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dePackh

[11] 3,719,893
[45] March 6, 1973

[54] **SYSTEM AND METHOD FOR ACCELERATING CHARGED PARTICLES UTILIZING PULSED HOLLOW BEAM ELECTRONS**

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[73] Assignee: **The United States of America as represented by the Secretary of the Navy**

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[52] **U.S. Cl.** **328/233,** 313/63, 313/161, 315/111
[51] **Int. Cl.** **H01j 1/50,** H05h 1/00
[58] **Field of Search** ..313/63, 161; 328/233; 315/111

[56] **References Cited**

UNITED STATES PATENTS

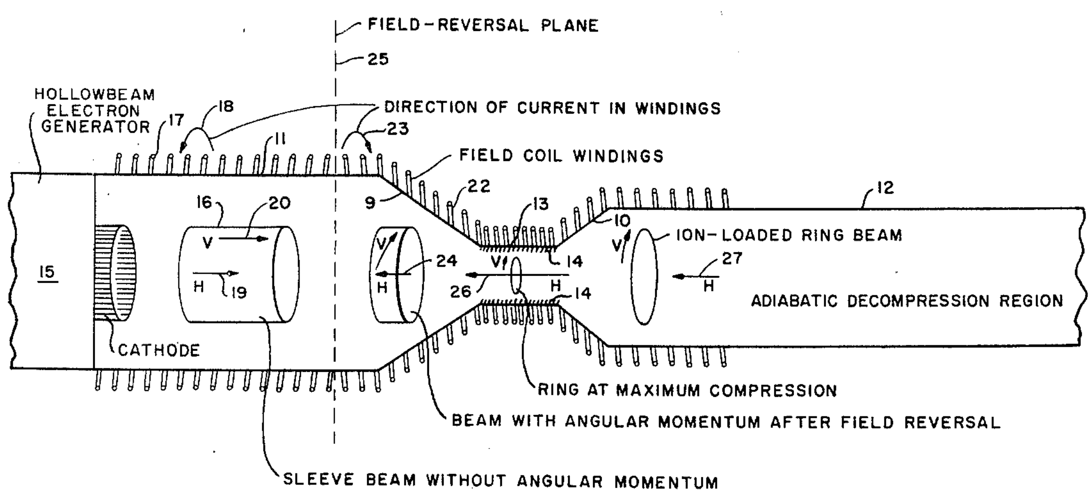
3,485,716 12/1969 Bodner ....313/161 X
3,626,305 12/1971 Furth et al. 328/233

Primary Examiner—Palmer C. Demeo
Attorney—R. S. Sciascia et al.

[57] **ABSTRACT**

This disclosure is directed to a system for accelerating protons or other positive ions along with acceleration of a hollow beam electron ring. Hollow beam electrons normally have a high tendency to fly apart due to their own space charge; however, the positive charge ions accelerated therewith coupled with organized angular momentum prevents blow-up of the electron rings during acceleration. The magnetic field used for acceleration is particularly shaped so that the ions are not lost and the positive charge ions are accelerated with the electrons.

4 Claims, 10 Drawing Figures



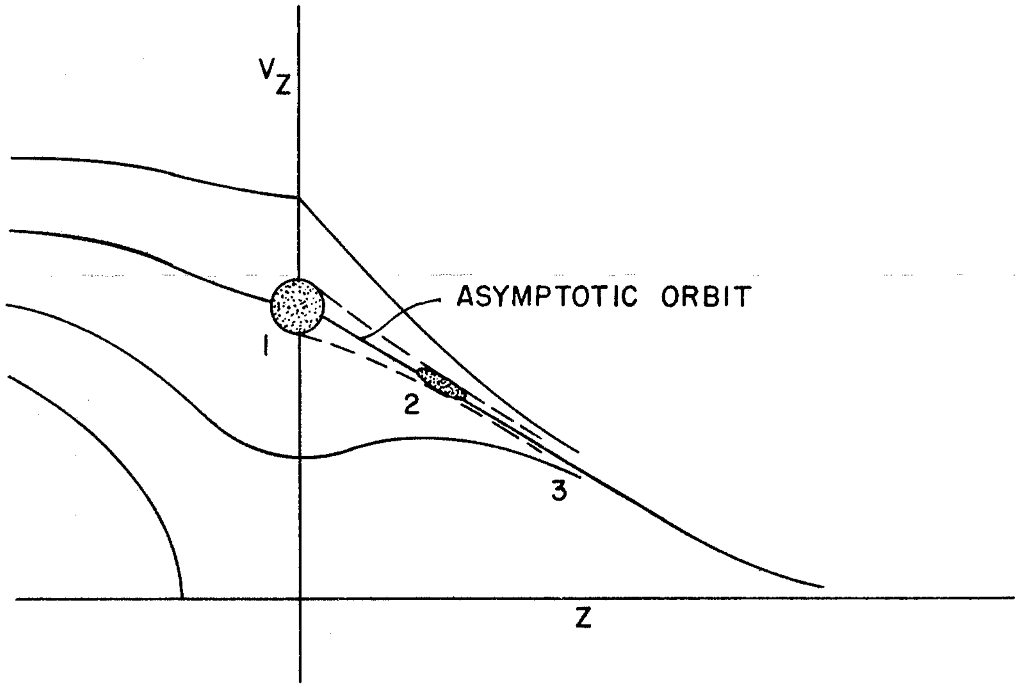

V_Z
ASYMPTOTIC ORBIT
1
2
3
Z

FIG. 1a.

