Feb. 12, 1935.

S. DARLINGTON

1,991,195

WAVE TRANSMISSION NETWORK

Filed Oct. 31, 1931

2 Sheets-Sheet 1

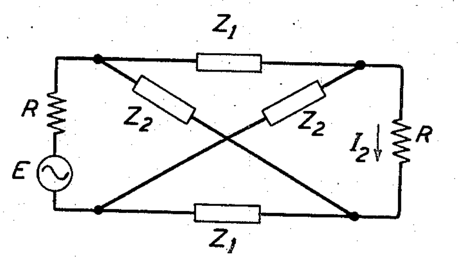

FIG. 1

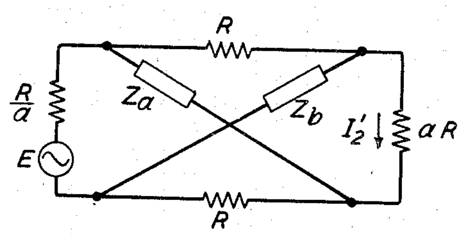

FIG. 2

FIG. 3

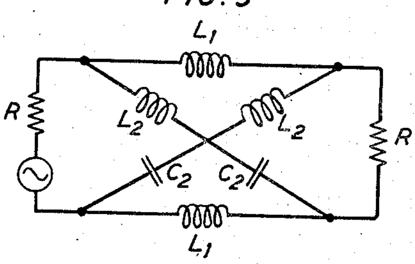

FIG. 4

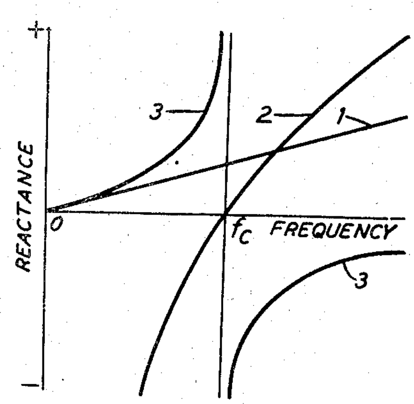

FIG. 5

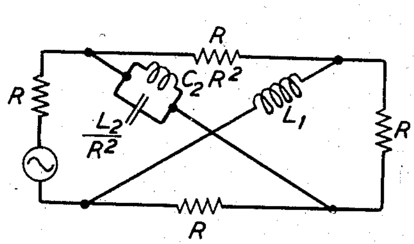

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FIG. 6

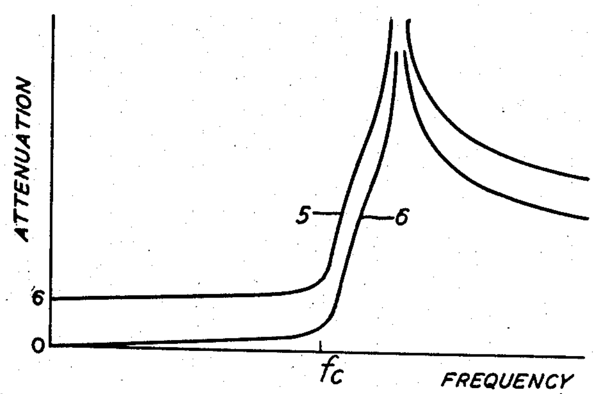


FIG. 7

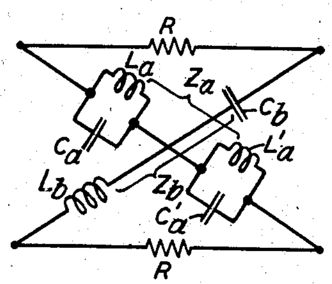


FIG. 8

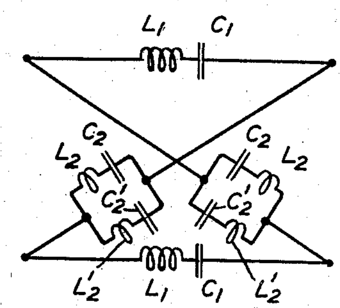
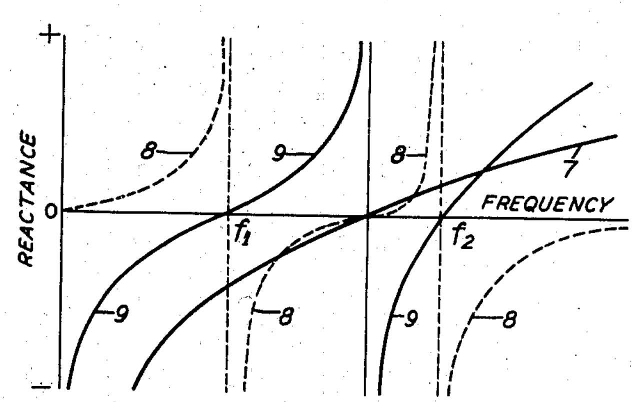


FIG. 9



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UNITED STATES PATENT OFFICE

1,991,195

WAVE TRANSMISSION NETWORK

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Application October 31, 1931, Serial No. 572,256

7 Claims. (Cl. 178—44)

This invention relates to wave transmission networks and more particularly to networks of the frequency selective type such as broad band wave filters.

It has for its principal object the economy of elements in the construction of wave filter networks and the like.

Another object is to facilitate the insertion of frequency selective networks between impedances of unequal value.

In their general form the networks of the invention comprise a pair of reactive impedances in combination with two equal resistances arranged as a Wheatstone bridge or lattice network, the two resistances forming one pair of opposing branches and the two reactive impedances forming the other pair. The networks have the property that, when connected between a wave source and a load having certain specified impedances, the frequency variation of the phase and the amplitude of the output current may be made to correspond, except for a constant attenuation loss, to that of any of the well-known broad-band wave filters, for example those of Campbell Patent 1,227,113, issued May 22, 1917.

The nature of the invention will be more fully understood from the following detailed description and from the accompanying drawings of which:

Figs. 1 and 2 are theoretical diagrams used in the explanation of the principles of the invention;

Fig. 3 is a network of known type;

