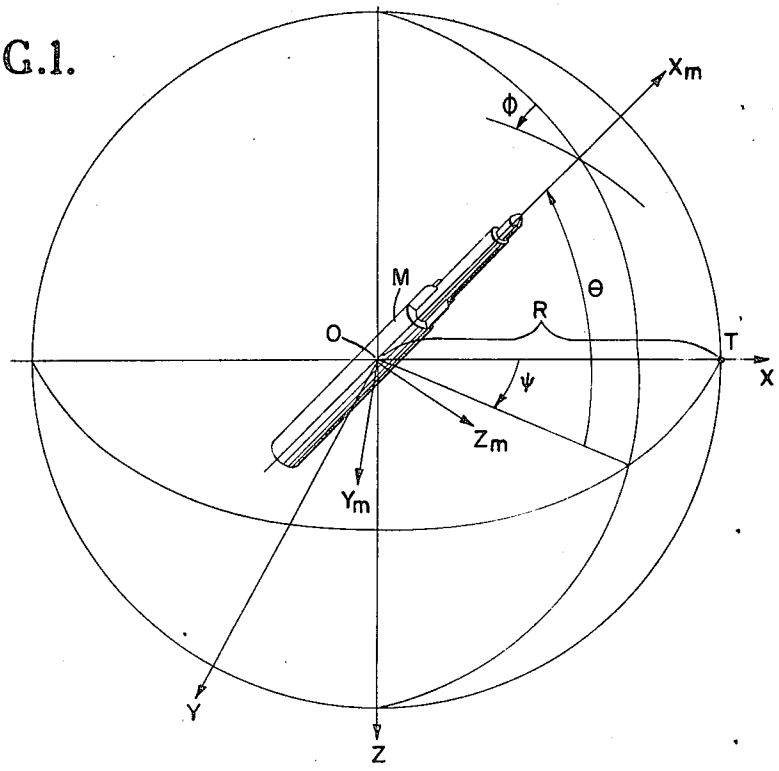
FIG.1.



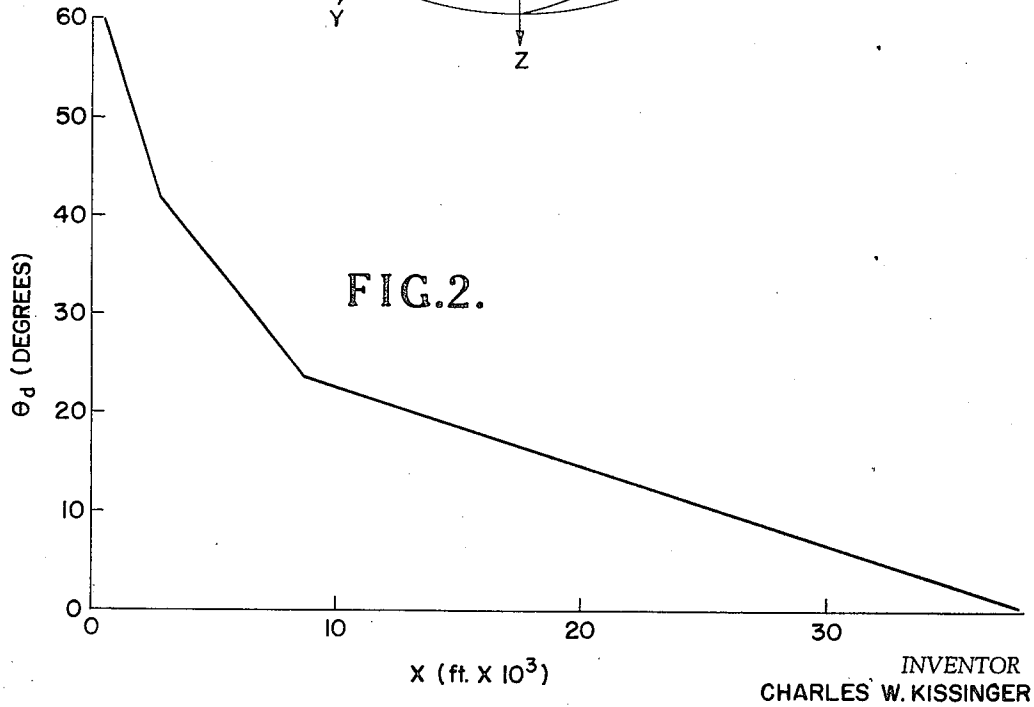


FIG.2.

