2LT}
z_{2LB}
z_{2RT}
z_{2RB}

Figure 41

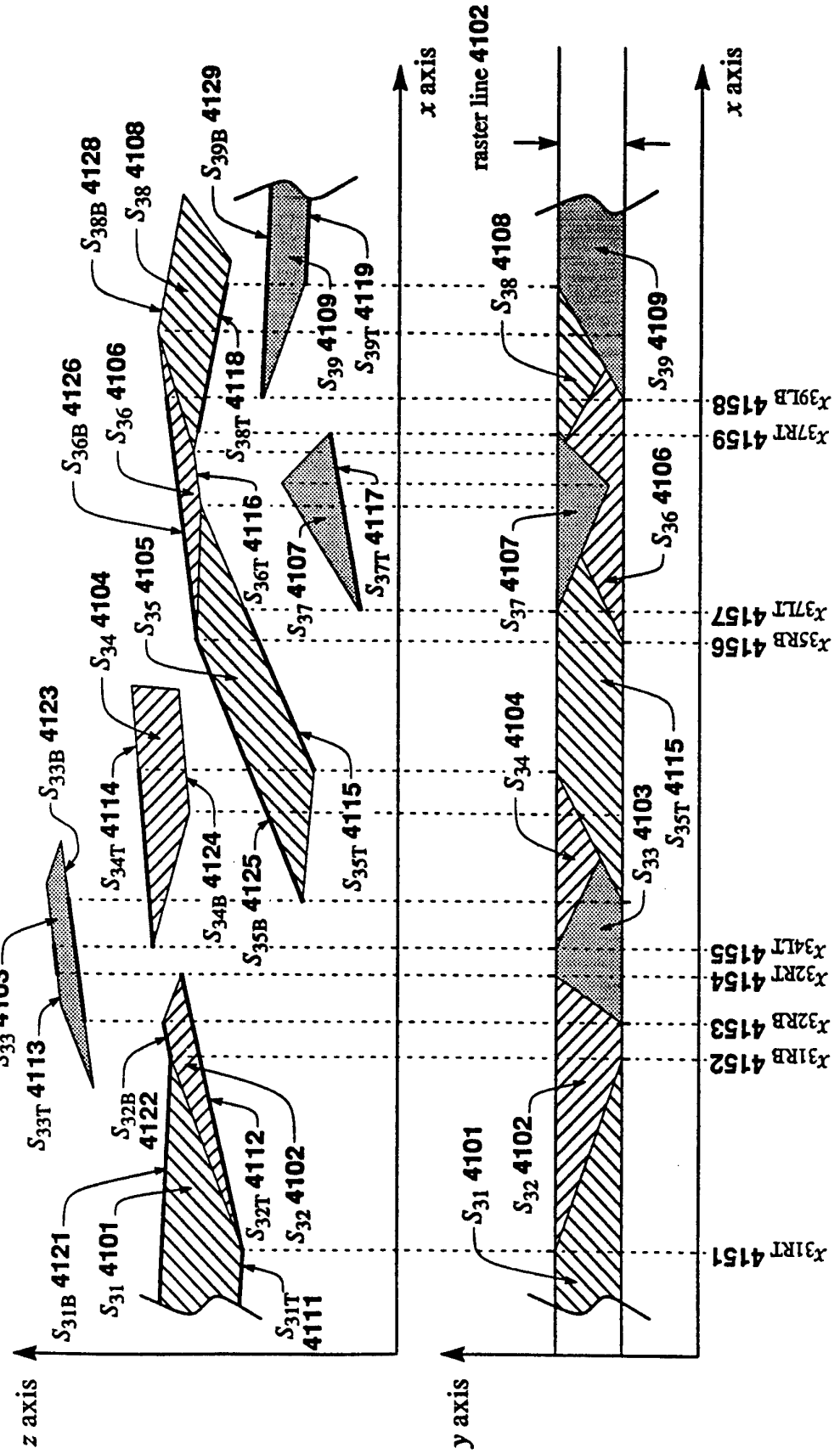

z axis
x axis
y axis
x axis
raster line 4102
S_{31B} 4121
S_{31} 4101
S_{31T} 4111
S_{32} 4102
S_{32T} 4112
S_{32B} 4122
S_{33} 4103
S_{33T} 4113
S_{33B} 4123
S_{34} 4104
S_{34T} 4114
S_{34B} 4124
S_{35} 4105
S_{35T} 4115
S_{35B} 4125
S_{36} 4106
S_{36T} 4116
S_{36B} 4126
S_{37} 4107
S_{37T} 4117
S_{38} 4108
S_{38T} 4118
S_{38B} 4128
S_{39} 4109
S_{39T} 4119
S_{39B} 4129
x_{31RT} 4151
x_{31RB} 4152
x_{32RB} 4153
x_{32RT} 4154
x_{34LT} 4155
x_{35RB} 4156
x_{37LT} 4157
x_{37RT} 4159
x_{39LB} 4158

