July 14, 1959 H. P. ROBINSON 2,894,688

ELECTRONIC ANALOG COMPUTER FOR DETERMINING RADIOACTIVE DISINTEGRATION

Filed April 28, 1955

2 Sheets-Sheet 1

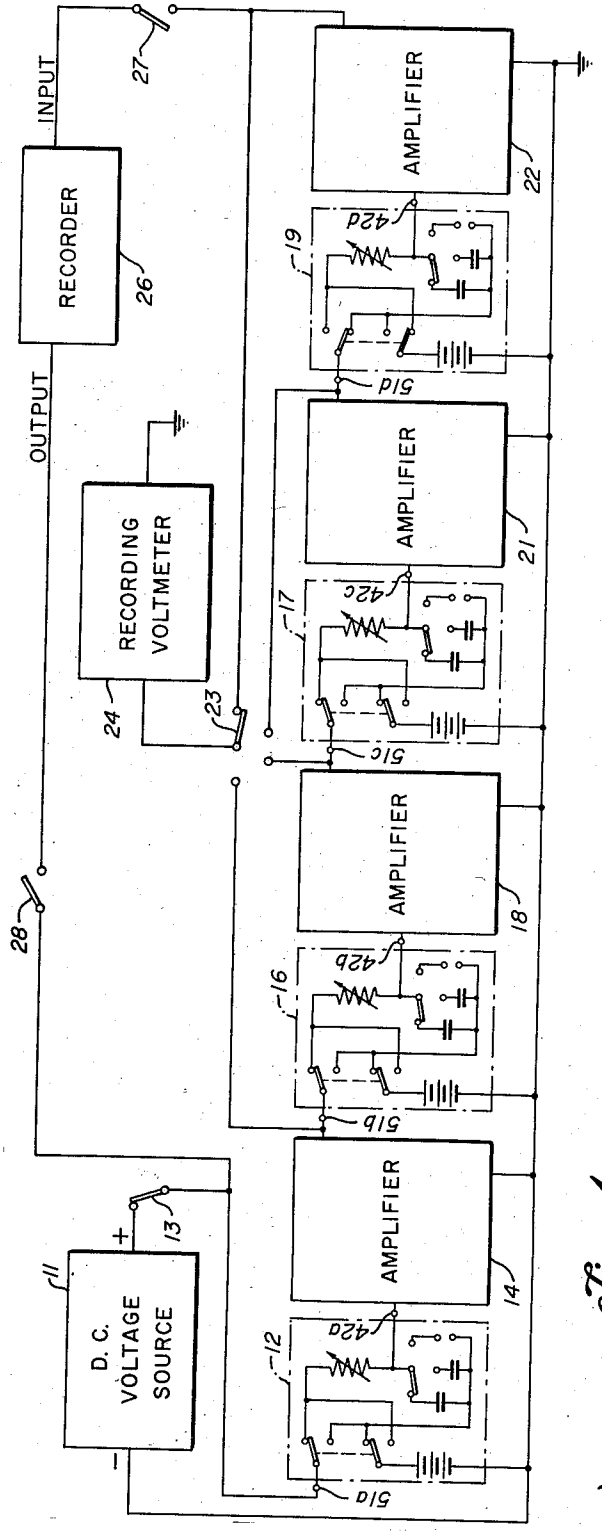
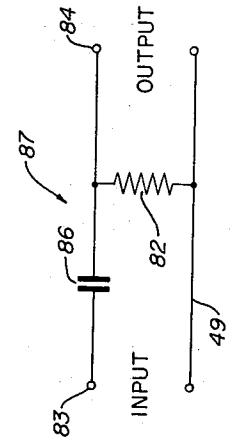


Fig. 1.



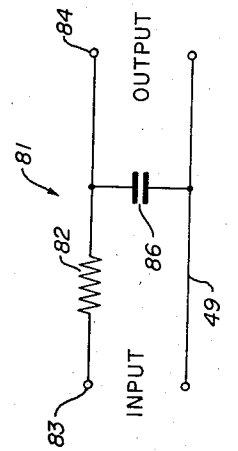


Fig. 3.