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Feb. 24, 1970 C. W. KISSINGER ET AL 3,497,161

METHOD FOR COMPENSATING A BALLISTIC MISSILE FOR ATMOSPHERIC PERTURBATIONS

Original Filed July 24, 1963 2 Sheets-Sheet 2

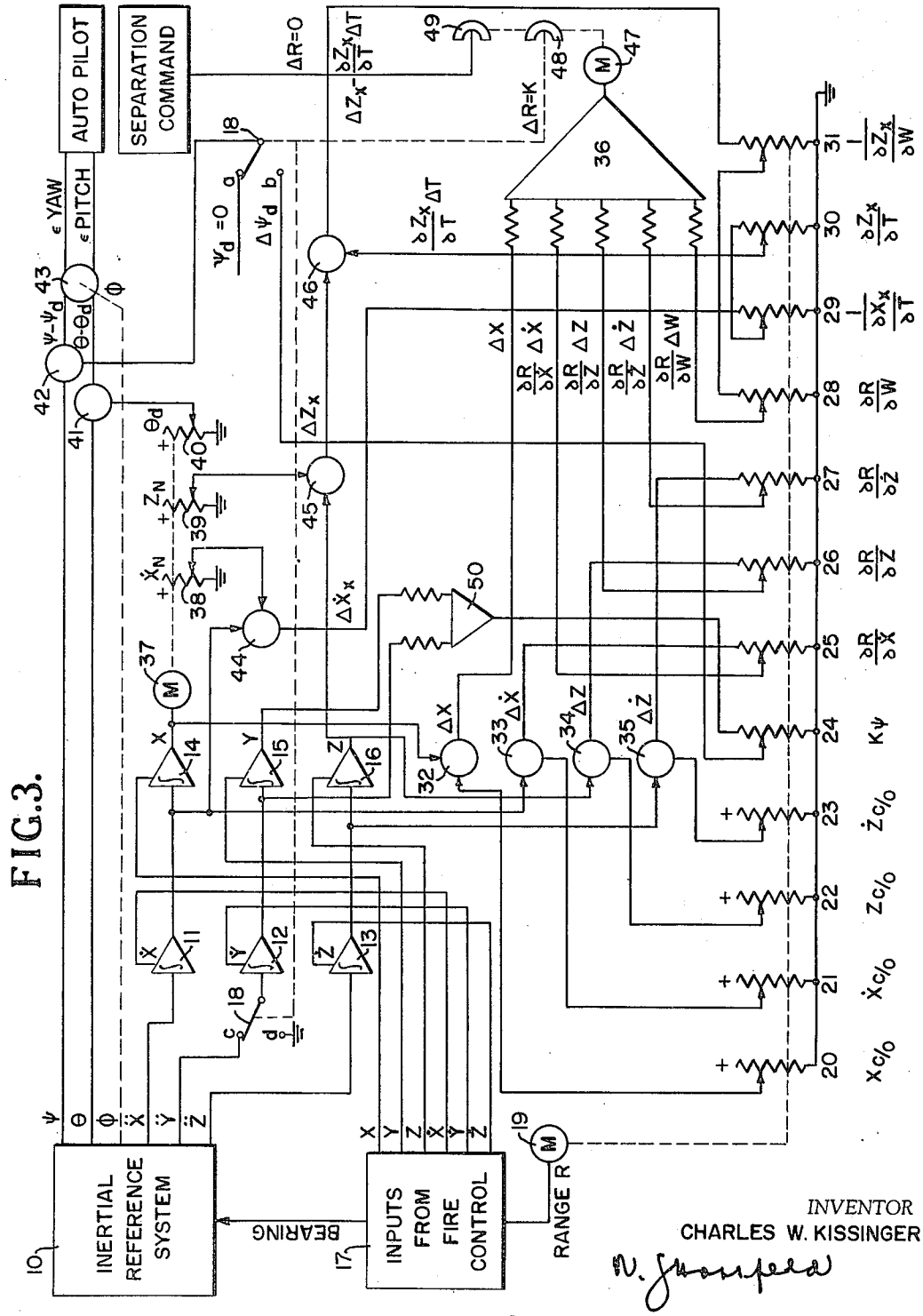


FIG. 3.

