

July 22, 1969

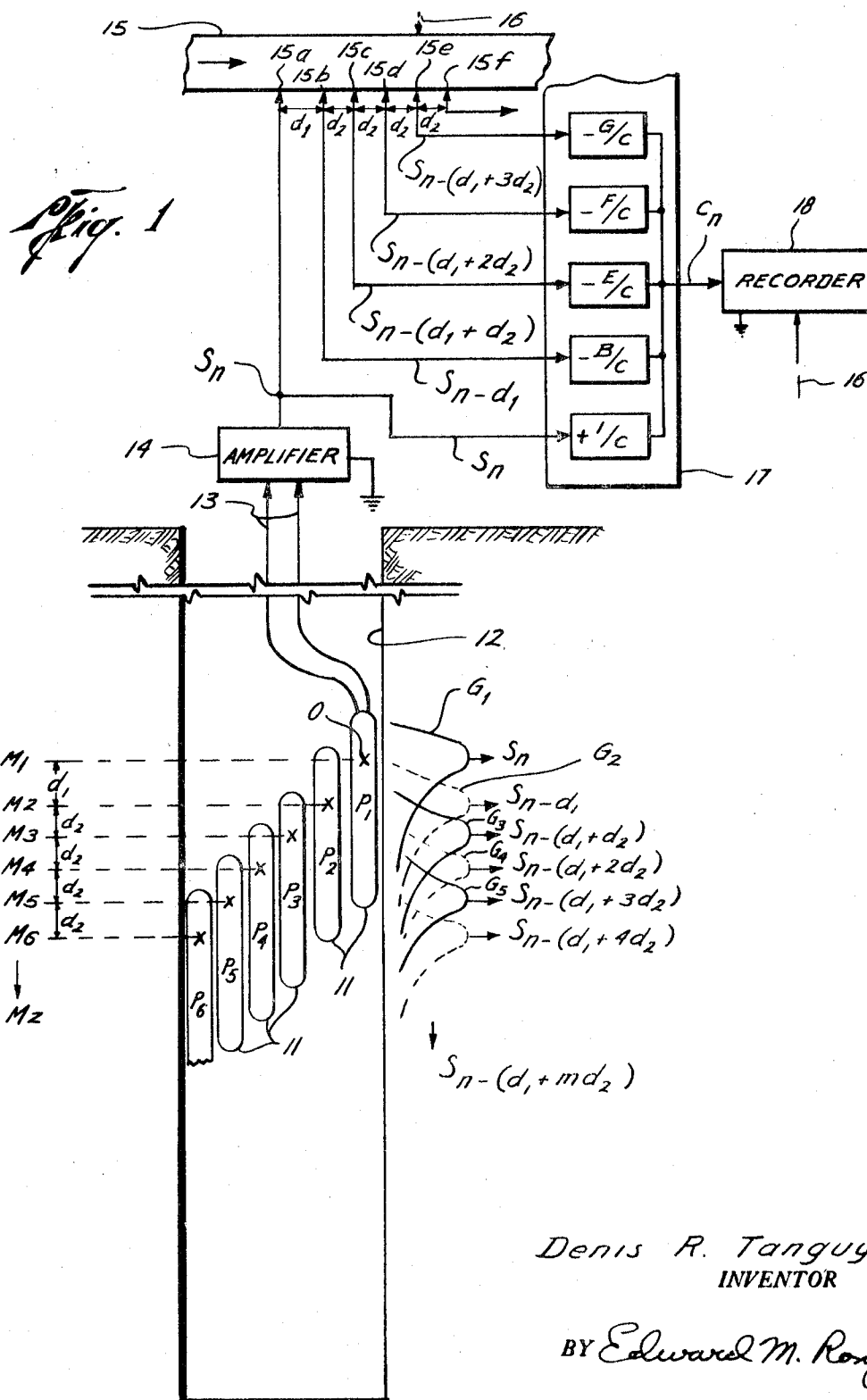
D. R. TANGUY

3,457,499

METHODS AND APPARATUS FOR IMPROVING THE RESOLUTION OF WELL LOGGING MEASUREMENTS

Filed June 5, 1967

8 Sheets-Sheet 1



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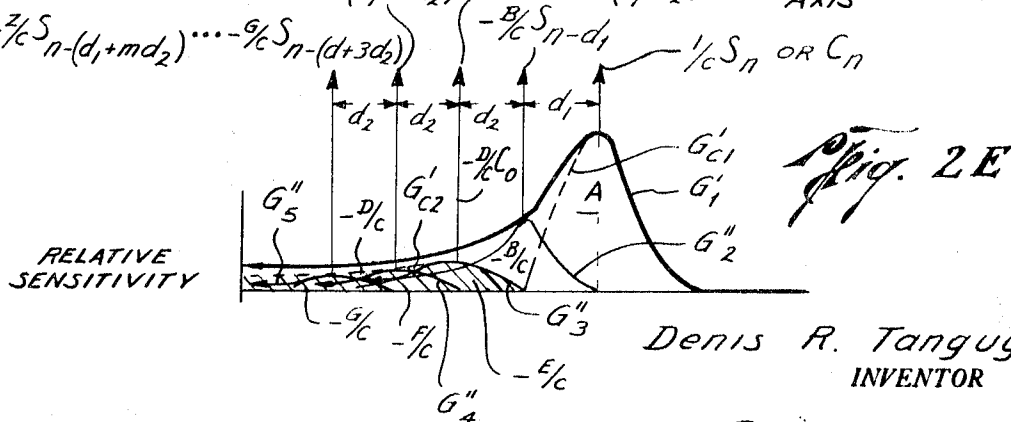
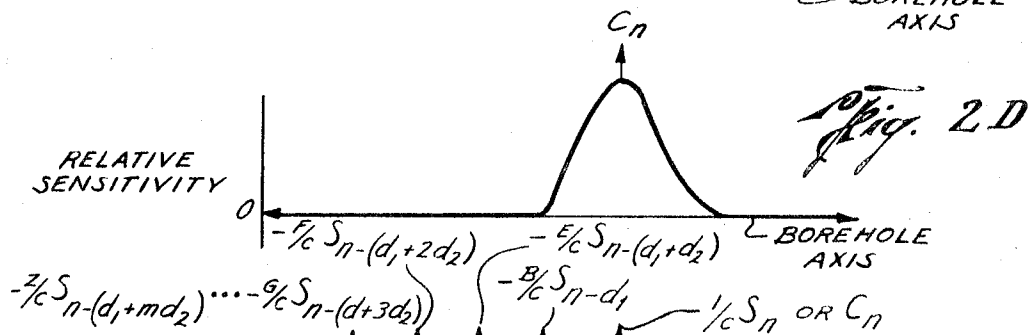
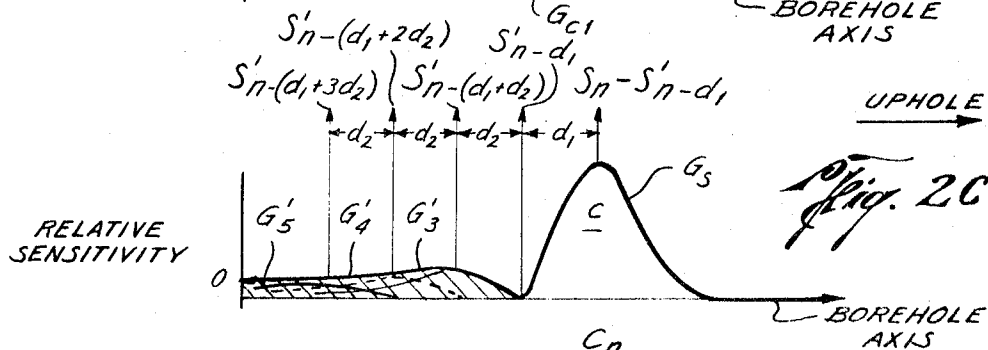
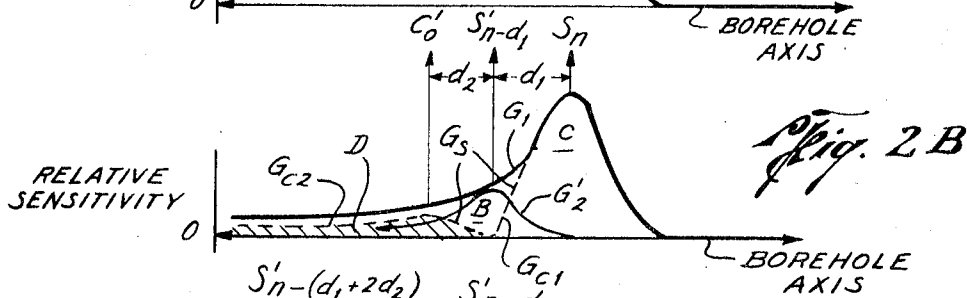
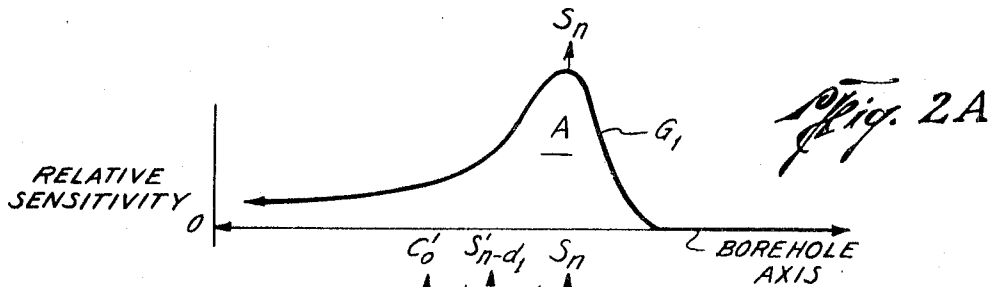
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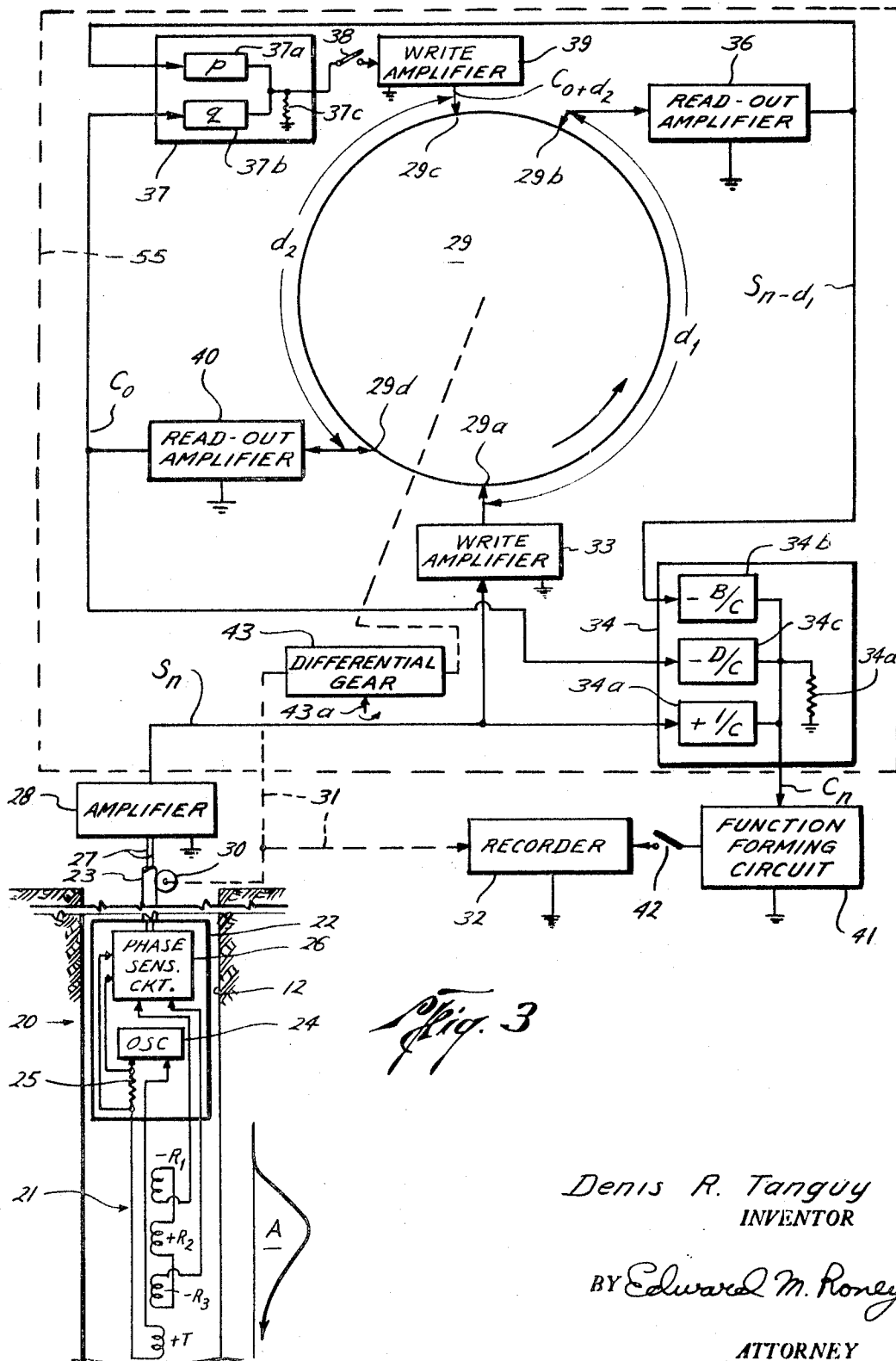