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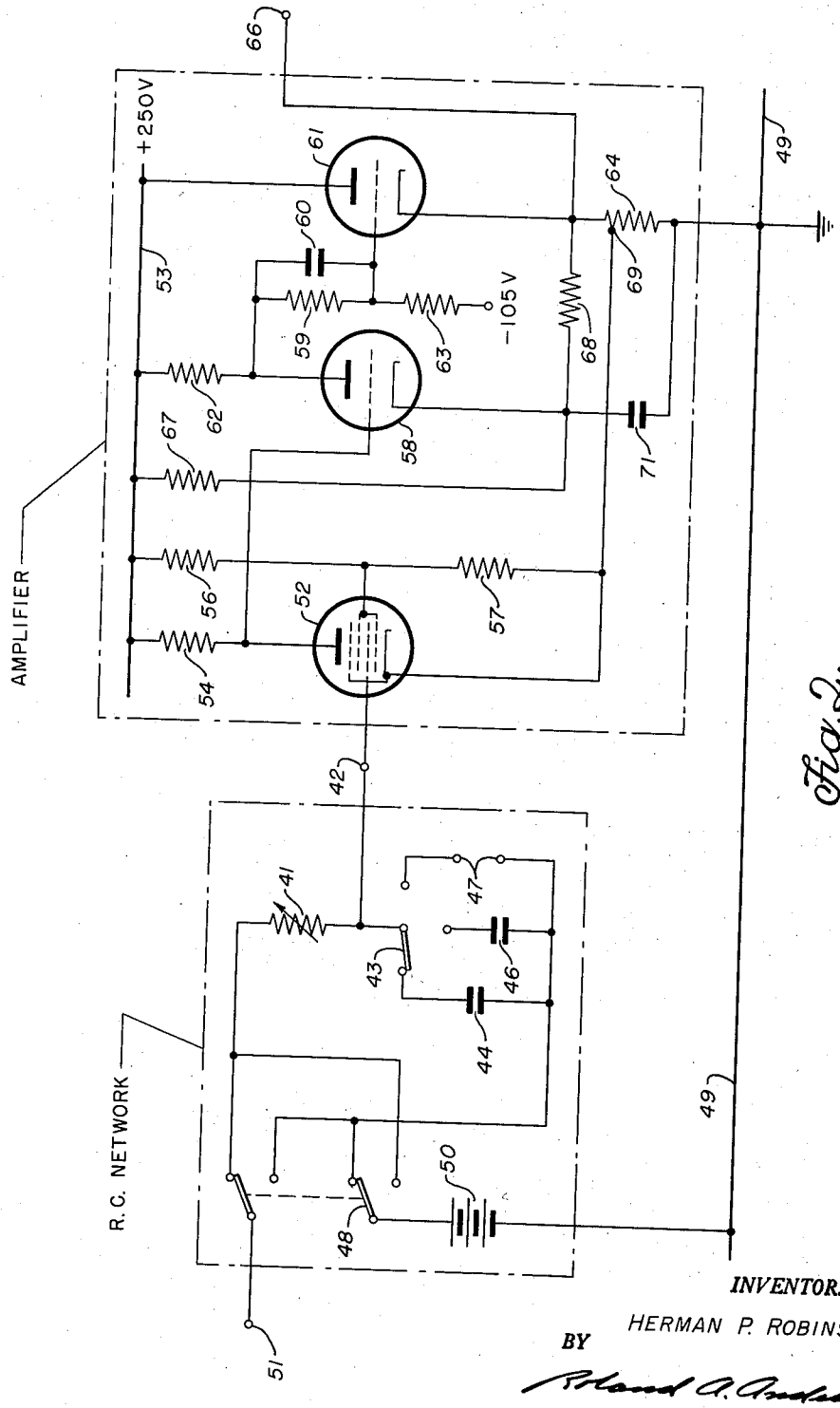


Fig. 2.

