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- (71) **Applicant:** KOFAX, INC. [US/US]; 15211 Laguna Canyon Road, Irvine, California 92618-3603 (US).
- (72) **Inventors:** MACCIOLA, Anthony; 15283 Glen Ridge Drive, Chino Hills, California 91709 (US). MA, Jiyong; 10507 Blue Granite Drive, San Diego, California 92127 (US). SHUSTOROVICH, Alexander; 39 Stonington Drive, Pittsford, New York 14534 (US). THRASHER, Christopher W.; 13 Woodruff Glen, Rochester, New York 14624 (US). AMTRUP, Jan; 4615 N. Park Avenue, #1211, Chevy Chase, Maryland 20815 (US).
- (74) **Agent:** KOTAB, Dominic M.; P.O. Box 721120, San Jose, California 95172 (US).

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(54) **Title:** SYSTEMS AND METHODS FOR THREE DIMENSIONAL GEOMETRIC RECONSTRUCTION OF CAPTURED IMAGE DATA

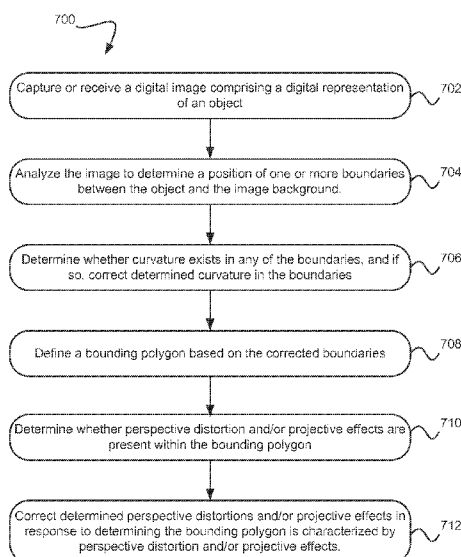


FIG. 7

(57) **Abstract:** In various embodiments, methods, systems, and computer program products for processing digital images captured by a mobile device are disclosed. Myriad features enable and/or facilitate processing of such digital images using a mobile device that would otherwise be technically impossible or impractical, and furthermore address unique challenges presented by images captured using a camera rather than a traditional flat-bed scanner, paper-feed scanner, or multifunction peripheral. Notably, the presently disclosed systems and techniques enable three-dimensional reconstruction of objects depicted in image captured using a camera of a mobile device. The reconstruction corrects or compensates for perspective distortion caused by camera-based capture.



SYSTEMS AND METHODS FOR THREE DIMENSIONAL GEOMETRIC RECONSTRUCTION OF CAPTURED IMAGE DATA

FIELD OF INVENTION

[0001] The present invention relates to digital image data capture and processing, and more particularly to rectifying image artifacts caused by distortional effects inherent to capturing an image using a camera.

BACKGROUND OF THE INVENTION

[0002] Digital images having depicted therein a document such as a letter, a check, a bill, an invoice, etc. have conventionally been captured and processed using a scanner or multifunction peripheral coupled to a computer workstation such as a laptop or desktop computer. Methods and systems capable of performing such capture and processing are well known in the art and well adapted to the tasks for which they are employed.

[0003] However, in an era where day-to-day activities, computing, and business are increasingly performed using mobile devices, it would be greatly beneficial to provide analogous document capture and processing systems and methods for deployment and use on mobile platforms, such as smart phones, digital cameras, tablet computers, etc.

[0004] A major challenge in transitioning conventional document capture and processing techniques is the limited processing power and image resolution achievable using hardware currently available in mobile devices. These limitations present a significant challenge because it is impossible or impractical to process images captured at resolutions typically much lower than achievable by a conventional scanner. As a result, conventional scanner-based processing algorithms typically perform poorly on digital images captured using a mobile device.

[0005] In addition, the limited processing and memory available on mobile devices makes conventional image processing algorithms employed for scanners prohibitively expensive in terms of computational cost. Attempting to process a conventional scanner-based image processing algorithm takes far too much time to be a practical application on modern mobile platforms.

[0006] A still further challenge is presented by the nature of mobile capture components (e.g. cameras on mobile phones, tablets, etc.). Where conventional scanners are capable of faithfully representing the physical document in a digital image, critically maintaining aspect

ratio, dimensions, and shape of the physical document in the digital image, mobile capture components are frequently incapable of producing such results.

[0007] Specifically, images of documents captured by a camera present a new line of processing issues not encountered when dealing with images captured by a scanner. This is in part due to the inherent differences in the way the document image is acquired, as well as the way the devices are constructed. The way that some scanners work is to use a transport mechanism that creates a relative movement between paper and a linear array of sensors. These sensors create pixel values of the document as it moves by, and the sequence of these captured pixel values forms an image. Accordingly, there is generally a horizontal or vertical consistency up to the noise in the sensor itself, and it is the same sensor that provides all the pixels in the line.

[0008] In contrast, cameras have many more sensors in a nonlinear array, e.g., typically arranged in a rectangle. Thus, all of these individual sensors are independent, and render image data that is not typically of horizontal or vertical consistency. In addition, cameras introduce a projective effect that is a function of the angle at which the picture is taken. For example, with a linear array like in a scanner, even if the transport of the paper is not perfectly orthogonal to the alignment of sensors and some skew is introduced, there is no projective effect like in a camera. Additionally, with camera capture, nonlinear distortions may be introduced because of the camera optics.

[0009] In view of the challenges presented above, it would be beneficial to provide an image capture and processing algorithm and applications thereof that compensate for and/or correct problems associated with image capture and processing using a mobile device, while maintaining a low computational cost via efficient processing methods.

SUMMARY OF THE INVENTION

[0010] In one embodiment, a method of reconstructing a digital image includes: receiving the digital image comprising a digital representation of an object bounded by a tetragon; correcting curvature in the tetragon to form a quadrilateral; and correcting projective effects in the quadrilateral to form a rectangle.

[0011] A system includes a processor configured to execute logic; and logic configured to receive a digital image comprising a digital representation of an object bounded by a tetragon; logic configured to correct curvature in the tetragon to form a quadrilateral; and logic configured to correct projective effects in the quadrilateral to form a rectangle.

[0012] A computer program product includes a computer readable storage medium having computer readable program code stored thereon. The computer readable program code includes code configured to receive a digital image comprising a digital representation of an object bounded by a tetragon; code configured to correct curvature in the tetragon to form a quadrilateral; and code configured to correct projective effects in the quadrilateral to form a rectangle.

BRIEF DESCRIPTION OF THE DRAWINGS

- [0013] FIG. 1 illustrates a network architecture, in accordance with one embodiment.
- [0014] FIG. 2 shows a representative hardware environment that may be associated with the servers and/or clients of FIG. 1, in accordance with one embodiment.
- [0015] FIG. 3A is a schematic representation of a digital image comprising a digital representation of a document, according to one embodiment.
- [0016] FIG. 3B is a schematic representation of a digital image comprising a digital representation of a document and a plurality of page detection analysis windows, according to one embodiment.
- [0017] FIG. 3C is a schematic representation of a digital image comprising a digital representation of a document characterized by a plurality of candidate edge points, according to one embodiment.
- [0018] FIG. 3D is a schematic representation of a large analysis window comprising a plurality of pixels of a digital image, and a small analysis window within the large analysis window, according to one embodiment.
- [0019] FIG. 4 is a schematic representation of a digital image comprising a digital representation of a document bounded by a target tetragon, according to one embodiment.
- [0020] FIG. 5A is a graphical representation of a first iteration of a page rectangularization algorithm, according to one embodiment.
- [0021] FIG. 5B is a graphical representation of an input to a page rectangularization algorithm, according to one embodiment.
- [0022] FIG. 6A is a simplified schematic showing a coordinate system for measuring capture angle, according to one embodiment.
- [0023] FIG. 6B depicts an exemplary schematic of a rectangular object captured using a capture angle normal to the object, according to one embodiment.
- [0024] FIG. 6C depicts an exemplary schematic of a rectangular object captured using a capture angle slightly skewed with respect to the object, according to one embodiment.
- [0025] FIG. 6D depicts an exemplary schematic of a rectangular object captured using a capture angle significantly skewed with respect to the object, according to one embodiment.
- [0026] FIG. 7 is a flowchart of a method, according to one embodiment.

DETAILED DESCRIPTION

[0027] The following description is made for the purpose of illustrating the general principles of the present invention and is not meant to limit the inventive concepts claimed herein. Further, particular features described herein can be used in combination with other described features in each of the various possible combinations and permutations.

[0028] Unless otherwise specifically defined herein, all terms are to be given their broadest possible interpretation including meanings implied from the specification as well as meanings understood by those skilled in the art and/or as defined in dictionaries, treatises, etc.

[0029] It must also be noted that, as used in the specification and the appended claims, the singular forms "a," "an" and "the" include plural referents unless otherwise specified.

[0030] The present application refers to image processing. In particular, the present application discloses systems, methods, and computer program products configured to transform objects depicted in digital images from a non-rectangular shape to a substantially rectangular shape, or preferably a rectangular shape. Even more preferably, this is accomplished by employing a two-step process where curvature in the object is corrected first, followed by correction of any projective effects in the image.

[0031] The following definitions will be useful in understanding the inventive concepts described herein, according to various embodiments. The following definitions are to be considered exemplary, and are offered for purposes of illustration to provide additional clarity to the present disclosures, but should not be deemed limiting on the scope of the inventive concepts disclosed herein.

[0032] As referred to henceforth, a "quadrilateral" is a four-sided figure where (1) each side is linear, and (2) adjacent sides form vertices at the intersection thereof. Exemplary quadrilaterals are depicted in FIGS. 6C and 6D below, according to two illustrative embodiments.

[0033] A "parallelogram" is a special type of quadrilateral, i.e. a four-sided figure where (1) each side is linear, (2) opposite sides are parallel, and (3) adjacent sides are not necessarily perpendicular, such that vertices at the intersection of adjacent sides form angles having values that are not necessarily 90°.

[0034] A "rectangle" or "rectangular shape" is a special type of quadrilateral, which is defined as a four-sided figure, where (1) each side is linear, (2) opposite sides are parallel, and (3) adjacent sides are perpendicular, such that an interior angle formed at the vertex between

each pair of adjacent sides is a right-angle, i.e. a 90° angle. An exemplary rectangle is depicted in FIG. 6B, according to one illustrative embodiment.

[0035] Moreover, as referred-to herein “rectangles” and “rectangular shapes” are considered to include “substantially rectangular shapes”, which are defined as a four-sided shape where (1) each side is predominantly linear (e.g. at least 90%, 95%, or 99% of each side’s length, in various embodiments, is characterized by a first-order polynomial (such as $y = mx + b$), (2) each pair of adjacent sides form an interior angle having a value θ , where θ is approximately 90° (e.g. θ satisfies the relationship: $85^\circ \leq \theta \leq 95^\circ$)) at either (a) a vertex between two adjacent sides, (b) a vertex between a projection of the predominantly linear portion of one side and an adjacent side, or (c) a vertex between a projection of the predominantly linear portion of one side and a projection of the predominantly linear portion of an adjacent side. An exemplary “substantially rectangular shape” is depicted below in FIG. 7 (note the corners of the depicted driver license are curved, such that there is no discrete vertex formed by the respective adjacent sides, but a projection of each adjacent side would produce a vertex at the intersection thereof having an angle θ of approximately 90°).

[0036] A “non-rectangular shape” as referred to herein includes any shape that is not either a “rectangular shape” or a “substantially rectangular shape” as defined above. In preferred embodiments, a “non-rectangular shape” is a “tetragon,” which as referred to herein is a four-sided figure, where: (1) each side is characterized in whole or in part by an equation selected from a chosen class of functions (e.g. selected from a class of polynomials preferably ranging from zeroth order to fifth order, more preferably first order to third order polynomials, and even more preferably first order to second order polynomials), and (2) adjacent sides of the figure form vertices at the intersection thereof. An exemplary tetragon as referred to herein is depicted in FIG. 4, according to one illustrative embodiment.

[0037] In one general embodiment of the presently disclosed inventive concepts, a method of reconstructing a digital image includes: receiving the digital image comprising a digital representation of an object bounded by a tetragon; correcting curvature in the tetragon to form a quadrilateral; and correcting projective effects in the quadrilateral to form a rectangle.

[0038] In another general embodiment, a system includes a processor configured to execute logic; and logic configured to receive a digital image comprising a digital representation of an object bounded by a tetragon; logic configured to correct curvature in the tetragon to form a quadrilateral; and logic configured to correct projective effects in the quadrilateral to form a rectangle.

[0039] In yet another general embodiment, a computer program product includes a computer readable storage medium having computer readable program code stored thereon. The computer readable program code includes code configured to receive a digital image comprising a digital representation of an object bounded by a tetragon; code configured to correct curvature in the tetragon to form a quadrilateral; and code configured to correct projective effects in the quadrilateral to form a rectangle.

[0040] Previous methods of rectangularization, such as described in U.S. Pat. Appl. No. 13/740,127 (filed Jan. 11, 2013), nicely correct curvature effects observed in camera-captured images. However, when the pitch and/or roll of the camera are large enough (e.g. about 30 degrees or more) the technique's ability to correct for projective effects is often inadequate due to severe projective effects/artifacts being present in the image.

[0041] The combination of the previously described curvature-correction method and the presently described projective-effect correction method into a single dual-purpose procedure effectively combines the respective strengths of each approach. In one embodiment, the approaches may be combined as follows. First, the curvature-correction component corrects the curvature by mapping the curved tetragon bounding the object to a tetragon with the same corners but having straight sides (i.e. characterized by linear, or first-degree polynomials). Second, this (now straight-sided) tetragon is mapped to a target rectangle using a 4-point method such as described below.

[0042] As a significant advantage, despite the more accurate reconstruction of the rectangular representation, this dual procedure affects only the two-step mapping of the coordinates, while the actual transformation of the image happens only once. Since most of the processing time is spend manipulating two large images in memory and combining the four pixels surrounding a non-integer (x, y) coordinates pair rather than calculating the correct coordinates by whatever method, the dual method is only about 3% more expensive than the original with respect to computational cost and therefore runtime, despite improved accuracy in terms of relative pixel location in the reconstructed image as compared to a corresponding image obtained from a traditional flatbed scanner or similar device.

[0043] Experimental testing demonstrates that on a photograph with both pronounced curvature and large projective distortions, the dual method reduces the coordinate error (measured as the largest distance from the rectangularized pixel to the same pixel in a scanned image of the same document in the same resolution) by about 3x relative to the error of the coordinate-based method alone. In one embodiment, the residual error was about 5 pixels in a 500-DPI resolution, or about one hundredth of an inch.

[0044] Images (e.g. pictures, figures, graphical schematics, single frames of movies, videos, films, clips, etc.) are preferably digital images captured by cameras, especially cameras of mobile devices. As understood herein, a mobile device is any device capable of receiving data without having power supplied via a physical connection (e.g. wire, cord, cable, etc.) and capable of receiving data without a physical data connection (e.g. wire, cord, cable, etc.). Mobile devices within the scope of the present disclosures include exemplary devices such as a mobile telephone, smartphone, tablet, personal digital assistant, iPod®, iPad®, BLACKBERRY® device, etc.

[0045] However, as it will become apparent from the descriptions of various functionalities, the presently disclosed mobile image processing algorithms can be applied, sometimes with certain modifications, to images coming from scanners and multifunction peripherals (MFPs). Similarly, images processed using the presently disclosed processing algorithms may be further processed using conventional scanner processing algorithms, in some approaches.

[0046] Of course, the various embodiments set forth herein may be implemented utilizing hardware, software, or any desired combination thereof. For that matter, any type of logic may be utilized which is capable of implementing the various functionality set forth herein.

[0047] One benefit of using a mobile device is that with a data plan, image processing and information processing based on captured images can be done in a much more convenient, streamlined and integrated way than previous methods that relied on presence of a scanner. However, the use of mobile devices as document(s) capture and/or processing devices has heretofore been considered unfeasible for a variety of reasons.

[0048] In one approach, an image may be captured by a camera of a mobile device. The term “camera” should be broadly interpreted to include any type of device capable of capturing an image of a physical object external to the device, such as a piece of paper. The term “camera” does not encompass a peripheral scanner or multifunction device. Any type of camera may be used. Preferred embodiments may use cameras having a higher resolution, e.g. 8 MP or more, ideally 12 MP or more. The image may be captured in color, grayscale, black and white, or with any other known optical effect. The term “image” as referred to herein is meant to encompass any type of data corresponding to the output of the camera, including raw data, processed data, etc.

[0049] The description herein is presented to enable any person skilled in the art to make and use the invention and is provided in the context of particular applications of the invention and their requirements. Various modifications to the disclosed embodiments will be readily apparent to those skilled in the art and the general principles defined herein may be applied to other

embodiments and applications without departing from the spirit and scope of the present invention. Thus, the present invention is not intended to be limited to the embodiments shown, but is to be accorded the widest scope consistent with the principles and features disclosed herein.

[0050] In particular, various embodiments of the invention discussed herein are implemented using the Internet as a means of communicating among a plurality of computer systems. One skilled in the art will recognize that the present invention is not limited to the use of the Internet as a communication medium and that alternative methods of the invention may accommodate the use of a private intranet, a Local Area Network (LAN), a Wide Area Network (WAN) or other means of communication. In addition, various combinations of wired, wireless (e.g., radio frequency) and optical communication links may be utilized.

[0051] The program environment in which one embodiment of the invention may be executed illustratively incorporates one or more general-purpose computers or special-purpose devices such as hand-held computers. Details of such devices (e.g., processor, memory, data storage, input and output devices) are well known and are omitted for the sake of clarity.

[0052] It should also be understood that the techniques of the present invention might be implemented using a variety of technologies. For example, the methods described herein may be implemented in software running on a computer system, or implemented in hardware utilizing one or more processors and logic (hardware and/or software) for performing operations of the method, application specific integrated circuits, programmable logic devices such as Field Programmable Gate Arrays (FPGAs), and/or various combinations thereof. In one illustrative approach, methods described herein may be implemented by a series of computer-executable instructions residing on a storage medium such as a physical (e.g., non-transitory) computer-readable medium. In addition, although specific embodiments of the invention may employ object-oriented software programming concepts, the invention is not so limited and is easily adapted to employ other forms of directing the operation of a computer.

[0053] The invention can also be provided in the form of a computer program product comprising a computer readable storage or signal medium having computer code thereon, which may be executed by a computing device (e.g., a processor) and/or system. A computer readable storage medium can include any medium capable of storing computer code thereon for use by a computing device or system, including optical media such as read only and writeable CD and DVD, magnetic memory or medium (e.g., hard disk drive, tape), semiconductor memory (e.g., FLASH memory and other portable memory cards, etc.), firmware encoded in a chip, etc.

[0054] A computer readable signal medium is one that does not fit within the aforementioned storage medium class. For example, illustrative computer readable signal media communicate or otherwise transfer transitory signals within a system, between systems e.g., via a physical or virtual network, etc.

