Sept. 22, 1936. R. L. DIETZOLD 2,054,794

WAVE FILTER

Filed June 9, 1934

FIG. 1

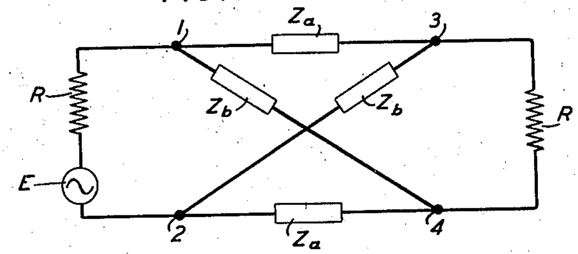


FIG. 2

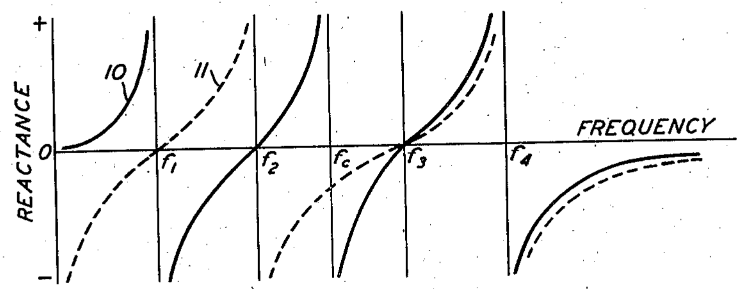


FIG. 3

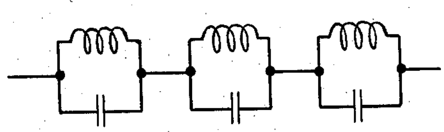


FIG. 4

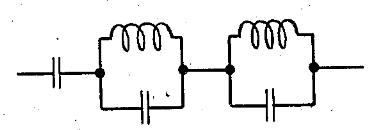
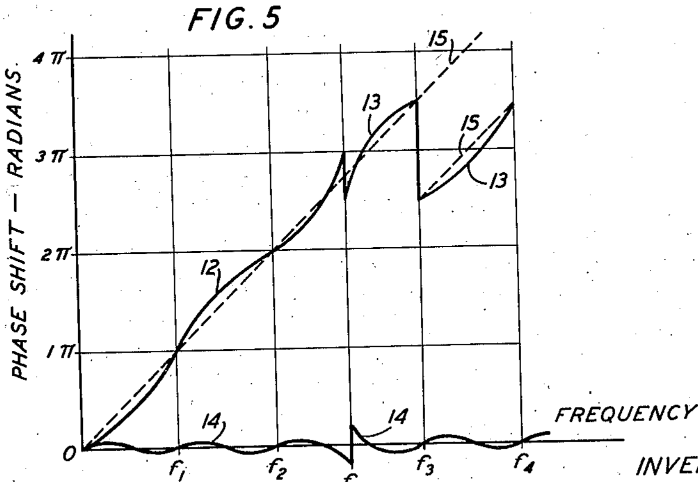


FIG. 5



INVENTOR
R. L. DIETZOLD
BY G H Stevenson
ATTORNEY

Patented Sept. 22, 1936 2,054,794

UNITED STATES PATENT OFFICE

2,054,794

WAVE FILTER

Robert L. Dietzold, New York, N. Y., assignor to Bell Telephone Laboratories, Incorporated, New York, N. Y., a corporation of New York

Application June 9, 1934, Serial No. 729,733

4 Claims. (Cl. 178—44)

This invention relates to frequency selective networks and more particularly to the control of the phase characteristics of broad band selective systems.

It has for its principal object the provision of a linear phase characteristic not only in the transmission band of a band selective system, but also through the band limits and into the attenuation ranges as far as may be desired.

This object is attained by the use in the branches of a filter network of multiple resonant impedances, the resonances and anti-resonances being distributed throughout the transmission band and the attenuation ranges in a particular manner as hereinafter described. The phase characteristic which is made linear in this way is the overall characteristic of the filter network in combination with its terminal impedances, which in practice will generally be fixed resistances. This overall characteristic involves not only the transfer constant of the filter network but also the wave reflection effects at the terminals and the desired linearity is obtained as the result of the proper coordination of the sum total of these reflection effects with the transfer characteristic of the filter network.

