May 5, 1964 M. B. SLATER 3,131,488

APPARATUS FOR TEACHING MATHEMATICS

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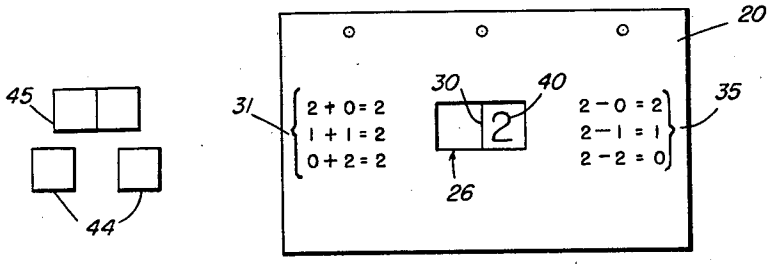


FIG. 1

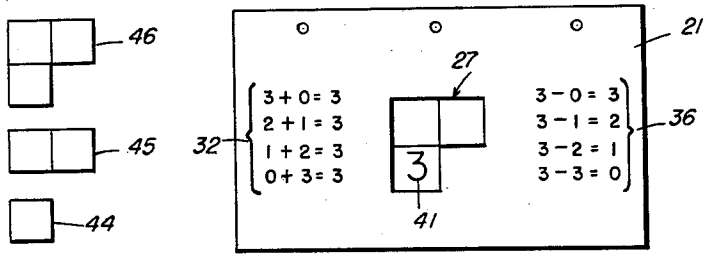


FIG. 2

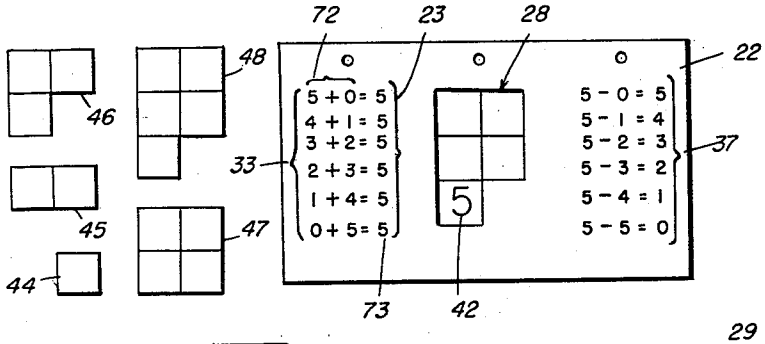


FIG. 3

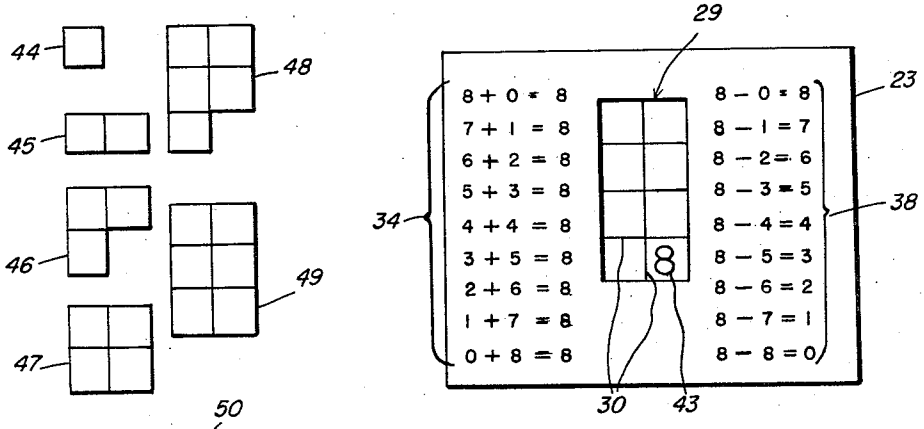


FIG. 4

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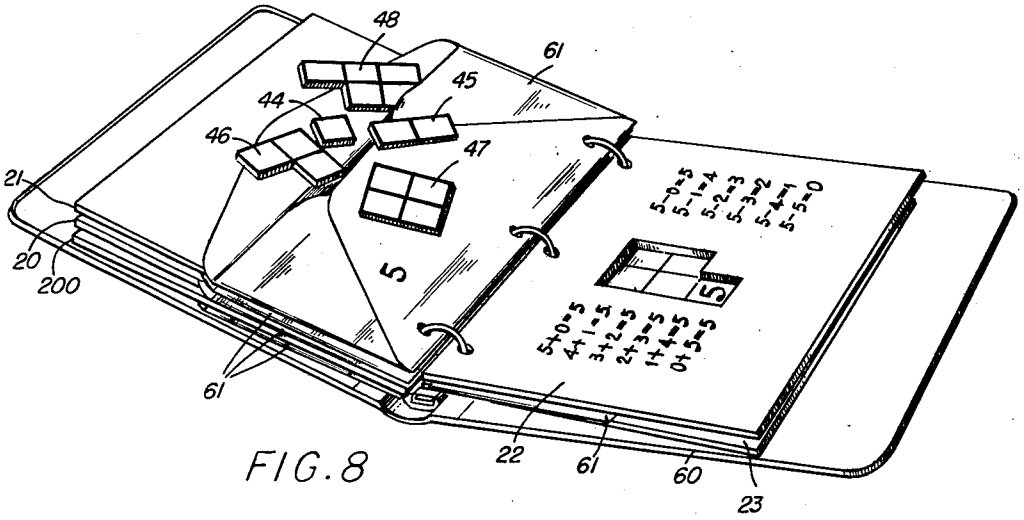