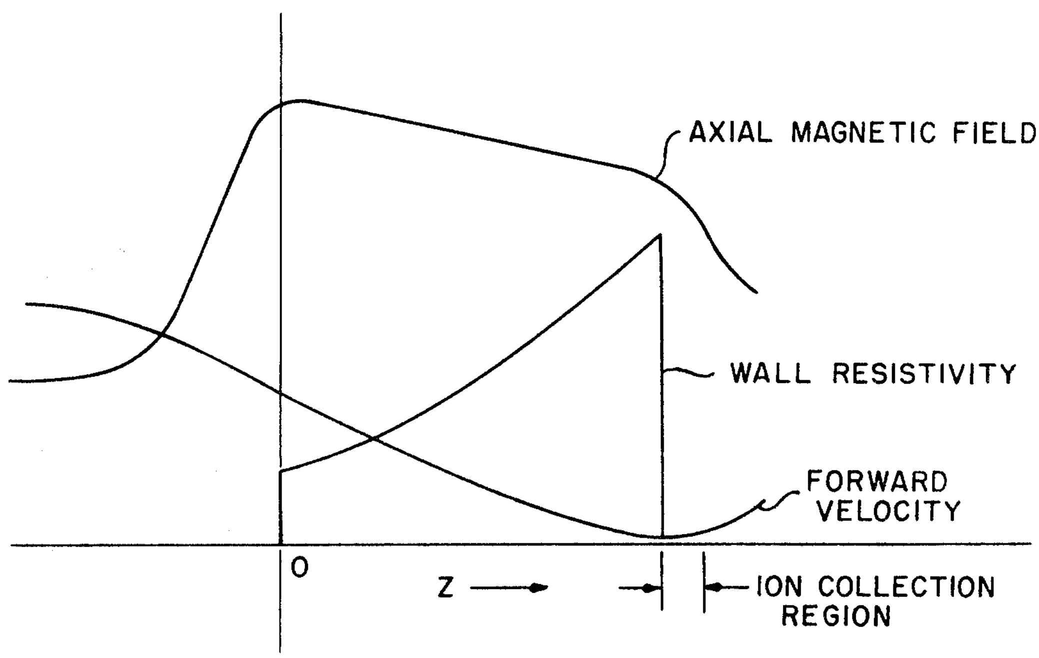

AXIAL MAGNETIC FIELD
WALL RESISTIVITY
FORWARD VELOCITY
0
Z
ION COLLECTION REGION

FIG. 1.

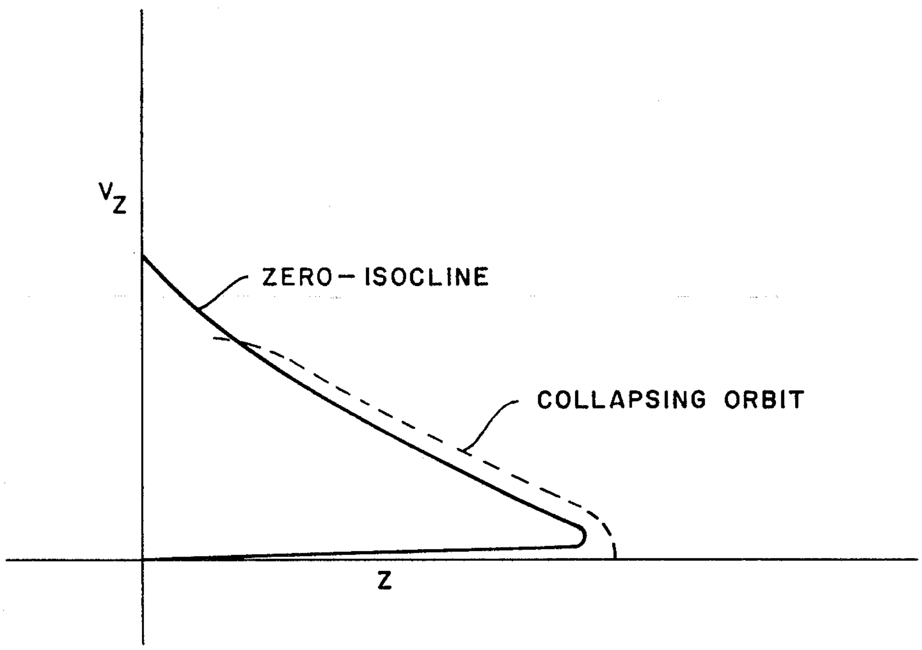


FIG. 2.

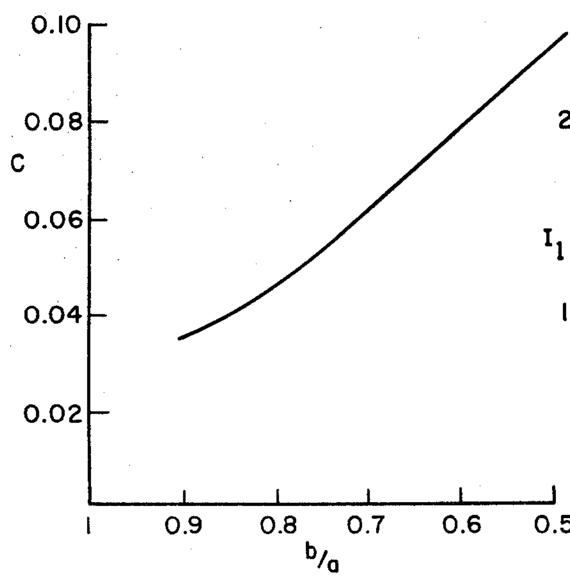


FIG. 4.

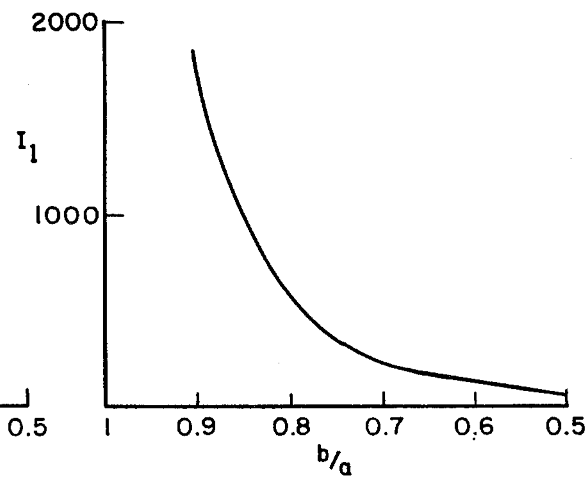


FIG. 3.

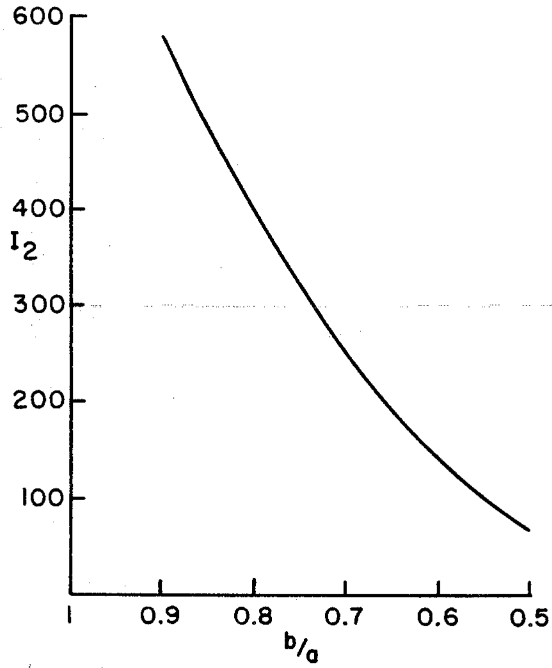


FIG. 5.