The nature of the invention will be more fully understood from the following detailed description and from the accompanying drawing of which:

Fig. 1 shows schematically a general type of network of the invention;

Fig. 2 is a reactance characteristic used in the explanation of the invention;

Figs. 3 and 4 illustrate the character of the impedances in a particular embodiment of the network of Fig. 1; and

Fig. 5 illustrates certain characteristics of the networks of the invention.

Referring to Fig. 1, the network illustrated comprises a symmetrical lattice having series and diagonal impedances Z_a and Z_b respectively, connected between equal terminal resistances R in series with one of which is a wave source E. The branch impedances may be of any degree of complexity, but should be substantially free from dissipation.

The properties of the symmetrical lattice are described at length in United States Patent 1,828,454, issued October 20, 1931 to H. W. Bode wherein it is shown that by particular allocation of the resonance and anti-resonance frequencies of the branch impedances, hereinafter designated critical frequencies, certain advantageous frequency characteristics of the image impedance and the transfer constant may be provided. The characteristics discussed in the Bode patent are those of the lattice per se, namely, its image impedance and transfer constant, as distinguished from the overall properties of the lattice plus the impedances between which it is connected. In the latter case the transmission characteristic of the system is not represented by the transfer constant alone but by this factor together with modifying factors representing the reflection effects at the junctions of the lattice and the external impedances. The total effect, which is a measure of the ratio of the currents at the receiving end of the system before and after the insertion of the lattice is termed the insertion transfer factor.

The present invention is concerned with the phase component of the insertion transfer factor, that is, with the sum of all the phase shifts in the system including those produced by reflection effects. The terms insertion phase shift and insertion phase characteristic are used to designate the phase component of the insertion transfer factor. In accordance with the invention this phase shift is made to have a linear variation with frequency not only within the transmission band but also through the attenuating ranges by a particular allocation of the critical frequencies of the branch impedances. This allocation is such that, except at each side of the cut-off frequencies, the critical frequencies are separated by a uniform interval both in the transmission band and in the attenuating ranges, the separation at each side of the cut-off frequencies being reduced to three quarters of the interval elsewhere.

The analysis which follows is directed to the demonstration of the linearity of the phase characteristic obtained by the simple frequency arrangement of the invention and to the determination of the lattice branch impedances so that the network will exhibit this linearity coupled with band selective properties.

The determination is further subject to the condition that the impedance arms be physically realizable. This condition imposes a certain functional form for the dependence upon the frequency, which may be quickly ascertained. The insertion transfer factor for the network is conveniently examined in terms of the image transfer constant, θ, and the image impedance, Z_I, which are related to the lattice impedances, Z_a and Z_b, say by the equations

$$\tan h\,\frac{\theta}{2}=\sqrt{\frac{Z_a}{Z_b}} \quad (1)$$

$$Z_I=\sqrt{Z_aZ_b} \quad (2)$$

Equation (1) shows that for free transmission, or for θ a pure imaginary, Z_a/Z_b must be negative. This result is achieved over an arbitrary frequency interval if Z_a and Z_b are reactances unlike in sign, that is, reactances of which the alternating resonances and anti-resonances correspond, a resonance in Z_a to an anti-resonance in Z_b and so son. Also, by Equation (1), the network attenuates in a frequency interval in which Z_a/Z_b is positive, for then θ is real. This ensues if Z_a and Z_b are alike in sign, or if resonances in Z_a correspond to resonances in Z_b, and so for anti-resonances. Since a condition for the physical realizability of a reactance is that its resonances and anti-resonances alternate, between an interval of transmission and an interval of suppression there must occur a critical frequency, the cut-off, in one impedance arm only.

The impedances Z_a and Z_b by their frequency variations and magnitudes completely determine the transmission properties of the network and for that reason may be termed characterizing impedances.