FIG. 8

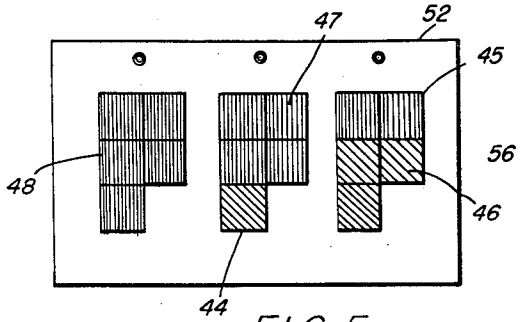


FIG. 5

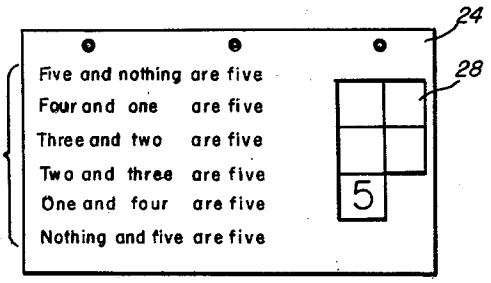


FIG. 6

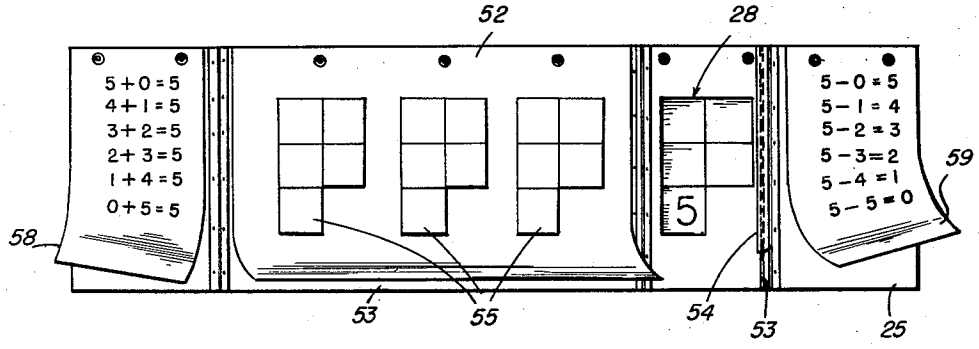


FIG. 7

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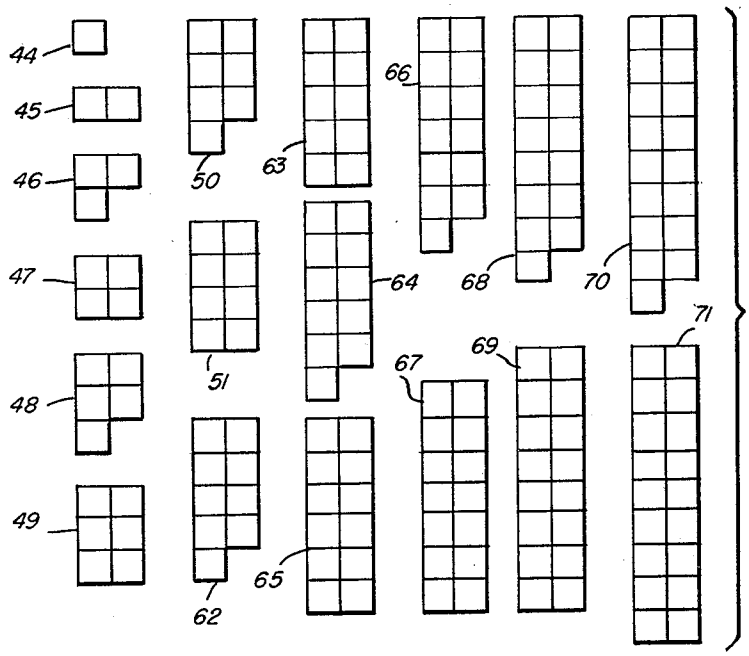


FIG. 9

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	74
1+0	2+0	3+0	4+0	5+0	6+0	7+0	8+0	9+0	10+0	11+0	12+0	13+0	14+0	15+0	16+0	17+0	18+0	
0+1	1+1	2+1	3+1	4+1	5+1	6+1	7+1	8+1	9+1	10+1	11+1	12+1	13+1	14+1	15+1	16+1	17+1	
1	0+2	1+2	2+2	3+2	4+2	5+2	6+2	7+2	8+2	9+2	10+2	11+2	12+2	13+2	14+2	15+2	16+2	
	3	0+3	1+3	2+3	3+3	4+3	5+3	6+3	7+3	8+3	9+3	10+3	11+3	12+3	13+3	14+3	15+3	
		3	0+4	1+4	2+4	3+4	4+4	5+4	6+4	7+4	8+4	9+4	10+4	11+4	12+4	13+4	14+4	
			5	0+5	1+5	2+5	3+5	4+5	5+5	6+5	7+5	8+5	9+5	10+5	11+5	12+5	13+5	
				5	0+6	1+6	2+6	3+6	4+6	5+6	6+6	7+6	8+6	9+6	10+6	11+6	12+6	
					7	0+7	1+7	2+7	3+7	4+7	5+7	6+7	7+7	8+7	9+7	10+7	11+7	
						7	0+8	1+8	2+8	3+8	4+8	5+8	6+8	7+8	8+8	9+8	10+8	
							9	0+9	1+9	2+9	3+9	4+9	5+9	6+9	7+9	8+9	9+9	
								9	0+10	1+10	2+10	3+10	4+10	5+10	6+10	7+10	8+10	
									11	0+11	1+11	2+11	3+11	4+11	5+11	6+11	7+11	
										11	0+12	1+12	2+12	3+12	4+12	5+12	6+12	
											13	0+13	1+13	2+13	3+13	4+13	5+13	
												13	0+14	1+14	2+14	3+14	4+14	
													15	0+15	1+15	2+15	3+15	
														15	0+16	1+16	2+16	
															17	0+17	1+17	
																17	0+18	
																	19	75

FIG. 10

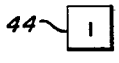
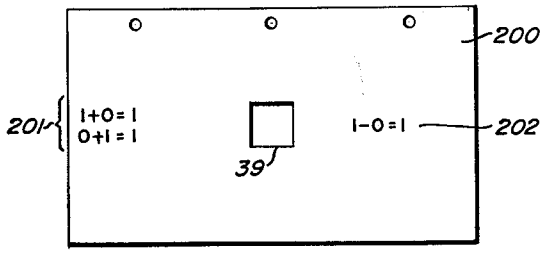


FIG. 11



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3,131,488

APPARATUS FOR TEACHING MATHEMATICS

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Filed Jan. 5, 1961, Ser. No. 80,940
8 Claims. (Cl. 35—31)

This invention relates to educational apparatus for learning mathematics, and more particularly the elementary facts of addition and subtraction in arithmetic.