Fig. 4 shows curves illustrating characteristics of the invention;

Fig. 5 is one example of a network of the invention related to the known network of Fig. 3;

Fig. 6 shows additional curves illustrating characteristics of the invention;

Fig. 7 shows schematically another embodiment of the invention;

Fig. 8 represents a known prototype of Fig. 7; and

Fig. 9 illustrates certain characteristics of the networks of Figs. 7 and 8.

A symmetrical lattice which may be regarded as a general prototype of the networks of the invention is shown in schematic form in Fig. 1. This network comprises two equal line impedances Z_1, which may be of any character whatsoever, and two lattice impedances Z_2, likewise unrestricted. To one end of the network is connected a resistance R in series with a wave source having an E. M. F. E, and to the other end is connected a resistance R representing a load impedance. This lattice network is well known, its general properties being discussed in a paper on the Physical Theory of the Electric Wave Fliter, by G. A. Campbell, Bell System Technical Journal, vol. I, No. 2, November 1922, and its use in wave filter networks being described in United States Patent 1,600,290, issued September 21, 1926 to W. T. Martin. Since it is possible, by the proper selection of the impedances, to make this network equivalent in its transmission properties to any other symmetrical form terminal network, its use as a prototype or as a standard of comparison will enable the properties of the networks of the invention to be more readily apprehended.

The general form of the networks of the invention which is shown schematically in Fig. 2, is that of a lattice network having two line branches constituted by resistances R and two lattice branches having impedances Z_a and Z_b respectively. The network is adapted to be connected between resistive impedances of magnitudes

$$\frac{R}{a}$$

at the input end and aR at the output end, the factor a being a simple numeric unrestricted in value. The transmission characteristics of the networks are determined by the character of the impedances Z_a and Z_b and by making these reactive, a wide range of frequency selective characteristics can be obtained.