[0055] FIG. 1 illustrates an architecture 100, in accordance with one embodiment. As shown in FIG. 1, a plurality of remote networks 102 are provided including a first remote network 104 and a second remote network 106. A gateway 101 may be coupled between the remote networks 102 and a proximate network 108. In the context of the present network architecture 100, the networks 104, 106 may each take any form including, but not limited to a LAN, a WAN such as the Internet, public switched telephone network (PSTN), internal telephone network, etc.

[0056] In use, the gateway 101 serves as an entrance point from the remote networks 102 to the proximate network 108. As such, the gateway 101 may function as a router, which is capable of directing a given packet of data that arrives at the gateway 101, and a switch, which furnishes the actual path in and out of the gateway 101 for a given packet.

[0057] Further included is at least one data server 114 coupled to the proximate network 108, and which is accessible from the remote networks 102 via the gateway 101. It should be noted that the data server(s) 114 may include any type of computing device/groupware. Coupled to each data server 114 is a plurality of user devices 116. Such user devices 116 may include a desktop computer, laptop computer, hand-held computer, printer or any other type of logic. It should be noted that a user device 111 may also be directly coupled to any of the networks, in one embodiment.

[0058] A peripheral 120 or series of peripherals 120, e.g. facsimile machines, printers, networked storage units, etc., may be coupled to one or more of the networks 104, 106, 108. It should be noted that databases, servers, and/or additional components may be utilized with, or integrated into, any type of network element coupled to the networks 104, 106, 108. In the context of the present description, a network element may refer to any component of a network.

[0059] According to some approaches, methods and systems described herein may be implemented with and/or on virtual systems and/or systems which emulate one or more other systems, such as a UNIX system which emulates a MAC OS environment, a UNIX system which virtually hosts a MICROSOFT WINDOWS environment, a MICROSOFT WINDOWS system which emulates a MAC OS environment, etc. This virtualization and/or emulation may be enhanced through the use of VMWARE software, in some embodiments.

[0060] In more approaches, one or more networks 104, 106, 108, may represent a cluster of systems commonly referred to as a "cloud." In cloud computing, shared resources, such as

processing power, peripherals, software, data processing and/or storage, servers, etc., are provided to any system in the cloud, preferably in an on-demand relationship, thereby allowing access and distribution of services across many computing systems. Cloud computing typically involves an Internet or other high speed connection (e.g., 4G LTE, fiber optic, etc.) between the systems operating in the cloud, but other techniques of connecting the systems may also be used.

[0061] FIG. 1 illustrates an architecture 100, in accordance with one embodiment. As shown in FIG. 1, a plurality of remote networks 102 are provided including a first remote network 104 and a second remote network 106. A gateway 101 may be coupled between the remote networks 102 and a proximate network 108. In the context of the present architecture 100, the networks 104, 106 may each take any form including, but not limited to a LAN, a WAN such as the Internet, public switched telephone network (PSTN), internal telephone network, etc.

[0062] In use, the gateway 101 serves as an entrance point from the remote networks 102 to the proximate network 108. As such, the gateway 101 may function as a router, which is capable of directing a given packet of data that arrives at the gateway 101, and a switch, which furnishes the actual path in and out of the gateway 101 for a given packet.

[0063] Further included is at least one data server 114 coupled to the proximate network 108, and which is accessible from the remote networks 102 via the gateway 101. It should be noted that the data server(s) 114 may include any type of computing device/groupware. Coupled to each data server 114 is a plurality of user devices 116. Such user devices 116 may include a desktop computer, lap-top computer, hand-held computer, printer or any other type of logic. It should be noted that a user device 111 may also be directly coupled to any of the networks, in one embodiment.

[0064] A peripheral 120 or series of peripherals 120, e.g., facsimile machines, printers, networked and/or local storage units or systems, etc., may be coupled to one or more of the networks 104, 106, 108. It should be noted that databases and/or additional components may be utilized with, or integrated into, any type of network element coupled to the networks 104, 106, 108. In the context of the present description, a network element may refer to any component of a network.

[0065] According to some approaches, methods and systems described herein may be implemented with and/or on virtual systems and/or systems which emulate one or more other systems, such as a UNIX system which emulates a MAC OS environment, a UNIX system which virtually hosts a MICROSOFT WINDOWS environment, a MICROSOFT WINDOWS system which emulates a MAC OS environment, etc. This virtualization and/or emulation may be enhanced through the use of VMWARE software, in some embodiments.

[0066] In more approaches, one or more networks 104, 106, 108, may represent a cluster of systems commonly referred to as a "cloud." In cloud computing, shared resources, such as processing power, peripherals, software, data processing and/or storage, servers, etc., are provided to any system in the cloud, preferably in an on-demand relationship, thereby allowing access and distribution of services across many computing systems. Cloud computing typically involves an Internet or other high speed connection (e.g., 4G LTE, fiber optic, etc.) between the systems operating in the cloud, but other techniques of connecting the systems may also be used.

[0067] FIG. 2 shows a representative hardware environment associated with a user device 116 and/or server 114 of FIG. 1, in accordance with one embodiment. Such figure illustrates a typical hardware configuration of a workstation having a central processing unit 210, such as a microprocessor, and a number of other units interconnected via a system bus 212.

[0068] The workstation shown in FIG. 2 includes a Random Access Memory (RAM) 214, Read Only Memory (ROM) 216, an I/O adapter 218 for connecting peripheral devices such as disk storage units 220 to the bus 212, a user interface adapter 222 for connecting a keyboard 224, a mouse 226, a speaker 228, a microphone 232, and/or other user interface devices such as a touch screen and a digital camera (not shown) to the bus 212, communication adapter 234 for connecting the workstation to a communication network 235 (e.g., a data processing network) and a display adapter 236 for connecting the bus 212 to a display device 238.

[0069] The workstation may have resident thereon an operating system such as the Microsoft Windows® Operating System (OS), a MAC OS, a UNIX OS, etc. It will be appreciated that a preferred embodiment may also be implemented on platforms and operating systems other than those mentioned. A preferred embodiment may be written using JAVA, XML, C, and/or C++ language, or other programming languages, along with an object oriented programming methodology. Object oriented programming (OOP), which has become increasingly used to develop complex applications, may be used.

[0070] Various Embodiments of a Mobile Image Capture and Processing Algorithm

[0071] Various embodiments of a Mobile Image Capture and Processing algorithm, as well as several mobile applications configured to facilitate use of such algorithmic processing within the scope of the present disclosures are described below. It is to be appreciated that each section below describes functionalities that may be employed in any combination with those disclosed in other sections, including any or up to all the functionalities described herein. Moreover, functionalities of the processing algorithm embodiments as well as the mobile application embodiments may be combined and/or distributed in any manner across a variety of computing resources and/or systems, in several approaches.

[0072] An application may be installed on the mobile device, e.g., stored in a nonvolatile memory of the device. In one approach, the application includes instructions to perform processing of an image on the mobile device. In another approach, the application includes instructions to send the image to one or more non-mobile devices, e.g. a remote server such as a network server, a remote workstation, a cloud computing environment, etc. as would be understood by one having ordinary skill in the art upon reading the present descriptions. In yet another approach, the application may include instructions to decide whether to perform some or all processing on the mobile device and/or send the image to the remote site. Examples of how an image may be processed are presented in more detail below.

[0073] In one embodiment, there may be no difference between the processing that may be performed on the mobile device and a remote server, other than speed of processing, constraints on memory available, etc. Moreover, there may be some or no difference between various user interfaces presented on a mobile device, e.g. as part of a mobile application, and corresponding user interfaces presented on a display in communication with the non-mobile device.

[0074] In other embodiments, a remote server may have higher processing power, more capabilities, more processing algorithms, etc. In yet further embodiments, the mobile device may have no image processing capability associated with the application, other than that required to send the image to the remote server. In yet another embodiment, the remote server may have no image processing capability relevant to the platforms presented herein, other than that required to receive the processed image from the remote server. Accordingly, the image may be processed partially or entirely on the mobile device, and/or partially or entirely on a remote server, and/or partially or entirely in a cloud, and/or partially or entirely in any part of the overall architecture in between. Moreover, some processing steps may be duplicated on different devices.

[0075] Which device performs which parts of the processing may be defined by a user, may be predetermined, may be determined on the fly, etc. Moreover, some processing steps may be re-performed, e.g., upon receiving a request from the user. Accordingly, the raw image data, partially processed image data, or fully processed image data may be transmitted from the mobile device, e.g., using a wireless data network, to a remote system. Image data as processed at a remote system may be returned to the mobile device for output and/or further processing.

[0076] In a further approach, the image may be partitioned, and the processing of the various parts may be allocated to various devices, e.g., 1/2 to the mobile device and 1/2 to the remote server, after which the processed halves are combined.

[0077] In one embodiment, selection of which device performs the processing may be based

at least in part on a relative speed of processing locally on the mobile device vs. communication with the server.

[0078] In one approach, a library of processing functions may be present, and the application on the mobile device or the application on a remote server simply makes calls to this library, and essentially the meaning of the calls defines what kind of processing to perform. The device then performs that processing and outputs the processed image, perhaps with some corresponding metadata.

[0079] Any type of image processing known in the art and/or as newly presented herein may be performed in any combination in various embodiments.

[0080] Referring now to illustrative image processing, the camera can be considered an area sensor that captures images, where the images may have any number of projective effects, and sometimes non-linear effects. The image may be processed to correct for such effects. Moreover, the position and boundaries of the document(s) in the image may be found during the processing, e.g., the boundaries of one or more actual pages of paper in the background surrounding the page(s). Because of the mobile nature of various embodiments, the sheet of paper may be lying on just about anything. This complicates image analysis in comparison to processing images of documents produced using a scanner, because scanner background properties are constant and typically known, whereas mobile capture backgrounds may vary almost infinitely according to the location of the document and the corresponding surrounding textures captured in the image background, as well as because of variable lighting conditions.

[0081] Accordingly, the non-uniformity of the background of the surface on which the piece of paper may be positioned for capture by the camera presents one challenge, and the non-linear and projective effects present additional challenges. Various embodiments overcome these challenges, as will soon become apparent.

[0082] In one exemplary mode of operation, an application on the mobile device may be initiated, e.g., in response to a user request to open the application. For example, a user-selection of an icon representing the application may be detected.

[0083] In some approaches, a user authentication may be requested and/or performed. For example, a user ID and password, or any other authentication information, may be requested and/or received from the user.

[0084] In further approaches, various tasks may be enabled via a graphical user interface of the application. For example, a list of tasks may be presented. In such case, a selection of one of the tasks by the user may be detected, and additional options may be presented to the user, a predefined task may be initiated, the camera may be initiated, etc.

[0085] An image may be captured by the camera of the mobile device, preferably upon receiving some type of user input such as detecting a tap on a screen of the mobile device, depression of a button on the mobile device, a voice command, a gesture, etc. Another possible scenario may involve some level of analysis of sequential frames, e.g. from a video stream. Sequential frame analysis may be followed by a switch to capturing a single high-resolution image frame, which may be triggered automatically or by a user, in some approaches. Moreover, the trigger may be based on information received from one or more mobile device sensors.

[0086] For example, in one embodiment an accelerometer in or coupled to the mobile device may indicate a stability of the camera, and the application may analyze low-resolution video frame(s) for a document. If a document is detected, the application may perform a focusing operation and acquire a high-resolution image of the detected document.

[0087] Either the low- or high-resolution image may be further processed, but preferred embodiments utilize the high-resolution image for subsequent processing. In more approaches, switching to single frame mode as discussed above may be unnecessary, particularly for smaller documents such as business cards and receipts. To increase processing rate and reduce consumption of processing resources, document type identification may facilitate determining whether or not to switch to single frame mode and/or capture a high-resolution image for processing. For the present discussion, assume an image of one or more documents is captured.

[0088] Given that mobile devices do not typically have the processing power of conventional non-mobile devices, one approach performs some limited processing on the mobile device, for example to let the user verify that the page(s) has been found correctly, that the image is not blurred, and/or that the lighting is adequate, e.g., a preview of sorts.

[0089] In one approach, the document(s) within the image captured by the camera may be found.

[0090] Additional methods of detecting one or more boundaries of the document(s) are also presented herein. If the document(s) in the image has nonlinearities or is not rectangular, correction processing may be applied.

[0091] Once the page(s) are found in the image, one embodiment performs a smooth transformation in order to make the page(s) rectangular, assuming of course the original piece of paper was rectangular. Another useful correction to the image may be mitigation of the unevenness of the illumination.

[0092] In one exemplary approach, page detection and rectangularization may be performed substantially as described below.

[0093] **Various Embodiments of Mobile Page Detection**

[0094] One exemplary embodiment illustrating an exemplary methodology for performing page detection will now be described with reference to FIGS. 3A-4. With reference to these descriptions, it will become more clear how the advantages implemented for a mobile processing algorithm as described herein handle images captured by area sensors (cameras) and compensate for the inherent difficulties presented thereby.

[0095] In one approach, and with particular reference to FIGS. 3A-3B, an edge detection algorithm proceeds from the boundaries of a digital image 300 toward a central region of the image 300, looking for points that are sufficiently different from what is known about the properties of the background. Notably, the background 304 in the images captured by even the same mobile device may be different every time, so a new technique to identify the document(s) in the image is provided.

[0096] Finding page edges within a camera-captured image according to the present disclosures helps to accommodate important differences in the properties of images captured using mobile devices as opposed, e.g., to scanners. For example, due to projective effects the image of a rectangular document in a photograph may not appear truly rectangular, and opposite sides of the document in the image may not have the same length. Second, even the best lenses have some non-linearity resulting in straight lines within an object, e.g. straight sides of a substantially rectangular document, appearing slightly curved in the captured image of that object. Third, images captured using cameras overwhelmingly tend to introduce uneven illumination effects in the captured image. This unevenness of illumination makes even a perfectly uniform background of the surface against which a document may be placed appear in the image with varied brightness, and often with shadows, especially around the page edges if the page is not perfectly flat.

[0097] In an exemplary approach, to avoid mistaking the variability within the background for page edges, the current algorithm utilizes one or more of the following functionalities.

[0098] In various embodiments, the frame of the image contains the digital representation of the document 302 with margins of the surrounding background 304. In the preferred implementation the search for individual page edges 306 may be performed on a step-over approach analyzing rows and columns of the image from outside in. In one embodiment, the step-over approach may define a plurality of analysis windows 308 within the digital image 300, such as shown in FIGS 3A-3B. As understood herein, analysis windows 308 may include one or more “background windows,” i.e. windows encompassing only pixels depicting the background 304 of the digital image 300, as well as one or more “test windows” i.e. windows encompassing

pixels depicting the background 304 of the digital image 300, the digital representation of the document 302, or both.

[0099] In a preferred embodiment, the digital representation of the document may be detected in the digital image by defining a first analysis window 308, i.e. a background analysis window, in a margin of the image corresponding to the background 304 of the surface upon which the document is placed. Within the first analysis window 308, a plurality of small analysis windows (e.g. test windows 312 as shown in FIG. 3D) may be defined within the first analysis window 308. Utilizing the plurality of test windows 312, one or more distributions of one or more statistical properties descriptive of the background 304 may be estimated.

[0100] With continuing reference to the preferred embodiment discussed immediately above, a next step in detecting boundaries of the digital representation of the document may include defining a plurality of test windows 312 within the digital image, and analyzing the corresponding regions of the digital image. For each test window 312 one or more statistical values descriptive of the corresponding region of the image may be calculated. Further, these statistical values may be compared to a corresponding distribution of statistics descriptive of the background 304.

[0101] In a preferred approach, the plurality of test windows 312 may be defined along a path, particularly a linear path. In a particularly preferred approach, the plurality of test windows 312 may be defined in a horizontal direction and/or a vertical direction, e.g. along rows and columns of the digital image. Moreover, a stepwise progression may be employed to define the test windows 312 along the path and/or between the rows and/or columns. In some embodiments, as will be appreciated by one having ordinary skill in the art upon reading the present descriptions, utilizing a stepwise progression may advantageously increase the computational efficiency of document detection processes.

[0102] Moreover, the magnitude of the starting step may be estimated based on the resolution or pixel size of the image, in some embodiments, but this step may be reduced if advantageous for reliable detection of document sides, as discussed further below.

[0103] In more embodiments, the algorithm estimates the distribution of several statistics descriptive of the image properties found in a large analysis window 308 placed within the background surrounding the document. In one approach a plurality of small windows 312 may be defined within the large analysis window 308, and distributions of statistics descriptive of the small test windows 312 may be estimated. In one embodiment, large analysis window 308 is defined in a background region of the digital image, such as a top-left corner of the image.

[00104] Statistics descriptive of the background pixels may include any statistical value that may be generated from digital image data, such as a minimum value, a maximum value, a median value, a mean value, a spread or range of values, a variance, a standard deviation, etc. as would be understood by one having ordinary skill in the art upon reading the present descriptions. Values may be sampled from any data descriptive of the digital image 300, such as brightness values in one or more color channels, e.g. red-green-blue or RGB, cyan-magenta, yellow, black or CMYK, hue saturation value or HSV, etc. as would be understood by one having ordinary skill in the art upon reading the present descriptions.

[00105] As shown in FIG. 3D, each of the small analysis windows 312 may comprise a subset of the plurality of pixels within the large analysis window 308. Moreover, small analysis windows 312 may be of any size and/or shape capable of fitting within the boundaries of large analysis window 308. In a preferred embodiment, small analysis windows 312 may be characterized by a rectangular shape, and even more preferably a rectangle characterized by being three pixels long in a first direction (e.g. height) and seven pixels long in a second direction (e.g. width). Of course, other small analysis window sizes, shapes, and dimensions are also suitable for implementation in the presently disclosed processing algorithms.

[00106] In one embodiment, test windows may be employed to analyze an image and detect the boundary of a digital representation of a document depicted in the image. Background windows are used for estimation of original statistical properties of the background and/or reestimation of local statistical properties of the background. Reestimation may be necessary and/or advantageous in order to address artifacts such as uneven illumination and/or background texture variations.

[00107] Preferably, statistical estimation may be performed over some or all of a plurality of small analysis window(s) 312 in a large analysis window 308 within the margin outside of the document page in some approaches. Such estimation may be performed using a stepwise movement of a small analysis window 312 within the large analysis window 308, and the stepwise movement may be made in any suitable increment so as to vary the number of samples taken for a given pixel. For example, to promote computational efficiency, an analysis process may define a number of small analysis windows 312 within large analysis window 308 sufficient to ensure each pixel 318 is sampled once. Thus the plurality of small analysis windows 312 defined in this computationally efficient approach would share common borders but not overlap.

[00108] In another approach designed to promote robustness of statistical estimations, the analysis process may define a number of small analysis windows 312 within large analysis

window 308 sufficient to ensure each pixel 318 is sampled a maximum number of times, e.g. by reducing the step to produce only a single pixel shift in a given direction between sequentially defined small analysis windows 312. Of course, any step increment may be employed in various embodiments of the presently disclosed processing algorithms, as would be understood by one having ordinary skill in the art upon reading the present descriptions.

[00109] The skilled artisan will appreciate that large analysis windows 308 utilized to reestimate statistics of local background in the digital image as well as test windows can be placed in the digital image in any which way desirable.