The first few steps in addition and subtraction are generally first learned by "counting," that is by tallying on one's fingers or using stones or other objects. This is similar to using an abacus. Often children are next taught to memorize most of the addition or subtraction facts solely by rote—by repetition of tables—with little, if any concept and connotations of the meaning of the words or figures they repeat orally or in writing.

In order to make progress in mathematics, it is necessary to advance both from rote and from the use of concrete aids in addition or subtraction to an understanding use of abstract symbols without tallying or counting. There has been a lack of apparatus for helping the teacher to convey to the child the relation between objects and abstractly stated addition and subtraction facts.

Unfortunately, many intelligent people never advance in the knowledge of addition beyond the stage of counting on one's fingers. Not only children in the higher grades, but a very large proportion of college students have given evidence, by unmistakable signs, of still using "counting" when anything arithmetical comes up. This lack of full understanding of the elementary facts of addition and subtraction is a great handicap when more complex mathematical operations must be dealt with in everyday life, and an ever greater handicap in work in the sciences.

The new educational apparatus which forms the subject of this invention facilitates the transition from adding by counting to a thorough knowledge of all the elementary addition and subtraction facts, and to use of abstract symbols therefor with understanding. It is useful in other mathematical connections.

When I speak of simple or elementary addition and subtraction facts, reference is to the classification, common in education, where not more than two quantities are added together or subtracted from one another. Any instance where three or more aggregates are added together is known as a multiple addition fact and is not an elementary addition fact.

One object of this invention is to provide teaching apparatus which helps the pupil make the transition from counting concrete objects to an understanding of abstract simple addition and subtraction facts and also of the facts about symbols.

Another object of the present invention is to provide apparatus to so guide the pupil's mind that he will of necessity learn the elementary addition and subtraction facts and their relation to each other and to the symbols therefor, so that the pupil will become so facile in handling these elements of mathematics that he can go on to higher mathematics without having to use counting or tallying.

Another object of the invention is to help the child to understand the meaning of the cardinal, as distinguished from the ordinal numbers.

Another object is to provide apparatus by the use of which a teacher can test the pupil and know whether the pupil has progressed to a knowledge of the relation of abstract addition and subtraction facts, as distinguished from mere rote repetition of tables.

It is characteristic of the apparatus when used for elementary addition and subtraction facts made in accordance with my invention that there is a workspace marked off in arbitrary units totalling the sum to be learned and tiles based on the same unit such that there is at least

one grouping of not more than two tiles which fill the workspace and also equal the sum to be learned.

In the drawings:

FIGURE 1 shows a plan view of a preferred form of a set of apparatus for teaching the elementary facts of addition and subtraction about the number 2.

FIG. 2 is a view similar to FIG. 1 of a set for teaching the facts about the number 3.

FIG. 3 is a view similar to FIGS. 1 and 2 for teaching the facts about the number 5.

FIG. 4 is a view similar to FIGS. 1–3 of apparatus for teaching the facts about the number 8.

FIG. 5 is a plan view of an additional element for learning the facts about the number 5 as shown in FIG. 3. It is a separate sheet or card carrying three extra representations of the workspace of FIG. 3 suitable for the pupil to fill in to show the different tile groupings equalling the number 5. They are shown filled in. This is called a semi-concrete element.

FIG. 6 is a plan view of a modified form of board showing a workspace for learning the number 5 and word statements of the addition facts about that number.

FIG. 7 is a plan view of a further set of elements for learning the facts about the number 5, these consisting of a workspace, number symbol statements of the facts and three extra blank representations of the workspace. This apparatus also differs from FIGS. 1 to 4 in that the board is composed of several parts detachably fastened together. The workspace, statements of facts etc. are on different parts.

FIG. 8 is a perspective view of an open loose leaf book including sets of elements for different numbers similar to those shown in FIGURES 1 to 4.

FIG. 9 is a plan view of an assortment of tiles having from one to eighteen units.

FIG. 10 is a chart showing a summary of addition and subtraction facts to be taught by means of apparatus such as shown in FIGS. 1–9 inclusive.

FIG. 11 is a view similar to FIG. 1 of a set for teaching the facts about the number 1.

The set of apparatus for each number comprises an element which is a surface having a workspace and an element comprising one or more tiles to fit into the workspace and fill it. Each workspace has one tile of the exact shape and size to fit it. This corresponds to the groupings in which one of the two quantities in a statement of fact is a zero. The other tiles are lesser in area.

In this description the two quantities or numbers and their relation to the sum or number to be learned are referred to as a statement of fact. Such a statement is divided into the sum or answer on the one hand and what is referred to as a grouping of the two numbers on the other. The character zero is considered as one member of the grouping in any statement where zero is involved.

The simple sets of elements of FIGS. 1 to 4 will first be described. There is a set of elements for each sum or number to be learned.

In FIG. 1 we have chosen to show the elements for learning the facts about the number two. There is a surface or board **20**, having thereon a workspace **26**. The boards and workspaces in FIGS. 11, 2, 3, and 4 for the numbers one, three, five and eight are numbered **200**, **21**, **22**, **23** and **39**, **27**, **28**, **29** respectively. Each workspace is different in area and represents the aggregate, number, sum or quantity to be taught.