For the general symmetrical lattice shown in Fig. 1 the output current, denoted by I_2, for an E. M. F. E in the input circuit is given by the formula:

$$I_2=\frac{E(Z_1-Z_2)}{2(R+Z_1)(R+Z_2)} \qquad (1)$$

In the case of the network shown in Fig. 2, the output current, denoted by I_2 for an E. M. F. E in the input circuit is given by the formula:

$$I_2'=\frac{a}{(1+a)^2}\cdot\frac{E(Z_aZ_b-R^2)}{R(R+Z_a)(R+Z_b)}. \qquad (2)$$

If now Z_b is made equal to Z_1 and Z_a

$$\frac{R^2}{Z_2},$$

Equation (2) becomes:

$$I_2'=\frac{a}{(1+a)^2}\cdot\frac{E(Z_1-Z_2)}{(R+Z_1)(R+Z_2)}, \qquad (3)$$

which indicates that under the conditions specified, the output current of the network of Fig. 2

will have the same frequency variation of phase and amplitude as that of Fig. 1, but will be diminished in amplitude in proportion to the constant factor

$$\frac{a}{(1+a)}^2.$$

If the quantity a is made unity, that is if the lattice of Fig. 2 is connected between equal terminal impedances resistance R, Equation (3) becomes:

$$I_2^1=\frac{E(Z_2-Z_1)}{4(R+Z_1)(R+Z_2)} \quad (4)$$

or just half the output current in the circuit of Fig. 1.

The insertion loss when a network of any kind is used to couple together a wave source and a load impedance is measured by the ratio of the power received by the load impedance when the network is absent to the power received when the network is present. The unit of loss is the decibel, the loss in decibels being equal to ten times the common logarithm of the power ratio.

From Equations (1) and (4) it follows that the insertion loss due to the network of Fig. 2 will be the equal to that of Fig. 1 when the impedances are related as described above, plus a fixed attenuation loss of about 6 decibels. The frequency selectivity and the phase displacement of the output currents will be the same for both networks.

The form of Equations (2) and (3) indicates that the frequency variation of the output current in the network of Fig. 2 is not affected by variation of the resistances of the terminal impedances so long as their product is equal to R^2. The actual value of the output current is modified in proportion to the factor $a \div (1+a)^2$, but the variations with frequency retain the same proportionate values. By virtue of this property the network may be designed to operate between impedances of unequal value without change of its selectivity.

The design formulæ for broad-band filter networks in accordance with the invention follow readily from the foregoing equations and from the principles discussed above and from the well-known design rules for lattice type filters employing reactive impedances only. The design of lattice type filters is described in the aforementioned United States Patent 1,600,290 of September 21, 1926 to W. T. Martin and also in United States Patent 1,828,454 to W. H. Bode, issued October 20, 1931. The procedure will be illustrated in connection with the design of the simple low-pass filter illustrated schematically in Fig. 5.

In Fig. 3 is shown a low-pass filter of the lattice type, the line branches of which comprise simple inductances L_1 and the lattice branches series resonant impedances L_2 C_2. The determination of the pass-band and the cut-off frequency for this filter is illustrated by the curves of Fig. 4 of which curve 1 represents the variation of the reactance of inductances L_1 with frequency, and curve 2 the variation of the reactance of the resonant impedances L_2 C_2. The theory of the lattice filter points out that a pass-band occurs in those frequency ranges where the line branch reactances are of opposite sign to the reactances of the lattice branches and that an attenuation band occurs when the reactances are of like sign. For the circuit of Fig. 3 the cut-off frequency evidently occurs at the resonance of L_2 C_2 and the pass-band in the range from this point to zero. In Fig. 4 the cut-off frequency is designated f_c.

The characteristic impedance K of the filter of Fig. 3 is given by

$$K=\sqrt{\frac{L_1}{C_2}}\sqrt{1-\frac{f^2}{f_c^2}} \quad (5)$$

which has the value

$$\sqrt{\frac{L_1}{C_2}}$$

at frequencies close to zero. To avoid transmission irregularities in the pass-band due to reflection at the terminal junctions, the terminal impedances R are usually given the value:

$$R=\sqrt{\frac{L_1}{C_2}} \quad (6)$$

which approximately matches the characteristic impedance over a substantial part of the band.