[00110] For example, according to one embodiment shown in FIG. 3A, the search for the left side edge in a given row i begins from the calculation of the above mentioned statistics in a large analysis window 308 adjacent to the frame boundary on the left side of the image centered around a given row i .

[00111] In still more embodiments, when encountering a possible non-background test window (e.g. a test window for which the estimated statistics are dissimilar from the distribution of statistics characteristic of the last known local background) as the algorithm progresses from the outer region(s) of the image towards the interior regions thereof, the algorithm may backtrack into a previously determined background region, form a new large analysis window 308 and re-estimate the distribution of background statistics in order to reevaluate the validity of the differences between the chosen statistics within the small analysis window 312 and the local distribution of corresponding statistics within the large analysis window 308, in some embodiments.

[00112] As will be appreciated by one having ordinary skill in the art upon reading the present descriptions, the algorithm may proceed from an outer region of the image 300 to an inner region of the image 300 in a variety of manners. For example, in one approach the algorithm proceeds defining test windows 312 in a substantially spiral pattern. In other approaches the pattern may be substantially serpentine along either a vertical or a horizontal direction. In still more approaches the pattern may be a substantially shingled pattern. The pattern may also be defined by a "sequence mask" laid over part or all of the digital image 300, such as a checkerboard pattern, a vertically, horizontally, or diagonally striped pattern, concentric shapes, etc. as would be understood by one having ordinary skill in the art upon reading the present descriptions. In other embodiments, analysis windows such as large analysis windows 308 and/or small analysis windows 312 may be defined throughout the digital image 300 in a random manner, a pseudo-random manner, stochastically, etc. according to some defined procedure, as would be understood by one having ordinary skill in the art upon reading

the present descriptions. The algorithm can proceed with a sequence of test windows in any desirable fashion as long as the path allows to backtrack into known background, and the path covers the whole image with desirable granularity.

[00113] Advantageously, recalculating statistics in this manner helps to accommodate for any illumination drift inherent to the digital image 300 and/or background 304, which may otherwise result in false identification of non-background points in the image (e.g. outlier candidate edge points 316 as shown in FIG. 3C).

[00114] In still yet more embodiments, when the difference is statistically valid, the algorithm may jump a certain distance further along its path in order to check again and thus bypass small variations in the texture of the background 304, such as wood grain, scratches on a surface, patterns of a surface, small shadows, etc. as would be understood by one having ordinary skill in the art upon reading the present descriptions.

[00115] In additional and/or alternative embodiments, after a potential non-background point has been found, the algorithm determines whether the point lies on the edge of the shadow (a possibility especially if the edge of the page is raised above the background surface) and tries to get to the actual page edge. This process relies on the observation that shadows usually darken towards the real edge followed by an abrupt brightening of the image.

[00116] The above described approach to page edge detection was utilized because the use of standard edge detectors may be unnecessary and even undesirable, for several reasons. First, most standard edge detectors involve operations that are time consuming, and second, the instant algorithm is not concerned with additional requirements like monitoring how thin the edges are, which directions they follow, etc. Even more importantly, looking for page edges 306 does not necessarily involve edge detection per se, i.e. page edge detection according to the present disclosures may be performed in a manner that does not search for a document boundary (e.g. page edge 306), but rather searches for image characteristics associated with a transition from background to the document. For example, the transition may be characterized by flattening of the off-white brightness levels within a glossy paper, i.e. by changes in texture rather than in average gray or color levels.

[00117] As a result, it is possible to obtain candidate edge points (e.g. candidate edge points 314 as shown in FIG. 3C) that are essentially the first and the last non-background pixels in each row and column on a grid. In order to eliminate random outliers (e.g. outlier candidate edge points 316 as shown in FIG. 3C) and to determine which candidate edge points 314 correspond to each side of the page, it is useful in one approach to analyze neighboring candidate edge points.

[00118] In one embodiment, a “point” may be considered any region within the digital image, such as a pixel, a position between pixels (e.g. a point with fractional coordinates such as the center of a 2-pixel by 2-pixel square) a small window of pixels, etc. as would be understood by one having ordinary skill in the art upon reading the present descriptions. In a preferred embodiment, a candidate edge point is associated with the center of a test window (e.g. a 3-pixel x 7-pixel window) that has been found to be characterized by statistics that are determined to be different from the distribution of statistics descriptive of the local background.

[00119] As understood herein, a “neighboring” candidate edge point, or a “neighboring” pixel is considered to be a point or pixel, respectively, which is near or adjacent a point or pixel of interest (e.g. pixel 318), e.g. a point or pixel positioned at least in part along a boundary of the point or pixel of interest, a point or pixel positioned within a threshold distance of the point or pixel of interest (such as within 2, 10, 64 pixels, etc. in a given direction, within one row of the point or pixel of interest, within one column of the point or pixel of interest), etc. as would be understood by one having ordinary skill in the art upon reading the present descriptions. In preferred approaches, the “neighboring” point or pixel may be the closest candidate edge point to the point of interest along a particular direction, e.g. a horizontal direction and/or a vertical direction.

[00120] Each “good” edge point ideally has at least two immediate neighbors (one on each side) and does not deviate far from a straight line segment connecting these neighbors and the “good” edge point, e.g. the candidate edge point and the at least two immediately neighboring points may be fit to a linear regression, and the result may be characterized by a coefficient of determination (R^2) not less than 0.95. The angle of this segment with respect to one or more borders of the digital image, together with its relative location determines whether the edge point is assigned to top, left, right, or bottom side of the page. In a preferred embodiment, a candidate edge point and the two neighboring edge points may be assigned to respective corners of a triangle. If the angle of the triangle at the candidate edge point is close to 180 degrees, then the candidate edge point may be considered a “good” candidate edge point. If the angle of the triangle at the candidate edge point deviates far from 180 degrees by more than a threshold value (such as by 20 degrees or more), then the candidate edge point may be excluded from the set of “good” candidate edge points. The rationale behind this heuristic is based on the desire to throw out random errors in the determination of the first and last non-background pixels within rows and columns. These pixels are unlikely to exist in consistent lines, so checking the neighbors in terms of distance and direction is particularly advantageous in some approaches.

[00121] For speed, the step of this grid may start from a large number such as 32, but it may be reduced by a factor of two and the search for edge points repeated until there are enough of them to determine the Least Mean Squares (LMS) based equations of page sides (see below). If this process cannot determine the sides reliably even after using all rows and columns in the image, it gives up and the whole image is treated as the page.

[00122] The equations of page sides are determined as follows, in one embodiment. First, the algorithm fits the best LMS straight line to each of the sides using the strategy of throwing out worst outliers until all the remaining supporting edges lie within a small distance from the LMS line. For example, a point with the largest distance from a substantially straight line connecting a plurality of candidate edge points along a particular boundary of the document may be designated the “worst” outlier. This procedure may be repeated iteratively to designate and/or remove one or more “worst” outliers from the plurality of candidate edge point. In some approaches, the distance with which a candidate edge point may deviate from the line connecting the plurality of candidate edge points is based at least in part on the size and/or resolution of the digital image.

[00123] If this line is not well supported all along its stretch, the algorithm may attempt to fit the best second-degree polynomial (parabola) to the same original candidate points. The algorithmic difference between finding the best parabola vs. the best straight line is minor: instead of two unknown coefficients determining the direction and offset of the line there are three coefficients determining the curvature, direction, and offset of the parabola; however, in other respects the process is essentially the same, in one embodiment.

[00124] If the support of the parabola is stronger than that of the straight line, especially closer to the ends of the candidate edge span, the conclusion is that the algorithm should prefer the parabola as a better model of the page side in the image. Otherwise, the linear model is employed, in various approaches.

[00125] Intersections of the four found sides of the document may be calculated in order to find the corners of (possibly slightly curved) page tetragon, (e.g. tetragon 400 as shown in FIG. 4 and discussed in further detail below). In the preferred implementation in order to do this it is necessary to consider three cases: calculating intersections of two straight lines, calculating intersections of a straight line and a parabola, and calculating intersections of two parabolas.

[00126] In the first case there is a single solution (since top and bottom page edges 306 stretch mostly horizontally, while left and right page edges 306 stretch mostly vertically, the corresponding LMS lines cannot be parallel) and this solution determines the coordinates of the corresponding page corner.

[00127] The second case, calculating intersections of a straight line and a parabola, is slightly more complicated: there can be zero, one, or two solutions of the resulting quadratic equation. If there is no intersection, it may indicate a fatal problem with page detection, and its result may be rejected. A single solution is somewhat unlikely, but presents no further problems. Two intersections present a choice, in which case the intersection closer to the corresponding corner of the frame is a better candidate – in practice, the other solution of the equation may be very far away from the coordinate range of the image frame.

[00128] The third case, calculating intersections of two parabolas, results in a fourth degree polynomial equation that (in principle) may be solved analytically. However, in practice the number of calculations necessary to achieve a solution may be greater than in an approximate iterative algorithm that also guarantees the desired sub-pixel precision.

[00129] One exemplary procedure used for this purpose is described in detail below with reference to rectangularization of the digital representation of the document 302, according to one approach.

[00130] There are several constraints on the validity of the resulting target tetragon (e.g. tetragon 400 as discussed in further detail below with regard to FIG. 4). Namely, the tetragon is preferably not too small (e.g., below a predefined threshold of any desired value, such as 25% of the total area of the image), the corners of the tetragon preferably do not lie too far outside of the frame of the image (e.g. not more than 100 pixels away), and the corners themselves should preferably be interpretable as top-left, top-right, bottom-left and bottom-right with diagonals intersecting inside of the tetragon, etc. If these constraints are not met, a given page detection result may be rejected, in some embodiments.

[00131] In one illustrative embodiment where the detected tetragon of the digital representation of the document 302 is valid, the algorithm may determine a target rectangle. Target rectangle width and height may be set to the average of top and bottom sides of the tetragon and the average of left and right sides respectively.

[00132] In one embodiment, if skew correction is performed, the angle of skew of the target rectangle may be set to zero so that the page sides will become horizontal and vertical. Otherwise, the skew angle may be set to the average of the angles of top and bottom sides to the horizontal axis and those of the left and right sides to the vertical axis.

[00133] In a similar fashion, if crop correction is not performed, the center of the target rectangle may be designated so as to match the average of the coordinates of the four corners of the tetragon; otherwise the center may be calculated so that the target rectangle ends up in the top left of the image frame, in additional embodiments.

[00134] In some approaches, if page detection result is rejected for any reason, some or all steps of the process described herein may be repeated with a smaller step increment, in order to obtain more candidate edge points and, advantageously, achieve more plausible results. In a worst-case scenario where problems persist even with the minimum allowed step, the detected page may be set to the whole image frame and the original image may be left untouched.

[00135] Now with particular reference to an exemplary implementation of the inventive page detection embodiment described herein, in one approach page detection includes performing a method such as described below. As will be appreciated by one having ordinary skill in the art upon reading the present descriptions, the method may be performed in any environment, including those described herein and represented in any of the Figures provided with the present disclosures.

[00136] In one embodiment, the method includes operation , where a plurality of candidate edge points corresponding to a transition from a digital image background to the digital representation of the document are defined.

[00137] In various embodiments, defining the plurality of candidate edge points in operation may include one or more additional operations such as operations -, described below.

[00138] In one operation, and according to one embodiment, a large analysis window (e.g. large analysis window 308 as shown in FIGS 3A-3B and 3D is defined within the digital image 300. Preferably, a first large analysis window is defined in a region depicting a plurality of pixels of the digital image background 304, but not depicting the non-background (e.g. the digital representation of the document 302) in order to obtain information characteristic of the digital image background 304 for comparison and contrast to information characteristic of the non-background (e.g. the digital representation of the document 302, such as background statistics discussed in further detail below with reference to operation). For example, the first large analysis window 308 may be defined in a corner (such as a top-left corner) of the digital image 300. Of course, the first large analysis window may be defined in any part of the digital image 300 without departing from the scope of the present disclosures.

[00139] Moreover, as will be understood by one having ordinary skill in the art upon reading the present descriptions, the large analysis window 308 may be any size and/or characterized by any suitable dimensions, but in preferred embodiments the large analysis window 308 is approximately forty pixels high and approximately forty pixels wide.

[00140] In particularly preferred approaches, the large analysis window 308 may be defined in a corner region of the digital image. For example, with reference to FIG. 3A, a digital image 300 is shown, the digital image 300 comprising a digital representation of a document 302

having a plurality of sides 306 and a background 304. As described above with reference to operation , the large analysis window 308 may be defined in a region comprising a plurality of background pixels and not including pixels corresponding to the digital representation of the document 302. Moreover, the large analysis window 308 may be defined in the corner of the digital image 300, in some approaches.

[00141] In another operation, according to one embodiment, a plurality of small analysis windows 312 may be defined within the digital image 300, such as within the large analysis window 308. The small analysis windows 312 may overlap at least in part with one or more other small analysis windows 312 such as to be characterized by comprising one or more overlap regions 320 as shown in FIG. 3D. In a preferred approach all possible small analysis windows 312 are defined within the large analysis window 308. Of course, small analysis windows may be defined within any portion of the digital image, such as shown in FIG. 3B, and preferably small analysis windows may be defined such that each small analysis window is characterized by a single center pixel.

[00142] In still another operation, according to one embodiment, one or more statistics are calculated for one or more small analysis windows 312 (e.g. one or more small analysis windows 312 within a large analysis window 308) and one or more distributions of corresponding statistics are estimated (e.g. a distribution of statistics estimated across a plurality of small analysis windows 312). In another embodiment, distributions of statistics may be estimated across one or more large analysis window(s) 308 and optionally merged.

[00143] Moreover, values may be descriptive of any feature associated with the background of the digital image, such as background brightness values, background color channel values, background texture values, background tint values, background contrast values, background sharpness values, etc. as would be understood by one having ordinary skill in the art upon reading the present descriptions. Moreover still, statistics may include a minimum, a maximum and/or a range of brightness values in one or more color channels of the plurality of pixels depicting the digital image background 304 over the plurality of small windows 312 within the large analysis window 308.

[00144] In yet another operation, and again according to one exemplary embodiment, one or more distributions of background statistics are estimated. By estimating the distribution(s) of statistics, one may obtain descriptive distribution(s) that characterize the properties of the background 304 of the digital image 300 within, for example, a large analysis window 308.

[00145] The distribution(s) preferably correspond to the background statistics calculated for each small analysis window, and may include, for example, a distribution of brightness minima,

a distribution of brightness maxima, etc., from which one may obtain distribution statistical descriptors such as the minimum and/or maximum of minimum brightness values, the minimum and/or maximum of minimum brightness values, minimum and/or maximum spread of brightness values, minimum and/or maximum of minimum color channel values, minimum and/or maximum of maximum color channel values, minimum and/or maximum spread of color channel values etc. as would be appreciated by one having ordinary skill in the art upon reading the present descriptions. Of course, any of the calculated background statistics (e.g. for brightness values, color channel values, contrast values, texture values, tint values, sharpness values, etc.) may be assembled into a distribution and any value descriptive of the distribution may be employed without departing from the scope of the present disclosures.

[00146] In still yet another operation, according to one embodiment, a large analysis window, such as analysis window 308 as shown in FIGS. 3A-3B is defined within the digital image 300.

[00147] Moreover, window shapes may be defined by positively setting the boundaries of the window as a portion of the digital image 300, may be defined by negatively, e.g. by applying a mask to the digital image 300 and defining the regions of the digital image 300 not masked as the analysis window. Moreover still, windows may be defined according to a pattern, especially in embodiments where windows are negatively defined by applying a mask to the digital image 300. Of course, other manners for defining the windows may be employed without departing from the scope of the present disclosures.

[00148] In more embodiments, the method may include performing an operation where one or more statistics are calculated for the analysis window 312. Moreover, in preferred embodiments each analysis window statistic corresponds to a distribution of background statistics estimated for the large analysis window 308 in operation. For example, in one embodiment maximum brightness corresponds to distribution of background brightness maxima, minimum brightness corresponds to distribution of background brightness minima, brightness spread corresponds to distribution of background brightness spreads, etc. as would be understood by one having ordinary skill in the art upon reading the present descriptions.

[00149] In more embodiments, the method include determining whether a statistically significant difference exists between at least one analysis window statistic and the corresponding distribution of background statistics. As will be appreciated by one having ordinary skill in the art upon reading the present descriptions, determining whether a statistically significant difference exists may be performed using any known statistical significance evaluation method or metric, such as a p-value, a z-test, a chi-squared correlation, etc. as would be appreciated by a skilled artisan reading the present descriptions.

[00150] In additional and/or alternative approaches, the method includes designating one or more points (e.g. the centermost pixel 318 or point) in the analysis window for which a statistically significant difference exists between a value describing the pixel 318 and the corresponding distribution of background statistics is designated as a candidate edge point. The designating may be accomplished by any suitable method known in the art, such as setting a flag corresponding to the pixel, storing coordinates of the pixel, making an array of pixel coordinates, altering one or more values describing the pixel 318 (such as brightness, hue, contrast, etc.), or any other suitable means.

[00151] In operation, according to one embodiment, one or more of operations - may be repeated one or more times. In a preferred embodiment, a plurality of such repetitions may be performed, wherein each repetition is performed on a different portion of the digital image. Preferably, the repetitions may be performed until each side of the digital representation of the document has been evaluated. In various approaches, defining the analysis windows 308, 312 may result in a plurality of analysis windows 308, 312 which share one or more borders, which overlap in whole or in part, and/or which do not share any common border and do not overlap, etc. as would be understood by one having ordinary skill in the art upon reading the present descriptions.

[00152] In a particularly preferred embodiment, the plurality of repetitions may be performed in a manner directed to reestimate local background statistics upon detecting a potentially non-background window (e.g. a window containing a candidate edge point or a window containing an artifact such as uneven illumination, background texture variation, etc.).

[00153] In operation, according to one embodiment, four sides of a tetragon 400 are defined based on the plurality of candidate edge points. Preferably, the sides of the tetragon 400 encompass the edges 306 of a digital representation of a document 302 in a digital image 300. Defining the sides of the tetragon 400 may include, in some approaches, performing one or more least-mean-squares (LMS) approximations.

[00154] In more approaches, defining the sides of the tetragon 400 may include identifying one or more outlier candidate edge points, and removing one or more outlier candidate edge points from the plurality of candidate edge points. Further, defining the sides of the tetragon 400 may include performing at least one additional LMS approximation excluding the one or more outlier candidate edge points.

[00155] Further still, in one embodiment each side of the tetragon 400 is characterized by an equation chosen from a class of functions, and performing the at least one LMS approximation comprises determining one or more coefficients for each equation, such as best coefficients of

second degree polynomials in a preferred implementation. According to these approaches, defining the sides of the tetragon 400 may include determining whether each side of the digital representation of the document falls within a given class of functions, such as second degree polynomials or simpler functions such as linear functions instead of second degree polynomials.

[00156] In preferred approaches, performing method may accurately define a tetragon around the four dominant sides of a document while ignoring one or more deviations from the dominant sides of the document, such as a rip 310 and/or a tab 320 as depicted in FIGS. 3A-3C and 4.