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3,497,161
Patented Feb. 24, 1970

3,497,161

METHOD FOR COMPENSATING A BALLISTIC MISSILE FOR ATMOSPHERIC PERTURBATIONS

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Continuation of application Ser. No. 297,468, July 24, 1963. This application Oct. 10, 1966, Ser. No. 586,009
Int. Cl. G05d 1/10; G06f 15/50
U.S. Cl. 244—3.2 1 Claim

ABSTRACT OF THE DISCLOSURE

A method of compensating the trajectory of a ballistic missile just prior to thrust termination by comparing the normal trajectory with the actual flight parameters encountered during the powered stage of the flight and introducing compensating corrections to provide for an accurate ballistic flight. The comparison is made by storing the nominal kinematic parameters and comparing thereto the actual flight parameters obtained from the inertial guidance system.

This invention described herein may be manufactured and used by or for the Government of the United States of America for governmental purposes without the payment of any royalties thereon or therefor.

This is a continuation of application Ser. No. 297,468, filed July 24, 1963, and now abandoned.

The present invention relates to the guidance of long and short range ballistic missiles and more particularly to flight compensation of these missiles taking into account the effects of atmospheric perturbations thereon.

A ballistic missile as considered herein is defined as a missile which is propelled by a thrust producing device, such as a rocket motor, during the powered stage of the flight and allowed to fly ballistically, that is without power or guidance control, during the remainder of the flight. If the power stage and the ballistic stage encounter mere nominal atmospheric conditions, the missile will fly through its predetermined programmed flight and be directed to the desired target impact point. However, should the atmospheric conditions produce substantial perturbations, the missile would fly to an undesired impact point remote from the target and possibly outside the kill-power range of the missile. Such perturbations include high velocity winds, variation in density, humidity, temperature, and atmospheric pressure.

In the field of missile guidance, it has been the general practice to employ various methods to perform the compensation of the guided missile to nullify the effect of atmospheric perturbations. One method which has been employed is to provide some form of terminal guidance to provide compensation after the rocket motor stage has jettisoned. The apparatus necessary for such terminal guidance adds to the complexity and cost of such a missile and renders the missile a wholly guided missile rather than a ballistic missile.

Another method of nullifying the effects of the atmospheric perturbations is to measure the atmospheric perturbations prior to the launch of the missile and use these measurements as a basis for inserting corrections in the flight program. The thrust of the missile which is controlled by the flight program is terminated in accordance with these measurements. This method requires atmospheric measurements which are not always possible to obtain, for example, in the case of a missile launched from a submerged submarine.

Still another method of nullifying effects of atmospheric perturbations is to measure directly the perturbing influences, such as wind, by means of instrumentation carried by the missile as it flies through the powered stage. Internal compensation based on these direct measurements is necessary to correct the point of thrust termination. This method similarly introduces considerable problems in the form of complex instrumentation, cost, and missile borne compensation. Although such methods and devices to implement these methods set forth hereinabove have served the purpose, they have not proven entirely satsfactory under all conditions of service for the reasons that considerable difficulty has been experienced in obtaining the data necessary for flight correction as well as the increased probability of error associated with the complex instrumentation required.

The general purpose of this invention is to provide a method and apparatus embracing all of the advantages similarly employed in guidance compensation methods and devices and which produces none of the aforedescribed disadvantages. To attain, this, the present invention contemplates the new method of adjusting the kinematic parameters at thrust termination so that the desired target impact point is attained in the presence of atmospheric perturbations occurring during the missile flight.

An object of the present invention is the provision of a new, reliable method of compensating for atmospheric perturbations experienced by a ballistic missile during flight.