Therefore, filter properties are obtainable from a physically realizable lattice network if only the arms are reactances having the appropriate type of correspondence between their respective natural frequencies. This is illustrated in the case of the low-pass filter, in which the branch impedances Z_a and Z_b are of the types shown in Figs. 3 and 4 respectively, by the reactance expressions,

$$Z_a=iK_af\frac{\left(1-\frac{f^2}{f_2^2}\right)\left(1-\frac{f^2}{f_4^2}\right)}{\left(1-\frac{f^2}{f_1^2}\right)\left(1-\frac{f^2}{f_3^2}\right)}$$

$$Z_b=\frac{1}{iK_bf}\frac{\left(1-\frac{f^2}{f_1^2}\right)\left(1-\frac{f^2}{f_c^2}\right)\left(1-\frac{f^2}{f_4^2}\right)}{\left(1-\frac{f^2}{f_2^2}\right)\left(1-\frac{f^2}{f_3^2}\right)}$$

where K_a and K_b are constants and where f_1 and f_2 are critical frequencies in the transmitting band, f_c a cut-off intermediate between f_2 and f_3, and f_4 and f_5 critical frequencies in the attenuating band. For convenience the critical frequencies representing resonances are termed zeros and those representing anti-resonances are termed poles. Within the transmission band of a filter the zeros and poles are inversely coincident, that is, the zeros of the one impedance are coincident with the poles of the other impedances, while in the attenuation ranges the zeros and the poles of the two impedances are directly coincident, zeros with zeros and poles with poles. Plots of the impedances, showing the manner of coincidence of the resonant frequencies, are given by Fig. 2 in which full line curve **10** represents the frequency variation of the reactance of Z_a and dotted line curve **11** represents the variation of Z_b. With these values for Z_a and Z_b, the Equations (1) and (2) become

$$\tan h\,\frac{\theta}{2}=i\sqrt{K_aK_b}\,f\frac{\left(1-\frac{f^2}{f_2^2}\right)}{\left(1-\frac{f^2}{f_1^2}\right)\sqrt{1-\frac{f^2}{f_c^2}}} \quad (1a)$$

and

$$Z_I=\sqrt{\frac{K_a}{K_b}}\sqrt{1-\frac{f^2}{f_c^2}}\frac{\left(1-\frac{f^2}{f_4^2}\right)}{\left(1-\frac{f^2}{f_3^2}\right)} \quad (2a)$$

It is seen that θ is imaginary and Z_I real for $f<f_c$, whereas for $f>f_c$, θ is real and Z_I imaginary, corresponding to the case of the low-pass filter. It will expedite the discussion to confine the attention to this case, subsequently extending the results to high-pass and band-pass filters.

The relations (1*a*) and (2*a*) then indicate the form which the dependence of the image parameters upon the frequency must take in order that the network may be a physically realizable low-pass filter. Evidently no restriction is placed upon the number of transfer-constant controlling frequencies (f_1 and f_2 in the example) nor upon the number of impedance controlling frequencies (f_3 and f_4 in the example). The cut-off factor,

$$\sqrt{1-\frac{f^2}{f_c^2}}$$

may appear either in the numerator or denominator in the image impedance and transfer-constant expressions, provided only that zeros and infinities alternate. Since it is equally easy to proportion the elements for the desired effects whatever the number of critical frequencies, we shall first establish the general conditions, and later complete the specification of the example. Thus if transfer-constant controlling frequencies be distinguished by the subscript *a*, the impedance controlling frequencies by *b*, the general form may be written

$$\tan h\,\frac{\theta}{2}=\sqrt{\frac{Z_a}{Z_b}}=iK_1f\frac{\left(1-\frac{f^2}{f_{a_2}^2}\right)\ldots\left(1-\frac{f^2}{f_{a_{n-1}}^2}\right)}{\left(1-\frac{f^2}{f_{a_1}^2}\right)\ldots\left(1-\frac{f^2}{f_{a_n}^2}\right)}\sqrt{1-\frac{f^2}{f_c^2}} \quad (1b)$$

$$Z_I=\sqrt{Z_aZ_b}=K_2\sqrt{1-\frac{f^2}{f_c^2}}\frac{\left(1-\frac{f^2}{f_{b_2}^2}\right)\ldots\left(1-\frac{f^2}{f_{b_{m-1}}^2}\right)}{\left(1-\frac{f^2}{f_{b_1}^2}\right)\ldots\left(1-\frac{f^2}{f_{b_m}^2}\right)} \quad (2b)$$

wherein K_1 and K_2 are constant real quantities. The solution of (1b) and (2b) always yields physically realizable expressions for Z_a and Z_b if the K's are positive and

$$f_{a_1}<\ldots<f_{a_n}<f_c<f_{b_1}<\ldots<f_{b_m}.$$

The adjustment of the lattice elements according to the invention is equivalent to the determination of the K's, and f_a's, and the f_b's so that the network, when operating between constant resistance terminations, will introduce a rotation in phase linear with frequency over an arbitrary frequency interval. This interval is supposed to include the entire transmitting band and an adjacent part of the attenuating band. The region between the two bands in the neighborhood of the cut-off, where the characteristics change from the one type to the other, may be called the transition band.

