Oct. 16, 1956 | O. J. ZOBEL | 2,767,380

IMPEDANCE TRANSFORMER

Filed Sept. 30, 1952

FIG. 1

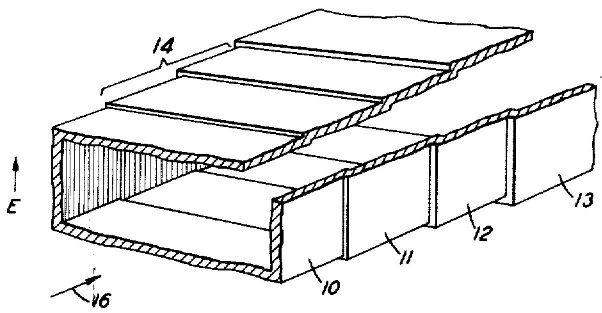


FIG. 2

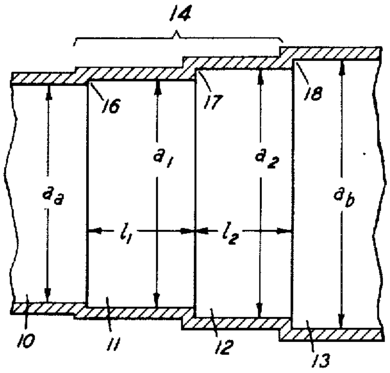


FIG. 3

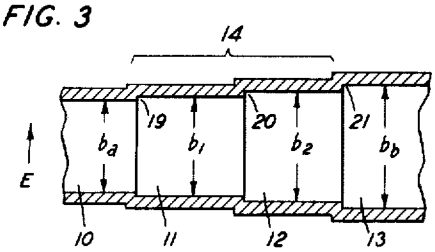


FIG. 4

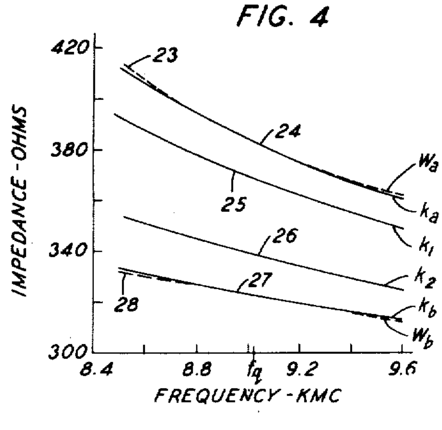


FIG. 5

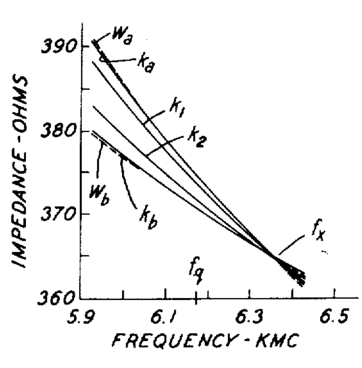
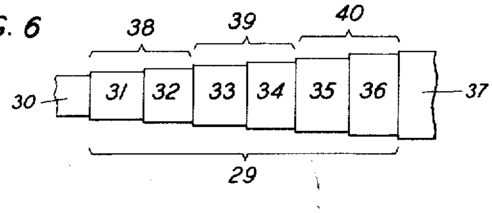


FIG. 6



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IMPEDANCE TRANSFORMER

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10 Claims. (Cl. 333—35)

This invention relates to wave transmission networks and more particularly to broad-band microwave impedance transformers.

An object of the invention is to connect without mismatch two resistive impedances which differ in the magnitude and slope of their impedance-frequency characteristics.

A more specific object is to match, over a band of frequencies, two wave guides of rectangular cross section which differs in one or both transverse dimensions.

In microwave systems, it is sometimes necessary to connect two wave guides having real characteristic impedances which differ from each other over the frequency band it is desired to transmit. The difference may be in magnitude only, in slope only, or in both magnitude and slope. For efficient transmission of energy from one wave guide to the other over a broad band of frequencies, they must be joined through a transducer, or impedance transformer, having image impedances which substantially match the characteristic impedances, respectively, of the guides throughout the band.