Another object of the present invention is to provide a new method of adjusting the kinematic parameters of a ballistic missile at thrust termination wherein no additional instrumentation, such as atmospheric condition sensors, is required.

A further object of the present invention is the provision of a method of compensating a ballistic missile for atmospheric perturbations wherein inertial guidance data which is already present and available in the missile is used in determining that point of thrust termination which will nullify the effects of the atmospheric perturbations.

Other objects, features and the attendant advantages of this invention will be readily appreciated as the same become better understood by reference to the following detailed description when considered in connection with the accompanying drawings wherein:

FIG. 1 is a perspective drawing of the terrestial sphere showing the orientation of a missile launched from an origin at the center of the sphere and the coordinates which define the orientation of the missile from the point of launch to the given target;

FIG. 2 is a graphical representation of the desired pitch program of the ballistic missile plotted against the distance the missile has traveled toward the target in the range direction; and

FIG. 3 is a block diagram representation of the analog computation apparatus used to implement the method of compensation of atmospheric perturbations of the present invention.

Referring now to the drawings wherein like reference characters designating like or corresponding parts throughout the several views, there is shown in FIG. 1 is a ballistic missile M positioned at the origin O of a terrestial sphere and coordinate system. The missile is assumed to be inertially guided during the powered stage of flight as is well known in the missile guidance art. Kinematic parameters ordinarily available and used for inertial guidance are the x, y, and z positions, the $\dot{x}$, $\dot{y}$, and $\dot{z}$ velocities, and ψ, θ, and φ, the modified Euler angles. The inertial coordinate system of FIG. 1 in which the above parameters determine the orientation of the missile is defined as an orthogonal right-handed system

having the origin at the launch point O, the x axis horizontal and positive in the direction extending from the origin O to the target T, and z axis vertical and positive in the downwardly direction. The Euler angles ψ, θ, and φ, are referenced from the right-handed coordinate system as shown in FIG. 1. The missile is assumed to be attitude stabilized during boost, according to the following error signals in azimuth and elevation, respectively;

$$E_{az}=K_1(\psi-\psi_d)+K_2r \quad (1)$$

$$E_{elev.}=K_3(\theta-\theta_d)+K_4q \quad (2)$$

where

K_1, K_2, K_3, K_4 are gain factors, ψ_d, θ_d are desired values of ψ and θ, q is the pitch angular rate, and r is the yaw angular rate.

The azimuth angle ψ_d is held at zero in the missile flight program to thereby define the direction from the origin O to the target T. θ_d Varies throughout the boost phase and defines a precomputed "pitch program," i.e., the manner in which the missile changes pitch attitude during boost. FIG. 2 represents one possible pitch program wherein the desired pitch θ_d is a function of an independent variable x, the distance along the range line R. However, the pitch program can be based on any number of a different independent variables such as time, $\dot{x}$, z, $\dot{z}$, etc. The factors governing the choice of the independent variable will be discussed in greater detail hereinafter.

It should be understood that although the discussion which is to follow concerns compensation for head-winds, tail-winds, and cross-wind type perturbations, a like consideration can be made for other types of perturbations such as humidity changes, temperature changes, and density changes. Considering, for purposes of illustration, those perturbations caused by wind perturbations, it can be seen from FIG. 1 that a wind blowing crosswise to the direction of the flight of the ballistic missile from origin O to target T will result in the development of both a y and $\dot{y}$ error. The magnitude of the errors can be used as a measure of the cross-wind experienced by the ballistic missile during the powered stage. Corrective action can be taken which will reduce the cross range error at impact. Such corrective action can reduce the impact error substantially since the assumption can reasonably be made that the winds experienced during the descending leg of the trajectory are approximately the same as those experienced on the ascending leg up to the point at which corrective action is taken. Obviously, the shorter the range of the missile, the better this assumption becomes. Also, if cross-wind determination occurs on the ascending leg of the trajectory before the maximum altitude or apogee is reached, the total wind effect will not be detected and corrective action cannot be initiated for those winds which are encountered between the thrust termination and maximum altitude. Therefore, the degree of successful compensation is enhanced when the boost phase or powered stage of flight covers the greatest possible portion of the ascending leg of the trajectory.