The surface may be any convenient surface such for example as a portable board, or table top, part of a floor or part of a wall. In the preferred form the surfaces are sheets of cardboard, plastic or light metal such as aluminum or magnesium. If desired the board can be divided into parts detachable from each other. Each part may contain a separate element of the combination. A con-

venient size for a sheet is 8½ by 11 inches. The sheets may, if desired, be perforated for including in a loose leaf binder (see FIG. 8).

The surface may be of any size and may be of any material such as wood, cardboard, metal or synthetic materials.

The workspace may be, for example, a depression in the board, a raised area or a space having its perimeter defined in any desired manner from the rest of the board. This workspace is divided by partition lines or marks 30 into one or more arbitrary area units totaling the aggregate sum about which the facts of addition or subtraction are to be learned. In FIGURE 1 the aggregate sum is two, and the workspace is divided into two arbitrary units. If desired an abstract symbol such as the numeral being taught may be placed on the final one of the arbitrary units. In FIG. 1 this is identified by the sum reference character 40. This workspace is the first element in the set for a number to be taught.

The second element in each set is the removable tiles. For each set there are one or more tiles whose area also is based on the same arbitrary unit and shaped correspondingly. In FIG. 1 three tiles 44, 45 are provided. The arbitrary unit or area into which the workspace and the tiles are divided can be considered as corresponding to the smallest whole number. The workspace plus the tiles make up a basic set of apparatus for teaching one aggregate sum. This also applies even if the mathematical fact to be learned is not elementary addition or subtraction. In such a case, the smallest whole number might be some other aggregate or symbol.

The tiles can be of the same material as the board, or of different material. The chief requirement for the tiles is that they be of a size and weight such that the child or other pupil who is learning can easily take them in his hand.

There are two discrete tiles 44 in the set for teaching the facts about the quantity two (FIG. 1), and one tile 45. Each tile 44 is the same size and shape as the basic area unit of the workspace. The tile 45 comprises two such area units placed in the same relation to each other as the two units in the workspace. Except in the tiles for the aggregate sums or quantity 2, there is no set of elements in which there are two tiles consisting of a single discrete unit.

FIG. 2 is a set of apparatus for teaching the elementary addition and subtraction facts related to the quantity 3. The abstract symbol consisting of the number 3 in the workspace in this figure of the drawings is marked with the reference character 41. The set of elements includes one tile 44 comprising the single unit, one tile 45 comprising 2 units and a tile 46 comprising 3 units and having the same size and shape as the workspace 27 on the board in this figure.

FIG. 3 comprises a set for teaching the facts about the sum 5. The number 5 in the workspace 28 bears the reference character 42. In addition to the board 22 having the workspace 28 and associated statements of addition and subtraction facts 33 and 37 respectively for aggregate sum 5, this set includes tiles 44, 45, 46 like those in the preceding sets, a tile 47 having 4 units, and a tile 48 having 5 units arranged in the same order and shape as the units in the workspace of this figure.

FIG. 4 shows a set for teaching the facts about the aggregate sum 8. It includes the board 23, the workspace 29 having eight units and a total of 9 tiles including one each of tiles 44, 45, 46 and 48, and two of tiles 47 like the tiles shown in the preceding figures, and tiles 49, 50 and 51 having 6, 7, and 8 units respectively. The number 8 in the workspace is identified by the number 43. The addition and subtraction facts are shown at 34 and 38 respectively.

While in the preferred example given in FIGURES 1 to 4 an embodiment has been used which involves three elements—workspace, tiles and the number symbol statements of the addition and subtraction facts, it should be noted that the first two elements alone have some of the values of the invention without the number symbols.

FIGURE 8 is a perspective view of a loose leaf book 60 containing a multiplicity of sets each including a board and a set of tiles for teaching a given aggregate quantity or sum, one or pairs of which tiles will illustrate each of the simple addition facts and subtraction facts for that aggregate sum.

FIG. 8 shows boards 200, 20, 21, 22 like those in FIGURES 11, 1, 2, and 3 for teaching aggregate sums 1, 2, 3, and 5 respectively. Associated with each board is a container or envelope 61 in which to keep the tiles needed to make the groupings of aggregates to teach the addition facts of the aggregate sum being taught by that set. FIG. 8 shows the book open at the set for teaching the aggregate sum 5 and the tiles 44 to 48 inclusive are shown partly removed from the envelope 61.

It may be noted, that when learning the addition or subtraction facts with regard to the number or aggregate sum 1, only one tile 44 having one unit is needed and the workspace 39 also is equal to only one unit. See FIG. 11. However, in this specification a tile having only one unit is considered an "aggregate" and is treated in the same way as aggregates which are composed of two or more of the basic units. The sets for teaching odd numbers require the same number of tiles as the number representing the aggregate sum being taught. Thus aggregate sum 3 requires 3 tiles 44, 45 and 46. Aggregate sum 5 requires 5 tiles 44, 45, 46, 47 and 48.

Sets teaching even numbers require one additional tile in view of the fact that two tiles are needed having half the units of the workspace i.e. of the aggregate sum.

Obviously in the apparatus such as that shown in FIG. 8, there could be provided a workspace for every consecutive number or aggregate sum.

The set for number or aggregate sum 4, which would go in between the sets shown in FIGS. 2 and 3, would comprise a board having a workspace shaped like the tile 47. The set would include 5 tiles, namely, a one-unit tile 44, a three-unit tile 46 and two two-unit tiles 45.

Similarly between the set for teaching the aggregate sum 5 shown in FIG. 3 and the set for teaching the aggregate sum 8 shown in FIG. 4 there would be sets for the aggregate sums 6 and 7 respectively.

In the drawings FIGS. 1–4 illustrate only selected aggregate sums but a set of the apparatus to teach all the elementary addition facts would cover aggregate sums from 1 to 18, or at least 1 to 9.