The insertion constant of the network is defined by

$$e^{-\gamma}=\frac{I_r}{I_r'}$$

where I_r' and I_r are the received currents before and after the insertion of the network. When expressed in terms of the image parameters and the terminating resistance, R, γ is found to be a sum of the transfer constant and the reflection and interaction constants. These latter are defined respectively by

$$e^{\theta_r}=\frac{\left(1+\frac{Z_I}{R}\right)^2}{4\frac{Z_I}{R}} \quad (3)$$

and

$$e^{\theta_i}=1-\frac{\left(1-\frac{Z_I}{R}\right)^2}{\left(1+\frac{Z_I}{R}\right)^2}e^{-2\theta} \quad (4)$$

It may be noted in passing that the interaction constant defined by Equation 4 represents the repeated reflection of the initially reflected part of the current or wave as it passes back and forth between the terminal impedances and infinite number of times. The convenience of this form of expression becomes manifest when one examines the variation in the phase shift separately in the three intervals, the transmitting band, the attenuating band, and the transition band. In so doing is established the distribution of the critical frequencies f_{a_i} and f_{b_i} corresponding to linear phase shift.

Transmitting band.—From Equation (2b) it is seen that

$$\frac{Z_I}{R}$$

tends to 1 as f tends toward zero, if K_2 be taken equal to R. Furthermore, the form of the function is such that Z_I differs but little from R in this interval, the immediate vicinity of the cut-off having been set aside for the transition interval. On this account in the pass band the contributions of the reflection and interaction factors to the phase shift are negligible and the transfer constant represents substantially the total insertion loss. This is readily seen from Equations (3) and (4), the right-hand sides of which converge to the value unity as Z_I approaches the value R, corresponding to negligibly small values of the reflection and interaction constants θ_r and θ_i. Since this condition holds throughout the interval in question, the phase characteristic there is determined substantially wholly by the transfer constant alone. If $\theta=\alpha+j\beta$, where j is the imaginary unit, then by (1b),

$$\tan\frac{\beta}{2}=K_1 f\frac{\left(1-\frac{f^2}{f_{a_2}^2}\right)\cdots\left(1-\frac{f^2}{f_{a_{n-1}}^2}\right)}{\left(1-\frac{f^2}{f_{a_1}^2}\right)\cdots\left(1-\frac{f^2}{f_{a_n}^2}\right)}\sqrt{1-\frac{f^2}{f_c^2}} \quad (5)$$

when β increases by π as f varies from one critical frequency to the next. In order that the slope be constant throughout the interval, it is therefore necessary that the critical frequencies f_{a_i} be uniformly spaced. If this spacing is Δf, then the phase shift undulates about the chord

$$\frac{\beta}{2}=\frac{\pi}{2}\frac{f}{\Delta f}$$

having its ideal value at least at each critical frequency.

Attenuating band.—In this interval, the imaginary part of the transfer constant is either zero or π, while interaction effects are negligible on account of the factor $e^{-2\theta}$ in (4), with θ real. The part of the phase-shift dependent upon the frequency is therefore the imaginary part of the reflection constant, θ_r. Since Z_I is reactive in this range, the phase of the denominator of (3) is

$$\pm\frac{\pi}{2}$$

while that of the numerator is

$$2\arctan\frac{Z_I}{iR}$$

Thus

$$\beta_r=\mp\frac{\pi}{2}+2\arctan\frac{Z_I}{iR} \quad (6)$$

The significance of the constant term will appear presently. With the help of (2b), the second term is seen to be, in the attenuating band, a function of the same type as the transfer constant in the transmitting band. Hence, for the phase slope to be constant in the attenuating range, the impedance controlling factors also must constitute a chain of uniformly spaced resonances and anti-resonances. Since β_r increases by π between successive critical frequencies, the slope will be equal to the slope in the pass band if the uniform spacing is the same constant Δf in both ranges.

