July 6, 1965 C. F. HEMPSTEAD ETAL 3,193,734

SUPERCONDUCTING FLUX CONCENTRATOR

Filed March 22, 1962 2 Sheets-Sheet 1

FIG. 1

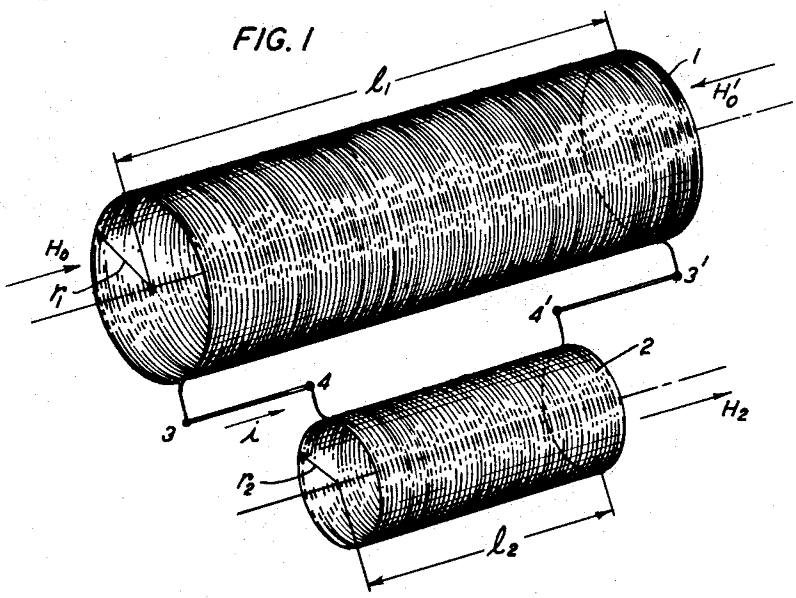

FIG. 2

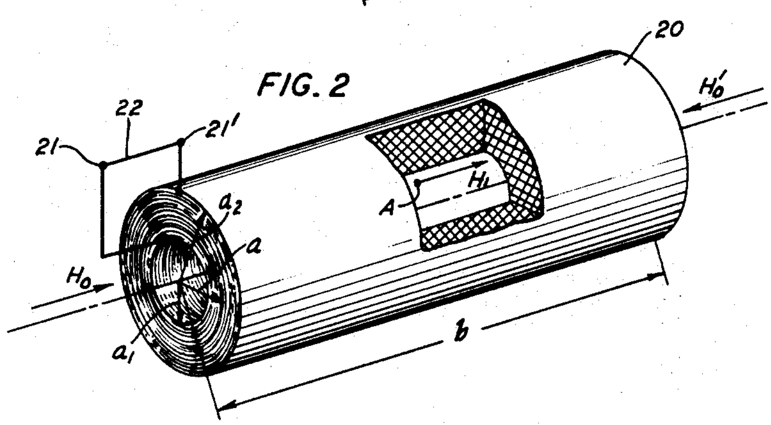

FIG. 3

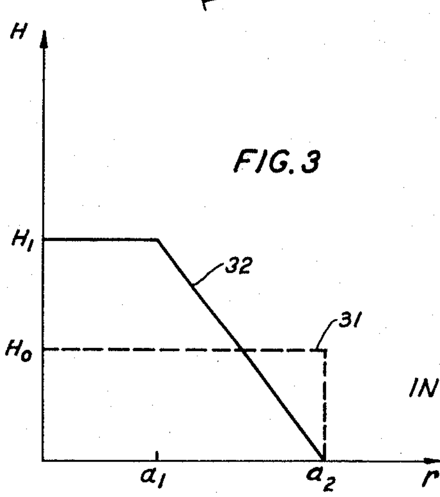

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FIG. 4

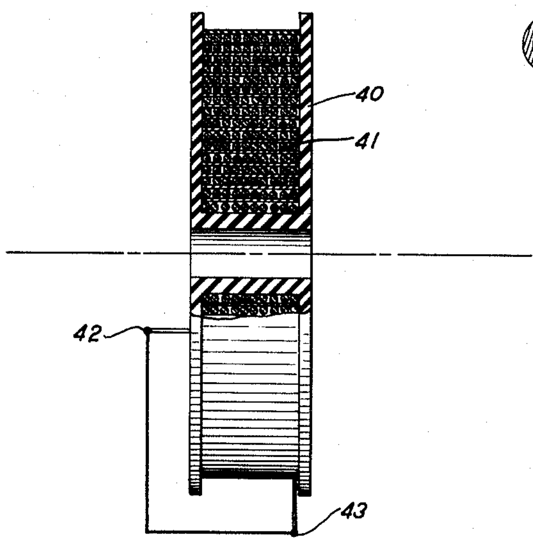


FIG. 4A

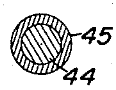
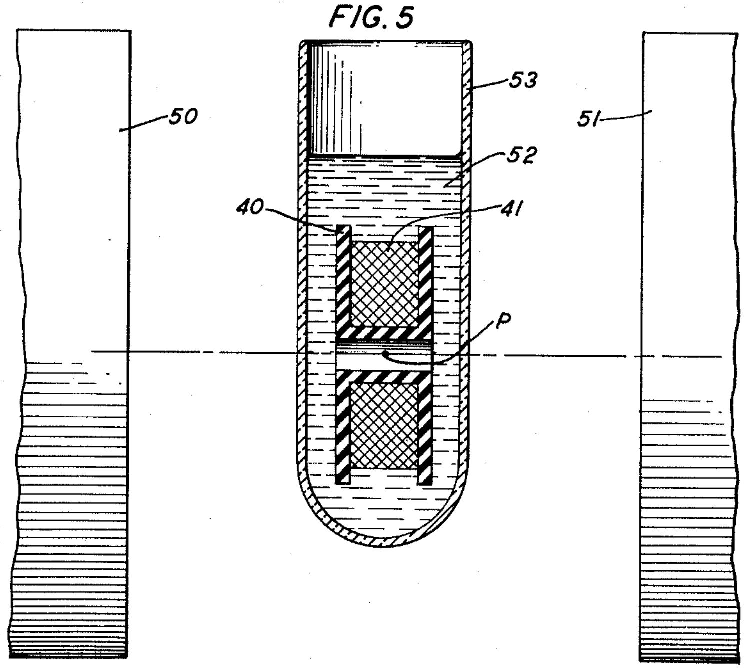


FIG. 5



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SUPERCONDUCTING FLUX CONCENTRATOR

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9 Claims. (Cl. 317—158)

This invention relates to magnetic circuits and more particularly to magnetic flux concentrators utilizing superconducting coils.

The need for very high magnetic fields has increased greatly in the past several years as a result of the rapid progress in many areas of physical research. Generally speaking, these high magnetic fields have, until recent years, been achieved in one of two ways. The first, the so-called "brute force" method, provides sustained magnetic fields at the cost of large and elaborate electromagnets and correspondingly large power and cooling apparatus. The second method provides short-duration pulsed magnetic fields with less elaborate magnetic coils but at the cost of shortened coil life. The first method is generally undesirable due to the prohibitive cost of the equipment, whereas the second method may be undesirable due to the short time duration of the magnetic field.