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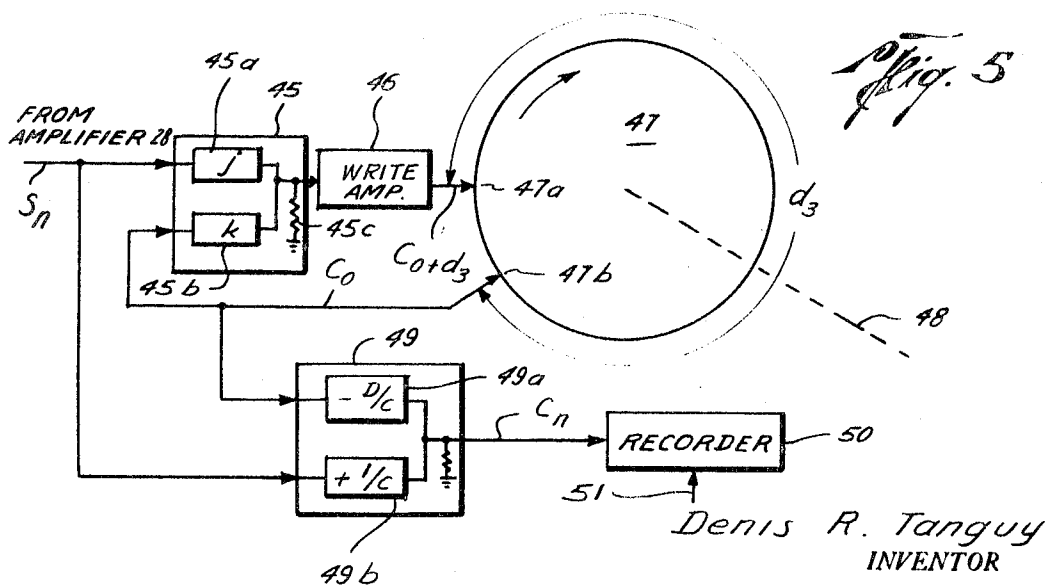
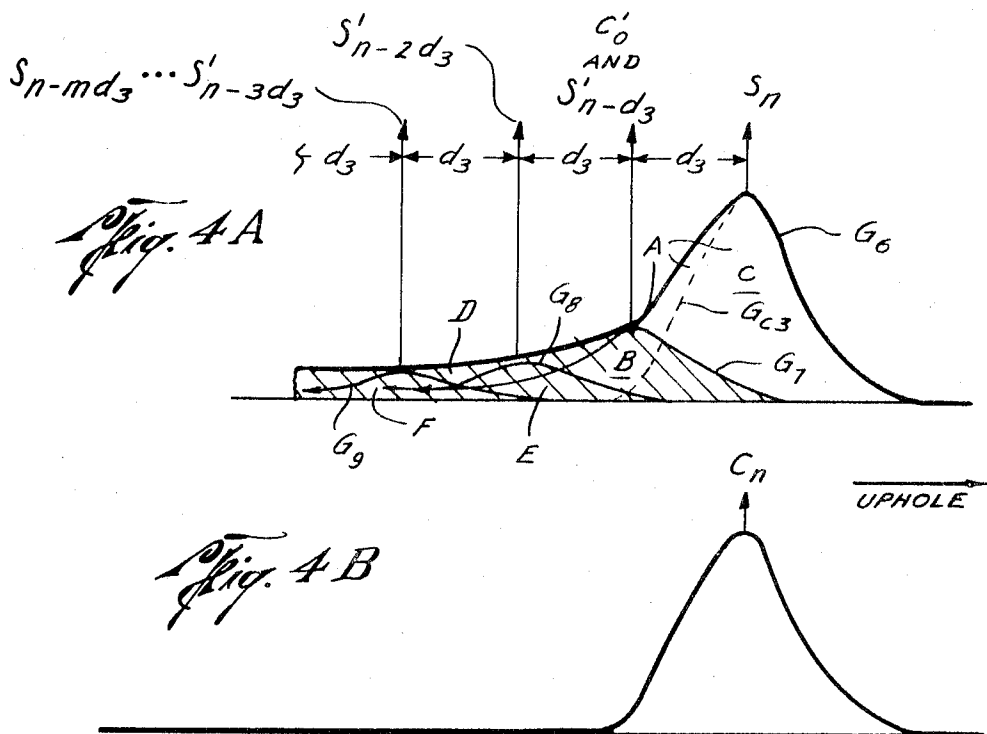
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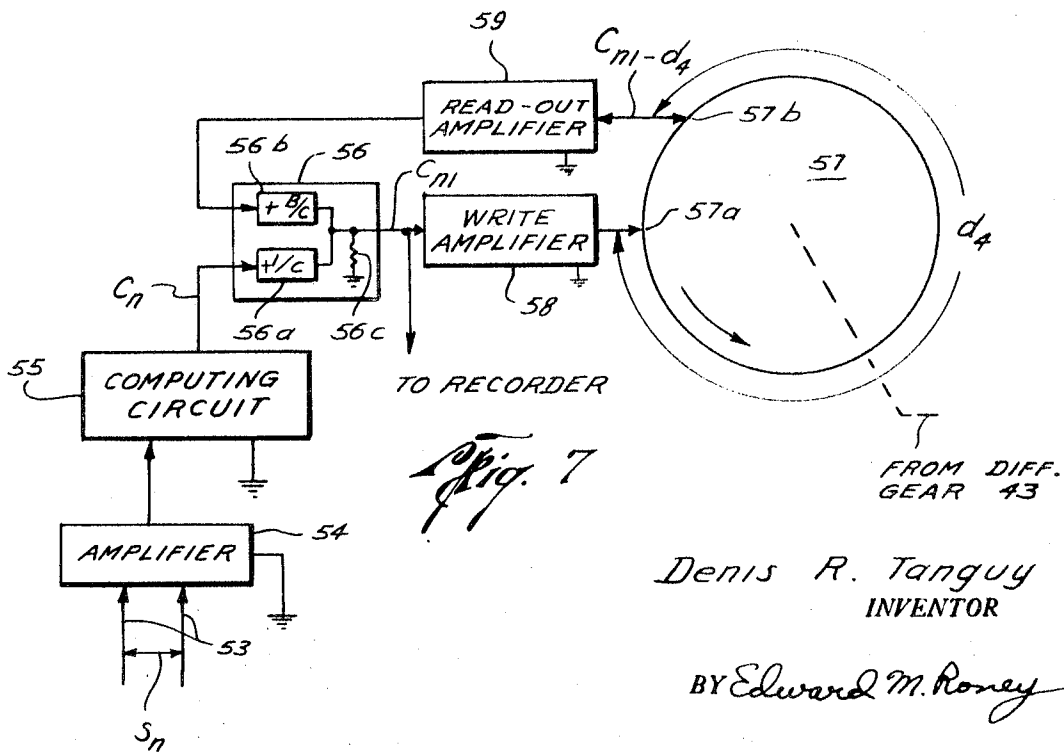
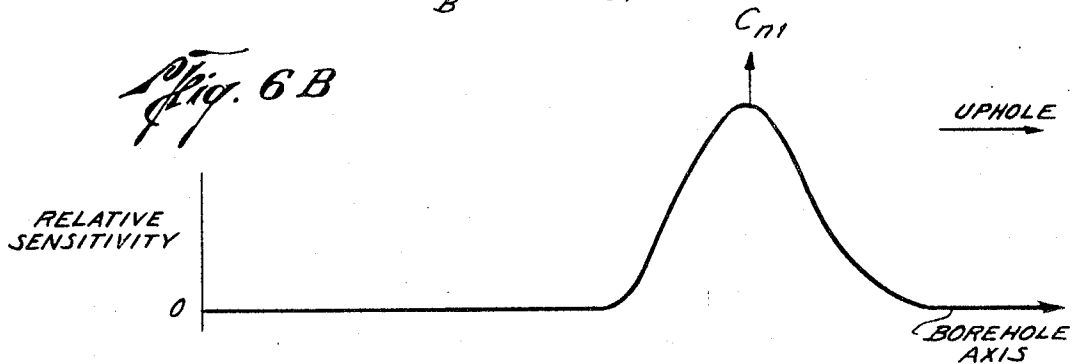
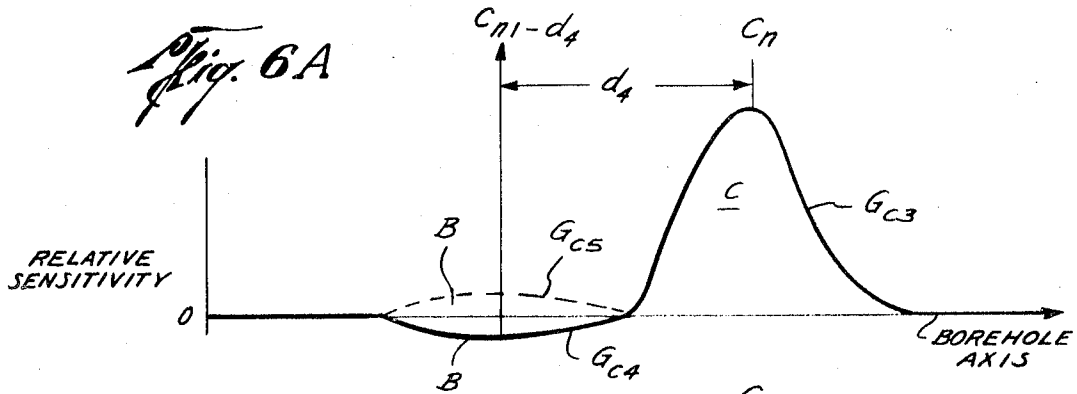
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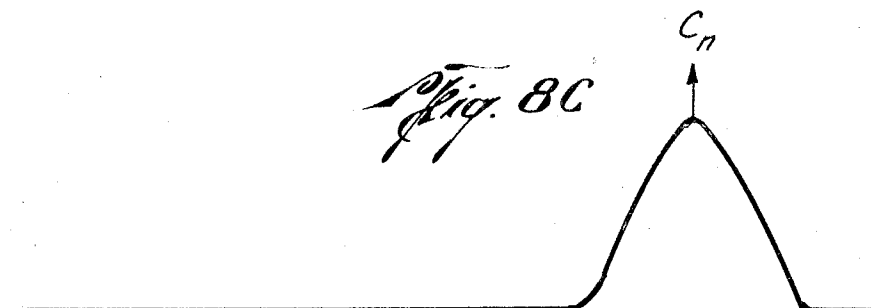
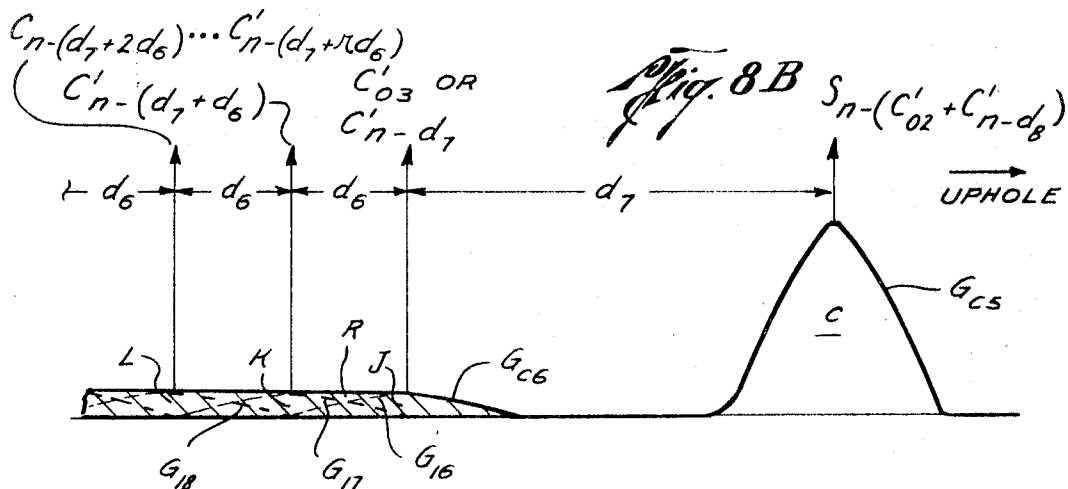
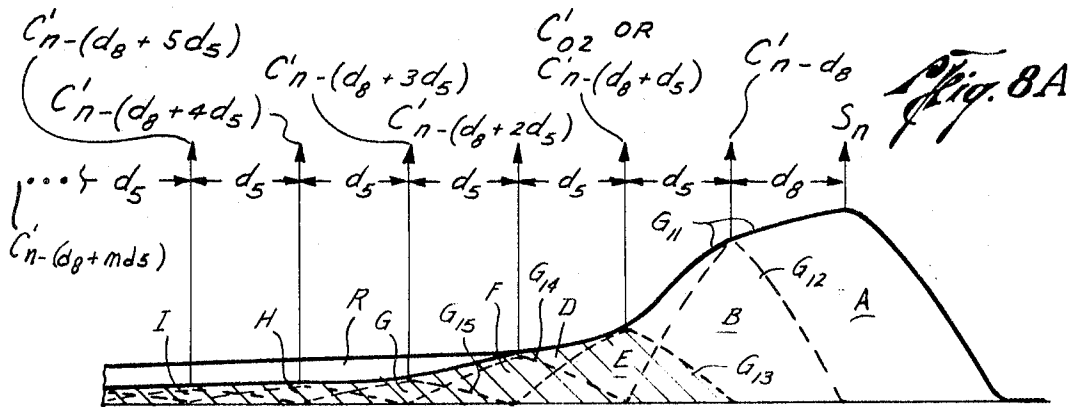
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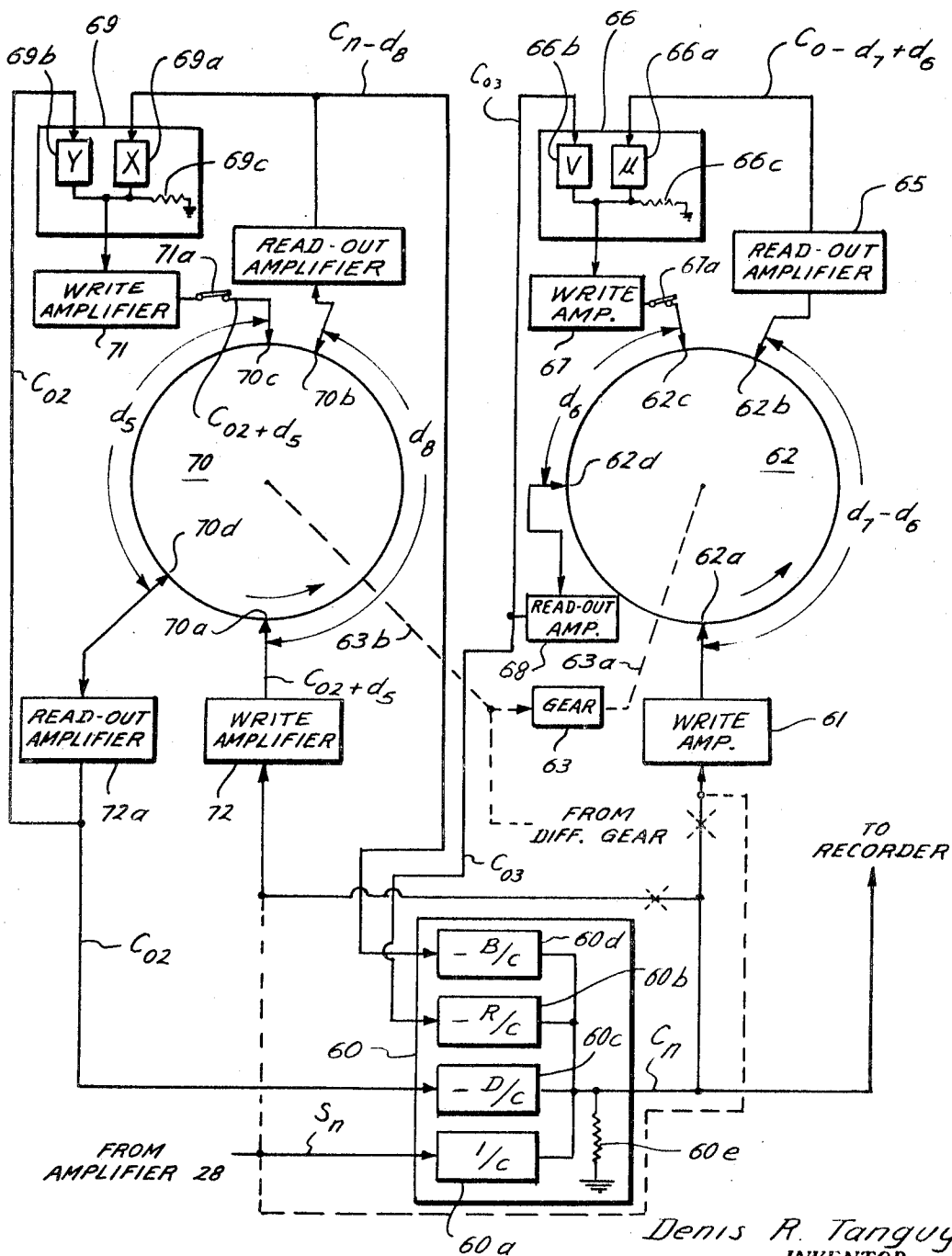