Figure 42

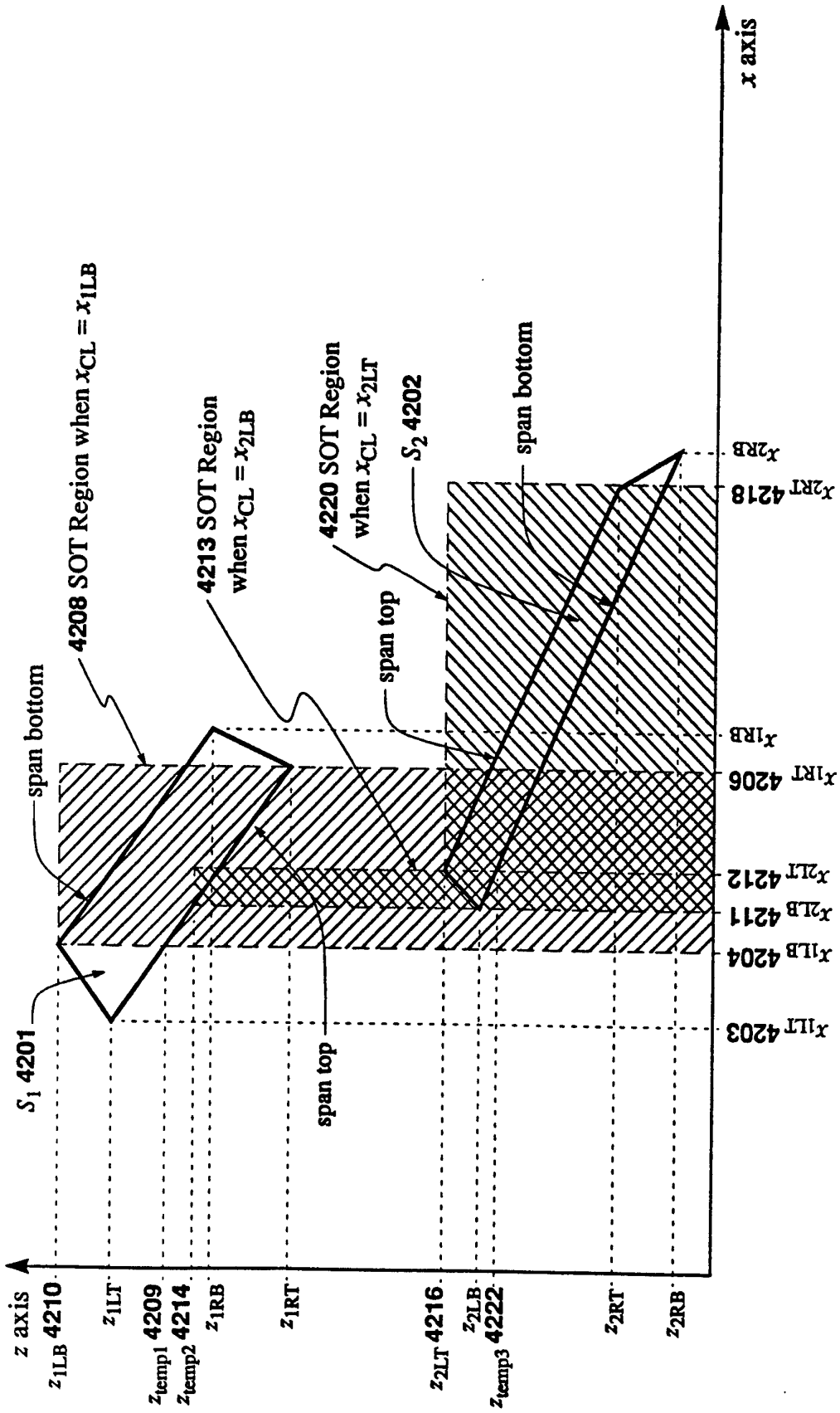

x axis
z axis
z_{1LB} 4210
z_{1LT}
z_{temp1} 4209
z_{temp2} 4214
z_{1RB}
z_{1RT}
z_{2LT} 4216
z_{2LB}
z_{temp3} 4222
z_{2RT}
z_{2RB}
x_{1LT} 4203
x_{1LB} 4204
x_{2LB} 4211
x_{2LT} 4212
x_{1RT} 4206
x_{1RB}
x_{2RT} 4218
x_{2RB}
S_1 4201
S_2 4202
span bottom
span top
span top
span bottom
4208 SOT Region when $x_{CL} = x_{1LB}$
4213 SOT Region when $x_{CL} = x_{2LB}$
4220 SOT Region when $x_{CL} = x_{2LT}$

