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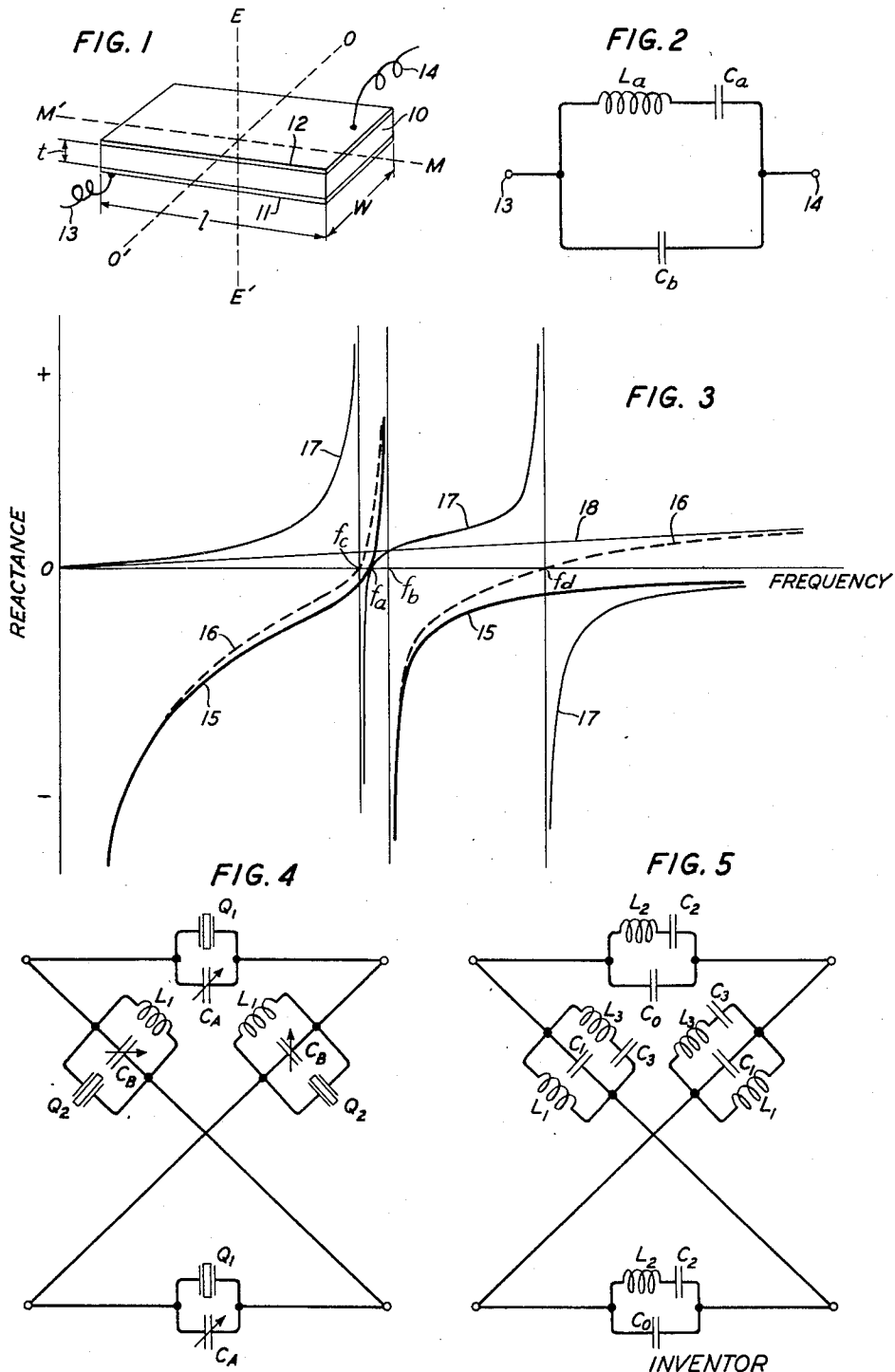
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1,921,035