FIG. 9 is a plan view of an assortment of 18 different tiles 44 to 51 and 62 to 71 inclusive, namely tiles having 1 to 18 units respectively and each corresponding in size, shape and arrangement to the arrangement of the units which a workspace for teaching the aggregate sums 1 to 18 respectively would have. A complete set of tiles for learning all the elementary addition facts about the number 18 would include two tiles like tile 62 representing the number 9.

FIG. 10 shows a chart which may be used as a sort of final check or aid for the pupil or the teacher to be sure that the pupil has acquired a fixed knowledge of the maximum number of relationships of the abstract number symbols which it is so necessary to substitute for the counting system in order to operate in the mathematics field with facility.

The chart shows a summary of addition facts to be taught by means of apparatus similar to that shown in FIGURES 1 to 4, 9 and 11 inclusive. Numerals 74 along the top line of FIGURE 10, represent the aggregate sums to be taught, i.e. 1 to 18 inclusive.

In a column immediately below each aggregate sum are listed the addition facts to be taught about that aggregate sum.

The numeral in the diagonal line 75 at the bottom of the column under each aggregate sum is a figure repre-

senting the number of tiles needed in the teaching of the simple addition or subtraction facts about that aggregate sum. For example, for aggregate sum 1, 1 tile is needed. To teach each of the aggregate sums 2 and 3 a total of 3 tiles will be needed and so forth.

It has been found that it frequently is helpful to the pupil if the number symbol of the aggregate sum being learned is put in the workspace of the final one of the area units formed by these partition lines. Thus, for example, in FIG. 1 the numeral 2 is found in the right hand unit of the workspace while in FIGS. 2 and 3 the numerals 3 and 5 respectively, are shown in the single lowermost basic area unit of the workspace in those two figures. These numerals are designated by the reference characters 40, 41 and 42. They may be located elsewhere. If desired, other symbols such for instance, as the letters A, B and C can be substituted for the number symbols used in the drawings of FIGS. 1, 2 and 3. This may be useful if the apparatus herein described with relation to addition and subtraction is to be used in teaching more complex mathematical facts.

Elementary addition and subtraction facts must be learned differently from multiplication which later process or operation deals solely with a varying number of units of like size. In learning simple addition and subtraction facts, the problem is conceived of as consisting in dealing with aggregations of different numbers of units, and consequently with tiles of different size. One of the disadvantages of trying to teach addition and subtraction facts by tallying or counting on one's fingers is that it is apt to concentrate on using only like units. This invention avoids that disadvantage. While the basic unit is always the same, tiles or aggregates have varying quantities of basic units. The area of each unit is the same but the tiles have different area sizes.

By the use of applicant's educational apparatus the pupil learns the relation of these different amounts of units to each other and finally the relations to the number symbols in which mathematical operations are carried out.

One of the characteristics of applicant's apparatus is what applicant terms groupings. Thus a grouping of aggregates equals the sum, or aggregate sum, being learned and when the equality is stated one has a statement of addition or subtraction fact. There are of course a multiplicity of addition and subtraction facts for each aggregate sum being learned, whether that aggregate sum happens to be part of an addition fact or a subtraction fact. For example, for the sum of 1 there is a group of addition facts 201 and a group of subtraction facts 202 in FIG. 11. The group of addition facts represented by the reference character 31 for the aggregate sum of 2 and the group of subtraction facts represented by the reference character 35 in FIG. 1 each include three facts. The addition facts for the aggregate sum of 3 in FIG. 2 contain four statements marked by the number 32 and the subtraction facts for that same aggregate sum in FIG. 2 are marked with the reference character 36. Each group of facts in this figure contains four statements.

FIG. 3 which is the preferred form of equipment for teaching the aggregate sum of 5 contains addition facts 33 of which there are six and subtraction facts 37 of which there are also six. Each statement of fact consists of a grouping 72, an equal sign, and the answer 73. The equality sign, while useful, may not always be essential as long as it is explained to the pupil in some way or other that there is equality between the grouping and the answer.

As thus far explained, the apparatus involves the workspace, the tiles of different size each called an aggregate, the tiles being of such size and shape that each tile, when placed with not more than one other particular tile of the set for the aggregate sum being learned, will fill the workspace. The tiles necessary to fill the workspace constitute a grouping of the aggregates thereby suggesting the equality of the grouping with the aggregate sum. The workspace is filled by not more than two tiles. When the statement of fact being learned involves a zero, only one tile is required to fill the space, i.e. a tile having the same number of units and the same size and shape as the corresponding workspace. Thus for example, in FIG. 1 the two-unit tile 45 fills the entire workspace and is the aggregate 2 in the grouping 2+0 and in the grouping 0+2, as well as the aggregate sum 2.

Further to assist the pupil in learning the addition facts from the tiles and from them the number symbols, it is desirable not only to have the basic unit area of a simple shape such as a square but also to arrange the tiles involving more than one unit area in a regular manner. Thus in the example shown in the drawings the arrangement of the tiles which have more than one unit of area has been made such that the pupil will also learn the differences between odd and even numbers. This arrangement of the area units shown in the examples in the drawings also conveys to the pupil a sense that all even numbers are similar in some respects and that all odd numbers also have similar aspects. Furthermore, this arrangement serves to facilitate the recognition of the tiles needed to fill the workspace. The arrangement employed in the drawings for the location of unit areas in the tiles having more than one unit of area consists essentially in arranging two unit areas side by side, and if there are more than two area units, placing them below the first pair in not more than a pair per level. If the number of area units is odd, the odd unit is placed below the lowermost pair and always on the same side. In the case of the example shown in the drawings, the odd unit, if there is one, is placed under the left unit of the bottom pair. Thus, for example, in FIG. 2 where the aggregate sum 3 is being learned, the workspace contains three basic area units, two arranged side by side and the third below the left hand unit. The tile 45 in this figure corresponds to the workspace arrangement, although in use a tile may fall in any position and the child may have to turn it to the proper position to fill the workspace.