Figure 43

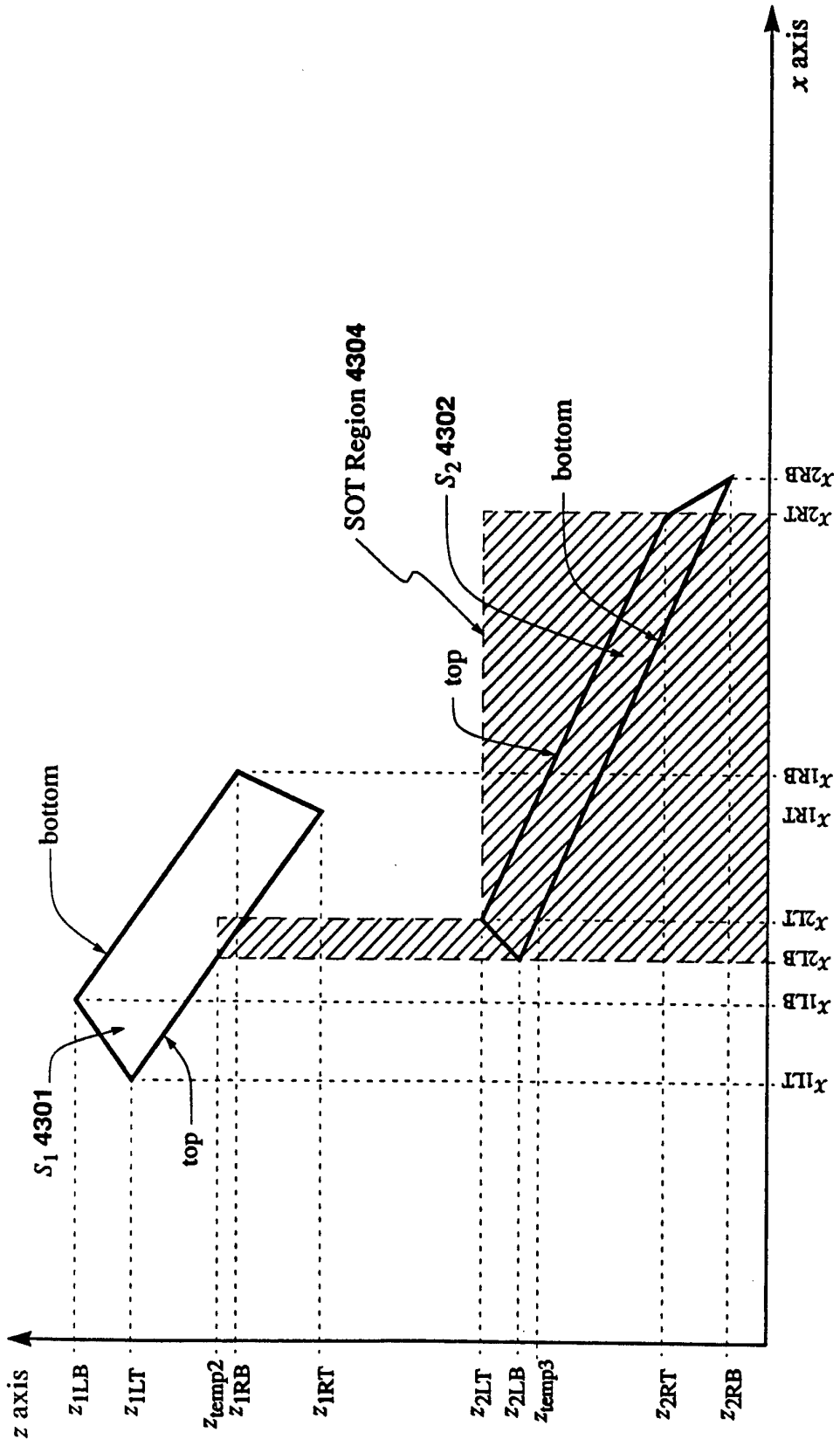

x axis
SOT Region 4304
S_2 4302
bottom
top
bottom
top
S_1 4301
x_{2RB}
x_{2RT}
x_{1RB}
x_{1RT}
x_{2LT}
x_{2LB}
x_{1LB}
x_{1LT}
z axis
z_{1LB}
z_{1LT}
z_{temp2}
z_{1RB}
z_{1RT}
z_{2LT}
z_{2LB}
z_{temp3}
z_{2RT}
z_{2RB}