The present invention is directed to a broad-band microwave impedance transformer for connecting two resistive impedances whose impedance-frequency characteristics differ from each other in magnitude, slope, or both. The transformer comprises one or more pairs of tandem-connected sections of rectangular wave guide which, in general, differ from each other in length and in both transverse dimensions. The design method employed is based on the assumption that, since each section alone can be treated mathematically like a length of non-dissipative smooth transmission line, the physically connected sections can be similarly treated, neglecting any small errors arising from physical changes of cross section at the junction points. Actual results obtained have shown that this assumption is entirely justified. Explicit design formulas are presented for determining the required length, width, and height of each section of a two-section transformer. These formulas are based on the magnitude and slope of each of the desired image impedances of the transformer at the selected mid-band frequency. Also, a simple procedure is outlined for applying the formulas to a transformer comprising two or more pairs of sections connected in tandem. Each of the sections has a phase shift approximately equal to $\pi/2$ radians at the mid-band frequency. The transformer is especially useful in connecting two dissimilar wave guides of rectangular cross section. In this case, the transformer can be designed to provide a very close impedance match over a comparatively broad band of frequencies.

The nature of the invention and its various objects, features, and advantages will appear more fully in the following detailed description of preferred embodiments illustrated in the accompanying drawing, of which

Fig. 1 is a perspective view, with one corner cut away, of a two-section microwave impedance transformer in accordance with the invention connecting two rectangular wave guides;

Figs. 2 and 3 are, respectively, a sectional top view and a sectional side view of the transformer shown in Fig. 1;

Figs. 4 and 5 present typical impedance-frequency characteristics showing the matches obtainable with a transformer of the type shown in Figs. 1, 2, and 3; and

Fig. 6 is a side view of a transformer in accordance with the invention comprising three pairs of sections connected in tandem.

In the embodiment shown in Figs. 1, 2, and 3, two hollow-pipe wave guides **10** and **13** of oblong cross section are connected by an impedance transformer **14** comprising two sections **11** and **12** of rectangular wave guide connected in tandem. The guide **10** has an inside width a_a and an inside height b_a. For the guide **13** the corresponding dimensions are a_b and b_b, in the section **11** they are a_1 and b_1, and in the section **12** they are a_2 and b_2. The sections **11** and **12** have respective lengths l_1 and l_2, each equal to a quarter wavelength within the section. The side walls are made continuous between the different sections of guide to prevent the escape of energy. In general, this results in small steps in the width, at the junction points **16**, **17**, and **18** in Fig. 2, and small steps in the height, at the points **19**, **20**, and **21** in Fig. 3.

It will be assumed that microwave energy is to be transmitted from the guide **10** through the transformer **14** to the guide **13**, as indicated by the arrow **16** in Fig. 1. The flow of energy may, of course, as well be in the reverse direction. For analysis, it will be assumed further that the mode of transmission is the $TE_{1,0}$, in which the electric field E is perpendicular to the longer transverse dimension of the guide, as indicated in Figs. 1 and 3.

The section **11** will transmit waves above the critical or cut-off frequency f_{c1}, given by the expression

$$f_{c1}=c/2a_1 \qquad (1)$$

where c is the velocity of light and is equal to 3×10^{10} centimeters per second. The phase constant, β_1, of the section at any frequency, f, is

$$\beta_1=\frac{2\pi f_{c1}}{c}\sqrt{(f/f_{c1})^2-1} \qquad (2)$$

and its characteristic impedance, k_1, is

$$k_1=k_{\infty 1}/\sqrt{1-(f_{c1}/f)^2} \qquad (3)$$

where $k_{\infty 1}$ is the characteristic impedance at infinite frequency and is given by

$$k_{\infty 1}=60\pi^2 b_1/a_1 \qquad (4)$$

The design formulas are greatly simplified by introducing a parameter D_1, defined as

$$D_1\equiv 1/[1-(f_{c1}/f_q)^2] \qquad (5)$$

where f_q is the quarter-wave frequency at which the phase shift B_1 of the section is $\pi/2$ radians. The frequency f_q is ordinarily chosen as the geometric mean of the limits of the desired transmission band. It will be noted that D_1 will have a value greater than unity, since f_q will always be greater than f_{c1}.