WAVE FILTER

Filed Sept. 30, 1931

2 Sheets-Sheet 1



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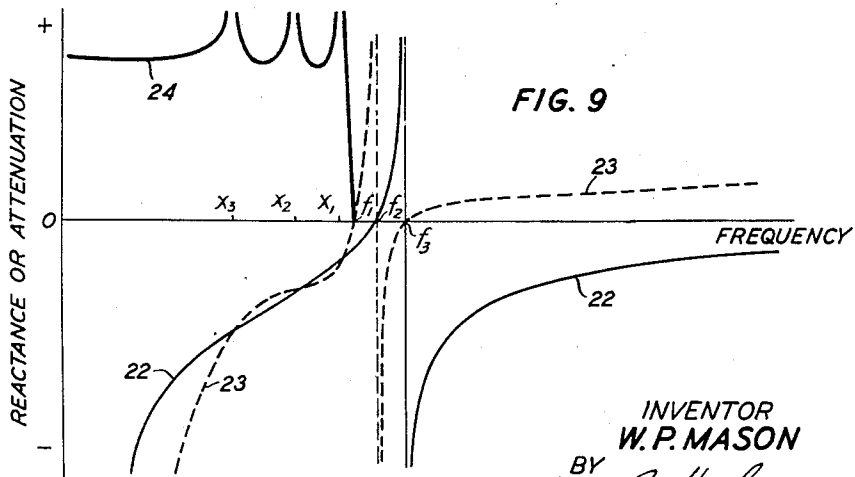
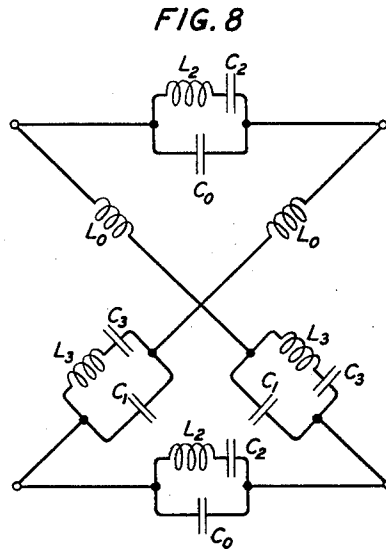
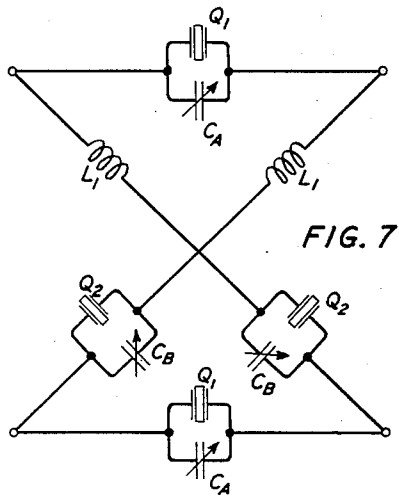
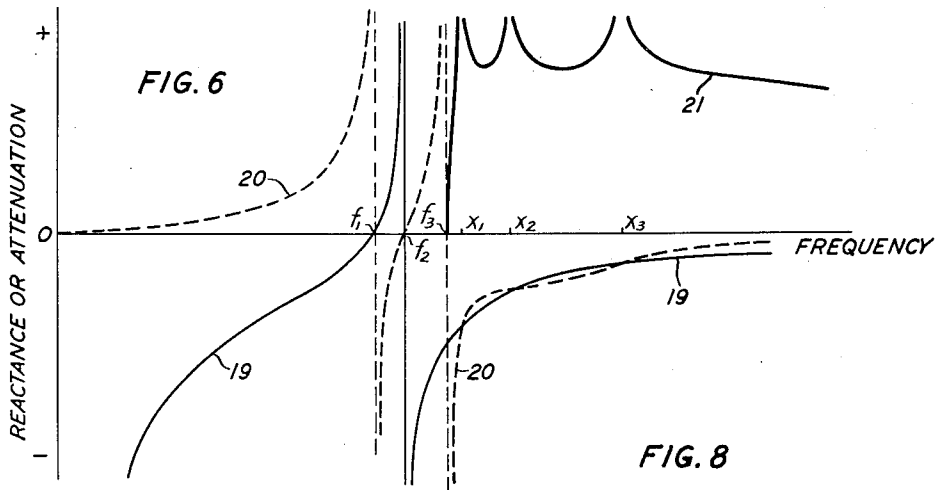
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2 Sheets-Sheet 2



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UNITED STATES PATENT OFFICE

1,921,035

WAVE FILTER

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Application September 30, 1931
Serial No. 566,031

5 Claims. (Cl. 178-44)

This invention relates to broad band wave filters in which piezo-electric crystals are used as impedance elements and more particularly to crystal filters of the low-pass type and of the high-pass type which are characterized by a single attenuation range and a single transmission range.