[00157] Additional and/or alternative embodiments of the presently disclosed tetragon 400 may be characterized by having four sides, and each side being characterized by one or more equations such as the polynomial functions discussed above. For example, embodiments where the sides of tetragon 400 are characterized by more than one equation may involve dividing one or more sides into a plurality of segments, each segment being characterized by an equation such as the polynomial functions discussed above.

[00158] Defining the tetragon 400 may, in various embodiments, alternatively and/or additionally include defining one or more corners of the tetragon 400. For example, tetragon 400 corners may be defined by calculating one or more intersections between adjacent sides of the tetragon 400, and designating an appropriate intersection from the one or more calculated intersections in cases where multiple intersections are calculated. In still more embodiments, defining the corners may include solving one or more equations, wherein each equation is characterized by belonging to a chosen class of functions such as N^{th} degree polynomials, etc. as would be understood by one having ordinary skill in the art upon reading the present descriptions.

[00159] In various embodiments, a corner of the tetragon 400 may be defined by one or more of: an intersection of two curved adjacent sides of the tetragon 400; an intersection of two substantially straight lines; and an intersection of one substantially straight line and one substantially curved line.

[00160] In even still more embodiments, the method may include an additional and/or alternative operation, where the digital representation of the document 302 and the tetragon 400 are output to a display of a mobile device. Outputting may be performed in any manner, and may depend upon the configuration of the mobile device hardware and/or software.

[00161] Moreover, outputting may be performed in various approaches so as to facilitate further processing and/or user interaction with the output. For example, in one embodiment the tetragon 400 may be displayed in a manner designed to distinguish the tetragon 400 from other features of the digital image 300, for example by displaying the tetragon 400 sides in a particular

color, pattern, illumination motif, as an animation, etc. as would be understood by one having ordinary skill in the art upon reading the present descriptions.

[00162] Further still, in some embodiments outputting the tetragon 400 and the digital representation of the document 302 may facilitate a user manually adjusting and/or defining the tetragon 400 in any suitable manner. For example, a user may interact with the display of the mobile device to translate the tetragon 400, i.e. to move the location of the tetragon 400 in one or more directions while maintaining the aspect ratio, shape, edge lengths, area, etc. of the tetragon 400. Additionally and/or alternatively, a user may interact with the display of the mobile device to manually define or adjust locations of tetragon 400 corners, e.g. tapping on a tetragon 400 corner and dragging the corner to a desired location within the digital image 300, such as a corner of the digital representation of the document 302.

[00163] Referring again to FIG. 4, one particular example of an ideal result of page detection is depicted, showing the digital representation of the document 302 within the digital image 300, and having a tetragon 400 that encompasses the edges of the digital representation of the document 302.

[00164] In some approaches, page detection methods such as described above may include one or more additional and/or alternative operations, such as will be described below.

[00165] In one approach, page detection may further include capturing one or more of the image data containing the digital representation of the document and audio data relating to the digital representation of the document. Capturing may be performed using one or more capture components coupled to the mobile device, such as a microphone, a camera, an accelerometer, a sensor, etc. as would be understood by one having ordinary skill in the art upon reading the present descriptions.

[00166] In another approach, page detection may include defining a new large analysis window 309 and reestimating the distribution of background statistics for the new large analysis window 309 upon determining that the statistically significant difference exists, i.e. essentially repeating operation 1908 and/or 1910 in a different region of the digital image 300 near a point where a potentially non-background point has been identified, such as near one of the edges 306 of the document.

[00167] In several exemplary embodiments, a large analysis window 308 may be positioned near or at the leftmost non-background pixel in a row or positioned near or at the rightmost non-background pixel in a row, positioned near or at the topmost non-background pixel in a column, positioned near or at bottommost non-background pixel in a column.

[00168] Approaches involving such reestimation may further include determining whether

the statistically significant difference exists between at least one small analysis window (e.g. a test window) statistic and the corresponding reestimated distribution of large analysis window statistics. In this manner, it is possible to obtain a higher-confidence determination of whether the statistically significant difference exists, and therefore better distinguish true transitions from the digital image background to the digital representation of the document as opposed to, for example, variations in texture, illumination anomalies, and/or other artifacts within the digital image.

[00169] Moreover, with or without performing reestimation as described above may facilitate the method avoiding one or more artifacts such as variations in illumination and/or background texture, etc. in the digital image, the artifacts not corresponding to a true transition from the digital image background to the digital representation of the document. In some approaches, avoiding artifacts may take the form of bypassing one or more regions (e.g. regions characterized by textures, variations, etc. that distinguish the region from the true background) of the digital image.

[00170] In some approaches, one or more regions may be bypassed upon determining a statistically significant difference exists between a statistical distribution estimated for the large analysis window 308 and a corresponding statistic calculated for the small analysis window 312, defining a new large analysis window near the small analysis window, reestimating the distribution of statistics for the new large analysis window, and determining that the statistically significant difference does not exist between the reestimated statistical distribution and the corresponding statistic calculated for the small analysis window 312.

[00171] In other approaches, bypassing may be accomplished by checking another analysis window 312 further along the path and resuming the search for a transition to non-background upon determining that the statistics of this checked window do not differ significantly from the known statistical properties of the background, e.g. as indicated by a test of statistical significance.

[00172] As will be appreciated by the skilled artisan upon reading the present disclosures, bypassing may be accomplished by checking another analysis window further along the path.

[00173] In still further approaches, page detection may additionally and/or alternatively include determining whether the tetragon 400 satisfies one or more quality control metrics; and rejecting the tetragon 400 upon determining the tetragon 400 does not satisfy one or more of the quality control metrics. Moreover, quality control metrics may include measures such as a LMS support metric, a minimum tetragon 400 area metric, a tetragon 400 corner location metric, and a tetragon 400 diagonal intersection location metric.

[00174] In practice, determining whether the tetragon 400 satisfies one or more of these metrics acts as a check on the performance of the method. For example, checks may include determining whether the tetragon 400 covers at least a threshold of the overall digital image area, e.g. whether the tetragon 400 comprises at least 25% of the total image area.

[00175] Furthermore, checks may include determining whether tetragon 400 diagonals intersect inside the boundaries of the tetragon 400, determining whether one or more of the LMS approximations were calculated from sufficient data to have robust confidence in the statistics derived therefrom, i.e. whether the LMS approximation has sufficient “support,” (such as an approximation calculated from at least five data points, or at least a quarter of the total number of data points, in various approaches), and/or determining whether tetragon 400 corner locations (as defined by equations characterizing each respective side of the tetragon 400) exist within a threshold distance of the edge of the digital image, e.g. whether tetragon 400 corners are located more than 100 pixels away from an edge of the digital image in a given direction. Of course, other quality metrics and/or checks may be employed without departing from the scope of these disclosures, as would be appreciated by one having ordinary skill in the art upon reading the present descriptions.

[00176] In one approach, quality metrics and/or checks may facilitate rejecting suboptimal tetragon 400 definitions, and further facilitate improving the definition of the tetragon 400 sides. For example, one approach involves receiving an indication that the defining the four sides of the tetragon 400 based on the plurality of candidate edge points failed to define a valid tetragon 400, i.e. failed to satisfy one or more of the quality control metrics; and redefining the plurality of candidate edge points. Notably, in this embodiment redefining the plurality of candidate edge points includes sampling a greater number of points within the digital image than a number of points sampled in the prior, failed attempt. This may be accomplished, in one approach, by reducing the step over one or more of rows or columns of the digital image and repeating all the steps of the algorithm in order to analyze a larger number of candidate edge points. The step may be decreased in a vertical direction, a horizontal direction, or both. Of course, other methods of redefining the candidate edge points and/or resampling points within the digital image may be utilized without departing from the scope of the present disclosures.

[00177] Further still, page detection may include designating the entire digital image as the digital representation of the document, particularly where multiple repetitions of the method failed to define a valid tetragon 400, even with significantly reduced step in progression through the digital image analysis. In one approach, designating the entire digital image as the digital representation of the document may include defining image corners as document corners,

defining image sides as document sides, etc. as would be understood by one having ordinary skill in the art upon reading the present descriptions.

[00178] As described herein, the diagonals of the tetragon 400 may be characterized by a first line connecting a calculated top left corner of the tetragon 400 to a calculated bottom right corner of the tetragon 400, and second line connecting a calculated top right corner of the tetragon 400 and a calculated bottom left corner of the tetragon 400. Moreover, the first line and the second line preferably intersect inside the tetragon 400.

[00179] In various approaches, one or more of the foregoing operations may be performed using a processor, and the processor may be part of a mobile device, particularly a mobile device having an integrated camera.

[00180] Curvature Correction

[00181] The presently described inventive concepts include correcting curvature in a digital image, and more particularly correcting curvature in sides of a digital representation of a document. Various approaches to correcting curvature will be described in detail below, with exemplary reference to FIGS. 5A-5B.

[00182] In one embodiment, the goal of a curvature correction algorithm is to smoothly transform a tetragon 400 into a quadrilateral. Notably, the tetragon 400 is characterized by a plurality of equations, each equation corresponding to a side of the tetragon 400 and being selected from a chosen class of functions. For example, each side of the tetragon 400 may be characterized by a first degree polynomial, second degree polynomial, third degree polynomial, etc. as would be appreciated by the skilled artisan upon reading the present descriptions.

[00183] In one approach, sides of the tetragon 400 may be described by equations, and in a preferred embodiment a left side of the tetragon 400 is characterized by a second degree polynomial equation: $x = a_2 * y^2 + a_1 * y + a_0$; a right side of the tetragon 400 is characterized by a second degree polynomial equation: $x = b_2 * y^2 + b_1 * y + b_0$; a top side of the tetragon 400 is characterized by a second degree polynomial equation: $y = c_2 * x^2 + c_1 * x + c_0$; and a bottom side of the tetragon 400 is characterized by a second degree polynomial equation: $y = d_2 * x^2 + d_1 * x + d_0$.

[00184] The description of curvature correction presented herein utilizes the definition of a plurality of tetragon-based intrinsic coordinate pairs (p, q) within the tetragon, each intrinsic coordinate pair (p, q) corresponding to an intersection of a top-to-bottom curve characterized by an equation obtained from the equations of its left and right sides by combining all corresponding coefficients in a top-to-bottom curve coefficient ratio of p to $1 - p$, and a left-to-right curve characterized by an equation obtained from the equations of its top and bottom sides

by combining all corresponding coefficients in a left-to-right curve coefficient ratio of q to $1 - q$, wherein $0 \leq p \leq 1$, and wherein $0 \leq q \leq 1$.

[00185] In a preferred embodiment where the sides of the tetragon 400 are characterized by second degree polynomial equations, the top-to-bottom curve corresponding to the intrinsic coordinate p will be characterized by the equation: $x = ((1 - p) * a_2 + p * b_2) * y^2 + ((1 - p) * a_1 + p * b_1) * y + ((1 - p) * a_0 + p * b_0)$, and the left-to-right curve corresponding to the intrinsic coordinate q will be characterized by the equation: $y = ((1 - q) * c_2 + q * d_2) * x^2 + ((1 - q) * c_1 + q * d_1) * x + ((1 - q) * c_0 + q * d_0)$. Of course, other equations may characterize any of the sides and/or curves described above, as would be appreciated by one having ordinary skill in the art upon reading the present descriptions.

[00186] For a parallelogram, the intrinsic coordinates become especially simple: within the parallelogram, each intrinsic coordinate pair (p, q) corresponds to an intersection of a line parallel to each of a left side of the parallelogram and a right side of the parallelogram, e.g. a line splitting both top and bottom sides in the proportion of p to $1 - p$; and a line parallel to each of a top side of the parallelogram and a bottom side of the parallelogram, e.g. a line splitting both top and bottom sides in the proportion of q to $1 - q$, wherein $0 \leq p \leq 1$, and wherein $0 \leq q \leq 1$. In another particular case, when the tetragon is a unit square, that is a square with sides of length 1, the intrinsic coordinates are exactly the ordinary Cartesian coordinates: a point with coordinates (p, q) is an intersection of a vertical line $x = p$ and a horizontal line $y = q$.

[00187] The goal of the curvature correction algorithm described below is to match each point in the curvature-corrected image to a corresponding point in the original image, and do it in such a way as to transform each of the four sides of the tetragon 400 into a substantially straight line connecting its existing corners; however, the same technique can smoothly transform any tetragon described by the equations of its four sides to any other such tetragon.

[00188] The main idea of the coordinate mapping algorithm described below is to achieve this goal by, first, calculating intrinsic coordinates (p, q) for each point **P** (not shown) in the destination image, second, matching these to the same pair (p, q) of intrinsic coordinates in the original image, third, calculating the coordinates of the intersection of the left-to-right and top-to-bottom curves corresponding to these intrinsic coordinates respectively, and finally, assigning the color or gray value at the found point in the original image to the point **P**.

[00189] Referring now to FIG. 5A, which depicts a graphical representation of a first iteration of a page curvature correction algorithm, according to one embodiment. As shown in FIG. 5A, each point in a digital image 500 may correspond to an intersection of a top-to-bottom curve 504 and a left-to-right curve 506 (a curve may include a straight line, a curved line, e.g. a parabola,

etc. as would be understood by one having ordinary skill in the art upon reading the present descriptions) corresponding to intrinsic coordinates (such as described above) associated with a point.

[00190] As will become apparent from the present descriptions, curvature correction may involve defining a plurality of such left-to-right lines 506 and top-to-bottom lines 504.

[00191] Moreover, curvature correction may include matching target intrinsic coordinates to original intrinsic coordinates of the digital representation of the document 502.

[00192] As shown in FIG. 5A, this matching may include iteratively searching for an intersection of a given left-to-right curve 506 and a given top-to-bottom curve 504. FIG. 5A shows the first iteration of an exemplary iterative search within the scope of the present disclosures.

[00193] The iterative search, according to one approach discussed in further detail below, includes designating a starting point 508 having coordinates (x_0, y_0) . The starting point 508 may be located anywhere within the digital representation of the document 502, but preferably is located at or near the center of the target tetragon.

[00194] The iterative search may include projecting the starting point 508 onto one of the two intersecting curves 504, 506. While the starting point may be projected onto either of the curves 504, 506, in one approach the first half of a first iteration in the iterative search includes projecting the starting point 508 onto the top-to-bottom curve to obtain x-coordinate (x_1) of the next point, the projection result represented in FIG. 5A by point 510, which has coordinates (x_1, y_0) . Similarly, in some embodiments the second half of a first iteration in the iterative search includes projecting the point 510 onto the left-to-right curve 506 to obtain y-coordinate (y_1) of the next point, the projection result represented in FIG. 5A by point 512, which has coordinates (x_1, y_1) .

[00195] FIG. 5B is a graphical representation of a starting point of a page curvature correction algorithm, after dividing the digital representation of the document 502 into a plurality of equally-sized sections defined by the plurality of top-to-bottom curves 504 and the plurality of left-to-right curves 506, according to one embodiment.

[00196] Further iterations may utilize a similar approach such as described in further detail below, in some embodiments.

[00197] With continuing reference to FIGS. 5A-5B, a method for modifying one or more spatial characteristics of a digital representation of a document in a digital image includes one or more of the following operations, according to one embodiment. As will be appreciated by one having ordinary skill in the art upon reading the present descriptions, the method may be

performed in any suitable environment, including those shown and/or described in the figures and corresponding descriptions of the present disclosures.

[00198] In one embodiment, the method includes an operation where a tetragon 400 is transformed into a quadrilateral. Notably, the tetragon 400 is characterized by a plurality of equations, each equation corresponding to a side of the tetragon 400 and being selected from a chosen class of functions. For example, each side of the tetragon 400 may be characterized by a first degree polynomial, second degree polynomial, third degree polynomial, etc. as would be appreciated by the skilled artisan upon reading the present descriptions.

[00199] In one embodiment, sides of the tetragon 400 may be described by equations, and in a preferred embodiment a left side of the tetragon 400 is characterized by a second degree polynomial equation: $x = a_2 * y^2 + a_1 * y + a_0$; a right side of the tetragon 400 is characterized by a second degree polynomial equation: $x = b_2 * y^2 + b_1 * y + b_0$; a top side of the tetragon 400 is characterized by a second degree polynomial equation: $y = c_2 * x^2 + c_1 * x + c_0$; and a bottom side of the tetragon 400 is characterized by a second degree polynomial equation: $y = d_2 * x^2 + d_1 * x + d_0$. Moreover, the top-to-bottom curve equation is: $x = ((1 - p) * a_2 + p * b_2) * y^2 + ((1 - p) * a_1 + p * b_1) * y + ((1 - p) * a_0 + p * b_0)$, and the left-to-right curve equation is: $y = ((1 - q) * c_2 + q * d_2) * x^2 + ((1 - q) * c_1 + q * d_1) * x + ((1 - q) * c_0 + q * d_0)$. Of course, other equations may characterize any of the sides and/or curves described above, as would be appreciated by one having ordinary skill in the art upon reading the present descriptions.

[00200] In one embodiment, curves 504, 506 may be described by exemplary polynomial functions fitting one or more of the following general forms.

$$x_1 = u_2 * y_0^2 + u_1 * y_0 + u_0;$$

$$y_1 = v_2 * x_1^2 + v_1 * x_1 + v_0,$$

where $u_i = (1 - p) * a_i + p * b_i$, and $v_i = (1 - q) * c_i + q * d_i$, and where, a_i are the coefficients in the equation of the left side of the tetragon, b_i are the coefficients in the equation of the right side of the tetragon, c_i are the coefficients in the equation of the top side of the tetragon, d_i are the coefficients in the equation of the bottom side of the tetragon, and p and q are the tetragon-based intrinsic coordinates corresponding to curves 504, 506. In some approaches, the coefficients such as a_i , b_i , c_i , d_i , etc. may be derived from calculations, estimations, and/or determinations achieved in the course of performing page detection, such as a page detection method as discussed above with reference to page detection.

[00201] Of course, as would be understood by one having ordinary skill in the art, transforming the tetragon 400 into a quadrilateral may include one or more additional operations, such as will be described in greater detail below.

[00202] In one embodiment, the method may additionally and/or alternatively includes stretching one or more regions of the tetragon 400 in a manner sufficiently smooth to avoid introducing additional artifacts (such as distortion of interior regions of the tetragon) into the resulting quadrilateral.

[00203] In some approaches, transforming the tetragon 400 into a rectangle may include determining a height of the rectangle, a width of the rectangle, a skew angle of the rectangle, and/or a center position of the rectangle. For example, such transforming may include defining a width of the target rectangle as the average of the width of the top side and the width of the bottom side of the tetragon 400; defining a height of the target rectangle as the average of the height of the left side and the height of the right side of the tetragon 400; defining a center of the target rectangle depending on the desired placement of the rectangle in the image; and defining an angle of skew of the target rectangle, e.g. in response to a user request to deskew the digital representation of the document.