Figure 44

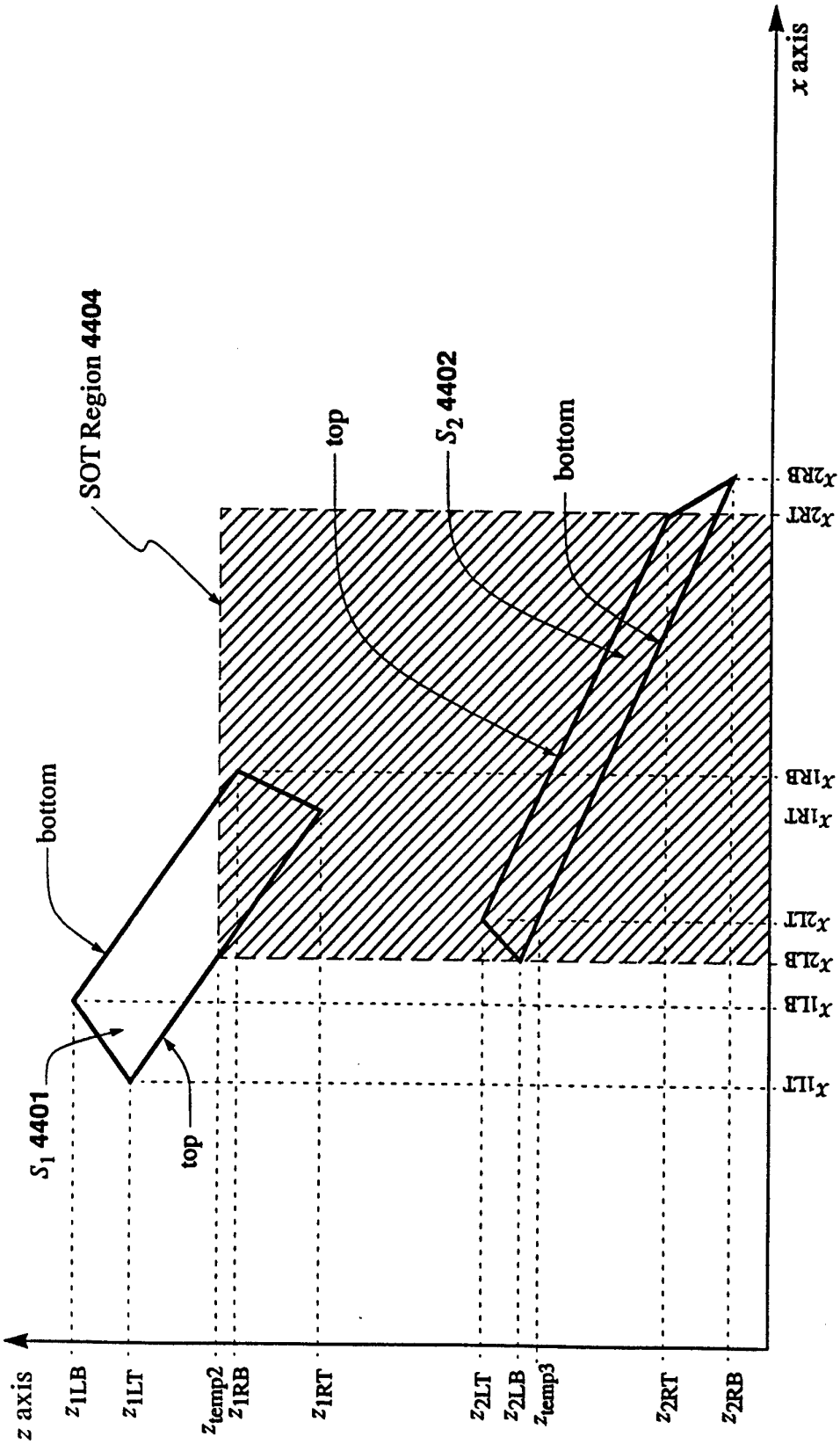

x axis
SOT Region 4404
top
S_2 4402
bottom
bottom
S_1 4401
top
z axis
z_{1LB}
z_{1LT}
z_{temp2}
z_{1RB}
z_{1RT}
z_{2LT}
z_{2LB}
z_{temp3}
z_{2RT}
z_{2RB}
x_{2RB}
x_{2RT}
x_{1RB}
x_{1RT}
x_{2LT}
x_{2LB}
x_{1LB}
x_{1LT}