Fig. 9

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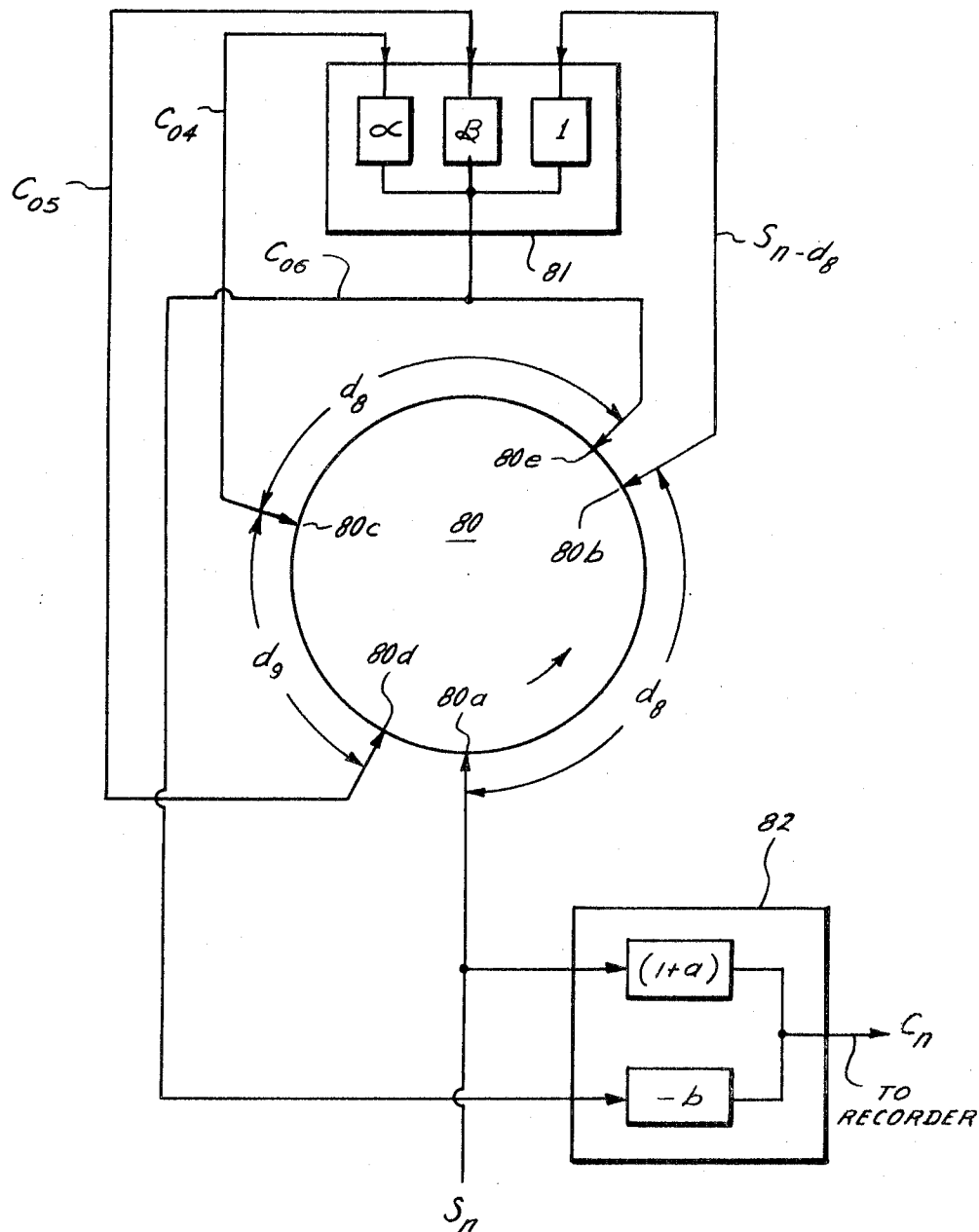


Fig. 10

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METHODS AND APPARATUS FOR IMPROVING THE RESOLUTION OF WELL LOGGING MEASUREMENTS

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Int. Cl. G01v 3/00, 3/18

U.S. Cl. 324—1

30 Claims 10

ABSTRACT OF THE DISCLOSURE

This disclosure describes techniques for processing derived well logging signals indicative of an investigated subsurface characteristic to arrive at improved indications of the subsurface characteristic. This is accomplished, in one manner, by a computational process wherein each derived signal is combined with a computed signal representing an infinite series of weighted derived signals to produce a new infinite series computed signal for subsequent combination with a later derived signal. Each infinite series computed signal is also combined with at least one derived signal to produce computed output signals for recording the computed value of the subsurface characteristic. Other disclosed manners of obtaining improved indications are to utilize the computed output signals to produce the infinite series computed signals, or to produce a plurality of infinite series computed signals having different parameters, or some combination of the above.

This invention relates to signal processing methods and apparatus for processing well logging measurement signals for providing improved indications of subsurface conditions or characteristics.

In the logging of subsurface earth formations surrounding a borehole drilled into the earth, investigating apparatus is moved through the borehole and investigates the surrounding earth formations to provide an output signal which varies in accordance with variations of the investigated characteristics of the adjoining earth formations. In electrical logging for example, the output signal varies in accordance with the electrical resistivity or conductivity of the subsurface earth formations. In any case, it is often desirable that the investigating apparatus respond to only a relatively limited portion of the formation material which is adjacent the apparatus at any given instant. For example, it is frequently desired that the vertical resolution of the investigating apparatus be sensitive to only a limited vertical interval of the adjoining earth formations. In this manner, earth formation beds can be more accurately investigated even under severe borehole conditions.

When speaking of vertical resolution of an investigating apparatus, the vertical geometrical factor (hereinafter called V.G.F.) is frequently utilized to more accurately describe this vertical resolution. The V.G.F. of an electrical logging type investigating apparatus for example, can be defined as the relative response of the investigating apparatus as a function of relative borehole depth as the investigating apparatus passes from $-\infty$ to $+\infty$ through a thin conductive bed extending radially outward from the borehole to infinity and surrounded by beds of zero conductivity. To make it easier to use, the V.G.F. is usually normalized to "1." Thus

$$\int_{-\infty}^{+\infty} X dz$$

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is made equal to "1" where X is the relative response and dz is a depth increment along the borehole axis. This same procedure can be used to find the V.G.F. if other formation characteristics than conductivity or its reciprocal resistivity are being investigated, i.e., if other than electrical logging type investigating apparatus is being utilized.

However, many investigating apparatus respond to a greater vertical region than desired (i.e., they do not have the most desirable V.G.F.). One technique for correcting this is to provide additional transducer elements in the downhole investigating apparatus to compensate for or to cancel the undesired portion of the response so that the effective vertical resolution of the apparatus is substantially improved. For example, in logging by electromagnetic principles, which is referred to as "induction logging," so-called "focusing" coils are added to the downhole investigating apparatus to cancel to a large extent the response of the apparatus to the so-called "shoulder" regions lying immediately above and below the desired vertical region of response of the investigating apparatus. However, other problems arise whenever additional transducer elements are added. One such problem is that more apparatus must be placed in the downhole investigating apparatus thus making the downhole investigating apparatus more complex and usually more expensive. Other problems concerning the quality of the measurement may also occur. For example, in induction logging as more coils are added to improve the vertical focusing, the depth of investigation of the apparatus in a horizontal or radial direction tends to decrease. Additionally, as more coils are added, the efficiency of the downhole investigating apparatus decreases, i.e. there is less received signal strength, thus increasing the problems of noise.

Another way of improving the effective vertical resolution of the downhole investigating apparatus is by utilizing the signal processing or computing techniques set forth in U.S. Patent No. 3,166,709 granted to H. G. Doll on Jan. 19, 1965. This Doll patent teaches the principle of temporarily storing or memorizing well logging signals obtained at various vertically spaced depth levels in the borehole. These stored signals are then combined in an appropriate manner to produce a resultant signal corresponding to the signal that would have been obtained with an investigating apparatus having better vertical resolution. This process is sometimes referred to as "computed focusing." The resultant signal is a computed signal and the relative depth levels corresponding to the stored signals which are being combined at any given instant are called computing stations. These computing stations are defined relative to the investigating apparatus and therefore effectively move through the borehole as the investigating apparatus moves through the borehole. The relative depth level to which the resultant signal is referenced is called the center point or recording point of the investigating system.

In following the teachings of the above-named Doll patent, it would sometimes be desirable to provide computing stations at a large number of measurement levels in the borehole such as in those cases where the total signal received by the investigating apparatus is made up of contributions from a relative great distance from the center point or recording point of the downhole investigating apparatus. For example, in some types of "induction logging" investigating apparatus, the V.G.F. never quite reaches zero response for an appreciable distance in the downhole direction from the apparatus center or recording point. However to provide such computing stations, a relatively large capacity memory would be re-

quired to store the necessary number of well logging measurement signal samples.

Another way of improving the vertical resolution by signal processing techniques is the technique shown in copending application Ser. No. 605,424 by Nick A. Schuster, filed on Dec. 28, 1966. Among other things, this technique enables the use of a much larger number of computing stations for a memory system of given capacity. In a more general sense, it enables more sophisticated forms of signal processing to be performed with a relatively small memory system of convenient size for use at the well site. The present application describes another technique of a different nature to obtain further improvements in certain situations.

It is an object of the invention therefore, to provide new and improved methods and apparatus for processing well logging measurement signals wherein more accurate measurements can be obtained.

It is another object of the invention to provide new and improved well logging signal processing methods and apparatus for providing more accurate well logging measurements utilizing relatively simple and inexpensive signal processing apparatus.

It is still another object of the present invention to provide new and improved well logging signal processing methods and apparatus for providing improved measurements of downhole earth formation characteristics with relatively small and inexpensive memory and computing devices.

In accordance with one feature of the present invention apparatus for processing well logging signals comprises means for deriving signals representative of a characteristic of earth formations surrounding a borehole at different depth levels in the borehole and first combining means for combining each derived signal with at least one other signal to provide first computed signals correlated in depth with the derived signals. The apparatus further comprises memory means for storing the first computed signals and means for reading out the first computed signals from the memory means at later times and supplying selected first computed signals to the first combining means to provide said at least one other signal. The apparatus further comprises second combining means for combining each derived signal with at least a selected signal read out from the memory means to provide second computed signals representative of the characteristics of the surrounding earth formations. By so doing, a selected formation response portion (i.e., a selected portion of the V.G.F.) of each derived signal is cancelled out to leave sharp resolution second computed signals.

In accordance with another feature of the present invention, apparatus for processing well logging signals comprises means for deriving signals representative of a characteristic of earth formations surrounding a borehole at different depth levels in the borehole and first combining means for combining each derived signal with at least one other signal to provide first computed signals which are more accurate representations of the characteristic of the surrounding earth formations. The apparatus further comprises second combining means for combining each first computed signal with at least one other signal to provide second computed signals and memory means for storing the second computed signals. The apparatus further comprises means for reading out the second computed signals from the memory means and supplying selected ones of the second computed signals to the first and second combining means. Again, high resolution signals are produced by this apparatus.

In accordance with still another feature of the present invention, a method of processing well logging signals comprises deriving signals, having a given vertical geometrical factor, representative of a characteristic of earth formations surrounding a borehole at different depth levels in the borehole and combining each derived signal with at least one other signal to provide first computed sig-

nals correlated in depth with the derived signals. The method further comprises storing the first computed signals and reading out the first computed signals at later times to be combined with each derived signal as said at least one other signal. The method further comprises combining each derived signal with at least a selected read out first computed signal having a given vertical geometrical factor (i.e., the tool which produces the signal has a given geometrical factor which, in effect, imparts this geometrical factor to the signal) to provide second computed signals, the vertical geometrical factor of said selected read out first computed signal cancelling an undesired portion of the derived signal vertical geometrical factor to provide each second computed signal with an improved vertical geometrical factor of improved vertical resolution, and recording the second computed signals as a function of depth to provide a computed log of the formation characteristic.

In accordance with yet another feature of the present invention, a method of processing well logging signals comprises deriving signals, having a given vertical geometrical factor, representative of a characteristic of earth formations surrounding a bore hole at different depth levels in the borehole and combining each derived signal with at least one other signal to provide first computed signals. The method further comprises combining each first computed signal with at least one other signal to provide second computed signals having a given vertical geometrical factor, and storing the second computed signals. The method further comprises reading out the second computed signals at later times and using selected second computed signals for combination with each derived signal and each first computed signal to produce the first and second computed signals, the vertical geometrical factor of the second computed signals cancelling undesired portions of the vertical geometrical factor of the derived signals to provide the first computed signals with an improved vertical geometrical factor of improved vertical resolution, and recording the first computed signals as a function of depth to provide a computed log of the investigated characteristic.

For a better understanding of the present invention, together with other and further objects thereof, reference is had to the following description taken in connection with the accompanying drawings, the scope of the invention being pointed out in the appended claims.