To make the filter of Fig. 5 have an insertion loss similar to that of Fig. 4, the two line branches are made resistances of value R. One lattice branch duplicates one of the line branches of Fig. 4, that is, it comprises a simple inductance L_1 and the other lattice branch is constituted by an impedance which is inverse to the series combination L_2 C_2 with respect to R^2. This inverse impedance, as shown in the figure consists of an inductance of value C_2R^2 in parallel with a capacity of value

$$\frac{L_2}{R^2}.$$

When R has the value given by Equation (6), the elements of the parallel combination have the values:

$$C_2R^2=L_1,$$

and

$$\frac{L_2}{R^2}=C_2\frac{L_2}{L_1}. \quad (7)$$

The frequency variation of the reactance of the parallel resonant lattice branch is represented by curve 3 of Fig. 4, the anti-resonance occurring at the same frequency as the resonance of L_2 and C_2 and the reactance having opposite sign to that of curve 2 at all frequencies. It follows that for the general lattice networks of the invention as illustrated by Fig. 2 a pass-band will exist when the two lattice branches Z_a and Z_b have reactances of the same sign and an attenuation band will exist when the reactances are of opposite sign. This is a converse of the rule defining the transmission bands of the wholly reactive network of Fig. 1.

The insertion loss characteristics of the low-pass network of Fig. 5 and of its prototype Fig. 4 are illustrated by curves 5 and 6 respectively of Fig. 6 in which the over-all attenuation is plotted against frequency, the attenuation being measured in decibels. The form of the two curves is the same but curve 5 has a vertical displacement of 6 decibels which represents the constant loss corresponding to the factor, $a \div (1+a)^2$. The attenuation peaks above the transmission band correspond to the crossing of curves 1 and 2 of Fig. 4. Such peaks occur in characteristics of the general lattice filters of Fig. 1 whenever Z_1 and Z_2 are equal and in the networks of the invention whenever the product Z_a Z_b of the impedances of the lattice branches is equal to R^2, the product of the resistance branch impedances. This is clear from equation (2) which shows that the output current is zero under this condition.

A typical band-pass filter of the invention is illustrated in Fig. 7, the reactance prototype being shown in Fig. 8. In this filter the lattice branches impedance Z_b comprises an inductance L_b and a capacity C_b in series and the other branch impedance Z_a comprises two parallel resonant combinations $L_a\ C_a$ and $L_a'\ C_a'$ connected in series. The frequency variation of the reactances of these branches are shown in Fig. 9 in which curve 7 represents the combination L_b and C_b and dotted curve 8 represents the combination L_a, C_a, L_a', C_a'. In order that there may be a single pass-band, the reactances of Z_a and Z_b must have the same sign in a single frequency range. This requires that the resonance frequency of Z_b shall coincide with that of Z_a. The limits of the band are determined by the two anti-resonance frequencies f_1 and f_2 of $L_a\ C_a$ and $L_a'\ C_a'$ respectively.

The prototype network of Fig. 8 comprises two line branches having simple series resonant combinations $L_1\ C_1$ therein and two lattice branches each constituted by two resonant combinations $L_2\ C_2$ and $L_2'\ C_2'$ connected in parallel. That the two networks may have similar insertion loss characteristics requires the following relationship between the elements;

$$L_b=L_1,$$
$$C_b=C_1,$$
$$L_a=C_2\ R^2,$$
$$L_a'=C_2'\ R^2,$$
$$C_a=\frac{L_2}{R^2}, \qquad (8)$$

and

$$C_a'=\frac{L_2'}{R^2}.$$

The networks will then have similar characteristics when inserted between terminal impedances of resistance R.

The reactances of the branches of the prototype filter are represented by curves 7 and 9 of Fig. 8 which correspond respectively to the line branches and the lattice branches. Curve 9 has a variation inverse to that of curve 8.

Equations (8) enable the band filter design to be arrived at in terms of the coefficients of the prototype filter, which latter may be designed in accordance with known principles as described, for example, in the aforementioned Patent 1,828,454 to H. W. Bode. In following this procedure the value of the resistance R is chosen as that of the terminal impedances which will give the best match with the characteristic impedance of the prototype filter throughout the transmission band. The filter networks of the invention may, however, be designed without reference to the prototype networks in accordance with the following procedure.