Figure 45

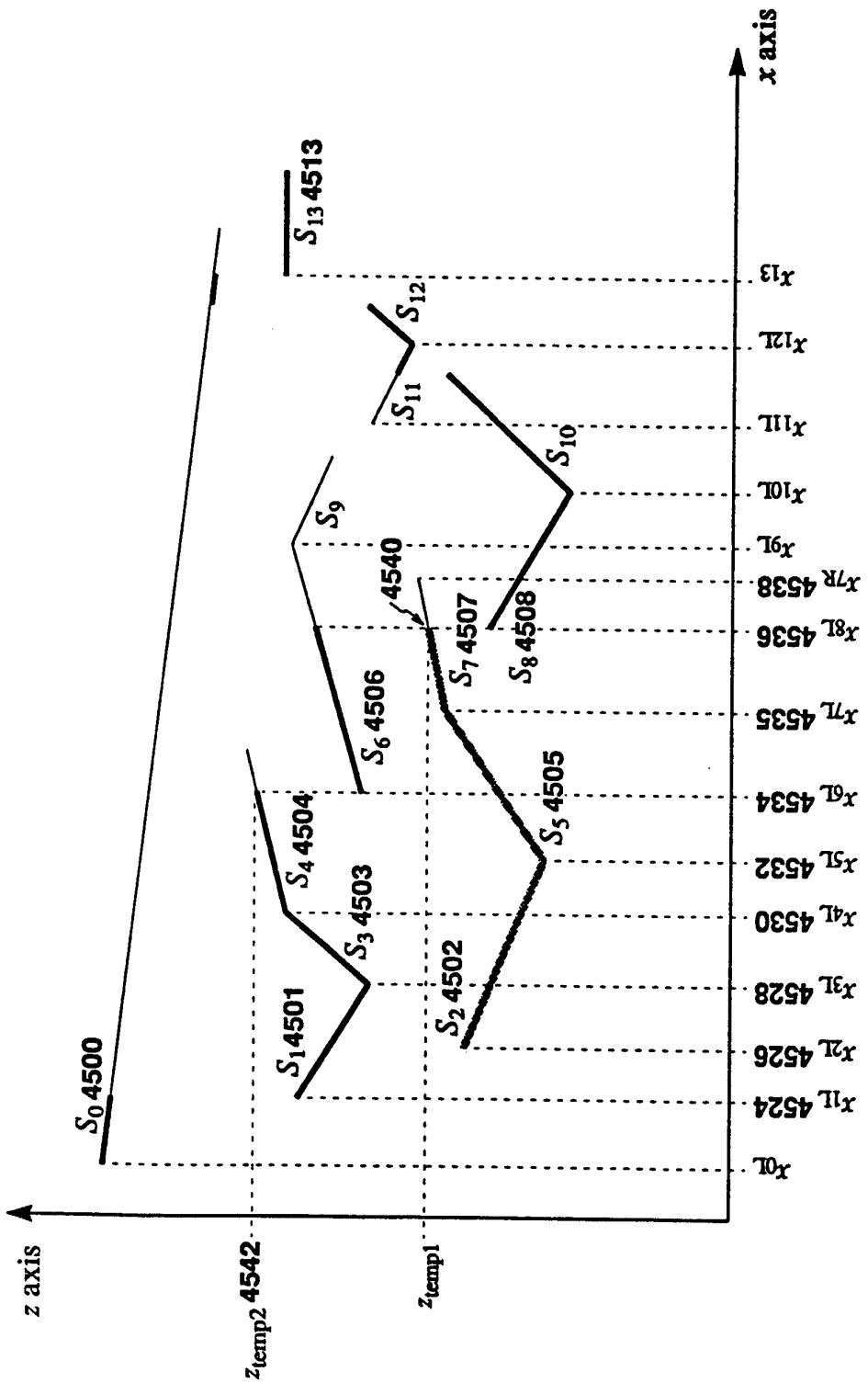

x axis
z axis
S_0 4500
S_1 4501
S_2 4502
S_3 4503
S_4 4504
S_5 4505
S_6 4506
S_7 4507
S_8 4508
S_9
S_{10}
S_{11}
S_{12}
S_{13} 4513
4540
x_{0L}
x_{1L} 4524
x_{2L} 4526
x_{3L} 4528
x_{4L} 4530
x_{5L} 4532
x_{6L} 4534
x_{7L} 4535
x_{8L} 4536
x_{7R} 4538
x_{9L}
x_{10L}
x_{11L}
x_{12L}
x_{13}
z_{temp2} 4542
z_{temp1}