Referring to the drawings:

FIGURE 1 shows a downhole investigating apparatus disposed at different depth levels in a borehole with a graphical representation of the investigating apparatus V.G.F. at a plurality of the depth levels along with a schematic representation of signal processing apparatus, for purposes of explaining the background of the present invention;

FIGURES 2a, 2b, 2c, 2d and 2e show graphical representations of V.G.F.'s useful in understanding the theory of the present invention;

FIGURE 3 shows an "induction logging" type investigating apparatus along with signal processing apparatus for performing the operation depicted in FIGURES 2a, 2b, 2c, 2d and 2e;

FIGURES 4a and 4b show other V.G.F.'s useful in explaining other features of the present invention;

FIGURE 5 shows apparatus for performing the operation depicted in FIGURES 4a and 4b;

FIGURES 6a and 6b show still other V.G.F.'s useful in understanding other features of the present invention;

FIGURE 7 shows apparatus for performing the operation depicted in FIGURES 6a and 6b;

FIGURES 8a, 8b and 8c show still other V.G.F.'s useful in understanding other features of the present invention;

FIGURE 9 shows apparatus for performing the operation depicted in FIGURES 8a, 8b and 8c along with other embodiments of the present invention; and

FIGURE 10 shows another embodiment of the present invention.

Referring to FIGURE 1, there is shown a downhole investigating apparatus 11 having a center or recording point O, located at a plurality of various depth positions in a borehole 12, designated $P_1, P_2, P_3, P_4 \dots P_z$ where the subscript z refers to the first apparatus position from which a signal was derived. The depth levels or measure points corresponding to the positions P_1, P_2 , etc. of the downhole investigating apparatus 11 are designated $M_1, M_2, M_3, M_4 \dots M_z$ respectively. The depth interval between measure points M_1 and M_2 is designated d_1 and the depth interval between each successive measure point thereafter is designated d_2 . To the right of the borehole 12 there are shown the V.G.F.'s $G_1, G_2, G_3, G_4 \dots G_z$ corresponding to each of the apparatus positions P_1, P_2, P_3 , etc. The derived signals corresponding to each of the V.G.F.'s G_1, G_2 , etc. are designated $S_n, S_{n-d_1}, S_{n-(d_1+d_2)}, S_{n-(d_1+2d_2)} \dots S_{n-(d_1+md_2)}$ respectively.

The derived signals from the downhole investigating apparatus 11 are supplied to the surface of the earth through a conductor pair 13 which passes through an armored multiconductor cable (not shown) which also supports the downhole investigating apparatus 11 in the borehole 12. These derived signals are supplied to an amplifier 14 which provides the ground reference potential for the uphole signal processing apparatus. Amplifier 14 could, for example, be a differential amplifier. The output signal S_n from amplifier 14 is written into a memory 15, which is considered to be an infinite memory moving to the right, at write-in contact 15a. The subscript n refers to the signal being presently derived, $n-d_1$ refers to the signal derived an interval d_1 downhole, etc. Memory 15 has a plurality of read-out contacts designated 15b, 15c, 15d, 15e, 15f $\dots$ 15z. The memory 15 is driven by a shaft 16 in accordance with the movement of the cable in and out of the borehole by suitable means (not shown).

Thus, the values written into the memory 15 at write-in contact 15a will move to the right in accordance with the movement of the downhole investigating apparatus 11 through the borehole. Located the interval d_1 (corresponding to depth interval d_1) from write-in contact 15a is a read-out contact 15b and located the interval d_2 from read-out contact 15b is a read-out contact 15c. The remainder of the read-out contacts 15c, 15d, 15e, 15f, etc. are located at intervals of d_2 from each other. All of these readout contacts are connected to a weighted adding network 17 to which also is supplied the derived signal S_n from the output of amplifier 14. The output signal from weighted adding network 17, designated C_n , is supplied to a recorder 18 which is driven by the shaft 16 so as to provide a log of C_n versus depth.

It can be seen that the signal derived when the investigating apparatus was in position P_3 at measure point M_3 has moved to read-out contact 15e when the investigating apparatus is in position P_1 , the signal derived at measure point M_4 has moved to read-out contact 15d, etc. Thus, the signals supplied to weighted adding network 17 from the read-out contacts 15a, 15b $\dots$ 15z are designated $S_n, S_{n-d_1}, S_{n-(d_1+d_2)}, S_{n-(d_1+2d_2)}, S_{n-(d_1+3d_2)} \dots S_{n-(d_1+(m+1)d_2)}$, respectively.

It is the purpose of the FIGURE 1 apparatus to substantially improve the V.G.F. of the downhole investigating apparatus 11, i.e., to substantially improve the vertical resolution of the system. Looking now at FIGURE 2a, there is shown the V.G.F. G_1 of the FIGURE 1 investigating apparatus 11. This V.G.F. G_1 has an area A. FIGURE 2d shows the V.G.F. resulting from the computing process of FIGURE 1 corresponding to the computed signal C_n . That is, the computing signal C_n is deemed to represent the well logging measurements as if the downhole investigating apparatus had the V.G.F. of FIGURE 2d.

Before proceeding with the discussion of how the FIGURE 1 apparatus converted the V.G.F. G_1 of FIGURE 2a to the V.G.F. of FIGURE 2d, it would be helpful to first

discuss what a V.G.F. is. As defined earlier, the V.G.F. is the relative response of the investigating apparatus to a thin conductive bed extending radially to infinity and surrounded by nonconductive shoulder beds, as the investigating apparatus passes from $-\infty$ to $+\infty$ across the conductive bed. The resulting curve obtained represents the signal that would be indicated on a meter as the investigating apparatus transgresses the conductive bed. This meter reading plotted versus depth gives the V.G.F. curve. This V.G.F. has a vertical point which is designated the center or recording point and in this application, the center or recording point for any given V.G.F. will be designated by an arrow with the corresponding signal which that particular V.G.F. produces (e.g. S_n, S_{n-d_1} , etc.) The V.G.F. is fixed with respect to the downhole investigating apparatus and thus, in effect, moves through the borehole with the investigating apparatus. For normalization, the area under the V.G.F. curve must be equal to 1. Another point to be noted about V.G.F.'s is that, if a signal is increased or reduced in magnitude, the V.G.F. which produced that signal can also be considered to be increased or reduced in magnitude by the same proportion.

Turning now to FIGURE 2b, assume that the V.G.F. G_2 corresponding to the signal S_{n-d_1} is reduced in magnitude from its original area A to an area B. (Note that the V.G.F.'s derived from the downhole investigating apparatus for each depth level will have the shape of G_1 and area A.) The new V.G.F. is represented in FIGURE 2b by the V.G.F. G'_2 , having an area B. This V.G.F. G'_2 is subtracted from V.G.F. G_1 but is shown in an upright position to more clearly show how it subtracts from V.G.F. G_1 . The weighted signal corresponding to V.G.F. G'_2 is designated S'_{n-d_1} . The V.G.F. resulting from the subtraction of the V.G.F. G'_2 from V.G.F. G_1 is shown in FIGURE 2b as the dotted line V.G.F. G_3 , having two component V.G.F.'s G_{c1} and G_{c2} of area C and D respectively (D is the hatched line area).

Looking at FIGURE 2c, this V.G.F. G_3 is represented by the solid line V.G.F. G_3 . The component V.G.F. G_{c2} , of area D, of V.G.F. G_3 is made-up of an infinite series of V.G.F.'s $G'_3, G'_4, G'_5 \dots G'_z$ having areas E, F, G $\dots$ Z respectively, only V.G.F.'s G'_3, G'_4 , and G'_5 being shown. The V.G.F. G'_4 is shown as a dotted line curve to help distinguish it from V.G.F.'s G'_3 and G'_5 . These V.G.F.'s G'_3, G'_4 , etc. represent weighted versions of the V.G.F.'s G_3, G_4 , etc. respectively, which V.G.F.'s G_3, G_4 , etc. correspond to the stored derived well logging signals $S_{n-(d_1+d_2)}, S_{n-(d_1+2d_2)}, S_{n-(d_1+3d_2)}$, etc. respectively. The signals corresponding to these weighted V.G.F.'s are designated $S'_{n-(d_1+d_2)}, S'_{n-(d_1+2d_2)}, S'_{n-(d_1+3d_2)}$, etc. These V.G.F.'s G'_3, G'_4, G'_5 , etc. which are subtracted from the portion of V.G.F. G_3 of area D, are also shown in an upright position to more clearly show how they combine to subtract the V.G.F. G_3 . The resulting V.G.F., after subtracting G'_2, G'_3 , etc. from the V.G.F. G_3 , is shown in FIGURE 2d.

The area A of V.G.F. G_1 is made-up of the component areas B, C, and D. Thus,

$$A=B+C+D \quad (1)$$

where,

$$D=E+F+G+\dots+Z \quad (2)$$

Thus, combining Equations 1 and 2, the area C is:

$$C=A-B-E-F-G-\dots-Z \quad (3)$$

Since each of the V.G.F.'s G_1, G_2 , etc., were of area A, when the respective signals S_n, S_{n-d_1} , etc. were derived, it is clear that the amount each V.G.F. is reduced, is the ratio of its new area to its original area A. Thus, as stated earlier, since a reduction of a signal produces an equal reduction in the V.G.F., each derived signal must be weighted by the ratio of its new area to its original

area A to provide the particular V.G.F.'s desired. Thus, writing Equation 3 for V.G.F. areas in terms of signals,

$$\frac{C}{A} C_n = \frac{A}{A} S_n - \frac{B}{A} S_{n-d_1} - \frac{E}{A} S_{n-(d_1+d_2)} - \frac{F}{A} S_{n-(d_1+2d_2)} - \dots - \frac{Z}{A} S_{n-(d_1+md_2)} \quad (4)$$

where

$$\frac{C}{A} C_n$$

is the signal that would result from the V.G.F. component, having an area C, of V.G.F. G_s . This V.G.F. of area C is the one that remains after subtracting the V.G.F.'s G'_2 , G'_3 , etc. from V.G.F. G_1 and is the shape of the computed V.G.F. of the computed signal C_n . Likewise, since the V.G.F. of area D is made-up of the areas $E+F+\dots+Z$, Equation 4 can be written as:

$$\frac{C}{A} C_n = \frac{A}{A} S_n - \frac{B}{A} S_{n-d_1} - \frac{D}{A} C_0 \quad (5)$$

where

$$\frac{D}{A} C_0 = \frac{E}{A} S_{n-(d_1+d_2)} + \frac{F}{A} S_{n-(d_1+2d_2)} + \dots + \frac{Z}{A} S_{n-(d_1+md_2)} \quad (6)$$

Equation 6 thus defines C_0 as the signal produced from a V.G.F. having the same shape as the V.G.F. component G_{c2} , but having an area of magnitude A. (This will be explained in more detail later.) Since the area D is made-up of an infinite series of V.G.F.'s, C_0 will hereinafter be called an "infinite series computed signal."

Now, for normalization, the area of this computed V.G.F. must be equal in magnitude to the area A of the original V.G.F. G_1 . As stated earlier, the area A is normalized to "one," i.e. $\int X dz = 1$. Thus, to normalize the computed signal to "one," both sides of Equation 4 are divided by C. The A in Equation 4 can be dropped since it is equal to "one." Thus, the normalized Equation 4 can be written as:

$$C_n = \frac{1}{C} S_n - \frac{B}{C} S_{n-d_1} - \frac{E}{C} S_{n-(d_1+d_2)} - \frac{F}{C} S_{n-(d_1+2d_2)} - \dots - \frac{Z}{C} S_{n-(d_1+md_2)} \quad (7)$$

and the normalized Equation 5 can be written as:

$$C_n = \frac{1}{C} S_n - \frac{B}{C} S_{n-d_1} - \frac{D}{C} C_0 \quad (8)$$

Looking now at FIGURE 2e, there is shown the V.G.F.'s of FIGURES 2a-2d after normalization. The original apparatus V.G.F. G_1 , now designated G'_1 , has an area of $1/C$. The original V.G.F.'s G_2 , G_3 , $G_4 \dots G_s$, now designated G''_2 , G''_3 , $G''_4 \dots G''_s$, have areas of $-B/C$, $-E/C$, $-F/C$, $-G/C \dots Z/C$ respectively. The area of V.G.F. G'_{c2} (the hatched line portion) is $-D/C$. The final computed V.G.F., designated G'_{c1} , now has an area of A, A being equal to 1. (I.e. it is now normalized to 1.) The signals corresponding to these V.G.F.'s G'_1 , G'_2 , G'_3 , etc. of FIGURE 2e are

$$\frac{1}{C} S_n, -\frac{B}{C} S_{n-d_1}, -\frac{E}{C} S_{n-(d_1+d_2)}, -\frac{F}{C} S_{n-(d_1+2d_2)} \dots -\frac{Z}{C} S_{n-(d_1+md_2)}$$

The signals corresponding to the V.G.F.'s G'_{c1} and G'_{c2} are C_n and

$$-\frac{D}{C} C_0$$

respectively.

Now, concerning how Equation 7 is solved by the FIGURE 1 apparatus, the signals S_n , S_{n-d_1} , etc. of Equation 7 are supplied to weighted adding network 17 to be weighted by the respective weights $1/C$, $-B/C$, $-E/C$, etc. of Equation 7 to provide the new computed signal C_n for recordation by recorder 18. It can thus be seen that the computed V.G.F. of FIGURE 2d can be produced from the original apparatus V.G.F. of FIGURE 2a by providing an infinite memory having an infinite number of read-out contacts for supplying an infinite number of signals to an infinite number of weighting function circuits.

Now referring to FIGURE 3, there is shown apparatus for solving Equation 7 with much less apparatus. In FIGURE 3, there is shown a downhole investigating apparatus 20 which comprises a central support member (not shown) which supports a coil array 21 and a fluid-tight housing 22 which contains the downhole electrical circuitry. The central support member is supported by a cable 23 which passes to the surface of the earth through the borehole 12. The coil array 21 comprises, in order from top to bottom, a first receiver coil $-R_1$, a second receiver coil $+R_2$, a third receiver coil $-R_3$, and a transmitter coil $+T$. Suitably, the distance between $-R_1$ and $+R_2$ could be approximately 30 inches, $+R_2$ to $-R_3$ could be approximately 18 inches and $-R_3$ to T could be approximately 22 inches, for example. The number of turns for the coils $-R_1$, $+R_2$, $-R_3$ and T could be 125 turns, 100 turns, 16 turns, and 100 turns respectively, for example. This coil array would approximately give the V.G.F. shown to the right of the coil array in FIGURE 3. It is to be understood that this coil array is only exemplary and any suitable coil array could be utilized. Furthermore, the present invention is not limited to "induction logging" tools and any well logging tool could be utilized, as for example, sonic or nuclear logging tools.

