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(54) **Title**: PROCESS FOR CODING DATA IN IMAGE OR VIDEO AND APPARATUS THEREOF

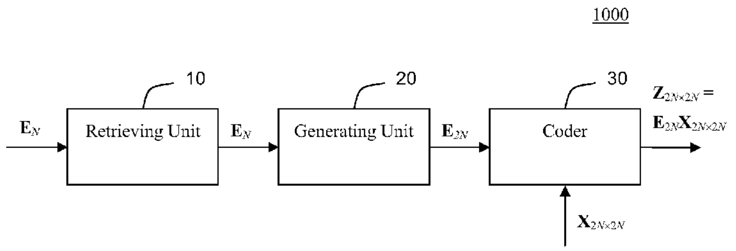

FIG.1

(57) **Abstract**: Provided are a method and apparatus for coding data in an image or video. The method may comprise a step of deriving an order-2N integer transform from an order-N integer transform for $N = 2^n$ where n is a positive integer greater than or equal to 2. The derived order-2N integer transform tends to produce small magnitude and high frequency transformed coefficients, and thus achieve high compressibility.

PROCESS FOR CODING DATA IN IMAGE OR VIDEO AND APPARATUS THEREOF

CROSS REFERENCE OF RELATED APPLICATION

【0001】 The application claims the benefit of the US application No. 61/605,478, filed on March 1, 2013, which is incorporated by reference in this entirety as part of this application.

TECHNICAL FIELD

【0002】 The present application relates to an integer transform of encoding and decoding for image or video signals, specifically, to a process for coding data in image or video and an apparatus thereof. The present application also relates to a process and a system for deriving an order-2*N* integer transform from an order-*N* integer transform in image or video coding.

BACKGROUND

【0003】 Nowadays, there are various applications for digital video communication and storage, and corresponding international standards have been and are continuing to be developed. To achieve low bit rate communications, MPEG-2, MPEG-4 Part 2, and H.263 standards divide a picture into 16 x 16 non-overlapping blocks called macro block and then use 2 dimensional (2D) order-8 discrete cosine transform (DCT) in digital video compression algorithms used in the coding standards. Newer video coding standards, such as H.264/AVC, AVS and SMPTE 421M etc, have adopted to use 2D order-4 or order-8 transforms to provide a better compressibility than the video standards based on the 2D order-8 DCT. In those standards, the 2D order-4 or order-8 transforms are used so as to seek a trade-off between computational efficiency and coding efficiency.

【0004】 Recently, in the development of standard HEVC, 16×16, 32×32 and higher order transforms has been proposed. A paper issued by *Siwei Ma* and *C.-C Jay Kuo* proposed a super-macroblock coding scheme with a high coding gain, see the paper "*High-definition Video Coding with Super-macroblocks*", SPIE-IS &T/Vol. 650816 (1-12), 2007. In this paper, Siwei Ma proposed an order-16 transform, which can be decomposed and implemented with existing order-8 transform modules. US Patent Application Publication No. 2007/0223590 filed by *Siwei Ma* discloses an apparatus for processing a $2N\times 2N$ integer transform by means of a unit that generates an $N\times N$ transform. Both the Paper and the US patent publication deal with reduction of computational complexity of the image and video coding, failure to address the issue of improvement of compression rates.

SUMMARY

【0005】 According to one aspect of the invention, there is provided a process for coding data in image or video, comprising:

retrieving a data matrix $X_{2N\times 2N}$ to be transformed; and

coding the retrieved data matrix by a rule of

$$\mathbf{Z}_{2N\times 2N} = \mathbf{E}_{2N}\mathbf{X}_{2N\times 2N}\mathbf{E}_{2N}^{T}$$

wherein $Z_{2N\times 2N}$ represents a resultant matrix through the coding, E_{2N} is an order-$2N$ integer transform and $N = 2^n$ where n is a positive integer no less than 2.

【0006】 According to the other aspect of the invention, there is provided an apparatus for coding data in image or video, comprising:

a retrieving unit for retrieving a data matrix $X_{2N\times 2N}$ to be transformed; and

a coder for coding the retrieved data matrix by a rule of

$$\mathbf{Z}_{2N\times 2N} = \mathbf{E}_{2N}\mathbf{X}_{2N\times 2N}\mathbf{E}_{2N}^{T}$$

wherein $Z_{2N\times 2N}$ represents a resultant matrix through the coding, E_{2N} is an order-$2N$ integer transform and $N = 2^n$ where n is a positive integer no less than 2.

【0007】 According to the other aspect of the invention, there is provided a process for deriving an order-2N integer transform from an order-N integer transform in image or video coding, comprising:

retrieving an order-N transform E_N; and

generating E_{2N} in response to the retrieved E_N by performing a transform of

$$\mathbf{E}_{2N} = \mathbf{P}_{2N}\mathbf{M}_{2N}\begin{bmatrix}\mathbf{E}_N & \mathbf{0}_N \\ \mathbf{0}_N & \mathbf{R}_N\end{bmatrix}\begin{bmatrix}\mathbf{I}_N & \mathbf{J}_N \\ \mathbf{I}_N & -\mathbf{J}_N\end{bmatrix} \text{ or}$$

$$E_{2N} = P_{2N}\begin{bmatrix}E_N & 0_N \\ 0_N & M_{2No}R_N\end{bmatrix}\begin{bmatrix}\mathbf{I}_N & \mathbf{J}_N \\ \mathbf{I}_N & -\mathbf{J}_N\end{bmatrix}.$$

where

$\mathbf{P}_{2N}$ is a permutation matrix,

M_{2N} is a diagonal matrix with integer elements to make every basis norms in E_{2N} similar,

I_N is an order-N identity matrix,

J_N is a mirrored version of I_N, and

R_N is a recursive matrix from an integer matrix $\mathbf{R}_{\frac{N}{2}}$.

【0008】 According to the other aspect of the invention, there is provided a system for deriving an order-2N integer transform from an order-N integer transform in image and video coding, comprising:

means for retrieving an order-N transform E_N; and

means for generating E_{2N} in response the retrieved E_N by performing a transform of

$$\mathbf{E}_{2N} = \mathbf{P}_{2N}\mathbf{M}_{2N}\begin{bmatrix}\mathbf{E}_N & \mathbf{0}_N \\ \mathbf{0}_N & \mathbf{R}_N\end{bmatrix}\begin{bmatrix}\mathbf{I}_N & \mathbf{J}_N \\ \mathbf{I}_N & -\mathbf{J}_N\end{bmatrix} \text{ or}$$

$$E_{2N} = P_{2N}\begin{bmatrix}E_N & 0_N \\ 0_N & M_{2No}R_N\end{bmatrix}\begin{bmatrix}\mathbf{I}_N & \mathbf{J}_N \\ \mathbf{I}_N & -\mathbf{J}_N\end{bmatrix}.$$

where

$\mathbf{P}_{2N}$ is a permutation matrix,

M_{2N} is a diagonal matrix with integer elements to make every basis norms in E_{2N} similar,

I_N is an order-N identity matrix,

J_N is a mirrored version of I_N, and

R_N is a recursive matrix from an integer matrix $\mathbf{R}_{\frac{N}{2}}$

【0009】 The detailed technology and preferred embodiments implemented for the subject invention are described in the following paragraphs accompanying the appended drawings for those skilled in this field to well appreciate the features of the claimed invention.

BRIEF DESCRIPTION OF THE DRAWINGS

【0010】 *FIG.* 1 is a block diagram of an apparatus for coding data in image and video according to one embodiment of the present application.

【0011】 *FIG.* 2 illustrates exemplarily a method for coding data in image and video according to one embodiment of the present application.

DETAILED DESCRIPTION

【0012】 Embodiments of the present application will be described with reference to the accompanying drawings, but these drawings are presented only for the illustrative purpose and in no respect, are intended to limit the present invention anymore.

【0013】 Referring now to *FIG.* 1, an apparatus 1000 for coding data in image and video according to one embodiment of the present application is illustrated. As shown, the apparatus 1000 includes a retrieving unit 10, a generating unit 20 and a coder 30.

【0014】 The retrieving unit 10 is utilized to retrieve an order-N transform E_N, which may be stored in a memory (not shown) in the apparatus 1000 or any storage means external to the apparatus 1000 so long as the apparatus 1000 may obtain it if necessary. According to one embodiment, the order-N transform may be input into the retrieving unit 10, for example, through a key board or the like. According to one embodiment, E_N may be an order-N integer transform in image or video coding in the prior art. For example, E_N may be an order-8 transform E_{AVC} adopted in H.264/AVC, or an order-8 transform E_{AVS} adopted in AVS. It should be understood that those skilled in the art could select other E_N if required based on the disclosures of the application.

【0015】 The generating unit 20 unitizes the retrieved order-N transform E_N from the unit 10 to generate an order-2N transform E_{2N} by computing the transform E_N, which will be detailed later.

【0016】 The coder 30 has two inputs, E_{2N} from the generating unit 20 and a data matrix $X_{2N\times 2N}$ to be transformed. The coder 30 operates to code the data matrix $X_{2N\times 2N}$ into a data matrix $Z_{2N\times 2N}$ by a rule of

$$\mathbf{Z}_{2N\times 2N} = \mathbf{E}_{2N}\mathbf{X}_{2N\times 2N}\mathbf{E}_{2N}^{T}. \qquad 1)$$

【0017】 According to one embodiment of the application, the coder 30 may carry out a 2D transform through two cascading 1D transforms to obtain $Z_{2N\times 2N}$ by a rule of

$$\begin{aligned}
\vec{U}_{2N,k} &= \mathbf{E}_{2N}\vec{X}_{2N,k} \\
\mathbf{V}_{2N\times 2N} &= \left[\vec{U}_{2N,0} \quad \vec{U}_{2N,1} \quad \cdots \quad \vec{U}_{2N,2N-1}\right] \\
\vec{Z}_{2N,k} &= \mathbf{E}_{2N}\vec{V}_{2N,k}^{T} \\
\mathbf{Z}_{2N\times 2N} &= \left[\vec{Z}_{2N,0}^{T}; \quad \vec{Z}_{2N,1}^{T}; \quad \cdots \quad \vec{Z}_{2N,2N-1}^{T}\right]
\end{aligned} \qquad 2)$$

where

$\vec{X}_{2N,k}$ is the k-th column vector in $X_{2N\times 2N}$

$V_{2N\times 2N}$ is a $2N\times 2N$ matrix built by $\vec{U}_{2N,k}$ for $k \in \{0,1,\ldots,2N-1\}$

$\vec{V}_{2N,k}$ is the k-th row vector in $V_{2N\times 2N}$ for $k \in \{0,1,\ldots,2N-1\}$.

【0018】 Hereinafter, a process for generating an order-2N transform E_{2N} from an order-N transform E_N according to the embodiment of the application will be discussed.

【0019】 E_{2N} can be constructed by the generating unit 20 by

$$\mathbf{E}_{2N} = \mathbf{P}_{2N}\mathbf{M}_{2N}\begin{bmatrix}\mathbf{E}_N & \mathbf{0}_N \\ \mathbf{0}_N & \mathbf{R}_N\end{bmatrix}\begin{bmatrix}\mathbf{I}_N & \mathbf{J}_N \\ \mathbf{I}_N & -\mathbf{J}_N\end{bmatrix} \qquad 3)$$

wherein

E_N is an order-N transform,

$\mathbf{P}_{2N}$ is a permutation matrix,

M_{2N} is a diagonal matrix with integer elements to make every basis norms in E_{2N} similar,

I_N is an order-N identity matrix,

J_N is a mirrored version of I_N, and

R_N is a recursive matrix from an integer matrix $\mathbf{R}_{\frac{N}{2}}$.

【0020】 In this embodiment, R_N may be constructed by a rule of

$$R_N = \begin{bmatrix} a_{N,1} & & & & & & & a_{N,2N-1} \\ & a_{N,3} & & & & & a_{N,2N-3} & \\ & & \ddots & & & \iddots & & \\ & & & a_{N,N-1} & a_{N,N+1} & & & \\ & & & -a_{N,N+1} & a_{N,N-1} & & & \\ & & \iddots & & & \ddots & & \\ & -a_{N,2N-3} & & & & & a_{N,3} & \\ -a_{N,2N-1} & & & & & & & a_{N,1} \end{bmatrix} \times \begin{bmatrix} \mathbf{R}_{\frac{N}{2}} & \mathbf{0}_{\frac{N}{2}} \\ \mathbf{0}_{\frac{N}{2}} & \mathbf{R}_{\frac{N}{2}} \end{bmatrix} \begin{bmatrix} 1 & 1 & & & & & \\ & & 1 & 1 & & & \\ & & & & \ddots & & \\ & & & & & 1 & 1 \\ 1 & -1 & & & & & \\ & & -1 & 1 & & & \\ & & & & \ddots & & \\ & & & & & -1 & 1 \end{bmatrix} \qquad 4)$$

wherein $a_{N,1}$, $a_{N,3}$, $\ldots a_{N,2N-1}$ are positive integers.

【0021】 According to one embodiment, R_N may be constructed by the generating unit 20 or may be retrieved from the memory internal or external to the apparatus 1000.

【0022】 To ensure a good coding performance, $a_{N,k}$ are selected by a rule of

$$\frac{a_{N,k}}{a_{N,2N-k}} \approx \frac{\cos\left(\frac{k\pi}{4N}\right)}{\cos\left(\frac{(2N-k)\pi}{4N}\right)} \text{ for } k = 1,3,\ldots,N-1 \qquad 5)$$

【0023】 A list of $a_{N,k}$ in co-prime pairs are listed in Table 1, Table 2A-B, Table 3A-D and Table 4A-H for different N. In one embodiment, Table 1, Table 2A-B, Table 3A-D and Table

4A-H may be stored in a memory (not shown) in the apparatus 1000 or any storage means external to the apparatus 1000 so long as the apparatus 1000 may obtain it when necessary. ($a_{N,k}$, $a_{N,2N-k}$) can be selected as a pair in any column in these tables. Larger value of $a_{N,k}$ usually means that the resultant E_{2N} has a higher coding performance. But this also means that the transform process requires more computation and the resultant transform coefficients have larger dynamic ranges.