[00204] Upon obtaining a (straight-sided) quadrilateral the presently disclosed algorithms may proceed with a projection correction component that transforms the quadrilateral into the target rectangle, advantageously with very low error as measured by comparing pixel location of various object elements in the corrected image with corresponding pixel locations of the respective object elements in a scanned image. In preferred embodiments, the residual error may be about 5 pixels or less in an image having resolution of about 500 dots-per-inch (DPI). This corresponds to no pixel on the “corrected image” of the object being more than 5 pixels away, in any direction, from the corresponding location of the same pixel in a scanned image of the object.

[00205] Projection Correction

[00206] Projection correction as described herein essentially includes transforming the quadrilateral produced by the curvature correction algorithm described above into a true rectangle, in one approach. Preferably, the rectangle substantially represents the actual dimensions, aspect ratio, etc. of the object captured in the digital image when viewed from a particular perspective (e.g. at an angle normal to the object, such as would be the capture angle if scanning the object in a traditional flatbed scanner, multifunction device, etc. as would be understood by one having ordinary skill in the art upon reading the present descriptions).

[00207] Various capture angles, and the associated projective effects are demonstrated schematically in FIGS. 6A-6D.

[00208] In some approaches, the projection correction may include applying an algorithm such as a four-point algorithm to the image data. In various embodiments, such algorithms may

or may not rely on one or more of the following assumptions: 1) from the perspective of the capture angle, the thickness of the 3D object is zero, and the size of the captured 3D object is nonzero along each of the width and height dimensions; 2) the aspect ratio of the width and height dimensions is known. The value of the aspect ratio does not need to be known exactly, it may tolerate small measurement errors, which may influence 3D reconstruction errors. In preferred embodiments, measure characterized by error of less than about 10% is acceptable (e.g. an aspect ratio corresponding to predetermined document type such as letter, legal, A4, A5, driver license, credit card, sales receipt, business card, etc. as would be understood by one having ordinary skill in the art upon reading the present descriptions); 3) 2D pixel positions of four corner pixels in the captured image are estimable using an image segmentation technique, such as utilized in page detection as described herein; 4) the position of object corners in the captured image and the reference image (which correspond to the “real-world coordinates” of the object) are described by a pre-established correspondences of four pixels/corners in the reference image and the captured image; and 5) 3D reconstruction achieves pixel positions in the reconstructed image that are substantially the same as those observed from a particular perspective of the real object, e.g. as observed in a 500 DPI image captured using a capture angle normal to the object such as would be created by scanning a 2D representation of the object from that same perspective.

[00209] In one embodiment, the presently described page detection algorithm may be utilized to estimate 2D pixel positions of the document corners. The intersections of the four found sides of the document are calculated in order to find the corners of (possibly a slightly curved) page tetragon, (e.g. tetragon 400 as shown in FIG. 4).

[00210] In another embodiment, and with particular reference to the correspondence between reference image and captured image pixel coordinates/positions (especially corner coordinates/positions), the coordinates of an object (e.g. document) left top corner, left bottom corner, right bottom corner and right top corner in the reference image preferably correspond to the respective object corner coordinates/positions in the captured image. Determining the precise relationship in each correspondence may use textual and/or image features as reference points in the determination. For instance, in one embodiment the text orientation and document aspect ratio may be used to determine an orientation of the captured document.

[00211] For exemplary purposes only, the following descriptions will illustrate one embodiment of perspective correction performed on a digital image of a driver license. A reference image of the driver license may be captured, preferably using a scanner, multi-function printer, or other device known in the art not to introduce perspective skew or distortion

into images captured therewith. The reference image may preferably have been captured using a scanner at a known resolution, most preferably a resolution of approximately 500 DPI, and a known capture angle, most preferably an angle normal to the document (e.g. a capture angle of 90 degrees such as shown above in FIG. 6B).

[00212] Using the reference image, and preferably as supplemented by *a priori* knowledge regarding the “true” shape, size, dimensions, texture, etc. of an object, it is possible to reconstruct that object in a manner such that the object represented in the reconstructed image has identical or nearly identical characteristics as the reference image. In some embodiments, the reconstructed image and reference image may be of different but proportional scale. In such cases, applying a scaling operation to the reconstructed image may eliminate any difference in scale such that the reconstructed object has identical or nearly identical characteristics as the object depicted in the reference image.

[00213] A user may capture an image of their driver’s license using a mobile device, and potentially at a steep capture angle (e.g. a capture angle deviating from normal by about 30 degrees or more). As a result, the representation of the driver license in the captured image is characterized by 3D perspective distortions, causing the substantially rectangular document to appear trapezoidal in shape. In extreme cases, such as observed when using a capture angle more than 30 degrees away from normal, the length of the edge farthest from the capture device may appear shorter than the length of the edge nearest the capture device, etc. as would be understood by one having ordinary skill in the art upon reading the present descriptions. In one embodiment, the length of the far edge may appear significantly shorter, e.g. as little as 50% of the length of the near edge, particularly when using steep capture angles (e.g. 30 degrees or greater deviation from normal).

[00214] In various embodiments, 3D reconstruction preferably minimizes introducing any distortions in reconstructing the original image to generate a rectangular representation of the captured object/document.

[00215] In one embodiment, perspective correction may include capturing an image using a mobile device, identifying four points, pixels, etc. within the captured image, each point/pixel corresponding to a potential corner of the detected driver’s license (or other tetragonal document) and constructing a 3D transformation based at least in part on and four corner pixels. The positions of the four pixels can be estimated, when the four-sided polygon which forms as the boundary of the image to be segmented.

[00216] A planar homography/projective transform is a non-singular linear relation between two planes. In this case, the homography transform defines a linear mapping of four

corner pixels/positions between the captured image and the image plane. The calculation of the camera parameters may utilize an estimation of the homography transform H , such as shown in Equation (1), in some approaches.

$$\lambda \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \underbrace{\begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{pmatrix}}_{\text{homography } H} \begin{pmatrix} X \\ Y \\ 1 \end{pmatrix}, \quad (1)$$

As depicted above in Equation (1):

- λ is the focal depth of position (X, Y, Z) in the “reference” or “real-world” coordinate system, (e.g. a coordinate system derived from a reference image, such as shown in FIG 6B and 7 above). Put another way, λ may be considered the linear distance between a point (X,Y,Z) in the reference coordinate system and the capture device;
- (x, y, z) are the coordinates of a given pixel position in the captured image; and
- H is a (3×3) matrix having elements h_{ij} , where, i and j define the corresponding row and column index, respectively.

[00217] In one approach, the (x, y) coordinates and (X, Y) coordinates depicted in Equation 1 correspond to coordinates of respective points in the captured image plane and the reference image (e.g. as shown in FIG. 6B). The Z coordinate is set to 0, corresponding to the assumption that the object depicted in each lies along a single (e.g. X - Y) plane with zero thickness. We may, in one embodiment, omit the z value in Equation 1 from the above calculations because it does not necessarily play any role in determining the homography matrix.

[00218] Thus, the homography H can be estimated by detecting four point-correspondences $p_i \leftrightarrow P_i'$ with $p_i = (x_i, y_i, 1)^T$ being four corner positions in the captured image plane; and $P_i' = (X_i, Y_i, 1)^T$ being the coordinates of the corresponding four corner points, where i is point index value with range from 1 to n in the following discussion. Using the previously introduced notation, Equation (1) may be written as shown in Equation (2) below.

$$\lambda p_i = H P_i', \quad (2)$$

[00219] In order to eliminate a scaling factor, in one embodiment it is possible to calculate the cross product of each term of Equation (2), as shown in Equation (3):

$$p_i \times (\lambda p_i) = p_i \times (H P_i'), \quad (3)$$

[00220] Since $p_i \times p_i = 0_3$, Equation (3) may be written as shown below in Equation (4).

$$p_i \times H P_i' = 0_3. \quad (4)$$

[00221] Thus, the matrix product $H P_i'$ may be expressed as in Equation (5).

$$HP'_i = \begin{bmatrix} h^{1T} P'_i \\ h^{2T} P'_i \\ h^{3T} P'_i \end{bmatrix}, \quad (5)$$

[00222] where h^{mT} is the transpose of the m^{th} row of H (e.g. h^{1T} is the transpose of the first row of H , h^{2T} is the transpose of the second row of H , etc.). Accordingly, it is possible to rework Equation (4) as:

$$p_i \times HP'_i = \begin{pmatrix} x_i \\ y_i \\ 1 \end{pmatrix} \times \begin{bmatrix} h^{1T} P'_i \\ h^{2T} P'_i \\ h^{3T} P'_i \end{bmatrix} = \begin{bmatrix} y_i h^{2T} P'_i - h^{2T} P'_i \\ h^{1T} P'_i - x_i h^{1T} P'_i \\ x_i h^{2T} P'_i - y_i h^{1T} P'_i \end{bmatrix} = 0_3. \quad (6)$$

[00223] Notably, Equation (6) is linear in h^{mT} and $h^{mT} P'_i = P_i^T h^m$. Thus, Equation (6) may be reformulated as shown below in Equation (7):

$$\begin{bmatrix} 0_3^T & -P_i^T & y_i P_i^T \\ P_i^T & 0_3^T & -x_i P_i^T \\ -y_i P_i^T & x_i P_i^T & 0_3^T \end{bmatrix} \begin{bmatrix} h^1 \\ h^2 \\ h^3 \end{bmatrix} = 0_9. \quad (7)$$

[00224] Note that the rows of the matrix shown in Equation (7) are not linearly independent. For example, in one embodiment the third row is the sum of $-x_i$ times the first row and $-y_i$ times the second row. Thus, for each point-correspondence, Equation (7) provides two linearly independent equations. The two first rows are preferably used for solving H . Because the homography transform is written using homogeneous coordinates, in one embodiment the homography H may be defined using 8 parameters plus a homogeneous scaling factor (which may be viewed as a free 9th parameter). In such embodiments, at least 4 point-correspondences providing 8 equations may be used to compute the homography. In practice, and according to one exemplary embodiment, a larger number of correspondences is preferably employed so that an over-determined linear system is obtained, resulting in a more robust result (e.g. lower error in relative pixel-position). By rewriting H in a vector form as $h =$

$[h_{11}, h_{12}, h_{13}, h_{21}, h_{22}, h_{23}, h_{31}, h_{32}, h_{33}]^T$, n pairs of point-correspondences enable the construction of a $2n \times 9$ linear system, which is expressed by Equation (8)

$$\underbrace{\begin{pmatrix} 0 & 0 & 0 & -X_1 & -Y_1 & -1 & y_1 X_1 & y_1 X_1 & y_1 \\ X_1 & Y_1 & 1 & 0 & 0 & 0 & -x_1 X_1 & -x_1 Y_1 & -x_1 \\ 0 & 0 & 0 & -X_2 & -Y_2 & -1 & y_2 X_2 & y_2 X_2 & y_2 \\ X_2 & Y_2 & 1 & 0 & 0 & 0 & -x_2 X_2 & -x_2 Y_2 & -x_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & -X_n & -Y_n & -1 & y_n X_n & y_n X_n & y_n \\ X_n & Y_n & 1 & 0 & 0 & 0 & -x_n X_n & -x_n Y_n & -x_n \end{pmatrix}}_C \begin{pmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33} \end{pmatrix} = 0_9. \quad (8)$$

[00225] As shown in Equation 8, the first two rows correspond to the first feature point, as indicated by the subscript value of coordinates X, Y, x, y - in this case the subscript value is 1. The

second two rows correspond to the second feature point, as indicated by the subscript value 2, the last two rows correspond to the n-th feature point. For four-point algorithm, the n is 4, and the feature points are the four corners of a document page.

[00226] Solving this linear system involves the calculation of a *Singular Value Decomposition* (SVD). Such an SVD corresponds to reworking the matrix to the form of the matrix product $C = UDV^T$, where the solution h corresponds to the eigenvector of the smallest eigenvalue of matrix C , which in one embodiment may be located at the last column of the matrix V when the eigenvalues are sorted in descendant order.

[00227] It is worth noting that the matrix C is different from the typical matrix utilized in an eight-point algorithm to estimate the essential matrix when two or more cameras are used, such as conventionally performed for stereoscopic machine vision. More specifically, while the elements conventionally used in eight-point algorithm consist of feature points projected on two camera planes, the elements in the presently described matrix C consist of feature points projected on only a single camera plane and the corresponding feature points on 3D objects.

[00228] In one embodiment, to avoid numerical instabilities, the coordinates of point-correspondences may preferably be normalized. This may be accomplished, for example, using a technique known as the normalized *Direct Linear Transformation* (DLT) algorithm. For example, in one embodiment, after the homography matrix is estimated, Equation 1 may be used to compute each pixel position (x,y) for a given value of (X,Y) . In practical applications the challenge involves computing (X,Y) when the values of (x, y) are given or known *a priori*. As shown in Equation 1, and in preferred embodiments, (x, y) and (X, Y) are symmetrical (i.e. when the values of (x, y) and (X, Y) are switched, the validity of Equation 1 holds true). In this case, the “inverse” homography matrix may be estimated, and this “inverse” homography matrix may be used to reconstruct 3D (i.e. “reference” or “real-world”) coordinates of an object given the corresponding 2D coordinates of the object as depicted in the captured image, e.g. in the camera view.

[00229] Based on the foregoing, it is possible to implement the presently described four-point algorithm (as well as any equivalent variation and/or modification thereof that would be appreciated by a skilled artisan upon reading these descriptions) which may be utilized in various embodiments to efficiently and effectively reconstruct digital images characterized by at least some perspective distortion into corrected digital images exempting any such perspective distortion, where the corrected image is characterized by a pixel location error of about 5 pixels or less.

[00230] Various embodiments may additionally and/or alternatively include utilizing the

foregoing data, calculations, results, and/or concepts to derive further useful information regarding the captured image, object, etc. For example, in various embodiments it is possible to determine the distance between the captured object and the capture device, the pitch and/or roll angle of the capture device, etc. as would be understood by one having ordinary skill in the art upon reading the present descriptions.

[00231] After (X, Y) values are estimated, the expression in Equation 1 may be described as follows:

$$\lambda = h_{31}X + h_{32}Y + h_{33} \quad (9)$$

[00232] Accordingly, in one embodiment the focal depth, also known as the distance between each point (X, Y, Z) in the 3D (i.e. “reference” or “real world”) coordinate system and the capture device, may be computed using Equation 9 above.

[00233] **The determination of the rotation matrix of the object.**

[00234] After estimating the position of the 3D object we have (X, Y) and λ for each pixel in the captured image. Note that (X, Y) are the coordinates in the world coordinate system, while λ is the distance to the point (X, Y) in the camera coordinate system. If the 3D object is assumed to be a rigid body, we will present an algorithm to estimate the rotation matrix from the world coordinate system to the camera coordinate system. The following equation holds for rotation and translation of the point $(X, Y, 0)$:

$$\begin{pmatrix} X_c \\ Y_c \\ Z_c \end{pmatrix} = R \begin{pmatrix} X \\ Y \\ 0 \end{pmatrix} + t \quad (10)$$

where (X_c, Y_c, Z_c) are the coordinates relative to camera coordinate system, which are derived by rotating a point (X, Y, Z) in the world coordinate system with rotation matrix R , and a translation vector of t , where t is a constant independent of (X, Y) . Note that the value of Z_c is the same as the value of λ , as previously estimated using equation 9.

[00235] Considering the relationships of homography matrix H and intrinsic camera parameter matrix A and r_1, r_2 , where r_1, r_2 are the first and second column vectors respectively, reveals the following relationship:

$$H = \sigma A(r_1, r_2, t) \quad (11)$$

where σ is a constant and A is the intrinsic camera parameter matrix, defined as:

$$A = \begin{pmatrix} a & c & d \\ & b & e \\ & & 1 \end{pmatrix} \quad (12)$$

where a and b are scaling factors which comprise of the camera focal length information, $a=f/dx$, and $b=f/dy$, where f is the focal length, while dx, dy are scaling factors of the image; c is the skew parameter about two image axes, and (d, e) are the coordinates of the corresponding

principal point.

[00236] After estimation of homography matrix H, the matrix A can be estimated as follows:

$$a = \sqrt{w/B_{11}}; \quad (12.1)$$

$$b = \sqrt{wB_{11}(B_{11}B_{22} - B_{12}^2)}; \quad (12.2)$$

$$c = -B_{12}a^2b/w; d = \frac{vv_0}{b} - B_{13}a^2/w; \quad (12.3)$$

$$v = -B_{12}a^2b/w; \quad (12.4)$$

$$e = (B_{12}B_{13} - B_{11}B_{23})/(B_{11}B_{22} - B_{12}^2); \quad (12.5)$$

$$w = B_{33} - (B_{13}^2 + e(B_{12}B_{13} - B_{11}B_{23}))/B_{11}. \quad (12.6)$$

[00237] In the above relationships, the unknown parameters are B_{ij} . These values are estimated by the following equations:

$$\begin{pmatrix} v_{12}^t \\ (v_{11} - v_{22})^t \end{pmatrix} G = 0, \quad (12.7)$$

where G is the solution of the above equation, alternatively expressed as:

$$G = (B_{11}, B_{12}, B_{22}, B_{13}, B_{23}, B_{33})^t, \quad (12.8)$$

$$\text{where } v_{ij} = (h_{i1}h_{j1}, h_{i1}h_{j2} + h_{i2}h_{j1}, h_{i2}h_{j2}, h_{i3}h_{j1} + h_{i1}h_{j3}, h_{i3}h_{j2} + h_{i2}h_{j3}, h_{i3}h_{j3})^t \quad (12.9)$$

[00238] Note that in a conventional four-points algorithm, since it is possible to accurately estimate scaling factors a, b, the skew factor c is assumed to be zero, which means that one may ignore camera's skew distortion. It is further useful, in one embodiment, to assume that d and e have zero values (d=0, e=0).

[00239] From equation (11), $B = (r1 \ r2 \ t)$, where $\sigma^{-1} A^{-1}H = B$. Utilizing this relationship enables a new approach to estimate r1, r2 from the equation $C = (r1 \ r2 \ 0)$ where the first and second column vectors of C are the first and second column vectors of B, and the third column vector of C is 0.

[00240] First, decompose matrix C with SVD (*Singular Value Decomposition*) method, $C = U\Sigma V^t$, where U is 3 by 3 orthogonal matrix, where V is 3 by 3 orthogonal matrix. Then r1 and r2 are estimated by the following equation:

$$(r1 \ r2 \ 0) = U \begin{pmatrix} W \\ 0 \end{pmatrix} \quad (13)$$

[00241] where W is a 2 by 3 matrix whose first and second row vectors are the first and second row vectors of V^t respectively. In the above computation, assume σ is 1. This scaling factor does not influence the value of U and W and therefore does not influence the estimation of r1 and r2. After r1, r2 are estimated (e.g. using Equation 13), it is useful to leverage the fact that R is a rotation matrix to estimate r3, which is the cross product of r1 and r2 with a sign to be determined (either 1 or -1). There are two possible solutions of R. In one example using a right-

hand coordinate system, the r_3 value is the cross-product value of r_1 and r_2 .

[00242] Determining yaw, pitch, and roll from a rotation matrix.