Although the use of superconductors in the construction of electromagnets has been known for half a century, it has only been relatively recently that work in this field was begun in earnest. This upsurge is, to a large degree, attributable to recent discoveries and improvements in wire which retain their superconductivity at useful current densities in the presence of magnetic fields in the order of 100 kilogauss and greater. As used herein, the term "superconducting wire" is understood to refer to wire formed of a material displaying zero resistivity to a current flowing therein at temperatures below a point referred to as the superconducting transition temperature. The attractiveness of magnets such as solenoidal coils wound of superconducting wire resides in the fact that such structures require no power to sustain the magnetic field once it is established. Thus it is now possible, through the use of superconducting wire, to produce large magnetic fields that can be sustained indefinitely with relatively small and inexpensive associated equipment.

Such magnets, however, still require means for establishing the initial current and for regulating it once it is established. Heretofore this requirement has necessitated current conducting elements between the low temperature superconducting coil and the high temperature power supply. As a result, heat is introduced into the low temperature environment surrounding the superconducting magnet both by resistive heating of, and by heat conduction through, the current conducting elements. This, in turn, necessitates additional refrigeration in order that the necessary low temperature be maintained.

It is therefore an object of the present invention to provide sustained high magnetic fields with superconducting coils requiring no external current source.

It is another object of the present invention to provide sustained high magnetic fields by concentrating the flux derived from relatively low magnetic fields.