Table 1

$N = 2$										
$a_{2,1}$	2	5	7	8	9	11	11	12	13	14
$a_{2,3}$	1	2	3	3	4	4	5	5	5	5

Table 2A

$N = 4$										
$a_{4,1}$	5	6	9	11	13	14	16	17	17	19
$a_{4,7}$	1	1	2	2	3	3	3	3	4	4

Table 2B

$N = 4$										
$a_{4,3}$	1	3	4	5	5	7	6	7	8	9
$a_{4,5}$	1	2	3	3	4	4	5	5	5	7

Table 3A

$N = 8$										
$a_{8,1}$	9	10	11	12	17	19	21	23	25	26
$a_{8,15}$	1	1	1	1	2	2	2	2	3	3

Table 3B

$N = 8$												
$a_{8,3}$	3	7	8	10	11	11	13	15	14	16	17	18
$a_{8,13}$	1	2	3	3	3	4	4	4	5	5	5	5

Table 3C

$N = 8$												
$a_{8,5}$	2	3	5	7	8	9	11	11	13	11	12	13

$a_{8,11}$	1	2	3	4	5	5	5	6	6	7	7	7

Table 3D

$N = 8$																
$a_{8,7}$	1	4	5	6	7	7	8	9	10	9	11	10	11	13	11	13
$a_{8,9}$	1	3	4	5	5	6	7	7	7	8	8	9	9	9	10	10

Table 4A

$N = 16$																
$a_{16,1}$	17	18	19	20	21	22	23	24	33	35	37	39	41	43	45	47
$a_{16,31}$	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2

Table 4B

$N = 16$															
$a_{16,3}$	6	7	8	11	13	15	17	19	20	22	23	23	25	27	29
$a_{16,29}$	1	1	1	2	2	2	3	3	3	3	3	4	4	4	4

Table 4C

$N = 16$																
$a_{16,5}$	4	7	9	10	11	13	14	13	15	17	19	16	17	18	19	21
$a_{16,27}$	1	2	2	3	3	3	3	4	4	4	4	5	5	5	5	5

Table 4D

$N = 16$																
$a_{16,7}$	3	5	7	8	10	9	11	13	12	13	14	16	17	19	16	17
$a_{16,25}$	1	2	3	3	3	4	4	4	5	5	5	5	6	6	7	7

Table 4E

$N = 16$																
$a_{16,9}$	2	5	7	7	9	9	11	12	11	13	12	13	15	17	15	17
$a_{16,23}$	1	2	3	4	4	5	5	5	6	6	7	7	7	7	8	8

Table 4F

$N = 16$																	
$a_{16,11}$	2	3	5	7	7	9	11	10	11	12	13	11	13	15	13	14	16
$a_{16,21}$	1	2	3	4	5	5	6	7	7	7	7	8	8	8	9	9	9

Table 4G

$N = 16$															
$a_{16,13}$	3	4	5	6	7	8	7	8	9	10	11	9	11	10	11
$a_{16,19}$	2	3	4	5	5	5	6	7	7	7	7	8	8	9	9

Table 4H

$N = 16$															
$a_{16,15}$	1	5	6	7	8	9	9	10	11	11	13	12	13	14	13
$a_{16,17}$	1	4	5	6	7	7	8	9	9	10	10	11	11	11	12

【0024】 The proposed transform E_{2N} can be implemented with additions, subtractions and multiplications with small integers only.

【0025】 Hereinafter, a process 2000 for coding data in image or video according to one embodiment of the present application will be discussed.

【0026】 At step S101, a data matrix $X_{2N\times 2N}$ to be transformed is retrieved. As mentioned in the above, according to one embodiment, E_N may be an order-N integer transform in image and video coding in the prior art. For example, E_N may be an order-8 transform E_{AVC} adopted in H.264/AVC, or an order-8 transform E_{AVS} adopted in AVS. It should be also understood that those skilled in the art could select other E_N if required based on the disclosures of the application.

【0027】 At step S102, an order-N transform E_N is retrieved and then E_{2N} *is* generated from the retrieved E_N by performing a transform according to the above mentioned equations 3)~5). And then at step S103, the retrieved data matrix is coded according to the above mentioned equation 1). As discussed in the above, the coding may further comprise a 2D transform carried out by two cascading 1D transforms according to a rule as set in equation 2).

【0028】 According to one embodiment, the process 2000 may be implemented by one or more processor of a computer using instructions encoded in computer readable storage medium. Alternatively, the steps S101~103 may be respectively implemented by the retrieving unit 10, the generating unit 20 and the coder 30 as discussed in the above. It shall be also understood that each of units 10-30 may include electronic circuits to carry out the corresponding functions or utilize the programs stored therein to carry out the corresponding functions.

【0029】 Hereinafter, some examples according to the embodiment of the application will be described. These examples are present only for the illustrative purpose and in no respect, are intended to limit the present invention anymore.

<u>Example 1: Order-4 Transform</u>

【0030】 Herein, starting from $N = 2$, E_2 and R_2 are built by rules

$$\mathbf{E}_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \text{ ; and}$$

$$\mathbf{R}_2 = \begin{bmatrix} a_{2,1} & a_{2,3} \\ -a_{2,3} & a_{2,1} \end{bmatrix}$$

$a_{2,1}$ and $a_{2,3}$ are taken as 5 and 2 respectively.

【0031】 As a result, E_4 is built as

$$\begin{aligned}
\mathbf{E}_4 &= \mathbf{P}_4\mathbf{M}_4 \begin{bmatrix} \mathbf{E}_2 & \mathbf{0}_2 \\ \mathbf{0}_2 & \mathbf{R}_2 \end{bmatrix} \begin{bmatrix} \mathbf{I}_2 & \mathbf{J}_2 \\ \mathbf{I}_2 & -\mathbf{J}_2 \end{bmatrix} \\
&= \begin{bmatrix} 1 & & & \\ & & 1 & \\ & 1 & & \\ & & & 1 \end{bmatrix} \begin{bmatrix} 4 & & & \\ & 4 & & \\ & & 1 & \\ & & & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & & \\ 1 & -1 & & \\ & & 5 & 2 \\ & & -2 & 5 \end{bmatrix} \begin{bmatrix} 1 & & & 1 \\ & 1 & 1 & \\ 1 & & & -1 \\ & 1 & -1 & \end{bmatrix} \\
&= \begin{bmatrix} 4 & 4 & 4 & 4 \\ 5 & -2 & -2 & 5 \\ 4 & -4 & -4 & 4 \\ 2 & -5 & 5 & -2 \end{bmatrix}
\end{aligned}$$

<u>Example 2: Order-8 Transform</u>

【0032】 Take $a_{4,1}$, $a_{4,3}$, $a_{4,5}$ and $a_{4,7}$ as 5, 3, 2 and 1, respectively, R_4 is built as

$$\mathbf{R}_4 = \begin{bmatrix} a_{4,1} & & & a_{4,7} \\ & a_{4,3} & a_{4,5} & \\ & -a_{4,5} & a_{4,3} & \\ -a_{4,7} & & & a_{4,1} \end{bmatrix} \begin{bmatrix} \mathbf{R}_2 & \mathbf{0}_2 \\ \mathbf{0}_2 & \mathbf{R}_2 \end{bmatrix} \begin{bmatrix} 1 & 1 & & \\ & & 1 & 1 \\ 1 & -1 & & \\ & & -1 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 27 & 23 & 15 & 5 \\ 16 & -4 & -19 & -11 \\ 11 & -19 & 4 & 16 \\ 5 & -15 & 23 & -27 \end{bmatrix}$$

【0033】 And then E_8 is built as

$$\mathbf{E}_8 = \mathbf{P}_8\mathbf{M}_8 \begin{bmatrix} \mathbf{E}_4 & \mathbf{0}_4 \\ \mathbf{0}_4 & \mathbf{R}_4 \end{bmatrix} \begin{bmatrix} \mathbf{I}_4 & \mathbf{J}_4 \\ \mathbf{I}_4 & -\mathbf{J}_4 \end{bmatrix}$$
$$= \begin{bmatrix} 22 & 22 & 22 & 22 & 22 & 22 & 22 & 22 \\ 27 & 23 & 15 & 5 & -5 & -15 & -23 & -27 \\ 25 & 10 & -10 & -25 & -25 & -10 & 10 & 25 \\ 16 & -4 & -19 & -11 & 11 & 19 & 4 & -16 \\ 22 & -22 & -22 & 22 & 22 & -22 & -22 & 22 \\ 11 & -19 & 4 & 16 & -16 & -4 & 19 & -11 \\ 10 & -25 & 25 & -10 & -10 & 25 & -25 & 10 \\ 5 & -15 & 23 & -27 & 27 & -23 & 15 & -5 \end{bmatrix}$$

wherein

$$\mathbf{P}_8 = \begin{bmatrix} 1 & & & & & & & \\ & & & & 1 & & & \\ & 1 & & & & & & \\ & & & & & 1 & & \\ & & 1 & & & & & \\ & & & & & & 1 & \\ & & & 1 & & & & \\ & & & & & & & 1 \end{bmatrix} \text{and } \mathbf{M}_8 = \begin{bmatrix} 22 & & & & & & & \\ & 22 & & & & & & \\ & & 5 & & & & & \\ & & & 5 & & & & \\ & & & & 1 & & & \\ & & & & & 1 & & \\ & & & & & & 1 & \\ & & & & & & & 1 \end{bmatrix}$$

<u>Example 3: Order-16 Transform</u>

【0034】 Take $a_{8,1}$, $a_{8,3}$, $a_{8,5}$, $a_{8,7}$ $a_{8,9}$, $a_{8,11}$, $a_{8,13}$ and $a_{8,15}$ as 10, 17, 15, 10, 8, 8, 5 and 1 respectively. Accordingly, E_8 will be built as

$$\mathbf{E}_{16} = \mathbf{P}_{16}\mathbf{M}_{16}\begin{bmatrix}\mathbf{E}_8 & \mathbf{0}_8 \\ \mathbf{0}_8 & \mathbf{R}_8\end{bmatrix}\begin{bmatrix}\mathbf{I}_8 & \mathbf{J}_8 \\ \mathbf{I}_8 & -\mathbf{J}_8\end{bmatrix}$$

$$= \begin{bmatrix}
256 & 256 & 256 & 256 & 256 & 256 & 256 & 256 & 256 & \cdots & 256 & 256 \\
275 & 265 & 245 & 215 & 173 & 127 & 77 & 23 & -23 & \cdots & -265 & -275 \\
324 & 276 & 180 & 60 & -16 & -180 & -276 & -324 & -324 & \cdots & 276 & 324 \\
327 & 217 & 27 & -163 & -303 & -343 & -267 & -107 & 107 & \cdots & -217 & -327 \\
320 & 128 & -128 & -320 & -320 & -128 & 128 & 320 & 320 & \cdots & 128 & 320 \\
293 & 37 & -253 & -317 & -92 & 212 & 328 & 152 & -152 & \cdots & -37 & -293 \\
288 & -72 & -342 & -198 & 198 & 342 & 72 & -288 & -288 & \cdots & -72 & 288 \\
266 & -166 & -334 & 34 & 350 & 110 & -310 & -230 & 230 & \cdots & 166 & -266 \\
256 & -256 & -256 & 256 & 256 & -256 & -256 & 256 & 256 & \cdots & -256 & 256 \\
230 & -310 & -110 & 350 & -34 & -334 & 166 & 266 & -266 & \cdots & 310 & -230 \\
198 & -342 & 72 & 288 & -288 & -72 & 342 & -198 & -198 & \cdots & -342 & 198 \\
152 & -328 & 212 & 92 & -317 & 253 & 37 & -293 & 293 & \cdots & 328 & -152 \\
128 & -320 & 320 & -128 & -128 & 320 & -320 & 128 & 128 & \cdots & -320 & 128 \\
107 & -267 & 343 & -303 & 163 & 27 & -217 & 327 & -327 & \cdots & 267 & -107 \\
60 & -180 & 276 & -324 & 324 & -276 & 180 & -60 & -60 & \cdots & -180 & 60 \\
23 & -77 & 127 & -173 & 215 & -245 & 265 & -275 & 275 & \cdots & 77 & -23
\end{bmatrix}$$

<u>Example 4 : Order-32 Transform</u>

【0035】 Take $a_{16,1}$, $a_{16,3}$, ... $a_{16,31}$ as 20, 13, 16, 14, 13, 13, 12, 15, 14, 9, 8, 6, 5, 4, 2 and 1 respectively. Accordingly, E_8 is built as

$$\mathbf{E}_{32} = \mathbf{P}_{32}\mathbf{M}_{32}\begin{bmatrix}\mathbf{E}_{16} & \mathbf{0}_{16} \\ \mathbf{0}_{16} & \mathbf{R}_{16}\end{bmatrix}\begin{bmatrix}\mathbf{I}_{16} & \mathbf{J}_{16} \\ \mathbf{I}_{16} & -\mathbf{J}_{16}\end{bmatrix}$$

$$= \left[\begin{array}{cccccccccccc}
4096 & 4096 & 4096 & 4096 & 4096 & 4096 & 4096 & 4096 & 4096 & \cdots & 4096 \\
5523 & 5477 & 5377 & 5223 & 5027 & 4773 & 4473 & 4127 & 3675 & \cdots & 5523 \\
5500 & 5300 & 4900 & 4300 & 3460 & 2540 & 1540 & 460 & -460 & \cdots & 5500 \\
4465 & 4037 & 3355 & 2287 & 1037 & -335 & -1513 & -2725 & -3613 & \cdots & 4465 \\
5616 & 4784 & 3120 & 1040 & -1040 & -3120 & -4784 & -5616 & -5616 & \cdots & 5616 \\
5296 & 4080 & 1904 & -720 & -3200 & -4896 & -5440 & -4704 & -2740 & \cdots & 5296 \\
\vdots & & & & & & & & & & \vdots \\
4195 & -3505 & -4865 & 2555 & 5335 & -1525 & 5065 & 415 & 5647 & \cdots & 4195 \\
\hdashline
4096 & -4096 & -4096 & 4096 & 4096 & -4096 & -4096 & 4096 & 4096 & \cdots & 4096 \\
\vdots & & & & & & & & & & \vdots \\
1712 & -4272 & 5488 & -4848 & 2608 & 432 & -3472 & 5232 & -5232 & \cdots & 1712 \\
1260 & -3604 & 5100 & -5396 & 4404 & -2380 & -204 & 2740 & -4704 & \cdots & 1260 \\
1040 & -3120 & 4784 & -5616 & 5616 & -4784 & 3120 & -1040 & -1040 & \cdots & 1040 \\
737 & -2045 & 3037 & -3905 & 4405 & -4513 & 4265 & -3613 & 2725 & \cdots & 737 \\
460 & -1540 & 2540 & -3460 & 4300 & -4900 & 5300 & -5500 & 5500 & \cdots & 460 \\
185 & -735 & 1275 & -1805 & 2295 & -2785 & 3245 & -3675 & 4127 & \cdots & 185
\end{array}\right]$$

【0036】 Alternatively, we may merge the diagonal matrix M_N into the integer transform kernel.