[00243] The yaw, pitch and roll (denoted by the α , β and γ respectively) are also known as Euler's angles, which are defined as the rotation angles around z, y, and x axes respectively, in one embodiment. According to this approach, the rotation matrix R in Equation 10 can be denoted as:

$$R = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix} \quad (14)$$

where each r is an element of the matrix R .

[00244] It is often convenient to determine the α , β and γ parameters directly from a given rotation matrix R . The roll, in one embodiment, may be estimated by the following equation (e.g. when r_{33} is not equal to zero):

$$\gamma = \text{atan2}(r_{32}, r_{33}) \quad (15)$$

[00245] Similarly, in another approach the pitch may be estimated by the following equation:

$$\beta = \text{atan2}(-r_{31}, \sqrt{r_{11}^2 + r_{21}^2}) \quad (16)$$

[00246] In still more approaches, the yaw may be estimated by the following equation (e.g. when r_{11} is nonzero)

$$\alpha = \text{atan2}(r_{21}, r_{11}) \quad (17)$$

[00247] Notably, in some approaches when r_{11} , r_{33} or $\sqrt{r_{11}^2 + r_{21}^2}$ are near in value to zero (e.g. $0 < r_{11} < \epsilon$, $0 < r_{33} < \epsilon$, or $0 < \sqrt{r_{11}^2 + r_{21}^2} < \epsilon$, where the value ϵ is set to a reasonable value for considering the numerical stability, such as $0 < \epsilon \leq 0.01$, in one embodiment, and $\epsilon = 0.0001$ in a particularly preferred embodiment. In general, the value of ϵ may be determined in whole or in part based on limited computer word length, etc. as would be understood by one having ordinary skill in the art upon reading the present descriptions), this corresponds to the degenerate of rotation matrix R , special formulae are used to estimate the values of yaw, pitch and roll.

[00248] Estimating Distance Between Object and Capture Device

[00249] In still more embodiments, it is possible to estimate the distance between an object and a capture device even without the knowledge of the object size, using information such as a camera's intrinsic parameters (e.g. focal length, scale factors of (u,v) in image plane).

[00250] The requirements of this algorithm, in one approach, may be summarized as follows:

1) The camera's focal length for the captured image can be provided and accessed by an API call of the device (for instance, an android device provides an API call to get focal length

information for the captured image); 2) The scale factors of dx and dy are estimated by the algorithm in the equations 12.1 and 12.2.

[00251] This means that we can estimate the scale factors dx, dy for a type of device and we do not need to estimate them for each device individually. For instance, in one exemplary embodiment utilizing an Apple iPhone® 4 smartphone, it is possible, using the algorithm presented above, to estimate the scale factors using an object with a known size. The two scaling factors may thereafter be assumed to be identical for the same device type.

[00252] The algorithm to estimate object distance to camera, according to one illustrative approach, is described as follows: We normalize (u,v) , (X,Y) in the equation below

$$\lambda \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = H \begin{pmatrix} X \\ Y \\ 1 \end{pmatrix} \quad (18)$$

[00253] Note that Equation 18 is equivalent to Equation 1, except that we use (u,v) in Equation 18 to replace the (x,y) in Equation 1.

[00254] Suppose that $\tilde{u} = u / L_u$, $\tilde{v} = v / L_v$; $\tilde{x} = X / L_x$; $\tilde{y} = Y / L_y$; where L_u, L_v are image size in coordinates u and v and L_x, L_y are the object size to be determined.

[00255] Then Equation 18 may be expressed as:

$$\lambda \begin{pmatrix} \tilde{u} \\ \tilde{v} \\ 1 \end{pmatrix} = \tilde{H} \begin{pmatrix} \tilde{x} \\ \tilde{y} \\ 1 \end{pmatrix}, \quad (19)$$

where

$$\tilde{H} = \begin{pmatrix} 1/L_u & & \\ & 1/L_v & \\ & & 1 \end{pmatrix} H \begin{pmatrix} L_x & & \\ & L_y & \\ & & 1 \end{pmatrix} \quad (20)$$

Normalized homography matrix \tilde{H} can be estimated by equation (20). Note that from equation 11, we have

$$H = \sigma A \begin{pmatrix} r_1 & r_2 & t \end{pmatrix} \quad (21)$$

and the intrinsic parameter matrix of the camera is assumed with the following simple form:

$$A = \begin{pmatrix} f/dx & c & d \\ & f/dy & e \\ & & 1 \end{pmatrix} \quad (22)$$

where f is the camera focal length, dx , dy are scaling factors of the camera, which are estimated by the algorithm presented from section [00182] to [00209].

From equations (19), (20) and (21), thus:

$$\sigma A \begin{pmatrix} r_1 & r_2 & t \end{pmatrix} \begin{pmatrix} L_x & & \\ & L_y & \\ & & 1 \end{pmatrix} = \tilde{\tilde{H}} \quad (23)$$

$$\text{where } \tilde{\tilde{H}} = \begin{pmatrix} L_u & & \\ & L_v & \\ & & 1 \end{pmatrix} \tilde{H}$$

Because A is known, from equation (23) we have:

$$\sigma \begin{pmatrix} r_1 & r_2 & t \end{pmatrix} \begin{pmatrix} L_x & & \\ & L_y & \\ & & 1 \end{pmatrix} = A^{-1} \tilde{\tilde{H}} \quad (24)$$

Denote $K = A^{-1} \tilde{\tilde{H}}$, $K = (k_1, k_2, k_3)$, from equation (24) we have:

$$\sigma r_1 L_x = k_1 \quad (25)$$

$$\sigma r_2 L_y = k_2 \quad (26)$$

$$\sigma t = k_3 \quad (27)$$

because t in equation (27) is the translation vector of the object relative to camera. The L2 norm (Euclidean norm) of t is as follows:

$$\|t\| = \|k_3\| / \sigma \quad (28)$$

is the distance of left-top corner of the object to the camera.

Because $\|r_1\| = \|r_2\| = 1$, from equation (8) and (9), we have

$$L_x = \|k_1\| / \sigma \quad (29)$$

$$L_y = \|k_2\| / \sigma \quad (30)$$

[00256] Equations (29) and (30) may be used to estimate the document size along X and Y coordinates. The scaling factor may remain unknown, using this approach.

[00257] Note that the algorithm to estimate rotation matrix described above does not need the scaling factor σ . Rather, in some approaches it is suitable to assume $\sigma = 1$. We can estimate roll, pitch, and yaw with the algorithm presented above. From equations (29) and (30), we can also estimate the aspect ratio of the object as:

$$\text{aspectratio} = L_x / L_y = \|k_1\| / \|k_2\| \quad (31)$$

[00258] Estimation of Pitch and Roll from Assumed Rectangle.

[00259] In practice the most common case is the camera capture of rectangular documents, such as sheets of paper of standard sizes, business cards, driver and other licenses, etc. Since the focal distance of the camera does not change, and since the knowledge of the yaw is irrelevant for the discussed types of document image processing, it is necessary only to determine roll and pitch of the camera relative to the plane of the document in order to rectangularize the corresponding image of the document.

[00260] The idea of the algorithm is simply that one can calculate the object coordinates of the document corresponding to the tetragon found in the picture (up to scale, rotation, and shift) for any relative pitch-roll combination. This calculated tetragon in object coordinates is characterized by 90-degree angles when the correct values of pitch and roll are used, and the deviation can be characterized by the sum of squares of the four angle differences. This criterion is useful because it is smooth and effectively penalizes individual large deviations.

[00261] A gradient descent procedure based on this criterion can find a good pitch-roll pair in a matter of milliseconds. This has been experimentally verified for instances where the tetragon in the picture was correctly determined. This approach uses yaw equal zero and an arbitrary fixed value of the distance to the object because changes in these values only add an additional orthogonal transform of the object coordinates. The approach also uses the known focal distance of the camera in the calculations of the coordinate transform, but if all four corners have been found and there are three independent angles, then the same criterion and a slightly more complex gradient descent procedure can be used to estimate the focal distance in addition to pitch and roll – this may be useful for server-based processing, when incoming pictures may or may not have any information about what camera they were taken with.

[00262] Interestingly, when the page detection is wrong, even the optimal pitch-roll pair leaves sizeable residual angle errors (of 1 degree or more), or, at least, if the page was just cropped-in parallel to itself, the aspect ratio derived from the found object coordinates does not match the real one.

[00263] Additionally, it is possible to apply this algorithm even when a location of one of the detected sides of the document is suspect or missing entirely (e.g. that side of the document is partially or completely obstructed, not depicted, or is blurred beyond recognition, etc.). In order to accomplish the desired result it is useful to modify the above defined criterion to use only two angles, for example those adjacent to the bottom side, in a gradient descent procedure. In this manner, the algorithm may still be utilized to estimate pitch and roll from a picture tetragon with bogus and/or undetectable top-left and top-right corners.

[00264] In one example, arbitrary points on the left and right sides closer to the top of the

image frame can be designated as top-left and top-right corners. The best estimated pitch-roll will create equally bogus top-left and top-right corners in the object coordinates, but the document will still be correctly rectangularized. The direction of a missing (e.g. top) side of the document can be reconstructed since it should be substantially parallel to the opposite (e.g. bottom) side, and orthogonal to adjacent (e.g. left and/or right) side(s).

[00265] The remaining question is where to place the missing side in the context of the image as a whole, and if the aspect ratio is known then the offset of the missing side can be nicely estimated, and if not, then it can be pushed to the edge of the frame, just not to lose any data. This variation of the algorithm can resolve an important user case when the picture contains only a part of the document along one of its sides, for example, the bottom of an invoice containing a deposit slip. In a situation like this the bottom, left and right sides of the document can be correctly determined and used to estimate pitch and roll; these angles together with the focal distance can be used to rectangularize the visible part of the document.

[00266] Thus, in one general approach exemplified by method 700 as depicted in FIG. 7, reconstruction includes capturing or receiving a digital image comprising a digital representation of an object, preferably a document or other object having known characteristics (size, texture, color profile, etc.) in operation 702.

[00267] In operation 704, the captured or received image is analyzed to determine a position of one or more boundaries separating the digital representation of the object from an image background or other objects represented in the image.

[00268] In operation 706, the boundaries are analyzed to determine whether any curvature (i.e. regions of non-linearity) exists in one or more of the boundaries. Curvature may be determined to exist in one of the boundaries, e.g. by determining a polynomial expression characterizing the boundary fits a particular class of function such as a first, second, third, fourth, etc. order polynomial, each of which may each be a different class of function. If curvature is determined to exist, it is preferably corrected to generate a boundary having substantially linear characteristics along the entirety of the boundary's length.

[00269] Once any determined curvature is corrected, in operation 708 the linear-edged boundaries are analyzed and/or extrapolated to define a bounding polygon, preferably a bounding tetragon, and even more preferably a bounding parallelogram, trapezoid, or rectangle.

[00270] In operation 710, the digital image and/or bounding polygon is analyzed to determine whether any perspective distortion and/or projective effects are present within the bounding polygon.

[00271] In operation 712, and in response to determining perspective distortion and/or

projective effects exist within the bounding polygon, the digital image, etc., the perspective distortion and/or projective effects are corrected to generate a reconstructed polygon. Preferably, the bounding polygon is a quadrilateral and the reconstructed polygon is a rectangle.

[00272] Of course, the foregoing disclosure of an exemplary method 700 may be embodied as a system configured to execute logic, and/or a computer program product comprising computer readable program code configured to perform functions substantially similar to any of those described herein.

[00273] Similarly, all the inventive concepts, features, techniques, components, systems, products, etc. discussed herein should be considered modular, and may be combined in any suitable manner that would be appreciated by one having ordinary skill in the art upon reading these descriptions. Notably, these combinations include any possible set of features disclosed above, with the sole exception of those features that skilled artisans would recognize upon reading these descriptions as being mutually exclusive or incompatible. Thus, various embodiments of the present inventive concepts may be defined by reciting any number of features disclosed above, inclusive or exclusive of any individual feature(s).

[00274] For instance, in one approach, a method of reconstructing a digital image includes receiving the digital image comprising a digital representation of an object bounded by a tetragon; correcting curvature in the tetragon to form a quadrilateral; and correcting projective effects in the quadrilateral to form a rectangle. The quadrilateral may be a parallelogram. Correcting the curvature may include transforming one or more portions of the tetragon based at least in part on intrinsic tetragon coordinates; and correcting the projective effects may include transforming the tetragon using a four-point algorithm. More specifically, correcting projective effects may include one or more of singular value decomposition (SVD), direct linear transformation (DLT), non-linear refinement (NLR), normalizing coordinates of point-correspondences between the digital image and a reconstructed image, and/or estimating a homography transform (H). Estimating H may include detecting one or more point correspondences $p_i \leftrightarrow P'_i$ with $p_i = (x_i, y_i, 1)^T$ and/or detecting four point correspondences, each point correspondence $p_i \leftrightarrow P'_i$ corresponding to a corner position p_i of the tetragon and a respective corner position P'_i of the rectangle. The digital image is optionally characterized by a capture angle of about 30 degrees or more deviation from normal with respect to the object. H preferably satisfies the expression $\lambda p_i = H P'_i$, where λ is a focal distance between a three-dimensional reference coordinate position (X, Y, Z) and a corresponding reference coordinate position of a capture device; $p_i = (x_i, y_i, 1)^T$; $P'_i = (X_i, Y_i, 1)^T$, and i is the respective index feature of the point p or P' in the corresponding image. The method may also include outputting the

reconstructed digital image, wherein the reconstructed digital image is characterized by a pixel location error of about 5 pixels or less relative to a scanned 500-DPI image of the object captured using a capture angle normal to the object. Sides of the tetragon may be characterized by a polynomial, which may be a second degree polynomial, or a polynomial of higher order than second degree. Most preferably, the object is a document characterized by a known height-to-width ratio.

[00275] Exemplary systems may include one or more processors, image capture devices, and corresponding functional logic configured to perform any one or more of the operations set forth in the immediately preceding paragraph, and also to leverage any number of the specific features such as the object being a document with rectangular shape and known height-to-width ratio, λ , p_i , P'_i , etc. as would be understood by one having ordinary skill in the art upon reading the present descriptions.

[00276] Exemplary computer program products may include a computer readable storage medium having embodied thereon computer code to perform any one or more of the operations set forth in the immediately preceding paragraph, and also to leverage any number of the specific features such as the object being a document with rectangular shape and known height-to-width ratio, λ , p_i , P'_i , etc. as would be understood by one having ordinary skill in the art upon reading the present descriptions.

[00277] While various embodiments have been described above, it should be understood that they have been presented by way of example only, and not limitation. Thus, the breadth and scope of an embodiment of the present invention should not be limited by any of the above-described exemplary embodiments, but should be defined only in accordance with the following claims and their equivalents.

CLAIMS

What is claimed is:

1. A method of reconstructing a digital image, the method comprising:
receiving the digital image comprising a digital representation of an object bounded by a tetragon;
correcting curvature in the tetragon to form a quadrilateral; and
correcting projective effects in the quadrilateral to form a rectangle.
2. The method as recited in claim 1, wherein the quadrilateral is a parallelogram.
3. The method as recited in claim 1, wherein correcting the curvature comprises transforming one or more portions of the tetragon based at least in part on intrinsic tetragon coordinates.
4. The method as recited in claim 1, wherein correcting the projective effects comprises transforming the tetragon using a four-point algorithm.
5. The method as recited in claim 1, wherein the digital image is characterized by a capture angle of about 30 degrees or more deviation from normal with respect to the object.
6. The method as recited in claim 1, wherein correcting the projective effects comprises singular value decomposition (SVD).
7. The method as recited in claim 1, wherein correcting the projective effects comprises direct linear transformation (DLT).
8. The method as recited in claim 1, wherein correcting the projective effects comprises estimating a homography transform H .
9. The method as recited in claim 8, wherein estimating H comprises detecting one or more point correspondences $p_i \leftrightarrow P_i'$ with $p_i = (x_i, y_i, 1)^T$.

10. The method as recited in claim 9, wherein estimating H comprises detecting four point correspondences, each point correspondence $p_i \leftrightarrow P'_i$ corresponding to a corner position p_i of the tetragon and a respective corner position P'_i of the rectangle.
11. The method as recited in claim 8, wherein H satisfies the expression $\lambda p_i = HP'_i$, where λ = a focal distance between a three-dimensional reference coordinate position (X, Y, Z) and a corresponding reference coordinate position of a capture device;

$$p_i = (x_i, y_i, 1)^T;$$

$$P'_i = (X_i, Y_i, 1)^T, \text{ and}$$

$$i \text{ is the respective index feature of the point } p \text{ or } P' \text{ in the corresponding image.}$$
12. The method as recited in claim 1, wherein correcting the projective effects comprises performing a non-linear refinement.
13. The method as recited in claim 1, further comprising outputting the reconstructed digital image, wherein the reconstructed digital image is characterized by a pixel location error of about 5 pixels or less relative to a scanned 500-DPI image of the object captured using a capture angle normal to the object.
14. The method as recited in claim 1, wherein at least one side of the tetragon is characterized by a second degree or higher order polynomial.
15. The method as recited in claim 14, wherein each side of the tetragon is characterized by a second degree or higher order polynomial.
16. The method as recited in claim 1, wherein correcting the projective effects comprises normalizing coordinates of point-correspondences between the digital image and a reconstructed image.
17. The method as recited in claim 1, wherein the object is a document characterized by a known height-to-width ratio.
18. A system, comprising:
 - a processor configured to execute logic; and

logic configured to receive a digital image comprising a digital representation of an object bounded by a tetragon;
logic configured to correct curvature in the tetragon to form a quadrilateral; and
logic configured to correct projective effects in the quadrilateral to form a rectangle.

19. A computer program product comprising a computer readable storage medium having computer readable program code stored thereon, the computer readable program code comprising:
computer readable program code configured to receive a digital image comprising a digital representation of an object bounded by a tetragon;
computer readable program code configured to correct curvature in the tetragon to form a quadrilateral; and
computer readable program code configured to correct projective effects in the quadrilateral to form a rectangle.