The above objects are accomplished, in accordance with the principles of the present invention, through the use of shorted superconducting coils. It is well known that an electric current can be established in a closed conducting loop by induction. This law, first observed by Faraday, states that an E.M.F. (electromotive force) is set up in a circuit when the magnetic flux linking the circuit is changed in any manner. The magnitude of the E.M.F. is proportional to the time rate of change of flux-linkages with the circuit.

If the flux-linkages through a shorted coil are increased while it is in its normal conducting state, a current is induced in the coil. However, since circuit resistance gives rise to power dissipation, the induced current eventually dies out. If the coil is then cooled to a point below its superconducting transition point and the means by which the initial flux was established removed, another current is induced; however, since there are no longer any circuit losses, this second induced current is sustained. According to Lenz's law the direction of the induced E.M.F. is such that any current it produces tends to oppose the change of flux that produces it. The sustained current therefore causes the total flux linkages through the coil to remain substantially constant in both number and direction.

Although the total flux is conserved, the flux distribution can be varied. In accordance with the principles of the present invention, the conserved flux is distributed throughout a smaller volume by utilizing certain coil geometries so that a higher flux density is achieved.

In order that the invention may be clearly understood and readily carried into effect it will now be described with reference, by way of example, to the accompanying drawings in which:

FIG. 1 is a perspective view of one embodiment of the present invention utilizing two solenoidal superconducting coils;

FIG. 2 is a perspective view, partially broken away, of another embodiment of the present invention utilizing a single multilayer solenoidal coil;

FIG. 3 is a graphical representation of the flux distribution in the embodiment of FIG. 2;

FIG. 4 is a view, partially in cross-section, of still another embodiment of the present invention utilizing a coil of "pancake" geometry;

FIG. 4A is a cross-sectional view of a superconducting wire having an outer coating of normal conducting substance useful in practicing the present invention; and

FIG. 5 is a cross-sectional view showing the "pancake" coil of FIG. 4 in a test setup.

Referring more specifically to the drawings, there is shown in the perspective view of FIG. 1 one embodiment useful in explaining the principles of the present invention. FIG. 1 shows two single-layer solenoidal coils **1** and **2**. Coil **1** has a length l_1 and radius r_1 and is wound of superconducting wire with n_1 turns per unit length. Coil **2** has a length l_2, radius r_2, and is wound of superconducting wire with n_2 turns per unit length. The ends, **3–3'** of the winding of coil **1** are connected to ends **4–4'**, respectively, of coil **2** also by means of superconducting wire. Such wire can be formed of Nb-Zr, Nb_3Sn or any of the other superconducting materials known in the art.

In the course of the following analysis, it will be assumed that the lengths of coils **1** and **2** are long enough, with respect to their diameters, that any magnetic fields produced within the coils is substantially uniform. This assumption is verified by practice and results in an error of only a few percent.

While coil **1** is at a temperature above its superconducting transition temperature, a uniform external magnetic field H_0 is established through it in a direction parallel to its axis. As a result of the establishment of field H_0 a current is induced in coil **1**. However, this current quickly decays due to the resistance of the winding, which is in its normal conducting state.

With H_0 still applied, coils **1** and **2** and the connecting wires, are cooled to a temperature below their superconducting transition point by suitable refrigerating means not shown. The external magnetic field H_0 is then removed.

The flux linkage λ_1 in coil **1** is given by the equation

$$\lambda_1 = \pi r_1^2 H_0 n_1 l_1 \qquad (1)$$

The current i, induced in coil 1 when H_0 is removed, which current also flows through coil 2, has a magnitude given by

$$i=\frac{\lambda_1}{L_1+L_2} \tag{2}$$

where L_1 and L_2 are the self-inductances of coils 1 and 2, respectively, and are equal to

$$L_1=\mu(\pi r_1^2)l_1n_1^2 \tag{3}$$

$$L_2=\mu(\pi r_2^2)l_2n_2^2 \tag{4}$$

Substituting in Equation 2, the nondecaying induced current i is

$$i=\frac{\pi r_1^2 l_1 n_1 H_0}{\mu(\pi r_1^2 l_1 n_1^2+\pi r_2^2 l_2 n_2^2)} \tag{5}$$

It is recognized, however, that the quantities $(\pi r_1^2 l_1)$ and $(\pi r_2^2 l_2)$ in Equation 5 are the volumes enclosed by the windings of coils 1 and 2 and are designated V_2 and V_2, respectively.