From Equation (2) it follows that, if the two lattice branch impedances Z_a and Z_b of the general network of Fig. 2 are made equal, the output current will be given by

$$I_2'=\frac{a}{(1+a)^2}\cdot\frac{E(Z-R)}{R(Z+R)} \qquad (9)$$

where Z is the common value of Z_a and Z_b. So long as Z is a pure reactance the output current given by Equation (9) will have a constant amplitude, the only change with frequency being a variation of its phase. If, now, the two impedances Z_a and Z_b, in addition to having their resonance frequencies so assigned as to provide a transmission band in a desired frequency range, are proportioned also so that throughout the major part of this range they are substantially equal, then the output current will have the substantially constant amplitude

$$\frac{a}{(1+a)^2}\ \frac{E}{R}$$

throughout the band.

The values of the elements to give approximate equality of the impedances throughout the band may be arrived at mathematically, for example by assigning a sufficient number of frequencies at which the impedances shall be exactly equal and equating the expressions for the reactances at these frequencies. A somewhat simpler procedure, however, is the following: Having assigned the resonance and anti-resonance frequencies in terms of the band location, the impedances may have arbitrary values assigned to their elements consistent with these frequencies. A plot of the reactances at various frequencies like that of Fig. 9 will show the extent and the nature of the disparity of the reactances in the band and will indicate in what proportion the ordinates of one or the other of the curves should be increased or decreased to make it match the other curve in an optimum manner. The required element values are then found by increasing or decreasing the arbitrarily chosen inductances of one of the impedances in the proportion indicated and changing the capacities in an inverse relation.

Having thus determined a set of impedance coefficients which make the two impedances substantially equal throughout the assigned band, the network may be used between terminal impedances of any resistance values provided that the resistances of the line branches are made equal to the geometric means of the terminal resistances. The uniformity of the output current in the transmission band range will not be much affected by changes in the terminal impedances but considerable changes may result in the attenuation outside the band. For given values of the terminal impedances, control of the attenuation characteristic outside the band may be effected by increasing or decreasing the reactances of the lattice branches in the same ratio, their resonance frequencies being kept constant.

The foregoing examples of networks of the invention serve to illustrate their general characteristics and the manner of their design. It is clear from the principles set forth that the types are not limited to these examples but may comprehend equivalents of all symmetrical lattice networks including high-pass filters, phase correctors or delay networks and attenuation equalizers.

What is claimed is:

1. A four-terminal wave transmission network comprising four-impedance branches arranged in lattice formation, two similarly disposed arms of the lattice being constituted by equal resistances and the other two branches being constituted by reactances, said reactances being of the same sign in a preassigned frequency range defining a transmission band and being of opposite sign at other frequencies.

2. A network in accordance with claim 1 in which the said reactances have substantially equal values in the frequency range defining a transmission band.

3. A lattice network having an insertion loss characteristic when inserted between terminal impedances of resistance R equivalent except for a constant additional attenuation loss to that of a prototype symmetrical lattice network having branch impedances of values Z_1 and Z_2, re-

spectively, comprising two line branches constituted by equal resistances of value R and lattice branches having impedances of value Z_1 and

$$\frac{R^2}{Z_2}$$

respectively.

4. In combination with a wave source and a load having resistance impedances, a four-terminal network connected therebetween comprising four-impedance branches arranged in lattice formation, two similarly disposed branches of the lattice comprising equal resistances of value equal to the geometric mean of the resistances of said wave source and said load and the other two branches of the lattice being constituted by unlike reactances, said reactances being proportioned to have the same sign in a preassigned frequency range whereby substantially uniform transmission between said source and said load is obtained in said range and to have opposite signs at all other frequencies whereby currents of frequencies outside said preassigned range are subject to a high degree of attenuation.

5. A combination in accordance with claim 4 in which the reactance branches of the lattice network have impedances of substantially equal value throughout the preassigned frequency range of uniform attenuation.

6. A band-pass filter network comprising four-impedance branches connected in lattice formation, two of said branches comprising equal resistances and being disposed symmetrically in said network and the other two branches being constituted by resonant reactance combinations, the resonance frequencies of said reactances being so assigned that the reactances are of the same sign at all frequencies between two finite frequencies defining the limits of a transmission band and are of opposite sign at all other frequencies.

7. A low-pass filter network comprising four-impedance branches connected in lattice formation, two of said branches comprising equal resistances and being symmetrically disposed in said network and the other two branches being constituted by reactances proportioned with respect to each other to have the same sign at all frequencies below a definite value and to have opposite signs at all higher frequencies.

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