FIG. 3 shows a set for teaching the aggregate sum 5 and the correspondence between the workspace and the tiles is carried forward. The set shown in this figure includes a workspace of five units, with tiles of one unit, of two units, of three units like those shown in previous figures, and also four units and tile 48 with five basic units. In each case the odd area unit or representation is under the left one of the lowermost pair in each tile. The tiles 45 and 47 having an even number of area units are even on the bottom and this differentiation between odd and even has been found to be a facile method of recognition for the pupil. Thus by using a small square for each unit with the total area equal to any given aggregate always arranged in the same shape and that shape always presented by the workspace in the same position, concrete physical representation fosters comprehension. It also fosters the idea that certain aggregates are odd numbers and certain aggregates are even numbers, as any representation which has an uneven bottom is an odd number.

To date the element of the invention known as the workspace and the element defined as the tiles have been discussed. The number symbol element which can be included in the statements of addition and subtraction facts will now be described in more detail. The workspace and the tiles have prepared the pupil's mind for the grouping and these statements of fact and generally a teacher is able to establish the relation between the concrete representations, the workspace and the tiles on the one hand and the corresponding number symbols in the groupings and the resulting statements of addition and subtraction facts on the other. By arranging the statements of addition facts in columnar form, a pupil can see that there are series of addition facts which lead to the same aggregate sum. The similarity of this abstract material to the concrete workspace and pairs of tiles forms a connecting link between the readily understood concrete groupings and abstractly stated addition facts. The relation between the

tiles and the workspace and the different addition facts will enable the pupil to sense the significance of the number symbols with relation to the addition facts. It has been found that the subtraction facts are learned through the workspace and the tiles and then through the groupings and statements of subtraction facts similarly to the addition situation are sensed and related with facility.

A manner of using the preferred form of the novel apparatus will now be described.

In the present invention the pupil's mind is directed to learning the elementary addition facts or subtraction facts, or both, with regard to one aggregate sum at a time. Nothing concerned with any other sum should be allowed to divert the pupil's attention from the particular aggregate sum being learned and the addition and subtraction facts related thereto. Actually the pupil is generally taught to learn only one addition or subtraction fact even of that particular sum before he learns the next fact although generally there is no harm, and in many cases some advantage, in having all the addition facts connected with that aggregate sum visible to the pupil at the same time.

The normal order for teaching different aggregate sums is to start with one and progress upwardly.

Usually a child old enough to be in school will have a good idea of the quantity of "one" and will also understand the meaning of "none," "nothing" or "no," all of which are ways of expressing the idea of "zero."

With beginners the teacher can teach the sum one and the addition and subtraction facts concerning one, namely, one and nothing are one and nothing and one are one by concrete examples and by acting. For instance, with any object the teacher can show the object and say, "This is one" and can then demonstrate that one plus nothing equals one. This will be plain to the student. The normal child of school age has a clear idea of itself as one person. He will readily see that he is one child; one child and no other is still one child.

If the child of school age lacks the idea of himself as one person, apart from his school group or from his family group, there is evidence that he is either emotionally or mentally retarded. Under these circumstances, making the child aware of himself, apart from his group, is important to avoid later confusion in all subjects, not only in mathematics. It has been found, especially in teaching retarded children, that to avoid later confusion in mathematics it is important to teach the concept of one and the basic addition and subtraction facts in regard to one before going on to any other aggregate sum.

As soon as the addition facts of one are understood by means of concrete objects, the pupil can proceed to the abstract representation. He will have learned the name symbols of the addition facts from oral presentation by the teacher. If he has not already learned the abstract numeral sysmbols 1 and 0, he can be taught to read and write them at this stage.

When the pupil has learned the aggregate sum one, he is ready to go on to the next aggregate sum two. Within the beginning student each new aggregate sum should be demonstrated by means of well known objects. From these he can go on to accept the concrete tiles 44 as units. The word "unit" need not be mentioned, nor need most of the words which are used in this specification to describe the novel apparatus.

Working with the apparatus shown in FIG. 1 of the drawing, the teacher will first take up the tile 44 and say, "This is one" then tile 45 and say, "This is two." Then pointing to the concrete workspace 26, indicating both units, will say, "This is two" and finally indicating the numeral 2 (reference numeral 40), will say, "This is 2." The teacher will then take up the 2-unit tile 45 and say, "Two and nothing are two" and place the tile 45 in the workspace 26, the act being a concrete example of the grouping of the abstract terms 2+0=2. The teacher will then remove tile 45 and put it aside and will take up the two one-unit tiles 44 and will say and demonstrate to the pupil, "One and one are two," placing the tiles together side by side in the two-unit workspace 26. After one or more repetitions the teacher will encourage the pupil to go through the same procedure. Demonstrating the third addition fact, the teacher will say, "Nothing and two are two," (taking up tile 45) and will then place the two-unit tile 45 in the two-unit workspace 26.

When the child is thoroughly familiar with the concrete aspect of the addition facts in connection with the oral presentation of them, the teacher will direct his attention to the successive addition fact statements 31 which, in FIG. 1, appear on the left side of the workspace 26, thus teaching him to read these concrete statements aloud and will then teach him to write the corresponding statements in number symbols.

Similarly the child will learn the subtraction facts. When the two-unit tile is in workspace 26, the teacher will point to the tiles and say, "This is two" and then will say, for example, "I take nothing away, two less nothing is two."

Then the teacher will remove the two-unit tile 45 without comment and will put two one-unit tiles 44 into the workspace 26, also without comment. Then the teacher will remove one of the one-unit tiles, leaving the other in the workspace 26 and will say, "Two take away one leaves one."

Then replacing the tiles 44 in the workspace, the teacher will remove both tiles 44 and will say, "Two take away two leaves nothing." Variations of that subject can be repeated by putting the two-unit tile in the workspace, removing it, and then saying, "Two take away two leaves nothing. Nothing is in this space." The pupil performs the operation himself too.