As a result of current i flowing in coil 2, a magnetic field H_2 is produced therein. The magnitude of H_2 is given by

$$H_2=\mu n_2 i \tag{6}$$

Substituting for i and λ_1,

$$H_2=\frac{V_1n_1n_2H_0}{(V_1n_1^2+V_2n_2^2)} \tag{7}$$

The flux concentration factor β is defined as the ratio of H_2 to H_0 and is equal to

$$\beta=\frac{H_2}{H_0}=\frac{V_1n_1n_2}{V_1n_1^2+V_2n_2^2}=\frac{1}{\frac{n_1}{n_2}+\frac{V_2}{V_1}\frac{n_2}{n_1}} \tag{8}$$

From Equation 8, it is seen that as long as n_2 is greater than n_1, a flux concentration factor greater than unity can be achieved. For example, if n_1/n_2 equals 1/10 and V_2/V_1 equals 1/100, in the circuit of FIG. 1, the flux concentration factor β equals five.

FIG. 2 is a partially broken away perspective illustration of another embodiment of the present invention comprising a single multilayer solenoidal coil **20** wound of superconducting wire. Coil **20** has an outer radius a_2, an inner radius a_1, and a length b. The two ends **21** and **21'** of coil **20** are shorted together by means of a superconducting element **22** so that when coil **20** is cooled below its superconducting transition temperature, it defines a single current loop whose resistance is zero.

The operation of the flux concentrator of FIG. 2 can be explained if it is again assumed that the length b of coil **20** is quite long compared to its diameter and the winding has a substantially uniform turn density. Initially a uniform external magnetic field H_0 is established parallel to the axis of coil **20** in the direction shown while coil **20** is in its normal conducting state. The field H_0 is uniform throughout coil **20** from the axis to the outer radius. An initial current is induced in coil **20** which produces a field tending to oppose H_0. This current, however, decays rapidly, due to circuit resistance. Coil **20** is then cooled to its superconducting state and the external field H_0 removed. A new current i is induced in coil **20** which produces a magnetic field H_1 tending to conserve the total flux linkages λ_0 through the coil. The total flux linkages λ_0 can be calculated as follows:

The flux linkages with a turn at a distance r from the axis of coil **20** is given by

$$\lambda_r=\pi r^2 H_0 \tag{9}$$

Defining m as the number of turns per unit length along the axis of coil **20** and n as the number of layers of wire

per unit length in the radial direction, the total flux linkages through the coil is

$$\lambda_0=mb\int_{a_1}^{a_2}\pi r^2 H_0 n\,dr=\frac{\pi mbnH_0}{3}(a_2^3-a_1^3)=\frac{\pi mbnH_0}{3}(a_2-a_1)(a_2^2+a_2a_1+a_1^2) \tag{10}$$

In general, the electromotive force induced in a closed loop is given by

$$\text{E.M.F.}\frac{\partial\lambda}{\partial t}=iR$$

For a shorted coil in its superconducting state, however, $R=0$. Therefore,

$$\frac{\partial\lambda}{\partial t}=0 \text{ and } \lambda=\text{constant}$$

Although the total flux linkages λ_0 must remain constant after H_0 is removed, the distribution of the flux need not be uniform. In the multilayer solenoidal coil **20**, the flux density due to the induced current i is maximum in the center and decreases linearly from the inner radius a_1 to the outer radius a_2. At point A, in the mid-region of coil **20**, the resulting flux density can be calculated with the aid of the graph of FIG. 3.

In FIG. 3, the radius of coil **20** is plotted along the abscissa and the flux density along the ordinate. The dashed curve **31** represents the initial uniform flux density H_0 within coil **20**. The solid curve **32** shows the approximate distribution of flux density resulting from the induced current i. In practice, the sloping portion of curve **32** will depart from perfect linearity to the extent that the above-mentioned assumptions depart from reality.