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ELECTRONIC ANALOG COMPUTER FOR DETERMINING RADIOACTIVE DISINTEGRATION

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4 Claims. (Cl. 235—184)

The present invention relates to computers and more particularly to an analog computer for plotting the growth and decay of members of a radioactive disintegration chain.

In the study and utilization of nuclear energy, one important phenomenon to be considered is the disintegration of a non-stable isotope of an element through several successive steps until a stable end product is formed. For instance, in the neptunium series, Pu^{241} disintegrates through a chain of thirteen steps before becoming stable as Bi^{209}. The half lives of the various steps vary from a small portion of a second to over two million years.

A first unstable element will disintegrate to form a second element, while the second element in turn disintegrates to form a third element, etc. Frequently, it is desirable to know the rate at which, for instance, the eighth element in the chain will grow and then decay. Mathematical computation of such information is a very laborious and time consuming task.

The present invention is a computer for facilitating the computation of growth and decay curves of radioactive elements. Use is made of the fact that the voltage decay curve of a resistance-capacitance network originally charged to a unit potential is analogous to the decay curve of an unstable radioactive element. With several resistance-capacitance networks connected in series, the time constant of the first resistance-capacitance network is set to represent the half life of the first radioactive element, the time constant of a second resistance-capacitance network is set to represent the half life of the second element in the chain, etc. Each of the resistance-capacitance networks is isolated by unity gain amplifiers, designed especially for use with this computer. With the application of a direct current voltage to the first resistance-capacitance network, the various networks charge and discharge in relation to the growth and decay of the elements they represent. It should be noted that a comparable system might be set up using resistance-inductance networks.

The usefulness of the device is not limited to growth and decay problems. It is equally applicable to the solution of problems involving pile bombardments in which elements are built up by succesive neutron capture. For neutron capture problems, the time constants of the resistance-capacitance networks are set proportional to the reciprocals of the capture cross sections. The computer then gives a response that describes accurately the growth and decay of the elements of the chain.

The instrument can be used to solve for an intermediate half life or cross section if the other constants are known merely by trying various values until the computer produces a result in agreement with experiment.

It is then an object of the present invention to facilitate mathematical computations associated with the study of growth and decay of radioactive substances.

It is another object to provide an analog computer wherein resistance-capacitance networks are utilized to simulate the growth and decay of radioactive substances.

It is a further object to provide a direct current amplifier having low output impedance and precise reproduction of an input signal, whereby the study of radioactive substances is facilitated.

The invention, both as to its organization and method of operation, together with further objects and advantages thereof, will be better understood by reference to the following specification taken in conjunction with the accompanying drawing in which:

Figure 1 is a circuit diagram of one embodiment of the present invention;

Figure 2 is a schematic diagram of a preferred embodiment of a resistance-capacitance network and isolation amplifier represented in block form in Figure 1; and

Figure 3 is a schematic diagram of two types of resistance-capacitance networks.

Referring now to Fig. 1 there is shown a circuit diagram of the present invention. A direct current voltage source **11** supplies an initiating voltage to the input terminal **51***a* of a first resistance-capacitance network **12** when an initiating voltage switch **13** is closed. The direct current voltage source **11** preferably has a low output impedance and supplies a well regulated potential. The internal connections of the resistance-capacitance networks will be subsequently described in detail with regard to Figure 2.