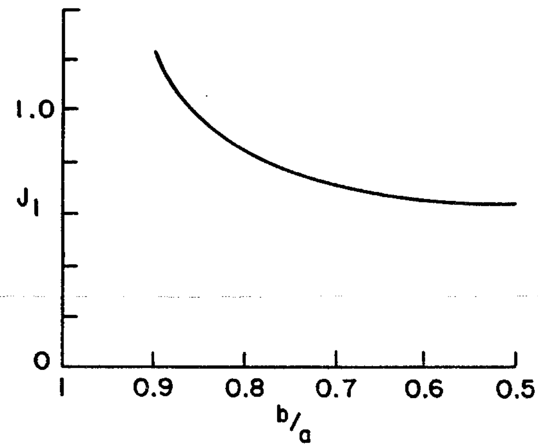


FIG. 6.

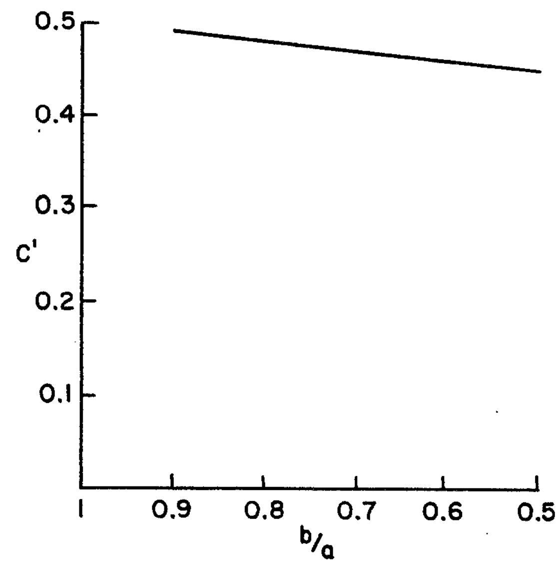


FIG. 7.

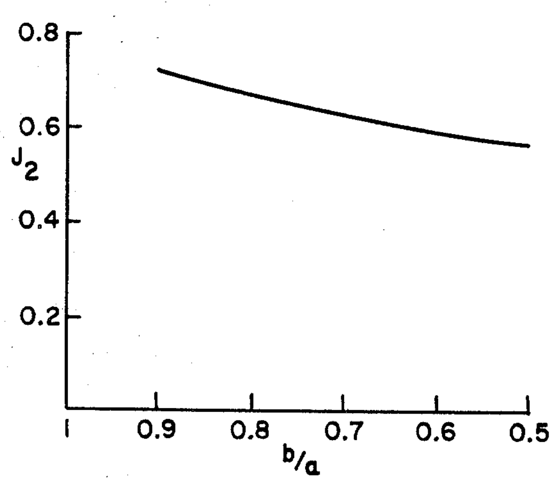


FIG. 8.

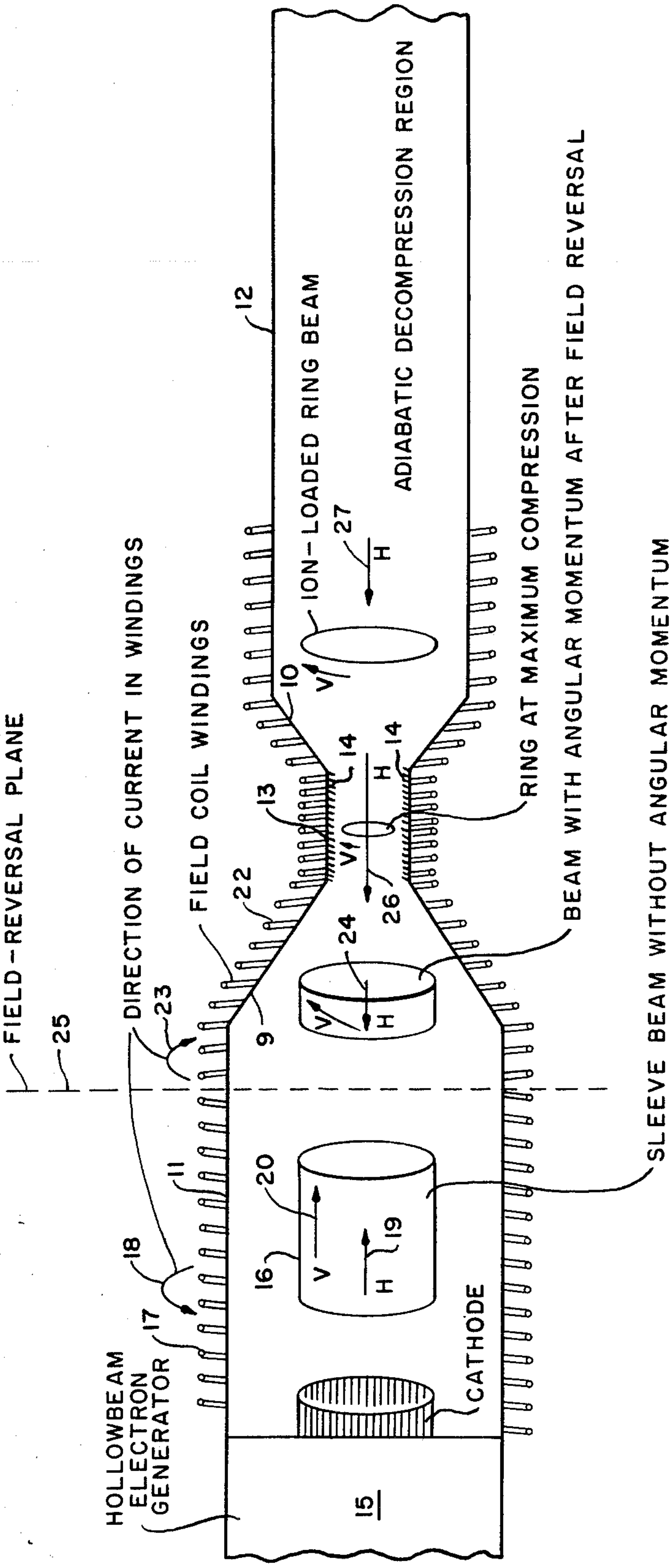

ADIABATIC DECOMPRESSION REGION
ION-LOADED RING BEAM
RING AT MAXIMUM COMPRESSION
BEAM WITH ANGULAR MOMENTUM AFTER FIELD REVERSAL
SLEEVE BEAM WITHOUT ANGULAR MOMENTUM
FIELD-REVERSAL PLANE
DIRECTION OF CURRENT IN WINDINGS
FIELD COIL WINDINGS
HOLLOWBEAM ELECTRON GENERATOR
CATHODE
12
27
10
14
13
22
23
25
24
26
9
11
20
19
18
16
17
15

FIG. 9.

SYSTEM AND METHOD FOR ACCELERATING CHARGED PARTICLES UTILIZING PULSED HOLLOW BEAM ELECTRONS

BACKGROUND OF THE INVENTION

This invention relates to particle accelerators and more particularly to the acceleration of positive protons or positive ions along with electrons.