When the pupil has arrived at the point where he knows abstractly all the addition and subtraction facts for any given aggregate sum, the board containing those statements of the addition and subtraction facts for that aggregate sum is returned to the teacher, and the pupil will be introduced to a set of a new board and tiles for a next aggregate sum when he has learned the preceding one.

When the aggregate sum 4 has been learned, the pupil will go on the apparatus shown in FIG. 3 for teaching the aggregate sum 5.

In teaching the aggregate sum 5, the teacher will take the tile 48 and show it to the child and say, "This is five." Pointing to the workspace 28, the teacher will say, "This is five." Then taking tile 48, the teacher will say, "Five and nothing are five," and either place the tile 48 in the workspace 28 or let the child perform the recital and the act. The teacher can then also indicate the first mathematical statement in the list 33 of FIG. 3 and pointing to the abstract numerical symbol say again, "Five and nothing are five."

With each mathematical statement it is desirable that the child work on that statement alone until he has mastered it, both in the concrete and the abstract.

For the sake of the future understanding of mathematics, it is important that the child concentrate on one fact only until it is learned thoroughly before he goes on to the next fact. With bright or precocious children this may be almost instantaneous. Some of the more advanced students can go on immediately and by themselves correlate the concrete and abstract facts, namely, fitting the four-unit tile 47 and the one-unit tile 44 into the workspace 28.

With slower children, the child may have to spend some time on the grouping in order to fully learn first the concrete fact and then to associate it with the abstract representation of the fact.

With the slower child, the teacher will demonstrate each one of the addition facts of the aggregate sum 5 by means of the several groupings of two tiles in the workspace 28.

By thus doing the addition facts in close relation to the concrete representations of the tiles, in addition to this

manipulation and the oral discussion with the teacher, the pupil will be able to grasp the relation of the abstract number symbols to the concrete symbols.

The number symbols used in the apparatus are preferably numerals, but the corresponding words may also be used as number symbols.

If further assistance is needed to substitute orderly perception of the meaning of the groupings and the addition and subtraction facts for the meaningless counting learned by rote, it is possible to use the constructions shown in FIGS. 5, 6 and 7 of the drawings. Here, in addition to the concrete facts or elements of the workspace and tiles, and the abstract number symbols, an intermediate element for learning is interposed, which may be called a semi-concrete element.

In FIGS. 5 and 7 a sheet 52 is shown as a semi-concrete element. The sheet is shown in FIG. 7 as it would normally be given to the child, with three blank representations 55 of the concrete workspace 23 shown in FIG. 3, none of which has been filled in. The child is asked to color the representations to show the varying combinations of tiles which fit into and fill the workspace. FIG. 5 shows a similar sheet 52 on which the child has filled the representations with color to show the arrangements of tiles which represent the different addition and subtraction facts. One representation is lined to show that the child has filled it in with red to represent a five-unit tile 43 or abstractly to represent the statement 5+0=5 or 0+5=5. In the same figure the next space has been filled in with red to indicate a four-unit tile 47 and with green to indicate a one-unit tile 44 and to represent the abstract statements 4+1=5 or 1+4=5. The child has indicated in red at the third unit a two-unit tile 45 and in green a three-unit tile 46 to represent the abstract statements 3+2=5 and 2+3=5. It may be preferred not to have any number symbols in the workspaces.

In the process of teaching or learning the concrete elements of the apparatus and their use, the pupil will be led to color, i.e. fill in these squares in each blank representation of the workspace to correspond with the concrete tiles needed to fill the concrete workspace. The number of representations of the workspace provided is equal to the number of addition facts for the aggregate sum being learned but preferably eliminating any duplication of facts such as mere reversal of the order of the aggregates in the groupings of the statements. The pupil will learn to color or mark in the combinations of tiles corresponding to the aggregates which will give the correct aggregate sum or answer.

FIG. 6 is an embodiment of another modified form of a separate sheet 24 for use in connection with the quantity or sum 5. It differs from the surface 22 shown in FIG. 3 in that the representations of subtraction facts are omitted, and the representations 56 of the addition facts are shown in words.

If the element being learned is the written or printed words which represent the numbers, those words having been heard orally in discussion with the teacher can be mentally associated in groupings to give equality to the aggregate sum being learned.

In FIG. 7 the addition and subtraction facts are not directly on the surface of the board 25. Cooperating removable sheets 58 and 59 are provided embodying the respective addition and subtraction facts. There also is shown a removable sheet 52 embodying semi-concrete representations 55 of aggregate sum 5 similar to that shown in FIG. 5, detachably associated on the surface 25. These elements of the workspace and the numerical statements of the addition and subtraction facts are on portions of the board 25 which are detachable from each other. They are shown fastened together by simple tongues 53 and grooves 54. The use of detachable means makes it possible to rearrange the order of the elements or to omit one or more as necessary. Any sheet or part of the board removed is shown punched for transfer to the pupil's ring book 60, shown in FIG. 8.

From what has already been said, those skilled in the art will appreciate that as the facts of each new aggregate, quantity or sum is taught, a few new addition and subtraction facts are added to those which have already been taught until at last when 18 is the aggregate sum, the pupil will have learned all the addition facts about all the preceding aggregate sums 1 to 17 inclusive. The initial concrete recognition of the quantity 18 may or may not be new at this time. It is possible that before this the pupil will have learned that there is a quantity 18 by means of counting objects.

Many of the addition facts for this aggregate sum will be learned almost automatically by analogy with the addition facts for the preceding sums. Probably the learning of 18+0=18, 17+1=18, 16+2=18, 15+3=18, 14+4=18, 13+5=18, 12+6=18, 11+7=18, 10+8=18 will be relatively simple. The only really new thing in teaching the aggregate sum 18 is 9+9=18. This is the critical addition fact in connection with the aggregate sum 18 and it is the last of the simple addition facts which the child needs to learn before going on into multiple addition or simple multiplication.