A first unity gain isolation amplifier **14** couples the output terminal **42***a* of the first resistance-capacitance network **12** to the input terminal **51***b* of a second resistance-capacitance network **16**. The first isolation amplifier **14** must have a very high input impedance so that no power is taken from the first resistance-capacitance network **12**. The output impedance of the first isolation amplifier **14** must be very low so that the internal resistance of the amplifier **14** does not contribute appreciably to the resistance of the resistance-capacitance network **16**. A third resistance-capacitance network **17** is coupled to the second resistance-capacitance network **16** by a second isolation amplifier **18** in the same manner that the second resistance-capacitance network **16** is coupled to the first resistance-capacitance network **12**. Similarly, a fourth resistance-capacitance network **19** is coupled to the third resistance-capacitance network **17** by a third isolation amplifier **21**. A fourth isolation amplifier **22** couples the output terminal **42***d* of the fourth resistance-capacitance network **19** to a contact on an output switch **23**. The output switch **23** has a plurality of contacts, each having a connection to the output of one of the isolation amplifiers. By rotation of the output switch **23** the output of any one of the isolation amplifiers may be selectively applied to a recording voltmeter **24**. The recording voltmeter **24** produces a permanent record of the charge and discharge curve of the resistance-capacitance network preceding the amplifier output selected by the output switch **23**.

It should be readily apparent that additional resistance-capacitance networks and amplifiers may be added in series with those shown here, thereby making the device more convenient for use with problems involving radioactive chains having many steps. However, if it is necessary to solve a problem having more steps than there are resistance-capacitance networks, it is still possible to obtain a solution. For example, suppose that with the computer of Figure 1 it is desired to find the growth and decay curve for the seventh element in a chain. The time constants of the resistance-capacitance networks **12**, **16**, **17** and **19** are set to represent the first, second, third and fourth steps respectively in the chain and a curve then obtained for the fourth element in the chain. The time constants of the resistance-capacitance networks **12**, **16** and **17** are next set to represent the fifth, sixth and seventh elements of the chain and a curve is obtained

representing the output of the third isolation amplifier 21. Such latter curve and the curve obtained for the fourth element are then combined mathematically by a convolution integral to obtain a final curve for the seventh element. Another system for accomplishing the same result involves recording the curve for the fourth element in the chain with a tape recorder 26 having an input selectively connected to the output of the fourth isolation amplifier 22 through a record switch 27. After setting the computer for the fifth, sixth and seventh elements in the radioactive chain, the tape recorder 26 output is connected through a playback switch 28 to the input of the first resistance-capacitance network 12 and the previously recorded voltage curve of the fourth element is applied to the computer. The recording voltmeter 24 will then indicate a curve representing the growth and decay of the seventh element in the radioactive chain.

Referring now to Figure 2, there is shown a representative resistance-capacitance network and a D.C. isolation amplifier as indicated in Figure 1. A variable resistor 41 has one end connected to both an output terminal 42 and to the center arm of a capacitor switch 43. With the capacitor switch 43, either a first network capacitor 44, a second network capacitor 46, or a pair of network capacitor terminals 47 may be selectively connected in series with the variable resistor 41. Thus a wide range of resistance-capacitance network time constants may be selected with provision for additional plug-in type capacitors which may be connected between the capacitor terminal pair 47. Increased flexibility may be secured by adding means for inserting additional resistance in series with the variable resistor 41. The capacitors used in the resistance-capacitance network preferably have a very low hysteresis loss such as is afforded by those having a polystyrene dielectric.

With a double-pole, double-throw switch 48, either end of the resistance-capacitance network may be connected to a ground bus 49 through a bias source 50, shown here as a battery, while the opposite end of the resistance-capacitance network is connected to an input terminal 51. The input terminal 51 is provided for ready connection to the output of the preceding isolation amplifier or the D.C. voltage source 11 and the initiating voltage switch 13, depending upon the position of the resistance-capacitance network in the computer. The input terminal 51 is selectively connected to either the variable resistor 41 or (with the capacitor switch 43 in the position shown) to the first network capacitor 44, depending upon the type of problem being solved and the position of the output switch 23 in Fig. 1 (hereinafter discussed in more detail).