Heretofore, various persons have conducted research on and proposed different systems for acceleration of positive charged ions with electron rings. Two such articles are "Static-Field acceleration of Electron Rings" by H. P. Furth and M.N. Rosenbluth; Symposium on Electron Ring Accelerators, UCRL Report 18103, pp 210–218; and "Some Preliminary Thoughts cn Ion Drag Accelerators", by W. Bennett Lewis, same Report 18103, pp 6–10. A system described in U.S. Pat. No. 3,506,866 sets forth a hollow beam electron ring generator.

SUMMARY OF THE INVENTION

This invention is directed to an accelerator in which high current beams in the form of an electron ring in a vacuum environment with a strong focusing solenoid guide field accelerates protons or other positive ions. A high current electron ring accelerator generates pulses of electron rings which are accelerated into a vacuum environment having an axially aligned magnetic field. The electron ring pulses are accelerated into a field reversal region in which the electron beam is given angular momentum, compressed, and then into a compression region, where the beam is retarded. In this region the protons or positive ions are captured by the electron ring and accelerated along with the electron ring. Subsequent to the compression region, the combined electron ring and ions are accelerated into an adiabatic decompression region within which the ion loaded ring beam expands into a larger ring and is accelerated by a magnetic field gradient or by electron energy increase. Any well known electron and ion separators may be used to separate the ions from the electrons subsequent to acceleration.

STATEMENT OF THE OBJECTS

It is an object of the present invention to accelerate a ring of electrons in such a manner that positive protons or ions are accelerated with the electron ring.

Another object is to provide an accelerator which has proper control fields for binding ions to an electron ring for subsequent acceleration therewith.

Still another object is to impart angular momentum to a ring beam of electrons to pickup and accelerate protons or positive ions.

Yet another object is to provide a linear accelerator which accelerates electron rings without blow-up of the electrons and without ions loose from the electron rings.

Still another object is to provide an accelerator which will acquire and accelerate a beam in a static magnetic-field.

While still another object is to provide an accelerator in which retardation of the beam is accomplished in a controlled manner by use of resistive walls.

BRIEF DESCRIPTION OF THE DRAWINGS

FIG. 1 is a schematic drawing of the resistive-wall ring retardation system.

FIG. 1*a* represents phase focusing due to the resistive wall for $Z>o$.

FIG. 2 illustrates orbit collapse due to drag proportional to $1/V_Z$.

FIG. 3 represents ring current, I, as a function of the ratio of the ring radius, *b*, to the wall radius, *a*.

FIG. 4 represents a constant C as a function of the ratio of ring radius, *b*, to wall radius, *a*.

FIG. 5 represents wall current I_2 as a function of the ratio of ring radius, *b*, to wall radius, *a*.

FIG. 6 represents angular momentum, J_1, as a function of the ratio of ring radius, *b*, to wall radius, *a*.

FIG. 7 represents a constant, C^1, as a function of the ratio of ring radius, *b*, to wall radius, *a*.

FIG. 8 represents angular momentum, J_2, as a function of the ratio of ring radius, *b*, to wall radius, *a*, and

FIG. 9 illustrates a schematic diagram of the accelerator.

Now referring to the drawings there is illustrated a linear accelerator section for carrying out the invention. As shown, the system includes a cylindrical housing made of pyrex or any other suitable material which is provided with a means not shown for evacuation of the housing. The inlet end **11** of the housing is larger in diameter than the outlet end **12** and includes a central section of much less diameter joined to the end sections by conical sections **9** and **10** to form a constriction section or mirror region **13** from which the electron rings and ions are accelerated. The constrictive section **13** is coated on the inner surface with a resistive coating **14** the purpose of which will be explained later. An electron ring or electron sleeve beam generator-accelerator **15** is secured to the inlet end for injecting pulses of electron sleeve beam **16** without angular momentum into a static magnetic field. The static magnetic field is produced by coil windings **17** to which a direct current denoted by arrow **18** is applied for providing an axial magnetic field **19** in the direction of the electron direction **20** as shown by the arrows.

Adjacent the coil windings **17** there are coil windings **22** which surround the end of housing section **11**. Along the conical sections **9** and **10**, the central constricture section **13** and the adjacent end **12** of the outlet section D.C. current denoted by arrow **23** is passed through coil **22** in the reverse direction from that of coil **17** produce an axial static magnetic field **24** in the opposite direction and equal magnitude. The magnetic field-reversal plane is indicated by the dotted line **25**. The magnetic field strength along the central housing section denoted by arrow **26** is much stronger than that of the field at arrow **24** in order to form a compression region. The magnetic field toward the outlet end is decreased from that of the central region and is shown by arrow **27**. The length of the arrows denoting the direction of the magnetic fields represent the comparative field strengths. The arrows indicated by a V indicate the electron direction, and H, the magnetic field direction. Lengths of H-arrows indicate relative magnetic field strengths.

In operation, the system is evacuated, the magnetic fields are developed and the electron sleeve beam generator-accelerator is made operational to produce electron sleeve beam pulses of typically 10 Mev that are injected into the field reversal section of the system as rapidly as possible. The sleeve beams are coaxially aligned within the housing without an angular momentum as they are accelerated through magnetic field **19**. This requires that the cathode of the electron source be within the magnetic field. The sleeve beams are accelerated through the first magnetic field area into the second magnetic field which is in reverse direction to the first magnetic field. As the sleeve beams are accelerated into the area of the reversed magnetic field, the sleeve beams are slowed down, contracted into shorter-length cylinders and an angular momentum is imparted to the electron sleeve after field reversal. The electron sleeve beams are further contracted and compressed into a smaller-diameter shorter cylindrical shape as the beams enter into the central section **13** which has a much greater magnetic field. The walls along this section are resistive to provide controlled beam retardation. Since the magnetic field strength along section **13** is much greater and angular momentum is imparted to the electron beams, the forward velocity of the electron beams in this section will be very low. Therefore, the beams will spend a long time in this section. It is in this section that the ions in the residual gas are captured by the electron beams and subsequently accelerated with the beams. The electron ring is prevented from flying apart by the ions to be accelerated and by the angular momentum of the electron rings. The angular momentum permits the number of positive charges to be materially less than the number of electrons.

From the compressed area, acceleration of the electron ring and the captured ions must be done carefully to avoid shaking the ions loose from the electron rings. Therefore, final acceleration may take place by simple adiabatic decompression. Additional acceleration may be provided by means of an accelerator following the adiabatic decompression region. In moving from the compressed central area where there is a strong magnetic field the electron ring-ions will move into a magnetic field of less intensity as the particles move from the compressed area. The electron ring will expand in diameter and the combined particles will be accelerated to a desired velocity and separated by electron and ion collectors which are well known in the art.

The successful operation of this system as an electron ring-ion accelerator depends on the control of the electron sleeve beam in the field reversal region. Therefore, the magnetic field must be carefully shaped, controlled and applied in order to prevent the electron ring and ions from being forced back toward the electron accelerator.