<u>Example 5: Another Order-4 Transform</u>

【0037】 E_4 is built as

$$
\begin{aligned}
\mathbf{E}_4 &= \mathbf{P}_4\mathbf{M}_4\begin{bmatrix}\mathbf{E}_2 & \mathbf{0}_2\\ \mathbf{0}_2 & \mathbf{R}_2\end{bmatrix}\begin{bmatrix}\mathbf{I}_2 & \mathbf{J}_2\\ \mathbf{I}_2 & -\mathbf{J}_2\end{bmatrix}\\
&= \begin{bmatrix}1 & & & \\ & & 1 & \\ & 1 & & \\ & & & 1\end{bmatrix}\begin{bmatrix}b_{1,1} & b_{1,1} & & \\ b_{1,1} & -b_{1,1} & & \\ & & b_{2,1} & b_{2,3}\\ & & -b_{2,3} & b_{2,1}\end{bmatrix}\begin{bmatrix}1 & & & 1\\ & 1 & 1 & \\ 1 & & & -1\\ & 1 & -1 & \end{bmatrix}\\
&= \begin{bmatrix}b_{1,1} & b_{1,1} & b_{1,1} & b_{1,1}\\ b_{2,1} & b_{2,3} & -b_{2,3} & -b_{2,1}\\ b_{1,1} & -b_{1,1} & -b_{1,1} & b_{1,1}\\ b_{2,3} & -b_{2,1} & b_{2,1} & -b_{2,3}\end{bmatrix}.
\end{aligned}
$$

<u>Example 6: Another Order-8 Transform</u>

【0038】 E_8 is built as:

$$
\begin{aligned}
E_8 &= P_8 M_8\begin{bmatrix}E_4 & 0_4\\ 0_4 & R_4\end{bmatrix}\begin{bmatrix}\mathbf{I}_4 & \mathbf{J}_4\\ \mathbf{I}_4 & -\mathbf{J}_4\end{bmatrix}\\
&= P_8\begin{bmatrix}E_4 & 0_4\\ 0_4 & M_{8o}R_4\end{bmatrix}\begin{bmatrix}\mathbf{I}_4 & \mathbf{J}_4\\ \mathbf{I}_4 & -\mathbf{J}_4\end{bmatrix}.
\end{aligned}
$$

【0039】 $M_{8o}R_4$ is built as:

$$
\begin{aligned}
M_{8o}R_4 &= \begin{bmatrix}m_0 & & & \\ & m_1 & & \\ & & m_1 & \\ & & & m_0\end{bmatrix}\begin{bmatrix}a_{4,1} & & & a_{4,7}\\ & a_{4,3} & a_{4,5} & \\ & -a_{4,5} & a_{4,3} & \\ -a_{4,7} & & & a_{4,1}\end{bmatrix}\begin{bmatrix}\mathbf{R}_2 & \mathbf{0}_2\\ \mathbf{0}_2 & \mathbf{R}_2\end{bmatrix}\begin{bmatrix}1 & 1 & & \\ & & 1 & 1\\ 1 & -1 & & \\ & & -1 & 1\end{bmatrix}\\
&= \begin{bmatrix}b_{4,1} & & & b_{4,7}\\ & b_{4,3} & b_{4,5} & \\ & -b_{4,5} & b_{4,3} & \\ -b_{4,7} & & & b_{4,1}\end{bmatrix}\begin{bmatrix}\mathbf{R}_2 & \mathbf{0}_2\\ \mathbf{0}_2 & \mathbf{R}_2\end{bmatrix}\begin{bmatrix}1 & 1 & & \\ & & 1 & 1\\ 1 & -1 & & \\ & & -1 & 1\end{bmatrix}
\end{aligned}
$$

【0040】 Recursively, E_{2N} is built as:

$$
E_{2N} = P_{2N}\begin{bmatrix}E_N & 0_N\\ 0_N & M_{2No}R_N\end{bmatrix}\begin{bmatrix}\mathbf{I}_N & \mathbf{J}_N\\ \mathbf{I}_N & -\mathbf{J}_N\end{bmatrix}.
$$

【0041】 $M_{(2N)o}R_N$ is built as:

$$
M_{(2N)o}R_N = \begin{bmatrix}b_{N,1} & & & & & b_{N,2N-1}\\ & \ddots & & & \iddots & \\ & & b_{N,N-1} & b_{N,N+1} & & \\ & & -b_{N,N+1} & b_{N,N-1} & & \\ & \iddots & & & \ddots & \\ -b_{N,2N-1} & & & & & b_{N,1}\end{bmatrix}\begin{bmatrix}\mathbf{R}_{\frac{N}{2}} & \mathbf{0}_{\frac{N}{2}}\\ \mathbf{0}_{\frac{N}{2}} & \mathbf{R}_{\frac{N}{2}}\end{bmatrix}\left[\begin{array}{cccc}1 & 1 & & \\ & & \ddots & \\ & & 1 & 1\\ \hdashline 1 & -1 & & \\ & & \ddots & \\ & & -1 & 1\end{array}\right].
$$

E_{2N} is orthogonal (up to scale) for any real $b_{N,i}$.

【0042】 The values of $b_{N,i}$ should be chosen: (1) to maximize the transform coding gain, and (2) to minimize the differences among the basis vector norms of the transform itself.

【0043】 According to the above requirements (1) and (2), $b_{N,i}$ are integers that satisfy:

$$B_{N,i}^{\min} \le b_{N,i}^2 + b_{N,2N-i}^2 \le B_{N,i}^{\max} \text{ and } R_{N,i}^{\min} \le {b_{N,i}}\big/{b_{N,2N-i}} \le R_{N,i}^{\max} \text{ for } N \ge 2 \text{ and } b_{1,1} \le B.$$

where B is a matrix consisting of $b_{N,i}$,

$B_{N,i}^{\min}$ is a minimal value of the sum of $b^2_{N,i}$ and $b^2_{N,2N-i}$,

$B_{N,1}^{\max}$ is a maximal value of the sum of $b^2_{N,i}$ and $b^2_{N,2N-i}$,

$R_{N,i}^{\min}$ is a minimal ratio between $b_{N,i}$ and $b_{N,2N-i}$,

$R_{N,i}^{\max}$ is a maximal ratio between $b_{N,i}$ and $b_{N,2N-i}$.

【0044】 According to the above claim, when N=2:

B	$B_{2,1}^{\min}$	$B_{2,1}^{\max}$	$R_{2,1}^{\min}$	$R_{2,1}^{\max}$
8	122	134	1.0000	13.000
16	488	533	1.0000	23.000
32	1949	2146	1.0000	46.000
64	7785	8597	1.0000	92.000
128	31138	34394	1.0000	185.00
256	124525	137605	1.0000	370.00
512	498074	550493	1.0000	741.00
1024	1992296	2202001	1.0000	1483.0
2048	7969184	8808029	1.0000	2967.0
4096	31876721	35232149	1.0000	5935.0

【0045】 when N=4 and $(a_{2,1}, a_{2,3}) = (2, 1)$

B	$B_{4,1}^{\min}$	$B_{4,1}^{\max}$	$R_{4,1}^{\min}$	$R_{4,1}^{\max}$

16	101	104	5.0000	10.000
32	397	425	2.3750	20.000
64	1557	1717	2.3750	41.000
128	6228	6868	2.3750	82.000
256	24905	27521	2.3593	165.00
512	99620	110097	2.3593	331.00
1024	398480	440397	2.3593	663.00
2048	1593841	1761605	2.3592	1327.0
4096	6375346	7046425	2.3591	2654.0

B	$B_{4,3}^{\min}$	$B_{4,3}^{\max}$	$R_{4,3}^{\min}$	$R_{4,3}^{\max}$
16	98	106	1.0000	10.000
32	392	425	1.0000	20.000
64	1557	1717	1.0000	41.000
128	6228	6877	1.0000	82.000
256	24905	27521	1.0000	165.00
512	99620	110098	1.0000	331.00
1024	398465	440397	1.0000	663.00
2048	1593841	1761605	1.0000	1327.0
4096	6375346	7046426	1.0000	2654.0

【0046】 when N=4 and $(a_{2,1}, a_{2,3}) = (5, 2)$

B	$B_{4,1}^{\min}$	$B_{4,1}^{\max}$	$R_{4,1}^{\min}$	$R_{4,1}^{\max}$
32	68	73	2.6666	4.0000
64	272	293	2.6666	17.000
128	1082	1181	2.3846	34.000
256	4297	4745	2.2962	68.000
512	17177	18980	2.2909	137.00
1024	68708	75925	2.2870	275.00
2048	274801	303725	2.2857	551.00
4096	1099201	1214901	2.2857	1102.0

B	$B_{4,3}^{\min}$	$B_{4,3}^{\max}$	$R_{4,3}^{\min}$	$R_{4,3}^{\max}$
32	72	74	1.0000	4.0000
64	269	296	1.0000	17.000
128	1076	1186	1.0000	34.000

256	4297	4745	1.0000	68.000
512	17177	18980	1.0000	137.00
1024	68708	75925	1.0000	275.00
2048	274801	303725	1.0000	551.00
4096	1099201	1214901	1.0000	1102.0

【0047】 when N=4 and $(a_{2,1}, a_{2,3}) = (7, 3)$

B	$B_{4,1}^{\min}$	$B_{4,1}^{\max}$	$R_{4,1}^{\min}$	$R_{4,1}^{\max}$
32	34	37	3.0000	6.0000
64	274	296	2.7500	12.000
128	1076	1186	2.4444	24.000
256	4304	4744	2.3157	48.000
512	17186	18980	2.2894	97.000
1024	68708	75920	2.2894	194.00
2048	274810	303722	2.2884	389.00
4096	1099204	1214900	2.2884	779.00

B	$B_{4,3}^{\min}$	$B_{4,3}^{\max}$	$R_{4,3}^{\min}$	$R_{4,3}^{\max}$
32	68	74	1.6666	6.0000
64	272	296	1.1250	12.000
128	1076	1186	1.0000	24.000
256	4304	4744	1.0000	48.000
512	17186	18980	1.0000	97.000
1024	68708	75920	1.0000	194.00
2048	274810	303722	1.0000	389.00
4096	1099204	1214900	1.0000	779.00

【0048】 when N=4 and $(a_{2,1}, a_{2,3}) = (12, 5)$

B	$B_{4,1}^{\min}$	$B_{4,1}^{\max}$	$R_{4,1}^{\min}$	$R_{4,1}^{\max}$
64	46	50	3.5000	7.0000
128	185	200	2.6000	14.000
256	738	810	2.3636	28.000
512	2952	3257	2.3181	57.000
1024	11801	13028	2.2888	114.00
2048	47162	52112	2.2857	228.00

4096	188621	208469	2.2857	456.00

B	$B_{4,3}^{\min}$	$B_{4,3}^{\max}$	$R_{4,3}^{\min}$	$R_{4,3}^{\max}$
64	46	52	1.0000	7.0000
128	185	202	1.0000	14.000
256	738	810	1.0000	28.000
512	2952	3257	1.0000	57.000
1024	11789	13028	1.0000	114.00
2048	47156	52112	1.0000	228.00
4096	188621	208469	1.0000	456.00

【0049】 when N=8 and $(a_{2,1}, a_{2,3}) = (2, 1)$ and $(a_{4,1}, a_{4,3}, a_{4,5}, a_{4,7}) = (5, 3, 2, 1)$

B	$B_{8,1}^{\min}$	$B_{8,1}^{\max}$	$R_{8,1}^{\min}$	$R_{8,1}^{\max}$
64	61	65	4.0000	8.0000
128	241	260	3.7500	16.000
256	962	1049	3.7500	32.000
512	3842	4234	3.5294	65.000
1024	15354	16936	3.5142	130.00
2048	61321	67745	3.5142	260.00
4096	245218	271010	3.5140	520.00