In my copending application, Serial No. 489,263, filed October 17, 1930, it is pointed out that in filters using piezo-electric crystals alone it is possible to obtain only very narrow transmission bands. For quartz crystals the maximum obtainable band width is about 0.7 per cent of the mean band frequency. In the same application filter networks are described in which by using piezo-electric crystals in combination with electrical inductances and capacities very much broader bands are obtained without impairing the sharp selectivity which is characteristic of piezo-electric crystals.

In accordance with the present invention piezo-electric crystal filters are provided in which the transmission band is so widened that one of the attenuation ranges is eliminated, the resulting transmission characteristic being either of the low-pass type or of the high-pass type.

The general form of the filters of the invention is similar to that of the filters described in my above mentioned copending application in that the crystal elements are combined with electrical inductances and capacities and that the branches of the networks are disposed in a symmetrical lattice formation.

The nature of the invention will be more fully apprehended from the following detailed description of networks representing typical embodiments thereof and by reference to the accompanying drawings of which:

Fig. 1 illustrates a form of crystal suitable for use in the networks of the invention;

Figs. 2 and 3 are diagrams illustrating the principles of the invention;

Fig. 4 shows schematically a low-pass filter embodying the invention;

Figs. 5 and 6 are diagrams illustrating the properties of the filter of Fig. 4;

Fig. 7 shows schematically a high-pass filter embodying the invention; and

Figs. 8 and 9 are diagrams illustrating the properties of the filter of Fig. 7.

The form and the cut of a crystal suitable for use in the frequency range up to about 500 kilocycles per second is shown in Fig. 1 in which 10 represents a crystal plate, preferably of quartz, having its length l parallel to the mechanical

axis MM' , its width w parallel to the optical axis OO' , and its thickness t parallel to the electrical axis EE' . Electrodes 11 and 12 are applied to the large faces of the crystal preferably by the electrical deposition of a layer of silver or other metal to secure an intimate contact over the whole surface. Leads 13 and 14 are connected to the electrodes by soldering.

If the length of the crystal is great in comparison with the width, its electrical impedance for frequencies up to and well above the first resonance is of a simple character and corresponds to that of the electrical circuit shown in Fig. 2. Other resonances, representing changes in the mode of vibration are practically eliminated if the length is made about three times as great as the width.

The equivalent electrical circuit, as shown in Fig. 2, comprises a parallel branch network between terminals 13 and 14, one branch consisting of an inductance L_a in series with a capacity C_a and the other branch comprising a simple capacity C_b . For a quartz crystal of the type shown in Fig. 1 the values of the elements of the equivalent electrical circuit are given in terms of the crystal dimensions measured in centimeters by the following formulæ:

$$L_a = \frac{118 \, l t}{w} \text{ Henries,} \quad 85$$

$$C_a = \frac{0.289 \, w l \, 10^{-14}}{t} \text{ Farads,} \quad 1.$$

$$C_b = \frac{40.5 \, w l \, 10^{-14}}{t} \text{ Farads,} \quad 90$$

The capacity C_b is the simple electrostatic capacity between the electrodes and its value is independent of the piezo-electric effect. Inductance L_a and capacity C_a have values depending not only upon the crystal dimensions but also upon its piezo-electric and elastic constants. These elements represent the piezo-electric property of the crystal. Capacity C_b is 140 times as great as capacity C_a and, except at frequencies close to the resonance of L_a and C_a , is the dominating factor in the crystal impedance.

Fig. 3 shows a typical crystal impedance characteristic and also shows the effect of combining a small inductance with the crystal. In this figure the curves show the variation of reactance, plotted as ordinates, with frequency, plotted as abscissa. The heavy line curve 15 represents the impedance of a crystal alone; dotted line curve 10 illustrates the impedance obtained by adding a

small inductance in series with the crystal; and the light line curve 17 represents the impedance obtained with the same small inductance in parallel with the crystal. The reactance of the added inductance is represented by straight line 18.