The system has been shown with resistive walls **14** along the constrictive section **13**. These resistive walls provide a means of controlling the ring motion in this region. For a correct choice of the different parameters, resistive drag is proportional to ring velocity. The effect of such a velocity dependence is to render the minimum velocity, where ion trapping occurs, less sensitive to residual errors in electron energy, magnetic field, and other parameters than would otherwise be the case. If, for example, the ring is handled altogether conservatively and adiabatically from the field reversal region onward, one has

$$\beta z^2 = 1 - \gamma^{-2} - \beta\phi^2 = 1 - \gamma^{-2} - \gamma^{-2}(e/mc^2)^2(R^2H_z)H_z,$$

where R^2H_z is an adiabatic invariant. Thus at minimum $_z$ (subscript c) one has

$$\beta c^2 = 1 - \gamma^{-2}[1 + (e/mc^2)^2 R_i^2 H_i H_c],$$

where H_i is H_z just beyond reversal and H_c is the field at the mirror. Ions are subject to an accelerating field of the order of 0.6 Mv/cm for nominal ring parameters (minor radius = 1 mm, i = 1000 amp), which corresponds to an energy of 0.06 Mev → β= 0.01 for protons and correspondingly lower for heavier ions at the ring boundary. Thus $\beta c^2 \sim 10^{-4}$; if γ , R^2, H_i, H_c are supposed independent, the same order of accuracy is required of all these quantities. This, of course, is a statement of the fact that the mirror point is unstable.

In considering the motion of such an elementary charge ring one can imagine the force to be made up of a number of components; thus one can write.

$$dv_z/dt = -AH_z' - Bv_2 - C/v_z$$

where the accent means a partial z-derivative and A, B, C are constants. This equation represents the most important external forces acting on the ring. The first term is the dipole force due to the external field gradient, the second proceeds from electric or magnetic images in partially conducting walls; the third may have a similar origin, and it also includes any space-charge drag forces produced by positive ions which are left behind. The detailed character of these forces will be taken up later. Here we consider the general effect of each term.

With respect to the first term one sees that the axial field acts like a potential. If $B = C = 0$, the equation integrates once to give

$$v_z = \pm(v_o^2 - 2AH_z)^{1/2}.$$

If v_f is the final velocity, at the peak field, then

$$\gamma v_f/\gamma v_o = v_o/v_f;$$

i.e., if $\beta_o \sim 1$, $\beta_f \sim 0.01$, requires that

$$\Delta\beta_o \sim 0.01\Delta_f \sim 10^{-4}.$$

A real beam with a spread in both z and β_z goes through a minimum extension in z before reaching the mirror. One expects the velocity spread to be gradually translated into a spread in position Δz roughly according to the approximate relation

$$\Delta z \sim \bar{t}\Delta v_z \sim (\Delta v_o/\bar{v}_z)z_o,$$

where $\bar{t}$ is the mean time for the ring to pass into the mirror, $\bar{v}_z$ is the average velocity in this region, and z_o is an effective length of the order of the mirror thickness. As the axial spread due to initial Δv_z grows with the approach to the mirror, the spread due to the original axial extension diminishes. From this cause

$$\Delta z = \Delta z_o\, v_f/v_o.$$

The minimum extension should occur where these two Δz's are equal, thus where

$$\Delta z_o\, v_f/v_o = z_o \Delta v_o/\bar{v}_z.$$

One can roughly take $v_f \sim \bar{v}_z$ since the ring spends most of its time near the region of least velocity. This gives

$$v_f^2 \cong (z_o/\Delta z_o)\,(v_o \Delta v_o),$$

$$\Delta z_{min} \cong (z_o \Delta\, z_o)\tfrac{1}{2}(\Delta v_o/v_o)\ .$$

All these results suggest that if $\Delta v_o/v_o$ is kept to the order of 10^{-4}, the resulting ring has a satisfactory behavior at a final velocity of order $0.01c$, but if the final velocity in the conservative region could be increased to $0.2c$, the tolerance could be increased to ~ 4 percent.

If the term $-Bv_z$ is included in the equation of motion, the system is of course no longer conservative. One notes first that if the rate of slowing down of the ring is small, the velocity is asymptotic to the value

$$v_z = -(A/B)H_z'.$$

Since A, B, and v_z are all positive, one requires $H_z'<0$. Thus if the drag force corresponding to the term $-Bv_z$ is to be used to control the velocity, the principal retardation by this effect occurs on the far side of the mirror.

On emergence from the cusp, β_z is still rather near 1. This follows from the fact that if large orbit perturbations due to transmission through the cusp are to be avoided, β_ϕ after transmission should not exceed about 0.5. On the other hand, if rather large energy losses are not to take place, β_z on entry into the resistive drag region should not exceed about 0.5. Thus for $z<0$, say, $B = 0$ (perfectly conducting walls). One then matches the wall resistance and field gradient to the condition (for $\beta_z = 0.5$ going into the drag region)

$$0.5c = -(A/B)H_z'$$

for z just > 0. Then H_z' can be graded down to correspond to $v_z = 0.01c$ just before the collection of ions (FIG. 1). At this velocity one would expect all ions to be trapped so that drag due to abandoned ions offers no problem. However, if there is say 2 percent neutralization by protons with $\gamma = 20$, the ratio of ion mass to electron transverse mass is

$$0.02\text{x}1840/20 = 1.84,$$

so that conservation of forward momentum requires that after ion collection v_z is

$$0.01\text{X}(1/2.82)c \sim 0.003c.$$

If it is desirable to offset this effect, the axial field can be reduced in the collection region, though there is no obvious reason for doing so.

The nature of the phase focusing provided by the resistive drag can be understood by consideration of the asymptotic orbit for the equation

$dv_z/dt = -AH_z' - Bv_z$ which approaches zero velocity as $A \rightarrow 0$. In first approximation this is just

$$v_z = -(A/B)H_z'.$$

In the next approximation one can substitute this value of v_z on the lhs of the differential equation to get a second approximation

$$v_z = -(AH_z'/B)\,(1 - AH_z'/B^3).$$

The particular orbit which is approached by this successive approximation procedure is asymptotically approached by other orbits in its neighborhood (FIG. 1*a*). Thus, phase space injected near this orbit becomes concentrated on it, as the figure schematically shows. Since there is then ideally negligible phase space at right angles to this curve, one then expects the approximately 50-fold shortening in the z-direction predicted by simple theory in going from $\beta_z = 0.5$ to $\beta_z = 0.01$ for the example chosen.