B	$B_{8,3}^{\min}$	$B_{8,3}^{\max}$	$R_{8,3}^{\min}$	$R_{8,3}^{\max}$
64	122	130	1.0000	11.000
128	481	522	1.0000	22.000
256	1921	2117	1.0000	46.000
512	7666	8469	1.0000	92.000
1024	30658	33876	1.0000	184.00
2048	122609	135505	1.0000	368.00
4096	490417	542026	1.0000	736.00

B	$B_{8,5}^{\min}$	$B_{8,5}^{\max}$	$R_{8,5}^{\min}$	$R_{8,5}^{\max}$
64	122	130	1.0000	11.000
128	481	522	1.0000	22.000
256	1921	2117	1.0000	46.000
512	7666	8469	1.0000	92.000

1024	30658	33876	1.0000	184.00
2048	122609	135505	1.0000	368.00
4096	490417	542026	1.0000	736.00

B	$B_{8,7}^{\min}$	$B_{8,7}^{\max}$	$R_{8,7}^{\min}$	$R_{8,7}^{\max}$
64	61	65	1.2000	8.0000
128	241	261	1.0000	16.000
256	962	1058	1.0000	32.000
512	3833	4234	1.0000	65.000
1024	15329	16938	1.0000	130.00
2048	61309	67752	1.0000	260.00
4096	245209	271013	1.0000	520.00

【0050】 when N=8 and $(a_{2,1}, a_{2,3}) = (5, 2)$ and $(a_{4,1}, a_{4,3}, a_{4,5}, a_{4,7}) = (5, 3, 2, 1)$

B	$B_{8,1}^{\min}$	$B_{8,1}^{\max}$	$R_{8,1}^{\min}$	$R_{8,1}^{\max}$
256	170	178	4.3333	13.000
512	661	730	3.5714	27.000
1024	2644	2920	3.3333	54.000
2048	10573	11680	3.2903	108.00
4096	42281	46720	3.2622	216.00

B	$B_{8,3}^{\min}$	$B_{8,3}^{\max}$	$R_{8,3}^{\min}$	$R_{8,3}^{\max}$
256	333	365	1.0000	19.000
512	1322	1460	1.0000	38.000
1024	5288	5840	1.0000	76.000
2048	21141	23362	1.0000	152.00
4096	84562	93448	1.0000	305.00

B	$B_{8,5}^{\min}$	$B_{8,5}^{\max}$	$R_{8,5}^{\min}$	$R_{8,5}^{\max}$
256	333	369	1.0000	19.000
512	1322	1460	1.0000	38.000
1024	5288	5840	1.0000	76.000
2048	21141	23362	1.0000	152.00
4096	84562	93448	1.0000	305.00

B	$B_{8,7}^{\min}$	$B_{8,7}^{\max}$	$R_{8,7}^{\min}$	$R_{8,7}^{\max}$
256	338	362	1.1111	13.000
512	1322	1460	1.0000	27.000
1024	5288	5840	1.0000	54.000
2048	21146	23362	1.0000	108.00
4096	84562	93448	1.0000	216.00

【0051】 when N=8 and $(a_{2,1}, a_{2,3}) = (7, 3)$ and $(a_{4,1}, a_{4,3}, a_{4,5}, a_{4,7}) = (5, 3, 2, 1)$

B	$B_{8,1}^{\min}$	$B_{8,1}^{\max}$	$R_{8,1}^{\min}$	$R_{8,1}^{\max}$
256	82	97	4.5000	9.0000
512	333	365	3.6000	19.000
1024	1325	1460	3.2727	38.000
2048	5297	5840	3.2727	76.000
4096	21146	23360	3.2727	152.00

B	$B_{8,3}^{\min}$	$B_{8,3}^{\max}$	$R_{8,3}^{\min}$	$R_{8,3}^{\max}$
256	169	181	1.1111	13.000
512	661	730	1.0000	27.000
1024	2644	2920	1.0000	54.000
2048	10573	11681	1.0000	108.00
4096	42281	46724	1.0000	216.00

B	$B_{8,5}^{\min}$	$B_{8,5}^{\max}$	$R_{8,5}^{\min}$	$R_{8,5}^{\max}$
256	169	181	1.1111	13.000
512	661	730	1.0000	27.000
1024	2644	2920	1.0000	54.000
2048	10573	11681	1.0000	108.00
4096	42281	46724	1.0000	216.00

B	$B_{8,7}^{\min}$	$B_{8,7}^{\max}$	$R_{8,7}^{\min}$	$R_{8,7}^{\max}$
256	85	90	1.1666	4.5000
512	333	365	1.0000	19.000
1024	1322	1460	1.0000	38.000
2048	5288	5840	1.0000	76.000
4096	21141	23362	1.0000	152.00

【0052】 when N=8 and $(a_{2,1}, a_{2,3}) = (12, 5)$ and $(a_{4,1}, a_{4,3}, a_{4,5}, a_{4,7}) = (5, 3, 2, 1)$

B	$B_{8,1}^{\min}$	$B_{8,1}^{\max}$	$R_{8,1}^{\min}$	$R_{8,1}^{\max}$
512	122	125	5.5000	11.000
1024	457	500	3.5000	22.000
2048	1825	2000	3.4166	44.000
4096	7261	8010	3.2692	89.000

B	$B_{8,3}^{\min}$	$B_{8,3}^{\max}$	$R_{8,3}^{\min}$	$R_{8,3}^{\max}$
512	229	250	1.0000	15.000
1024	909	1000	1.0000	31.000
2048	3636	4005	1.0000	63.000
4096	14517	16034	1.0000	126.00

B	$B_{8,5}^{\min}$	$B_{8,5}^{\max}$	$R_{8,5}^{\min}$	$R_{8,5}^{\max}$
512	229	250	1.0000	7.5000
1024	909	1000	1.0000	31.000
2048	3636	4005	1.0000	63.000
4096	14517	16034	1.0000	126.00

B	$B_{8,7}^{\min}$	$B_{8,7}^{\max}$	$R_{8,7}^{\min}$	$R_{8,7}^{\max}$
512	229	250	1.0000	7.5000
1024	909	1000	1.0000	31.000
2048	3636	4005	1.0000	63.000
4096	14517	16034	1.0000	126.00

【0053】 when N=16 and $(a_{2,1}, a_{2,3}) = (2, 1)$ and $(a_{4,1}, a_{4,3}, a_{4,5}, a_{4,7}) = (5, 3, 2, 1)$ and $(a_{8,1}, a_{8,3}, a_{8,5}, a_{8,7}, a_{8,9}, a_{8,11}, a_{8,13}, a_{8,15}) = (10, 10, 9, 6, 5, 5, 3, 1)$

B	$B_{16,1}^{\min}$	$B_{16,1}^{\max}$	$R_{16,1}^{\min}$	$R_{16,1}^{\max}$
2048	612	661	4.0000	25.000
4096	2437	2682	3.6153	51.000

B	$B_{16,3}^{\min}$	$B_{16,3}^{\max}$	$R_{16,3}^{\min}$	$R_{16,3}^{\max}$
2048	1125	1241	1.31746	106.00

4096	4500	4969	1.31451	213.00

B	$B_{16,5}^{\min}$	$B_{16,5}^{\max}$	$R_{16,5}^{\min}$	$R_{16,5}^{\max}$
2048	1157	1277	1.0000	35.000
4096	4628	5113	1.0000	71.000

B	$B_{16,7}^{\min}$	$B_{16,7}^{\max}$	$R_{16,7}^{\min}$	$R_{16,7}^{\max}$
2048	1009	1109	1.0000	33.000
4096	4021	4442	1.0000	66.000

B	$B_{16,9}^{\min}$	$B_{16,9}^{\max}$	$R_{16,9}^{\min}$	$R_{16,9}^{\max}$
2048	1009	1109	1.0000	33.000
4096	4021	4442	1.0000	66.000

B	$B_{16,11}^{\min}$	$B_{16,11}^{\max}$	$R_{16,11}^{\min}$	$R_{16,11}^{\max}$
2048	1157	1277	1.0000	35.000
4096	4628	5113	1.0000	71.000

B	$B_{16,13}^{\min}$	$B_{16,13}^{\max}$	$R_{16,13}^{\min}$	$R_{16,13}^{\max}$
2048	1125	1241	1.0000	35.000
4096	4500	4969	1.0000	70.000

B	$B_{16,15}^{\min}$	$B_{16,15}^{\max}$	$R_{16,15}^{\min}$	$R_{16,15}^{\max}$
2048	610	666	1.0000	25.000
4096	2434	2682	1.0000	51.000

【0054】 when N=16 and $(a_{2,1}, a_{2,3}) = (5, 2)$ and $(a_{4,1}, a_{4,3}, a_{4,5}, a_{4,7}) = (5, 3, 2, 1)$ and $(a_{8,1}, a_{8,3}, a_{8,5}, a_{8,7}, a_{8,9}, a_{8,11}, a_{8,13}, a_{8,15}) = (10, 10, 9, 6, 5, 5, 3, 1)$

B	$B_{16,1}^{\min}$	$B_{16,1}^{\max}$	$R_{16,1}^{\min}$	$R_{16,1}^{\max}$
2048	101	122	3.3333	5.0000
4096	425	457	3.3333	21.000

B	$B_{16,3}^{\min}$	$B_{16,3}^{\max}$	$R_{16,3}^{\min}$	$R_{16,3}^{\max}$

2048	194	212	1.2222	14.000
4096	776	857	1.0327	88.000

B	$B_{16,5}^{\min}$	$B_{16,5}^{\max}$	$R_{16,5}^{\min}$	$R_{16,5}^{\max}$
2048	200	218	1.0000	7.0000
4096	800	881	1.0000	29.000

B	$B_{16,7}^{\min}$	$B_{16,7}^{\max}$	$R_{16,7}^{\min}$	$R_{16,7}^{\max}$
2048	178	185	1.1111	4.3333
4096	697	765	1.0000	27.000

B	$B_{16,9}^{\min}$	$B_{16,9}^{\max}$	$R_{16,9}^{\min}$	$R_{16,9}^{\max}$
2048	178	185	1.1111	4.3333
4096	697	765	1.0000	27.000

B	$B_{16,11}^{\min}$	$B_{16,11}^{\max}$	$R_{16,11}^{\min}$	$R_{16,11}^{\max}$
2048	200	218	1.0000	7.0000
4096	800	881	1.0000	29.000

B	$B_{16,13}^{\min}$	$B_{16,13}^{\max}$	$R_{16,13}^{\min}$	$R_{16,13}^{\max}$
2048	194	212	1.0000	14.000
4096	776	857	1.0000	29.000

B	$B_{16,15}^{\min}$	$B_{16,15}^{\max}$	$R_{16,15}^{\min}$	$R_{16,15}^{\max}$
2048	85	137	1.1428	5.0000
4096	425	457	1.0000	21.000

【0055】 when N=16 and $(a_{2,1}, a_{2,3}) = (7, 3)$ and $(a_{4,1}, a_{4,3}, a_{4,5}, a_{4,7}) = (5, 3, 2, 1)$ and $(a_{8,1}, a_{8,3}, a_{8,5}, a_{8,7}, a_{8,9}, a_{8,11}, a_{8,13}, a_{8,15}) = (10, 10, 9, 6, 5, 5, 3, 1)$

B	$B_{16,1}^{\min}$	$B_{16,1}^{\max}$	$R_{16,1}^{\min}$	$R_{16,1}^{\max}$
2048	37	73	2.6666	6.0000
4096	212	229	3.5000	4.6667

B	$B_{16,3}^{\min}$	$B_{16,3}^{\max}$	$R_{16,3}^{\min}$	$R_{16,3}^{\max}$
2048	97	106	1.0476	31.000
4096	388	425	1.0232	62.000

B	$B_{16,5}^{\min}$	$B_{16,5}^{\max}$	$R_{16,5}^{\min}$	$R_{16,5}^{\max}$
2048	100	109	1.3333	10.000
4096	400	436	1.0714	20.000

B	$B_{16,7}^{\min}$	$B_{16,7}^{\max}$	$R_{16,7}^{\min}$	$R_{16,7}^{\max}$
2048	89	90	1.6000	3.0000
4096	349	377	1.0769	19.000

B	$B_{16,9}^{\min}$	$B_{16,9}^{\max}$	$R_{16,9}^{\min}$	$R_{16,9}^{\max}$
2048	89	90	1.6000	3.0000
4096	349	377	1.0769	19.000

B	$B_{16,11}^{\min}$	$B_{16,11}^{\max}$	$R_{16,11}^{\min}$	$R_{16,11}^{\max}$
2048	100	109	1.3333	10.000
4096	400	436	1.0714	20.000

B	$B_{16,13}^{\min}$	$B_{16,13}^{\max}$	$R_{16,13}^{\min}$	$R_{16,13}^{\max}$
2048	97	106	1.0000	10.000
4096	388	425	1.0000	20.000

B	$B_{16,15}^{\min}$	$B_{16,15}^{\max}$	$R_{16,15}^{\min}$	$R_{16,15}^{\max}$
2048	37	73	2.6666	6.0000
4096	212	229	1.1000	15.000

【0056】 when N=16 and $(a_{2,1}, a_{2,3}) = (2, 1)$ and $(a_{4,1}, a_{4,3}, a_{4,5}, a_{4,7}) = (5, 3, 2, 1)$ and $(a_{8,1}, a_{8,3}, a_{8,5}, a_{8,7}, a_{8,9}, a_{8,11}, a_{8,13}, a_{8,15}) = (10, 13, 13, 6, 5, 5, 3, 1)$

B	$B_{16,1}^{\min}$	$B_{16,1}^{\max}$	$R_{16,1}^{\min}$	$R_{16,1}^{\max}$
2048	612	661	4.0000	25.000
4096	2437	2682	3.6153	51.000
8192	9748	10728	1.0000	103.00