Under quiescent circuit conditions, the voltage applied to the input terminal 51 from the preceding isolation amplifier has approximately a twenty volt direct current bias. With the variable resistor 41 connected to the input terminal 51, the output terminal 42 also has a twenty-volt bias. If the double-pole, double-throw switch 48 is in the alternate position, the bias source 50 maintains the twenty volt quiescent potential at the output terminal 42 so that an amplifier coupled to the output terminal 42 receives a constant quiescent direct current voltage level. Otherwise the potential at the output terminal 42 would decrease to zero with the double-pole, double-throw switch in the alternate position.

Referring now to the amplifier portion of Fig. 2, the control grid of an input tube 52 is connected to the output terminal 42 of the resistance-capacitance network. In the input tube 52 the control grid is surrounded by a screen grid for minimum control grid current, the input tube 52 being operated as an electrometer tube. The plate is coupled to a B plus bus 53 by a first plate resistor 54 having a typical value of 30 megohms. With a potential (from any conventional power supply) of 250 volts supplied to the B plus bus 53, the plate potential of the input tube 52 will be about ten volts plate to cathode. Similarly, the screen grid of the input tube 52 is also held at approximately a ten volt screen to cathode potential by a connection to the juncture of a first screen grid resistor 56 and a second screen grid resistor 57 connected in series between the B plus bus 53 and the cathode of the input tube 52.

Voltage variations appearing at the plate of the input tube 52 are applied to the control grid of a phase inverter tube 58 by a direct connection. The plate of the phase inverter tube 58 is coupled through a grid resistor 59, in parallel with a peaking capacitor 60, to the control grid of a cathode follower tube 61, the plate also being coupled through a plate resistor 62 to the B plus bus 53. A bias resistor 63 couples the control grid of the cathode follower tube 61 to a convenient supply of negative control grid bias (not shown). The plate of the cathode follower tube 61 is connected directly to the B plus 53 and the cathode is coupled to the ground bus 49 through a feedback resistor 64. An amplifier output terminal 66 provided for convenient coupling to a succeeding resistance-capacitance network is connected to the cathode of the cathode follower tube 61. A first voltage divider resistor 67 is connected between the B plus bus 53 and the cathode of the phase inverter tube 58. A second voltage divider resistor 68 is connected between the cathode of the phase inverter tube 58 and the cathode of the cathode follower tube 61. The first and second voltage divider resistors 67 and 68 serve to provide proper operating potential for the cathode of the phase inverter tube 58 and the second voltage divider resistor 68 also acts as a positive feedback path from the amplifier output terminal 66 to the cathode of the phase inverter tube 58. A negative feedback path is formed by a connection from the cathode of the input tube 52 to a movable tap 69 on the feedback resistor 64. A cathode capacitor 71 connected from the cathode of the phase inverter tube 58 to the ground potential bus 49, together with the peaking capacitor 60, provides for stability in the amplifier.

Considering now the operation of the amplifier, assume that a unit of positive potential is applied to the control grid of the input tube 52 from the output terminal 42 of the resistance-capacitance network. Electron flow through the input tube 52 is then increased and the potential at the plate and also at the control grid of the phase inverter tube 58 decreases. The potential at the plate of the phase inverter tube 58 and at the control grid of the cathode follower tube 61 then increases positively causing the potential at the cathode of the cathode follower tube 61 to also rise. Owing to the connection to the movable tap 69 near the cathode end of the feedback resistor, a portion of the potential increase at the cathode of the cathode follower tube 61 is applied to the cathode of the input tube 52.

Since the control grid of the input tube 52 originally was impressed with a positive signal voltage, the rise in cathode potential at the input tube 52 restores the tube to nearly the original no-input-signal operating condition, the potential on the elements having been increased by nearly the same amount. Thus the plate of the input tube 52 and the control grid of the phase inverter tube 58 are more positive by nearly the amount of the original input signal potential. The positive voltage at the cathode of the cathode follower tube 61 is also applied to the cathode of the phase inverter tube 58 as positive feedback, approximately matching the rise in the control grid potential, thus holding the phase inverter tube 58 within operating range, but with the plate at a higher potential necessary to stabilize the cathode follower 61 at the new operating condition. The circuit then is comprised of a positive feedback loop within a negative feedback loop, the positive feedback loop having a longer time constant than the negative feedback loop.