Transition band.—It remains to determine the frequency spacings adjoining the cut-off so that the phase curves in the transmitting and attenuating bands are joined through the transition band by a chord of the same slope. In this interval, which we suppose to be bounded by the last uniformly spaced critical frequencies in the transfer constant and impedance controlling chains and to contain only the cut-off frequency, neither the reflection nor interaction effects are negligible. In fact, for this method of decomposing the total insertion loss, these components become oppositely infinite at the cut-off. However, the interaction factor introduces no net change of phase over the interval, since it vanishes at one edge in virtue of Z_I equal to R very nearly and at the other in virtue of $e^{-2\theta}$ being very small. It may therefore be ignored in evaluating the total change in phase through this interval.

In the space adjoining the cut-off on the transmitting side, the phase of the transfer constant increases by π. In the space adjoining the cut-off on the attenuating side, the phase of the reflection factor increases by π. At the cut-off, however, where the image impedance changes from real to imaginary, the reflection factor introduces an abrupt change in phase of

$$-\frac{\pi}{2}$$

represented by the first term of (6). Therefore the net change in the transition interval is

$$\frac{3\pi}{2}$$

radians, and the interval must contain 3/2 uniform spaces if the average slope is to be correct. Considerations of symmetry require that the cut-off be the center of the interval, which thus comprises two three-quarter spaces.

These observations establish necessary conditions upon the frequency pattern corresponding to the requirement of linear phase shift in both transmitting and attenuating bands. The sufficiency of these conditions, when appropriate values have been assigned to K_1 and K_2 in Equations (1*b*) and (2*b*), may be verified by direct computation. For this purpose the formulae for the reflection and interaction factors are not useful because of the indeterminacy at the cut-off. This difficulty is avoided by expressing Z_I and θ in terms of the lattice impedances, in which event

$$e^{\gamma}=\frac{1+\frac{Z_aZ_b}{R^2}+\frac{Z_a}{R}+\frac{Z_b}{R}}{\frac{Z_a}{R}-\frac{Z_b}{R}}$$

If jX_a and jY_b be written for Z_a and Z_b, the insertion loss and phase shift, $A\gamma$ and $B\gamma$, are given by

$$e^{A\gamma}=\frac{\sqrt{1+\frac{X_a^2}{R^2}}\sqrt{1+\frac{X_b^2}{R^2}}}{\frac{X_a}{R}-\frac{X_b}{R}} \qquad (7)$$

and

$$\tan B\gamma=\frac{\frac{X_aX_b}{R^2}-1}{\frac{X_a}{R}+\frac{X_b}{R}} \qquad (8)$$

These formulæ contain the reflection and interaction effects and enable the total transmission to be conveniently calculated. It is instructive, however, to investigate the manner in which these effects combine with the transfer constant to produce the required performance. We have already noticed that in the three-quarter space adjoining the cut-off in the theoretical pass band, the transfer constant phase increases by π. At the cut-off the reflection factor acquires an imaginary component with Z_I, introducing first a phase rotation of

$$-\frac{\pi}{2},$$

which is increased by π radians in the three-quarter interval on the attenuation side of the cut-off. The contribution of the interaction constant must, of course, remove this phase discontinuity at the cut-off. In the pass band, its imaginary part is

$$\operatorname{arc}\left[1-\frac{\left(1-\frac{Z_I}{R}\right)^2}{\left(1+\frac{Z_I}{R}\right)}\underline{|2\beta}\right]$$

where Z_I is real. The limiting value of this angle as the cut-off is approached through frequencies in the pass band can be determined by expressing it in terms of the lattice impedances. At the cut-off, either X_a or X_b is either zero or infinite. Suppose that X_a is zero. Then this limit is arctan

$$\left(-\frac{R}{X_b}\right)_{f=f_c}$$

On the attenuation side of the cut-off, the imaginary part of the interaction constant is

$$\operatorname{arc}\left[1-e^{-2\alpha}\underline{|4\arctan\frac{Z_I}{jR}}\right]$$

where Z_I is imaginary. The limiting value of this expression, as the cut-off is approached through frequencies in the attenuation band, is found by the same method to be

$$\arctan\left(\frac{X_b}{R}\right)_{f=f_c}.$$

Thus the discontinuity in β_I at the cut-off is

$$\frac{\pi}{2}$$

as required to remove the discontinuity at this point introduced by the reflection effect.