16384	38992	42912	1.0000	207.00
32768	155968	171648	1.0000	414.00

B	$B_{16,3}^{\min}$	$B_{16,3}^{\max}$	$R_{16,3}^{\min}$	$R_{16,3}^{\max}$
2048	390	432	1.00000	20.000
4096	1561	1726	1.00000	41.000
8192	6244	6904	1.00000	83.000
16384	24976	27616	1.00000	166.00
32768	99904	110464	1.00000	332.00

B	$B_{16,5}^{\min}$	$B_{16,5}^{\max}$	$R_{16,5}^{\min}$	$R_{16,5}^{\max}$
2048	562	621	1.0000	24.000
4096	2250	2487	1.0000	49.000
8192	9000	9948	1.0000	99.000
16384	36000	39792	1.0000	199.00
32768	14400	159168	1.0000	398.00

B	$B_{16,7}^{\min}$	$B_{16,7}^{\max}$	$R_{16,7}^{\min}$	$R_{16,7}^{\max}$
2048	1009	1109	1.0000	33.000
4096	4021	4442	1.0000	66.000
8192	16084	17768	1.0000	133.00
16384	64336	71072	1.0000	266.00
32768	257344	284288	1.0000	533.00

B	$B_{16,9}^{\min}$	$B_{16,9}^{\max}$	$R_{16,9}^{\min}$	$R_{16,9}^{\max}$
2048	1009	1109	1.0000	33.000
4096	4021	4442	1.0000	66.000
8192	16084	17768	1.0000	133.00
16384	64336	71072	1.0000	266.00
32768	257344	284288	1.0000	533.00

B	$B_{16,11}^{\min}$	$B_{16,11}^{\max}$	$R_{16,11}^{\min}$	$R_{16,11}^{\max}$
2048	562	621	1.0000	24.000
4096	2250	2487	1.0000	49.000
8192	9000	9948	1.0000	99.000

16384	36000	39792	1.0000	199.00
32768	14400	159168	1.0000	398.00

B	$B_{16,13}^{min}$	$B_{16,13}^{max}$	$R_{16,13}^{min}$	$R_{16,13}^{max}$
2048	390	432	1.00000	20.000
4096	1561	1726	1.00000	41.000
8192	6244	6904	1.00000	83.000
16384	24976	27616	1.00000	166.00
32768	99904	110464	1.00000	332.00

B	$B_{16,15}^{min}$	$B_{16,15}^{max}$	$R_{16,15}^{min}$	$R_{16,15}^{max}$
2048	612	661	1.0000	25.000
4096	2437	2682	1.0000	51.000
8192	9748	10728	1.0000	103.00
16384	38992	42912	1.0000	207.00
32768	155968	171648	1.0000	414.00

【0057】 when N=16 and $(a_{2,1}, a_{2,3}) = (5, 2)$ and $(a_{4,1}, a_{4,3}, a_{4,5}, a_{4,7}) = (5, 3, 2, 1)$ and $(a_{8,1}, a_{8,3}, a_{8,5}, a_{8,7}, a_{8,9}, a_{8,11}, a_{8,13}, a_{8,15}) = (10, 13, 13, 6, 5, 5, 3, 1)$

B	$B_{16,1}^{min}$	$B_{16,1}^{max}$	$R_{16,1}^{min}$	$R_{16,1}^{max}$
2048	101	122	1.1428	10.000
4096	425	457	1.0000	4.0000
8192	1700	1828	1.0000	8.2000
16384	6800	7312	1.0000	9.1111
32768	27200	29248	1.0000	9.1111

B	$B_{16,3}^{min}$	$B_{16,3}^{max}$	$R_{16,3}^{min}$	$R_{16,3}^{max}$
2048	114	125	1.5000	11.000
4096	457	505	1.0667	22.000
8192	1828	2020	1.0000	44.000
16384	7312	8080	1.0000	89.000
32768	29248	32320	1.0000	179.000

B	$B_{16,5}^{min}$	$B_{16,5}^{max}$	$R_{16,5}^{min}$	$R_{16,5}^{max}$
2048	97	106	1.0000	10.000

4096	388	429	1.0000	20.000
8192	1552	1716	1.0000	41.000
16384	6208	6864	1.0000	82.000
32768	24832	27456	1.0000	165.00

B	$B_{16,7}^{\min}$	$B_{16,7}^{\max}$	$R_{16,7}^{\min}$	$R_{16,7}^{\max}$
2048	178	185	1.1111	4.3333
4096	697	765	1.0000	27.000
8192	2788	765	1.0000	55.000
16384	11152	12240	1.0000	110.00
32768	44608	48960	1.0000	221.00

B	$B_{16,9}^{\min}$	$B_{16,9}^{\max}$	$R_{16,9}^{\min}$	$R_{16,9}^{\max}$
2048	178	185	1.1111	4.3333
4096	697	765	1.0000	27.000
8192	2788	765	1.0000	55.000
16384	11152	12240	1.0000	110.00
32768	44608	48960	1.0000	221.00

B	$B_{16,11}^{\min}$	$B_{16,11}^{\max}$	$R_{16,11}^{\min}$	$R_{16,11}^{\max}$
2048	97	106	1.0000	10.000
4096	388	429	1.0000	20.000
8192	1552	1716	1.0000	41.000
16384	6208	6864	1.0000	82.000
32768	24832	27456	1.0000	165.00

B	$B_{16,13}^{\min}$	$B_{16,13}^{\max}$	$R_{16,13}^{\min}$	$R_{16,13}^{\max}$
2048	114	125	1.5000	11.000
4096	457	505	1.0667	22.000
8192	1828	2020	1.0000	44.000
16384	7312	8080	1.0000	89.000
32768	29248	32320	1.0000	179.000

B	$B_{16,15}^{\min}$	$B_{16,15}^{\max}$	$R_{16,15}^{\min}$	$R_{16,15}^{\max}$
2048	85	137	1.0000	11.000

4096	425	457	1.0000	4.0000
8192	1700	1828	1.0000	8.2000
16384	6800	7312	1.0000	9.1111
32768	27200	29248	1.0000	9.1111

【0058】 when N=16 and $(a_{2,1}, a_{2,3}) = (7, 3)$ and $(a_{4,1}, a_{4,3}, a_{4,5}, a_{4,7}) = (5, 3, 2, 1)$ and $(a_{8,1}, a_{8,3}, a_{8,5}, a_{8,7}, a_{8,9}, a_{8,11}, a_{8,13}, a_{8,15}) = (10, 13, 13, 6, 5, 5, 3, 1)$

B	$B_{16,1}^{\min}$	$B_{16,1}^{\max}$	$R_{16,1}^{\min}$	$R_{16,1}^{\max}$
2048	37	73	1.0000	8.0000
4096	212	229	1.0000	15.000
8192	848	916	1.0000	30.000
16384	3392	3664	1.0000	60.000
32768	13568	14656	1.0000	121.00

B	$B_{16,3}^{\min}$	$B_{16,3}^{\max}$	$R_{16,3}^{\min}$	$R_{16,3}^{\max}$
2048	57	63	1.2000	2.3333
4096	228	251	1.0000	7.5000
8192	912	1004	1.0000	31.000
16384	3648	4016	1.0000	63.000
32768	14592	16064	1.0000	126.00

B	$B_{16,5}^{\min}$	$B_{16,5}^{\max}$	$R_{16,5}^{\min}$	$R_{16,5}^{\max}$
2048	48	53	1.0000	7.0000
4096	194	212	1.0000	14.000
8192	776	848	1.0000	29.000
16384	3104	3392	1.0000	58.000
32768	12416	13568	1.0000	116.00

B	$B_{16,7}^{\min}$	$B_{16,7}^{\max}$	$R_{16,7}^{\min}$	$R_{16,7}^{\max}$
2048	88	90	1.6000	3.0000
4096	169	184	1.1111	13.000
8192	676	736	1.0000	27.000
16384	2704	2944	1.0000	54.000
32768	10816	11776	1.0000	108.00

B	$B_{16,9}^{\min}$	$B_{16,9}^{\max}$	$R_{16,9}^{\min}$	$R_{16,9}^{\max}$
2048	88	90	1.6000	3.0000
4096	169	184	1.1111	13.000
8192	676	736	1.0000	27.000
16384	2704	2944	1.0000	54.000
32768	10816	11776	1.0000	108.00

B	$B_{16,11}^{\min}$	$B_{16,11}^{\max}$	$R_{16,11}^{\min}$	$R_{16,11}^{\max}$
2048	48	53	1.0000	7.0000
4096	194	212	1.0000	14.000
8192	776	848	1.0000	29.000
16384	3104	3392	1.0000	58.000
32768	12416	13568	1.0000	116.00

B	$B_{16,13}^{\min}$	$B_{16,13}^{\max}$	$R_{16,13}^{\min}$	$R_{16,13}^{\max}$
2048	57	63	1.2000	2.3333
4096	228	251	1.0000	7.5000
8192	912	1004	1.0000	31.000
16384	3648	4016	1.0000	63.000
32768	14592	16064	1.0000	126.00

B	$B_{16,15}^{\min}$	$B_{16,15}^{\max}$	$R_{16,15}^{\min}$	$R_{16,15}^{\max}$
2048	37	73	1.0000	8.0000
4096	212	229	1.0000	15.000
8192	848	916	1.0000	30.000
16384	3392	3664	1.0000	60.000
32768	13568	14656	1.0000	121.00

【0059】 Some suggested value of $b_{N,i}$ pairs are listed below but does not limit to these. When $N = 2$:

B	Suggested Values	
8	$(b_{2,1}, b_{2,3})$	(10, 5), (11,3), (11, 2), (9, 7), (11, 1)
16	$(b_{2,1}, b_{2,3})$	(20,11), (22,6), (22, 5), (21, 8), (19,12)
32	$(b_{2,1}, b_{2,3})$	(42,17), (41, 19), (40,21), (43, 14), (39,23)
64	$(b_{2,1}, b_{2,3})$	(83,36), (84, 34), (85,31), (86, 28), (79,44)

128	$(b_{2,1}, b_{2,3})$	(166,72), (167, 70), (169,65), (170, 62), (162,81)
256	$(b_{2,1}, b_{2,3})$	(333,142), (330, 149), (336,135), (329, 151), (339,127)
512	$(b_{2,1}, b_{2,3})$	(663, 291), (669, 277), (659,300), (675, 262), (652,315)
1024	$(b_{2,1}, b_{2,3})$	(1330, 573), (1333, 566), (1338,554), (1323, 589), (1318,600)
2048	$(b_{2,1}, b_{2,3})$	(2671, 1120), (2676, 1108), (2641,1189), (2685, 1086), (2687,1081)
4096	$(b_{2,1}, b_{2,3})$	(5316, 2301), (5331, 2266), (5306, 2324), (5342, 2240), (5347, 2228)

When N=4, $(a_{2,1}, a_{2,3}) = (2, 1)$:

	B	Suggested Values
$(b_{4,1}, b_{4,7})$	16	(10, 2), (10, 1)
	32	(20, 4), (20, 3), (20, 2), (20, 1), (19, 6)
	64	(40, 7), (40, 6), (39, 10), (40, 5), (39, 11)
	128	(80, 14), (79, 17), (80, 13), (79, 18), (80, 11)
	256	(159, 31), (159, 30), (159, 32), (159, 29), (158, 35)
	512	(319, 61), (318, 62), (318, 61), (317, 62), (317, 61)
	1024	(636, 123), (636, 124), (636, 122), (635, 125), (636, 121)
	2048	(1272, 245), (1272, 244), (1271, 249), (1272, 243), (1271, 250)
	4096	(2545, 484), (2544, 489), (2543, 494), (2542, 499), (2544, 488)
$(b_{4,3}, b_{4,5})$	16	(9, 5), (8, 6), (7, 7), (10, 2), (10, 1)
	32	(17, 11), (18, 9), (19, 7), (20, 4), (20, 3)
	64	(34, 22), (31, 26), (38, 14), (39, 11), (40, 6)
	128	(72, 37), (73, 35), (62, 52), (76, 28), (77, 25)
	256	(139, 83), (143, 76), (144, 74), (145, 72), (146, 70)
	512	(273, 174), (275, 171), (272, 176), (271, 177), (277, 168)
	1024	(550, 342), (545, 350), (543, 353), (553, 337), (541, 356)
	2048	(1093, 695), (1103, 679), (1086, 706), (1084, 709), (1082, 712)
	4096	(2193, 1379), (2198, 1371), (2203, 1363), (2179, 1401), (2214,

		1345)

When N=4, ($a_{2,1}$, $a_{2,3}$) = (5, 2):

	B	Suggested Values
$(b_{4,1}, b_{4,7})$	16	(4, 1)
	32	(8, 2)
	64	(16, 4), (17, 2), (16, 5)
	128	(33, 6), (33, 7), (33, 8), (33, 5), (33, 4)
	256	(66, 13), (66, 12), (66, 14), (66, 11), (65, 16)
	512	(132, 26), (132, 25), (131, 30), (133, 20), (133, 19)
	1024	(264, 51), (263, 56), (265, 46), (266, 40), (266, 39)
	2048	(529, 97), (526, 112), (525, 117), (524, 121), (532, 79)
	4096	(1058, 194), (1051, 229), (1060, 183), (1061, 177), (1049, 238)
$(b_{4,3}, b_{4,5})$	16	(4, 1), (3, 3)
	32	(7, 5), (6, 6), (8, 3), (8, 2)
	64	(14, 9), (15, 8), (12, 12), (16, 5), (17, 1)
	128	(29, 17), (27, 20), (30, 15), (31, 13), (32, 10)
	256	(58, 34), (54, 40), (50, 45), (62, 26), (49, 46)
	512	(116, 68), (108, 80), (105, 84), (99, 91), (124, 52)
	1024	(229, 141), (232, 136), (219, 156), (237, 127), (210, 168)
	2048	(451, 293), (453, 290), (449, 296), (447, 299), (445, 302)
	4096	(904, 583), (902, 586), (911, 572), (916, 564), (888, 607)