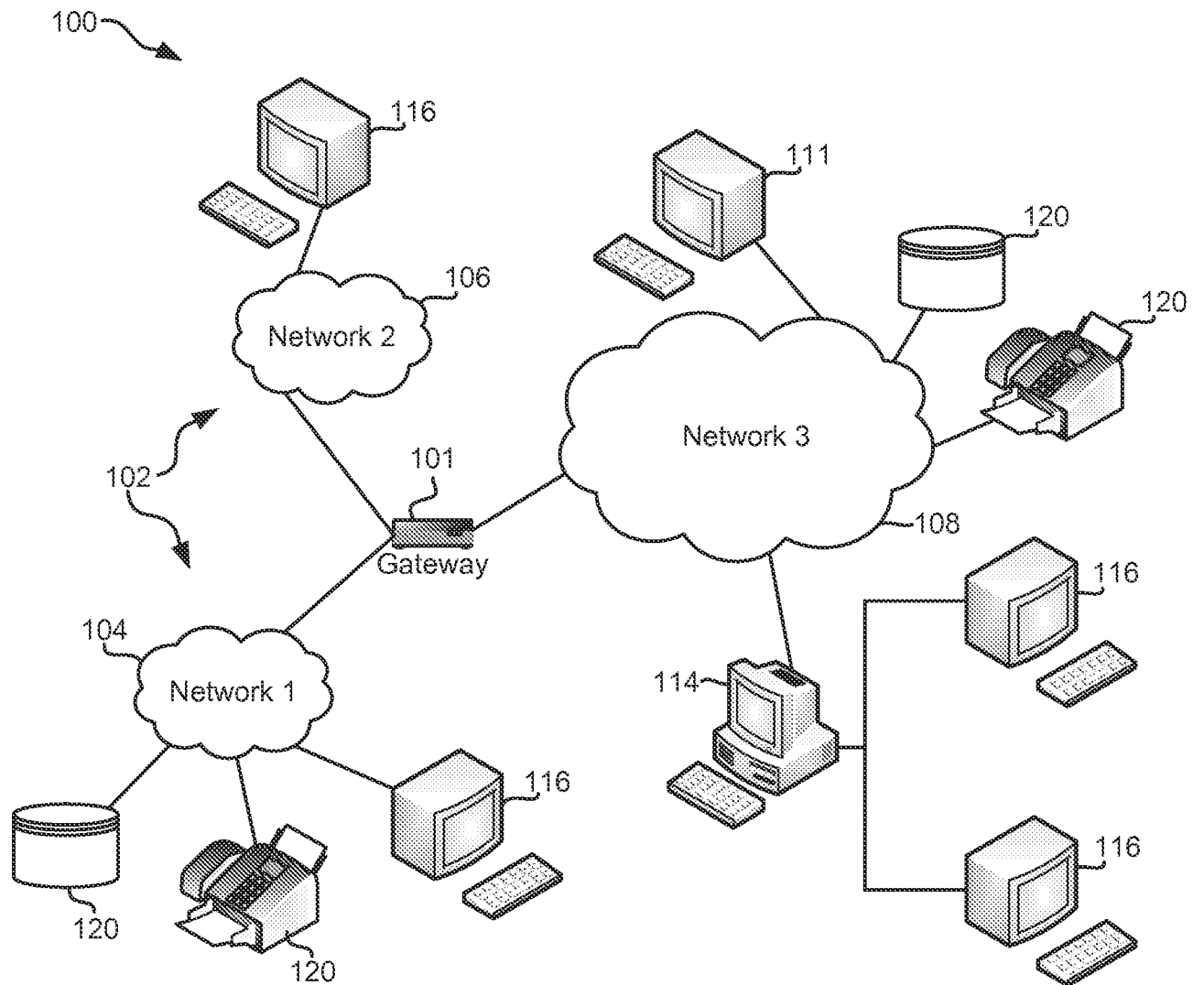


FIG. 1

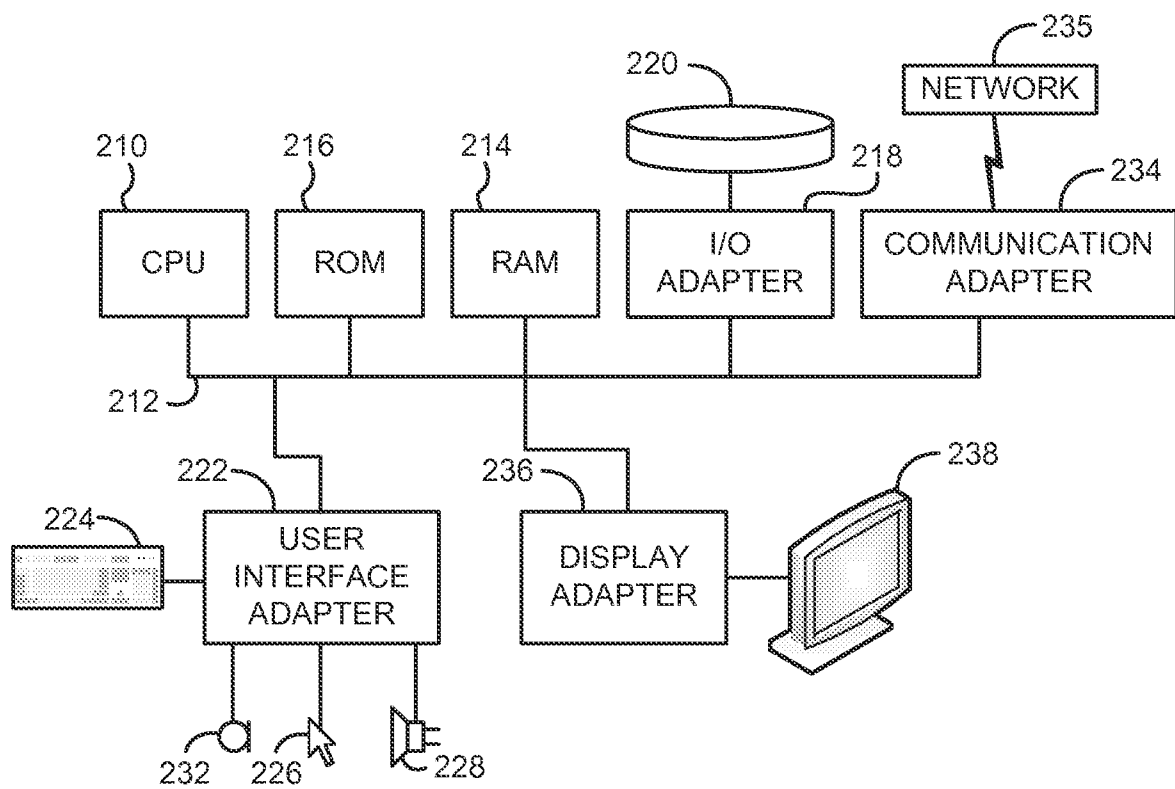


FIG. 2

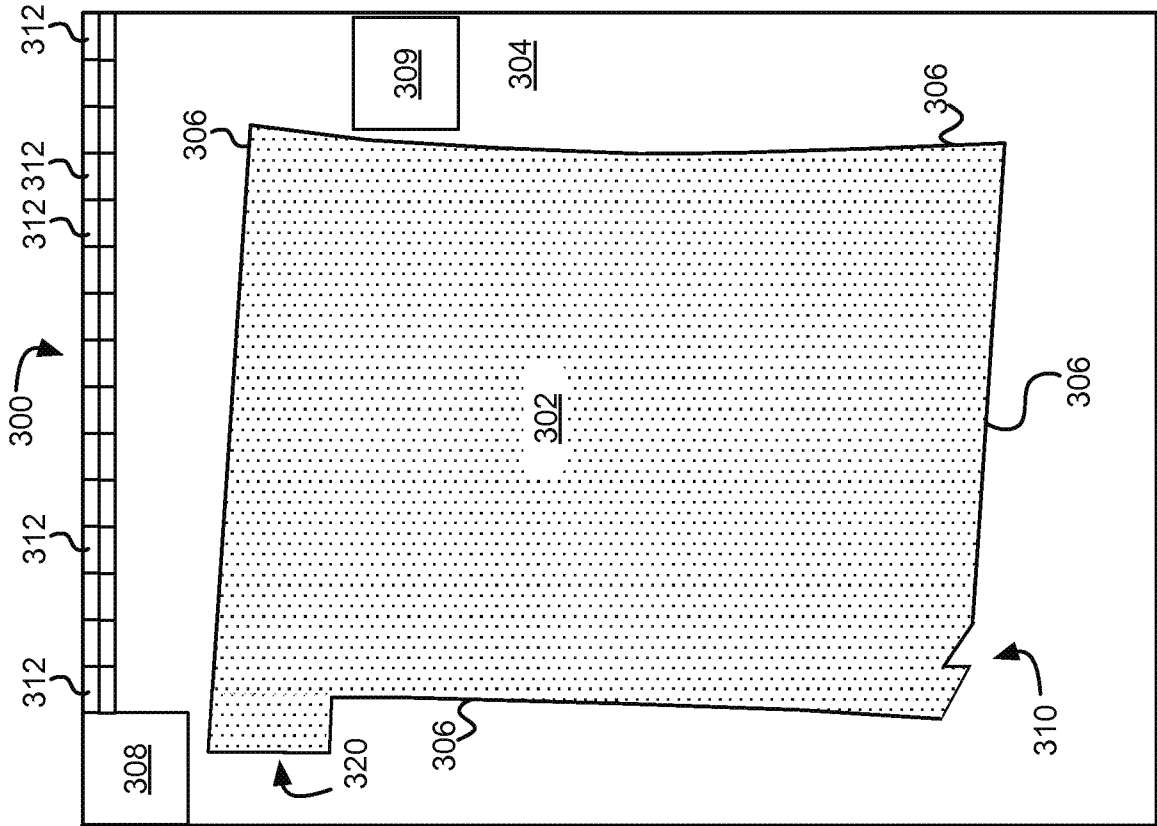


FIG. 3B

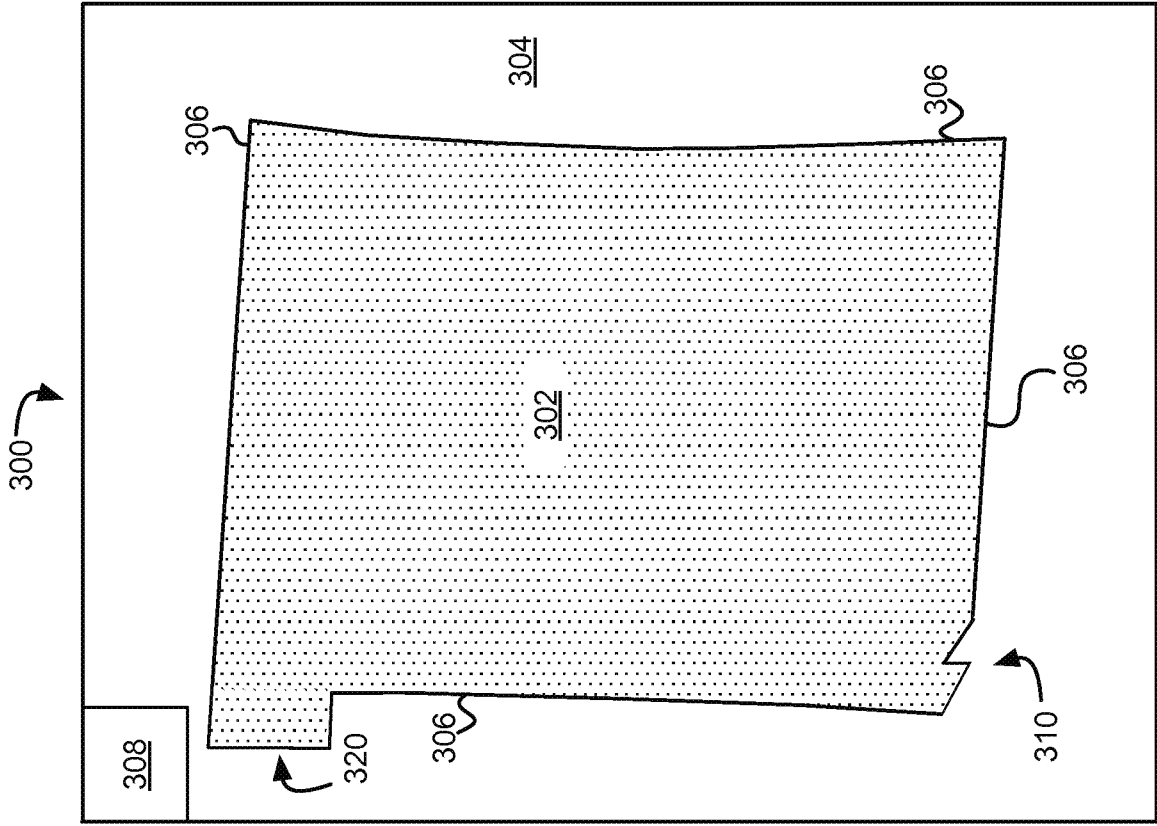


FIG. 3A

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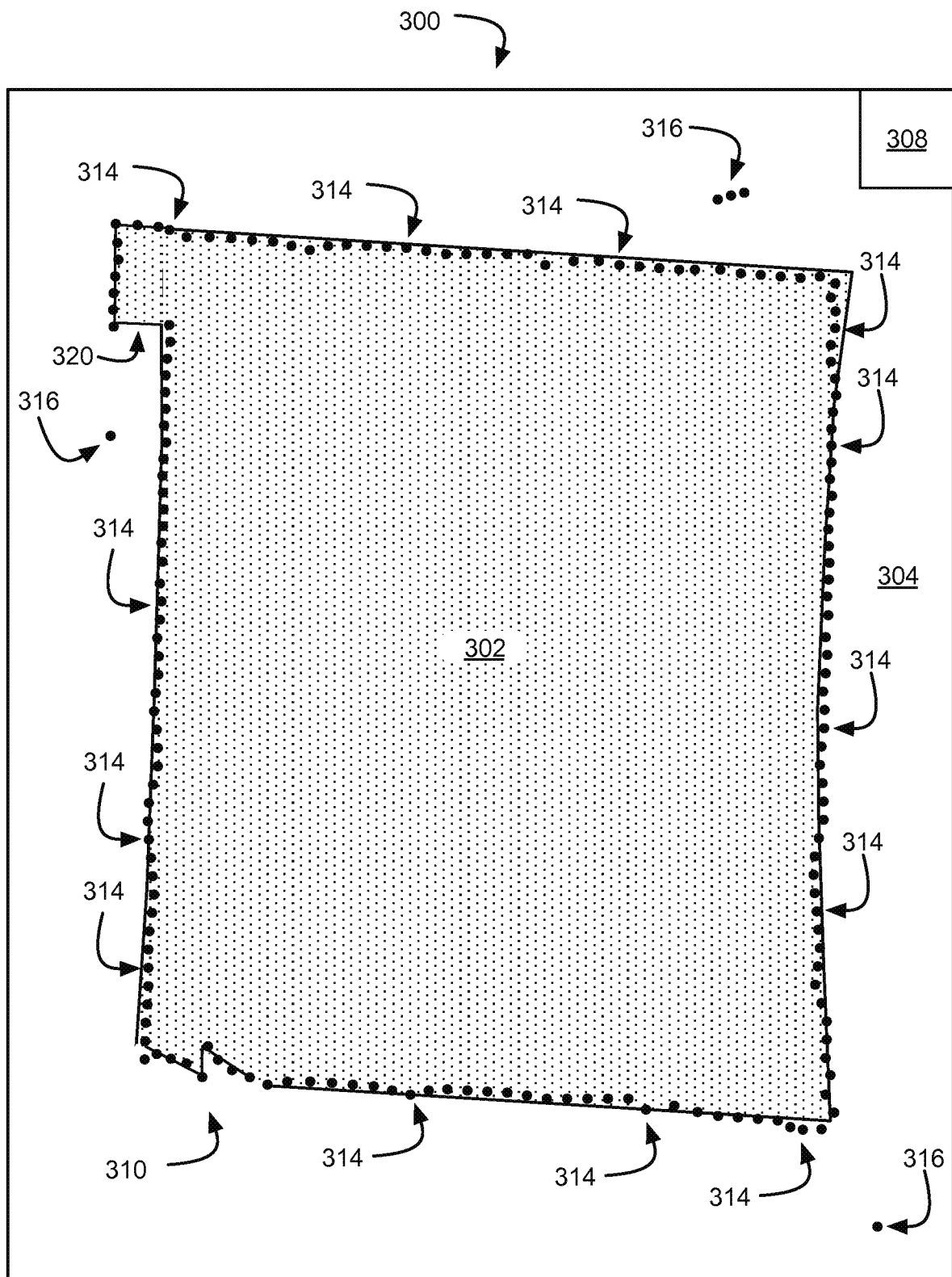