It will have been noted that in the preferred embodiment of the apparatus the basic unit is always shown in the same size and shape in each aggregate sum from 1 to 18. Also any aggregate sum for each quantity 1 to 18 is always shown in the same size and shape wherever it is met, and the relation of the units is the same each time it appears.

It has been found useful in elementary presentation of the simple addition and subtraction facts to begin with the concrete and lead to the abstract as set forth above.

It will have been seen that from one point of view the object of the invention is to teach arithmetic on an abstract level with special clarification of addition and subtraction. This is important because tallying or counting—using stones, sticks, fingers or an abacus—for too long delays and blocks the majority of pupils from progressing satisfactorily in mathematics beyond its primary forms. Often the teacher heretofore has attempted to jump from counting to teaching abstract mathematical statements by rote rather than by helping the pupil to understand the relation between the concrete objects, the abstract number symbols and the abstract expressions of addition and subtraction facts.

The child should firmly grasp the relation between the concrete and abstract, and between the cardinal and ordinal meanings of number symbols, e.g. 5 as an aggregate rather than the fifth unit in counting, before going on to multiple additions, and to multiplication as a process dealing with identical aggregates.

In the novel apparatus the workspace and the separate tiles teach that the unit is a recognizable part of each aggregate and of each aggregate sum. The workspace of the aggregate sum one and the tile 44 show that one is a unit. The pupil is thus led to learn that aggregates of units are definite quantities. He also learns that the workspace represents a combination of quantities, which lead him into the subject of groupings. The concept that the one or two tiles fill the entire workspace relates the units and the aggregates to the aggregate sum, all this giving more meaning to the addition facts. The perception and meaning of the subtraction facts are taught along with the addition facts as their inverse. If the abstract number relations are learned in this manner, the idea of counting as the only meaning attached to the numbers can be counteracted if not substituted for. Including the number symbols in words or numerals in writing in the apparatus generally is sufficient to work in with the oral use of number symbols in words as given by the teacher and used by the pupil to enable the pupil to proceed from the concrete workspace and tiles to the abstract number symbols.

This invention is particularly useful with those who have comparatively little knowledge of arithmetic such, for instance, as mentally retarded children, although its usefulness for any one will be clear from the foregoing description.

While it is preferred to arrange the units as shown in the workspaces and tiles illustrated in the figures of this application. units of other shape and workspaces of other dimensions can be used.

Other embodiments of the invention will occur to those skilled in the art.

What is claimed is:

1. Educational apparatus for the teaching of elementary addition or subtraction facts about a consecutive series of cardinal numbers, comprising

(*a*) a plurality of workspaces each of a different area corresponding to a different cardinal number in said series, but all based on the same area unit,

(*b*) in combination with tiles whose areas are based on the same unit,

(*c*) there being a set of tiles for each workspace, including

(*d*) a tile of the same size and shape as said workspace and other tiles of lesser size than the workspace,

(*e*) not more than two of said lesser tiles having the same size and shape,

(*f*) and each lesser tile being of a size and shape such that when put in a grouping with one other lesser tile fills the workspace,

(*g*) in which the number of lesser tiles for each set equals not more than the cardinal number about which facts are being taught and not fewer than one less than said number;

whereby each elementary addition fact for each of said cardinal numbers can be taught by putting not more than two tiles in the workspace corresponding to said number.

2. Educational apparatus according to claim 1 in which each workspace is on a separate surface.

3. Educational apparatus according to claim 1 in which the consecutive series of cardinal numbers represented is within the numbers 1 to 18.

4. Educational apparatus according to claim 1 in which each workspace and tile is visibly marked into said area units.

5. Educational apparatus for the teaching of elementary addition or subtraction facts about a consecutive series of cardinal numbers, comprising

(*a*) a plurality of workspaces each of a different area corresponding to a different cardinal number in said series, but all based on the same area unit,

(*b*) in combination with tiles whose areas are based on the same unit, comprising

(*c*) at least two tiles the same size and shape as each workspace corresponding to a cardinal number in the lower half of the series and

(*d*) at least one tile the same size and shape as each of the other workspaces in the series,

said tiles providing, for any cardinal number in said series, a set of tiles for the workspace for said number representing each elementary addition fact for said number, said set consisting of a tile of the same size and shape as the workspace and other tiles of lesser size than said workspace, such that each lesser tile when put in a grouping with a single additional lesser tile fills the workspace, not more than two of the said lesser tiles being of the same size and shape.

6. Educational apparatus according to claim 5 in which each workspace is on a separate surface.

7. Educational apparatus for the teaching of elementary addition or subtraction facts about a cardinal number, said apparatus comprising

(*a*) a plurality of workspaces each of a different area corresponding to a different cardinal number in a consecutive series, but all based on the same area unit,

(*b*) at least one of said workspaces having its area visibly marked into area units totaling the cardinal number about which facts are being taught, in combination with

(*c*) tiles whose areas are based on the same unit, and are visibly marked into said area units,

one tile being of the same size and shape as said workspace and the other tiles of lesser size than said workspace,

not more than two of said lesser tiles having the same size and shape,

and each lesser tile being of a size and shape such that when put in a grouping with one other lesser tile fills said workspace, and

in which the total number of lesser tiles for said workspace equals not more than the cardinal number about which facts are being taught and not fewer than one less than said number,

whereby each elementary addition fact for said cardinal number can be taught by putting not more than two tiles in said workspace.

8. Educational apparatus according to claim 7

(*d*) in which there are associated with said workspace statements of the elementary addition or subtraction facts for said cardinal number, and

(*e*) a plurality of semi-concrete representations the same size and shape as the work space and visibly marked the same as the workspace and adapted to be further marked by the learner to show tile groupings or elementary mathematical facts which relate to the cardinal number,

whereby a learner can progress from a concrete to an abstract understanding of said number.

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