If it is desirable to introduce the corrective action to the guidance system just prior to separation of the rocket motor, the amount of change may be expressed as follows:

$$\Delta\psi_d=(K_5(y-y_{nom})+K_6(\dot{y}-\dot{y}_{nom}))K \quad (3)$$

where

$\Delta\psi_d$ is the change in desired heading.

$K\psi$, K_5, K_6 are the gain constants.

$y-y_{nom}$ is the y error existing at initiation of the corrective action, and is equal to the difference between actual displacement, y, and the displacement under nominal (no-wind) conditions y_{nom}.

$\dot{y}-\dot{y}_{nom}$ is the $\dot{y}$ error existing at initiation of the corrective action and is equal to the difference between the actual $\dot{y}$ velocity and the velocity, $\dot{y}_{nom}$, under nominal (no-wind) conditions.

By employing this method of action, the cross-wind is sensed up to the time at which $\Delta\psi_d$ is introduced. $\Delta\psi_d$ remains fixed for the remainder of the boost stage. The value of K_5 and K_6 can be chosen such that effective compensation would be obtained for essentially all of the wind profiles (i.e. the relation between altitude and wind velocity) likely to occur on a statistical basis.

In order to compensate for impact point errors resulting from head or tail-winds as distinguished from cross-winds hereinabove considered, the missile is controlled in pitch during the boost phase in accordance with Equation 2. The variables x, $\dot{x}$, z, $\dot{z}$ exhibit a certain relationship dependent upon the magnitude of the head or tail-wind as the missile proceeds through the boost phase. A quite different relationship of these same parameters is encountered when the missile proceeds through the boost phase under nominal or no-wind conditions. In order to detect the effect of such head or tail-winds by sensing changes in these parameters it is desirable to choose both the pitch program and the functional relation used for detection in such a way that wind perturbations are readily separable from perturbations produced by other causes, e.g. variations in thrust, air density, launch conditions, etc. A choice of the appropriate relations to be used as a basis for the method can best be made by using a computer which can calculate trajectories and the effects of perturbing influences. Such a study has shown that a suitable mechanization is to define θ_d, the desired missile attitude in the vertical plane, as a function of x, as shown in FIG. 2. Winds are detected by their perturbation of the nominal relation of z versus x. For example a head or tail-wind causes the altitude z at a given range x to be higher or lower then nominal, respectively. In general this relation may be expressed as follows:

$$\Delta z_x=\frac{\partial z_x}{\partial w}\Delta w+\frac{\partial z_x}{\partial T}\Delta T \quad (4)$$

where

Δz_x is the actual altitude z minus the nominal altitude z at a given x range.

$z_x/\partial w$, $\partial z_x/\partial T$ are the partial derivatives of z (at a given x range) with respect to wind and thrust, respectively.

Δw is the number representing an effective wind speed present during the powered stage up to the time when the corrective action is to be taken.

$\Delta T=\frac{T}{T_{nom}}-1=$variation of thrust from nominal thrust averaged over the boost phase

Obviously there may be additional terms required on the right-hand side of Equation 4 above and where significant they should be included. However, in the illustrative embodiment set forth herein only a select number of terms are included. Equation 4 can be solved for Δw, the quantity which is essential in determining the desired compensation. Thrust variations which are necessary in the solution of Equation 4 may be detected by the direct measurement of rocket motor chamber pressure, or by the effects of thrust variations on trajectory parameters. For example, trajectory calculations show that the function $\dot{x}_x$ versus x is strongly sensitive to variations in missile thrust and essentially independent of wind. Therefore, thrust variations may be detected by the following equation:

$$\Delta T=\frac{\Delta\dot{x}_x}{\frac{\partial\dot{x}_x}{\partial T}} \quad (5)$$

where

$\Delta\dot{x}_x$ is the actual $\dot{x}$ minus the nominal $\dot{x}$ at a given x range.

$\partial R/\partial x$ . . . are the partial derivatives of range with T, at a given x range.