For the section **12**, the cut-off frequency, f_{c2}, the phase constant, β_2, the characteristic impedance, k_2, its value, $k_{\infty 2}$, at infinite frequency, and the value of a second parameter, D_2, may be found from expressions similar, respectively, to (1), (2), (3), (4), and (5) except that the subscripts 1 are all changed to 2.

The composite transformer **14**, comprising the sections **11** and **12** connected in tandem, has a real image impedance W_a at the left end and a real image impedance W_b at the right end. The slops of the image impedance-frequency characteristics are, respectively, W_a' and W_b'. In accordance with the invention, the impedances W_a and W_b may be chosen arbitrarily at the frequency f_q, and their slopes W_a' and W_b' may be given any desired negative values at f_q. This imposes four conditions. There are also two more conditions, namely, that the phase shift B_1 in the section **11** and the phase shift B_2 in the section **12** shall each be $\pi/2$ radians at f_q; that is,

$$B_1 \equiv \beta_1 l_1 = B_2 = \beta_2 l_2 = \pi/2 \tag{6}$$

Since the transformer **14** has six variables, the dimensions a_1, a_2, b_1, b_2, l_1, and l_2, these six conditions can all be satisfied at the frequency f_q.

A recommended design procedure and explicit design formulas will now be presented, using the known quantities W_a, W_b, W_a', and W_b'. As the derivation of the formulas is lengthy and somewhat involved, it will be omitted.

First, the parameter D_1 is obtained as the root, greater than unity, of the cubic expression

$$D_1^3+\left[s+\frac{s(1-\rho)}{4(1+\rho)}+t-1\right]D_1^2+s\left[\frac{s(1-\rho)}{4(1+\rho)}+t-1\right]D_1+\frac{\rho s^2}{(1+\rho)^2}\left[t=\frac{s}{2}-1=0\right. \tag{7}$$

where the symbols ρ, s, and t are defined as

$$\rho \equiv \sqrt{W_b' W_a'} \tag{8}$$

$$s \equiv \frac{f_q}{2}\left[\frac{W_a'}{W_a}-\frac{W_b'}{W_b}\right] \tag{9}$$

and

$$t \equiv \frac{f_q}{4}\left[3\frac{w_a'}{w_a}+\frac{w_b'}{w_b}\right] \tag{10}$$

Next, evaluate the parameter D_2, which is given by

$$D_2 = D_1 + s \tag{11}$$

Now, the characteristic impedance k_1 of the section **11** is found from the formula

$$k_1 = W_a^{3/4} W_b^{1/4} U \tag{12}$$

where

$$U=\sqrt{\frac{D_1+\rho D_2}{\rho D_1+D_2}} \tag{13}$$

Then, the dimensions of the section **11** are found from the formulas

$$a_1=\frac{2}{2f_q}\sqrt{\frac{D_1}{D_1-1}} \tag{14}$$

$$b_1 = ck_1/120\pi^2 f_q\sqrt{D_1-1} \tag{15}$$

and

$$l_1=\frac{c}{4f_q}\sqrt{D_1} \tag{16}$$

Also, convenient expressions for the cut-off frequency f_{c1} of the section and its characteristic impedance $k_{\infty 1}$ at infinite frequency may be written in terms of the parameter D_1 as follows:

$$f_{c1}=f_q\sqrt{\frac{D_1-1}{D_1}} \tag{17}$$

and

$$k_{\infty 1}=k_1/\sqrt{D_1} \tag{18}$$

For the section **12**, the characteristic impedance k_2 is given by

$$k_2 = W_a^{1/4} W_b^{3/4} U \tag{19}$$

The dimensions a_2, b_2, and l_2, the cut-off f_{c2}, and the impedance $k_{\infty 2}$ may now be found from the following formulas:

$$a_2=\frac{c}{2f_q}\sqrt{\frac{D_2}{D_2-1}} \tag{20}$$

$$b_2 = ck_2/120\pi^2 f_q\sqrt{D_2-1} \tag{21}$$

$$l_2=\frac{c}{4f_q}\sqrt{D_2} \tag{22}$$

$$f_{c2}=f_q\sqrt{\frac{D_2-1}{D_2}} \tag{23}$$

and

$$k_{\infty 2}=k_2/\sqrt{D_2} \tag{24}$$

The transformer **14** will give a particularly close impedance match over a comparatively wide band if the image impedances W_a and W_b are to match the characteristic impedances of rectangular wave guides such as **10** and **13**. It will be assumed that the guide **10** has a cut-off frequency f_{ca} and, at the frequency f_q, a characteristic impedance k_a with a slope k_a'. For the guide **13**, the corresponding quantities are f_{cb}, k_b, and k_b'. The dimensions of the sections **11** and **12** are found from the formulas (14), (15), (16), (20), (21), and (22), but in using formula (7) to find D_1, the expression for ρ becomes

$$\rho=\sqrt{k_b/k_a} \tag{25}$$

and the expressions for s and t can be simplified to

$$s=D_b-D_a)/2 \tag{26}$$

and

$$t=1-(3D_a+D_b)/4 \tag{27}$$

where

$$D_a=1/[1-(f_{ca}/f_q)^2] \tag{28}$$

and

$$D_b=1/[1-(f_{cb}/f_q)^2] \tag{29}$$

This simplification can be made since, at f_q, the following relationships hold:

$$\frac{k_a'}{k_a}=\frac{1-D_a}{f_q} \tag{30}$$

and

$$\frac{k_b'}{k_b}=\frac{1-D_b}{f_q} \tag{31}$$

In this case, the characteristic impedances k_1 and k_2 are given by the formulas

$$k_1=k_a^{3/4}k_b^{1/4}U \tag{32}$$

and

$$k_2=k_a^{1/4}k_b^{3/4}U \tag{33}$$

The impedance-frequency characteristics in Fig. 4 show the match obtained with a two-section transformer **14** in accordance with the invention designed to connect a rectangular wave guide **10** in which a_a is 0.900 inch and b_a is 0.400 inch to a rectangular wave guide **13** in which a_b is 1.122 inches and b_b is 0.497 inch. The band to be transmitted extends between 8.5 and 9.6 kilomegacycles. The quarter-wave frequency f_q is taken as 9.033 kilomegacycles, the geometric mean of these frequencies. The required dimensions in inches of the sections **11** and **12**, found from the design formulas given above, are as follows:

$a_1=0.929$
$b_1=0.410$
$l_1=0.460$
$a_2=1.027$
$b_2=0.452$
$l_2=0.424$

The solid-line curves **24** and **27** show the characteristic impedances k_a and k_b, respectively, of the guides **10** and **13** to be matched. The curves **25** and **26** show the characteristic impedances k_1 and k_2, respectively, of the transformer sections **11** and **12**. The broken-line curves **23** and **28** show the image impedances W_a and W_b, respectively, of the transformer **14** at the left end and at the right end. It is apparent that an extremely close match is obtained, throughout a wide band, between the curves **23** and **24**, and also between the curves **27** and **28**. In fact, over a considerable portion of the range, the curves **23** and **24** are indistinguishable, and the same is true of the curves **27** and **28**.

The characteristics shown in **Fig.** 5 apply to another example of a two-section transformer in accordance with the invention, designed to connect a wave guide **10** in which a_a is 1.372 inches and b_a is 0.622 inch to a guide **13** in which a_b is 1.590 inches and b_b is 0.795 inch. The band to be passed extends between 5.925 and 6.425 kilomegacycles, and f_q is 6.170 kilomegacycles. The required dimensions in inches of the sections **11** and **12** are as follows:

$$a_1=1.408$$
$$b_1=0.652$$
$$l_1=0.652$$
$$a_2=1.512$$
$$b_2=0.735$$
$$l_2=0.618$$

The curves labelled W_a, W_b, k_a, k_b, k_1, and k_2 correspond, respectively, to the similarly designated curves in Fig. 4. In this case, also, the image impedances of the transformer **14** closely match the characteristic impedances of the guides **10** and **13**, as evidenced by the substantial coincidence of the curves for W_a and k_a, and also of the curves for W_b and k_b. This is an interesting example in that all of the curves cross at the frequency f_x. At this particular frequency, no impedance transformation is required, but the important point is that the transformer impedances match the guide impedances in slope.