The final flux density $H_{(r)}$ as seen from curve **32** can be written as

$$H_{(r)}=H_1 \text{ for } 0<r<a_1$$

$$=H_1\left(\frac{a_2}{a_2-a_1}\right)\left(1-\frac{r}{a_2}\right) \text{ for } a_1<r<a_2 \tag{11}$$

By utilizing the expressions of Equation 11 and integrating, the total flux λ_r' linking a single turn at an arbitrary radius r is

$$\lambda_r'=\frac{\pi H_1}{3(a_2-a_1)}(3a_2r^2-2r^3-a_1^3) \tag{12}$$

Equation 12, when integrated once again over the entire coil, yields an expression for the total flux linkages λ_1 due to current i.

$$\lambda_1=mb\int_{a_1}^{a_2}\frac{\pi H_1}{3(a_2-a_1)}(3a_2r^2-2r^3-a_1^3)n\,dr=\frac{\pi mbnH_1}{6}(a_2-a_1)(a_2^2+2a_2a_1+3a_1^2) \tag{13}$$

Since the total flux linkages must remain constant, Equations 10 and 13 can be combined. When this is done and when the resulting expression is simplified, the flux concentration factor β is found to be

$$\beta=\frac{H_1}{H_0}=2\frac{(a_2^2+a_2a_1+a_1^2)}{(a_2+2a_2a_1+3a_1^2)} \tag{14}$$

Therefore, with a coil having a length that is quite long compared to its diameter, it is seen that a maximum flux concentration factor of two is obtained as the inner radius approaches zero.

If a solenoidal coil such as coil **20** of FIG. 2 has a length b that is not long compared to its diameter, the flux concentration factor β can be greater than the value calculated from Equation 14. This is due to the fact that the lines of magnetic flux diverge near the ends of the coil; and whereas this effect can be neglected in the case of a long coil, it must be considered in a complete analysis of a shorter coil.

Since the lines of magnetic flux in the region between the inner and outermost turns of the coil diverge, it follows that their contribution to the total flux linkages is less. However, since the total flux linkages through the coil must remain constant, this means that the flux density near the center of the coil must increase. Calculations based on tabular data indicate a maximum flux concentration factor of approximately 3.8 for a solenoidal coil having an inner radius approaching zero and a length to outside diameter ratio of approximately .05.

In accordance with the principles of the present invention, the magnitude of the flux density can be further increased by the subsequent reapplication of the uniform external field in a direction opposite to the first direction, while the coil is still in its superconducting state. The additional current induced in the coil as a result of this reapplication of an external field establishes an additional magnetic field component tending to oppose the new change in flux. The additional magnetic field component thereby established is therefore in the same direction as the previously established field.

Referring once again to FIG. 2, after H_1, the field due to superconducting current, has been established, a new uniform external magnetic field H_0' is applied along the axis of coil **20** as shown. The total current i is caused to increase as a result, until the total flux linkages through coil **20** equals the original flux λ_0 plus the new flux linkages. At a point A within the coil the resulting flux density is equal to H_1 plus the new flux density $\beta H_0'$ minus the external field flux density H_0'. The total flux at point A, designated H_1', is therefore given by the equation

$$H_1'=\beta H_0+H_0'(\beta-1) \qquad (15)$$

The same principle applies equaly well to the embodiment of FIG. 1. In that embodiment the new external field H_0' would be applied only to coil **1** in a direction opposite H_0 and the new flux in coil **2** would become equal to $\beta H_0+\beta H_0'$.

It is obvious that the new external field H_0' can be used as a "fine" adjustment means for the flux concentrators; and if applied in the original direction of H_0, can be used to decrease the degree of flux concentration as well as increase it as explained above.

A more practical embodiment, similar to that of FIG. 2, is shown in the perspective view of FIG. 4. In this embodiment, Nb_3Sn wire of the type described in the copending application of E. Buehler and J. E. Kunzler, Serial No. 81,400, filed January 9, 1961, now United States Patent No. 3,124,455 issued March 10, 1964, was utilized. In this embodiment, which is of the so-called "pancake" geometry, the wire, having a diameter of approximately .025 inch, was wound on a stainless steel form **40**. There were 39 layers of winding consisting of 19½ turns per layer. Each layer was separated by a .001 inch wrapper of stainless steel. The inside diameter of the coil **41** was .25 inch, the outside diameter was 2.276 inches and the length was .51 inch. As in the case of the previous embodiment, the ends **42** and **43** of the coil were shorted together.