Within the fluid-tight housing 22, an oscillator 24 supplies a constant current to a transmitter coil T through a low value resistor 25. The signal received by the receiver coils are supplied to a phase-sensitive circuit 26 to which also is supplied the phase-reference signal taken across resistor 25. The phase-sensitive circuit 26 supplies the derived well logging signals to the surface of the earth via a conductor pair 27 which passes through the armored cable 23. These derived signals comprise a varying DC signal proportional to that portion of the signal induced into the receiver coils which is in-phase with the phase-reference signal taken across resistor 25. These signals are therefore proportional to the conductivity of the adjoining earth formations.

At the surface of the earth, the conductor pair 27 is supplied to the input of an amplifier 28 which supplies the ground reference potential for the uphole circuitry. Amplifier 28 could thus be a differential amplifier, for example. The output signal S_n of amplifier 28 is supplied to the input of a weighting function circuit 34a having a weight of $1/C$, of a weighted adding network 34, and to the input of a write amplifier 33, the output of which is supplied to a write-in contact 29a of a rotating memory 29. Rotating memory 29 could comprise any suitable memory device, such as for example, the magnetic memory shown in the above-mentioned Doll patent, or the capacitor memory shown in U.S. Patent No. 3,230,445 granted to W. J. Slougher et al. on Jan. 18, 1966. In this application, the memories can be considered to be capacitor memories, and thus the write amplifiers will have low output impedances to quickly charge or discharge the capacitors to the proper voltage. The read-out amplifiers will have high input impedances so as to not disturb the charge on the capacitors for later read out, if desired. Rotating memory 29 is considered to rotate in a counterclockwise direction in accordance with the depth of the downhole investigating apparatus 20. This movement of the downhole investigating apparatus 20 is sup-

plied to the rotating memory 29 by means of a rotating wheel 30 which rotates in accordance with the movement of cable 23. The wheel 30 transfers motion to a shaft 31 which causes the rotation of rotating memory 29, through a differential gear 43, and the recording medium of the recorder 32 in accordance with the movement of the downhole investigating apparatus 20. A shaft 43a is supplied to the other input of the differential gear 43.

Located a counterclockwise interval d_1 from write-in contact 29a is a read-out contact 29b which is connected to the input of a read-out amplifier 36, whose output signal S_{n-d_1} is supplied to a weighted function circuit 34b having a weight of $-B/C$ of weighted adding network 34. The output signal S_{n-d_1} from read-out amplifier 36 is also supplied to the input of a weighting function circuit 37 having a weight of p , of weighted adding network 37. The output of weighted adding network 37, taken across a low value resistor 37c, is supplied through a single-throw switch 38 to a write amplifier 39, whose output designated C_{o+d_2} is supplied to a write-in contact 29c of memory 29. Write-in contact 29c is located a short counterclockwise interval from read-out contact 29c.

Located a counterclockwise interval d_2 from write-in contact 29c is another read-out contact 29d which is supplied to the input of a read-out amplifier 40, which supplies the output signal C_o to a weighting function circuit 34c having a weight $-D/C$, of weighted adding network 34, and to a weighting function circuit 37b having a weight q , of weighted adding network 37. The output from weighted adding network 34, which comprises the outputs of weighting function circuits 34a, 34b, and 34c, taken across a low value resistor 34d, is supplied to the input of a function forming circuit 41 which corrects for electrical skin effect phenomena. This function forming circuit 41 could be constructed in accordance with the teachings of U.S. Patent No. 3,226,633 granted to W. P. Schneider on Dec. 28, 1965. Alternatively, the skin effect correction scheme shown in U.S. Patent No. 3,230,445 granted to W. J. Slougher on Jan. 18, 1966, could be utilized. The output signal from function forming circuit 41 is supplied through a single-throw switch 42 to the recorder 32 for recordation as a function of depth.

While the details of weighted adding networks 34 and 37 have not been shown, it will be appreciated that they could comprise standard weighting circuits. For example, each weighting function circuit could comprise a suitable amplifier of the proper polarity and a high value resistor of desired weight. By this means, the currents produced by each of the weighting function circuits will be proportional to the respective weighted input signal, and these currents will sum across the low resistance resistor to produce a voltage proportional to the weighted input signals.

Concerning how the FIGURE 3 apparatus performs the operation depicted in FIGURES 2a-2e, the derived signal S_n applied to weighting function circuit 34a and written into memory 29 corresponds to the V.G.F. G_1 in FIGURES 2a and 2b. After weighting, this derived signal S_n corresponds to the V.G.F. G'_1 of FIGURE 2e, i.e.

$$\frac{1}{C} S_n$$

The signal S_{n-d_1} read out from the read-out contact 29c a depth interval d_1 from write-in contact 29a corresponds to the signal derived an interval d_1 in the downhole direction from the depth level at which S_n is derived and, after weighting, corresponds to the V.G.F. G'_2 of FIGURE 2e, i.e.

$$\frac{B}{C} S_{n-d_1}$$

The infinite series computed signal C_o read out at read-out

contact 29d, after weighting, corresponds to the V.G.F. G'_{e2} of FIGURE 2e, i.e.

$$\frac{D}{C} C_o$$

The recording point for this V.G.F. G'_{e2} , shown as the arrow C'_o in FIGURE 2b or the arrow

$$-\frac{D}{C} C_o$$

in FIGURE 2e, is located a depth interval d_2 in the downhole direction from the recording point S'_{n-d_1} of V.G.F. G'_2 .

In FIGURE 3, it can be seen that the read-out derived signal S_{n-d_1} is combined with the read-out computed signal C_o to produce the new computed signal C_{o+d_2} , which is immediately written into memory 29. (Note.—After the write-in signal C_{o+d_2} is stored for a depth interval d_2 , this same signal is designated $C_{o+d_2-d_2}$ or C_o the depth interval d_2 later.) Thus, since the signal C_{o+d_2} is written into memory 29 at, for all practical purposes, the same instant of time as the read-out derived signal S_{n-d_1} which, in part, produced it, the read-out computed signal C_o is located the depth interval d_2 from the read-out derived signal S_{n-d_1} . Thus, it can be seen that the computed signal C_n recorded in recorder 32 is equal to the sum of the signals S_n , S_{n-d_1} , and C_o weighted by the weighting factors $1/C$, $-B/C$, and $-D/C$ respectively of weighted adding network 34 in accordance with Equation 8. In V.G.F. terms, this is saying that V.G.F. G'_{e2} and G'_2 are subtracted from V.G.F. G'_1 to leave V.G.F. G'_{e1} in FIGURE 2e.

To start the FIGURE 3 apparatus at the bottom of the borehole, it is necessary to first store signals in all positions of the memory. To accomplish this, the investigating apparatus 20 is held stationary at the bottom of the borehole while supplying signals to the surface of the earth. Memory 29 is rotated by turning shaft 43a by hand while reading the derived signal S_n into all positions of the memory 29. Switches 38 and 42 are opened during this initial operation. In this manner, the derived signal S_n is stored in all positions of the memory. (Remember that the signals stored in all positions of memory during the operation discussed above are normalized to 1.) For memory 29 to have accurate readings in all positions of memory at this time, the assumption must be made that the earth formations at the bottom of the borehole are substantially homogeneous for the depth interval that the original apparatus V.G.F. response is somewhat greater than zero. However, in the event that this condition does not exist, any error will be quickly eliminated. This can be seen by noting in FIGURE 2b or 2e that the V.G.F.'s diminish in area as the downhole investigating apparatus moves uphole, thus diminishing any error also. Now, to initiate the logging operation, the switches 38 and 42 are closed, shaft 43a is no longer rotated, and the downhole investigating apparatus 20 begins moving uphole.

To better understand how the infinite series computed signal C_o is obtained, it would be desirable to follow several signals derived at several depth levels in the borehole through the apparatus of FIGURE 3 to see how the operation depicted in FIGURES 2b and 2c is performed. Thus, looking at FIGURES 1, 2e and 3, in conjunction, the signal derived at measure point M_3 in FIGURE 1 was written into memory 29 at write-in contact 29a of memory 29 and read out at read-out contact 29b for computation. (Each derived signal is, of course, combined with the read-out signals S_{n-d_1} and C_o as stated earlier, and this facet of the operation need not be repeated over again below.) When the investigating apparatus is at depth level or measure point M_2 , the signal derived at measure point M_3 is read out at read-out contact 29b, multiplied by the weighting factor p , and written back

into memory 29. (Each signal read out at read-out point 29b is combined with the signals S_n and C_0 and this facet of the operation need not be repeated again below.) After the downhole investigating apparatus has moved an interval d_2 to the measure point M_1 , the signal which was derived at measure point M_3 (now called $S_{n-(d_1+d_2)}$) since the investigating apparatus has moved a depth interval d_1+d_2 uphole from the measure point M_3 is read out at read-out contact 29d as a component $pS_{n-(d_1+d_2)}$ of the infinite series computed signal C_0 . The V.G.F. G_3'' of FIGURE 2e corresponds to this component signal $pS_{n-(d_1+d_2)}$, after weighting in weighted adding network 34 of FIGURE 3.

When the investigating apparatus is at measure point M_1 , the signal derived at measure point M_4 , now designated $S_{n-(d_1+2d_2)}$ has been read out of memory 29 at read-out contact 29b, multiplied times p , written back into memory 29 at write-in contact 29c, read out at read-out contact 29d as a portion of the infinite series computed signal C_0 , multiplied times q in weighted adding network 37 and written back into memory 29 at write-in contact point 29c. This signal is then, after a depth interval d_2 , read out again at read-out point 29e as the component $pqS_{n-(d_1+2d_2)}$ of the infinite series computed signal C_0 . This signal $pqS_{n-(d_1+2d_2)}$, after weighting in weighted adding network 34, corresponds to the V.G.F. G_4'' of FIGURE 2e.

The signal derived at measure point M_5 , tracing its past history, has been read out of memory 29 at read-out contact 29b, multiplied times p , written back into memory 29, read out and multiplied times q , written back into memory 29, read out again and multiplied times q again and written back into memory 29. This signal is then read out of memory 29 at read-out contact 29d as the weighted signal $pq^2S_{n-(d_1+3d_2)}$, corresponding to the V.G.F. G_5'' of FIGURE 2e. Thus, it can be seen that weighting function circuit 37 is, in effect, an infinite series weighting function circuit.

Now, putting the above into a mathematical expression for C_0 ,

$$C_0 = pS_{n-(d_1+d_2)} + pqS_{n-(d_1+2d_2)} + pq^2S_{n-(d_1+3d_2)} + \dots + pq^{m-1}S_{n-(d_1+md_2)} \quad (9)$$

Now, setting $p=E/D$ and $q=F/E$ and substituting these values for p and q into Equation 9, the new relationship for C_0 is:

$$C_0 = \frac{E}{D} S_{n-(d_1+d_2)} + \frac{F}{D} S_{n-(d_1+2d_2)} + \frac{F}{D} \cdot \frac{F}{E} S_{n-(d_1+3d_2)} + \frac{F}{D} \cdot \left(\frac{F}{E}\right)^2 S_{n-(d_1+4d_2)} + \dots + \frac{F}{D} \cdot \left(\frac{F}{E}\right)^{m-2} S_{n-(d_1+md_2)} \quad (10)$$

Now substituting Equation 10 into Equation 7, the expanded relationship for C_n in terms of V.G.F. areas can be written as:

$$C_n = \frac{1}{C} S_n - \frac{B}{C} S_{n-d_1} - \frac{E}{C} S_{n-(d_1+d_2)} - \frac{F}{C} S_{n-(d_1+2d_2)} - \frac{F}{C} \cdot \frac{F}{E} S_{n-(d_1+3d_2)} - \frac{F}{C} \cdot \left(\frac{F}{E}\right)^2 S_{n-(d_1+4d_2)} - \dots - \frac{F}{C} \cdot \left(\frac{F}{E}\right)^{m-2} S_{n-(d_1+md_2)} \quad (11)$$

Now comparing the weighting factors in Equation 11 with the weighting factors in Equation 7, it can be seen that the weighting factors applied to the signals S_n , S_{n-d_1} , $S_{n-(d_1+d_2)}$, and $S_{n-(d_1+2d_2)}$ are exactly equal to the desired weighting factors in Equation 7. For the weights to be applied to the remainder of the signals $S_{n-(d_1+3d_2)}$, $S_{n-(d_1+4d_2)}$, $\dots$, $S_{n-(d_1+md_2)}$, comparing the

weighting factors for these signals in Equation 11 with the weighting factors in Equation 7, it is clear that:

$$G = F \cdot \frac{F}{E}, H = F \cdot \left(\frac{F}{E}\right)^2 \dots Z = F \cdot \left(\frac{F}{E}\right)^{m-2} \quad (12)$$

In terms of q , Equation 12 can be written as:

$$G = Fq, H = Fq^2 \dots Z = Fq^{m-2} \quad (13)$$

Now it can be seen that the areas G , H , etc. can be determined from Equation 12. In practice, in selecting the circuit parameters, a trial and error method may be utilized to substantially cancel undesired portions of the original apparatus V.G.F., including providing a plurality of noninfinite series computed signals.

It should be noted here that the weighting functions for C_0 must equal 1 for normalization. That is, in terms of p and q from Equation 9,

$$p \left(1 + \sum_{i=2}^{\infty} q^{i-1} \right)$$

must equal 1. By utilizing the V.G.F. areas given above for p and q , i.e. $p=E/D$ and $q=F/E$, p and q will automatically be normalized.

From the preceding, a general technique can be formulated for determining the values of the weighting functions and the location of read-out contacts from memory with respect to the write-in contacts. To determine these factors, in connection with the infinite series computed signal, it is necessary to first graphically locate the infinite series of V.G.F.'s at the desired positions, spaced an equal distance apart, and with desired weights to cancel out the undesired portion of the original apparatus V.G.F. The interval between the recording points of these infinite series V.G.F.'s determines the depth interval between the write-in and read-out contacts for the infinite series computed signals. (I.e. 29c and 29d in FIGURE 3.) The depth interval between the recording point of the original apparatus V.G.F. and the recording point of the first V.G.F. (closest to the original apparatus V.G.F.) determines the depth interval between the write-in contact where the derived signal S_n is read into memory (if S_n is supplied directly to weighted adding network 34), and the read-out contact where the infinite series computed signal is read out of memory.