The variation with frequency of the several phase shift components is illustrated by the curves of Fig. 5 for the case of the low-pass filter having impedances Z_a and Z_b in accordance with Figs. 3 and 4 respectively, and having the critical frequencies spaced in the manner described. Curve **12** represents the transfer phase shift β, that is, the phase component of the transfer constant of the lattice per se, in the transmission range from zero frequency to the cut-off. This component increases by π in each of the intervals between the critical frequencies, including f_c, and undulates about the straight line **15** departing therefrom by $\pi/4$ at the cut-off. In the figure, the undulations of this curve, as well as those of the other curves are somewhat exaggerated in order that their character may be exhibited.

Curve **13** represents the reflection phase shift β_r in the attenuating range, this component being zero in the transmission band. By virtue of the critical frequency spacing the general slope of this curve is that of the line **15** but it is characterized first by a departure of $-\pi/4$ at the cut-off and a sudden change of π at the critical frequency f_3. Since this latter change simply amounts to a reversal of phase its effect in general is not material.

Curve **14** represents the interaction phase shift. This curve is characterized by undulations of half the period of those of the other curves and by a sudden change of $\pi/2$ at the cut-off.

The total phase shift in the system is obtained by adding the three curves together in which case it will be noted that the discontinuity of curve **14** at the cut-off just neutralizes that at the junction of curves **12** and **13**$_2$. The resultant phase shift will therefore show a smooth variation which is very close to linear through the whole range from zero to f_3 and which continues at the same slope, subject to reversals at the critical frequencies, in the higher range.

The pattern for the transfer constant and impedance controlling frequencies which has been found is sufficient to insure only that the phase shift has its linear value at each critical frequency, or that the average slope in each space be the same. In order that the slope may closely approximate to the average at every intermediate point, it is further necessary to determine the multipliers K_1 and K_2 of the transfer constant and image impedance expressions. We have already seen that K_2 should be taken equal to the terminating impedance, R, so as to obtain impedance match and vanishing interaction effects in the pass band. K_1 may be evaluated from

Equation (5), of which the principal part in the limit of small f is

$$\frac{\beta}{2}=K_1 f$$

The chord with which the phase characteristic should coincide is

$$\frac{\beta}{2}=\frac{\pi}{2}\frac{f}{\Delta f}$$

whence

$$K_1=\frac{\pi}{2\Delta f}$$

in order that the phase curve have the proper slope at the origin. This relates the multiplier K_1 to the uniform spacing, Δf.

These conditions, applied to the case illustrated in Figs. 2, 3, and 4, serve completely to determine the elements of the lattice impedances. For the transfer constant controlling frequencies, f_1 and f_2, must be uniformly spaced, falling at Δf and $2\Delta f$. The cut-off, f_c, is separated from f_2 by three-quarters a uniform interval, and from the first of the uniformly spaced impedance controlling frequencies by a like interval. Thus, Equations (1a) and (2a) become

$$\tan h\ \frac{\theta}{2}=i\frac{\pi}{2}\frac{f}{\Delta f}\frac{\left(1-\frac{f^2}{(2\Delta f)^2}\right)}{\left(1-\frac{f^2}{\Delta f^2}\right)\sqrt{1-\frac{f^2}{(2.75\Delta f)^2}}} \qquad (1c)$$

and

$$Z_I=R\sqrt{1-\frac{f^2}{(2.75\Delta f)^2}}\frac{\left(1-\frac{f^2}{(4.5\Delta f)^2}\right)}{\left(1-\frac{f^2}{(3.5\Delta f)^2}\right)} \qquad (2c)$$

in which Δf may be selected to bring the cut-off to any desired point on the frequency scale. The solutions of these relations for the lattice impedances are then