By the determination of ΔT from Equation 5 and substitution thereof into Equation 4 the measure of the wind experienced during the boost phase, Δw, is achieved. Having achieved this measure of the wind experienced during the boost phase, it is necessary to adjust the point of thrust termination of the rocket motor and jettison thereof such that the desired impact point will be reached. A cut-off criterion such as the following will provide such a result:

$$\Delta R=\frac{\partial R}{\partial_x}\Delta x+\frac{\partial R}{\partial \dot{x}}\Delta \dot{x}+\frac{\partial R}{\partial z}\Delta z+\frac{\partial R}{\partial \dot{z}}\Delta \dot{z}+\frac{\partial R}{\partial w}\Delta w+\frac{\partial R}{\partial p}\Delta \rho+\cdots \quad (6)$$

where

ΔR is the actual range of the impact or target position minus the desired range.

$\partial R/\partial x$ . . . are the partial derivatives of range with respect to the indicated variable.

Δx is the actual value of x minus the value of x at thrust termination under nominal condition.

$\Delta \dot{x}$ is the actual value of $\dot{x}$ minus the value of $\dot{x}$ at thrust termination under nominal conditions.

Δz is the actual value of z minus the value of z at thrust termination under nominal conditions.

$\Delta \dot{z}$ is the actual value of $\dot{z}$ minus the value of $\dot{z}$ at thrust termination under nominal conditions.

Δw is the wind as determined from Equation 4.

$\Delta \rho$ is the variation in the air density which is equal to

$$\frac{\rho}{\rho_{nom}}-1$$

averaged over the boost phase.

The mechanization and solution of Equation 6 as practiced by the present invention is computed by the circuitry to be set forth hereinafter in the ballistic missile, and thrust is terminated when ΔR goes to zero. The partial derivatives as well as the nominal values of x, $\dot{x}$, z, and $\dot{z}$ must be inserted into the missile prior to launch. These quantities can be obtained from trajectory calculations. Δw is determined during the flight from Equation 4 and the remaining variable, density, can be estimated on the basis of location and season, or determined from atmospheric pressure and temperature. Since temperature variations are more influential than pressure variations upon the density, a temperature measurement on the missile would permit a sufficiently accurate determination of density. Should the required degree of accuracy of wind compensation permit, an average temperature based on location and season could be inserted by fire control, thereby obviating the need for the temperature measurement device.

A schematic block diagram of one possible mechanization of the method described hereinabove is shown in FIG. 3. The computing components used to perform the functions of integration, multiplication, summation, etc., are shown as analog type devices and are well known in the field of analog computation. The illustrative embodiment shown in FIG. 3 is not necessarily optimum with regard to the number of elements required. Any details of the mechanization and instrumentation would obviously vary with the particular application.

The inertial reference system **10** incorporates accelerometers with a stable platform and a system of free-gyros as is well known in the inertial guidance systems art. It is assumed that the output information for system **10** includes the angles ψ, θ and ϕ and accelerations $\ddot{x}$, $\ddot{y}$, and$\ddot{z}$. The accelerations $\ddot{x}$, $\ddot{y}$ and $\ddot{z}$ are integrated once by integrators **11**, **12** and **13** respectively. The output signals from the integrators **11**, **12** and **13** provide the velocities $\dot{x}$, $\dot{y}$ and $\dot{z}$, respectively and a second integration by integrators **14**, **15** and **16** yield the displacements x, y and z, respectively.