For the crystal alone the impedance is characterized by a resonance frequency f_a and an anti-resonance frequency f_b which is slightly higher. The values of f_a and f_b in terms of the crystal dimensions are given by the following equations which are readily obtainable from the values given by Equation 1:

$$f_a = \frac{10^6}{3.68L},$$

$$f_b = 1.0035f_a.$$

The extremely small separation of the two frequencies is due to the high ratio of C_b to C_a .

The addition of a small inductance in series has the effect of moving the resonance frequency down to a slightly lower value, indicated by f_c in the figure, introducing a new resonance frequency f_a at a considerably higher frequency. When the inductance is added in parallel the impedance exhibits anti-resonances at f_c and f_a together with an intermediate resonance at the original frequency f_a . If the added inductance is varied in magnitude, the upper frequency f_a is shifted a large amount while the lower frequency f_c is not greatly altered. Likewise if capacity is added in parallel with C_b the upper frequency f_a is moved closer to the crystal resonance while the lower frequency f_c is not greatly affected.

The application of the foregoing principles to the design of a low-pass filter of the type shown schematically in Fig. 4 will now be considered. In this figure a lattice type network is shown the line branches of which contain similar crystals Q_1 shunted by adjustable capacities C_A and the lattice branches of which contain similar crystals Q_2 shunted by inductances L_1 and adjustable capacities C_B . The equivalent electrical circuit is shown in Fig. 5 wherein the resonant circuits L_2C_2 and L_3C_3 correspond to the crystal resonances of Q_1 and Q_2 respectively, C_0 represents the combination of C_A with the electrode capacity of crystal Q_1 and C_1 represents the corresponding combinations for crystal Q_2 .

Fig. 6 shows certain characteristics of the filter of Fig. 4 and illustrates the requirements that must be met in order that the low-pass characteristic may be obtained. In this figure curves 19 and 20 represent the reactance-frequency characteristics of the line and the lattice branches respectively and curve 21 is a typical attenuation-frequency characteristic. The impedance of the line branches, comprising crystals Q_1 and shunt capacities C_A , is similar to that of a crystal alone, being characterized by a single resonance at frequency f_1 and a single anti-resonance frequency f_2 . The effect of the added capacity is simply to move the anti-resonance frequency closer to the resonance frequency; in general, it is desirable that the added shunt capacity in the line branches should be small.

The formation of a single low-pass band requires that the impedance of the shunt branches should be of opposite sign to that of the line branches at all frequencies below the cut-off and of the same sign at all frequencies above the cut-off. If the crystals of the lattice branches are so proportioned as to have their piezo-electric resonances at the frequency f_2 and if they are shunted by small inductances an impedance char-

acteristic for the lattice branches like curve 20 can be obtained and by proper proportioning of the inductance an anti-resonance can be produced at the frequency f_1 at which the line branch crystals are resonant. A second anti-resonance of the lattice branches occurs at a higher frequency f_3 which, as inspection of the diagram shows, becomes the filter cut-off frequency.

In order that the cut-off may be sharp, it is desirable that the crystal resonances should be quite close to the cut-off frequency. From Fig. 3 it is seen that the addition of a small shunt inductance alone to a crystal tends to place the second anti-resonance frequency some distance above the crystal resonance; however, as explained in connection with Fig. 3 the further addition of a shunt capacity brings this frequency quite close to the crystal resonance without substantially affecting the position of the first anti-resonance. By the proper choice of the shunting inductance and the shunting capacity, which, if desired, may be made empirically with the help of calculated impedance curves, the cut-off frequency may be brought as close as desired to the crystal resonances. A further control of the adjustment is provided by the small capacities C_A shunting the line branch crystals.

In Fig. 6 the two reactance curves 19 and 20 are shown crossing at three points above the cut-off frequency, the crossing frequencies being designated by X_1 , X_2 and X_3 , respectively. Since the lattice corresponds to a bridge network which is balanced when the branch impedances are equal, these frequencies correspond to points of infinite attenuation and in the particular case illustrated give rise to an attenuation characteristic of the type shown by curve 20. Various conditions are possible; for example, if the total shunting capacity C_1 in the lattice branches is much smaller than the corresponding capacity C_0 of the line branches the curves may not cross at all and the attenuation characteristic would then have no peaks. This, however, would only occur in a filter having a cut-off considerably above the crystal resonances and would represent a case in which the crystals contribute little to the filter selectivity. In the type where the cut-off is brought close to the crystal resonances by the addition of shunt capacity to the lattice branches there will generally be three peaks as illustrated.