$$Z_A=i\frac{\pi}{2}\frac{f}{\Delta f}R\frac{\left(1-\frac{f^2}{(2\Delta f)^2}\right)\left(1-\frac{f^2}{(4.5\Delta f)^2}\right)}{\left(1-\frac{f^2}{\Delta f^2}\right)\left(1-\frac{f^2}{(3.5\Delta f)^2}\right)}$$

and

$$Z_B=\frac{R}{i\frac{\pi}{2}\frac{f}{\Delta f}}\frac{\left(1-\frac{f^2}{\Delta f^2}\right)\left(1-\frac{f^2}{(2.75\Delta f)^2}\right)\left(1-\frac{f^2}{(4.5\Delta f)^2}\right)}{\left(1-\frac{f^2}{(2\Delta f)^2}\right)\left(1-\frac{f^2}{(3.5\Delta f)^2}\right)}$$

The element values for the impedances are readily found by expanding these expressions in partial fractions, after the manner described by R. M. Foster, "A reactance theorem", Bell System Technical Journal, v. 3, No. 2, April, 1924.

With this choice of parameters the greatest deviation of the phase slope from the average is found by computation to be of the order of 1 per cent. This approximation is satisfactory for most practical purposes. Since all the parameters of the network have been determined with an eye to the phase characteristic, this is accompanied by a unique loss characteristic. The loss characteristic is marked by reflection peaks at each impedance controlling frequency, where the lattice impedances are zero or infinite together. At these frequencies the image impedance changes sign, and therefore also the constant term of Equation (6). Thus, although the phase slope is uniform throughout the attenuating range, the phase characteristic itself has discontinuities of π radians at each impedance controlling frequency. This is the interpretation of the constant term of Equation (6). Whether this is an increase or a decrease of π radians is not distinguishable for a non-dissipative network. When parasitic dissipation of energy in the network elements is taken into account, the reflection peaks of loss have finite maxima and the phase in the neighborhood increases or decreases by π according as the line- or cross-arm of the lattice has the smaller resistance component at the peak frequency. The infinite peak at this frequency, and the associated abrupt change in phase, can evidently be restored by adding a lumped resistance to the smaller impedance so as to bring the arms into balance. This observation is of importance in considering the effect of dissipation on the phase shift.

When the network is constructed of physical elements its performance characteristics will be somewhat changed from those computed upon the assumption of pure reactance lattice arms. However, the relations subsisting between the real and imaginary parts of any analytic function such as the insertion constant enable these changes to be readily computed so long as the dissipation can be regarded as uniformly distributed among the elements. In fact, if d is the average ratio of resistance to reactance in the elements, and ΔA and ΔB are the variations in the insertion loss and phase shift due to the introduction of dissipation, we have approximately

$$\Delta A=\omega d\frac{\partial B}{\partial \omega} \qquad (9)$$

and

$$\Delta B=-\omega d\frac{\partial A}{\partial \omega} \qquad (10)$$

where the derivatives are computed for the network of pure reactances. The frequency variable ω is $2\pi f$. Now the dissipation is ordinarily concentrated chiefly in the coils, so that ωd is constant if the coil resistances are constant. Then the effect of dissipation upon the loss characteristic in a linear phase shift network is simply the addition of a uniform loss.

Moreover, throughout the transmitting band, in which

$$\frac{\partial A}{\partial \omega}=0$$

there is by Equation (10) no first order change in the phase characteristic. But in the transition interval, when the loss is increasing, the phase curve is displaced through dissipation from the ideal straight line. This effect may be compensated in two ways. It depends upon the dissipation being uniformly distributed among the resonant combinations of which the network is composed, and is modified if that distribution is modified. In particular if lumped resistance be added to the meshes resonating at the first impedance controlling frequency in such a way as to balance the lattice at this frequency, the phase curve will be restored to linearity, as predicted above.

The effect of dissipation on the phase characteristic in the transition interval may be otherwise corrected for by small variations in the ideal frequency pattern. By diminishing slightly the two three-quarter space intervals in the transition band, the non-dissipative phase slope may be caused progressively to increase through the band so that change due to parasitic dissipation displaces the characteristic toward, rather than away from, the ideal straight line. Since to shorten the cut-off spacing increases the selectivity of the network, the attenuation characteristic is improved by increase of dissipation in the impedances together with compensating modification of the frequency spacing in this way. The appropriate variations of the critical frequencies from their theoretical locations are best determined by trial.