Resistive drag produces a reduction of the compression achievable in a system without it. The compression ahead of the mirror (conservative) is

$$(1 - \beta_e^2)^{1/2}/\beta_{\phi i},$$

where β_e is β_z at the entrance to the resistive region (peak H_z) and $\beta_{\phi i}$ is β_ϕ at the emergence from the cusp. This is a rather small reduction with respect to the $1/\beta_{\phi i}$ which would result from slowing the ring down conservatively to a very small forward velocity for any reasonable value of β_e. Because the velocity is given approximately by $-(A/B)H_z'$, the principal slowing being caused by a graded increase of B, there is a definite drop in the field produced by nonvanishing H_z'. The time required to approach the asymptotic orbit is, however, independent of H_z', being given approximately by $1/B$; thus, the characteristic distance for approach to the orbit is v_z/B. On the other hand the system length L is of the order $-A\Delta H_z/B_{v_z}$, as follows from the first approximation to the asymptotic velocity. It is required that L should be much greater than the characteristic distance for approach to the asymptotic orbit; thus

$$-A\Delta H_z \gg v_z^2.$$

This relation indicates that the field drops in the drag region by twice the amount by which it would increase if the slowdown were carried out conservatively (since conservatively

$$\tfrac{1}{2}\, dv_z^2/dz = -AH_z',\ A\Delta H_z = -\tfrac{1}{2}\Delta v_z^2).$$

Now it will be shown that the energy loss in stopping the ring is given approximately by

$$-\Delta\gamma = \tfrac{1}{2}\gamma_z^2\,\gamma;$$

also $A = b^2 i_o/N\gamma m$, where b is the ring major radius and i_o is the current. One has $\Delta b/b = \Delta\gamma/\gamma - \Delta H_z/H_z = \tfrac{1}{2}\beta_z^2 + v_z^2/AH_z = -\tfrac{1}{2}\beta_z^2 + \tfrac{1}{2}\beta_z^2 = 0$ if one takes $\beta = 1$ and uses the limiting value of $-A\Delta H_z$ given by v_z^2. In practice $-A\Delta H_z$ must be several times larger than this so that $\Delta b/b$ should be several times β_e^2. If β_e is as much as 0.5 this could result in an appreciable increase in radius.

The third term in the force equation $-C/v_z$ represents an unstable drag force since it increases without bound as the velocity diminishes. It is therefore important to know how large C can be before this force seriously affects the motion. Orbits which collapse toward zero velocity exist outside the zero-isocline given by

$$-AH_z' = Bv_z + C/v_z$$

(See FIG. 2). The minimum value of $-AH_z'$ is

$$-AH_z' = 2(BC)^{1/2};$$

i.e., if $-AH_2'$ falls below the value, velocity collapse is to be expected. If the C-term is small except in this neighborhood, one has

$$-AH_z' \sim Bv_z,$$

so that approximately one must satisfy

$$v_z \gg 2(C/B)$$

to insure freedom from abandoned-ion effects.

In the following analysis, the magnetic and electric-drag forces will be considered. From the following analysis, it will be shown that it is desirable to suppress magnetic images in the wall through the use of longitudinal slots.

MAGNETIC DRAG

In considering magnetic drag one supposes a filamentory current ring of radius b in a tube of radius a. The ring moves along the z-axis with a velocity $v_z \ll c$ and is centered in the tube. The components of magnetic field and flux are numbered 1 or 2 according as they result from ring or wall current. The primary flux produced by the ring at $z_r(t)$ and linked by an elementary ring of the wall at z is $\Phi 1 = i_o M(b,a,z,z_r) \equiv i_o(ab)^{1/2} m(b/a,(z\text{-}z_r)/a) \equiv i_o(ab)\ m(b/a,\xi)$, where i_o is the ring current. M is the mutual inductance, and m is the normalized mutual inductance. Tables of m in terms of elliptic integrals may be found in inductance calculations by F. W. Glover, Dover, New York 1962, p. 77.

The secondary flux produced by all wall rings and linked by a ring of radius r at axial position z is

$$\Phi_2(r/a) = \int_{-\infty}^{\infty} j(z',\ t) m(r/a,\ (z-z')/a)\,(ar)^{1/2} dz'$$

$$= a(ar)^{1/2} \int_{-\infty}^{\infty} j(u) m(r/a,\ u-\zeta)\,du$$

where $u \equiv (z - z_r)/a$ and j is the surface current density at the wall.

Combining Ohm's law and the continuity equation gives

$$j = -(\sigma/2\pi a)\ (\gamma\Phi_1/\gamma t + \gamma\Phi_2(1)\ /\gamma t),$$

where σ is the surface conductivity. One is concerned with a quasi-static treatment in which v_z is taken as approximately constant and the field variables depend on $z - z_r$ explicitly. To exhibit this, one writes the preceding equation in the form

$\gamma\Phi_2(1)/\gamma a z_r + (2\pi a/\gamma v_z) j = -\gamma\Phi_1/\gamma z_r$. The longitudinal force on the ring is

$$f_{z_r} = -i_o(\partial\Phi_2(b/a)/a)/\partial z)_{z=z_r}$$

$$= -i_o(ab)^{1/2} \int_{-\infty}^{\infty} m'(b/a,\ u) j(u)\,du$$

$$= i_o(ab)^{1/2} \int_{-\infty}^{\infty} j'(u) m(b/a,\ u)\,du$$

where m' means $\gamma m/\gamma\xi$. One can of course also calculate the Force on an isolated subring ($z \neq z_r$) if one wants to consider the focal properties of the system, it is sufficient at this point to note that the force is greatest at the ring position so that the focal equilibrium is neutral as far as the drag forces are concerned. It will appear below that, although the drag forces (resulting from nonvanishing wall resistivity) have a neutral equilibrium, the image contribution from the perfectly conducting wall terms does have focal properties.

Case a: σ is small.

$$j = (\sigma v_z/2\,\pi a) i_o (b/a)\ ^{1\!/\!2}m,\ (b/a,\ \xi),$$

$$f_{z_r} = -i_0^2(\sigma v_z/2\pi)(b/a) \int_{-\infty}^{\infty} m'^2(b/a,\ \zeta)\,d\zeta$$

$$\equiv -i_0^2(\sigma v_z/2\pi) I_1(b/a)$$

I_1 is shown in FIG. 3.

Case b. : σ is large.