The weight of the infinite series weighting function circuit to which the read-out derived signal (S_{n-d_1} in FIGURE 3) is applied, is equal to the ratio of the area of the first V.G.F. of the infinite series to the total area of all of the V.G.F.'s of the infinite series. The weight of the weighting function circuit to which the read-out infinite series computed signal C_0 is applied, is equal to the ratio of the area of the second V.G.F. of the infinite series of V.G.F.'s to the area of the first V.G.F. thereof. Thus, it can be seen that these first two V.G.F.'s of the infinite series determine the areas of the remainder of the infinite series V.G.F.'s, as well as the depth interval between each successive V.G.F. For this reason, it may at times be desirable to utilize read-out derived signals not connected with the infinite series computed signal, like S_{n-d_1} in FIGURE 3, for combination with the derived signal S_n and the read-out infinite series computed signal C_0 to produce the final computed signal C_n . As many of these noninfinite series signals as desired may be used. Finally, the weighting functions of the weighted adding network 34 which produces the final computed signal C_n are determined in the same manner discussed in connection with Equation 8.

Looking now at FIGURE 4a, there is shown a graphical example of cancelling out undesired portions of an original apparatus V.G.F. G_6 by utilizing an infinite series cancellation without a "noninfinite series" computing station. The arrow designated S_n represents the recording

point of the original apparatus V.G.F. G_6 which produces the derived signal S_n . Located at intervals d_3 apart in the downhole direction from the arrow designated S_n , are a plurality of V.G.F.'s designated G_7, G_8, G_9 , etc. extending to infinity. The signal designation $S'_{n-d_3}, S'_{n-2d_3}, S'_{n-3d_3} \dots S'_{n-md_3}$ correspond to the weighted signals corresponding to these V.G.F.'s G_7, G_8 , etc. The areas D, E, F $\dots$ Z are represented by the hatched line portion of the V.G.F. G_6 and have an area D. (The designations A, B, C, etc. are the same in FIGURE 4a as in some of the other figures, but this does not necessarily mean that they are equal thereto.) After subtracting the hatched line portion of area D from the original V.G.F. G_6 , the V.G.F. G_{c6} having an area C remains. This final computed V.G.F. is shown in FIGURE 4b. (Note.—The subtracted V.G.F.'s G_7, G_8 , etc. are reversed in polarity in FIGURE 4a to more clearly show how they subtract from the V.G.F. G_6 .)

Now looking at FIGURE 5, there is shown apparatus for performing the operation depicted in FIGURES 4a and 4b. The derived signal from the surface amplifier (like amplifier 28 of FIGURE 3) is supplied to a weighting function circuit 49b having a weight of $1/C$, of a weighted adding network 49 and to a weighting function circuit 45a having a weight j , of a weighted adding network 45. The output from weighted adding network 45, taken across a low value resistor 45c, is supplied through a suitable write amplifier 46 to a write in contact 47a of a suitable memory 47, similar to the memory 29 of FIGURE 3. Memory 47 is considered to rotate in a clockwise direction in accordance with depth, which depth function is supplied from a shaft 48 as in the FIGURE 3 apparatus. Located a clockwise interval d_3 from write in contact 47a is a read-out contact 47b which supplies the read-out computed signal C_0 to a weighting function circuit 45b having a weight k , of weighted adding network 45, and to a weighting function circuit 49a having a weight $-B/C$, of weighted adding network 49. The output signal C_n from weighted adding network 49 is supplied to a suitable recorder 50 driven by shaft 51 in accordance with depth as in the FIGURE 3 apparatus.

To determine the parameters of the FIGURE 5 apparatus, the procedure set forth above can be utilized. Since there is no readout derived signals to be combined with S_n and C_0 to produce C_n in FIGURES 4a and 4b, the derived signal S_n need not be stored in memory. Thus, the derived signal S_n is combined with the read-out infinite series computed signal C_0 to produce the new infinite series computed signal C_{0+d_3} . Thus, in accordance with the technique discussed earlier, the interval d_3 between the recording points of the original apparatus V.G.F. G_6 and the first infinite series V.G.F. determines the interval between the write in and read-out contacts 47a and 47b.

To obtain the values of the weighting functions for the infinite series computed signal of FIGURES 4a, 4b and 5, the same procedure discussed above is utilized. The weight to be applied to the derived signal S_n is determined by the ratio of the area of the first infinite series V.G.F. G_7 to the area of all of the infinite series of V.G.F.'s. Thus, $j=B/D$. The weight to be applied to the read-out computed signal C_0 is the ratio of the area E of the second successive infinite series V.G.F. G_8 to the area B of the first infinite series V.G.F. G_7 . Thus, $k=E/B$. Likewise, the weights of weighted adding network 49 are determined in the same manner as in FIGURES 2a-2e and 3. That is, the weight to be applied to the infinite series computed signal C_0 is determined by the area of the V.G.F.'s of the infinite series of V.G.F.'s which since this infinite series of V.G.F.'s is subtracted from the original apparatus V.G.F. G_6 , is negative, i.e. $-D$. The weight to be applied to the derived signal S_n is again A, which equals 1. For normalization, $-D$ and $+1$ are divided by the area C of final computed V.G.F.'s normalization. Thus, the weight applied to the signals S_n and C_0 are $+1/C$ and $-D/C$ respectively.

The equation for the computed signal C_n for the FIGURE 5 apparatus is:

$$C_n = \frac{1}{C} S_n - \frac{D}{C} C_0 \quad (14)$$

where

$$C_0 = jS_{n-d_3} + jkS_{n-2d_3} + jk^2S_{n-3d_3} + \dots + jk^{m-1}S_{n-md_3} \quad (15)$$

The values for j and k could be inserted in Equation 15 to write Equation 15 in terms of V.G.F. areas. Again

$$j \left(1 + \sum_{k=2}^{\infty} k^{m-1} \right)$$

must equal "one" for normalization which automatically occurs by using the above technique for selecting the infinite series weights j and k . The FIGURE 5 apparatus is started at the bottom of the borehole in the same manner as the FIGURE 3 apparatus.

Now referring to FIGURES 6a and 6b, there is shown another feature of the present invention. It may happen that the infinite series cancellation of undesired portions of the original apparatus V.G.F., or infinite series cancellation in combination with noninfinite series cancellation, as shown above, will not completely cancel all undesired portions of the original apparatus V.G.F. Such a case is shown in FIGURES 6a and 6b where the desired final computed V.G.F. is shown in FIGURE 6b. The solid line V.G.F. of FIGURE 6a represents the final computed V.G.F. produced from the infinite series cancellation or infinite series plus noninfinite series cancellation discussed above. The signal corresponding to this solid line V.G.F. of FIGURE 6a is the term designated C_n , its recording point being the location of the arrow designated C_n . This solid line V.G.F. of FIGURE 6a has two component V.G.F.'s G_{c3} and G_{c4} of areas C and B respectively. To provide the final computed V.G.F. of FIGURE 6b, it can be seen that the component V.G.F. G_{c4} must be cancelled out. In accordance with the teachings of the above mentioned copending Schuster application, the signal C_n corresponding to the final computed V.G.F. of FIGURE 6b is stored in memory and subsequently read out to cancel the undesired component V.G.F. G_{c4} . This read-out final computed V.G.F. is represented by the dotted line V.G.F. G_{c5} in FIGURE 6a, which is shown in its proper polarity (positive). The signal corresponding to this V.G.F. G_{c5} is designated C_{n1-d_4} , its recording point being a depth interval d_4 from the arrow C_n in FIGURE 6a.

Now referring to FIGURE 7, there is shown apparatus for performing the operation depicted in FIGURES 6a and 6b. In FIGURE 7, the derived signal S_n from the downhole investigating apparatus (not shown) is supplied via the conductor pair 53 to an amplifier 54, which supplies the ground reference potential in the same manner as amplifier 27 in FIGURE 3. The output of amplifier 54 supplies the referenced derived signal S_n to a computing circuit 55 which is constructed similar to the computing circuit enclosed by the dotted line box 55 in FIGURE 3. The output signal C_n from this computing circuit, corresponding to the solid line V.G.F. of FIGURE 6a, is supplied to a weighting function circuit 56a having a weight $1/C$, of a weighted adding network 56. The output signal C_{n1} from weighted adding network 56, taken across a low value resistor 56c, is supplied through a write amplifier 58 to a write in contact 57a of a rotating memory 57 which rotates in a counterclockwise direction. The signal C_{n1} is also supplied to a suitable recorder (not shown) for a recordation as a function of depth, in the same manner as in the FIGURE 3 apparatus. Located a counterclockwise interval d_4 from the write in point 57a is a read-out point 57b which supplies the read-out computed signal C_{n1-d_4} through a suitable read-out amplifier 59 to a weighting function circuit 56b

having a weight of $+B/C$ of weighted adding network 56.

Looking at FIGURES 6a, 6b and 7 in conjunction, the solid line V.G.F. of FIGURE 6a corresponds to the output signal C_n from computing circuit 55. The output signal C_{n1} from weighted adding network 56 corresponds to the final computed V.G.F. in FIGURE 6b. This signal C_{n1} is written into memory 57 at write-in point 57a, and read out an interval d_4 later at read-out point 57b to be combined with the signal C_n in weighted adding network 56 so as to cancel out the undesired V.G.F. portion G_{c4} of the V.G.F. corresponding to the signal C_n , thus leaving the V.G.F. of FIGURE 6b. This computed signal C_{n1} is recorded in recorder 32 in the same manner as before. Assuming that the computed signal C_n is already normalized to 1 (i.e. $B+C=1$), the weights to be applied to the signals C_n and C_{n1-d4} are $+1/C$ and $+B/C$ respectively. It is to be understood that other computed signals could be read out of memory 57 at other read-out contacts (other depth levels) if required. The weighting functions for these additional signals would be $\pm D/C$, $\pm E/C$, etc. where D, E, etc. are the areas of the V.G.F.'s corresponding to the other read-out computed signals.

Referring now to FIGURES 8a, 8b and 8c, there are shown graphical representations of other V.G.F.'s in accordance with other features of the present invention. In FIGURE 8a, there is shown an original apparatus V.G.F. G_{11} having an area A (the letter symbols used in connection with this embodiment do not necessarily have the same magnitudes as those used in other embodiments). There is also shown in FIGURE 8a a noninfinite series V.G.F. G_{12} of area B and an infinite series of V.G.F.'s G_{13} , G_{14} , G_{15} , etc. having areas of E, F, H, etc. respectively. All of these V.G.F.'s G_{12} , G_{13} , G_{14} , G_{15} , etc. have the same shape but not magnitude. The combined areas of all of these V.G.F.'s of the infinite series are represented by the hatched line area D. Considering the center point of each of these V.G.F.'s G_{12} , G_{13} , etc. to be the peak thereof, the arrows emanating from the peaks of each of these V.G.F.'s designate the recording points thereof. The signal designations C'_{n-d8} , $C'_{n-(d8+d5)}$, $C'_{n-(d8+2d5)}$, $\dots$, $C'_{n-(d8+md5)}$ represents the signals produced by each of these V.G.F.'s. (Again, the "prime" designates the fact that these are weighted signals corresponding to the weighted V.G.F.'s of FIGURE 8a.) The depth interval between S_n and C'_{n-d8} is d_8 and between each of the V.G.F.'s of the infinite series of V.G.F.'s is d_5 .

As in the preceding embodiments, the arrow emanating from the center point of the first V.G.F. G_{13} of the infinite series of V.G.F.'s also designates the recording point of the infinite series computed signal corresponding to the hatched line area D, and is designated C'_{o2} . After subtracting this hatched line area D and the V.G.F. G_{12} of area B from the area A of the original V.G.F. G_{11} , the resulting V.G.F. is shown in FIGURE 8b as the solid line V.G.F. having an uphole portion G_{c5} of area C and a downhole portion G_{c6} of area R. Since this solid line V.G.F. is produced by the subtraction of the areas B and D from the area A in FIGURE 8a, corresponding to the subtraction of C'_{o2} and C'_{n-8} from S_n , the signal designation for this solid line V.G.F. of FIGURE 8b is $S_n - (C'_{o2} + C'_{n-8})$, whose recording point will be the same as that of the derived signal S_n .

Since the V.G.F. portion G_{c5} is the desired final computed V.G.F. shape, the portion G_{c6} must be cancelled out. To accomplish this, another infinite series of V.G.F.'s, different from the series represented by the area D in FIGURE 8a, is utilized. This second infinite series of V.G.F.'s are the dotted line V.G.F.'s G_{16} , G_{17} , G_{18} whose recording points are a depth interval d_6 apart, having weights of j, k, l, etc., in FIGURE 8b. The hatched line portion of the V.G.F. portion G_{c6} of area R represents the total area of this second infinite series of V.G.F.'s.

The depth interval between the recording point of the original apparatus V.G.F. $G_{11}(S_n)$ and the recording point of the first V.G.F. $G_{16}(C'_{n-d7})$ of this second infinite series of V.G.F.'s is d_7 . The signals corresponding to each of these V.G.F.'s of the second infinite series of V.G.F.'s are designated

$$C'_{n-d7}, C'_{n-(d7+d6)}, C'_{n-(d7+2d6)} \dots C'_{n-(d7+rd6)}$$

The recording point of the first V.G.F. G_{16} of the second infinite series of V.G.F.'s also designates the recording point of the hatched line V.G.F. portion of area R corresponding to the signal designated C'_{o3} . After subtracting out the hatched line V.G.F. portion of area R, the V.G.F. of FIGURE 8c remains. This is the final computed V.G.F. whose recording point is designated by the arrow emanating from the peak thereof. The signal produced from this final computed V.G.F. is designated C_n .

Now, in retrospect, it can be seen that the V.G.F.'s in the noninfinite series V.G.F. and both of the infinite series of V.G.F.'s that were utilized to cancel out the undesired portions of the original apparatus V.G.F. G_{11} , are made up of individual V.G.F.'s having the same shape and vertical borehole axis extent, though not necessarily magnitude, as the final computed V.G.F. of FIGURE 8c. Thus, as in the FIGURE 1 situation, the computed signals C_n could be stored in a memory of infinite extent or length and read out and weighted in accordance with the weights and intervals between readout stations as represented in FIGURES 8a and 8b. Again, the V.G.F. which subtract from original V.G.F. G_{11} are shown of opposite polarity to more clearly show how they subtract from G_{11} .

Now referring to FIGURE 9, there is shown apparatus for performing the operation depicted in FIGURES 8a-8c wherein relatively short memories and a minimum of apparatus need be utilized in accordance with the present invention. The derived signal S_n from amplifier 28 is supplied to a weighting function circuit 60a of a weighted adding network 60. The output signal C_n from weighted adding network 60, taken across a low value resistor 60a, is supplied through a suitable write amplifier 61 to a write in contact 62a of a suitable memory 62 and to a suitable recorder (not shown) for recordation as a function of depth. Memory 62 is driven in a counterclockwise direction by a shaft 63a which is coupled through a suitable gear 63 from a differential gear (not shown) like the one in FIGURE 3.