It is possible in other ways to obtain a measure of control over the loss characteristic by means of slight variations in the ideal values of the parameters. For example, the loss may be increased at the cost of some degradation of the phase property by varying the constants K_1 and K_2.

Since the spacing of impedance controlling frequencies must be uniform over that portion of the attenuating band in which the phase slope is to be uniform, the extension of this condition over the infinite attenuating band of a low-pass filter would result in an infinite network. In practice the phase slope is seldom of interest very far into the attenuating band, so that the chain of uniformly spaced impedance controlling frequencies may be soon terminated. If the phase requirement ends at a frequency f_d, uniform spacing must be maintained through f_d. Then the infinite chain of uniformly spaced critical frequencies greater than f_d may be replaced by one or more critical frequencies so located that the corresponding factors approximate in the range below f_d to the factors associated with the omitted infinite sequence. The numerical determination of the terminating critical frequencies is simple, since a close approximation is obtained by use of one, or at most two, of them at somewhat extended spacings.

The foregoing discussion has for simplicity been confined to the case of the low-pass filter. Similar observations may be made in respect to band-pass and high-pass filters. For the band-pass filter we must have a chain of uniformly spaced critical frequencies in the pass band with cut-offs at three-quarter spacing at both edges. Uniform spacing of impedance controlling frequencies in both attenuating bands is resumed after three-quarter intervals beyond the cut-offs. Since the lower cut-off factor replaces the factor,

$$\frac{\pi f}{2\alpha}$$

of the transfer constant expression in the frequency range above the lower cut-off, the theoretical constant multiplier is unity. The multiplier of the image impedance expression is determined to make the impedance R at the mean of the cut-off frequencies.

The high-pass filter may be regarded as the limiting case of the band-pass filter as the upper cut-off recedes toward infinity. The preservation of linear phase shift over this infinite pass band would require an infinite network on account of the necessity of uniform spacing of transfer constant controlling frequencies, but if there is a frequency, f_d, beyond which the phase shift is not of interest, the high-pass filter may be realized in a finite network by terminating the chain of critical frequencies beyond this point in the manner described above for impedance controlling frequencies.

What is claimed is:

1. In a broad band selective system comprising a symmetrical reactance network having multiple resonant characterizing impedances Z_a and Z_b, and equal resistive terminal impedances connected to the input and the output terminals of the network, the method of producing a linear phase shift throughout the band and beyond the limits thereof which comprises spacing the critical frequencies of the characterizing impedances at uniform intervals throughout the greater portion of the transmission band and in a portion of an attenuation range beyond a band limit and spacing the critical frequencies on each side of said band limit at intervals therefrom substantially equal to three-quarters of the uniform interval elsewhere.

2. A broad band selective system comprising a symmetrical four-terminal reactance network having characterizing impedances Z_a and Z_b, and equal resistive terminal impedances connected to the input and the output terminals of said network, said characterizing impedances each having a plurality of critical frequencies which are spaced at uniform intervals throughout the greater portion of the transmission band and in an attenuation range beyond a cut-off frequency, and which on each side of the cut-off frequency are spaced at intervals substantially equal to three-quarters of the uniform spacing elsewhere whereby the insertion phase characteristic is linear throughout the band and a portion of the attenuation range.

3. A broad band selective system comprising a symmetrical four-terminal reactance network having characterizing impedances Z_a and Z_b, and equal resistive terminal impedances connected to the input and the output terminals of said network, said characterizing impedances having a plurality of critical frequencies certain of which lie within the transmission band and others of which lie outside the band and one of which determines a band limit, said critical frequencies being spaced at uniform intervals throughout the transmission band and in a portion of the attenuation range beyond said band limit and having a spacing on each side of said band limit substantially equal to three-quarters of the uniform spacing elsewhere whereby the insertion phase characteristic of the network is a substantially linear function of the frequency throughout the band and through the cut-off frequency.

4. A system in accordance with claim 3 in which the poles and zeros of the impedance Z_a are inversely coincident with the poles and zeros of the impedance Z_b within the band and are directly coincident with the poles and zeros of impedance Z_b outside the band.

ROBERT L. DIETZOLD.