Figure 46

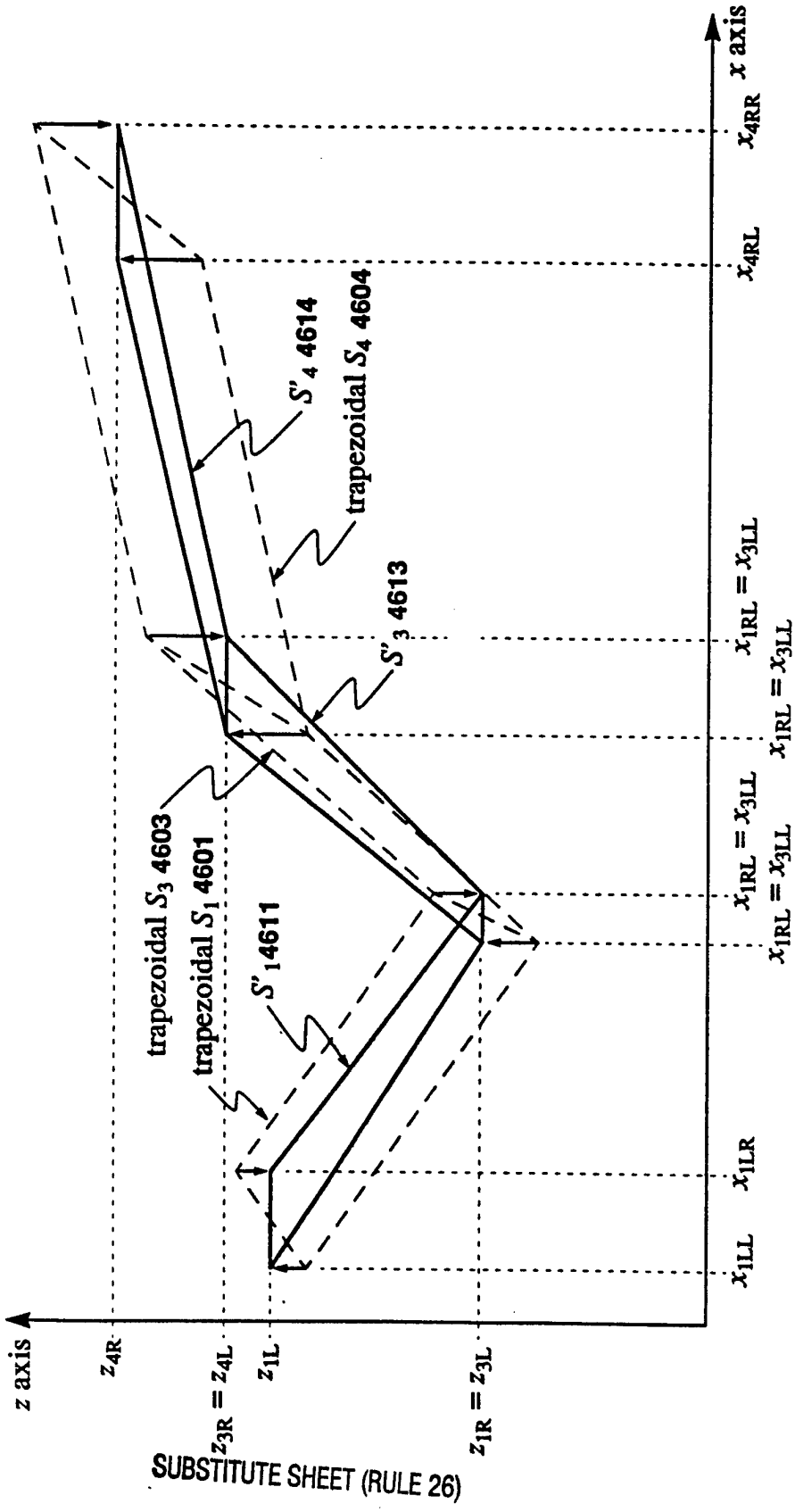
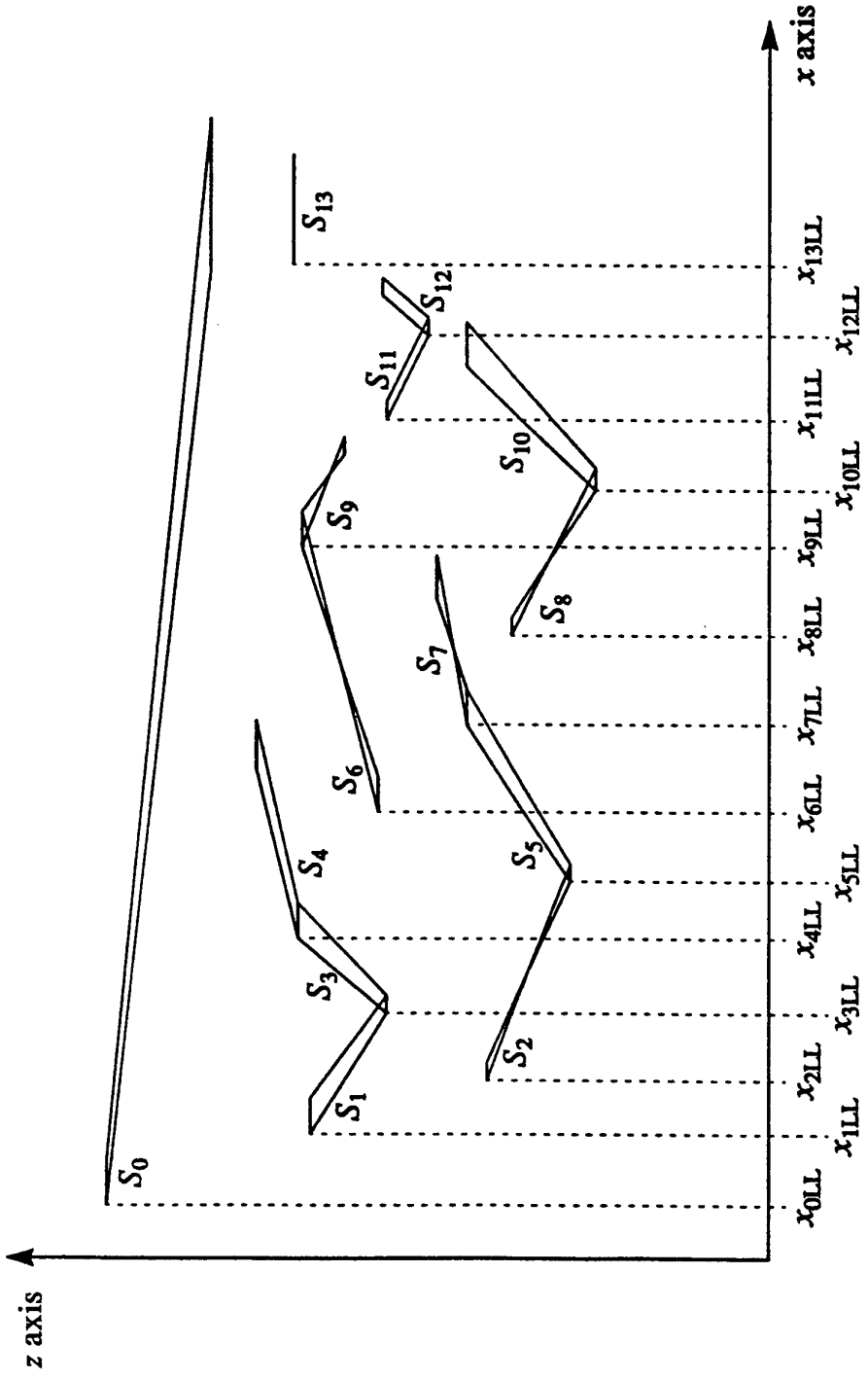

x axis
x_{4RR}
x_{4RL}
$x_{1RL} = x_{3LL}$
$x_{1RL} = x_{3LL}$
$x_{1RL} = x_{3LL}$
$x_{1RL} = x_{3LL}$
x_{1LR}
x_{1LL}
trapezoidal S_4 4604
S'_4 4614
S'_3 4613
trapezoidal S_3 4603
trapezoidal S_1 4601
S'_1 4611
z axis
z_{4R}
$z_{3R} = z_{4L}$
z_{1L}
$z_{1R} = z_{3L}$