In each of the examples disclosed herein, the insertion loss within the desired transmission band is not more than 0.00004 decibel greater than that of an ideal transformer. The corresponding voltage standing wave ratio is less than 0.053. This comparison neglects the effects of dissipation, which ordinarily will be very small in the short lengths of wave guide employed.

It should be pointed out that a two-section transformer in accordance with the present invention has a considerable advantage over a tapered transformer in that the former is easier to construct and is generally shorter in physical length.

In accordance with an extension of the invention, if the impedances k_a and k_b to be connected differ considerably, their difference is divided into two or more ranges. The formulas presented above are then used to design a two-section transformer for each of the ranges and the transformers are connected in tandem. Thus, by using two or more pairs of transformer sections, good impedance matches will be obtained over a broad frequency band even in this case.

Fig. 6 shows a side view of such a composite transformer **29** connecting a wave guide **30** of characteristic impedance k_a to a wave guide **37** of characteristic impedance k_b. The transformer **29** comprises three two-section transformers **38**, **39**, and **40** connected in tandem. Each of these component transformers is of the type shown in Figs. 1, 2, and 3, comprising two sections of wave guide connected in tandem. In the transformer **38** the sections are designated **31** and **32**, in the transformer **39** they are **33** and **34**, and in the transformer **40** they are **35** and **36**.

If the impedance range between k_a and k_b is to be covered in two equal step-up or step-down ratios, there will be an auxiliary dividing impedance $k_{d,1}$ such that

$$\frac{k_{d,1}}{k_a}=\frac{k_b}{k_{d,1}} \quad (34)$$

giving

$$k_{d,1}=\sqrt{k_a k_b} \quad (35)$$

Assuming that the impedance $k_{d,1}$ follows the law of wave guides, the design parameter $D_{d,1}$ is given by

$$D_{d,1}=(D_a+D_b)/2 \quad (36)$$

For this case, the composite transformer **29** will consist of the two two-section component transformers **38** and **39**. The required dimensions of the sections **31** and **32** are found from the design formulas given above, except that, in determining ρ, s, and t, $k_{d,1}$ is substituted for k_b in Equation 25 and $D_{d,1}$ is substituted for D_b in Equations 26 and 27. The required dimensions of the sections **33** and **34** are found in the same way, except that $k_{d,1}$ is substituted for k_a in Equation 25 and $D_{d,1}$ for D_a in Equations 26 and 27.

If there are n equal step-up or step-down ratios, the corresponding expressions for the jth dividing impedance $k_{d,j}$ and the jth design parameter $D_{d,j}$ are

$$k_{d,j}=k_a^{1-j/n}k_b^{j/n} \quad (37)$$

and

$$D_{d,j}=(1-j/n)D_a+(j/n)D_b \quad (38)$$

from which the dimensions of the various pairs of transformer sections may be found by applying the design formulas, after making the proper substitutions.

It is to be understood that the above-described arrangements are illustrative of the application of the principles of the invention. Numerous other arrangements may be devised by those skilled in the art without departing from the spirit and scope of the invention.

What is claimed is:

1. In combination, two wave guides and a transducer connecting said guides, said guides having characteristic impedances which differ in magnitude and slope at a selected frequency, said transducer comprising two sections of rectangular wave guide connected in tandem, each of said sections having a phase shift approximately equal to $\pi/2$ radians at said frequency, the transverse dimensions of one of said sections differing, respectively, from the transverse dimensions of the other of said sections, and said dimensions being so chosen that the image impedance of said transducer at said frequency substantially match said characteristic impedances, respectively, both in magnitude and in slope.

2. In combination, two wave guides and a transducer connecting said guides, said guides having characteristic impedances which differ in magnitude and slope at a selected frequency, said transducer comprising two sections of rectangular wave guide connected in tandem, the length and transverse dimensions of one of said sections differing, respectively, from the length and transverse dimensions of the other of said sections, and said dimensions being so chosen that the image impedances of said transducer at said frequency substantially match said characteristic impedances, respectively, both in magnitude and in slope.