The current density j and the secondary flux are divided into zeroth and first order components; the zeroth order part is the contribution associated with a perfectly conducting wall. Then

$$\Phi_{20}(1) = -\Phi_1,$$

$$\gamma\Phi_{21}(1)/\gamma z_r - \gamma\Phi_{21}(1)/\gamma_z = (-2\pi a/\sigma\ v_z)\ j_o.$$

One needs the solution of the Integral equation

$$\int_{-\infty}^{\infty} j_0(u) m(1,\ u-\zeta)\,du = -(i_0/a)(b/a)^{1/2} m(b/a,\zeta)$$

for the "equilibrium". The solution is to an adequate approximation

$$j_o = -C(b/a)\ (i_o/a)(b/a)\ \ m(b/a,\xi),$$

where $C(b/a)$ is shown in FIG. 4. The perturbation requires a solution for j_1. One has

$$\partial\Phi_{21}/\partial_{z_r} = -(ab)^{1/2} \int_{-\infty}^{\infty} j_1'(u) m(1,\ u-\zeta)\,du$$

$$= -(2\pi a/\sigma v_z) j_0$$

This has the same form as the preceding integral equation; therefor

$$j_1' = C(b/a)\ (2\pi/\sigma v_z)\ (aa/b)\ j_o = -C^2(2\pi/\sigma v_z)\ (i_o/a) m\ (b/a,\xi),$$

and

$$f_{z_r} = -C^2 i_0^2 (2\pi/\sigma v_z)(b/a)^{1/2} \int_{-\infty}^{\infty} m^2(b/a,\ \zeta)\,d\zeta$$

$$\equiv -i_0^2(2\pi/\sigma v_z) C^2 I_2(b/a)$$

I_2 is displayed in FIG. 5. Thus, the magnetic drag has the correct velocity-dependent properties for sufficiently low conductivity. To make this definite one can define a transition value of conductivity at which the asymptotic force formulas are equal. This occurs for

$$\sigma v_z = 2\pi C\ (I_2/I_1)\ .$$

Using this value, one finds an upper bound to the force to be

$$f_{z_r} < i_o{}^2 C(I_1 I_2\ .$$

As a typical value one might take $b/a = 0.8$, which makes the upper limit

$$f_{z_r} < 20\ i_o{}^2 \text{ dynes}$$

for the current in emu; the corresponding value of v_z is about 0.2. Thus, if $v_z = 0.05c$, the surface conductivity at the transition is 1.3×10^{-10} emu = 0.13 mho. To insure a drag force proportional to velocity the conductivity should be substantially below this value. The distance required to stop the ring, using the upper bound on the drag and a current of 1000 amp = 100 emu and an initial velocity of $0.05c$, with 10^{12} electrons in the ring and γ=20, is

$$1/2N\gamma m v_0^2/f_{z_r} = 0.012 \text{ cm}.$$

Thus, the magnetic drag is entirely adequate to slow the ring in a reasonable distance by a suitable choice of resistivity.

ELECTRIC DRAG

The axial electric field produced by the ring charge Q_o at z_r at the wall point (z,a) is

$$E_1 = (Q_o/a^2)\, d'\,(b/a, \xi),$$

where

$$d = (2/\pi)K(k)/(\xi bv2 + (1+b/a)^2)\ , k^2 = 4(b/a)/(\xi^2 + (1+\mathbf{b/a})^{2a.)}.$$

The axial field produced by the whole wall charge at the point (r,z) is

$$E_2(r/a) = -\int_{-\infty}^{\infty} S(u)\, d'(r/a,\ u-\zeta)\, du/a$$

where S is the wall charge per unit axial length. If the surface resistivity is ρ, Ohm's law and continuity give

$$\gamma E_2(1)/\gamma z + \rho v_z/2\pi a)\gamma S/\gamma z_r = -\gamma E_1/\gamma z,$$

where dz_r/v_z has been written for dt. With the quasi-static approximation $\gamma/\gamma z \sim \gamma/\gamma z_r$ this equation immediately integrates to give

$$(\rho v_z/2\pi a)S - E_2(1) = E_1.$$

The longitudinal force on the ring is

$$f_{z_r} = Q_o E_2(b/a).$$

Case a: ρ is large.

$$S = (2\pi a/\rho v_z)\, E_1$$

$$f_{z_r} = -(Q_0/a)^2(2\pi/\rho v_z)\int_{-\infty}^{\infty} d'^2(b/a,\ \zeta)\, d\zeta$$
$$\equiv -(Q_0/a)^2(2\pi/\rho v_z)\, J_1$$

Case b: ρ is small.

$$E_{20}(1) = -E_1 \text{ (zero order)}$$

$$E_{21}(1) = (\rho v_z/2\pi a)\, S_o \text{ (first order)};$$

S_o solves the integral equation

$$\int_{-\infty}^{\infty} S_0(u)\, d'(1,\ u-\zeta)\, du = (Q_0/a)\, d'(b/a,\ \zeta)$$

(The surface charge in zero order could of course also be found by setting it equal to ¼π times the primary radial field at the wall, thereby avoiding the integral equation. With the approximations used here there is no gain in introducing the radial field.) The perturbation requires a solution for S_1. One has

$$-\int_{-\infty}^{\infty} S_1(u)\, d'(b/a,\ u-\zeta)\, du/a = -(\rho v_z/2\pi a)\, S_0$$

This has the same form as the preceding integral equation; after appropriate integrations,

$$S_o = -C'(b/a)\,(Q_o/a)\, d(b/a, \xi$$
$$S_1' = -C'^2(b/a)\,(Q_0/a)\,(\rho v_z/2\pi)\, d(b/a,\ \zeta).$$

The force is (after integrating by parts)

$$f_{z_r} = \int_{-\infty}^{\infty} S_1'(u)\, d(b/a,\ u)\, Q_0 du/a$$
$$= -C'^2(Q_0/a)^2(\rho v_z/2\pi)\int_{-\infty}^{\infty} d^2(b/a)\, d$$
$$= -(Q_0/a)^2(\rho v_z/2\pi)\, J_2$$

The quantities J_1, C', and J_2 are shown in FIGS. **6**, **7**, and **8**.

The transition between high and low resistivity is at

$$\rho v_z = 2\pi\cdot(J_1/J_2)\ ,$$

and the upper bound on the force is given by

$$f_{z_r} < (Q_o/a)^2(J_1J_2)\ .$$

If one again uses $b/a = 0.8$ as an example, setting $Q_o = 2\pi i_o b$, one finds for the upper bound

$$f_{z_r} < 17\, i_o^2.$$

The magnitude of this force is therefore the same as the magnetic force. The resistivity at transition for $v_z = 0.05c$ is about 5×10^{-9} esu or about 4,500 ohms/square. To insure an electric drag force proportional to the velocity the resistivity should be materially below this value. It was shown that the same velocity dependence could be achieved by means of magnetic drag (for the same velocity) if the resistivity exceeded about 7 ohms/square; there is therefore a range of resistivities in which both forces have the correct behavior. One also notes, however, that if the resistivity is set at the geometric mean of values, or about 200 ohms/square, the stopping distance is only about

$$½ \times (4500/200) \times 0.012 \sim 0.1 \text{ cm},$$

which is obviously prohibitively short. It therefore appears to be desirable to use one of the two mechanisms while suppressing the other by means of slots in the coating.