The inputs from fire control are derived at **17** and provide the target range and bearing as well as the initial conditions for integrators **11** through **16**. Target bearing is used to align the inertial reference system in azimuth, so that the azimuth angle ψ is equal to zero along the range line from the origin O to target T. Thus, the desired azimuth angle ψ_d is maintained at zero during that portion of the boost phase prior to the initiation of the azimuth corrective action. This is indicated at contact a of switch **18**. The voltage from input **17** representing target range R, drives a servo motor **19** which rotates an output shaft an amount proportional to the target range. This shaft drives the potentiometers **20** through **31**. Each of the individual potentiometers of this potentiometers of this potentiometer bank provides a variable voltage which is a non-linear function of desired range. The output voltages of potentiometers **20**, **21**, **22** and **23** represent the nominal or no-wind values of x, $\dot{x}$, z, and $\dot{z}$, respectively at the programmed thrust termination point. This is denoted in FIG. 3 by the subscript c/o. The differences between these nominal cut-off voltages and the instantaneous voltages representing the instantaneous values of x, $\dot{x}$, z and $\dot{z}$, are obtained at summing points **32**, **33**, **34** and **35**, respectively. The output voltages of the summing devices represent Δx, $\Delta \dot{x}$, Δz, and $\Delta \dot{z}$ which are used in the solution of Equation 6.

To obtain the first four terms of Equation 6, the terms must be multiplied by their associated partial derivatives. With the exception of $\partial R/\partial x$, which is always unity by definition in the coordinate system under consideration, these partial derivatives vary as the ballistic missile progresses through a normal boost phase. As a first order of approximation, it is sufficiently accurate to use that value of the partial derivative which applies at the point of normal boost phase corresponding to thrust termination for the desired range. These values for the derivatives

$$\frac{\partial R}{\partial \dot{x}}\ \frac{\partial R}{\partial z},\ \text{and}\ \frac{\partial R}{\partial \dot{z}}$$

as functions of the desired range are determined by the top settings on potentiometers **25**, **26** and **27**, respectively. It should be understood for the purposes of illustration that these partial derivatives are less than unity and are shown as being generated by potentiometers. However, where it is necessary to provide voltages which represent partial derivative values greater than unity, an amplifier can be inserted at a convenient point in the signal path to provide the necessary gain. This insertion of a conventional amplifier is necessary since a potentiometer multiplies a voltage only by a factor less than unity. The first four terms of Equation 6 thus obtained are fed to summing amplifier **36**. The term

$$\frac{\partial R}{\partial \rho}\Delta \rho$$

is not shown in the circuit of FIG. 3, it being assumed that this term is computed in fire control and entered as a correction to the desired range, R. Therefore, for the solution of Equation 6 it remains only to compute the term

$$\frac{\partial R}{\partial w}\Delta w$$

The manner in which this is accomplished is set forth directly hereinafter.

The output of x integrator **14** in addition to being fed to summing point **32** drives a servo motor **37** which in turn drives a mechanical shaft through an angular rotation proportional to the value of x. Non-linear potentiometers **38**, **39** and **40** are constructed so as to generate the desired variables as functions of x. These potentiometers are mechanically linked to the shaft being rotated by servo-motor **37**. The output of potentiometer **38** is the nominal value of $\dot{x}$ as a function of x as is required for the solution of Equation 5. The output potentiometer

39 is the nominal value of z as a function of x as is required for the solution of Equation 4. The potentiometer 40 generates the desired elevation attitude θ_d which is fed to summing point 41 and there compared with the actual elevation angle θ. The output of summing point 41 is the elevation error signal which along with the azimuth error signal derived at summing point 42 is resolved through the roll angle ϕ by a conventional resolver 43. The output signals of resolver 43 are values, in the missile coordinate system, for the pitch and yaw error signals which are used to control the autopilot and thereby the flight of the missile.

The $\Delta\dot{x}_x$ of Equation 5 is obtained by taking the difference between the actual $\dot{x}$ appearing as the output of integrator 11 and the value of $\dot{x}$ appearing as the output of nominal $\dot{x}$ potentiometer 38. This is accomplished by the summing amplifier 44. The term $\Delta\dot{x}_x$ thus obtained is divided by $\partial\dot{x}_x/\partial T$ by means of potentiometer 29 to yield the quantity ΔT of Equation 5. This quantity ΔT in turn is multiplied by $\partial z_x/\partial T$ at potentiometer 30 to yield the term

$$\frac{\partial z_x}{\partial T}\Delta T$$

of Equation 4. It should be understood that separate potentiometers 29 and 30 are shown for the purposes of clarity and that these two potentiometers could be combined into one potentiometer providing the desired multiplication and division.