When N=4, ($a_{2,1}$, $a_{2,3}$) = (7, 3):

	B	Suggested Values
$(b_{4,1}, b_{4,7})$	32	(6, 1)
	64	(12, 2)

	128	(23, 5), (23, 4), (24, 4), (23, 6), (24, 3)
	256	(47, 9), (46, 10), (47, 8), (46, 11), (47, 7)
	512	(93, 19), (93, 20), (94, 15), (94, 14), (94, 13)
	1024	(187, 35), (186, 39), (187, 34), (186, 40), (185, 44)
	2048	(373, 74), (373, 75), (374, 69), (374, 68), (372, 79)
	4096	(746, 148), (747, 144), (746, 149), (747, 143), (745, 153)
$(b_{4,3}, b_{4,5})$	32	(5, 3), (6, 1)
	64	(10, 6), (9, 8), (11, 5), (11, 4), (12, 1)
	128	(20, 13), (19, 14), (21, 11), (22, 9), (23, 6)
	256	(40, 26), (39, 27), (41, 24), (42, 22), (37, 30)
	512	(79, 53), (82, 48), (76, 57), (73, 61), (72, 62)
	1024	(161, 101), (163, 98), (155, 110), (153, 113), (167, 91)
	2048	(319, 207), (317, 210), (321, 204), (315, 213), (313, 216)
	4096	(640, 411), (626, 432), (650, 395), (653, 390), (619, 442)

When N=4, $(a_{2,1}, a_{2,3})$ = (12, 5):

	B	Suggested Values
$(b_{4,1}, b_{4,7})$	64	(7, 1)
	128	(14, 2)
	256	(27, 5), (28, 5), (27, 6), (27, 4), (28, 4)
	512	(55, 8), (54, 13), (55, 9), (55, 10), (55, 11)
	1024	(108, 27), (110, 17), (110, 18), (109, 23), (109, 22)
	2048	(216, 55), (216, 54), (220, 34), (217, 51), (220, 35)
	4096	(437, 87), (438, 82), (436, 92), (439, 76), (435, 97)
$(b_{4,3}, b_{4,5})$	64	(5, 5), (7, 1)
	128	(12, 7), (11, 9), (10, 10), (13, 5), (14, 2)
	256	(23, 16), (24, 14), (22, 17), (25, 12), (21, 18)

	512	(47, 30), (48, 28), (45, 33), (44, 34), (51, 22)
	1024	(92, 63), (95, 58), (89, 67), (98, 53), (99, 51)
	2048	(187, 121), (189, 118), (183, 127), (181, 130), (192, 113)
	4096	(378, 236), (381, 231), (364, 257), (384, 226), (359, 264)

When N=8, $(a_{2,1}, a_{2,3}) = (2, 1)$, $(a_{4,1}, a_{4,3}, a_{4,5}, a_{4,7}) = (5, 3, 2, 1)$:

	B	Suggested Values
$(b_{8,1}, b_{8,15})$	128	(16, 2), (16, 1)
	256	(32, 3), (32, 4), (32,2), (31, 5), (32, 1)
	512	(63, 6), (63, 7), (63, 5), (63, 8), (63, 9)
	1024	(126, 14), (126, 15), (127, 9), (126, 16), (127, 8)
	2048	(253, 24), (253, 25), (253, 23), (253, 22), (253, 21)
	4096	(506, 46), (506, 45), (506, 55), (505, 56), (507, 34)
$(b_{8,3}, b_{8,13})$	128	(21, 8), (22, 5), (22, 4), (22, 3), (20, 10)
	256	(43, 13), (42, 16), (44, 10), (44, 9), (44, 8)
	512	(86, 26), (85, 25), (87, 22), (84, 32), (88, 18)
	1024	(172, 52), (171, 55), (173, 48), (170, 58), (169, 61)
	2048	(343, 107), (345, 100), (342, 110), (346, 97), (341, 113)
	4096	(688, 207), (685, 217), (684, 220), (691, 197), (683, 223)
$(b_{8,5}, b_{8,11})$	128	(20, 10), (19, 12), (21, 8), (22, 5), (22, 4)
	256	(39, 22), (41, 18), (38, 24), (42, 16), (36, 27)
	512	(85, 29), (86, 26), (71, 55), (88, 18), (65, 62)
	1024	(156, 89), (165, 71), (170, 58), (171, 55), (145, 106)
	2048	(317, 169), (325, 153), (326, 151), (330, 142), (303, 193)
	4096	(634, 338), (651, 304), (628, 349), (656, 293), (620, 363)
$(b_{8,7}, b_{8,9})$	128	(13, 9), (14, 7), (15, 5), (16, 2), (16, 1)
	256	(24, 21), (26, 18), (23, 22), (27, 17), (28, 15)

	512	(53, 35), (56, 30), (57, 28), (58, 26), (60, 21)
	1024	(101, 77), (97, 82), (104, 73), (106, 70), (112, 60)
	2048	(202, 154), (205, 150), (194, 164), (210, 143), (212, 140)
	4096	(397, 317), (393, 332), (410, 300), (388, 328), (372, 346)

When N=8, $(a_{2,1}, a_{2,3}) = (5, 2)$, $(a_{4,1}, a_{4,3}, a_{4,5}, a_{4,7}) = (5, 3, 2, 1)$:

	B	Suggested Values
$(b_{8,1}, b_{8,15})$	256	(13, 2), (13, 1)
	512	(26, 3), (26, 4), (26, 2), (26, 1), (27, 1)
	1024	(52, 7), (53, 3), (52, 8), (53, 2), (52, 9)
	2048	(105, 10), (105, 11), (105, 9), (105, 12), (105, 8)
	4096	(210, 21), (210, 20), (210, 19), (209, 28), (209, 29)
$(b_{8,3}, b_{8,13})$	256	(18, 5), (18, 6), (18, 4), (17, 7), (17, 8)
	512	(36, 10), (36, 9), (35, 13), (34, 15), (37, 5)
	1024	(72, 20), (71, 23), (72, 19), (70, 26), (73, 15)
	2048	(143, 42), (142, 46), (144, 39), (141, 49), (145, 35)
	4096	(286, 85), (285, 88), (288, 78), (289, 74), (290, 70)
$(b_{8,5}, b_{8,11})$	256	(17, 8), (16, 10), (15, 11), (18, 5), (14, 12)
	512	(32, 19), (34, 15), (35, 13), (30, 22), (36, 10)
	1024	(67, 33), (63, 40), (61, 43), (70, 26), (71, 23)
	2048	(129, 75), (137, 59), (124, 83), (139, 54), (119, 90)
	4096	(264, 139), (265, 137), (266, 135), (167, 133), (259, 148)
$(b_{8,7}, b_{8,9})$	256	(11, 7), (12, 5), (13, 3), (13, 2), (13, 1)
	512	(21, 16), (20, 17), (23, 13), (24, 11), (25, 8)
	1024	(41, 33), (42, 32), (44, 29), (46, 26), (47, 24)
	2048	(83, 65), (79, 70), (86, 61), (78, 71), (77, 72)
	4096	(163, 134), (170, 125), (156, 142), (180, 110), (183, 105)

When N=8, $(a_{2,1}, a_{2,3})$ = (7, 3), $(a_{4,1}, a_{4,3}, a_{4,5}, a_{4,7})$ = (5, 3, 2, 1):

	B	Suggested Values
$(b_{8,1}, b_{8,15})$	256	(9, 2)
	512	(19, 2), (19, 1), (18, 3), (18, 4), (18, 5)
	1024	(37, 4), (37, 3), (37, 5), (36, 9), (36, 10)
	2048	(74, 7), (74, 8), (74, 6), (74, 9), (74, 10)
	4096	(148, 18), (148, 19), (149, 9), (149, 8), (149, 7)
$(b_{8,3}, b_{8,13})$	256	(13, 3), (12, 5), (13, 2), (13, 1), (11, 7)
	512	(25, 8), (26, 5), (24, 11), (26, 4), (23, 13)
	1024	(50, 17), (51, 13), (52, 9), (48, 22), (52, 8)
	2048	(102, 27), (103, 23), (98, 39), (104, 18), (104, 17)
	4096	(201, 64), (200, 67), (199, 70), (196, 78), (208, 35)
$(b_{8,5}, b_{8,11})$	256	(11, 7), (12, 5), (13, 3), (13, 2), (13, 1)
	512	(23, 13), (24, 11), (21, 16), (25, 8), (20, 17)
	1024	(47, 24), (46, 26), (48, 22), (44, 29), (50, 17)
	2048	(94, 48), (95, 46), (90, 55), (88, 58), (98, 39)
	4096	(183, 105), (180, 110), (196, 78), (199, 70), (200, 67)
$(b_{8,7}, b_{8,9})$	256	(7, 6), (8, 5), (9, 3), (9, 2)
	512	(14, 12), (15, 11), (16, 10), (17, 8), (18, 5)
	1024	(30, 22), (31, 21), (32, 19), (33, 17), (34, 15)
	2048	(58, 47), (57, 48), (61, 43), (63, 40), (67, 33)
	4096	(115, 95), (119, 90), (124, 83), (107, 104), (106, 105)

When N=8, $(a_{2,1}, a_{2,3})$ = (12, 5), $(a_{4,1}, a_{4,3}, a_{4,5}, a_{4,7})$ = (5, 3, 2, 1):

	B	Suggested Values
$(b_{8,1}, b_{8,15})$	512	(11, 1), (11, 2)

	1024	(22, 2), (22, 3), (22, 1), (21, 5), (21, 6)
	2048	(43, 6), (43, 7), (44, 1), (43, 8), (43, 9)
	4096	(87, 9), (87, 8), (87, 10), (87, 7), (87, 6)
$(b_{8,3}, b_{8,13})$	512	(15, 4), (15, 3), (14, 6), (14, 7), (13, 8)
	1024	(30, 8), (30, 7), (29, 11), (28, 13), (27, 15)
	2048	(59, 18), (60, 15), (58, 21), (57, 24), (61, 10)
	4096	(118, 37), (119, 33), (117, 40), (120, 30), (115, 45)
$(b_{8,5}, b_{8,11})$	512	(14, 7), (13, 8), (14, 6), (12, 10), (15, 4)
	1024	(27, 15), (28, 13), (29, 11), (25, 18), (30, 8)
	2048	(55, 28), (54, 30), (56, 26), (53, 32), (57, 24)
	4096	(109, 58), (108, 60), (107, 62), (112, 52), (105, 65)
$(b_{8,7}, b_{8,9})$	512	(9, 6), (10, 5), (10, 4), (11, 2), (11, 1)
	1024	(16, 15), (17, 14), (18, 12), (19, 11), (20, 9)
	2048	(35, 26), (36, 25), (32, 30), (31, 31), (37, 23)
	4096	(68, 55), (67, 56), (71, 51), (73, 48), (75, 45)

When N=16, $(a_{2,1}, a_{2,3})$ = (2, 1), $(a_{4,1}, a_{4,3}, a_{4,5}, a_{4,7})$ = (5, 3, 2, 1), $(a_{8,1}, a_{8,3}, a_{8,5}, a_{8,7}, a_{8,9}, a_{8,11}, a_{8,13}, a_{8,15})$ = (10, 10, 9, 6, 5, 5, 3, 1):

	B	Suggested Values
$(b_{16,1}, b_{16,31})$	2048	(25, 1), (25, 2), (25, 3), (25, 4), (25, 5)
	4096	(50, 6), (50, 7), (50, 8), (50, 9), (49, 12)
$(b_{16,3}, b_{16,29})$	2048	(34, 5), (34, 6), (33, 10), (32, 13), (31, 15)
	4096	(68, 10), (68, 11), (67, 15), (67, 16), (66, 19)
$(b_{16,5}, b_{16,27})$	2048	(33, 10), (34, 6), (34, 5), (32, 13), (31, 15)
	4096	(68, 16), (68, 15), (67, 19), (67, 20), (69, 11)
$(b_{16,7}, b_{16,25})$	2048	(31, 10), (29, 15), (32, 6), (32, 5), (27, 18)
	4096	(62, 20), (60, 25), (63, 16), (64, 12), (64, 11)

$(b_{16,9}, b_{16,23})$	2048	(29, 15), (31, 10), (27, 18), (32, 6), (25, 21)
	4096	(60, 25), (56, 33), (63, 16), (52, 39), (47, 45)
$(b_{16,11}, b_{16,21})$	2048	(30, 18), (31, 16), (28, 21), (32, 14), (27, 22)
	4096	(61, 34), (62, 32), (63, 30), (64, 28), (55, 43)
$(b_{16,13}, b_{16,19})$	2048	(28, 20), (30, 17), (31, 15), (32, 13), (33, 10)
	4096	(56, 40), (58, 37), (53, 44), (52, 45), (61, 32)
$(b_{16,15}, b_{16,17})$	2048	(18, 18), (21, 14), (23, 10), (24, 8), (25, 4)
	4096	(39, 32), (40, 31), (42, 28), (44, 25), (46, 21)

When N=16, $(a_{2,1}, a_{2,3}) = (5, 2)$, $(a_{4,1}, a_{4,3,} a_{4,5}, a_{4,7}) = (5, 3, 2, 1)$, $(a_{8,1}, a_{8,3}, a_{8,5}, a_{8,7}, a_{8,9}, a_{8,11}, a_{8,13}, a_{8,15}) = (10, 10, 9, 6, 5, 5, 3, 1)$:

	B	Suggested Values
$(b_{16,1}, b_{16,31})$	2048	(10, 3)
	4096	(21, 1), (21, 2), (21, 3), (21, 4), (20, 5)
$(b_{16,3}, b_{16,29})$	2048	(14, 4), (14, 2), (14, 3), (13, 6), (12, 8)
	4096	(28, 5), (28, 6), (27, 9), (26, 12), (23, 17)
$(b_{16,5}, b_{16,27})$	2048	(14, 4), (14, 3), (13, 6), (13, 7), (12, 8)
	4096	(28, 7), (28, 8), (29, 2), (29, 1), (26, 13)
$(b_{16,7}, b_{16,25})$	2048	(13, 4), (13, 3), (12, 6), (11, 8), (10, 9)
	4096	(25, 10), (26, 7), (23, 14), (27, 2), (27, 1)
$(b_{16,9}, b_{16,23})$	2048	(12, 6), (13, 4), (11, 8), (13, 3), (10, 9)
	4096	(25, 10), (23, 14), (26, 7), (21, 17), (20, 18)
$(b_{16,11}, b_{16,21})$	2048	(12, 8), (13, 7), (13, 6), (11, 9), (14, 4)
	4096	(24, 16), (26, 13), (22, 19), (21, 20), (28, 8)
$(b_{16,13}, b_{16,19})$	2048	(11, 9), (12, 8), (10, 10), (13, 6), (14, 3)
	4096	(23, 17) , (22, 18), (25, 14), (26, 12), (27, 9)
$(b_{16,15}, b_{16,17})$	2048	(8, 7), (9, 5), (10, 3)

	4096	(18, 11), (19, 9), (20, 7), (20, 6), (21, 2)

When N=16, $(a_{2,1}, a_{2,3}) = (7, 3)$, $(a_{4,1}, a_{4,3}, a_{4,5}, a_{4,7}) = (5, 3, 2, 1)$, $(a_{8,1}, a_{8,3}, a_{8,5}, a_{8,7}, a_{8,9}, a_{8,11}, a_{8,13}, a_{8,15}) = (10, 10, 9, 6, 5, 5, 3, 1)$:

	B	Suggested Values
$(b_{16,1}, b_{16,31})$	2048	(7, 2)
	4096	(15, 1), (15, 2), (14, 4)
$(b_{16,3}, b_{16,29})$	2048	(10, 1), (10, 2), (9, 4), (9,5), (8, 6)
	4096	(20, 3), (20, 2), (19, 7), (18, 9), (17, 11)
$(b_{16,5}, b_{16,27})$	2048	(10, 2), (10, 3), (10, 1), (9,5), (8, 6)
	4096	(20, 5), (20, 4), (19, 8), (18, 10), (16, 13)
$(b_{16,7}, b_{16,25})$	2048	(9, 3), (8, 5)
	4096	(18, 6), (17, 9), (19, 3), (19, 2), (19, 1)
$(b_{16,9}, b_{16,23})$	2048	(9, 3), (8, 5)
	4096	(17, 9), (18, 6), (15, 12), (19, 3), (14, 13)
$(b_{16,11}, b_{16,21})$	2048	(9, 5), (8, 6), (10, 3), (10, 2), (10, 1)
	4096	(18, 10), (19, 8), (16, 13), (15, 14), (20, 5)
$(b_{16,13}, b_{16,19})$	2048	(8, 6), (9, 5), (7, 7), (10, 2), (10, 1)
	4096	(17, 11), (18, 9), (19, 7), (20, 4), (20, 3)
$(b_{16,15}, b_{16,17})$	2048	(7, 2)
	4096	(11, 10), (12, 9), (13, 7), (14, 5), (15, 1)

<u>Example 6: Order-8 Transform</u>

【0060】 Taking $b_{1,1} = 128$, $(a_{2,1}, a_{2,3}) = (5, 2)$, $(b_{2,1}, b_{2,3}) = (166, 72)$ and $(b_{4,1}, b_{4,3}, b_{4,5}\ b_{4,7}) = (33, 29, 17, 6)$, An order-8 transform is built:

$$E_4 = P_4 \begin{bmatrix} 128 & 128 & 0 & 0 \\ 128 & -128 & 0 & 0 \\ 0 & 0 & 72 & 166 \\ 0 & 0 & -166 & 72 \end{bmatrix} \begin{bmatrix} \mathbf{I}_2 & \mathbf{J}_2 \\ \mathbf{I}_2 & -\mathbf{J}_2 \end{bmatrix}.$$

$$F_4 = \begin{bmatrix} 33 & 0 & 0 & 6 \\ 0 & 29 & 17 & 0 \\ 0 & -17 & 29 & 0 \\ -6 & 0 & 0 & 33 \end{bmatrix} \begin{bmatrix} 2 & 5 & 0 & 0 \\ -5 & 2 & 0 & 0 \\ 0 & 0 & 2 & 5 \\ 0 & 0 & -5 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}.$$

$$E_8 = P_8 \left[\begin{array}{c:c} E_4 & \mathbf{0}_4 \\ \hdashline \mathbf{0}_4 & F_4 \end{array}\right] \begin{bmatrix} \mathbf{I}_4 & \mathbf{J}_4 \\ \mathbf{I}_4 & -\mathbf{J}_4 \end{bmatrix}$$

$$= \begin{bmatrix} 128 & 128 & 128 & 128 & 128 & 128 & 128 & 128 \\ 177 & 153 & 96 & 36 & -36 & -96 & -153 & -177 \\ 166 & 72 & -72 & -166 & -166 & -72 & 72 & 166 \\ 143 & -27 & -179 & -111 & 111 & 179 & 27 & -143 \\ 128 & -128 & -128 & 128 & 128 & -128 & -128 & 128 \\ 111 & -179 & 27 & 143 & -143 & -27 & 179 & -111 \\ 72 & -166 & 166 & -72 & -72 & 166 & -166 & 72 \\ 36 & -96 & 153 & -177 & 177 & -153 & 96 & -36 \end{bmatrix}.$$

【0061】 While we have hereinbefore described the embodiments of this application, it is understood that our basic constructions can be altered to provide other embodiments which utilize the processes and compositions of this application. Consequently, it will be appreciated that the scope of this application is to be defined by the claims appended hereto rather than by the specific embodiments which have been presented hereinbefore by way of examples.

What is claimed is:

1. A process for coding data in image or video, comprising:

retrieving a data matrix $X_{2N\times2N}$ to be coded; and

coding the retrieved data matrix by a rule of

$$\mathbf{Z}_{2N\times2N} = \mathbf{E}_{2N}\mathbf{X}_{2N\times2N}\mathbf{E}_{2N}^{T}$$

wherein $Z_{2N\times2N}$ represents a matrix for the coded data, E_{2N} is an order-$2N$ integer transform and $N = 2^n$ where n is a positive integer no less than 2.

2. The process according to claim 1, further comprising:

generating E_{2N} from an order-N transform E_N.

3. The process according to claim 2, wherein the generating further comprises:

generating E_{2N} from an order-N transform E_N by performing a transform of

$$\mathbf{E}_{2N} = \mathbf{P}_{2N}\mathbf{M}_{2N}\begin{bmatrix}\mathbf{E}_N & \mathbf{0}_N \\ \mathbf{0}_N & \mathbf{R}_N\end{bmatrix}\begin{bmatrix}\mathbf{I}_N & \mathbf{J}_N \\ \mathbf{I}_N & -\mathbf{J}_N\end{bmatrix} \quad \text{or}$$

$$E_{2N} = P_{2N}\begin{bmatrix}E_N & 0_N \\ 0_N & M_{2No}R_N\end{bmatrix}\begin{bmatrix}\mathbf{I}_N & \mathbf{J}_N \\ \mathbf{I}_N & -\mathbf{J}_N\end{bmatrix}.$$

where

$\mathbf{P}_{2N}$ is a permutation matrix,

M_{2N} is a diagonal matrix with integer elements to make every basis norms in E_{2N} similar,

I_N is an order-N identity matrix,

J_N is a mirrored version of I_N, and

R_N is a recursive matrix from an integer matrix $\mathbf{R}_{\frac{N}{2}}$.

4. The process according to claim 3, wherein $M_{2N}R_N$ is constructed by a rule of

$$M_{(2N)o}R_N = \begin{bmatrix} b_{N,1} & & & & & b_{N,2N-1} \\ & \ddots & & & \iddots & \\ & & b_{N,N-1} & b_{N,N+1} & & \\ & & -b_{N,N+1} & b_{N,N-1} & & \\ & \iddots & & & \ddots & \\ -b_{N,2N-1} & & & & & b_{N,1} \end{bmatrix}\begin{bmatrix}\mathbf{R}_{\frac{N}{2}} & \mathbf{0}_{\frac{N}{2}} \\ \mathbf{0}_{\frac{N}{2}} & \mathbf{R}_{\frac{N}{2}}\end{bmatrix}\begin{bmatrix} 1 & 1 & & & \\ & & \ddots & & \\ & & & 1 & 1 \\ \hdashline 1 & -1 & & & \\ & & \ddots & & \\ & & & -1 & 1 \end{bmatrix}.$$

wherein, the process further comprises:

selecting values of $b_{N,i}$ such that a transform coding gain for the process is maximized, and differences among a basis vector norms of the transform itself is to minimized.

5. The process according to claim 4, wherein, $b_{N,i}$ are integers that satisfy:

$$B_{N,i}^{\min} \le b_{N,i}^2 + b_{N,2N-i}^2 \le B_{N,i}^{\max} \text{ and } R_{N,i}^{\min} \le b_{N,i} / b_{N,2N-i} \le R_{N,i}^{\max} \text{ for } N \ge 2 \text{ and } b_{1,1} \le B.$$

where B is a matrix consisting of $b_{N,i}$,

$B_{N,i}^{\min}$ is a minimal value of the sum of $b^2_{N,i}$ and $b^2_{N,2N-i}$,

$B_{N,1}^{\max}$ is a maximal value of the sum of $b^2_{N,i}$ and $b^2_{N,2N-i}$,

$R_{N,i}^{\min}$ is a minimal ratio between $b_{N,i}$ and $b_{N,2N-i}$,

$R_{N,i}^{\max}$ is a maximal ratio between $b_{N,i}$ and $b_{N,2N-i}$.

6. The process according to claim 2, wherein R_N is constructed by a rule of

$$R_N = \begin{bmatrix} a_{N,1} & & & & & & a_{N,2N-1} \\ & a_{N,3} & & & & a_{N,2N-3} & \\ & & \ddots & & \iddots & & \\ & & & a_{N,N-1} \quad a_{N,N+1} & & & \\ & & & -a_{N,N+1} \quad a_{N,N-1} & & & \\ & & \iddots & & \ddots & & \\ & -a_{N,2N-3} & & & & a_{N,3} & \\ -a_{N,2N-1} & & & & & & a_{N,1} \end{bmatrix} \times$$

$$\begin{bmatrix} \mathbf{R}_{\frac{N}{2}} & \mathbf{0}_{\frac{N}{2}} \\ \mathbf{0}_{\frac{N}{2}} & \mathbf{R}_{\frac{N}{2}} \end{bmatrix} \begin{bmatrix} 1 & 1 & & & & & \\ & & 1 & 1 & & & \\ & & & & \ddots & & \\ & & & & & 1 & 1 \\ 1 & -1 & & & & & \\ & & -1 & 1 & & & \\ & & & & \ddots & & \\ & & & & & -1 & 1 \end{bmatrix}$$

where $a_{N,1}$, $a_{N,3}$, $\ldots a_{N,2N-1}$ are positive integers.

7. The process according to claim 6, wherein $a_{N,,\ k}$, and $a_{N,2N\text{-}k}$ are prime pairs

$for\ k = 1,3,...,N-1$.

8. The process according to claim 7, wherein

$$\frac{a_{N,k}}{a_{N,2N-k}} \approx \frac{\cos\left(\frac{k\pi}{4N}\right)}{\cos\left(\frac{(2N-k)\pi}{4N}\right)} \text{ for } k = 1,3,...,N-1.$$

9. The process according to claim 1, wherein the coding further comprises a 2D transform carried out by two cascading 1D transforms according to a rule of

$$\vec{U}_{2N,k} = \mathbf{E}_{2N}\vec{X}_{2N,k}$$
$$\mathbf{V}_{2N\times 2N} = \left[\vec{U}_{2N,0} \quad \vec{U}_{2N,1} \quad \cdots \quad \vec{U}_{2N,2N-1}\right]$$
$$\vec{Z}_{2N,k} = \mathbf{E}_{2N}\vec{V}^{T}_{2N,k}$$
$$\mathbf{Z}_{2N\times 2N} = \left[\vec{Z}^{T}_{2N,0}; \quad \vec{Z}^{T}_{2N,1}; \quad \cdots \quad \vec{Z}^{T}_{2N,2N-1}\right]$$

where

$\vec{X}_{2N,k}$ is a k-th column vector in $X_{2N\times 2N}$,

$V_{2N\times 2N}$ is a $2N\times 2N$ matrix built by $\vec{U}_{2N,k}$ for $k \in \{0,1,...,2N-1\}$, and

$\vec{V}_{2N,k}$ is a k-th row vector in $V_{2N\times 2N}$ for $k \in \{0,1,...,2N-1\}$.

10. The process according to claim 2, wherein E_N is one selected from a group consisting of an order-8 transform, an order-16 transform and an order-32 transform t adopted in H.264/AVC.

11. The process according to claim 2, wherein E_N is an order-8 transform, an order-16 transform and an order-32 transform adopted in AVS.

12. The process according to claim 1, wherein the process is implemented by one or more processor executing instructions encoded in computer readable storage medium.