To develop explicit formulae for the design of a filter of this type, it is necessary that a sufficient number of parameters be specified. The most fundamental design parameters are the cut-off frequency and the characteristic impedance at zero frequency, but if these alone are specified a wide range of designs is possible which vary in respect of their attenuation characteristics or their phase characteristics in the transmission band. If the three frequencies of infinite attenuation are specified in addition to the cut-off frequency and the characteristic impedance, the design becomes fixed and explicit formulae for the elements can be found. Such formulae do not give the crystal dimensions directly but give the values of the various inductances and capacities indicated in Fig. 5 from which the crystal dimensions may be computed by means of equation 1.

The following formulae, which may be arrived at by the same general procedure as is described in my aforementioned copending application in connection with the design of band pass filters,

are applicable to the design of a filter in accordance with Figs. 4 and 5:

$$L_1 = \frac{Z_0 B}{2\pi f_c D}$$

$$L_2 = \frac{Z_0 (A+D)^2}{2\pi f_c (AB-D)}$$

$$L_3 = \frac{Z_0 (1+B)^2 B}{2\pi f_c (AB-D)}$$

$$C_0 = \frac{1+B}{2\pi f_c Z_0 (A+D)}$$

$$C_1 = \frac{A+D}{2\pi f_c Z_0 (1+B)}$$

$$C_2 = \frac{AB-D}{2\pi f_c Z_0 (A+D) D}$$

$$C_3 = \frac{AB-D}{2\pi f_c Z_0 (1+B) B^2}$$

in which:

$$A = a_1 + a_2 + a_3,$$

$$B = a_1 a_2 + a_2 a_3 + a_3 a_1,$$

$$C = a_1 a_2 a_3$$

$$a_n = \sqrt{\frac{X_n^2}{X_n^2 - f_c^2}}, n=1, 2, 3,$$

f_c is the cut-off frequency, and

Z_0 is the characteristic impedance at zero frequency.

In using these formulæ it may be found that the choice of the peak frequencies X_1 , X_2 and X_3 gives rise to designs which are not physically realizable by means of crystals due to the fact that the capacities C_0 and C_1 are less than about 140 times the capacities C_2 and C_3 respectively. This limits the choice of the peak frequencies, but it is a simple matter to make a preliminary check by computing the capacity ratios for a given choice of peak frequencies before proceeding with the detail calculations. The capacity ratios are given by

$$\frac{C_0}{C_2} = \frac{(1+B)D}{AB-D}$$

$$\frac{C_1}{C_3} = \frac{(A+D)B^2}{AB-D}$$

and

So long as these ratios exceed 140 the filter will be physically realizable. As a general rule the peak frequencies should be quite close to the cut-off frequency, preferably all within two per cent.

A high-pass filter in accordance with the invention is shown schematically in Fig. 7. The same notation is used in this figure as in Fig. 4, the difference between the two circuits being that the added inductances L_1 are connected in series with the lattice branch crystals instead of in shunt therewith. The equivalent electrical circuit is shown in Fig. 8 in which, as in Fig. 5, the capacities C_0 and C_1 represent the combinations of the added capacities C_A and C_B with the crystal electrode capacities.

The effect of adding an inductance in series with a crystal has already been described in connection with Fig. 3. If the added inductance is small the new resonance introduced by the inductance may be located a frequency considerably above the anti-resonance frequency while the lower resonance of the combination will be very little below the crystal resonance. The further addition of capacity in shunt to the crystal has the effect of bringing the upper resonance much closer to the crystal anti-resonance without sub-

stantially affecting the position of the lower resonance.

The formation of a high-pass band with a network of the type shown in Fig. 7 is illustrated by the curves of Fig. 9 which show the required variation of the two branch impedances and the type of attenuation characteristic obtained. Curve 22 corresponds to the reactance of the line branch impedances comprising crystals Q_1 and shunting capacities C_A . Curve 23 corresponds to the lattice branch impedances comprising crystals Q_2 shunted by capacities C_B and having inductances L_1 in series. The line branch impedances are resonant at a frequency f_2 and anti-resonant at a slightly higher frequency f_3 . The lattice branches are resonant at a frequency f_1 lower than f_2 and are anti-resonant and resonant at frequencies f_2 and f_3 respectively. Above f_1 which is the cut-off of the filter the impedances are always of opposite sign indicating a transmission band, while below f_1 they are of the same sign indicating an attenuation band.