FIG. 3C

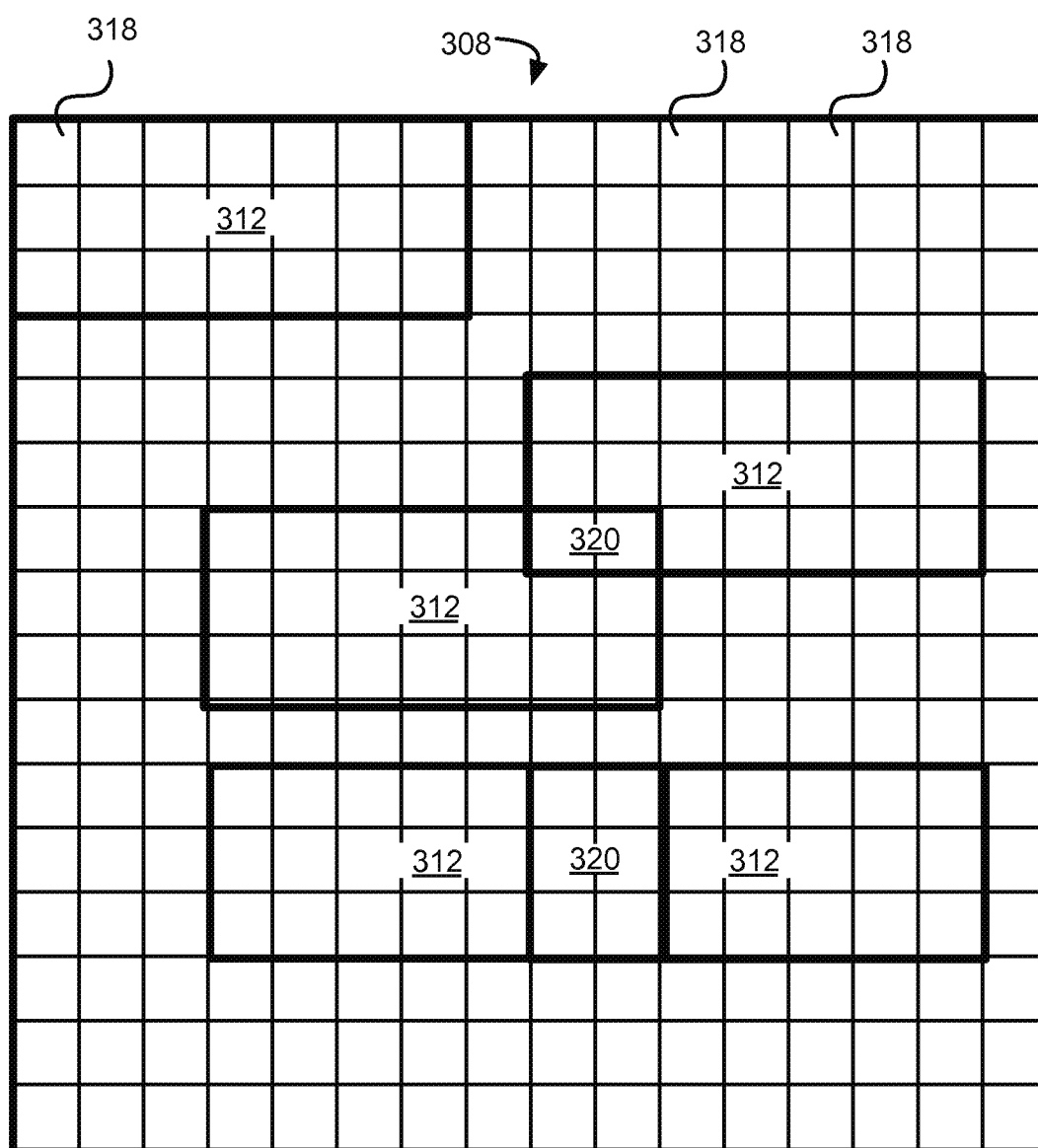


FIG. 3D

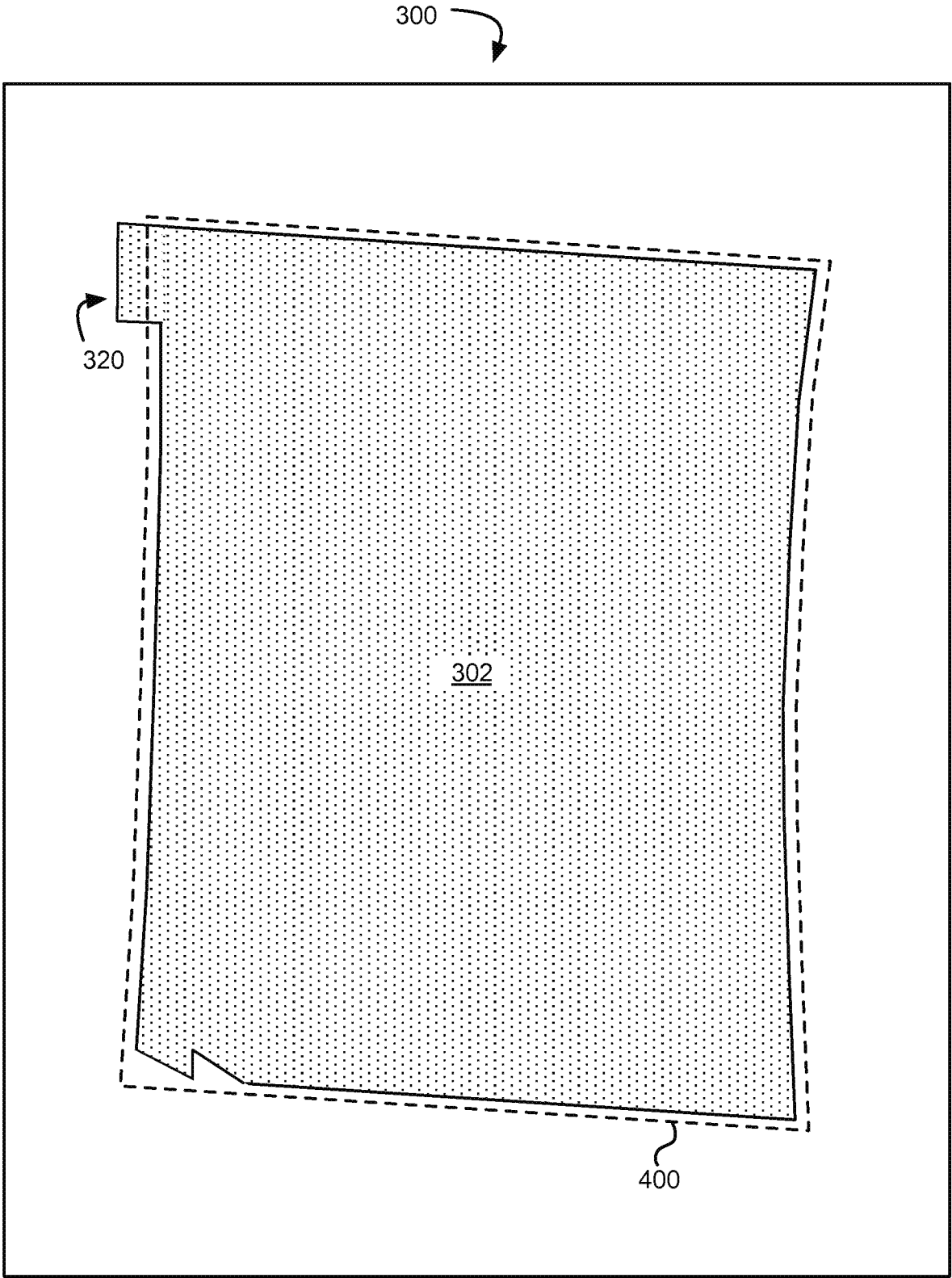


FIG. 4

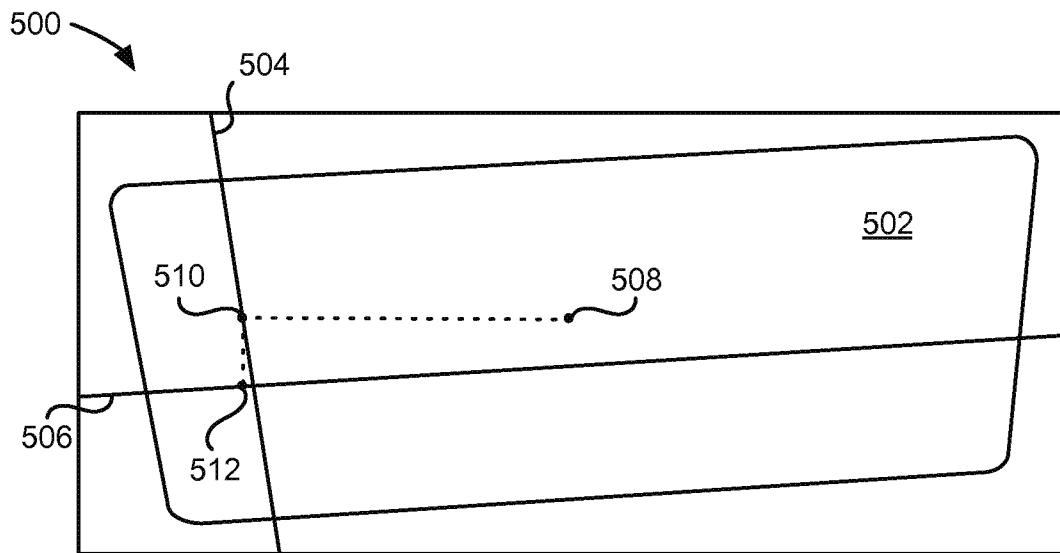


FIG. 5A

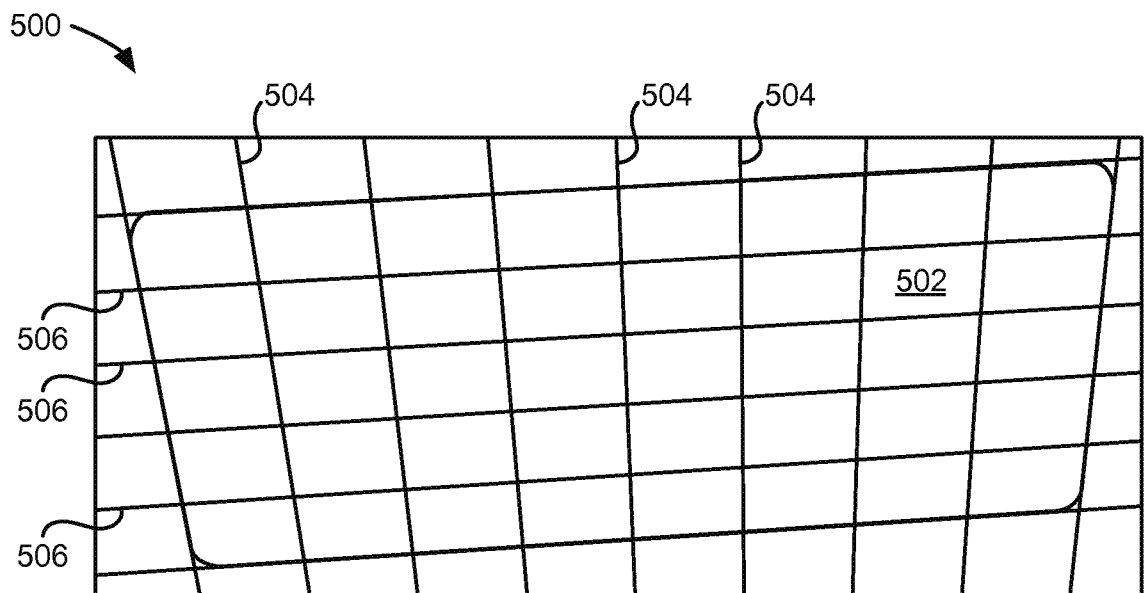


FIG. 5B

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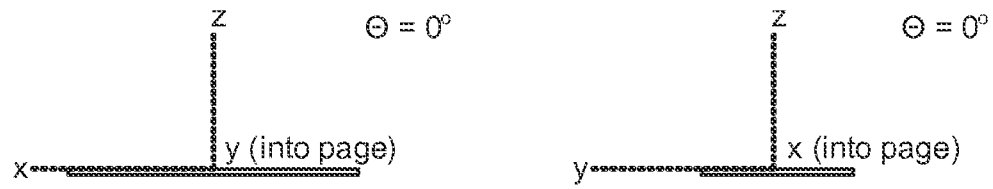


FIG. 6A

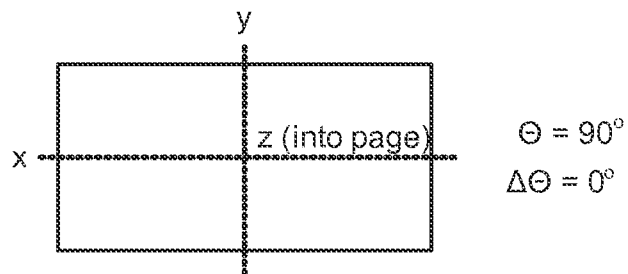


FIG. 6B

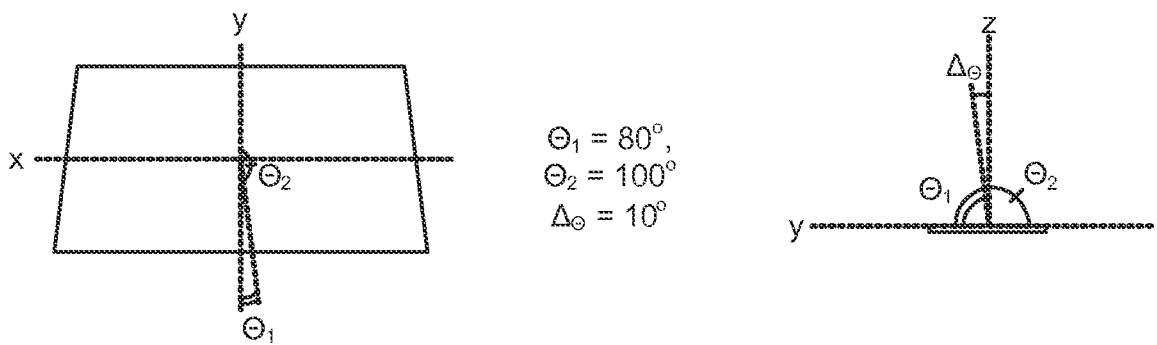


FIG. 6C

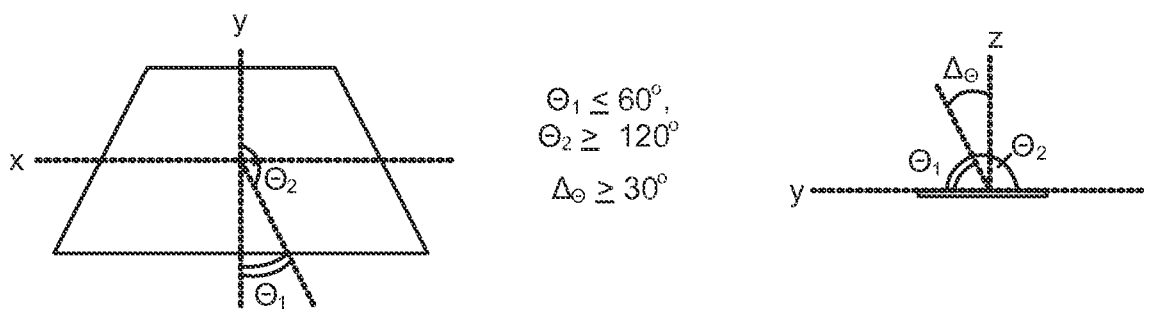
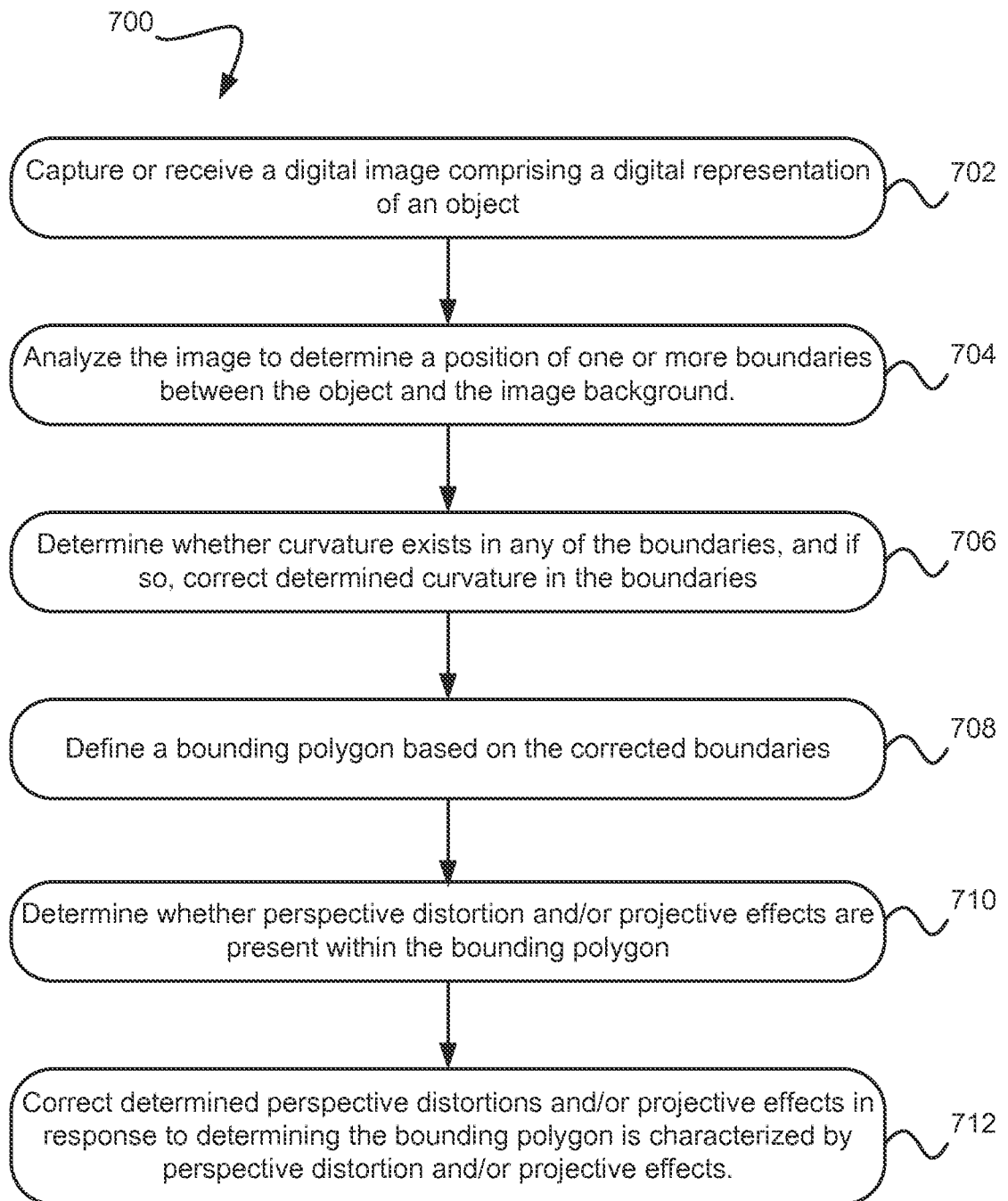


FIG. 6D

**FIG. 7**

INTERNATIONAL SEARCH REPORT

International application No.

PCT/US2014/057065

A. CLASSIFICATION OF SUBJECT MATTER

IPC(8) - G06F 15/00 (2014.01)

CPC - G06K 15/02 (2014.09)

According to International Patent Classification (IPC) or to both national classification and IPC

B. FIELDS SEARCHED

Minimum documentation searched (classification system followed by classification symbols)

IPC(8) - G06F 15/00; G06K 1/00; G01T 5/00 (2014.01)

USPC - 358/1.9, 3.26, 3.27, 520, 522, 537, 538

Documentation searched other than minimum documentation to the extent that such documents are included in the fields searched
CPC - G06K 15/02; H04N 1/00002, 1/4092 (2014.09) (keyword delimited)

Electronic data base consulted during the international search (name of data base and, where practicable, search terms used)

Orbit, Google Patents, ProQuest

Search terms used: digital image, curvature, tetragon, rectangle, correction, parallelogram

C. DOCUMENTS CONSIDERED TO BE RELEVANT

Category*	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
Y	US 2013/0182973 A1 (MACCIOLA et al) 18 July 2013 (18.07.2013) entire document	1-19
Y	US 2011/0279456 A1 (HIRANUMA et al) 17 November 2011 (17.11.2011) entire document	1-19
Y	US 2009/0225180 A1 (MARUYAMA et al) 10 September 2009 (10.09.2009) entire document	2
Y	US 2011/0090337 A1 (KLOMP et al) 21 April 2011 (21.04.2011) entire document	4
Y	US 2008/0068452 A1 (NAKAO et al) 20 March 2008 (20.03.2008) entire document	5
Y	US 6,729,733 B1 (RASKAR et al) 04 May 2004 (04.05.2004) entire document	6
Y	US 2009/0324062 A1 (LIM et al) 31 December 2009 (31.12.2009) entire document	7
Y	US 2012/0236019 A1 (OH et al) 20 September 2012 (20.09.2012) entire document	8-11
Y	US 2012/0301011 A1 (GRZECHNIK) 29 November 2012 (29.11.2012) entire document	11
Y	US 2012/0300020 A1 (ARTH et al) 29 November 2012 (29.11.2012) entire document	12
Y	US 4,651,287 A (TSAO) 17 March 1987 (17.03.1987) entire document	13
A	US 2004/0019274 A1 (GALLOWAY et al) 29 January 2004 (29.01.2004) entire document	1-19
A	US 2008/0298718 A1 (LIU et al) 04 December 2008 (04.12.2008) entire document	1-19

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Date of the actual completion of the international search

18 November 2014

Date of mailing of the international search report

30 DEC 2014

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Mail Stop PCT, Attn: ISA/US, Commissioner for Patents
P.O. Box 1450, Alexandria, Virginia 22313-1450

Facsimile No. 571-273-3201

Authorized officer:

Blaine R. Copenheaver

PCT Helpdesk: 571-272-4300

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