ENERGY LOSS

The loss of energy is easily calculated by means of energy and momentum balance. Thus we have

$$Nmc^2\, d\,\gamma/dt = -f_{z_r}\,\beta zc,$$
$$Nmc\, d(\beta_z\gamma)/dt = -f_{z_r}$$

whence after an integration

$$\beta_z^2 = ; -\gamma_o^2/\gamma^2.$$

The electron energy loss is therefore small as long as $\beta_z \ll \beta$. Even for $\beta_z = 0.5$ the loss is only 13 percent, and it drops with the square of the velocity.

Drag Due to Abandoned Ions

The number of ions formed per cm of electron track is $Nn\sigma$, where n is the neutral density and σ is the ionization cross section. There, the number per axial cm is $Nn\sigma/\beta z$. To simplify matters one assumes that there is a range of force L determined by the geometry, including the effect of the wall, so that the whole number of ions which are active in retarding the ring is $LNn\,\sigma/\beta z$. One simplifies further by regarding the ions as constituting a line charge which therefore gives rise to an electric field $2\,\lambda_i Ze/z$, where z is the axial distance from the equivalent ion ring charge, Z is the charge state, and λ_i is the number of ions per cm of the ion ring periphery, given by

$$\lambda_i = LNn\,\sigma/2\,\pi b\,\beta_z.$$

The total drag force is therefore

$$e^2 \cdot 4\pi bZ\,\lambda_i\lambda_e/z;$$

if in order of magnitude we take $z = L$, the drag force then becomes

$$ZN^2e^2n\sigma/\pi b\beta_z.$$

The ion drag coefficient C is then given by

$$C = ZN^2e^2cn\sigma/\pi b\gamma m.$$

If one supposes that electric drag is the chosen mechanism, one has for the coefficient B

$$B = N^2e^2\rho J_2/2\pi ma^2,$$

so that the condition for freedom from abandoned-ion effects becomes

$$v_z \gg 2(C/B) = 2(c/\rho)\ (2nZ\sigma a^2/bJ_2)\ .$$

If one takes $\sigma = 5 \times 10^{-20}\text{cm}^2$, $\sigma = (1/9)\times^{-11}\times 2$ ohms/square, $Z = 1$, and a pressure of 10^{-6} mm, one finds for this requirement

$$v_z \gg 3 \times 10^8 \text{ cm/sec}.$$

Accordingly if one is to reduce the ring velocity to about this value, it is desirable to have a pressure of an order or so lower, i.e., about 10^{-7} mm. This calculation should be done with greater accuracy if exact pressure limits are to be established.

Focal Effects in the Resistive Region

For an unnuetralized ring beam close to the wall one can estimate the zero-order (perfect-conductor) force on a sub-ring using simple image considerations. If the center of gravity of the ring section at (r,O) is at a distance s from the wall and one considers the z-component of force on a filamentary sub-ring at (r,ϵ) with charge density λ'e per unit length, and if in addition one supposes the wall to be longitudinally slotted so that there is no magnetic image, then the axial force per unit azimuthal beam length is

$$\lambda\lambda'e^2/^2\epsilon - \lambda\lambda'e^2\epsilon/(\epsilon^2+4s^2) = (\lambda\lambda'e^2/\epsilon)\ (\gamma^{-2} - \epsilon^2/(\epsilon^2+4s^2)).$$

Thus, electrostatic defocusing dominates at distance

$$2s/(\gamma^2-1)\ ;$$

there is a short-range axial defocusing followed by axial focusing at greater distances. The whole ring would of course be radially unstable because of the attractive force of the image except for the external magnetic field. On comparing the radial force on a sub-ring (charge linear density λ', position $(r+\epsilon', 0)$) due to the combined effects of the image and the main ring $(\lambda, (r,0))$ on the one hand with that produced by the external magnetic field on the other, one has for the first force per unit azimuthal length a quantity of order.

$$\lambda\lambda'e^2/x$$

for ϵ' of the order of s (short-range defocusing of course always exists); for the second force one has

$$\lambda' eH_z.$$

The first can thus be neglected if

$$\gamma/\gamma \ll s/b$$

on using the relations $\gamma = \lambda e^2/mc^2$, $H_z b = \gamma mc^2/e$. This requirement should be rather easily satisfied.

The effect of short-range defocusing in both axial and radial directions is to give the ring a nonvanishing cross section. The radial part is included in a more general radial smearing-out which takes in the acceptance phase space, the scalloping due to the cusp, and the like. The axial part is included in the axial extension due principally to energy spread. If the above numbers are accepted as order-of-magnitude estimates, then

$$\Delta r \sim (\gamma/\gamma)b,\ \Delta z \sim 2s/\gamma.$$

Both of these are well below what would be expected of the normal phase space spread.

The magnetic image is defocusing in the axial direction, and its use is therefore not indicated. The use of a longitudinally slotted wall, because of its suppression of azimuthal currents, should be of some effect in the suppression of azimuthal resistive-wall instability.

What is claimed and desired to be secured by Letters Patent of the United States is:

1. A system for accelerating positive ions along with acceleration of a hollow beam electron ring which comprises:

a housing having an inlet and an outlet,

said housing including first and second cylindrical sections separated by a smaller diameter cylindrical section joined thereto by conical sections said first section being the inlet end of said housing,

means relative to said first section for producing first axially aligned magnetic field along said first section whose direction is toward the outlet end,

means relative to said conical sections, said smaller diameter section and said second section for producing second axially aligned magnetic field in a reverse direction from that in said first section,

said second magnetic field increasing in strength along the conical section on the inlet side of said smaller diameter while decreasing from said smaller diameter section toward the outlet end

said second magnetic-field strength along said smaller diameter being of uniform strength and much greater than said first magnetic field, and

means for injecting pulses of hollow beam electrons into said inlet end, into said first magnetic field for acceleration by said first magnetic field into said second magnetic field.

2. An accelerator system as claimed in claim 1; wherein,

the inner wall surface of said smaller diameter section of said housing has a resistive coating thereon.

3. An accelerator system as claimed in claim 2; wherein,

said outlet section provides an adiabatic decompression region through which positive ions are accelerated along with hollow beam electrons.

4. A method of accelerating positive ions along with pulses of hollow beam electrons which comprises,

evacuating an accelerator housing,

injecting pulses of hollow beam electrons into first axially aligned magnetic field with the direction of the magnetic in the same direction as the direction of movement of said hollow beam electrons,

directing said hollow beam electrons into a second axially aligned magnetic field whose direction is reversed from said first magnetic field and opposite to the direction of travel of said electron beam whereby angular momentum is given said electron beam,

increasing the magnetic field strength toward a smaller housing section,

maintaining said magnetic field at said increased strength along said smaller housing section and adding ions to said electron beam confined by said magnetic field along the smaller housing section,

decreasing said second magnetic field in an axial direction from said smaller housing section to a larger outlet section which forms an adiabatic decompression regions,

maintaining said magnetic field steady along said outlet section and directing said combined ion-hollow beam electron pulses into said adiabatic decompression region from which the ions and electrons are accelerated.

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