Any change in the potential applied to the control grid

of the input tube 52 is matched by a corresponding change in the cathode and screen grid potentials. This permits the use of an electrometer type tube with attendant low grid current, a condition necessary for obtaining accurate answers from the computer. The input tube 52 retains the characteristics of an electrometer type tube with input voltages over a 60 volt range.

At the amplifier output terminal 66 the impedance presented to the succeeding resistance-capacitance network is necessarily of a low value so that the internal impedance of the amplifier does not affect the time constant of the resistance-capacitance network. The amplifier described here has an output impedance less than ten ohms. The voltage gain of the amplifier must be unity so that quantitative measurements made of the various elements in the radioactive chain are on the same scale. If the cathode of the input tube 52 is connected direct to the amplifier output terminal 66 the gain of the amplifier is slightly less than unit. By connecting the cathode of the input tube 52 to a movable tap 69, a slightly lower negative feedback voltage is applied to the cathode of the input tube 52 and the circuit may be adjusted to obtain exactly unity gain.

By the application of Laplace transforms to the differential equations which form a solution to the growth and decay of the radioactive elements, it is found that:

$$A \xrightarrow{\lambda_1} B \xrightarrow{\lambda_2} C \xrightarrow{\lambda_3} \cdots N \xrightarrow{\lambda_n}$$

$$V_N=\frac{1}{s}\cdot\frac{\lambda_1}{s+\lambda_1}\cdot\frac{\lambda_2}{s+\lambda_2}\cdots\frac{s}{s+\lambda_n}$$

where

A, B, C . . . N represent various elements in a radioactive chain

V_N=transform of amount of N expressed in atoms or equivalent, based on unit amount of A at time zero.

s=Operator

λ=Decay rate constant of a radioactive element, the subscript indicating the position of the element in the chain.

Referring now to Fig. 3 there is shown a resistance-capacitance network 81 in which a resistor 82 is connected between an input terminal 83 and an output terminal 84. A capacitor 86 is connected between the output terminal 84 and the ground bus 49 (neglecting the bias source 50 of Fig. 1). The resistance-capacitance network 81 may be rearranged to form a resistance-capacitance network 87 wherein the capacitor 86 is connected between the input terminal 83 and the output terminal 84 with the resistor 82 connected between the output terminal 84 and the ground bus 49. The resistance-capacitance networks 81 and 87 may be described by their transfer functions which can be written as Laplace transforms provided there is no load on the circuits. Thus the resistance-capacitance network 81 has the transfer function

$$\frac{1}{RCs+1}=\frac{1/RC}{s+1/RC}$$

and the resistance-capacitance network 87 has

$$\frac{RCs}{RCs+1}=\frac{s}{s+1/RC}$$

where RC=time constant of the resistance-capacitance network.

The quantity RC is analogous to $1/\lambda$, therefore it may be seen by reference to the equation for V_N that for each factor except $1/s$ a resistance-capacitance network may be substituted. The factor $1/s$ may be represented by a unit step function such as is produced by turning on the initiating voltage switch 13 and applying voltage from the D.C. voltage source 11 to the resistance-capacitance network 12 in Fig. 1.

To set up an analog to measure the growth and decay of the nth component, a series of resistance-capacitance values proportional to the respective $1/\lambda$ values is chosen, noting that the last resistance-capacitance network is reversed from the others, controlled by the position of the double-pole, double-throw switch 48 of Fig. 2. With all the capacitors initially at equilibrium voltage, the initiating voltage switch 13 is closed and voltage growth and decay of the nth component recorded as a function of time.