13. The process according to claim 1, wherein the retrieving and coding are respectively implemented by a retrieving unit and a coding unit in a coding system, each of units

including electronic circuits.

14. An apparatus for coding data in image or video, comprising:

a retrieving unit for retrieving a data matrix $X_{2N\times 2N}$ to be coded; and

a coder for coding the retrieved data matrix by a rule of

$$\mathbf{Z}_{2N\times 2N} = \mathbf{E}_{2N}\mathbf{X}_{2N\times 2N}\mathbf{E}_{2N}^{T}$$

wherein $Z_{2N\times 2N}$ represents a matrix for the coded data, E_{2N} is an order-2N integer transform and $N = 2^n$ where n is a positive integer no less than 2.

15. The apparatus according to claim 14, wherein the retrieving unit is configured to further retrieve an order-N transform E_N; and the system further comprises:

a generating unit for generating E_{2N} from the retrieved E_N.

16. The apparatus according to claim 15, wherein the generating unit generates E_{2N} by performing a transform of

$$\mathbf{E}_{2N} = \mathbf{P}_{2N}\mathbf{M}_{2N}\begin{bmatrix}\mathbf{E}_N & \mathbf{0}_N \\ \mathbf{0}_N & \mathbf{R}_N\end{bmatrix}\begin{bmatrix}\mathbf{I}_N & \mathbf{J}_N \\ \mathbf{I}_N & -\mathbf{J}_N\end{bmatrix}, \text{ or}$$

$$E_{2N} = P_{2N}\begin{bmatrix}E_N & 0_N \\ 0_N & M_{2No}R_N\end{bmatrix}\begin{bmatrix}\mathbf{I}_N & \mathbf{J}_N \\ \mathbf{I}_N & -\mathbf{J}_N\end{bmatrix}.$$

where

$\mathbf{P}_{2N}$ is a permutation matrix,

M_{2N} is a diagonal matrix with integer elements to make every basis norms in E_{2N} similar,

I_N is an order-N identity matrix,

J_N is a mirrored version of I_N, and

R_N is a recursive matrix from an integer matrix $\mathbf{R}_{\frac{N}{2}}$.

17. The apparatus according to claim 16, wherein $M_{2N}R_N$ is constructed by a rule of

$$M_{(2N)o}R_N = \begin{bmatrix} b_{N,1} & & & & & b_{N,2N-1} \\ & \ddots & & & \iddots & \\ & & b_{N,N-1} & b_{N,N+1} & & \\ & & -b_{N,N+1} & b_{N,N-1} & & \\ & \iddots & & & \ddots & \\ -b_{N,2N-1} & & & & & b_{N,1} \end{bmatrix} \begin{bmatrix} \mathbf{R}_{\frac{N}{2}} & \mathbf{0}_{\frac{N}{2}} \\ \mathbf{0}_{\frac{N}{2}} & \mathbf{R}_{\frac{N}{2}} \end{bmatrix} \left[\begin{array}{cccc} 1 & 1 & & \\ & & \ddots & \\ & & 1 & 1 \\ \hdashline 1 & -1 & & \\ & & \ddots & \\ & & -1 & 1 \end{array}\right].$$

where, the values of $b_{N,i}$ are chosen such that a transform coding gain for the process is maximized, and the differences among a basis vector norms of the transform itself is to minimized.

18. The apparatus according to claim 17, wherein, $b_{N,i}$ are integers that satisfy:

$B_{N,i}^{\min} \le b_{N,i}^2 + b_{N,2N-i}^2 \le B_{N,i}^{\max}$ and $R_{N,i}^{\min} \le b_{N,i} / b_{N,2N-i} \le R_{N,i}^{\max}$ for $N \ge 2$ and $b_{1,1} \le B$.

where B is a matrix consisting of $b_{N,i}$,

$B_{N,i}^{\min}$ is a minimal value of the sum of $b^2{}_{N,i}$ and $b^2{}_{N,2N-i}$,

$B_{N,1}^{\max}$ is a maximal value of the sum of $b^2{}_{N,i}$ and $b^2{}_{N,2N-i}$,

$R_{N,i}^{\min}$ is a minimal ratio between $b_{N,i}$ and $b_{N,2N-i}$,

$R_{N,i}^{\max}$ is a maximal ratio between $b_{N,i}$ and $b_{N,2N-i}$.

19. The apparatus according to claim 14, wherein the generating unit is configured to generate R_N by a rule of

$$R_N = \begin{bmatrix} a_{N,1} & & & & & & & a_{N,2N-1} \\ & a_{N,3} & & & & & a_{N,2N-3} & \\ & & \ddots & & & \iddots & & \\ & & & a_{N,N-1} & a_{N,N+1} & & & \\ & & & -a_{N,N+1} & a_{N,N-1} & & & \\ & & \iddots & & & \ddots & & \\ & -a_{N,2N-3} & & & & & a_{N,3} & \\ -a_{N,2N-1} & & & & & & & a_{N,1} \end{bmatrix} \times$$

$$\begin{bmatrix} \mathbf{R}_{\frac{N}{2}} & \mathbf{0}_{\frac{N}{2}} \\ \mathbf{0}_{\frac{N}{2}} & \mathbf{R}_{\frac{N}{2}} \end{bmatrix} \begin{bmatrix} 1 & 1 & & & & \\ & & 1 & 1 & & \\ & & & & \ddots & \\ & & & & & 1 \quad 1 \\ 1 & -1 & & & & \\ & & -1 & 1 & & \\ & & & & \ddots & \\ & & & & & -1 \quad 1 \end{bmatrix}$$

where $a_{N,1}$, $a_{N,3}$, $\ldots a_{N,2N\text{-}1}$ are positive integers.

20. The apparatus according to claim 19, wherein the generating unit is further configured to select $a_{N,,\mathrm{k}}$, and $a_{N,2N\text{-k}}$ from prime pairs $for\ k = 1,3,\ldots,N-1$.

21. The apparatus according to claim 20, wherein

$$\frac{a_{N,k}}{a_{N,2N-k}} \approx \frac{\cos\left(\frac{k\pi}{4N}\right)}{\cos\left(\frac{(2N-k)\pi}{4N}\right)}\ for\ k = 1,3,\ldots,N-1.$$

22. The apparatus according to claim 14, wherein the coder is further configured to carry out a 2D transform through two cascading 1D transforms to obtain $Z_{2N\times 2N}$ by a rules of

$$\vec{U}_{2N,k} = \mathbf{E}_{2N}\vec{X}_{2N,k}$$
$$\mathbf{V}_{2N\times 2N} = \left[\vec{U}_{2N,0} \quad \vec{U}_{2N,1} \quad \cdots \quad \vec{U}_{2N,2N-1}\right]$$
$$\vec{Z}_{2N,k} = \mathbf{E}_{2N}\vec{V}^{T}_{2N,k}$$
$$\mathbf{Z}_{2N\times 2N} = \left[\vec{Z}^{T}_{2N,0}; \quad \vec{Z}^{T}_{2N,1}; \quad \cdots \quad \vec{Z}^{T}_{2N,2N-1}\right]$$

where

$\vec{X}_{2N,k}$ is a k-th column vector in $X_{2N\times 2N}$

$V_{2N\times 2N}$ is a $2N\times 2N$ matrix built by $\vec{U}_{2N,k}$ for $k \in \{0,1,\ldots,2N-1\}$

$\vec{V}_{2N,k}$ is a k-th row vector in $V_{2N\times 2N}$ for $k \in \{0,1,\ldots,2N-1\}$.

23. The apparatus according to claim 15, wherein E_N is an order-8 transform, an order-16 transform and an order-32 transform adopted in H.264/AVC.

24. The apparatus according to claim 12, wherein E_N is an order-8 transform, an order-16 transform and an order-32 transform adopted in AVS.

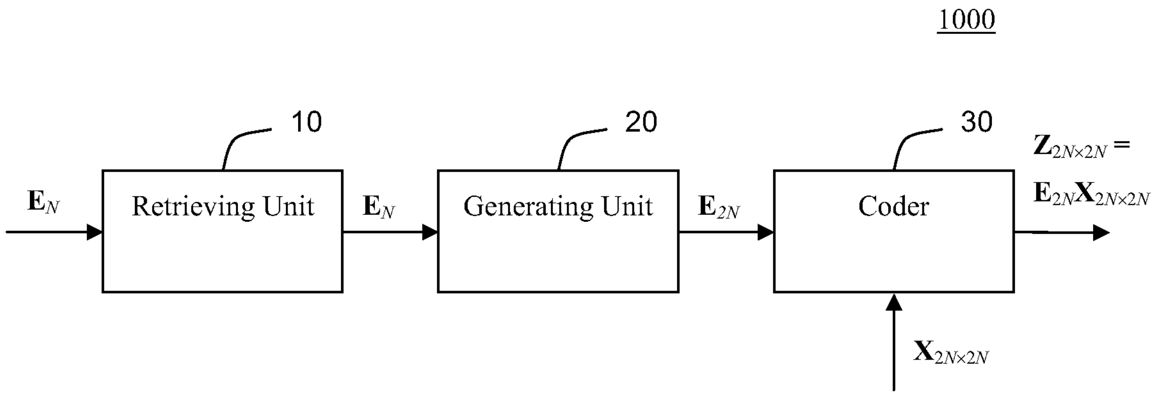
1000
10
20
30
$\mathbf{E}_N$
Retrieving Unit
$\mathbf{E}_N$
Generating Unit
$\mathbf{E}_{2N}$
Coder
$\mathbf{Z}_{2N\times 2N} = \mathbf{E}_{2N}\mathbf{X}_{2N\times 2N}$
$\mathbf{X}_{2N\times 2N}$

FIG.1

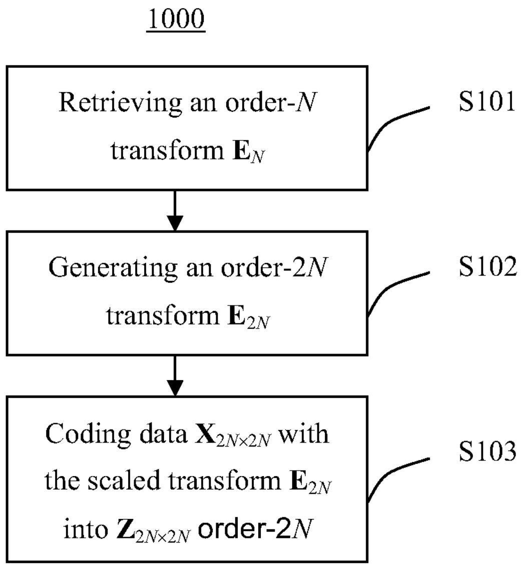
1000
Retrieving an order-N transform E_N
S101
Generating an order-2N transform E_2N
S102
Coding data X_2N×2N with the scaled transform E_2N into Z_2N×2N order-2N
S103

FIG. 2

INTERNATIONAL SEARCH REPORT

International application No.
PCT/CN2013/072084

A. CLASSIFICATION OF SUBJECT MATTER

H04N 7/00 (2011.01) i

According to International Patent Classification (IPC) or to both national classification and IPC

B. FIELDS SEARCHED

Minimum documentation searched (classification system followed by classification symbols)

IPC: H04N, G06T

Documentation searched other than minimum documentation to the extent that such documents are included in the fields searched

Electronic data base consulted during the international search (name of data base and, where practicable, search terms used)

CNPAT, WPI, EPODOC, CNKI: integer, transform, cod+, image, video, matrix, 2n, n, order, permutation, diagonal, identity, mirror+, recursive, sum, ratio, positive, column, row, vector, AVS, AVC, H.264

C. DOCUMENTS CONSIDERED TO BE RELEVANT

Category*	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
X	US 2009141796 A1 (HONG KONG APPLIED SCI&TECHNOLOGY RES) 04 June 2009 (04.06.2009) description, paragraphs [0015] to [0029], and figure 6	1,2,10-15,23,24
A		3-9,16-22
X	US 2009257504 A1 (UNIV CHINESE HONG KONG) 15 October 2009 (15.10.2009) description, paragraphs [0025] to [0045]	1,2,10-15,23,24
A		3-9,16-22
A	CN 1461150 A (UNIV ZHEJIANG) 10 December 2003 (10.12.2003) the whole document	1-24
A	CN 101042691 A (MEDIATEK INC) 26 September 2007 (26.09.2007) the whole document	1-24

☒ Further documents are listed in the continuation of Box C. ☒ See patent family annex.

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"E" earlier application or patent but published on or after the international filing date

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"O" document referring to an oral disclosure, use, exhibition or other means

"P" document published prior to the international filing date but later than the priority date claimed

"T" later document published after the international filing date or priority date and not in conflict with the application but cited to understand the principle or theory underlying the invention

"X" document of particular relevance; the claimed invention cannot be considered novel or cannot be considered to involve an inventive step when the document is taken alone

"Y" document of particular relevance; the claimed invention cannot be considered to involve an inventive step when the document is combined with one or more other such documents, such combination being obvious to a person skilled in the art

"&"document member of the same patent family

Date of the actual completion of the international search	Date of mailing of the international search report
29 May 2013 (29.05.2013)	**13 Jun. 2013 (13.06.2013)**

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INTERNATIONAL SEARCH REPORT

International application No.
PCT/CN2013/072084

C (Continuation). DOCUMENTS CONSIDERED TO BE RELEVANT

Category*	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
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A	CN 101990763 A (UNIV CHINESE HONG KONG) 23 March 2011 (23.03.2011) the whole document	1-24
A	CN 101232625 A (UNIV JILIN) 30 July 2008 (30.07.2008) the whole document	1-24

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Information on patent family members

International application No.
PCT/CN2013/072084

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CN 101232625 A	30.07.2008	None	