3. In combination, two wave guides and a transducer connecting said guides, said guides having characteristic impedances which differ in magnitude and slope at a selected frequency, said transducer comprising a plurality of pairs of sections of rectangular wave guide connected in tandem, each of said sections having a phase shift approximately equal to $\pi/2$ radians at said frequency, said sections differing from section to section in both transverse dimensions, and said dimensions being so chosen that the image impedances of said transducer at said frequency substantially match said characteristic impedances, respectively, both in magnitude and in slope.

4. The comibnation in accordance with claim 3 in which said pairs of sections have substantially equal impedance step-up ratios at said frequency.

5. The combination in accordance with claim 3 in which the number of said pairs of sections exceeds two.

6. In combination, two wave guides and a transducer connecting said guides, said guides having characteristic impedances which differ in magnitude and slope at a selected frequency, said transducer comprising a plurality of pairs of sections of rectangular wave guide connected in tandem, said sections differing from section to section in length and in both transverse dimensions, and the lengths and transverse dimensions of said sections being

so chosen that the image impedance of said transducer at said frequency substantially match said characteristic impedances, respectively, both in magnitude and in slope.

7. The combination in accordance with claim 6 in which said pairs of sections have substantially equal impedance step-up ratios at said frequency.

8. The combination in accordance with claim 6 in which the number of said pairs of sections exceeds two.

9. In combination, a broad-band microwave impedance transformer and two wave guides, said transformer comprising two sections of rectangular wave guide connected in tandem, each of said sections having a phase shift equal to $\pi/2$ radians at a selected frequency, and the transverse dimensions of one of said sections differing, respectively, from the transverse dimensions of the other of said sections, whereby at said frequency the transformer has at one end a real image impedance of preselected magnitude and slope and at its other end a real image impedance of different preselected magnitude and slope, one of said guides being connected to said one end of said transformer and having a characteristic impedance which substantially matches, in magnitude and slope, the image impedance of said transformer at said one end, and the other of said guides being connected to said other end of said transformer and having a characteristic impedance which substantially matches, in magnitude and slope, the image impedance of said transformer at said other end.

10. In combination, two dissimilar wave guides and a transformer connected therebetween, one of said guides having a cut-off frequency f_{ca} and a characteristic impedance of magnitude k_a at a selected frequency f_q, the other of said guides having a cut-off frequency f_{cb} and a characteristic impedance of magnitude k_b at the frequency f_q, said transformer comprising two sections of rectangular wave guide connected in tandem, one of said sections having a length l_1 and transverse dimensions a_1 and b_1, the other of said sections having a length l_2 and transverse dimensions a_2 and b_2, and said dimensions having approximately the following values:

$$a_1=\frac{c}{2f_q}\sqrt{\frac{D_1}{D_1-1}}$$

$$b_1=ck_1/120\pi^2f_q\sqrt{D_1-1}$$

$$l_1=\frac{c}{4f_q}\sqrt{D_1}$$

$$a_2=\frac{c}{2f_q}\sqrt{\frac{D_2}{D_2-1}}$$

$$b_2=ck_2/120\pi^2f_q\sqrt{D_2-1}$$

$$l_2=\frac{c}{4f_q}\sqrt{D_2}$$

where c is the velocity of light and D_1 is the root, greater than unity, of the cubic

$$D_1^3+\left[s+\frac{s(1-\rho)}{4(1+\rho)}+t-1\right]D_1^2+s\left[\frac{s(1-\rho)}{4(1+\rho)}+t-1\right]D_1+\frac{\rho s^2}{(1+\rho)^2}\left[t-\frac{s}{2}-1\right]=0$$

where

$\rho=\sqrt{k_b/k_a}$
$s=(D_b-D_a)/2$
$t=1-(3D_a+D_b)/4$
$D_a=1/[-(f_{ca}/f_q)^2]$
$D_b=1/[1-(f_{cb}/f_q)^2]$
$D_2=D_1+s$
$k_1=k_a^{3/4}k_b^{1/4}U$
$k_2=k_a^{1/4}k_b^{3/4}U$

and

$$U=\sqrt{\frac{D_1+\rho D_2}{\rho D_1+D_2}}$$

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