Figure 47


z axis
x axis
S_0
S_1
S_2
S_3
S_4
S_5
S_6
S_7
S_8
S_9
S_{10}
S_{11}
S_{12}
S_{13}
x_{0LL}
x_{1LL}
x_{2LL}
x_{3LL}
x_{4LL}
x_{5LL}
x_{6LL}
x_{7LL}
x_{8LL}
x_{9LL}
x_{10LL}
x_{11LL}
x_{12LL}
x_{13LL}

INTERNATIONAL SEARCH REPORT

International application No.
PCT/US96/12301

A. CLASSIFICATION OF SUBJECT MATTER

IPC(6) :G06T 15/00
US CL :395/119

According to International Patent Classification (IPC) or to both national classification and IPC

B. FIELDS SEARCHED

Minimum documentation searched (classification system followed by classification symbols)

U.S. : 395/119

Documentation searched other than minimum documentation to the extent that such documents are included in the fields searched

Electronic data base consulted during the international search (name of data base and, where practicable, search terms used)

APS, DIALOG, IEEE

C. DOCUMENTS CONSIDERED TO BE RELEVANT

Category*	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
Y	FOLEY et al, "Computer Graphics: Principles and Practices", published in 1990 by Addison-Wesley Publishing Co., see pages 99, 108, 883-6, 903-4 and 980.	1-20
Y, P	US, A, 5,493,644 (THAYER ET AL) 20 February 1996, see entire document, especially column 3, line 52 to column 4, line 53.	1-20
Y	US, A, 5,123,085 (WELLS ET AL) 16 June 1992, see entire document, especially column 5, lines 3 - 61, 52 to column 8, line 5 to column 10, line 64.	1-20
Y	US, A, 4,996,666 (DULUK, JR.) 26 February 1991, see entire document.	1-20

[X] Further documents are listed in the continuation of Box C. [ ] See patent family annex.

* Special categories of cited documents:

"A" document defining the general state of the art which is not considered to be part of particular relevance

"E" earlier document published on or after the international filing date

"L" document which may throw doubts on priority claim(s) or which is cited to establish the publication date of another citation or other special reason (as specified)

"O" document referring to an oral disclosure, use, exhibition or other means

"P" document published prior to the international filing date but later than the priority date claimed

"T" later document published after the international filing date or priority date and not in conflict with the application but cited to understand the principle or theory underlying the invention

"X" document of particular relevance; the claimed invention cannot be considered novel or cannot be considered to involve an inventive step when the document is taken alone

"Y" document of particular relevance; the claimed invention cannot be considered to involve an inventive step when the document is combined with one or more other such documents, such combination being obvious to a person skilled in the art

"&" document member of the same patent family

Date of the actual completion of the international search	Date of mailing of the international search report
26 SEPTEMBER 1996	04 NOV 1996
Name and mailing address of the ISA/US Commissioner of Patents and Trademarks Box PCT Washington, D.C. 20231 Facsimile No. (703) 305-3230	Authorized officer RUDOLPH J. BUCHEL Telephone No. (703) 305-3800

Form PCT/ISA/210 (second sheet)(July 1992)★

INTERNATIONAL SEARCH REPORT

International application No.
PCT/US96/12301

C (Continuation). DOCUMENTS CONSIDERED TO BE RELEVANT

Category*	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
A	US, A, 5,278,948 (LUKEN, JR.) 11 January 1994, see entire document, especially column 2, line 31 to column 3 line 19 and column 6, line 38 to column 15, line 40.	1-20
Y	US, A, 5,129,060 (PFEIFFER ET AL) 07 July 1992, see entire document, especially column 2, line 38 to column 4 line 9 and column 7, line 10 to column 68, line 39.	1-20
Y	US, A, 5,146,592 (PFEIFFER ET AL) 08 September 1992, see entire document, especially column 2, line 40 to column 4 line 10 and column 7, line 14 to column 68, line 45.	1-20
A	US, A, 4,825,391 (MERZ) 25 April 1989, see entire document.	1-20