The quantity Δz_x is obtained by taking the output z of integrator 16 and feeding it to summing point 45 where it is compared to z_n, the nominal value of z derived from potentiometer 39. The output

$$\frac{\partial z_x}{\partial T}\Delta T$$

of potentiometer 30 is subtracted from Δz_x at summing point 46. The voltage output of summing point 46 is divided by $\partial z_x/\partial w$ at potentiometer 31, yielding the desired unknown quantity Δw which inturn is multiplied by $\partial R/\partial w$ at potentiometer 28. The resultant output voltage of potentiometer 28 has the value

$$\frac{\partial R}{\partial w}\Delta w$$

which is the remaining term of Equation 6 to be determined. This term

$$\frac{\partial R}{\partial w}\Delta w$$

is summed with the other terms of Equation 6 by means of summing amplifier 36 to provide the change in range ΔR of Equation 6.

The output voltage of amplifier 36 drives the servomotor 47 which in turn drives a mechanical shaft to thereby control the operation of the actuators 48 and 49. Actuator 48 is set to operate when ΔR is some value other than zero occurring prior to thrust termination. Through the operation of actuator 48 the contacts of switch 18 are switched from the *a* position to the *b* position. Switch 18 being a ganged switch, the positions *c* and *d* are also controlled by switch 18. Operation of actuator 48 to change the positions of the ganged switch, initiates the azimuth maneuver which corrects for cross range error due to cross-winds as set forth hereinabove. Prior to the time of operation of actuator 48, contact 18*a* is closed sending the value $\psi_d=0$ to summing point 42. However, after operation of switch 48, contact 18*b* is closed, sending the value $\Delta\psi_d$ to summing point 42.

This value $\Delta\psi_d$ is obtained by summing K_5y and $K_6\dot{y}$ at summing amplifier 50 and multiplying this sum by $K\psi$ at potentiometer 24. Actuator 48 also serves the dual

purpose of removing the $\dot{y}$ input from integrator 12 by breaking contact 18*c* and making contact 18*d*. This is necessary to insure that $\Delta\psi_d$ remains constant throughout the azimuth maneuver.

When $\Delta R=0$, actuator 49 operates thereby initiating thrust termination by providing a separation command signal. Motor separation and thrust termination occurs when Equation 6 is satisfied by the left-hand side, ΔR, being equal to zero. Ballistic flight then begins with the assurance that compensation for the atmospheric wind perturbations has been carried out.

Thus it may be seen by the use of purely inertial information which is already present during the guided boost phase of a ballistic missile it is possible to detect and measure the effects of atmospheric perturbations on the flight of a ballistic missile during the powered stage. The information thus gained is used to compensate for these perturbations by comparing certain known-relations of kinematic parameters for nominal atmospheric conditions to the relations of these same parameters under actual flight conditions. Availability of these actual flight parameters in the inertial guidance system is thereby utilized to avoid complex instrumentation which is necessary where compensation of atmospheric perturbations depends upon direct measurements thereof.

Obviously many modifications and variations of the present invention may be made possbile in the light of the above-teachings.

What is claimed is:

1. The method of compensating an inertially guided ballistic missile for atmospheric perturbations on both the boost phase and the ballistic phase of the missile flight with a single correction at the end of the boost phase comprising the steps of:

launching the missile toward a target in a direction determined by the predetermined trajectory parameters;

generating data signals having values corresponding to the kinematic parameters defining the predetermined trajectory of the missile under nominal atmospheric conditions;

sensing the effect of atmospheric perturbations upon the missile flight range and bearing parameters during the boost stage of the flight;

comparing said nominal atmospheric conditions data signals with actual atmospheric conditions data signals obtained during the boost stage;

computing a range and bearing flight correction program for the ballistic phase of the missile flight which doubly compensates for any deviation between said nominal atmospheric conditions signals and actual atmospheric conditions signals at the end of the boost stage;

correcting the actual trajectory once only for the entire missile flight during the period at the end of the boost stage to correspond with said flight correction program; and

terminating the thrust supplied by a booster motor immediately after the correction of the trajectory has been introduced.

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