In practice, it is sometimes expedient to vary the operation of the computer. For instance, as shown in Fig. 1, if growth and decay of the fourth step in a chain is being measured, normally all but the fourth-resistance-capacitance network in the series is of the type designated as resistance-capacitance network 81 of Fig. 3, the fourth being connected as the resistance-capacitance network 87. However, if the fourth resistance-capacitance network has a very short time constant, the voltage applied to the recording voltmeter 24 will be quite small and the resulting curve will have a very low amplitude. If, for instance, the second resistance-capacitance network has a longer time constant than any other in the series being measured, then it may be changed to the type of resistance-capacitance network 87 and the fourth element may be represented by the type of resistance-capacitance network 81. In this way the amplitude of the curve drawn by the recording voltmeter 24 is multiplied by the factor,

$$\frac{\text{RC of second network}}{\text{RC of fourth network}}$$

and the characteristics of the curve are more easily observed.

While the invention has been disclosed with respect of a single preferred embodiment, it will be apparent to those skilled in the art that numerous variations and modifications may be made within the spirit and scope of the invention, and thus it is not intended to limit the invention except as defined in the following claims.

What is claimed is:

1. In a computer, the combination comprising a plurality of resistance-capacitance networks connected in a cascade and having adjustable time constants each adjustable in proportion to the decay rate constant of an element in a radioactive chain, the last one of said resistance-capacitance network in said cascade having a series connected capacitance and a parallel connected resistance across the output, the others of said resistance-capacitance networks in said cascade having a series connected resistor and a parallel connected capacitance across the output, a plurality of unity gain isolating amplifiers, said amplifiers being alternated with said resistance-capacitance networks in series arrangement therewith, a voltmeter, switch means for coupling said voltmeter to a selected one of said resistance-capacitance networks, and a direct current voltage supply connectable to the first of said series connected resistance-capacitance networks in said cascade.

2. In an analog computer, the combination comprising a plurality of series connected resistance-capacitance networks, the last such network in said series being of the differentiator class, the others of said resistance-capacitance networks in said series being of the integrator class, a plurality of unity gain direct coupled amplifiers one connected between each successive pair of said resistance-capacitance networks in series therewith, said amplifiers each having a high impedance input stage and a low impedance output stage, an initiating voltage supply coupled to a first of said series connected resistance-capacitance networks, and means connected to graphically record voltage fluctuations occurring in at least one of said resistance-capacitance networks.

3. In an analog computer, the combination comprising a plurality of series connected resistance-capacitance networks having controllably variable time constants adjustable in proportion to the growth-decay rate constant

of an element in a radioactive chain, one of said resistance-capacitance networks in said series being selected to represent the longest growth-decay rate and having a series connected capacitance and a parallel connected resistance across the output, the others of resistance-capacitance networks in said series having a series connected resistor and a parallel connected capacitance across the output, a plurality of unity gain isolating amplifiers one connected between each successive pair of said resistance-capacitance networks in series arrangement therewith, a voltmeter selectively connectable with a particular one of said resistance-capacitance networks, and a direct current voltage supply selectively coupled to the first of said series connected resistance-capacitance networks.

4. In an analog computer, the combination comprising a plurality of cascade-coupled resistance-capacitance networks having variable time constants adjustable into proportionality with growth-decay constants of elements in a radioactive growth-decay series, a plurality of isolating amplifiers one connected in series between each adjacent pair of said resistance-capacitance networks, a voltmeter adapted to be coupled to any selected one of said resistance-capacitance networks, a direct current supply, switch means connecting said current supply with the first of said cascade coupled resistance-capacitance networks, recording means having an input connectable with the last resistance-capacitance network in said cascade and having an output connectable with the first resistance-capacitance network in said cascade whereby the capacity of said computer may be extended by delayed application of a previously recorded potential from the last resistance-capacitance network in said cascade to the first resistance-capacitance network in said cascade after adjustment of said resistance-capacitance networks to represent subsequent elements in said radioactive growth-decay series.

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