Located a counterclockwise interval d_7-d_6 from write-in contact 62a is a read-out contact 62b which supplies the read-out computed signal $C_{n-d7+d6}$ representative of the investigated characteristic an interval d_7-d_6 downhole from the present depth position of the downhole investigating apparatus, through a suitable read-out amplifier 65 to a weighting function circuit 66a of weight u of a weighted adding network 66. The output from weighted adding network 66, taken across a low value resistor 66c, is supplied through a suitable write amplifier 67 and single-throw switch 67a to a write-in contact 62c, of memory 62, located a short clockwise interval from read-out contact 62b. Located a clockwise interval d_6 from write-in contact 62c is a read-out contact 62d, which supplies the read-out infinite series computed signal C_{o3} to a weighting function circuit 66b of weight v , of weighted adding network 66 and to a weighting function circuit 60b, having a weight of $-R/C$, of the weighted adding network 60.

The output signal C_n from weighted adding network 60 is also supplied through a write amplifier 72 to a write-in contact 70a of a rotating memory 70, which is driven in a counterclockwise direction by a shaft 63b from a suitable differential gear. Located a counterclockwise interval d_8 from write-in contact 70a is a read-out contact 70b which supplies the read-out computed signal C_{n-d8} to a weighting function circuit 60d, of weight $-B/C$, of weighted adding network 60, and to a weighting function circuit 69a, of weight x , of a weighted adding network 69. The output signal C_{o2+d5} of weighted adding network 69,

taken across a low value resistor 69c, is written in to write-in contact 70c of memory 70 via a read-in amplifier 71 and single-throw switch 71a. Located a counterclockwise interval d_5 from write-in contact 70c is a read-out contact 70d which reads out the infinite series computed signal C_{02} to a read-out amplifier 72. The output of read-out amplifier 72 is supplied to a weighting function circuit 60c, of weight $-D/C$, of weighted adding network 60, and to a weighting function circuit 69b of weight y , of weighted adding network 69.

Before proceeding with the operation of the apparatus of FIGURE 9, it would be desirable to first determine the values of the various weighting function circuits of FIGURE 9 by looking at the V.G.F.'s of FIGURES 8a-8c. First, taking the weighting functions of the weighted adding network 60 which provides the final computed signal C_n , it is first noted that the area R of the hatched line portion of FIGURE 8b is the same in area as the non-hatched line portion of the original apparatus V.G.F. G_{11} from the peak of V.G.F. G_{14} to infinity in the downhole direction. Thus, since the area A is equal to the sum of the areas $B+C+D+R$, it can be seen that

$$C=A-B-D-R$$

Since the V.G.F. G_{11} is considered to be normalized, i.e., $A=1$, this relationship becomes $C=1-B-D-R$. Now, for normalization of the final computed V.G.F., the above relationship is divided by C to give the weights $1/C$, $-B/C$, $-D/C$, and $-R/C$ to be applied to the signals S_n , C_{n-d_8} , C_{02} , and C_{03} to which these areas A, B, D, and R correspond.

Now, to determine the weights x and y in accordance with the previously discussed procedure, the ratio of the area E of the first V.G.F. G_{13} of the first infinite series to the total area D of all of the V.G.F.'s of this first infinite series gives the weight for the weighting function x to which the read-out computed signal C_{n-d_8} is applied (i.e., $x=E/D$). For the weight of the weighting function y , which the read-out first infinite series computed signal C_{02} is weighted by, the ratio of the area F of the second V.G.F. G_{14} of this first infinite series to the area E of the first V.G.F. G_{13} gives the value for y (i.e., $y=F/E$).

Now for the values of the weighting functions u and v for the second infinite series, the weighting function u to which the computed signal $C_{n-d_7+d_6}$ is weighted by, is determined in accordance with the prior procedure by the ratio of the area J of the first V.G.F. G_{16} of the second infinite series to the total area R of all of the V.G.F.'s of the second infinite series (i.e., $u=J/R$). The value of the weighting function v to which the read-out signal C_{03} is weighted by, is determined in accordance with the prior procedure by taking the ratio of the area K of the second V.G.F. G_{17} to the area J of the first V.G.F. G_{16} of the second infinite series, i.e., $v=K/J$.

Now, concerning how the apparatus of FIGURE 9 performs the operation depicted in FIGURES 8a-8c, the derived signal S_n which is supplied to the weighting function circuit 60a corresponds to the V.G.F. G_{11} of area A in FIGURE 8a. Concerning the discussion of the remainder of the operation of the FIGURE 9 apparatus in connection with the FIGURES 8a-8c V.G.F.'s, it is to be understood that the signals stored in memories 62 and 70 are normalized signals, that is, weighted adding networks 60, 66 and 69 provide normalized computed signals. The original apparatus V.G.F. G_{11} in FIGURE 8a is normalized, but the remainder of the V.G.F.'s shown in FIGURES 8a and 8b are weighted V.G.F.'s relative to V.G.F. G_{11} to show how the cancellation of undesired portions of the original V.G.F. G_{11} takes place. However by dividing all of the V.G.F. areas of FIGURES 8a and 8b by the area C of the final computed V.G.F. G_{05} , as in FIGURE 2e, the normalized weighted V.G.F.'s will result. Thus, the V.G.F.'s shown in FIGURES 8a and 8b are proportional to the normalized V.G.F.'s by the factor $1/C$.

The final computed signal C_n , having the V.G.F. shape of FIGURE 8c is stored in memory 70 until the downhole investigating apparatus has moved an interval d_8 , at which time it is read out as the computed signal C'_{n-d_8} to be combined with the read-out infinite series computed signal C_{02} in weighted adding network 69 to produce the new infinite series computed signal C_{02+d_5} which is stored in memory 70. After a depth interval d_5 , the read-out infinite series computed signal C_{02} (Note.—After the interval d_5 , the write-in signal C_{02-d_5} becomes $C_{02+d_5-d_5}$ or C_{02}) is combined with the new derived signal S_n , the second infinite series computed signal C_{03} , and the read-out computed signal C_{n-d_8} in weighted adding network 60 to produce the new computed signal C_n . This first infinite series computed signal C_{02} proportionally corresponds to the hatched line V.G.F. of area D in FIGURE 8a.

Concerning the second infinite series computed signal C_{03} , the final computed signal C_n is stored in memory 62 until the downhole investigating apparatus has traveled a depth interval d_7-d_6 , at which time it is read out and combined with the read-out second infinite series computed signal C_{03} to produce the new second infinite series computed signal C_{03+d_6} . This new signal is then written back into memory 62 until the downhole investigating apparatus has moved uphole a depth interval d_6 , at which time it is read out as the second infinite series computed signal C_{03} and combined in weighted adding network 60 with S_n , C_{n-d_8} and C_{02} to produce the new computed signal C_n . The proportional V.G.F. corresponding to the second infinite series computed signal C_{03} , after weighting, is the hatched line V.G.F. of area R in FIGURE 8b. Thus, it can be seen how the V.G.F.'s of areas B, D, and R cancel out undesired portions of the original apparatus V.G.F. G_{11} to produce the final computed V.G.F. of FIGURE 8c.

The equation for the computed signal C_n is therefore:

$$C_n = \frac{1}{C} S_n - \frac{B}{C} C_{n-d_8} - \frac{D}{C} C_{02} - \frac{R}{C} C_{03} \quad (16)$$

where

$$C_{02} = x C_{n-d_8} + xy C_{n-(d_8+d_5)} + xy^2 C_{n-(d_8+2d_5)} + \dots + xy^m C_{n-md_5} \quad (17)$$

and

$$C_{03} = u C_{n-d_7} + uv C_{n-(d_7+d_6)} + uv^2 C_{n-(d_7+2d_6)} + \dots + uv^r C_{n-(d_7+rd_6)} \quad (18)$$

Again,

$$x \left(1 + \sum_1^{\infty} y^m \right) \text{ and } u \left(1 + \sum_1^{\infty} v^r \right)$$

must both equal 1 for normalization although this is automatically taken into account by the weights given for x , y , u , and v .

The apparatus of FIGURE 9 produces both of the infinite series computed signals C_{02} and C_{03} in the same general manner as the apparatus of FIGURE 3 provides the infinite series of FIGURES 2b and 2c, except that in FIGURE 9, the final computed signal C_n is utilized in generating each of the infinite series. The final computed signals C_n and corresponding V.G.F.'s for each of the various depth levels could be traced in the same manner as in the FIGURES 2a-2e and 3, to show how both of the infinite series of FIGURES 8a and 8b are produced. Since this has already been done in connection with FIGURES 2a-2e and 3, it need not be repeated in connection with FIGURES 8a-8c and 9. Additionally, the FIGURE 9 apparatus could be initiated at the bottom of the borehole in the same manner as in the FIGURE 3 apparatus, by opening switches 67a and 71a and rotating the memories by hand while supplying S_n to the surface signal processing apparatus.

It can thus be seen that in accordance with the teachings of the present invention, there can be a plurality of

infinite series and noninfinite series computed signals utilized. Furthermore, these infinite series of V.G.F.'s can be started at any depth interval from the present depth level of the downhole investigating apparatus. Additionally, any number of infinite series or noninfinite series computed signals could be utilized, as desired.

Additionally, a combination of infinite series computed signals derived from both computed signals and derived signals could be utilized (i.e. the combination of the principles of FIGURES 2a-2e and 3 and FIGURES 8a-8c and 9). This is represented by the dotted lines in FIGURE 9 coupling the derived signal S_n to the write amplifier 61 or to the write amplifier 72, the dotted x's indicating that the computed signal C_n would not be supplied to write amplifier 61 or write amplifier 72. It is also to be understood that a plurality of infinite series computed signals obtained from the derived signal S_n could be utilized. This could be carried out by supplying the derived signal S_n , instead of the computed signal C_n , to both write amplifiers 61 and 72. Obviously, in these alternative situations, the circuit parameters would be different from those obtained in accordance with the V.G.F. of FIGURES 8a-8c, but nevertheless, these circuit parameters could be obtained in accordance with the teachings of the present invention.

It may happen that the original apparatus V.G.F. does not have the most desirable original apparatus V.G.F. to which to apply the teachings of the present invention. In this event, the original apparatus V.G.F. could be "prepared" for the signal processing technique of the present invention. This "preparing apparatus" could utilize for example, the computing process disclosed in the above-named Doll patent, the computing process disclosed in the copending Schuster application, or any other signal processing technique. This may take the form of applying the derived signal S_n to the computing apparatus for preparing the original apparatus V.G.F. and then applying the normalized output signal from this "preparing type" computing apparatus to the apparatus constructed in accordance with the present invention.

In addition to the above examples, there will occur to one of ordinary skill in the art many other possibilities of using infinite series computed signals to provide more desirable V.G.F.'s. For example, a given V.G.F. may have an oscillatory function, having positive and negative excursions, and having reduced magnitudes approaching depth infinity (like a damping function). In accordance with the teachings of the present invention, an infinite series computed signal could be produced to substantially cancel this oscillatory V.G.F. Taking the FIGURE 3 apparatus as an example, the weighting function q would be negative and the weighting function p would be positive or negative depending on the polarity of the first V.G.F. portion of the oscillatory V.G.F.

It is to be understood that the weighting factors of the various weighting function circuits could be shifted around in a desired manner to arrive at the same result. For example, the normalizing factor could be provided in the weighting function network that provides the final computed signal C_n and not in the infinite series weighted adding networks. Additionally, the weighting factors could be determined by a computational process rather than the trial and error method described earlier. For example, in the FIGURES 4 and 5 embodiment, if the weighting function circuits 45a, 45b, 49b and 49a are assigned the weights 1, α , $(1+\alpha)$, and $-b$, the equation for C_n can be written as:

$$C_n = S_n(1+\alpha) - bC_o \quad (19)$$

where

$$C_o = S_{n-d_3} + \alpha S_{n-2d_3} + \alpha^2 S_{n-3d_3} \quad (20)$$

Considering a homogeneous case where

$$S_n = S_{n-d_3} = S_{n-2d_3} = \dots = S_{n-md_3}$$

and combining Equations 19 and 20 and rearranging:

$$C_n = S_n[1 + \alpha - b(1 + \alpha + \alpha^2 + \dots)] = S_n \left(1 + \alpha - \frac{b}{1-\alpha} \right) \quad (21)$$

Equation 21 is satisfied only if:

$$a = \frac{b}{1-\alpha} \quad (22)$$

α could now be computed for a given depth interval d_3 . Additionally, the depth interval d_3 and α could both be varied to arrive at the most desirable computed V.G.F.

Considering the embodiment shown in FIGURES 8a-8c and 9, it should also be pointed out that a single memory could be utilized in place of the two memories 62 and 70 shown in FIGURE 9. To illustrate this, refer to FIGURE 10 where there is shown a functional block diagram of this operation. In FIGURE 10, the derived signal S_n is written into a rotating memory 80 at a write-in contact 80a. Memory 80 has read-out contacts 80b, 80c and 80d, and a write-in contact 80e, read-out contact 80b being located an interval d_8 from write-in contact 80a, read-out contact 80c located an interval d_8 from write-in contact 80e, and read-out contact 80d located an interval d_8 from read-out contact 80c. The signals S_{n-d_8} , C_{o4} and C_{o5} are supplied from read-out contacts 80b, 80c and 80d respectively to the weighting functions 1, α , and β respectively of an infinite series weighted adding network 81. The resulting computed signal C_{o6} is written into memory 80 at write-in contact 80e and supplied to a weighted adding network 82, where it is weighted by the factor $-b$. The derived signal S_n is weighted by the factor $(1+\alpha)$ in weighted adding network 82 which provides the final computed signal C_n .

The relationship for C_n is thus:

$$C_n = (1+\alpha)S_n - bC_{o6} \quad (23)$$

where

$$C_{o6} = S_{n-d_8} + \alpha C_{o4} + \beta C_{o5} \quad (24)$$

Writing the equation for C_n in terms of derived signals in the same manner as done earlier:

$$C_n = S_n(1+\alpha) - b[S_{n-d_8} + \alpha S_{n-2d_8} + \alpha^2 S_{n-3d_8} + \dots + \alpha^{m-1} S_{n-md_8}] - b[S_{n-(2d_8+d_9)} + \beta^2 S_{n-(2d_8+2d_9)} + \dots + \beta^m S_{n-m(d_8+d_9)}] - b[\text{terms in } \alpha \text{ and } \beta] \quad (25)$$

This infinite series of Equation 25 can mathematically be made to fit a given original apparatus V.G.F. to provide a final computed V.G.F. as in the earlier embodiments. Additionally, α or β could be negative to produce an infinite series of a different nature. It is noted that the last term of Equation 25, i.e. the α and β terms could be neglected for many cases since α times β would usually be much less than "one."