Below the cut-off frequency the reactance curves cross at points corresponding to three frequencies X_1 , X_2 and X_3 at which, as pointed out in connection with Fig. 6, the attenuation of the filter becomes infinite. This is illustrated by curve 24 in which the ordinates represent attenuation. The adjustment of the crystal dimensions and those of the associated electrical impedances may be accomplished in the empirical manner indicated in connection with the design of a low-pass filter or the frequencies of infinite attenuation may be used as design parameters for the development of explicit design formulæ. As in the case of the low-pass filter, the three crossing frequencies are obtained when a relatively large capacity is used to shunt the lattice branch crystals, this also being the condition that brings all of the critical frequencies close to the cut-off. The following formulæ apply to the design of a high-pass filter of the type shown in Figs. 7 and 8:

$$L_1 = \frac{Z_0}{2\pi f_c A}$$

$$L_2 = \frac{Z_0 A^2}{2\pi f_c (AB-D)}$$

$$L_3 = \frac{Z_0 [A(1+B) - D]}{2\pi f_c AD(AB-D)}$$

$$C_0 = \frac{1}{2\pi f_c Z_0 A}$$

$$C_1 = \frac{A^2}{2\pi f_c Z_0 [A(1+B) - D]}$$

$$C_2 = \frac{AB-D}{2\pi f_c Z_0 A(A+D)}$$

$$C_3 = \frac{(AB-D)D}{2\pi f_c Z_0 (1+B)[A(1+B) - D]}$$

in which:

$$A = b_1 + b_2 + b_3,$$

$$B = b_1 b_2 + b_2 b_3 + b_3 b_1,$$

$$D = b_1 b_2 b_3,$$

$$b_n = \sqrt{1 - \frac{X_n^2}{f_c^2}}, n=1, 2, 3,$$

f_c is the cut-off frequency, and

Z_0 is the characteristic impedance at infinite frequency.

In using these formulæ it is generally desirable, as in the case of the low-pass filter, to make a

preliminary check on the values assumed for X₁, X₂ and X₃ to insure that the capacity ratios

$$\frac{C_0}{C_2} \text{ and } \frac{C_1}{C_3}$$

5 have values greater than 140, this being the requirement that the circuit may be physically realizable when crystals are used. The requirement is best met by placing all three of the peak
10 frequencies very close to the cut-off frequency.

What is claimed is:

1. A broad band wave filter network comprising four impedance branches equal in pairs and disposed to form a symmetrical lattice, one of
15 said pairs of branches including similar piezo-electric crystals in combination with equal inductances and the other of said branches including similar piezo-electric crystals and having impedances adjusted with respect to the impedances of the first pair of branches to provide
20 a single transmission band of frequencies and a single attenuation band.

2. A broad band wave filter network comprising four impedance branches equal in pairs and disposed to form a symmetrical lattice, one of
25 said pairs of branches comprising similar piezo-electric crystals and equal inductances connected in parallel and the others of said branches including similar piezo-electric crystals and having

impedances proportioned with respect to the impedances of the first pair of branches to provide a transmission band extending from zero to a finite frequency.

3. A wave filter in accordance with claim 2 in
80 which the branches including the shunting inductances include also shunting capacities whereby the filter cut-off frequency is brought close to the resonance frequency of the shunted
85 crystals.

4. A broad band wave filter network comprising four impedance branches equal in pairs and disposed to form a symmetrical lattice, one of
90 said pairs of branches comprising similar piezo-electric crystals and equal inductances connected in series and the others of said branches comprising similar piezo-electric crystals and having impedances proportioned with respect to the impedances of the first pair of branches to provide
95 a single attenuation band extending from zero to a finite frequency and a single transmission band.

5. A wave filter in accordance with claim 4 in which the branches including the series inductances include also capacities in shunt to the
100 crystals whereby the filter cut-off frequency is brought close to the resonance frequency of the crystals.

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