FIG. 5 is a simplified cross-sectional view of the coil of FIG. 4 installed in a typical test setup. Coil **40** is shown suspended in the air gap of a magnet having a controllable magnetic field. For reasons of clarity the suspension means for coil **40** have not been shown. Likewise, only the pole pieces **50** and **51** of the magnet have been illustrated. Coil **40** is oriented near the center of the air gap with its axis perpendicular to the faces of pole pieces **50** and **51**. Surrounding coil **40** is a Dewar flask **53**. A uniform magnetic field of 6.8 kilogauss was established through coil **40** while it was in its normal conducting state. The temperature was then reduced to a point below the critical temperature of the Nb_3Sn wire by adding liquid helium **52** to the flask and the field was removed. The magnetic flux density was measured at

point P in the center of coil **40** and found to be approximately 15 kilogauss and the resultant flux concentration factor approximately 2.2.

As mentioned above, the present invention can be practiced by utilizing any of the superconducting wire known in the art. The operation of the invention is enhanced, however, by utilizing wire having a core of superconducting material and an outer layer of normal or nonsuperconducting material. A cross-sectional view of such wire is shown in FIG. 4A wherein the superconducting core is designated by numeral **44** and the normal conducting outside coating is designated by numeral **45**. Typical of this type wire is the Nb_3Sn wire disclosed in the above-mentioned copending application of E. Buehler and J. E. Kunzler and the metallically insulated wire disclosed in the copending application of T. H. Geballe, Serial No. 52,409, filed August 29, 1960, now United States Patent No. 3,109,963 issued November 5, 1963.

As described in the last-mentioned copending application of T. H. Geballe, by utilizing superconducting wire having an outer layer of nonsuperconducting material of low resistivity, the stability of operation is improved. In case of a temporary excursion above the superconducting transition point, the current in the superconducting core of this wire is transferred to the nonsuperconducting outer layer. Once the superconductivity of the core is reestablished, the reverse process takes place and the current is coupled back into the superconducting core. In the meantime, some energy will have been lost due to resistive heating, depending upon the length of time the device was above the superconducting transition point. The amount of energy loss, however, is quite small for short excursions above the superconducting transition point.

In all cases it is understood that the above-described circuits represent only a limited number of embodiments of the present invention. Many other embodiments including those utilizing different coil geometries and core materials can be constructed by those skilled in the art without departing from the spirit and scope of the present invention.

What is claimed is:

1. A flux concentrator comprising, a first coil of superconducting wire having n_1 turns per unit length wound in the form of a cylinder having a volume V_1, a second coil of superconducting wire having n_2 turns per unit length wound in the form of a cylinder having a volume V_2, where $n_1<n_2$ and

$$\frac{V_2}{V_1}<\frac{n_1}{n_2}\left(1-\frac{n_1}{n_2}\right)$$

means for connecting the first and second ends of said first coil to the first and second ends of said second coil respectively, external means for establishing a constant magnetic field component along the axis of said first coil while said coil is in its normal conducting state, means for changing both of said coils to their superconducting state, and means for subsequently removing said externally applied magnetic field.

2. The flux concentrator according to claim **1** wherein said superconducting wire is Nb-Zr.

3. The flux concentrator according to claim **1** wherein said superconducting wire is Nb_3Sn.

4. The flux concentrator according to claim **1** wherein said superconducting wire has an outer coating of a normal conducting substance.

5. A flux concentrator comprising, in combination, a multilayer solenoidal coil wound of superconducting wire, means for shorting the ends of said coil, external means for establishing a substantially uniform magnetic field through said coil parallel to the axis thereof, said externally applied field being established while said coil is in its normal conducting state, means for changing said coil to its superconducting state, and means for subsequently removing said externally applied field.

6. The flux concentrator according to claim 5 wherein said superconducting wire is Nb-Zr.

7. The flux concentrator according to claim 5 wherein said superconducting wire is Nb_3Sn.

8. The flux concentrator according to claim 5 wherein said superconducting wire has an outer coating of a normal conducting substance.

9. The method of concentrating the flux of a magnetic field comprising the steps of, applying an external uniform magnetic field through a shorted multilayer coil wound of superconducting wire in a direction substantially parallel to the axis of said coil while said wire is in its normal conducting state, cooling said wire to a temperature below its superconducting transition temperature, removing said external field, and applying a controllable

external uniform magnetic field parallel to the axis of said coil in a direction opposite that of said first applied magnetic field.

References Cited by the Examiner

UNITED STATES PATENTS

3,088,077 4/63 Schmitt et al. ------------ 336—30

OTHER REFERENCES

McFee: Application of Superconductivity to the Generation and Distribution of Electric Power, Electrical Engineering, February 1962, pages 122–129.

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