Additionally, if a capacitor type memory is utilized in the various embodiments, it may be desirable to read out the derived signal S_n from memory for combination with the infinite series computed signals to produce the final computed signal C_n rather than supplying S_n directly to the weighted adding network. This is because the signals read out from memory in this case (capacitor memory) are in the form of pulses and a continuous derived signal S_n applied to the weighted adding network may produce erroneous results. This, of course, would cause a depth shift of the entire operation.

While there have been described what are at present considered to be preferred embodiments of this invention, it will be obvious to those skilled in the art that various changes and modifications may be made therein without departing from the invention, and it is, therefore, intended to cover all such changes and modifications as fall within the true spirit and scope of the invention.

What is claimed is:

1. Apparatus for processing well logging signals, comprising:

- (a) means for deriving signals representative of a characteristic of earth formations surrounding a borehole at different depth levels in the borehole, each of said derived signals being representative of the characteristic in a given portion of a formation;
- (b) first combining means for combining each derived signal with at least one other signal to provide first computed signals correlated in depth with the derived signals;
- (c) memory means for storing the first computed signals;
- (d) means for reading out the first computed signals from the memory means at later times and supplying selected first computed signals to the first combining means to provide said at least one other signal, said selected first computed signals corresponding to previously investigated depth levels; and
- (e) second combining means for combining each derived signal with at least one signal read out from the memory means to provide second computed signals representative of the characteristic of the surrounding earth formations, said read-out signals operating to cancel a selected portion of the vertical formation response of each derived signal so that the second computed signals will have a sharper vertical formation response than do the derived signals.

2. The apparatus of claim 1 wherein the combining means for combining each derived signal S_n with the read-out first computed signal C_0 to obtain the new first computed signal C_{0-d} is combined in accordance with the relationship:

$$C_{0-d} = gS_n + hC_0$$

where g and h are weighting functions of positive or negative polarity and C_0 represents an infinite series of weighted signals derived at depth levels extending back to the depth level at which a signal was initially derived.

3. The apparatus of claim 2 wherein the infinite series of weighted signals is an infinite power series, C_0 being expressed by the relationship:

$$C_0 = gS_{n-d} + ghS_{n-2d} + gh^2S_{n-3d} + \dots + gh^{m-1}S_{n-md}$$

where S_{n-d} , S_{n-2d} , etc., are the signals derived at the different depth levels and S_{n-md} is the first derived signal.

4. The apparatus of claim 3 wherein the means for providing second computed signals combines each derived signal S_n with the signal C_0 read out from the memory means to obtain each second computed signal C_n in accordance with the relationship:

$$C_n = XS_n - YC_0$$

where X and Y are weighting functions of positive or negative polarity.

5. The apparatus of claim 4 wherein the means for deriving signals includes:

- (1) downhole investigating means for investigating the surrounding earth formations and generating signals indicative of the investigated characteristic as the downhole investigating apparatus moves through the borehole; and
- (2) means for supplying the signals to the surface of the earth.

6. The apparatus of claim 5 and further including recorder means having a recording medium driven in accordance with the movement of the downhole investigating apparatus through the borehole and responsive to the second computed signals C_n for providing a log of the computed values of the investigated characteristic as a function of depth.

7. The apparatus of claim 1 and further including:

- (1) means for storing the derived signals; and
- (2) means for reading out the stored derived signals at later times and supplying said read-out derived

signals to the second combining means to produce the second computed signals.

8. The apparatus of claim 7 wherein the means for providing the second computed signals combines each derived signal S_n with each first computed signal C_0 and one read-out derived signal S_{n-e} to produce each second computed signal C_n in accordance with the relationship:

$$C_n = XS_n - YC_0 - ZS_{n-e}$$

where X , Y , and Z are weighting functions of positive or negative polarity.

9. The apparatus of claim 7 wherein the means for providing the second computed signals combines each derived signal S_n with each first computed signal C_0 and a plurality of read out derived signals $S_{n-e} \dots S_{n-e'}$ to produce each second computed signal C_n in accordance with the relationship:

$$C_n = XS_n - YC_0 - ZS_{n-e} - \dots - Z'S_{n-e'}$$

where X , Y , $Z \dots Z'$ are weighting functions of positive or negative polarity.

10. The apparatus of claim 7 wherein the combining means for combining each derived signal S_n with the read-out first computed signal C_0 to obtain the new first computed signal C_{0-d} combines these signals in accordance with the relationship:

$$C_{0-d} = gS_n + hC_0$$

where g and h are weighting functions of positive or negative polarity and C_0 represents an infinite series of weighted signals derived at depth levels extending back to the depth level at which the first signal was derived.

11. The apparatus of claim 10 wherein the infinite series of weighted signals is an infinite power series, C_0 being expressed by the power series:

$$C_0 = gS_{n-(d'+d)} + ghS_{n-(d'+2d)} + gh^2S_{n-(d'+3d)} + \dots + gh^{m-1}S_{n-(d'+md)}$$

where $S_{n-(d'+d)} \dots S_{n-(d'+md)}$ are the signals derived at the different depth levels and S_{n-md} represents the first derived signal.

12. The apparatus of claim 1 wherein at least two of the first computed signals are read out of the memory means to be combined with each derived signal.

13. Apparatus for processing well logging signals, comprising:

- (a) means for deriving signals representative of a characteristic of the earth formations surrounding a borehole at different depth levels in the borehole, each of said derived signals being representative of the characteristic in a given portion of a formation;
- (b) memory means for storing the derived signals;
- (c) means for reading out the stored derived signals at later times;
- (d) first combining means for combining each read-out derived signal with at least one other signal to provide first computed signals correlated in depth with the derived signals;
- (e) means for writing each first computed signal back into the memory element of the memory means from which the particular read-out derived signal which produced said first computed signal was stored;
- (f) means for reading out the first computed signals from the memory means and supplying selected first computed signals to the first combining means as said at least one other signal; and
- (g) means for combining each derived signal with a selected first computed signal and at least one stored derived signal read out of the memory means to provide second computed signals representative of the characteristic of the surrounding earth formations, the selected first computed signal and stored derived signal operating to cancel a selected portion of the vertical response of each derived signal so that the second computed signals will have a sharper vertical formation response than the derived signals.

14. Apparatus for processing well logging signals, comprising:

- (a) means for deriving signals representative of a characteristic of earth formations surrounding a borehole at different depth levels in the borehole, each of said derived signals being representative of the characteristic in a given portion of a formation;
- (b) first combining means for combining each derived signal with at least one other signal to provide first computed signals correlated in depth with the derived signals and which first computed signals are representative of the characteristic of the surrounding earth formations;
- (c) second combining means for combining each first computed signal with at least one other signal to provide second computed signals correlated in depth with the derived signals;
- (d) memory means for storing the second computed signals; and
- (e) means for reading out the second computed signals from the memory means and supplying selected ones of the second computed signals to the first and second combining means whereby the read-out computed signals supplied to the first combining means will operate to cancel out a selected portion of the formation response of the derived signals.

15. The apparatus of claim 14 and further including means responsive to the first computed signals for recording the first computed signals to provide a log of the computed characteristic of the surrounding earth formations.

16. The apparatus of claim 15 wherein the first combining means combines each derived signal S_n with a read-out second computed signal C_o to obtain each first computed signal C_n in accordance with the relationship:

$$C_n = gS_n + hC_o$$

where g and h are weighting functions of positive or negative polarity and C_o is an infinite series of weighted first computed signals representing the formation characteristic at depth levels extending back to the first investigated depth level.

17. The apparatus of claim 16 wherein the second combining means combines each first computed signal with a selected read-out second computed signal C_o to produce each new second computed signal C_{o-d} in accordance with the relationship:

$$C_{o-d} = sC_n + tC_o$$

where s and t are weighting functions of positive or negative polarity.

18. The apparatus of claim 17 wherein the infinite series of weighted first computed signals is an infinite power series, C_o being expressed by the power series:

$$C_o = sC_{n-d} + stC_{n-2d} + st^2C_{n-3d} + \dots + st^{m-1}C_{n-md}$$

where C_{n-d} , C_{n-2d} , $\dots$, C_{n-md} are the first computed signals representing the computed formation characteristic at depth levels extending back to the first investigated depth level.

19. The apparatus of claim 1 and further including:

- (1) third combining means for combining each derived signal with at least one other signal to produce third computed signals correlated in depth with the derived signals;
 - (2) means for storing the third computed signals; and
 - (3) means for reading out the third computed signals at later times for supplying one of said at least one other signals to the second and third combining means.
20. The apparatus of claim 14 and further including:
- (1) third combining means for combining each first computed signal with at least one other signal to produce third computed signals correlated in depth with the derived signals;
 - (2) means for storing the third computed signals; and
 - (3) means for reading out the third computed signals

at later times for supplying one of said at least one other signals to the first and third combining means.

21. The apparatus of claim 1 and further including:

- (1) third combining means for combining each second computed signal with at least one other signal to produce third computed signals correlated in depth with the derived signals;
- (2) means for storing the third computed signals; and
- (3) means for reading out the third computed signals at later times for supplying one of said at least one other signals to the second and third combining means.

22. The apparatus of claim 14 wherein at least two second computed signals are read out of the memory means and supplied to the second combining means for combination with each first computed signal.

23. A method of processing well logging signals, comprising:

- (a) deriving signals from an exploring device having a given vertical geometrical factor, said derived signals being representative of a characteristic of earth formations surrounding a borehole at different depth levels in the borehole;
- (b) combining each derived signal with at least one other signal to provide first computed signals correlated in depth with the derived signals;
- (c) storing the first computed signals;
- (d) reading out the first computed signals at later times for combination with each derived signal as said at least one other signal, said read-out first computed signals corresponding to previously investigated depth levels;
- (e) combining each derived signal with at least a selected read-out first computed signal corresponding to a given vertical geometrical factor to provide second computed signals, the vertical geometrical factor corresponding to said selected read-out first computed signal cancelling an undesired portion of the derived signal vertical geometrical factor to provide each second computed signal with a corresponding improved vertical geometrical factor of improved vertical resolution; and
- (f) recording the second computed signals as a function or depth to provide a computed log of the formation characteristic.

24. The method of claim 23 and further including:

- (1) combining each derived signal with at least one other signal to provide third computed signals which correspond to a given vertical geometrical factor and which are correlated in depth with the derived signals;
- (2) storing the third computed signals; and
- (3) reading out the third computed signals at later times for use as said at least one other signal which is combined with each derived signal to produce the third computed signals, and using said read-out third computed signals for combination with the derived signals and the first computed signals to produce the second computed signals, the vertical geometrical factors corresponding to the first and third computed signals cancelling undesired portions of the vertical geometrical factor corresponding to the derived signals to provide each second computed signal with a corresponding improved vertical geometrical factor of improved vertical resolution.

25. The method of claim 23 and further including:

- (1) combining each second computed signal with at least one other signal to produce third computed signals corresponding to a given vertical geometrical factor and which are correlated in depth with the derived signals;
- (2) storing the third computed signals; and
- (3) reading out the third computed signals at later times for use as said at least one other signal which is combined with each second computed signal to

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produce the third computed signals, and using said read-out third computed signals for combination with the derived signals and the first computed signals to produce the second computed signals, the vertical geometrical factors corresponding to the first and third computed signals cancelling undesired portions of the vertical geometrical factor corresponding to the derived signals to provide each second computed signal with a corresponding improved vertical geometrical factor of improved vertical resolution.

26. A method of processing well logging signals, comprising:

- (a) deriving signals from an exploring device having a given vertical geometrical factor, said derived signals being representative of a characteristic of earth formations surrounding a borehole at different depth levels in the borehole;
- (b) combining each derived signal with at least one other signal to provide first computed signals;
- (c) combining each first computed signal with at least one other signal to provide second computed signals having a corresponding given vertical geometrical factor and which second computed signals are correlated in depth with the derived signals;
- (d) storing the second computed signals;
- (e) reading out the second computed signals at later times and using selected second computed signals for combination with each derived signal and each first computed signal to produce the first and second computed signals, the vertical geometrical factor corresponding to the second computed signals cancelling undesired portions of the vertical geometrical factor corresponding to the derived signals to provide the first computed signals with a corresponding improved vertical geometrical factor of improved vertical resolution; and
- (f) recording the first computed signals as a function of depth to provide a computed log of the investigated characteristic.

27. The method of claim 26 and further including:

- (1) combining each first computed signal with at least one other signal to produce third computed signals having a given vertical geometrical factor and which are correlated in depth with the derived signals;
- (2) storing the third computed signals; and
- (3) reading out the third computed signals at later time and using selected read-out third computed signals for combination with the first computed signals to produce the third computed signals, and using selected third computed signals for combination with the derived signals and the second computed signals to produce the first computed signals, the vertical geometrical factors corresponding to the second and third computed signals cancelling out undesired portions of the derived signal vertical geometrical factor to provide improved vertical geometrical factors of improved vertical resolution for the first computed signals.

28. The method of claim 23 and further including:

- (1) storing each derived signal; and
- (2) reading out the stored derived signals at later times and using selected stored derived signals as another of said at least one other signal to be combined with

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the derived and first computed signals to produce the second computed signals.

29. A method of processing well logging signals, comprising:

- (a) deriving signals representative of a characteristic of earth formations surrounding a borehole at different depth levels in the borehole, each of said derived signals being representative of the characteristic in a given portion of a formation;
- (b) storing the derived signals;
- (c) reading out the stored derived signals at later times;
- (d) combining each derived signal with at least one other signal to provide first computed signals which are correlated in depth with the derived signals;
- (e) storing the first computed signals;
- (f) reading out the first computed signals from the memory means at later times to be combined with each derived signal as said at least one other signal;
- (g) combining each derived signal with at least a selected read-out first computed signal to provide second computed signals which are correlated in depth with the derived signals and have a sharper vertical formation response than the derived signals;
- (h) recording the second computed signals as a function of depth to provide a computed log of the formation characteristic.

30. A method of processing well logging signals, comprising:

- (a) deriving signals representative of a characteristic of earth formations surrounding a borehole at different depth levels in the borehole, each of said derived signals being representative of the characteristic in a given portion of a formation;
- (b) combining each derived signal with at least one other signal to provide first computed signals correlated in depth with the derived signals;
- (c) storing the first computed signals in a memory means;
- (d) reading out the first computed signals from the memory means;
- (e) combining each read-out first computed signal with at least one other signal to provide second computed signals correlated in depth with the derived signals;
- (f) storing the second computed signals; and
- (g) reading out the second computed signals at later times for combination with each derived signal and each first computed signal to produce the first and second computed signals whereby the vertical formation response of said first computed signals will be sharper than for the derived signals.

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